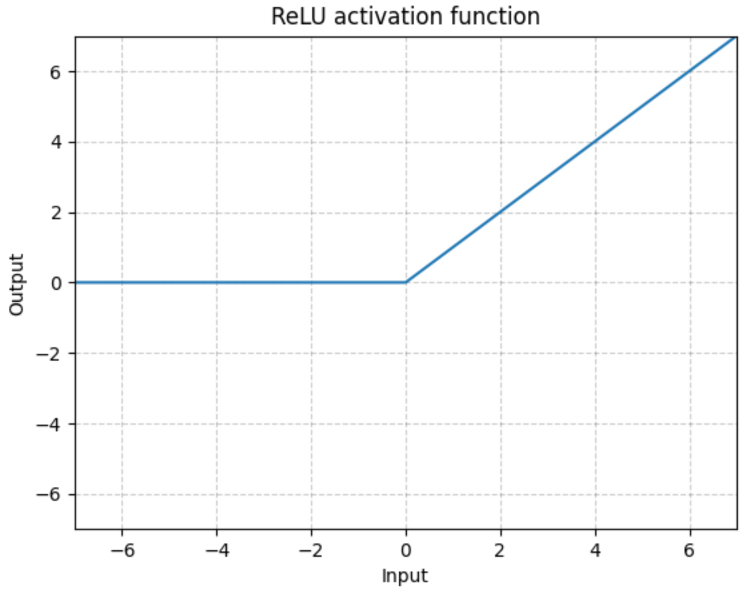
Activation Functions

In PyTorch & how to use them

ReLU – `nn.ReLU()`

$$\mathrm{ReLU}(x) = (x)^{+} = \max(0, x)$$



RReLU – `nn.RReLU()`

$$\mathrm{RReLU}(x) = \begin{cases} x & \text{if } x \geq 0 \\ ax & \text{otherwise} \end{cases}$$

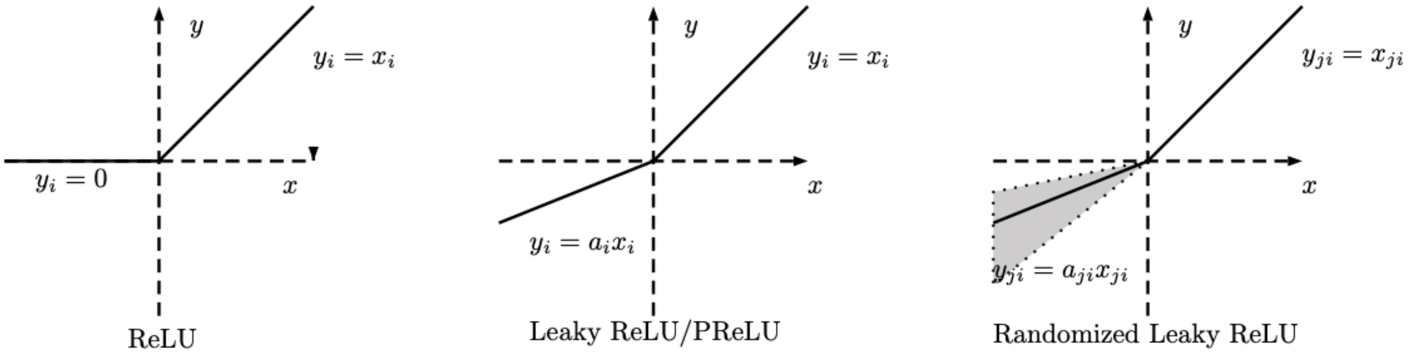
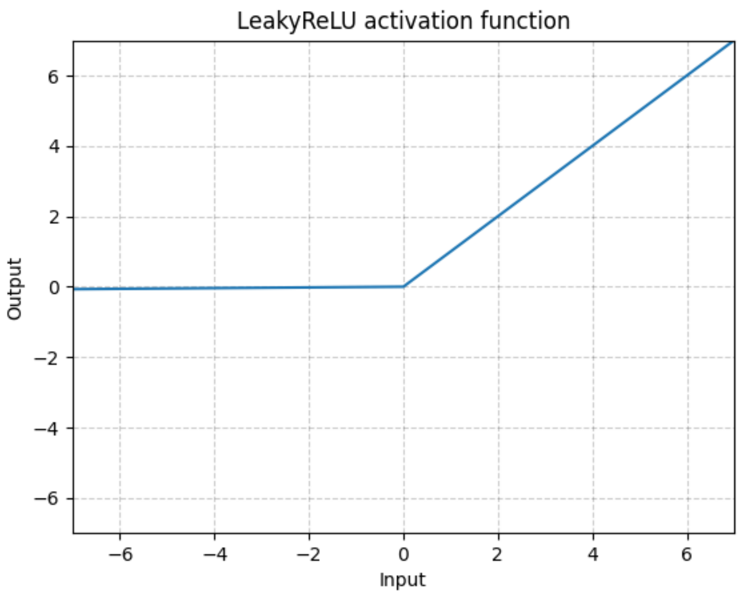


Figure 1: ReLU, Leaky ReLU, PReLU and RReLU. For PReLU, a_i is learned and for Leaky ReLU a_i is fixed. For RReLU, a_{ji} is a random variable keeps sampling in a given range, and remains fixed in testing.

LeakyReLU – `nn.LeakyReLU()`

Y. LeCun

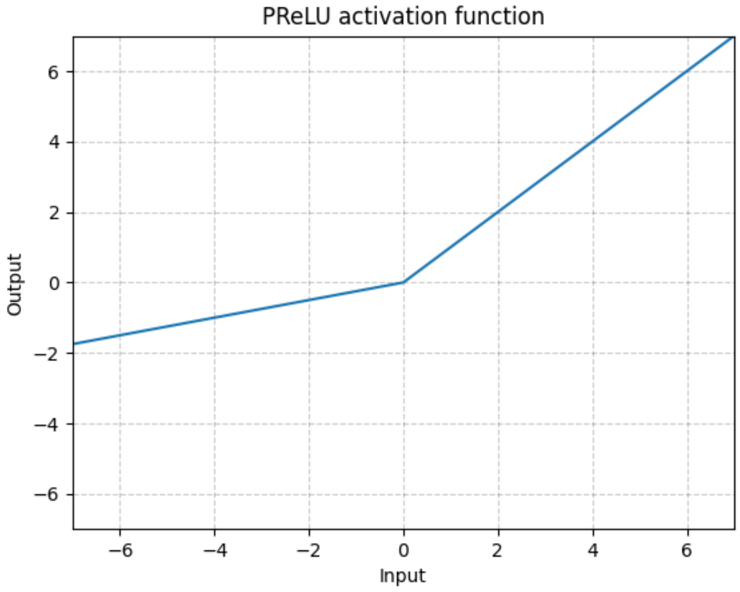
$$\text{LeakyRELU}(x) = \begin{cases} x, & \text{if } x \geq 0 \\ \text{negative\_slope} \times x, & \text{otherwise} \end{cases}$$



PReLU – `nn.PReLU()`

$$\text{PReLU}(x) = \begin{cases} x, & \text{if } x \geq 0 \\ ax, & \text{otherwise} \end{cases}$$

Here a is a learnable parameter. When called without arguments, *nn.PReLU()* uses a single parameter a across all input channels. If called with *nn.PReLU(nChannels)*, a separate a is used for each input channel.

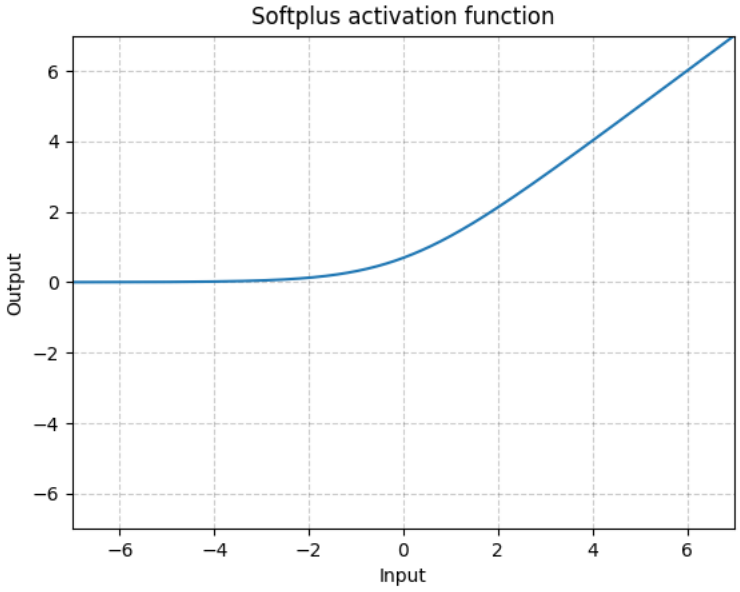


Softplus – `nn.Softplus()`

$$\text{Softplus}(x) = \frac{1}{\beta} * \log(1 + \exp(\beta * x))$$

SoftPlus is a smooth approximation to the ReLU function and can be used to constrain the output of a machine to always be positive.

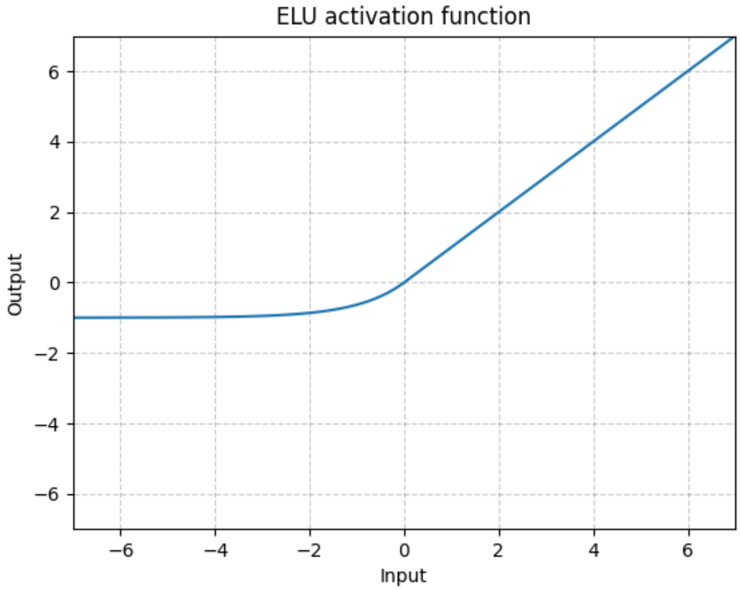
For numerical stability the implementation reverts to the linear function when $input \times \beta > threshold$.



ELU – `nn.ELU()`

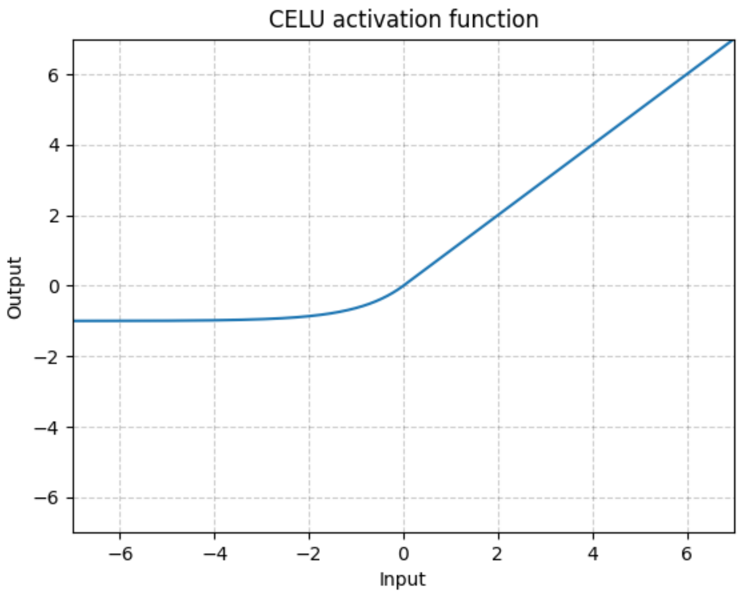
Applies the element-wise function:

$$\text{ELU}(x) = \max(0, x) + \min(0, \alpha * (\exp(x) - 1))$$



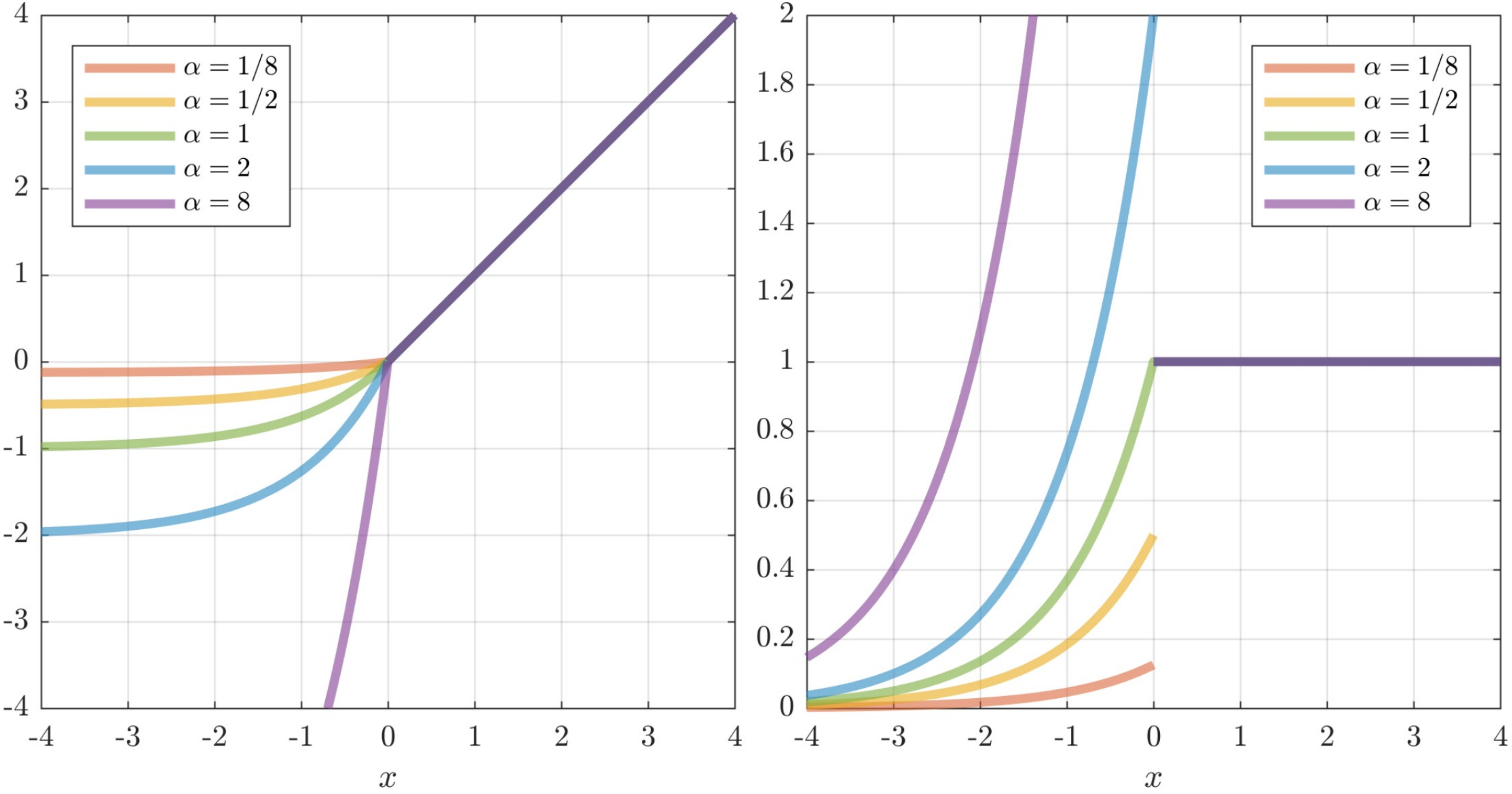
CELU – `nn.CELU()`

$$\mathrm{CELU}(x) = \max(0, x) + \min(0, \alpha * (\exp(x/\alpha) - 1))$$



Barron (2017) Continuously differentiable exponential linear units

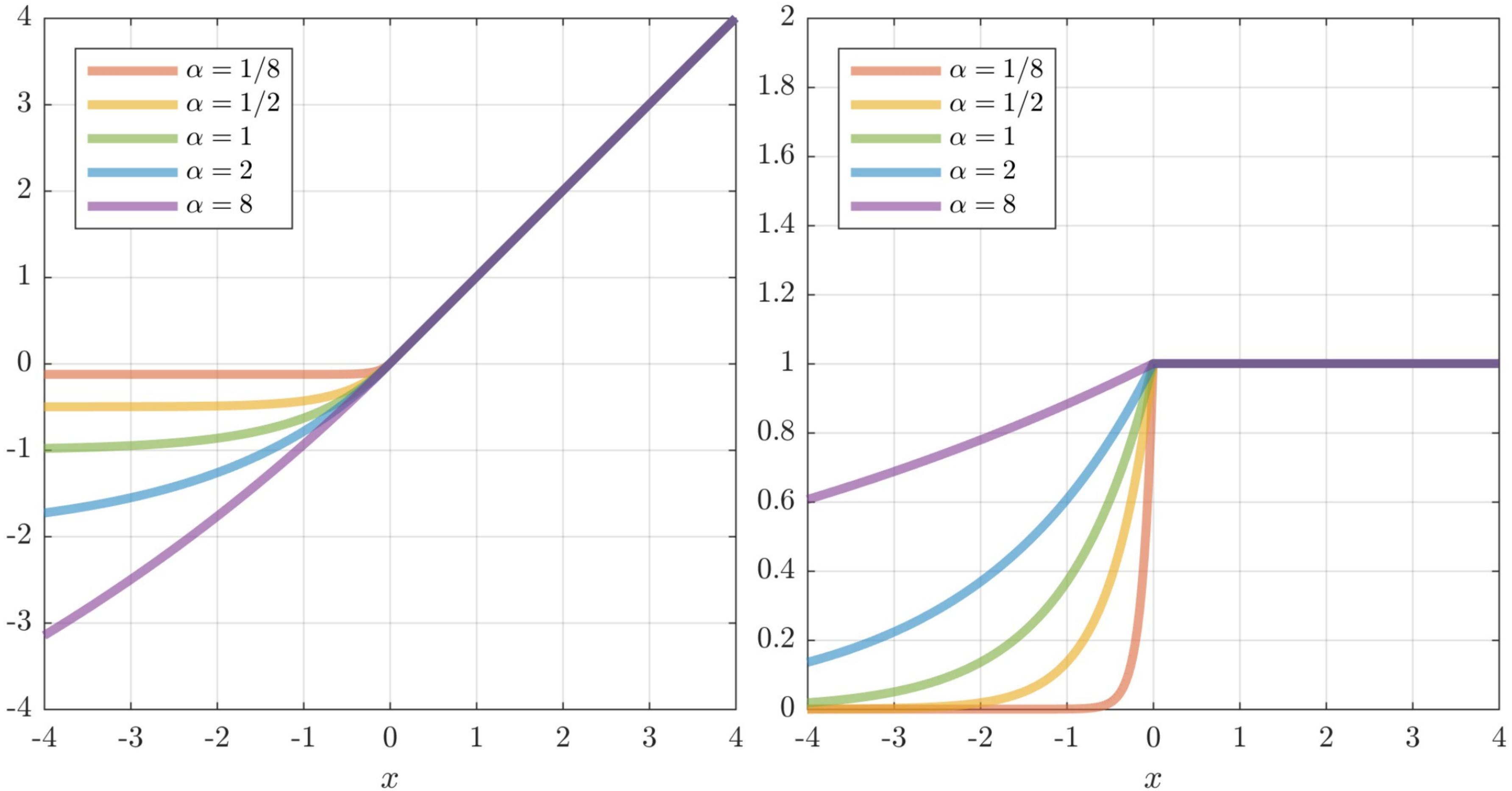
CELU – `nn.CELU()`



(a) $\mathrm{ELU}(x, \alpha)$

(b) $\frac{d}{dx}\mathrm{ELU}(x, \alpha)$

CELU – `nn.CELU()`



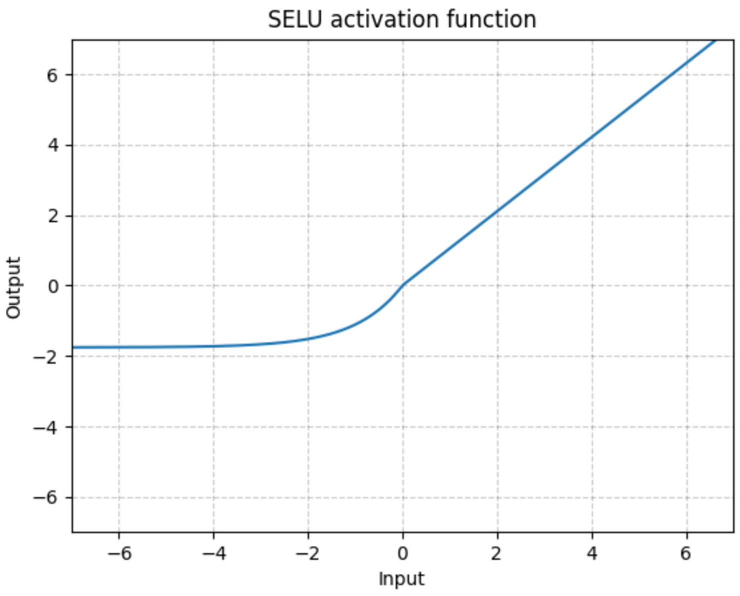
(c) $\mathrm{CELU}(x, \alpha)$

(d) $\frac{d}{dx}\,\mathrm{CELU}(x, \alpha)$

SELU – `nn.SELU()`

$$\text{SELU}(x) = \text{scale} * (\max(0, x) + \min(0, \alpha * (\exp(x) - 1)))$$

with $\alpha = 1.6732632423543772848170429916717$ and $\text{scale} = 1.0507009873554804934193349852946$.



SELU – `nn.SELU()`

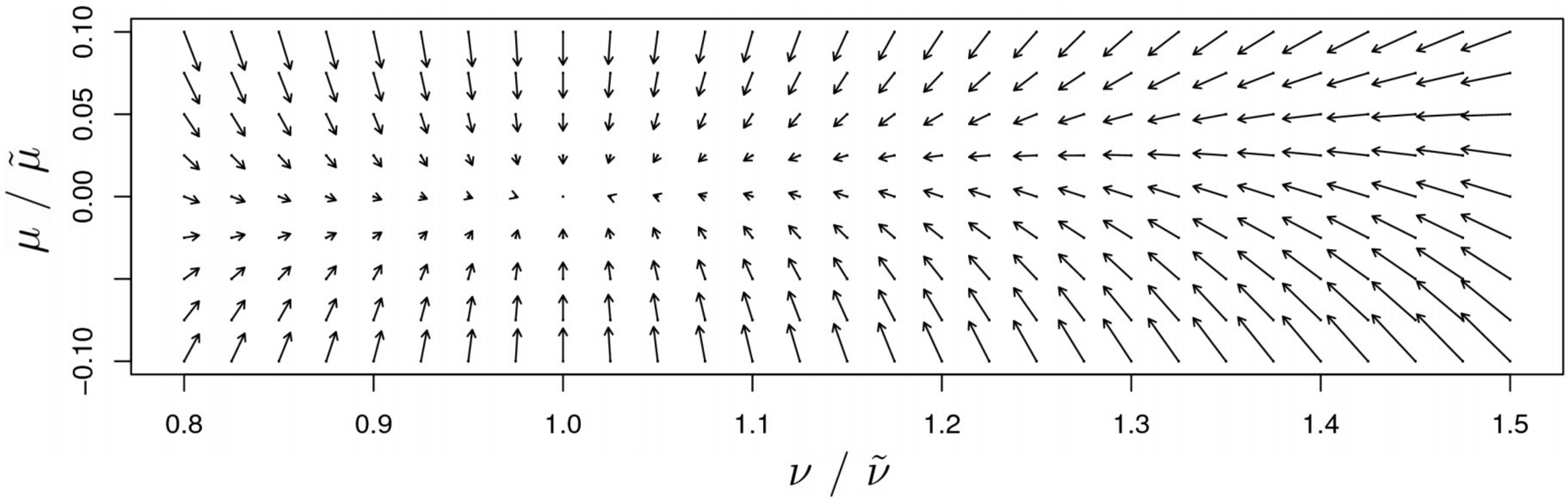


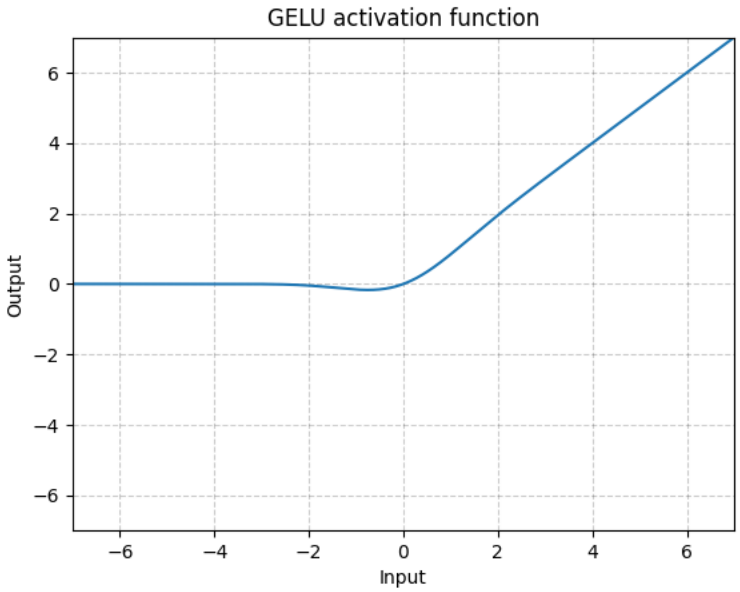
Figure 2: For $\omega = 0$ and $\tau = 1$, the mapping g of mean μ (x-axis) and variance ν (y-axis) to the next layer's mean $\tilde{\mu}$ and variance $\tilde{\nu}$ is depicted. Arrows show in which direction (μ, ν) is mapped by $g : (\mu, \nu) \mapsto (\tilde{\mu}, \tilde{\nu})$. The fixed point of the mapping g is $(0, 1)$.

Klambauer, Unterthiner, Mayr, Hochreiter (2017) Self-normalizing neural networks

GELU – `nn.GELU()`

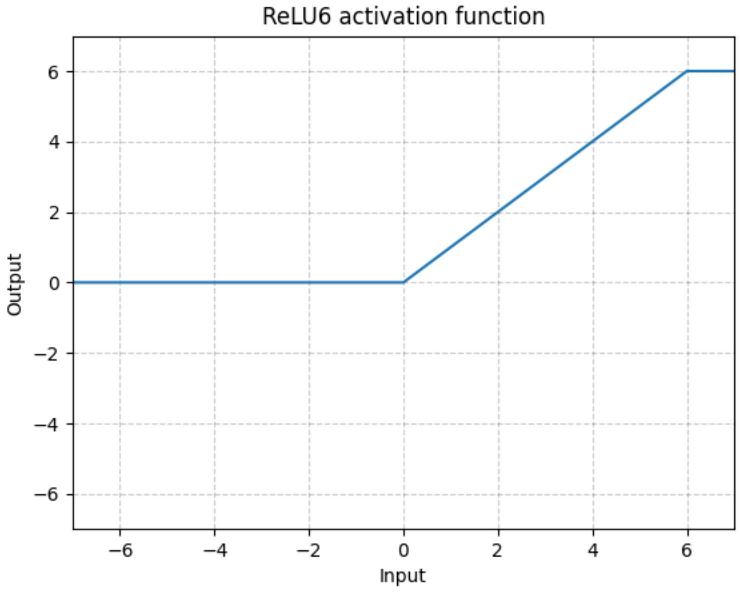
$$\text{GELU}(x) = x * \Phi(x)$$

where $\Phi(x)$ is the Cumulative Distribution Function for Gaussian Distribution.



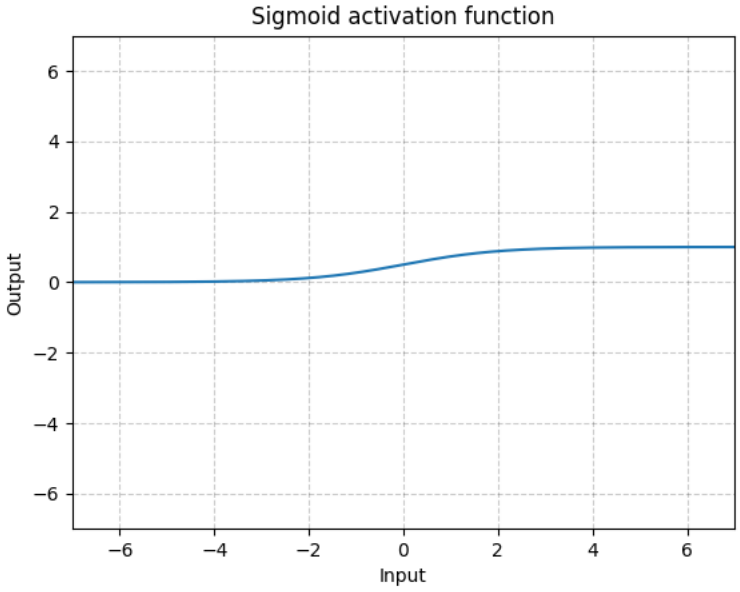
ReLU6 – `nn.ReLU6()`

$$\text{ReLU6}(x) = \min(\max(0, x), 6)$$



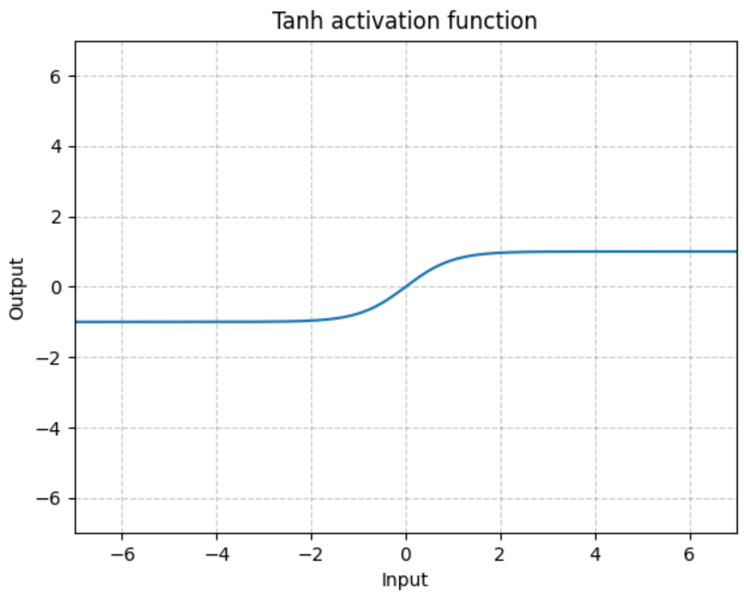
Sigmoid – `nn.Sigmoid()`

$$\text{Sigmoid}(x) = \sigma(x) = \frac{1}{1 + \exp(-x)}$$



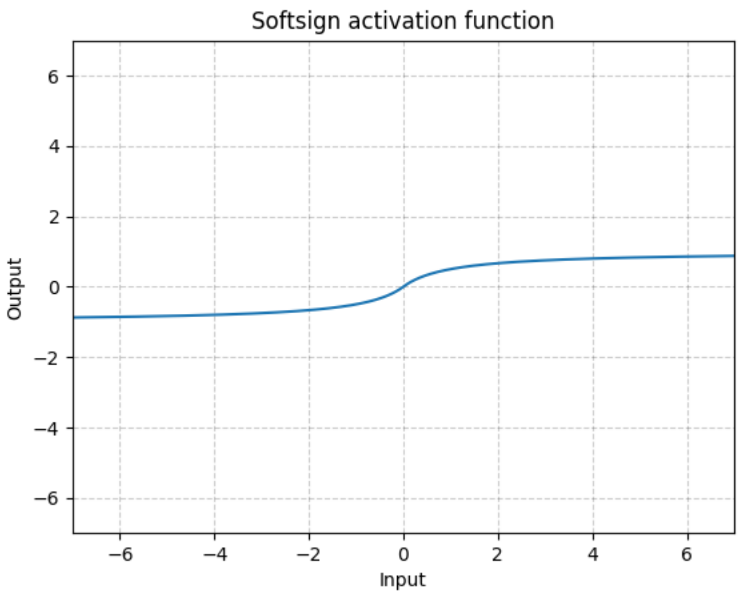
Tanh – `nn.Tanh()`

$$\text{Tanh}(x) = \tanh(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}$$



Softsign – `nn.Softsign()`

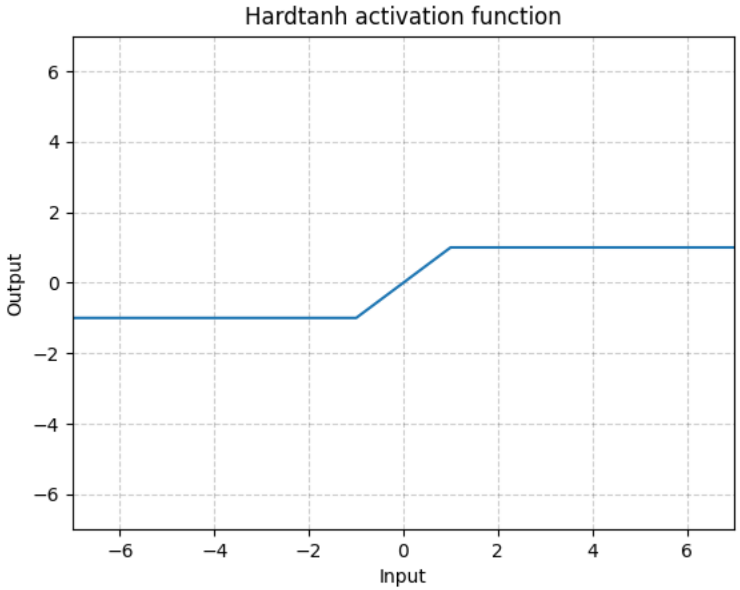
$$\mathrm{SoftSign}(x) = \frac{x}{1 + |x|}$$



Hardtanh – `nn.Hardtanh()`

$$\text{HardTanh}(x) = \begin{cases} 1 & \text{if } x > 1 \\ -1 & \text{if } x < -1 \\ x & \text{otherwise} \end{cases}$$

The range of the linear region $[-1, 1]$ can be adjusted using `min_val` and `max_val`.



Threshold – `nn.Threshold()`

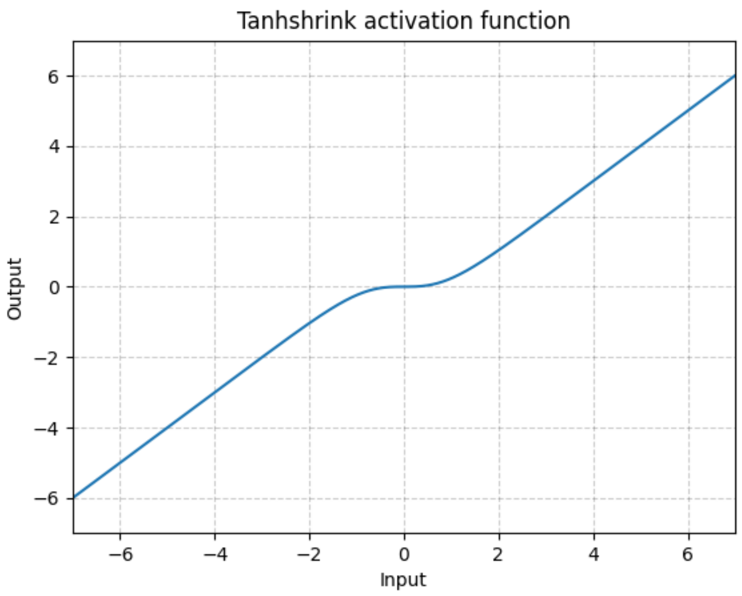
Y. LeCun

Threshold is defined as:

$$y = \begin{cases} x, & \text{if } x > \text{threshold} \\ \text{value}, & \text{otherwise} \end{cases}$$

Tanhshrink – `nn.Tanhshrink()`

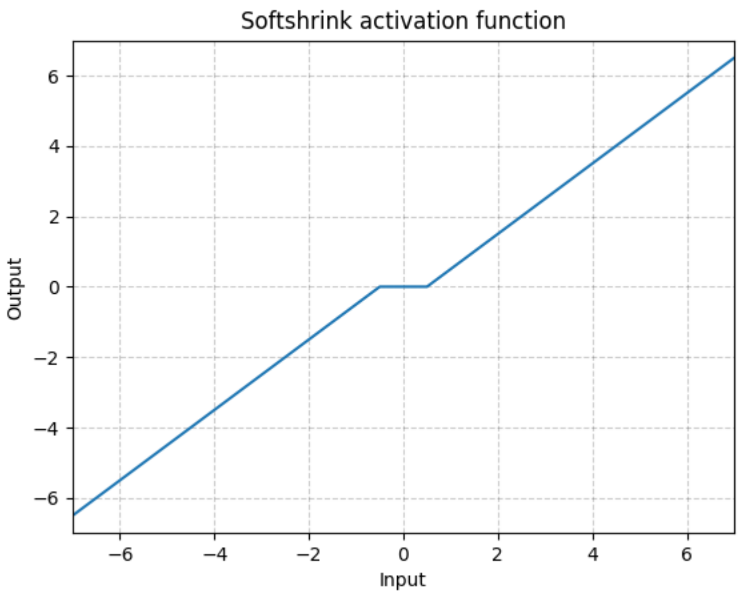
$$\text{Tanhshrink}(x) = x - \tanh(x)$$



Softshrink – `nn.Softshrink()`

Y. LeCun

$$\mathrm{SoftShrinkage}(x) = \begin{cases} x - \lambda, & \text{if } x > \lambda \\ x + \lambda, & \text{if } x < -\lambda \\ 0, & \text{otherwise} \end{cases}$$

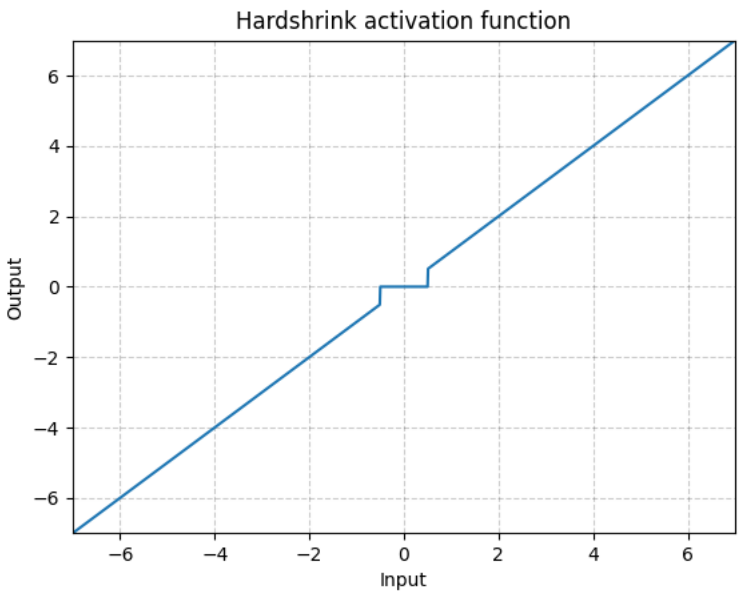


Hardshrink – `nn.Hardshrink()`

Y. LeCun

Applies the hard shrinkage function element-wise:

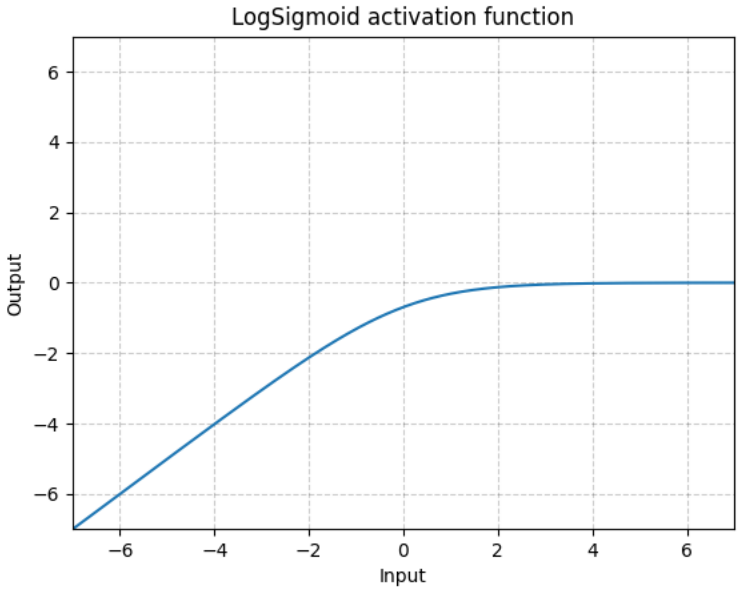
$$\text{HardShrink}(x) = \begin{cases} x, & \text{if } x > \lambda \\ x, & \text{if } x < -\lambda \\ 0, & \text{otherwise} \end{cases}$$



LogSigmoid – `nn.LogSigmoid()`

Applies the element-wise function:

$$\text{LogSigmoid}(x) = \log\left(\frac{1}{1 + \exp(-x)}\right)$$



Softmin – `nn.Softmin()`

Y. LeCun

Applies the Softmin function to an n-dimensional input Tensor rescaling them so that the elements of the n-dimensional output Tensor lie in the range $[0, 1]$ and sum to 1.

Softmin is defined as:

$$\text{Softmin}(x_i) = \frac{\exp(-x_i)}{\sum_j \exp(-x_j)}$$

Softmax – `nn.Softmax()`

Applies the Softmax function to an n-dimensional input Tensor rescaling them so that the elements of the n-dimensional output Tensor lie in the range [0,1] and sum to 1.

Softmax is defined as:

$$\text{Softmax}(x_i) = \frac{\exp(x_i)}{\sum_j \exp(x_j)}$$

LogSoftmax – `nn.LogSoftmax()`

Applies the $\log(\text{Softmax}(x))$ function to an n-dimensional input Tensor. The LogSoftmax formulation can be simplified as:

$$\text{LogSoftmax}(x_i) = \log\left(\frac{\exp(x_i)}{\sum_j \exp(x_j)}\right)$$