Inference

Eric He / eh1885 / N16436011

September 2019

1 Problem 1

a) The probability one child is a girl given you know there is at least one boy is $\frac{2}{3}$, since our sample space is uniform across GB, BB, and BG, and there are girls in two of these three events.

b) Since we saw a boy, we know GG is no longer a probability. However, there is a 100% chance of seeing a boy in the BB case, and only a 50% chance of seeing a boy in the BG and GB cases. Thus we have a 50% chance of seeing BB and a 50% chance of seeing BG. Then the probability there is a girl is only 50%.

2 Problem 2

a) The probability the suspect is guilty given he has a blood type match is not equivalent to the probability the suspect has a blood type match, which is what the prosecutor is implying. Presumably the prior probability the suspect is guilty is a very low number, and so it would not be possible to update the posterior to become exactly as large as the probability of a blood type match.

b) If there was no other evidence implicating the suspect, then the defender would be correct in stating that the suspect's probability of guilt is just 1 in 8000, since the sample space of guilty people now only consists of 8000 people with the blood type. However, there is presumably other evidence which would increase the chances the suspect committed the crime.

3 Problem 3

a) $p(X_2 = \text{Happy})$ is the probability Harry is happy on day 2. Since Harry is happy on day 1, the probability Harry is happy on day 2 is given by the probability of Harry staying happy given he is already happy, which is 90%.

b) $p(Y_2 = \text{Frown})$ has a 10% chance of frowning when happy, and a 60% chance of frowning when angry. Since there is a 90% chance of being happy on day 2 and a 10% chance of being angry on day 2, the answer is given by $.9 .1 + .1 .6 = .15$.

c) Given Harry frowned on the second day, this could have come from the $.9 .1 = 9\%$ chance of being happy and frowning, or it could have come from the $.1 .6 = 6\%$ chance of being angry and frowning. Then Harry's probability of being happy is given by $\frac{9}{9+6} = \frac{3}{5} = 60\%$.

d) The probability of Harry yelling on the 60th day is extremely close to the steady state probability of Harry yelling. Since in the limit, Harry will be equally likely to be angry as he is to be happy (since the Markov chain of happy-angry is symmetric), Harry's steady state probability of yelling is given by $[.1\ .2] \cdot [.5\ .5] = .1 .5 + .2 .5 = .15$. If you want the exact probability, you could diagonalize the happy-angry matrix, raise the diagonal to the 60th power, and left multiply the matrices by $[1\ 0]$ to get the probabilities of being happy and angry on the 60th day, which you would then substitute into $[.1\ .2] \cdot [p\ 1-p]$

e) Harry is guaranteed to be happy on the 1st day, so $X_1 = 1$ is known. Without needing Viterbi's algorithm, we know that if Harry is going to be angry, he had better transition on the 1st day, since frowning is much more probable in the angry state (.6 vs .1), and being angry in the next state is much more probable if Harry is already angry to begin with (.9 vs .1).

The probability of frowning on the second day while staying happy is given by $.9 .1 = 9\%$, while the probability of frowning on the second day while turning angry is given by $.6 .1 = 6\%$. Thus, in the one day case, Harry is more likely to have stayed happy even though he frowned; however, if he stays happy, then every day forth his probability of frowning while staying happy is multiplied by .06 again. On the other hand, if he did switch to being angry, his probability of staying angry and also frowning is given by $.9 .6 = .54$.

Clearly $.06 \cdot .54^4 > .06 \cdot .09^n \cdot .54^{3-n} > .09^5$ where $1 \leq n \leq 4$, so **with maximum a posteriori probability Harry was angry on all days possible.**