Note for IB Physics HL

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Contents

1 Wave

1.1 Simple harmonic motion

Defination Simple Harmonic Motion is motion in which net force on an object is propotional to negative of the displacement.

$$F \propto -x \tag{1}$$

Abstract Model Shown in Figure 1, the x-t relation is a model of simple harmonic motion. The line is rotating with angular speed ω and initial angle ϕ. The radius is x_0.

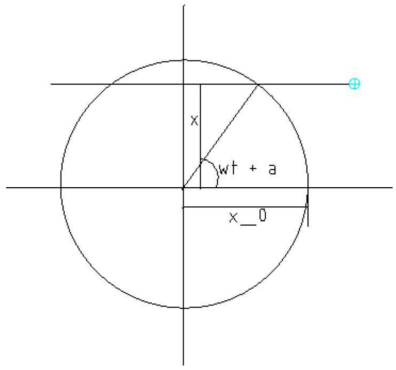

Figure 1: Model of simple harmonic motion

$$\begin{align} x &= x_0 \sin(\omega t + \phi) \tag{2} \\ v &= \frac{\mathrm{d}x}{\mathrm{d}t} = x_0 \omega \cos(\omega t + \phi) \tag{3} \\ a &= \frac{\mathrm{d}v}{\mathrm{d}t} = -x_0 \omega^2 \sin(\omega t + \phi) \tag{4} \\ &= -\omega^2 x \tag{5} \end{align}$$

Spring For an object on a spring, for displacement x, the net force act on it is $F = -kx$. Thus, it is SHM.

$$\begin{align} \begin{cases} a = \dfrac{F}{m} = \dfrac{-kx}{m} = -\omega^2 x \\ x = x_0 \sin(\omega t + \phi) \end{cases} &\Rightarrow \frac{-kx}{m} = -\omega^2 x \tag{6} \\ &\Rightarrow \omega = \sqrt{\frac{k}{m}} \tag{7} \end{align}$$

$$\begin{align} \therefore T &= \frac{2\pi}{\omega} \tag{8} \\ &= 2\pi \sqrt{\frac{m}{k}} \tag{9} \end{align}$$

Pendulum Similarly,

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{10}$$

Note: only for small angle which $\sin\theta \approx \theta$.

1.2 Intensity

Properties of wave Wave have following properties:

1. Speed v
2. Period T
3. Amplitude A
4. Wave length λ
5. Frequency f

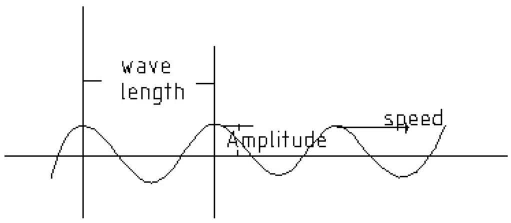

Figure 2: Properties of wave

$$f = \frac{1}{T} = \frac{v}{\lambda} \tag{11}$$

Intensity of wave Intensity $= \frac{\text{Power}}{\text{Area}}$
For example, for a wave source with total power P_0 radiate to all direction, at distance r from the center, as shwon in Figure 3,

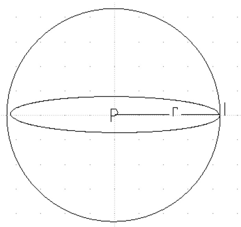

Figure 3: Intensity at distance r

the intensity is

$$I = \frac{P}{A} = \frac{P_0}{4\pi r^2} \tag{12}$$

1.3 Polarization

Direction of polarization For polarization grating, as shown in Figure 4 the polarized wave only with oscillate direction parallel to grating direction can fully pass.

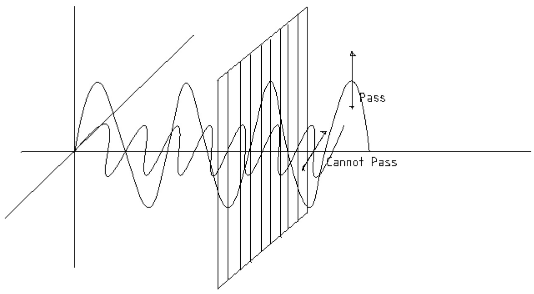

Figure 4: Polarization direction

Intensity after polarization For unpolarized wave with intensity I_0, the amplitude of the wave is

$$kA_0{}^2 = I_0 \tag{13}$$

$$\Rightarrow A_0 = \sqrt{\frac{I_0}{k}} \tag{14}$$

For a polarized wave with the angle between grating and oscillate direction is θ and intensity I_0, the intensity after polarized is

$$\begin{aligned} I_t &= kA_t{}^2 && (15)\\ &= k(\cos\theta A_0)^2 && (16)\\ &= (\cos\theta)^2 kA_0{}^2 && (17)\\ &= (\cos\theta)^2 I_0 && (18) \end{aligned}$$

Thus, for unpolarized wave after polarization, the intensity is

$$\begin{aligned} I &= \int_0^{2\pi} (\cos\theta)^2 \frac{I_0}{2\pi} \mathrm{d}\theta && (19)\\ &= \frac{1}{2} I_0 && (20) \end{aligned}$$

1.4 Refraction

Snell's law As shown in Figure 5, a light shoot from refraction index n_1 to refraction index n_2. The incident angle is θ_1, the refraction angle is θ_2.

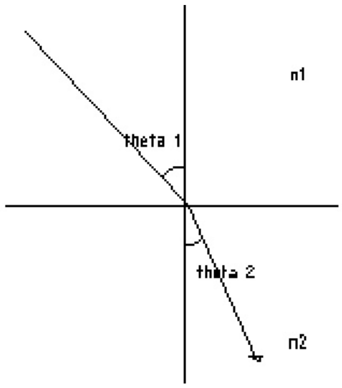

Figure 5: Refraction

Snell'law states that

$$\sin\theta_1 n_1 = \sin\theta_2 n_2 \tag{21}$$

Proof: Pass.

1.5 Doppler effect

Frequency of wave change due moving of observer and radiator.

Stationary source, approaching viewer In Figure 6, the wave have speed v and wave length λ. The viewer is approaching at speed v_v.

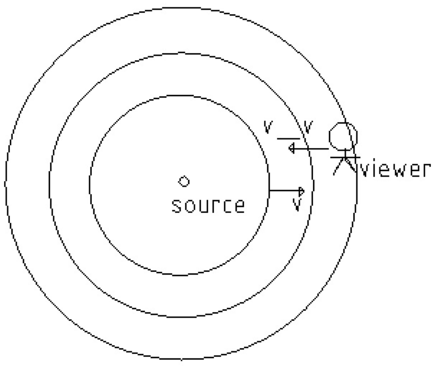

Figure 6: Stationary source, approaching viewer

The frequency of the wave is $f_0 = \frac{v}{\lambda}$
The frequency to viewer is

$$f' = \frac{v'}{\lambda'} \tag{22}$$
$$= \frac{v + v_v}{\lambda} \tag{23}$$
$$= f_0 \frac{v + v_v}{v} \tag{24}$$

Stationary source, leaving viewer Similar to approaching viewer, the frequency to viewer is

$$f' = \frac{v - v_v}{\lambda} \tag{25}$$
$$= f_0 \frac{v - v_v}{v} \tag{26}$$

Approaching source, stationary viewer In Figure 7, the wave have speed v and wave length λ. The source is approaching the viewer with speed v_s.

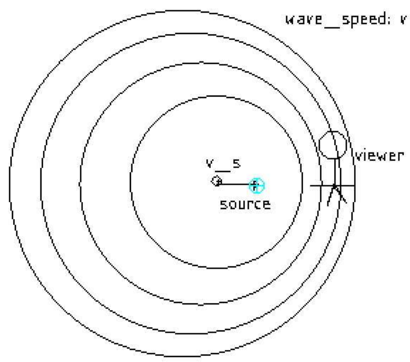

Figure 7: Approaching source, stationary viewer

The wave frequency is $f_0 = \frac{v}{\lambda}$
The frequency to the viewer is

$$f' = \frac{v'}{\lambda'} \tag{27}$$
$$= \frac{v}{\frac{v - v_s}{f_0}} \tag{28}$$
$$= f_0 \frac{v}{v - v_s} \tag{29}$$

Leaving source, stationary viewer Samilarly,

$$f' = f_0 \frac{v}{v + v_s} \tag{30}$$

Moving source and moving viewer Define the direction from source to viewer as position. The wave frequency is f_0 and speed is v. The source speed is v_s, the viewer speed is v_v.
The frequency observed at viewer is

$$f' = f_0 \frac{v - v_v}{v + v_s} \tag{31}$$

Doppler effect of EM wave The change in frequency Δf, with relative motion speed v between viewer and source (approaching is positive), is

$$\frac{\Delta f}{f_0} = \frac{\Delta \lambda}{\lambda_0} \tag{32}$$
$$\approx \frac{v}{c} \tag{33}$$

Thus, the observed frequency is

$$f' = f_0 + \Delta f \tag{34}$$
$$= f_0 + f_0\frac{v}{c} \tag{35}$$
$$= f_0(1 + \frac{v}{c}) \tag{36}$$

1.6 Wave shock

As shown in Figure 8, when the speed of the source v_s approach wave speed v, and then increase to $v + \tau, \tau \approx 0$, the wave shock.

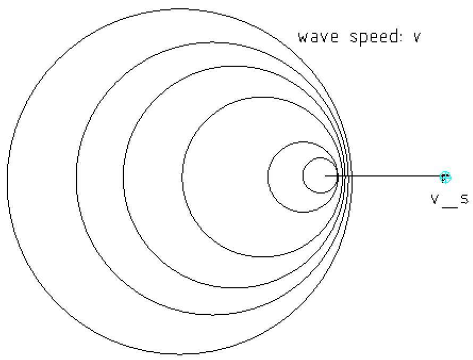

Figure 8: Wave shock

This is the cause of sound boom.

1.7 Wave interferance

Two wave meet and produce a new wave.

Constructive When the wave have same frequency and at the point the wave phase distance is $k\lambda, k \in N$, the point is constructive.

Destructive At point the wave phase difference is $(k + \frac{1}{2})\lambda, k \in N$, the point is destructive.

Double slit interferance As shown in Figure, light with wave length λ pass through two slits with distance d and form a interferance partern on screen at distance L.

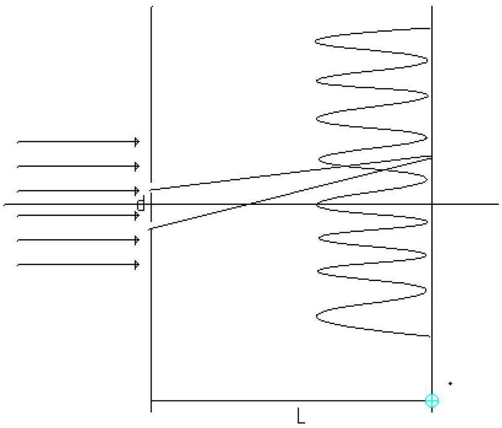

Figure 9: Double slit interferance

The angle of maximas are

$$\theta d \approx \sin\theta = n\lambda \tag{37}$$

$$\Rightarrow \theta \approx \sin\theta = \frac{n\lambda}{d} \tag{38}$$

Minimums are

$$\theta \approx \sin\theta = \frac{(n+\frac{1}{2})\lambda}{d} \tag{39}$$

Grating interferance Let the density of grating be $\rho = n/\mathrm{m}$.

$$d = \frac{1}{\rho} \tag{40}$$

Therefore angle of maximas are

$$\theta = \frac{n\lambda}{\frac{1}{\rho}} \tag{41}$$

$$= n\rho\lambda \tag{42}$$

Minimums are

$$\theta = (n+\frac{1}{2})\rho\lambda \tag{43}$$

1.8 Diffraction

Wave spread after passing small slit or hole.

Single slit diffraction As shown in Figure 10, light with wave length λ pass through a slit with width d and form a diffraction partern on screen at distance L.

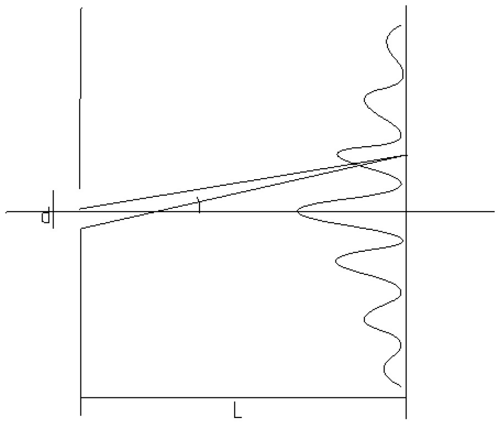

Figure 10: Single slit diffraction

Thus, the angle of maximums are

$$\sin\theta\frac{d}{2} = n\lambda, n \in N \tag{44}$$

$$\Rightarrow \theta \approx \sin\theta = \frac{2n\lambda}{d} \tag{45}$$

Angle of minimums

$$\theta \approx \sin\theta = \frac{(2n+1)\lambda}{d} \tag{46}$$

The angular width of central minimum is

$$\theta = \frac{2\lambda}{d} \tag{47}$$

1.9 Anti-reflection coating

Half-way loss When wave shoot from low refrection index to high refrection index, the reflected wave have a $\frac{1}{2}\lambda$ phase change from incident wave.
When shoot from low index to high, no such phase change.

AR-coating As shown in Figure 11, the equations of two reflected wave are f_1 and f_2. wave speed is v, thickness of the plate is d.

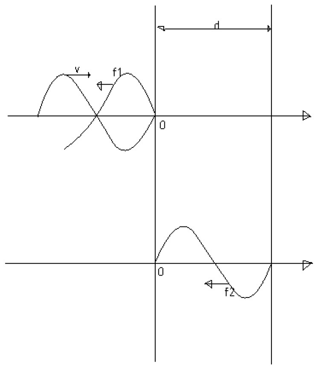

Figure 11: Anti reflection coating

The equation of first reflection is

$$f_1 = \sin(vt) \tag{48}$$

Equation of second reflection is

$$\begin{aligned} f_2 &= -\sin(d - vt + d) & (49) \\ &= \sin(vt - 2d) & (50) \end{aligned}$$

If constructive,

$$\begin{aligned} f_1 &= f_2 & (51) \\ &\Rightarrow 2d = n\lambda & (52) \end{aligned}$$

If destructive,

$$\begin{aligned} f_1 + f_2 &= 0 & (53) \\ \Rightarrow 2d &= (\frac{1}{2} + n)\lambda & (54) \end{aligned}$$

Thus, for anti reflection coating, the coating thickness is

$$d = (\frac{1}{4} + \frac{n}{2})\lambda \tag{55}$$

NOTE: The λ is the wave length inside the coating. Need to consider refrection index.

1.10 Resolution

Circular hole interferance The angular width of the interferance partern of circular hole is 1.22 times the single slit one.
First minimum:

$$\theta = 1.22\frac{\lambda}{d} \tag{56}$$

Resolution Figure 12 show two light (object) with wave length λ_1 and λ_2 pass through the circular slit and just resolve.

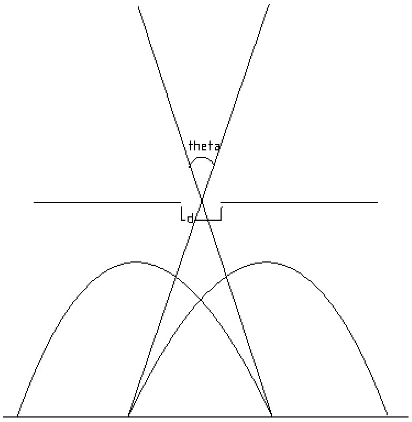

Figure 12: Just resolve

Let the minimum angle of seperation of two objects be *theta* to be resolve.

$$1.22\frac{\bar{\lambda}}{d} = \theta \tag{57}$$

$$\Rightarrow \theta = 1.22\frac{\lambda_1 + \lambda_2}{2d} \tag{58}$$

1.11 Standing wave

How to form A wave met and interferance with its reflected wave and form a new wave. The new wave is standing wave.

Period The standing wave have same period.

Standing wave in pip If both end closed, shown in Figure 13.

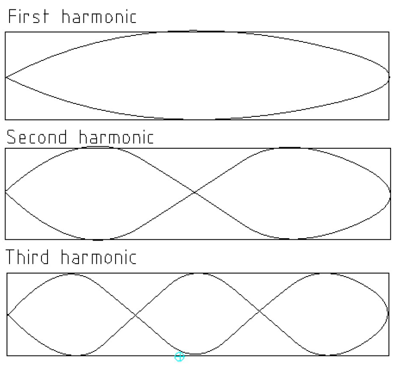

Figure 13: Harmonics in a pip with both end closed

The n^{th} harmonic have $\frac{n}{2}$'s period. Thus,

$$L = \frac{n}{2}\lambda \tag{59}$$

$$\Rightarrow \lambda = \frac{2}{n}L, n \in Z^+ \tag{60}$$

If one end closed and one end open, shown in Figure 14.

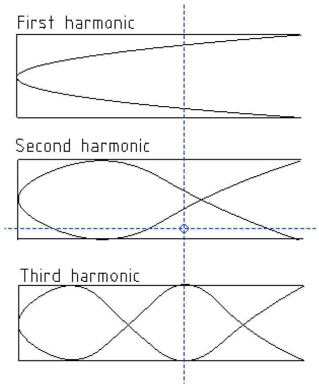

Figure 14: Harmonics in a pip with one end closed and one end open

The n^{th} harmonic have $\frac{1}{2}n - \frac{1}{4}$'s period. Thus,

$$L = (\frac{n}{2} - \frac{1}{4})\lambda \tag{61}$$
$$\Rightarrow \lambda = \frac{4L}{2n-1}, n \in Z^+ \tag{62}$$

If both end open, shown in Figure 15.

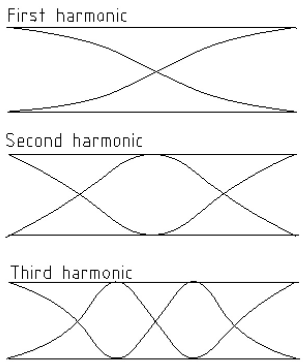

Figure 15: Harmonics in a pip with both end open

The n^{th} harmonic have $\frac{n}{2}$'s period. Thus

$$L = \frac{n}{2}\lambda \tag{63}$$
$$\Rightarrow \lambda = \frac{2L}{n} \tag{64}$$

Standing wave in string Same model as two-end-closed pipe.

2 Potential and field

2.1 Gravitational field

Gravitational force Force between mass m and M at distance r

$$F = \frac{GMm}{r^2} \tag{65}$$

G is gravitational constant.

Gravitational field strength Defined as

$$g = \frac{F}{m} \tag{66}$$

Gravitational potential energy Work done by an affact on a mass to take it to infinity distance wih constant speed.
Two body system with mass m and M. m 's gravitational energy at r is

$$\begin{aligned} U &= \int_r^\infty (-\frac{GMm}{x^2})\mathrm{d}x && (67)\\ &= -GMm(-r^{-1})|_r^\infty && (68)\\ &= -\frac{GMm}{r} && (69) \end{aligned}$$

Potential energy difference between at r and $r + \Delta h$ is $mg\Delta h$

$$\begin{aligned} \Delta U &= (-\frac{GMm}{r+\Delta h}) - (-\frac{GMm}{r}) && (70)\\ &= GMm(\frac{1}{r} - \frac{1}{r+\Delta h}) && (71)\\ &= mgr^2(\frac{1}{r} - \frac{1}{r+\Delta h}) && (72)\\ &= mg\frac{r\Delta h}{r+\Delta h} && (73)\\ &= mg\Delta h && (74) \end{aligned}$$

Gravitational potential Gravitational potential energy per unit of mass. Defined as

$$V = \frac{U}{m} = -\frac{GM}{r} \tag{75}$$

2.2 Electric field

Charge Electron have negative charge, proton have positive charge, neutron have zero charge.
An electron have charge -1.6×10^{-19}C. An proton have charge $+1.6 \times 10^{-19}$C.

Force beween charges Electric force,

$$F = \frac{kQq}{r^2} \tag{76}$$

$$k = \frac{1}{4\pi\epsilon_0} \tag{77}$$

$$= \frac{1}{4\pi \cdot 8.85 \times 10^{-12}} \tag{78}$$

$$= 8.99 \times 10^9 \tag{79}$$

Electric field strength

$$E = \frac{F}{q} \tag{80}$$

Electric potential energy Similar to gravitational potential energy.

$$U = \frac{kQq}{r} \tag{81}$$

Electric potential Similarly,

$$V = \frac{U}{q} = \frac{kQ}{r} \tag{82}$$

Charged ball shell In a ball shell: charge is 0.
Outside ball shell:
Gauss' flux theorem:

$$\oint E\mathrm{d}S = \frac{\Sigma q}{\epsilon_0} \tag{83}$$

$$\therefore E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0} \tag{84}$$

$$\therefore E = \frac{Q}{4\epsilon_0 \pi r^2} = k\frac{Q}{r^2} \tag{85}$$

2.3 Equal potential surface

Set of points that have equal potential

2.4 Field line

Direction Go along field line, potential decrease.

3 Electromagnetism Physics

3.1 Capacitor

Property of capacitor For parallel plate capacitor shown in Figure 16, the area of each plate is A, the distance between two plates is d, the permittivity of the filling is ϵ.

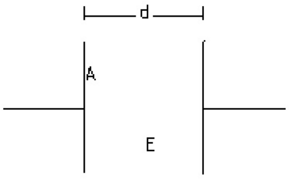

Figure 16: Structure of a capacitor

Permittivity is relative permittivity k times vacuum dielectric constant ϵ_0.

$$\begin{aligned} \epsilon &= k\epsilon_0 && (86) \\ &= 8.85 \times 10^{-12} k && (87) \end{aligned}$$

The capacitance is

$$C = \epsilon \frac{A}{d} \quad (88)$$

The relation of voltage between two plates U and charge on plates Q is

$$Q = CU \quad (89)$$

Connection of capacitor Parallel connection is shown in Figure 17

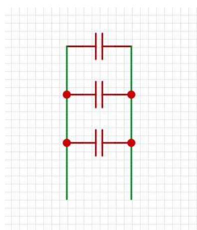

Figure 17: Capacitor parallel connection

The resultant capacitance is

$$\begin{aligned} C_T &= \frac{Q_T}{U_T} && (90) \\ &= \frac{\Sigma Q_i}{U_0} && (91) \\ &= \Sigma C_i && (92) \end{aligned}$$

Serial connection is shown in Figure 18

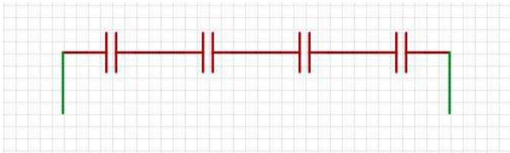

Figure 18: Capacitor serial connection

The resultant capacitance is

$$C_T = \frac{Q_T}{U_T} = \frac{Q_0}{\Sigma U_i} \tag{93}$$

$$\Rightarrow \frac{1}{C_T} = \frac{\Sigma U i}{Q_0} = \Sigma \frac{1}{C_i} \tag{94}$$

Charging the capacitor Figure 19 shows charging a capacitor with capacitance C through a resistance R with voltage ϵ.

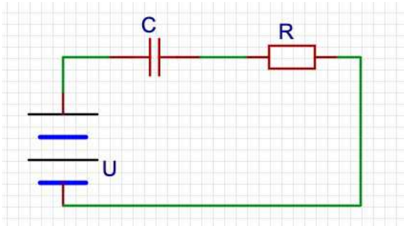

Figure 19: Charge capacitor

$$Q(t) = CU(t) \tag{95}$$

$$\epsilon = I(t)R + U(t) \tag{96}$$

$$Q(t) = \int_0^t I(t)\mathrm{d}t \tag{97}$$

From Equation 95 and Equation 97,

$$CU(t) = \int_0^t I(t)\mathrm{d}t \tag{98}$$

$$\therefore C\frac{\mathrm{d}U(t)}{\mathrm{d}t} = I(t) \tag{99}$$

From Equation 96,

$$U(t) = \epsilon - I(t)R \tag{100}$$

$$\therefore \frac{\mathrm{d}U(t)}{\mathrm{d}t} = -R\frac{\mathrm{d}I(t)}{\mathrm{d}t} \tag{101}$$

$$\therefore \frac{I(t)}{C} = -R\frac{\mathrm{d}I(t)}{\mathrm{d}t} \tag{102}$$

$$\therefore \frac{\mathrm{d}I(t)}{I(t)} = \frac{\mathrm{d}t}{-CR} \tag{103}$$

$$\therefore I(t) = Ae^{-\frac{t}{RC}} \tag{104}$$

$$\because \text{when } t = 0, I(t) = \frac{\epsilon}{R} \tag{105}$$

$$\therefore A = \frac{\epsilon}{R} \tag{106}$$

$$\therefore I(t) = \frac{\epsilon}{R}e^{-\frac{t}{RC}} \tag{107}$$

$$\therefore U(t) = \epsilon - I(t)R = \epsilon(1 - e^{-\frac{t}{RC}}) \tag{108}$$

$$Q(t) = CU(t) = \epsilon C(1 - e^{-\frac{t}{RC}}) \tag{109}$$

Discharging the capacitor Figure 20 shows discharging a capacitor with capacitance C and initial voltage U_0 across a resistance R.

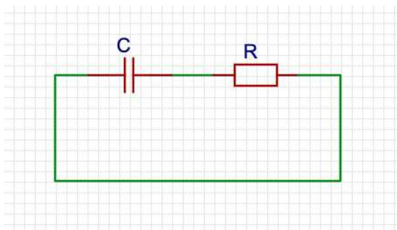

Figure 20: Discharge capacitor

$$Q(t) = CU(t) \tag{110}$$

$$U(t) = I(t)R \tag{111}$$

$$Q(t) = CU_0 - \int_0^t I(t)\mathrm{d}t \tag{112}$$

From Equation 110 and Equation 112,

$$CU(t) = CU_0 - \int I(t)\mathrm{d}t \tag{113}$$

Substitude Equation 111 into Equation 113,

$$CRI(t) = CU_0 - \int I(t)\mathrm{d}t \tag{114}$$

$$\therefore CR\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -I(t) \tag{115}$$

$$\therefore -CR\int \frac{1}{I(t)}\mathrm{d}t = \int \mathrm{d}t \tag{116}$$

$$\therefore I(t) = Ae^{-\frac{t}{RC}} \tag{117}$$

$$\because \text{when } t = 0, I(t) = \frac{U_0}{R} \tag{118}$$

$$\therefore I(t) = \frac{U_0}{R}e^{-\frac{t}{RC}} \tag{119}$$

$$\therefore U(t) = I(t)R = U_0 e^{-\frac{t}{RC}} \tag{120}$$

$$Q(t) = CU(t) = CU_0 e^{-\frac{t}{RC}} \tag{121}$$

$$= Q_0 e^{-\frac{t}{RC}} \tag{122}$$

Time constant Time constant, τ, defined as $\tau = RC$.
Define $t_{1/2}$ be the time to charge the capacitor to a half full. $\tau = \frac{t_{1/2}}{\ln 2}$. Proof:

$$U(t_{1/2}) = \epsilon(1 - e^{-\frac{t_{1/2}}{\tau}}) = \frac{1}{2}\epsilon \tag{123}$$

$$\Rightarrow e^{-\frac{t_{1/2}}{\tau}} = 2^{-1} \tag{124}$$

$$\Rightarrow -\frac{t_{1/2}}{\tau} = \ln(2^{-1}) \tag{125}$$

$$\Rightarrow \frac{t_{1/2}}{\tau} = \ln 2 \tag{126}$$

$$\Rightarrow \tau = \frac{t_{1/2}}{\ln 2} \tag{127}$$

3.2 Lorentz's force

Moving charge in a magnetic field is exerted with a Lorentz force. Is

$$\vec{F} = q\vec{v}\vec{B} \tag{128}$$

Since force vector is always perpendicular to speed vector, the charge will circular motion in uniform magnetic field.
In Figure 21, an tiny object with mass m and charge q shoot into a uniform magnetic field with speed v.

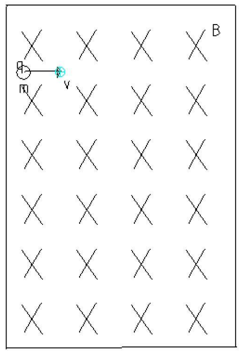

Figure 21: Charge exerted with Lorentz force

The radius of circular motion is

$$F = qvB = m\frac{v^2}{r} \tag{129}$$

$$\Rightarrow r = \frac{mv}{Bq} \tag{130}$$

3.3 Ampere's force

Current in magnetic field exerted with Ampere's force.

$$F = Q\vec{v}\vec{B} \tag{131}$$

$$= It\vec{v}\vec{B} \tag{132}$$

$$= I\vec{L}\vec{B} \tag{133}$$

3.4 Electronic Magnetic Induction

Conductor cut magnetic field line In Figure 22 is a conductor moving perpendicular to field line with speed v in magnetic field with strength B.

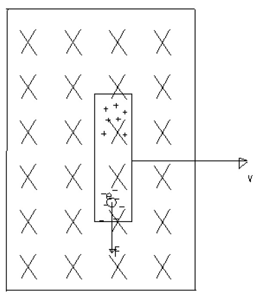

Figure 22: Conductor cut magnetic field line

The electrons in the conductor suffer Lorentz force and move down. As a resultant, upper end get positive charge and bottom end get negative charge.
The potential difference induced is called motional emf.

Magnetic flux ϕ, defined by Faraday in 1831,

$$\phi = B \cdot A \cdot \cos(\theta) \tag{134}$$

Here θ is the angle between the normal to the area and the magnetic field.
The unit is Tm^2 or Wb.
If there is N loops,

$$\phi = NBA\cos\theta \tag{135}$$

Faraday law

$$\epsilon \propto \frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{136}$$

For coil with N circles, the emf is

$$\epsilon = -N\frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{137}$$

Slide-conductor model In Figure 23, the length of conductor is l, sliding speed is v, magnetic field strength is B.

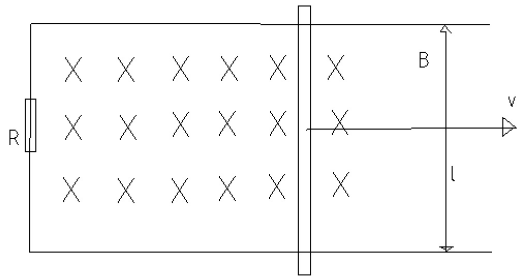

Figure 23: Slide conductor model

The motional emf is

$$\begin{aligned}\epsilon &= N\frac{\mathrm{d}\phi}{\mathrm{d}t} &(138)\\ &= NB\frac{\mathrm{d}A}{\mathrm{d}t} &(139)\\ &= NBlv &(140)\\ &= Blv &(141)\end{aligned}$$

The current in loop is

$$I = \frac{\epsilon}{R} \tag{142}$$

The induced resistance force is

$$\begin{aligned}F &= BIL &(143)\\ &= B\frac{\epsilon}{R}l &(144)\\ &= B\frac{Blv}{R}l &(145)\\ &= \frac{B^2l^2v}{R} &(146)\end{aligned}$$

Direction is opposite to direction of sliding. (Energy conservation)

3.5 Power transformation

Power generator Figure 24 shows a generator model.

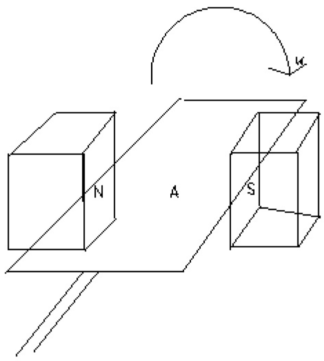

Figure 24: Generator

The rotor is rotate at angular speed ω. The cross-area of rotor is A_0. There are N loops in the rotor. The output voltage is

$$\begin{aligned}\epsilon &= NB\frac{\mathrm{d}A}{\mathrm{d}t} &(147)\\ &= NB\frac{\mathrm{d}(\sin(\omega t)A_0)}{\mathrm{d}t} &(148)\\ &= NB\omega A_0\cos(\omega t) &(149)\\ &\text{Equivalent to} &(150)\\ &NB\omega A_0\sin(\omega t) &(151)\end{aligned}$$

The power is

$$\begin{aligned}P &= \frac{{\epsilon_{\mathrm{rms}}}^2}{R} &(152)\\ &= \frac{(NB\omega A)^2}{2R} &(153)\end{aligned}$$

Alternating Current A.C., the relation of voltage to time is sine function.

$$U = U_0\sin(2\pi ft) \tag{154}$$

The root-mean-square voltage, $\sqrt{\bar{u^2}}$, is

$$U_{\mathrm{rms}} = \frac{U_0}{\sqrt{2}} \tag{155}$$

Power:

$$P = \frac{{U_{\text{rms}}}^2}{R} \tag{156}$$
$$= \frac{{U_0}^2}{2R} \tag{157}$$

Voltage transformer Figure 25 show a transformer. The input coil have N_1 loops, input voltage is U_1, output coil have N_2 loops, output voltage is U_2.

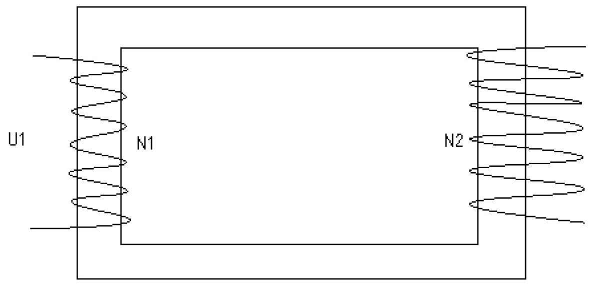

Figure 25: Transformer

$$\frac{U_1}{N_1} = \frac{U_2}{N_2} \tag{158}$$

Eddy current: current in iron that may leed to energy loss.
Ways to avoid energy loss:

1. Slice the iron into pieces and use insulator to seperate them.
2. Use higher voltage.

4 Thermal Physics

4.1 Heat transmition

Three ways of heat transmition:

1. Conduction
2. Radiation
3. Convection

Conduction This way need media. Fourier's law: as shown in Figure **??**, the rate of heat transfer is

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -\frac{A}{k}\frac{\mathrm{d}T}{\mathrm{d}l} \tag{159}$$

There is negative because the high-temperature end (which means $\mathrm{d}t > 0$) lose energy (which means $\mathrm{d}Q < 0$).

Convection Newton cooling law:

$$\frac{\mathrm{d}T}{\mathrm{d}t} \propto \Delta T \tag{160}$$

Temperature change rate is propotional to temperature difference.

Radiation This way does not need media. Energy transfered in form of EM wave.
Albedo rate:

$$a = \frac{P_{\text{out}}}{P_{\text{in}}} \tag{161}$$

4.2 Ideal gas

Properties

1. Gas molecules are point particles.
2. Molecules follow laws of mechanism.
3. Only collision force exist among molecules.
4. Collisions are elastic which means no energy loss.
5. Molecules move totally random.
6. Cannot be solidify or liquidify due to temperature and pressure change.

Ideal gas equation

$$PV = \gamma RT \tag{162}$$

P is pressure, V is volumn, γ is number of moles of the gas, R is gas constant, T is temperature. $R \approx 8.31$.
Boltzmann constant, k_B, is defined as

$$k_B = \frac{R}{N_A} \tag{163}$$

Isothermal process Temperature is constant. The internal energy of gas stay same. $P \propto \frac{1}{V}$.

Isobanic process Pressure is constant. $V \propto T$.

Isovolumn process Volumn is constant. $P \propto T$.

Pressure of gas Pressure is caused by gas molecules continously hitting on the wall of container. As shown in Figure 26, the velocity of the molecule is v and mass is m.

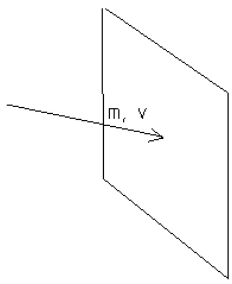

Figure 26: Gas molecules collide on wall of container

In small cube with side length d as shown in Figure 27, the time for one collide on a wall is

$$t = \frac{2d}{v} \tag{164}$$

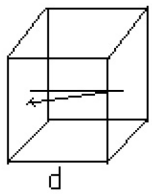

Figure 27: Time for one collide in small cube

Thus, the average force on that wall is

$$F = \frac{\Delta p}{t} \tag{165}$$

$$= \frac{2mv_x}{\frac{2d}{v_x}} \tag{166}$$

$$= \frac{m{v_x}^2}{d} \tag{167}$$

Here, v_x is the component of velocity perpendicular to the wall. Since it is randomly three-dimension motion, velocity component in each of three direction is same. Thus

$$\begin{cases} {v_x}^2 + {v_y}^2 + {v_z}^2 = v^2 \\ v_x = v_y = v_z \end{cases} \Rightarrow {v_x}^2 = \frac{1}{3}v^2 \tag{168}$$

$$F = \frac{mv^2}{3d} \tag{169}$$

Thus, the pressure is

$$P = \frac{F}{A} \tag{170}$$
$$= \frac{\frac{mv^2}{3d}}{d^2} \tag{171}$$
$$= \frac{mv^2}{3d^3} \tag{172}$$
$$= \frac{1}{3}\rho v^2 \tag{173}$$

Energy of molecule Total internal energy = Potential energy + Kinetic energy. In ideal gas model, there is no potential energy.
Energy of all molecules is

$$P = \frac{mv^2}{3d^3} = \frac{\gamma RT}{V} \tag{174}$$
$$\Rightarrow mv^2 = 3\gamma RT \tag{175}$$
$$\Rightarrow E = \frac{1}{2}mv^2 = \frac{3}{2}\gamma RT \tag{176}$$

Energy of a single molecule is

$$E_i = \frac{E}{\gamma N_A} \tag{177}$$
$$= \frac{3RT}{2N_A} \tag{178}$$

4.3 Black body radiation

Stefan-Boltzmann law

$$P = \sigma A T^4 \tag{179}$$

P: radiate power; A: cross-area; σ: Stefan-Boltzmann constant; T: temperature.

Wien's displacement law

$$\lambda_{\max} T = b \tag{180}$$
$$b = 2.9 \times 10^{-3} \tag{181}$$

$\lambda_{\max}$: Wave length in which radiate intensity is the maximum; b: Wien's displacement constant; T: temperature.

Emissivity e, Ratio of power emitted by an object to power emitted by black body with same dimension and temperature.
For real objects, radiate power

$$P = e\sigma A T^4 \tag{182}$$

5 Nuclear Physics

5.1 Structure of atom

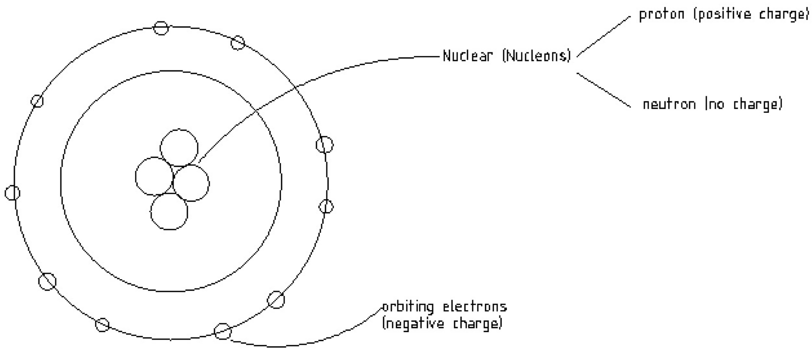

Figure 28: Structure of an atom

Isotope Isotope is atoms of a same element with different number of neutrons.

Atom symbol ^A_ZX. A: mass number, equal to number of protons plus number of neutrons. Z: atomic number, equal to number of protons.

Atom mass Atom mass unit, u, is

$$1u = \frac{1}{12} \times \text{mass of one atom of } ^{12}_{6}C \tag{183}$$

$$= \frac{10^{-3}\text{kg}}{N_A} \tag{184}$$

$$= 1.66 \times 10^{-27}\text{kg} \tag{185}$$

Particle	Symbol	Mass (u)	Charge (e)
Proton	^1_1p	1	1
Neutron	^1_0n	1	0
Electron	$^0_{-1}e$	0	-1
Positron	^0_1e	0	1
Photon	$^0_0\gamma$	0	0

Table 1: Data of common particles

5.2 Electronic Magnetic Spectrum

Emit spectrum Low pressure gas be emitted by electricity or radiation or heat and then radiate EM wave.
Energy of an atom is discrete. They are a specific set of values

$$E_n = \frac{E_1}{n^2},\ n \in Z^+ \tag{186}$$

Electrons at high energy state transit to low energy state. In this process EM wave radiated. The wave frequency is

$$f = \frac{E}{h} = \frac{E_{n1} - E_{n2}}{h} \tag{187}$$

$$= \frac{\frac{E_1}{{n_1}^2} - \frac{E_1}{{n_2}^2}}{h}, \ n_1, n_2 \in Z^+ \tag{188}$$

Absorb spectrum White light (mix of light with all different frequency) lit on atom. Electrons in atom absorb some photons to gain energy and transit to high energy state. Photons that are not absorbed are observed on screen.
The absorbed frequency is also discrete.
The path of energy level transittion is random.

5.3 Mass energy

Mass-Energy conservation Einstein: When body is in high energy state, it has more mass.

$$E = mc^2 \tag{189}$$

Binding Energy Binding energy is energy needed to seperate nucleons in an atom to infinity distance. Stability of atom is positive correlation to average binding energy per nucleon.

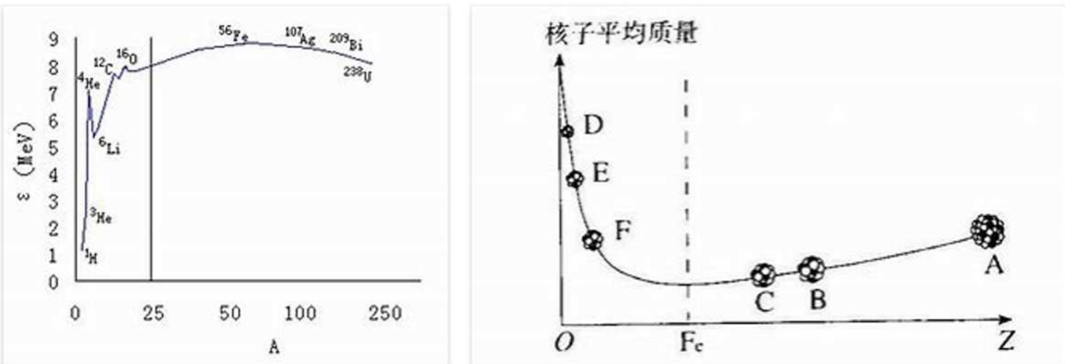

Figure 29: Average binding energy per nucleon and mass per nucleon against atomic number

Figure 29 shows the higher average binding energy is, the lower average mass. This is because mass-energy of a single nucleon is same. Thus the average binding energy $\bar{E}$ and average mass $\bar{M}$ should follow

$$Mc^2 + |E| = 1uc^2 \tag{190}$$

Mass defect Seperated nucleons weight more than when joined together in nucleus. This is because binding energy become smaller then mass become bigger.

5.4 Fission

Heavy element split to two parts and have lighter mass and release energy.

Four Fundamental Forces Four fundamental forces are Strong nuclear force (S), Weak nuclear force (W), electromagnetic force (M), Gravitation force (G).
In order:

$$S > W > M > G \tag{191}$$

Why more readily for heavy elements The forces in nuclear are Strong force and Electromagnetic force. Strong force is attraction, electromagnetic force here is repulsion (protons have same charge). Relation of electromagnetic force to distance is $F \propto \frac{1}{r^2}$. Strong force is almost 0 when distance $r > 1 \times 10^{-15}$m.
Thus, for heavy elements, the distance between nucleus is bigger. Thus strong force is smaller and electromagnetic force is bigger. Thus more easily to split.

Fission process for $U235$ Neutron collide with $U235$ → Form $U236$ → Split into two lighter atoms and release two high-energy neutrons and release energy → high-energy neutrons collide with other $U235$ atoms and chain reaction.
Split reaction is

$$U + n \rightarrow Sr + Xe + 2n \tag{192}$$

5.5 Fusion

Nucleons join together and release energy.

Why more readily for light elements For light elements, smaller distance, higher strong force, attraction force is bigger.
Also, according to Figure 29, for light elements, when combine which means number of nucleons increase, the average binding energy increase, thus energy released.

Process Isotope of H under high temperature combine and form He and release energy. Released energy keep the temperature high and then chain reaction.

5.6 Differences between fission and fussion

	Fission	Fussion
Defination	Atom split	Atom combine
Natural occurrence	Not likely	In stars, e.g. Sun
Byproduct	Many radioactive particles	Little
Condition	Critical mass and high-speed neutrons	High temperature
Energy required	Low	High
Energy released	Lower	Higher
Application	Atomic bomb, power station	Hydrogen bomb, Sun

Table 2: Compare of fission and fussion

5.7 Elementary particles

Three classes of elementary particles

1. Quarks
2. Leptons
3. Gauge bosons (strange particle)

<table>
<tr><th>Name</th><th>Symbol</th><th>Charge</th><th>Baryon number</th><th>Strangeness</th></tr>
<tr><td>up</td><td>u</td><td rowspan="3">$+\frac{2}{3}e$</td><td rowspan="6">$\frac{1}{3}$</td><td rowspan="4">0</td></tr>
<tr><td>charm</td><td>c</td></tr>
<tr><td>top</td><td>t</td></tr>
<tr><td>down</td><td>d</td><td rowspan="3">$-\frac{1}{3}e$</td></tr>
<tr><td>strange</td><td>s</td><td>-1</td></tr>
<tr><td>bottom</td><td>b</td><td>0</td></tr>
</table>

Table 3: Type of quarks

Type of quarks Hadron is particles made up of quarks or anti-quarks. It has two types:

1. Baryon, particle make up of three quarks or anti-quarks. E.g. proton, neutron
2. Meson, particle make up of one quark and one anti-quark. E.g. Pion $\pi^0[u, \bar{u}]$

<table>
<tr><th>Name</th><th>Symbol</th><th>Charge</th><th>L_e number</th><th>L_p number</th><th>L_γ number</th></tr>
<tr><td>Electron</td><td>e^-</td><td>-e</td><td rowspan="2">+1</td><td rowspan="2">0</td><td rowspan="4">0</td></tr>
<tr><td>Electron neutrino</td><td>V_e</td><td>0</td></tr>
<tr><td>Muon</td><td>μ^-</td><td>-e</td><td rowspan="4">0</td><td rowspan="2">+1</td></tr>
<tr><td>Muon neutrino</td><td>V_μ</td><td>0</td></tr>
<tr><td>Tau</td><td>τ^-</td><td>-e</td><td rowspan="2">0</td><td rowspan="2">+1</td></tr>
<tr><td>Tau neutrino</td><td>V_τ</td><td>0</td></tr>
</table>

Table 4: Type of leptons

Type of leptons

Structure of common particles Pion, π, type of meson, has three types:

1. $\pi^+[u, \bar{d}]$
2. $\pi^0[u, \bar{u}]$
3. π^-

Kaon, κ, type of meson, has structure $\kappa^0[d, \bar{s}]$
Proton, type of baryon, $p^1[u, u, d]$
Neutron, type of baryon, $n^0[u, d, d]$

Conservation of lapton Number of each type (family) of leptons have to be same before and after reaction.

Muon decay

$$\mu^\pm \to e^\pm + V + \bar{V} \tag{193}$$

Strange particle Three types: κ, Λ, Σ.
They behave strange:

1. Always produced in pairs and decay individually
2. They produce in fast rate and decay in slow rate
3. When produce, must follow strangeness conservation
4. When decay, do not need to follow strangeness conservation

5.8 Feynman Diagram

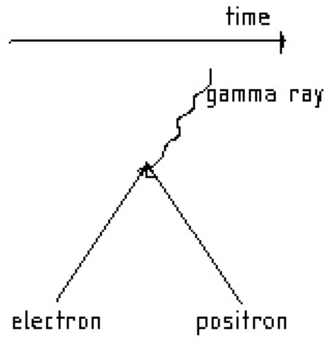

Figure 30: Example of Feynman Diagram

1. Particles before reaction point to one end of the line, particles after the reaction point to another end.
2. Anti particles along negative direction of time.
3. If Fermi particle engaged, use straight line. If photon engaged, use wave curve. If boson particle engaged, use dot line.
4. If is boson engaged, if total charge is position, use W^+. If is negative, use W^-, if no charge, use Z.

5.9 Nuclear density and radius

Measure radius of nuclear As shown in Figure 31, a He nuclear is shoot to a gold nuclear with speed v from infinity. The He nuclear stops at distance d from gold nuclear.

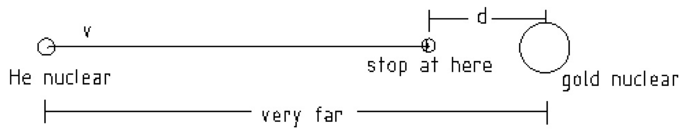

Figure 31: Experiment to measure nuclear radius

Thus,

$$\frac{1}{2}mv^2 = k\frac{2Ze^2}{d} \tag{194}$$

$$\Rightarrow d = \frac{4kZe^2}{mv^2} \tag{195}$$

Z is the atomic number of gold nuclear.
Therefore, the radius of gold nuclear r

$$r \approx d = \frac{4kZe^2}{mv^2} \tag{196}$$

Volumn of nucleon Volumn of nucleus V is propotional to number of nucleons A. Thus,

$$\begin{cases} V \propto A \\ V = \dfrac{4}{3}\pi R^3 \end{cases} \Rightarrow R^3 \propto A \tag{197}$$

$$\Rightarrow R \propto A^{\frac{1}{3}} \tag{198}$$

$$\Rightarrow R = R_0 A^{\frac{1}{3}} \tag{199}$$

R_0 is Fermi radius. $R_0 \approx 1.2 \times 10^{-15}\mathrm{m}$

Nuclear density

$$\rho = \frac{m}{V} = \frac{Au}{\frac{4}{3}\pi(R_0 A^{\frac{1}{3}})^3} \tag{200}$$

$$= \frac{Au}{\frac{4}{3}\pi {R_0}^3 A} \tag{201}$$

$$= \frac{u}{\frac{4}{3}\pi {R_0}^3} \tag{202}$$

$$= 2 \times 10^{17}\mathrm{kg/m^3} \tag{203}$$

5.10 Decay

Decay is the process a radioactive element turn into another element.

Properties

1. By random.
2. Spontinously. Do not need trigger.
3. Unpredictable

Decay rate A, the rate of decay,

$$A = \frac{\mathrm{d}N}{\mathrm{d}t} \propto N \tag{204}$$

Decay constant λ, defined as

$$A = -\lambda N \tag{205}$$

The higher decay constant is, the more radioactive.

Number of atoms left at time t Let the initial number be N_0, the decay constant be λ.

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N \tag{206}$$
$$\int \frac{\mathrm{d}N}{-\lambda N} = \int \mathrm{d}t \tag{207}$$
$$-\frac{1}{\lambda}\ln(N) = t + C \tag{208}$$
$$N = e^{-\lambda(t+C)} \tag{209}$$
$$= e^{-(\lambda+C)}e^{-\lambda t} \tag{210}$$
$$\because \text{when } t = 0, N = N_0 \tag{211}$$
$$\therefore N = N_0 e^{-\lambda t} \tag{212}$$

Decay rate at time t

$$A = -\lambda N = -\lambda N_0 e^{-\lambda t} \tag{213}$$
$$= A_0 e^{-\lambda t} \tag{214}$$

Half life $t_{\frac{1}{2}}$, Time needed for half of the atoms to decay.

$$\frac{1}{2}N_0 = N_0 e^{-\lambda t_{\frac{1}{2}}} \tag{215}$$
$$-\lambda t_{\frac{1}{2}} = \ln(\frac{1}{2}) \tag{216}$$
$$= -\ln(2) \tag{217}$$
$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} \tag{218}$$

Alpha decay α - decay,

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2} Y + {}^{4}_{2} He \tag{219}$$

Energy released by mass defect only turn into ${}^{4}_{2}He$'s kinetic energy. Thus the energy of ${}^{4}_{2}He$ is discrete.

Beta decay β^{+} decay:

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1} Y + {}^{0}_{1} e + {}^{0}_{0} V_e \tag{220}$$

β^{-} decay:

$${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1} Y + {}^{0}_{-1} e + {}^{0}_{0} \bar{V}_e \tag{221}$$

Energy released is shared by electron and neutrino. Thus kinetic energy of each is discrete.

5.11 Pair production

Process that high energy gamma ray turn into a positron and an electron. Follow mass-energy conservation.

5.12 Annihilation

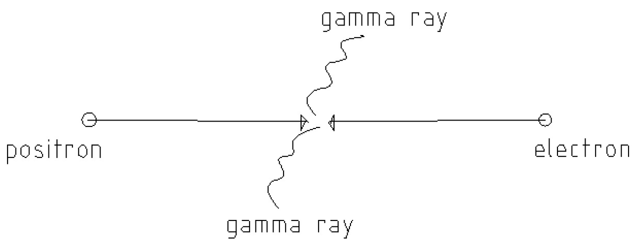

Figure 32: Annihilation

Process that a positron and an electron collide and turn into gamma ray. Opposite to pair production. Follow mass-energy conservation and momentum conservation.

6 Quantum Physics

6.1 Light: Particle and wave model

Wave model: double slit experiment. Light has properties of wave.
Particle model: photoelectric effect. Light emit to matal and electrons emit from metal.

6.2 Photoelectric effect

Light of specific range of frequency light on a metal plate and the plate emit electrons.

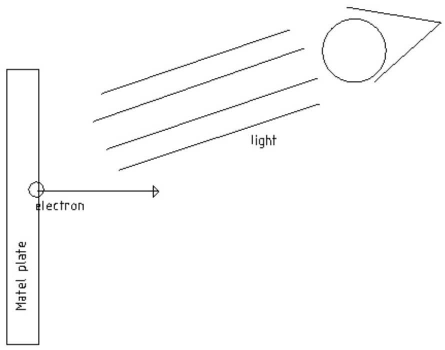

Figure 33: Diagram of photoelectric effect experiment

Why photoelectric effect can not be explained with wave model

1. For light with low frequency (which means low energy per photon), electrons can not emit even with long time lighting. This shows energy of light can not accumulate on the electrons.
2. There is not time delay for electrons to emit. If light is wave, it need time to transfer enough energy to emit the electrons. If light is particle, it has energy itself and can transfer its energy to electrons as it hit the electrons with no delay.

Work function Work function, ϕ, is the energy required for the electron transit to ground state. It depends on the material.

Threshold frequency Threshold frequency is the minimum frequency of light that can emit electrons. The energy of a photon with frequency f is $E_p = hf$. When $E_p > \phi$, the electron emit. Thus the threshold frequency is

$$f_{min} = \frac{\phi}{h} \tag{222}$$

Kinetic energy of emitted electron Some energy of photon is used for the electrons to transit to ground state. The rest energy turns to electrons' kinetic energy. Thus the kinetic energy of the emitted electron is

$$E_k = hf - \phi \tag{223}$$

This function is shown in Figure 34

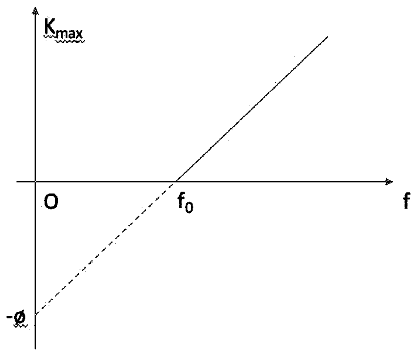

Figure 34: Kinetic energy against frequency

Photoelectric in circuit Figure 35 shows a photoelectric pannel serial connected in a circuit with a cell and an ampere meter.

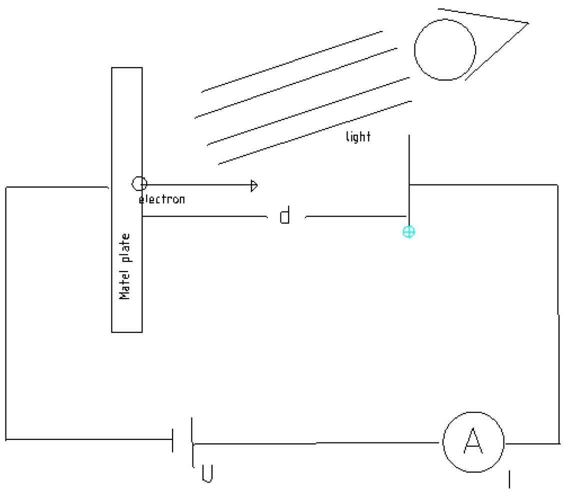

Figure 35: Photoelectric experiment in circuit

With cell voltage U, the energy required for the electron to emit is $\phi+eU$ [1]. Thus the threshold frequency is $\frac{\phi+eU}{h}$.
Stop voltage, U_0, is when the voltage of cell that the electrons just can not emit. It is calculated by

$$\phi + eU = hf \tag{224}$$
$$\Rightarrow U = \frac{hf - \phi}{e} \tag{225}$$

The current in the circuit, I, can be calculated by

$$I = \frac{Q}{t} \tag{226}$$
$$= \frac{eN}{t} \tag{227}$$
$$\tag{228}$$

[1][e]: amount of charge of one electron, $e = 1.6 \times 10^{-19}$C

Here N is the number of photons per second.

Relation to the intensity of light Defination of intensity: energy radiated per unit area.

$$I = \frac{P}{A} \tag{229}$$

Thus, intensity of light is

$$\begin{aligned} I &= \frac{P}{A} && (230) \\ &= \frac{\frac{Nhf}{t}}{A} && (231) \\ &= \frac{Nhf}{At} && (232) \end{aligned}$$

Thus number of photons emitted per second is

$$N = \frac{IAt}{hf} \tag{233}$$

Thus, with same intensity, the higher frequency is, the smaller number of photons emitted. Photon and electron is one-to-one. Thus higher frequency is, lower current in the circuit.

6.3 Mass wave

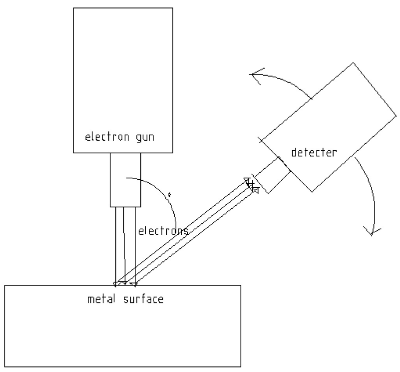

Figure 36: Davisson Germer experiment

Davisson Germer experiment In this experiment, change the angle between incident electrons and detecter, the intensity of electrons detected change with partern same as diffraction partern.
This is because the electrons are diffracted by the metal lattice. This shows electrons have properties of wave.

De Broglie Wave De Broglie found that matter is wave. It shows probability of the position of matter. De Broglie wave length is

$$\lambda = \frac{h}{p} \tag{234}$$

h is the Plank constant, p is the momentum of the matter.

6.4 Bohr model

Energy of electron The kinetic energy of spinning electron is

$$m\frac{v^2}{r} = k\frac{e^2}{r^2} \tag{235}$$
$$\Rightarrow v = \sqrt{\frac{ke^2}{mr}} \tag{236}$$
$$\Rightarrow E_k = \frac{1}{2}mv^2 = \frac{ke^2}{2r} \tag{237}$$

Thus, total energy of electron is

$$E_T = E_k + E_p \tag{238}$$
$$= -k\frac{e^2}{r} + \frac{ke^2}{2r} \tag{239}$$
$$= -k\frac{e^2}{2r} \tag{240}$$

Electron spin around the nucleus has angular momentum. It is

$$L = mvr \tag{241}$$
$$= n\frac{h}{2\pi}\ ,\ n \in Z^+ \tag{242}$$

From Equation 240 and Equation 242, the possible energy of electron is

$$E = -k\frac{e^2}{2\frac{nh}{2\pi mv}} \tag{243}$$
$$= -\frac{\pi mv}{nh}ke^2\ ,\ n \in Z^+ \tag{244}$$

Electron cloud Electron is a "cloud" that it randomly appears in somewhere by possibility.

Model for ^1_1H Restriction of Bohr's model is only applicable for atom of one-proton-one-electron, which is ^1_1H.
Speed of orbiting electron

$$\frac{ke^2}{r^2} = m\frac{v^2}{r} \Rightarrow r = \frac{ke^2}{mv^2} \tag{245}$$
$$mvr = \frac{nh}{2\pi} \Rightarrow r = \frac{nh}{2\pi mv} \tag{246}$$
$$\therefore \frac{nh}{2\pi mv} = \frac{ke^2}{mv^2} \tag{247}$$
$$\therefore v = \frac{2\pi ke^2}{nh}\ ,\ n \in Z^+ \tag{248}$$

Energy state of electrons in ^1_1H

$$E_T = -E_k = -\frac{1}{2}mv^2 \tag{249}$$
$$= -\frac{1}{2}m(\frac{2\pi ke^2}{nh})^2 \tag{250}$$
$$= -\frac{2\pi^2 mk^2e^4}{n^2h^2}\ ,\ n \in Z^+ \tag{251}$$

Substitude constants into Equation 251, the energy state of electron in ${}_1^1H$ is

$$E = -\frac{13.6}{n^2}\text{ev} \ , \ n \in Z^+ \tag{252}$$

6.5 Wave function

Wave function show the probability of the position of the electron. Wave function of electron in ${}_1^1H$:

$$ih\frac{\partial \Psi}{\partial t} = -\frac{h^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi \tag{253}$$

Probability for the electron to be at position x at time t:

$$P(x,t) = |[\Psi(x,t)]^2|\Delta V \tag{254}$$

Position of particle before and after measurement Before measurement the particle is at "Orthodox" position. It is at no where before measurement. The probability of position follows wave function.
After measurement, the wave function collapse into a particular position.

6.6 Heisenberg uncertainty principle

Position and momentum uncertainty

$$\Delta x \Delta p \geq \frac{h}{4\pi} \tag{255}$$

Here, Δx is the uncertainty of position and Δp is the uncertainty of momentum.

Energy and time uncertainty

$$\Delta E \Delta t \geq \frac{h}{4\pi} \tag{256}$$

Here, ΔE is the uncertainty of the energy state of the electron and Δt is the time period that the electron remains in that state.

6.7 Tuning effect

Small mass have a probability to go across a potentio barrier without enough energy.

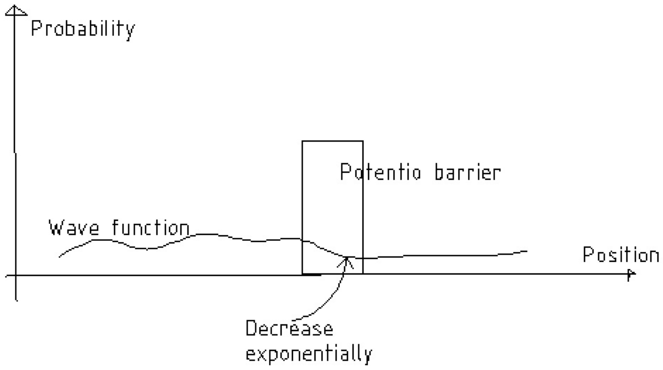

Figure 37: Probability of position with potentio barrier

Factors that influence probability of tuning

1. Mass of the particle
2. Thickness of the barrier
3. Height of the barrier
4. Energy difference between particle and barrier

With uncertainty principle For a particle with energy state E_s trapped in a potential well with barrier of energy state E_b, the time that it be trapped in the well is

$$\Delta t = \frac{h}{4\pi\Delta E} \tag{257}$$

$$= \frac{h}{4\pi(E_b - E_s)} \tag{258}$$