一、

1. 1 2. 1 3. 45 4. 2/3 5. 0.7

6. $f(z)=\frac{1}{\sqrt{2\pi}}e^{-(z+1)^2/2}, z\in R$

二、CADBDD

三、

解：用 A_1、A_2 分别表示某人参加“笔试”和“口试”的事件，A 表示“他能取得该种资格”。

由已知条件得 $P(A_1)=p$，$P(\overline{A}_1)=1-p$，$P(A_2|A_1)=p$，$P(A_2|\overline{A}_1)=\frac{p}{2}$.

所求概率为

(1) $P(A)=P(A_1\cup\overline{A}_1A_2)=P(A_1)+P(\overline{A}_1A_2)=p+P(A_2|\overline{A}_1)P(\overline{A}_1)$

$$=p+\frac{p}{2}(1-p)=\frac{3}{2}p-\frac{1}{2}p^2$$

(2) $P(A_1|A_2)=\frac{P(A_1A_2)}{P(A_2)}=\frac{P(A_2|A_1)P(A_1)}{P(A_2|A_1)P(A_1)+P(A_2|\overline{A}_1)P(\overline{A}_1)}=\frac{2p}{1+p}$

四、

解 (1) 由 $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}f(x,y)\mathrm{d}x\mathrm{d}y=1$，有

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x, y)\mathrm{d}x\mathrm{d}y = \int_0^{+\infty}\int_0^{+\infty} k\mathrm{e}^{-(2x+3y)}\mathrm{d}x\mathrm{d}y$$
$$= k\int_0^{+\infty}\mathrm{e}^{-2x}\mathrm{d}x\int_0^{+\infty}\mathrm{e}^{-3y}\mathrm{d}y$$
$$= k\left[-\frac{1}{2}\mathrm{e}^{-2x}\right]_0^{+\infty}\left[-\frac{1}{3}\mathrm{e}^{-3y}\right]_0^{+\infty}$$
$$= k\cdot\frac{1}{6} = 1$$

所以，$k = 6$.

(2)

$$F(x, y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f(u, v)\mathrm{d}u\mathrm{d}v$$
$$= \begin{cases} \int_0^{y}\int_0^{x} 6\mathrm{e}^{-(2u+3v)}\mathrm{d}u\mathrm{d}v = (1-\mathrm{e}^{-2x})(1-\mathrm{e}^{-3y}), & x > 0, y > 0 \\ 0, & \text{其他} \end{cases}$$

(3) $P\{X < Y\} = \iint\limits_{x<y} f(x, y)\mathrm{d}x\mathrm{d}y = \int_0^{+\infty}\left[\int_0^{y} 6\mathrm{e}^{-(2x+3y)}\mathrm{d}x\right]\mathrm{d}y$

$$= \int_0^{+\infty} 3\mathrm{e}^{-3y}[1-\mathrm{e}^{-2y}]\mathrm{d}y$$
$$= \int_0^{+\infty} 3\mathrm{e}^{-3y}\mathrm{d}y - \int_0^{+\infty} 3\mathrm{e}^{-5y}\mathrm{d}y$$
$$= 1 - \frac{3}{5} = \frac{2}{5}.$$

(4) X 与 Y 的边缘密度分别为

$$f_X(x) = \int_{-\infty}^{+\infty} f(x, y)\mathrm{d}y = \begin{cases} \int_0^{+\infty} 6\mathrm{e}^{-(2x+3y)}dy, & x > 0, \\ 0, & \text{其他}. \end{cases} = \begin{cases} 2\mathrm{e}^{-2x}, & x > 0 \\ 0, & \text{其他}. \end{cases}$$

$$f_Y(x) = \int_{-\infty}^{+\infty} f(x, y)\mathrm{d}x = \begin{cases} \int_0^{+\infty} 6\mathrm{e}^{-(2x+3y)}dx, & x > 0, \\ 0, & \text{其他}. \end{cases} = \begin{cases} 3\mathrm{e}^{-3x}, & y > 0 \\ 0, & \text{其他}. \end{cases}$$

显然，$f(x, y) = f_X(x)f_Y(y)$，所以 X 与 Y 相互独立.

五、

解、

ξ	0	1
p	$\frac{2}{3}$	$\frac{1}{3}$

η	-1	0	1
p	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$\xi\eta$	-1	0	1
p	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

$E\xi = \frac{1}{3} \qquad E\eta = 0$

$D\xi = \frac{2}{9} \qquad D\eta = \frac{2}{3}$

$$E(\xi - E\xi)(\eta - E\eta) = E\left(\xi\eta\right) - E\xi \cdot E\eta = 0 - 0 = 0$$

$\therefore \rho_{\xi\eta} = 0$

$$D(\xi-\eta) = E(\xi-\eta)^2 - \left[E(\xi-\eta)\right]^2$$
$$= E\left(\xi^2\right) - 2E\left(\xi\eta\right) + E\left(\eta^2\right) - \left(E\xi\right)^2 + 2E\xi \cdot E\eta - \left(E\eta\right)^2$$
$$= D\xi + D\eta = \frac{8}{9}.$$

六、

解：令 $h_i = \begin{cases} 1 & \text{第三粒为良种} \\ 0 & \text{第三种为非良种} \end{cases} \qquad i = 1,2,\cdots 180.$

则 $p(h_i = 1) = \frac{1}{6} = p. \ (q = 1 - p = \frac{5}{6})$

$$P(\left|\sum_{i=1}^{180} h_i / 180 - \frac{1}{6}\right| < C) = P(\left|\frac{\sum_{i=1}^{180} h_i - 180 \times \frac{1}{6}}{\sqrt{180 \times \frac{1}{6} \times \frac{5}{6}}}\right| < \frac{\sqrt{180} \cdot C}{\sqrt{\frac{1}{6} \times \frac{5}{6}}})$$

$$= P\left(\left|\frac{\sum_{i=1}^{180} h_i - 30}{5}\right| < 36C\right)$$

$$\approx \int_{-36C}^{36C} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 2\int_{-\infty}^{36C} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx - 1$$

$$= 2\Phi(36C) - 1 = 0.99$$

$\therefore \Phi(36C) = 0.995$ $\quad \therefore 36C = 2.48$ $\quad \therefore C = 2.48/36 = \frac{31}{450} = 0.07$