Precautionary Mismatch*

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Abstract

The overall productivity of an economy depends on how efficiently talents are allocated to productive activities. Using NLSY79 and O*NET, we first document that while the labor market as a whole features positive assortative matching (PAM) based on observables, the degree of sorting seems to vary by the amount of wealth owned by workers. Workers who enter the labor market with lower liquid wealth experience shorter unemployment period and greater skill mismatch. Wage data suggests that skill mismatch could lead to output loss as greater mismatch is associated with lower wages. To understand the macro implications of mismatch and its relationship with wealth, we construct an incomplete market model with heterogeneous workers and firms where skill mismatch arises due to search frictions. Precautionary saving motive induces wealth-poor workers to accept a wider range of jobs at the expense of potentially lower wages and match-specific output. We show that an improvement in workers' ability to smooth consumption leads to stronger sorting, which in turn increases average output per worker. This insight suggests a new channel through which policies such as unemployment insurance can affect aggregate productivity, i.e. an improvement in the allocative efficiency of the labor market.

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1 Introduction

There may not be a question in economics that receives more attention than "why are some countries so rich and others so poor". Large cross-country differences in income have been attributed to total factor productivity. The next question would naturally be "what are the underlying causes of large TFP differences". Since Hsieh and Klenow (2009), a large and growing literature has emphasized the role of "misallocation" of inputs across firms, which is usually conceptualized as the dispersion in marginal products, and its resulting negative effects on aggregate productivity. This literature largely focuses on capital allocation, presumably because capital is more homogeneous and hence there is a natural notion of marginal product. We believe equally important is the potential contribution from labor misallocation, which is much harder to study due to the presence of wild heterogeneity embedded in workers as well as the heterogeneity in jobs' requirements.

This paper aims at bridging the gap and stresses the allocation of talents. We first document using US data that labor misallocation, in the form of skill mismatch, is prevalent in the US economy and depends at least in part on workers' liquid wealth upon entering the labor market. Workers that are otherwise equally talented but experience greater skill mismatch receive lower wages. This suggests that an economy where more workers are allocated to "wrong" jobs might have lower overall productivity, even though the pool of talents remain the same. Next, we present an incomplete market model with heterogeneous workers and firms that matches our empirical findings. The nature of the production function between heterogeneous worker skills and job requirements determines the optimal allocation between talents and jobs that yield the highest possible aggregate output. The existence of labor market search frictions hinders the optimal allocation. The precautionary saving motives of risk averse agents further interact with the uninsured labor market risks, and shape the misallocation of labor. We use this framework to estimate the amount of labor misallocation as a result of search frictions and liquidity constraints, and help us conduct policy experiments that can potentially improve labor market allocative efficiency. We describe each part of our paper in more detail below.

First, we provide empirical evidence on how liquid wealth owned by workers affects allocations in the labor market. To characterize heterogeneous workers and jobs, we follow Lise and Postel-Vinay (2020) and estimate worker skills and job skill requirements based on direct measures of worker and occupational characteristics in NLSY79 and O*NET. Characterizing workers and jobs along the cognitive skill dimension, we show that overall the labor market exhibits positive sorting, while the strength of sorting increases with workers' liquid wealth. Based on the observed sorting patterns, we then define a notion of skills mismatch as the difference between worker skills and the skill requirements of matched jobs. We find that the extent of mismatch is negatively correlated with liquid wealth. In particular, low-wealth workers spend less time being unemployed but are more likely to be either under-qualified or over-qualified for their positions. Importantly, we also find that labor income exhibits an inverted U-shape in terms of mismatch: that is, being either over-qualified or under-qualified for a job reduces labor income. This finding confirms the predictions of sorting models proposed by Shimer and Smith (2000) and Eeckhout and Kircher (2011) and highlights the trade-off in job search. With limited ability to smooth consumption, low-wealth workers trade-off pay and productivity against the speed of finding a job. As a result, workers that have little savings but are otherwise equally skilled are likely to end up in jobs that are poor matches.

Next, we propose a theory of labor misallocation. While the empirical evidence shows us the consequences of low saving on the worker side, it is largely silent on the production side. To fully characterize the extent of misallocation and its implications on output, we need a structural theory of sorting. We propose an equilibrium search framework with two-sided heterogeneity, risk-averse agents and borrowing constraints to study our problem. At its core is Bewley (1977)-Huggett (1993)-Aiyagari (1994) meets Diamond-Mortensen-Pissarides, with two-sided heterogeneity in the spirit of Shimer and Smith (2000). On the worker side, unemployed workers draw down savings to maintain consumption and search for jobs. Lack of savings limits their ability to smooth consumption, inducing them to adjust job search strategies to self-insure, which effectively works as a "precautionary job search motive". Due to search frictions, a trade-off exists between wages and the speed of finding a job. For this reason, workers with lower savings would choose to match with a job more quickly, but at the cost of potentially lower wages. This trade-off might be particularly salient nowadays as a result of increasing student debt burden on young workers entering the labor market. On the firm side, vacant firms also face a trade-off between profits and the speed of filling a vacancy. This suggests that the equilibrium features a set of matching sets between workers and firms, for which the shapes depend on the amount of wealth owned by workers. Skill mismatch induces a tradeoff between job finding rates and output per worker, and this tradeoff exacerbates when more people are financially constrained.

We plan to extend the model to allow for quantitative analysis and study of policy counterfactuals. Using the estimated model, our goals is to estimate the extent of labor misallocation and productivity loss due to workers' liquidity constraints. We can use the model as a laboratory to study various policies that provide consumption insurance to workers, including unemployment insurance, credit stimulus, etc. As student debt has ballooned to astonishing figures in the US lately, we can also contribute to the debate on student loan policies by studying how much labor misallocation can be improved by adopting alternative repayment plans, such as extended grace period and income-based repayment.

Related Literature

Empirically, our paper is related to a large literature documenting relations between asset holdings and job search behavior (see, for example, Card, Chetty and Weber (2007), Rendon (2007), Lentz (2009), Chetty (2008), Herkenhoff, Phillips and Cohen-Cole (2017), among many others). These papers show overwhelmingly that increasing the ability to smooth consumption, either through unemployment insurance, wealth or access to credit, leads to longer unemployment duration and higher accepted wages. These findings provide us with an important guidance to think about the implications of the observed search behavior in the context of labor market sorting. A natural prediction from a longer unemployment duration is that the match quality of unemployed workers with new jobs also increases. To our knowledge, we are among the first papers to document the joint effect of worker assets and skills on allocations to jobs following an unemployment spell. Our approach to measure worker and job heterogeneity follows recent papers including Lise and Postel-Vinay (2020) and Guvenen et al. (2020), which also use observable worker and job characteristics from NLSY79 and N*NET to estimate skills mismatch and effects on wages. We extend their approach to include wealth heterogeneity and show that skills mismatch is likely to be influenced by precautionary saving motive.

The second literature we are contributing to extends the linear utility assumption in a standard job search framework initiated from McCall (1970) by incorporating risk averse agents in a Bewley (1977)-Huggett (1993)-Aiyagari (1994) type incomplete market model. The key insights arise from the standard exogenous income process being replaced by job search behavior that endogenizes uninsurable income risk. Using a Diamond-Mortensen-Pissarides framework with risk aversion, Krusell, Mukoyama and Şahin (2010) is the first to study an incomplete-markets model with labor-market frictions, which is used to evaluate a tax-financed unemployment insurance scheme. Lise (2013) introduces on-the-job search but focuses on a partial equilibrium, and generates an important asymmetry of saving behavior between the incremental wage increases generated by on-the-job search (climbing the wage ladder) and the drop in income associated with job loss (falling off the ladder). Recent updates including Eeckhout and Sepahsalari (2018), Chaumont and Shi (2018), Herkenhoff, Phillips and Cohen-Cole (2017) and Krusell, Luo and Rios-Rull (2019), instead study a directed search equilibrium model with risk-averse workers, where the key trade-off is the speed of finding a job versus the wage for workers (and similarly, the speed of filling a vacancy versus profits for firms). Griffy (2018) further introduces human capital accumulation to study the life-cycle inequality in earnings and wealth. Our framework is essentially an extension of Krusell, Mukoyama and Şahin (2010) by introducing two-sided skill heterogeneity in the spirit of Shimer and Smith (2000), which allows us to study the interaction between precautionary saving and mismatch. For ease of computation we adopt the continuous-time method introduced by Achdou et al. (2020), which cast rather complex optimization problems into systems of HJB equations that are easier to compute.

Third, there is a recent literature that starts to study multidimensional skills mismatch. Lindenlaub and Postel-Vinay (2020) characterizes sorting with random search when both workers and jobs have multi-dimensional heterogeneity. Their key theoretical insight is that multidimensional heterogeneity is in itself a source of sorting. They also argue that multi-dimensional sorting is empirically relevant in the sense that a single-index representation misses substantial features in the data. Lise and Postel-Vinay (2020) studies dynamic sorting by incorporating human capital accumulation via learning by doing. An interesting finding is that the half-life of skill accumulation varies quite a lot across different types of skills. According to their estimates, the half-year is 7.5 for cognitive skills, 1.7 for manual skills, and 55.8 for interpersonal skills. The message is that it is super hard to accumulate one's interpersonal skills. Closely related is Guvenen et al. (2020) who also examines multidimensional skill mismatch using similar empirical measures, but provides a different theoretical angle for the source of mismatch. In Lise and Postel-Vinay (2020), the source of mismatch is the (random) search frictions, while in Guvenen et al. (2020) it is the misperception about one's own abilities. Baley, Figueiredo and Ulbricht (2019) studies the business cyclic properties of mismatch.

Lastly, we contribute to the macro-development literature on misallocation, which is pioneered by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). The idea is so appealing – if we reallocate some production from a firm with a lower marginal product to a firm with a higher marginal product, we could achieve higher aggregate output even without accumulating any inputs. This literature, by its nature, pays more attention on the firm side and typically abstracts away from labor heterogeneity. Our paper digs deeper into the misallocation arising from the allocation of heterogeneous workers to heterogeneous firms.

The rest of the paper is organized as follows. In Section 2, we describe the data sets we use for empirical analysis and the methods to estimate worker and firm types. In Section 3 we present empirical evidence on the relationship between liquid wealth, skill mismatch and wages. In Section 4, we construct the backbone of a model which we will use to match the empirical findings and analyze labor misallocation. Section 7 concludes and provides a roadmap for the next steps.

2 Data

Our empirical analysis is based on a selected worker panel from the 1979 National Longitudinal Survey of Youth (NLSY79), a nationally representative survey conducted on individuals 14-22 years old when first interviewed in 1979. We merge the NLSY79 work history and asset information with data from the Occupational Information Network (O*NET), an occupation-level data set with scores on the skill contents of 974 occupations, so that we have a matched worker-job data set with joint worker and job characteristics.

2.1 NLSY79

We use the work history data from NLSY79 to construct a monthly panel, and focus on a cross-sectional sample of workers with no experience of serving in the military. We further exclude individuals who are already considered to be in the labor market at the beginning of the survey, where we consider an individual to be in the labor market if they work more than 30 hours per week or 1200 hours per year, or if they have finished their last schooling spell and started working. To minimize the effect of work experience gained during education on our estimation, we also exclude those who have more than 2 years of work experience before the end of his/her schooling spell.

Worker characteristics are extracted from the test scores of the Armed Services Vocational Aptitude Battery (ASVAB), a special survey conducted by the US Departments of Defense and Military Services in 1980 that evaluates individuals in 10 categories. To construct skill bundles for workers, we follow a procedure similar to the one used in Lise and Postel-Vinay (2020) (see Appendix for details): we run principal component analysis (PCA) on the 10 individual-level ASVAB test scores and keep the first two principal components. We then construct work skills along 2 dimensions, namely cognitive and manual skills by recombining the principal components so that they satisfy the following exclusion restrictions: (1) the ASVAB mathematicas knowledge score only loads on cognitive skill, and (2) the ASVAB automotive and shop information score only loads on manual skill. We rescale the skill measures to a unit-length interval $[0, 1]$. As the test was conducted before the majority of the respondents entered the labor market, we believe that this skill measure is mostly free from the endogeneity issue wherein jobs affect worker skills.

In addition to work history and test scores, we also obtain annual history on assets from NLSY79. Unfortunately, NLSY79 did not start extensively collecting assets information until 1985, when over half of the respondents had entered the labor market. Therefore our sample is heavily biased towards late-entrants when examining the relationship between liquid wealth and skill mismatch. We construct a measure of liquid wealth of individuals based on the sum of financial assets such as cash, deposit, mutual fund and money market accounts and other assets more than $500, net of debts that are not asset-backed. Since asset information is not updated in each round of survey for most respondents, we linearly interpolate the amount of assets for each individual to maximize the amount of information we can use in our empirical analysis.

2.2 O*NET

The O*NET data contains ratings of importance and level on hundreds of specific aspects, called "descriptors", of each occupation. The descriptors can be summarized by 9 broad categories: skills, knowledge, abilities, work activities, work context, education levels required, job interests, work styles and work values. Following Lise and Postel-Vinay (2020), we keep the level ratings related to descriptors from the first 6 categories, which add up to over 200 descriptors for each occupation. Similar to the procedure for worker skills construction, we reduce the descriptors to 2 dimensions using PCA and keep the first 2 components. Then, we recover cognitive and manual skill requirements by recombining the principal components in such a way that (1) the mathematics rating only loads on cognitive skill requirements, and (2) the mechanical knowledge rating only loads on manual skill requirements. We also rescale the skill requirements measures to a unit-length interval $[0, 1]$. Therefore, each job can be characterized by a bundle of skill requirements, in which a higher number in each dimension represents higher requirements of the corresponding skill.

For this paper, however, we only focus on sorting based on cognitive skills. While our structural model can easily account for multidimensional skill types in theory, solving and estimating such a model turns out to be computationally heavy.

3 Empirical Analysis

3.1 Descriptive Statistics

3.1.1 Skill Measures and Sorting

Our selected sample includes 3,285 individuals with substantial heterogeneity in levels of education, ranging from no degree to PhD. Presumably, our measure of worker skills and job skill requirements should reflect their relative rankings and productivity in the sample respectively. An obvious way to examine this presumption is to see how the two measures vary by levels of education. Table 1 shows average worker cognitive skills and job cognitive skill requirements by highest degrees at the time of initial labor market entry. Both measures are normalized to a unit-length range $[0,1]$, where a higher number represents higher cognitive skill/skill requirement. Comparison of the skill measures at the lowest and highest education levels (No Degree

Table 1: Average Worker Cognitive Skills and Job Skill Requirements Upon Labor Market Entry, by Level of Education

	No Degree	High School	Some College	2-yr College	4-yr College	Masters	PhD
Worker Skill (x)	0.266	0.388	0.456	0.543	0.684	0.714	0.770
Job Skill Req (y)	0.223	0.272	0.304	0.336	0.425	0.474	0.504
Observations	159	1053	233	402	500	323	99

Note: Both x and y are normalized to $[0,1]$.

and PhD) shows that education seems to account for a substantial amount of worker skill heterogeneity, and a modest amount of job skill heterogeneity. It is perhaps not surprising to find that both worker skills (first row) and job skill requirements (second row) increase monotonically with level of education. Therefore at least with respect to skill differences across education groups, our skill measures are able to capture the relative ranking of workers and jobs, as well as positive sorting. However, some questions yet to be answered are whether we can identify sorting beyond sorting on education using the cognitive skill measures, and whether sorting is still positive after controlling for education groups. To answer this question, we show the correlation between job skill requirements and worker skills in Table 2, with and without controlling for worker educations. We take one observation from each worker-employer match in the data and

Table 2: Skill Sorting Over Occupations

	(1) Job Skill Req (y)	(2) Job Skill Req (y)
Worker Skill (x)	0.691***	0.513***
	(0.020)	(0.027)
Education Level	No	Yes
Obs	35616	35616

Note: Standard errors are clustered on occupation level. The sample is taken from the first observations of each worker-employer match.

regress job cognitive skill requirements on worker cognitive skills. Column (1) shows that the correlation between worker skills and job skill requirements are 0.69, which is both statistically and economically significant. To isolate sorting on skills from sorting on education, we perform an additional regression controlling for dummies for years of education. After controlling for education, the remaining correlation is still large and highly significant at 0.51, suggesting a substantial amount of sorting on individual skills exists beyond sorting on education.

3.1.2 Initial Liquid Wealth and Worker Characteristics

There are 1,114 individuals with valid information about liquid financial wealth at the time when they entered labor market. We define net liquid wealth as the value of financial assets such as cash, deposit, mutual fund and money market accounts net of debts that are not asset-backed. This measure is supposed to reflect assets that workers can access in a relatively short period of time. Table 3 shows the characteristics of workers upon labor market entry, where the workers are divided into quintiles according to their liquid wealth during the first month of work.

Table 3: Worker Characteristics by Initial Wealth Quintile

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Net Financial Assets (1000s)	-7.971	0.346	1.764	5.356	31.83
Weekly Income	233.8	195.2	193.3	255.8	301.4
Years of Educ	15.91	14.54	15.38	15.88	16.25
Age	27.48	27.09	26.98	27.68	29.13
Male	0.416	0.405	0.368	0.446	0.350
PRTs Annual Income	19874.5	18343.5	23147.3	25479.3	25623.4
Observations	202	200	204	202	203

Note: liquid assets, weekly income and parents' annual income are in 1982 dollars

There are substantial heterogeneity in the level of liquid wealth upon labor market entry, ranging from \$-7,971 in the lowest quintile to \$31,830 in the highest quintile (in 1982 dollars), a difference of almost \$40,000. Workers who enter the labor market with higher liquid wealth tend to have higher income, more education, higher age and higher parental income. The only exception is the lowest quintile, where weekly income, years of education, age and parental income are all higher than those in the quintile above. A likely explanation is that the lowest liquid wealth quintile could consist of individuals who borrow substantial amount of debt for their higher education, thereby lowering their initial wealth. Note that the age of labor market entry is highly upward biased (most workers enter labor market in early 20s) because NLSY79 didn't start collecting wealth information until 1985, when half of the sample were above 25. This means that later when we analyze the effect of initial wealth, our sample is biased towards late entrants.

3.1.3 Initial Liquid Wealth and Unemployment Duration

The previous table shows that wealthier workers tend to find jobs with higher income upon labor market entry. There are several potential explanations. First, as wealthier workers are more likely to receive higher education, they have better cognitive skills on average and are therefore qualified for jobs that are more sophisticated and better-paid. Another explanation is that liquid wealth increases their ability to smooth consumption during unemployment, thereby enabling them to be more patient in job search. We would like to focus on the second explanation as it has direct implications to mismatch and allocative efficiency. To shed light on how liquid wealth affects workers' job search behavior while unemployed, we examine the effect of liquid wealth on unemployment duration in the first unemployment spell of each worker after labor market entry. Table 4 shows regression estimates where we regress weeks of unemployment in the first unemployment spell on workers' liquid wealth at labor market entry, along with other covariates.

Table 4: Duration of Unemployment and Liquid Wealth

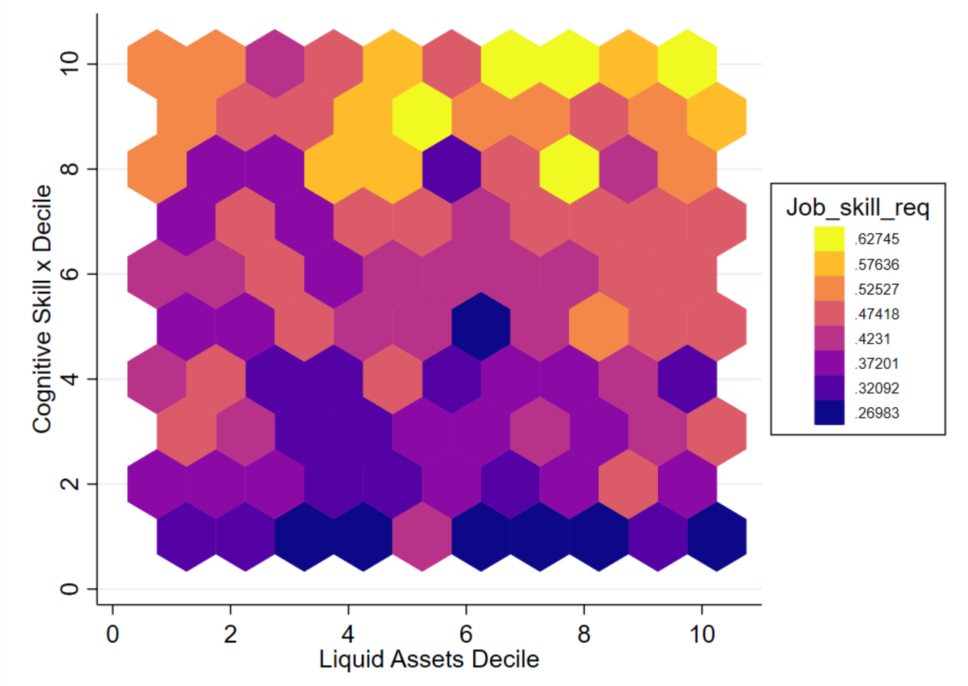
	(1)	(2)	(3)
Net Liquid Wealth	0.347*	0.506***	0.484**
	(0.182)	(0.171)	(0.223)
Demographic	No	Yes	Yes
Family background	No	No	Yes
Obs	849	849	674

Column (1) of Table 4 shows the coefficient of liquid wealth with it being the only regressor. In column (2), we control for a standard set of individual characteristics including sex, race, education, age and AFQT score (a proxy for ability). To further control for insurance that young workers may receive from their families, we additionally include parents' annual income and poverty status in column (3). The estimates imply that a \$10,000 increase in liquid wealth is associated with a 4- to 5-week increase in unemployment duration. Considering that fact that the difference in average wealth between the lowest and highest quintile is about \$40,000, and that the median unemployment duration is 14 weeks, the estimates imply that a substantial fraction of cross-sectional variation in unemployment duration exists due to wealth heterogeneity.

3.1.4 Initial Liquid Wealth and Sorting

Having shown that wealthier workers spend more time in unemployment, we then ask a natural follow-up question: does the job search strategy lead to different patterns of labor allocation for low- and high-wealth workers? In Figure 1, we show how labor market entrants of different skill and initial liquid wealth deciles sort into jobs with different skill requirements. Each hexagon represents a combination of liquid assets and skill decile, and the color corresponds to the average level of skill requirement within each bin.

Figure 1: Allocations to Occupations by Liquid Assets and Cognitive Skill Deciles



We can see that holding asset fixed, workers with higher skill sort into jobs with higher skill requirements, echoing our finding above that sorting is positive. In addition, the gradient of job skill requirements with respect to worker skills is more pronounced for high-wealth workers, suggesting that the degree of positive sorting between workers and jobs increases with wealth. This leads us to believe that an increase in liquid wealth induces workers to spend more time searching for jobs with higher returns to their skills, and therefore form more efficient matches.

So far we have not discussed the welfare implications of sorting, nor have we introduced the concept of skill mismatch. In the sections below, we will construct skill mismatch as a measure for match quality, and show how labor income varies with mismatch. In particular, we will demonstrate that labor income is inverted U-shaped: it is the highest when mismatch is minimized.

3.2 Skills Mismatch

We think of skill mismatch as a proxy for match quality that can potentially be output-relevant. Precisely speaking, it is defined as the distance between a worker's skill level to that required by the job she is matched with. Since we are only concerned about one skill dimension, a worker is over-qualified if her skill level is above that required by the job, and vice versa.

Definition 1. Mismatch measures

*Let x_i denote the skill level of individual i, and y_c denote the skill requirement of job c, then we define the **mismatch** between individual i and job c as*

$$m_{i,c} \equiv y_c - x_i \tag{1}$$

*$m_{i,c} > 0$ means that worker i is under-qualified (or over-employed) for job c, and vice versa. We define the **magnitude of mismatch** between individual i and job c as*

$$mm_{i,c} = |m_{i,c}| \tag{2}$$

*We also define **positive** and **negative mismatch** as*

$$m^{+}_{i,c} \equiv \max\left(m_{i,c}, 0\right), \text{ and } m^{-}_{i,c} \equiv \text{-}\min\left(m_{i,c}, 0\right) \tag{3}$$

so that the former is positive only if individual i is under-qualified for job cin terms of cognitive skills, and the latter is positive only if individual i is over-qualified in terms of cognitive skills. We add a negative sign to the negative mismatch so that both measures are positive numbers.

For ease of interpretation, we normalize each mismatch measure so that they have means of 0 and unit standard deviations. A measure of 0 positive mismatch, for example, means that the level of positive skill mismatch experienced by a particular worker is at the average level of positive mismatch in our sample. Alternatively, a measure of positive mismatch equal to 1 means that the worker experiences a level of positive mismatch that is 1 standard deviation above mean.

3.2.1 Initial Wealth and Mismatch

We now document the relationship between liquid wealth and skill mismatch. As we know from Table 3, initial wealth might be confounded by workers' levels of education, which could in turn also affect sorting. Therefore we control for level-of-education fixed effects when plotting the levels of mismatch. Figure 2 shows the relationship between liquid wealth and mismatch for labor market entrants. The top panel shows the magnitude of mismatch, defined by equation 2. The next 2 panels show mismatch in in positive and negative components respectively, defined by equation 3.

First of all, we see from the top panel of Figure 2 a consistent drop in the magnitude of skill mismatch with liquid wealth, from 0.1 standard deviations above average in the lowest wealth quintile to 0.2 standard deviations below average. This pattern holds true both for the positive component ("Mismatch Underqualified") and the negative component ("Mismatch Overqualified"), as shown by the bottom two panels. Nevertheless, changes in mismatch over wealth is more pronounced for the overqualification measure, which drops by about 0.3 standard deviations from the lowest to the highest wealth quintile. This shows that as workers become less constrained financially, they are matched with jobs that have skill requirements closer to their own skill levels, which helps to explain why the strength of sorting increases with liquid wealth.

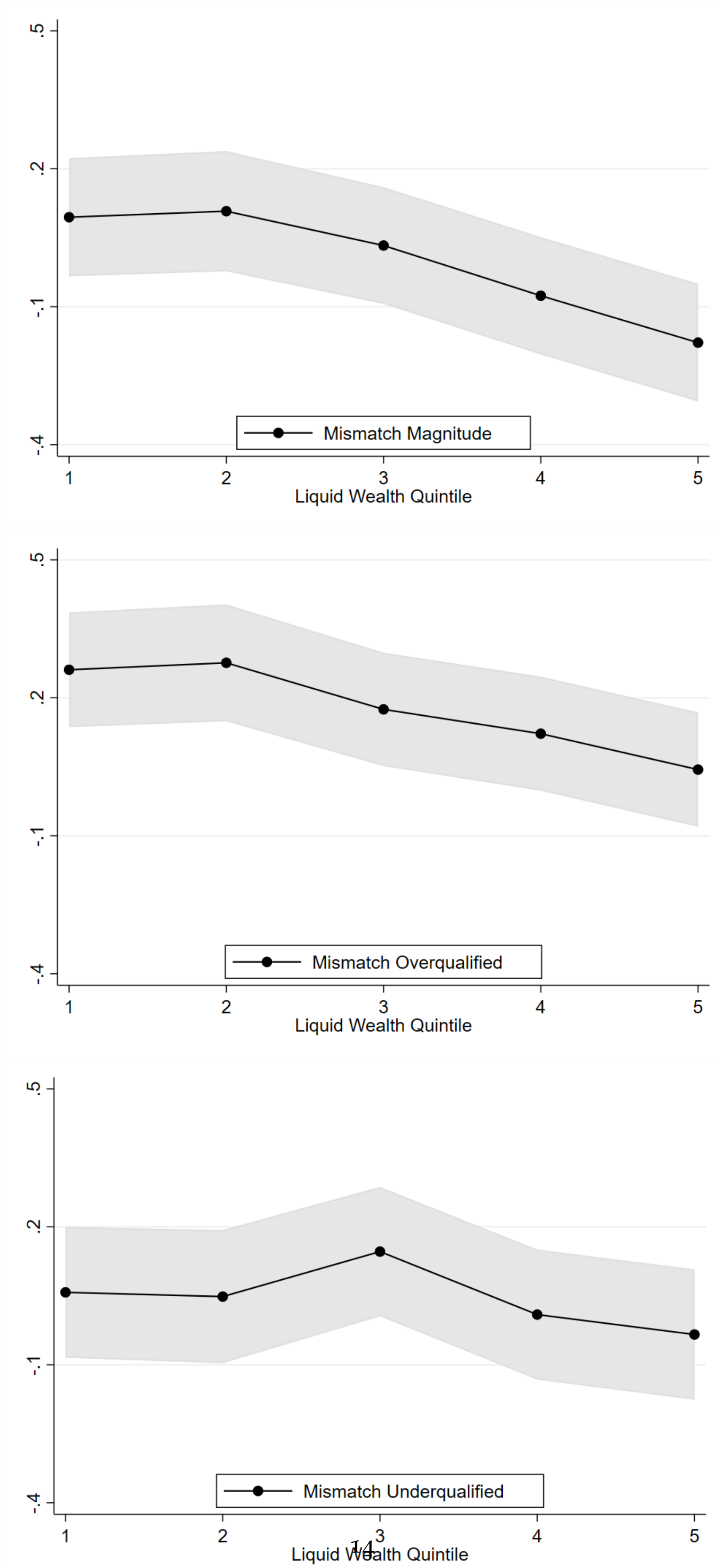
3.3 Mismatch and Wages

3.3.1 The (Inverted) U-Shape of Wages

To show how deviation from a job's skill requirement affects workers' welfare, we document a key fact in this section, mainly that labor income is (inverted) U-shaped in terms of skill deviation. That is, given a worker with a fixed skill level, a job with skill requirements that are either too high or too low gives her lower wages than one that is perfectly-matched with the worker. Such patterns would naturally arise in models such as those in Shimer and Smith (2000) and Eeckhout and Kircher (2011) for the following reason: if a worker is under-qualified for a job, she is paid lower wages as the firm demands a larger share to be willing to match; on the other hand, an over-qualified worker is also paid lower wages as the firm has low output and is thus unable to pay high wages.

Figure 3 shows non-parametric plots of log wages (in 1982 dollars) as a function of the deviation from job's skill requirement $(y - x)$, based on kernel smoothed local linear regressions. The left panel is based on raw wage data, while the right panel is based on residual wages by

Figure 2: Mismatch Measures by Initial Wealth Quintile

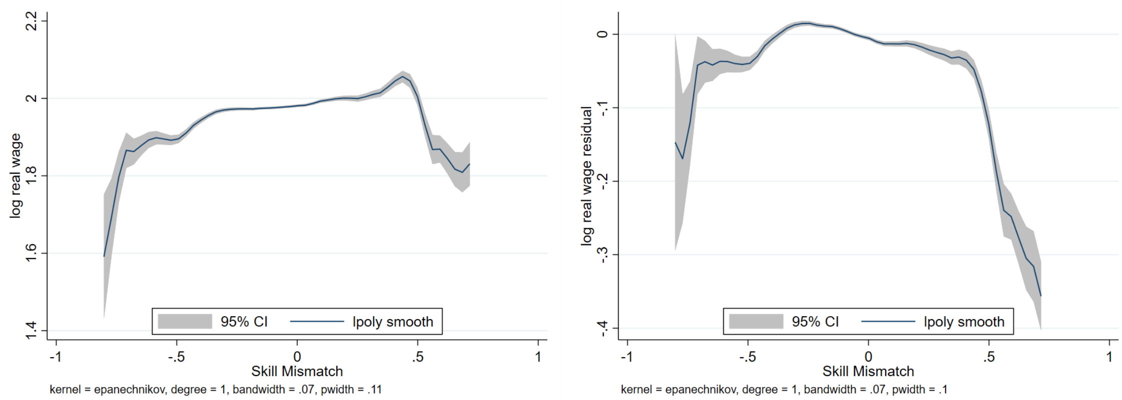


estimating the following wage regression

$$\ln w_{i,l,c,t} = X_{i,l,c,t}\beta + \epsilon_{i,l,c,t}$$

where $w_{i,l,c,t}$ is real wage of an individual i working with employer l in occupation c in period t and $\epsilon_{i,l,c,t}$ is the residual. The control variables in X includes race, sex, education fixed effects, quadratic functions of employer tenure, occupation tenure, labor market experience and age as well as 3-digit occupational fixed effect. While the scales in the two plots are not directly

Figure 3: Non-parametric Plot of Log Labor Income by Worker's Percentage Deviation from Job Skill Requirement



comparable because the right panel uses wage residuals, we can see that in both cases, wages tend to be higher when mismatch is close to 0, and lower when job skill requirement is either too high or too low relative to the worker's skill. This figure provides evidence for the aforementioned theories and suggests that skill mismatch is directly linked with the returns to worker skills.

3.3.2 Wage Regression

We examine the effects of skills mismatch on wages more formally using Mincer wage regression augmented with our mismatch measures. Suppose that a worker can be matched with a single employer at any given time, where each employer offers a job in a particular occupation (requiring a fixed bundle of skills). The (logged) wage of individual i working with employer l in occupation c at time t be be specified as:

$$\ln w_{i,l,c,t} = \alpha + \beta_0 m_{i,c} + \Phi\left(J_{i,l,t}\right) + \Phi\left(T_{i,c,t}\right) + \Phi\left(E_{i,t}\right) + \xi_c + X'_{i,t}\Gamma + \epsilon_{i,l,c,t}$$

where $m_{i,c}$ is a measure of mismatch between worker i and occupation c, $J_{i,l,t}$, $T_{i,c,t}$ and $E_{i,t}$ are worker's tenure at employer l, occupation c and total work experience respectively. Φ denotes a quadratic function. ξ_c represents occupation fixed-effect on a 3-digit level. $X_{i,t}$ represents a set of individual controls including sex, race, age, years of education.

We use a monthly panel of 2,167 individuals to estimate the wage regression. After dropping observations before the last schooling spell, the average length of the panel is 15.6 years. We keep observations within the initial 15 years since the sample becomes heavily biased towards workers with lower educations beyond this point. Table 5 shows the regression coefficients where we use different measures for $m_{i,c}$. The wages are multiplied by hours of work to reflect weekly labor income.

Table 5: Effect of Mismatch on Labor Income

	(1) Total	(2) Sign
Total Mismatch	-0.023***	
	(0.002)	
Mismatch Underqualified		-0.007***
		(0.002)
Mismatch Overqualified		-0.029***
		(0.004)
Obs	195605	195605

Note: Controls include: demographic characteristics, quadratic functions of employer experience, occupation, work experience and age and 3-digit occupation codes.The errors in the parentheses are robust standard errors.

In column (1), we examine how the magnitude of mismatch affects labor income. The coefficient indicates that a 1-standard-deviation increase in mismatch lowers labor income by 2.4 percent. Column (2) shows the effect of mismatch in either direction: being under-qualified and being over-qualified for a job both leads to lower income. However, being over-qualified has a much stronger impact on wages, as a 1-standard-deviation increase in the level of over-qualification leads to a 2.9% decrease in labor income. This possibly explains why over-qualification is much more responsive to changes in initial wealth, as shown by Figure 2.

The empirical evidence we have shown thus far informs us of a way to think about how liquid wealth, skill mismatch and labor misallocation are linked to each other. Intuitively, search frictions in the labor market lead to a trade-off between wages and the speed of finding a job. Having limited ability to smooth consumption, workers with lower liquid wealth favor jobs that are easier to find at the cost of lower wages. Under the reasonable assumption that firms provide higher wages to workers with higher productivity in their positions, and the assumption that production is jointly determined by firms' skill requirements and workers' skills, the job search trade-off would thus imply that low-wealth workers end up experiencing larger skill mismatch, leading to an allocation of talents that fails to maximize the overall return to skills.

4 Model

In this section, we propose a theory of labor market misallocation due to search frictions and incomplete market. The core of the model is a Bewley (1977)-Huggett (1993)-Aiyagari (1994) economy with search and two-sided heterogeneity.

4.1 Environment

Time is continuous.

Preference. Workers and jobs are forward looking and infinitely lived. They maximize expected present value according to a common discount rate ρ. Workers are risk averse with flow utility $u(c)$ and firms are risk neutral. The utility function $u(\cdot)$ exhibits common properties $u' > 0, u'' < 0$.

Production. Workers and jobs are heterogeneous. Workers are characterized by $x \in \mathbb{X}$ and jobs by $y \in \mathbb{Y}$. The production function of a matched pair is denoted $f(x, y) : \mathbb{X} \times \mathbb{Y} \to \mathbb{R}_{+}$. We impose technical assumptions on f to guarantee existence. Unemployed workers produce b (e.g., leisure, unemployment benefits, and home production).

Search and Matching. Labor markets are frictional. Search and matching is random via a *meeting* function $M(u, v)$ that is constant returns to scale (CRS), where u denotes unemployment and v vacancies. We denote by $\theta = v/u$ the labor market tightness. Due to CRS, the meeting rate for an unemployed worker can be written as $p(\theta) := M(u, v)/u = M(1, \theta)$. Similarly, the meeting rate for a vacancy can be written as $q(\theta) := M(u, v)/v = M(\theta^{-1}, 1)$. Note that $q(\theta) = p(\theta)/\theta$. The difference between *meetings* and successful *matches* is worth noting. Once a worker and a job meet, they can decide whether to start production or not. Some meetings may not end up with a succesful match if the agents prefer to continue searching. Jobs are destroyed exogenously with a Poisson rate σ. In the benchmark model, there is no on-the-job search. Wage is determined by Nash bargaining with worker bargaining power denoted η.

Incomplete Market. There is not a complete set of Arrow securities. Instead, there is only one asset that agents can lend (and borrow) at a risk-free rate, to smooth consumption against fluctuations in labor income. Workers face a borrowing constraint $\underline{a}$.

4.2 Characterization

4.2.1 Distribution

Before characterizing the value functions, it proves useful to define several relevant measures. The population distributions over worker types and job types are given by $d_w(x)$ and $d_j(y)$, respectively. For notational convenience, we refer to matches as m, employed workers e, unemployed workers u, producing jobs p, and vacant jobs v, all using the first letter the words. For example, the density function of producing matches is denoted $d_m(a,x,y) : \mathbb{R} \times \mathbb{X} \times \mathbb{Y} \to \mathbb{R}_+$. We could define other densities in a similar fashion, with density of employed workers $d_e(a,x) = \int d_m(a,x,y)\,\mathrm{d}y$, density of unemployed workers $d_u(a,x)$, density of producing jobs $d_p(y) = \iint d_m(a,x,y)\,\mathrm{d}a\mathrm{d}x$, and density of vacant jobs $d_v(y) = d_j(y) - d_p(y)$. Notice that the aggregate unemployment and vacancy are given by $u = \iint d_u(a,x)\,\mathrm{d}a\mathrm{d}x$ and $v = \int d_v(y)\,\mathrm{d}y$, respectively. These add-up properties are summarized in Table 6.

Table 6: Distribution Add-up Properties

Description	Add-up Property
Workers	$d_w(x) = \int d_u(a,x)\,da + \iint d_m(a,x,y)\,\mathrm{d}y da$
Total unemployment	$u = \iint d_u(a,x)\,\mathrm{d}x\mathrm{d}a$
Firms	$d_j(y) = d_v(y) + \iint d_m(a,x,y)\,\mathrm{d}x\mathrm{d}a$
Total vacancies	$v = \int d_v(y)\,\mathrm{d}y$

Notes: The table summarizes the aggregation properties relating densities $d_u(a,x)$, $d_v(y)$, u, v and the match density $d_m(a,x,y)$.

4.2.2 Hamilton-Jacobi-Bellman Equations

Let $U(a,x)$ denote the value of an unemployed worker of type x with with wealth a, and $W(a,x,y)$ the value of an employed worker of type x with asset a working at a firm of type y. The HJB equation for the value of being employed is:

$$\begin{aligned}\rho W(a,x,y) &= \max_{c} \mathrm{u}(c) + \sigma\left[U(a,x) - W(a,x,y)\right] + \dot{a}W_a(a,x,y) \qquad (4)\\ \text{s.t. } \dot{a} &= ra + \omega(a,x,y) - c\\ a &\geq \underline{a}\end{aligned}$$

where $[\bullet]^+ := \max\{\bullet, 0\}$. An employed receives flow interest ra and wage $\omega(a, x, y)$, and makes a consumption-saving decision, which gives a flow utility of $\mathfrak{u}(c)$. With Poisson rate σ, the worker loses the job and becomes unemployed. The optimal consumption-saving decision is characterized by the first order condition

$$\mathfrak{u}'(c^e) = W_a(a, x, y). \tag{5}$$

The Hamilton-Jacobi-Bellman (HJB) equation for the value of being unemployed is:

$$\begin{aligned}\rho U(a, x) = {} & \max_c \mathfrak{u}(c) + p(\theta) \int \frac{d_v(y)}{v} [W(a, x, y) - U(a, x)]^+ \mathrm{d}y + \dot{a} U_a(a, x) \\ \text{s.t. } \dot{a} = {} & ra + b - c \\ a \geq {} & \underline{a}\end{aligned} \tag{6}$$

An unemployed worker receives unemployment benefits b as well as flow interest ra. The unemployed worker meets a vacant job at rate $p(\theta)$, which is randomly sampled from the distribution of all vacancies. The first order condition for the consumption-saving decision is given by

$$\mathfrak{u}'(c^u) = U_a(a, x). \tag{7}$$

Let $V(y)$ denote the value of a vacant job of type y, and $J(a, x, y)$ the value of a producing job of type y, with an employee of type x who has asset a. The HJB equation for the producing job is

$$\rho J(a, x, y) = f(x, y) - \omega(a, x, y) + \sigma [V(y) - J(a, x, y)] + \dot{a}^e J_a(a, x, y), \tag{8}$$

where $\dot{a}^e := ra + \omega(a, x, y) - c^e(a, x, y)$ is the optimal saving policy of the employee. The firm retains the remaining output net of wage paid to the worker. With Poisson rate σ, the match is separated.

The vacant value is

$$\rho V(y) = q(\theta) \iint \frac{d_u(a, x)}{u} [J(a, x, y) - V(y)]^+ \, da dx. \tag{9}$$

The vacancy meets an unemployed worker at rate $q(\theta)$ that is randomly drawn from the distribution of all unemployed workers. Entrepreneurs pay an entry cost of c before the job type is realized. The free entry condition is given by

$$c = \int V(y) \, dy. \tag{10}$$

4.2.3 Wage Determination

The wage is determined by Nash bargaining, denoted by $\omega(a,x,y)$. Appendix A.1 proves that the Nash solution can be characterized by

$$\eta \frac{J(a,x,y) - V(y)}{1 - J_a(a,x,y)} = (1-\eta) \frac{W(a,x,y) - U(a,x)}{W_a(a,x,y)}, \tag{11}$$

where $\eta \in (0,1)$ represents the bargaining power of the worker.

To gain intuitions, it is useful to contrast our result with common Nash solutions in the case of linear utility. In an environment with linear utility, the match surplus is defined by the sum of worker's surplus and the job's surplus:

$$S(a,x,y) := W(a,x,y) - U(a,x) + J(a,x,y) - V(y).$$

Then Nash bargaining works in a way that the worker and the job are splitting the match surplus according to η. However, it does not make sense to directly add up the worker surplus and the firm surplus if they are not measured in the same units, as is in the case when we have curvature in the utility function. In particular, once we introduce curvature in the flow utility function, worker values are measured in present discounted util, while firm values are measured in present discounted numeraire. It turns out that W_a and $1 - J_a$ are the right adjustment terms so that we could add up the adjusted worker value and firm value. That is, consider the adjusted surplus

$$\hat{S}(a,x,y) := \frac{1}{W_a(a,x,y)} \left[W(a,x,y) - U(a,x)\right] + \frac{1}{1 - J_a(a,x,y)} \left[J(a,x,y) - V(y)\right]. \tag{12}$$

The worker and the firm are splitting the adjusted surplus according to the bargaining power η.

It is obvious that W_a properly measures the marginal value of a dollar to the worker. Now we illustrate the intuition why $1 - J_a$ is the right adjustment term for the firm. Think of the scenario of a marginal dollar transfer between the worker and the firm. If the worker transfers one additional dollar to the firm, there is a direct 1 dollar increase in firm's value and an indirect impact to the firm through asset decumulation of the worker, i.e., $-J_a$. Thus the total marginal value of additional dollar to the firm is properly captured by $1 - J_a$.

In the formal proof in Appendix A.1, we write down the full Nash problem by defining values of deviating wages with tilde notations, e.g., $\tilde{W}(w,a,x,y)$. We show that

$$\frac{-\tilde{J}_w(w,a,x,y)}{\tilde{W}_w(w,a,x,y)} = \frac{1 - J_a(a,x,y)}{W_a(a,x,y)},$$

which implies that the adjusted surplus could alternatively be written as

$$\hat{S}(a,x,y) := \frac{1}{\tilde{W}_w}\left[W(a,x,y) - U(a,x)\right] + \frac{1}{\left(-\tilde{J}_w\right)}\left[J(a,x,y) - V(y)\right].$$

This provides further intuition to the bargaining solution – the worker's surplus is adjusted by $\tilde{W}_w$ to the dollar value, and the firm's surplus is adjusted by $\left(-\tilde{J}_w\right)$ to the dollar value. Workers and firms split the adjusted surplus.

Finally, notice that as the curvature of the utility function goes to 0, i.e., as the utility function goes to linear, then $W_a = 1$ and $J_a = 0$. In this case, our adjusted surplus collapses to the standard definition of surplus.

4.2.4 Steady State

We consider a stationary equilibrium. The steady state could be characterized by two set of Kolmogorov Forward (KF) equations. The first one characterizes the inflow-outflow balancing equation for employed workers $d_m(a,x,y)$, i.e.,

$$0 = -\frac{\partial}{\partial a}\left[\dot{a}^e(a,x,y)\, d_m(a,x,y)\right] - \sigma d_m(a,x,y) + d_u(a,x) p(\theta)\frac{d_v(y)}{v}\Phi(a,x,y), \tag{13}$$

for all a, x, y. The second one characterizes the inflow-outflow balancing equation for unemployed workers $d_u(a,x)$, i.e.,

$$0 = -\frac{\partial}{\partial a}\left[\dot{a}^u(a,x)\, d_u(a,x)\right] - \int p(\theta)\frac{d_v(y)}{v}\Phi(a,x,y)\, d_u(a,x)\mathrm{d}y + \sigma\int d_m(a,x,y)\,\mathrm{d}y, \tag{14}$$

for all a, x. In addition, there is an add-up condition that density integrates to 1:

$$1 = \int_{\underline{a}}^{\infty} d_m(a,x,y)\, dadxdy + \int_{\underline{a}}^{\infty} d_u(a,x)\, dadx$$

as well as

$$\begin{aligned} d_x &= \int_{\underline{a}}^{\infty} d_m(a,x,y)\, dady + \int_{\underline{a}}^{\infty} d_u(a,x)\, da \\ d_y &= \int_{\underline{a}}^{\infty} d_m(a,x,y)\, dadx + d_v(y) \end{aligned}$$

4.3 Equilibrium

4.3.1 Formal Equilibrium Definition

A stationary search equilibrium consists of a set of value functions $\{W(a,x,y), U(a,x), J(a,x,y), V(y)\}$ for employed workers, unemployed workers, producing jobs, and vacant jobs, respectively; a set of policy functions including consumption policy $\{c^e(a,x,y), c^u(a,x)\}$ and matching acceptance decision conditonal on meeting $\Phi(a,x,y)$; a wage policy $\omega(a,x,y)$; and an invariant distribution of employed workers $d_m(a,x,y)$ and unemployed workers $d_u(a,x)$, and market tightness θ such that:

1. The value functions and policy functions solve worker and firm's optimization problem (4, 6, 8, 9);
2. Wage setting and matching acceptance decision satisfy Nash bargaining (11);
3. The stationary distributions satisfy the Kolmogorov Forward equations (13 and 14);
4. Market tightness adjusts so that free entry gives zero economic profits to vacant firms.

4.3.2 Model Outputs

This model provides a joint characterization of the labor market and wealth inequality.

First, it characterizes standard labor market variables of interest. Since the baseline model assumes an exogeneous separateion rate, it is thus the job losing rate in the economy $\pi_{eu} = \sigma$. The job finding rate (not job meeting) in the economy is

$$\pi_{ue} = p(\theta) \int \frac{d_v(y)}{v} \Phi(a,x,y)\, \mathrm{d}y.$$

The steady state unemployment rate is given by

$$u = \frac{\sigma}{\sigma + \pi_{ue}},$$

which is known as the Beveridge curve.

Second, the model allows for a joint characterization of wage inequality and wealth inequality. Specifically, the joint distribution of wealth and wage (among employed workers) is characterized by

$$h(a,w) = \frac{1}{e} \iint d_m(a,x,y)\, \mathbb{1}\{\omega(a,x,y) = w\}\, dx dy.$$

Third, the model is able to study how micro inequality affects the macro aggregate. The total output is

$$y = \iiint f(x, y)\, d_m(a, x, y)\, dadxdy,$$

and average output per employed (i.e. productivity) is $\bar{y} = y/e$. Our model allows us to study how the interaction between wealth inequality affect labor market sorting and labor misallocation, characterized by output loss or productivity loss relative to a benchmark economy (e.g. without search friction or with complete markets).

4.4 Algorithm

Consider grids $\{a_1, a_2, \ldots, a_{N_a}\}$ for asset, $\{x_1, x_2, \ldots, x_{N_x}\}$ for skills,, $\left\{y_1, y_2, \ldots, y_{N_y}\right\}$ for skill requirements. Suppose they are equally spaced (probably assets are logspaced) and $\Delta_a, \Delta_x, \Delta_y$ are the steps.

normalize d_x, d_y uniform $d_x = \int d_e(a, x, y)\, dady + \int d_u(a, x)\, da$

1. Guess θ and $d_v(y_k)/v$

   We can start by guessing, for example, that $\theta = 0.8$ and $d_v(y_k)/v = 1/N_y$ for all $k = 1, ..., N_y$.

2. Guess bargaining solution for each pair $w\left(a_i, x_j, y_k\right)$.

   We can start from $w(a, x, y) = \gamma f(x, y)$, a fraction of the flow profit.

   Solve the worker's problem using the implicit method as in Achdou et al. (2020) (see Appendix B for details).

3. Calculate the stationary distribution of workers.

   Discretize the Kolmogorov forward (KF) equation as

$$0 = -\frac{s_{i,F}^{jk,W+} d_{i,}^{jk,W} - s_{i-1,F}^{jk,W+} d_{i-1}^{jk,W}}{\Delta_a} - \frac{s_{i+1,B}^{jk,W-} d_{i+1}^{jk,W} - s_{i,B}^{jk,W+} d_{i}^{jk,W}}{\Delta_a} - \delta d_i^{jk,W} + p(\theta)\, d_v(k)\, \mathbb{1}_i^{jk} d_i^{j,U}$$
$$0 = -\frac{s_{i,F}^{j,U+} d_{i}^{j,U} - s_{i-1,F}^{j,U+} d_{i-1}^{j,U}}{\Delta_a} - \frac{s_{i+1,B}^{j,U-} d_{i+1}^{j,U} - s_{i,B}^{j,U-} d_{i}^{j,U}}{\Delta_a} - p(\theta) \sum_k d_v(k)\, \mathbb{1}_i^{jk} d_i^{j,U} + \delta \sum_k d_i^{jk,W}$$

   which leads to

$$\mathbf{A}\left(\mathbf{W}^n\right)' d = 0$$

   and the scale of the worker density is pinned down by the fact that d sums up to 1. $d_e(a, x, y)$ and $d_u(a, x)$. $d_v(y) = 1/N_y - \int d_e(a, x, y)\, \mathrm{d}a\mathrm{d}x$

4. Solve the firm's problem. The discretized HJB equation for firm is

$$\rho J_i^{jk} = f^{jk} - w_i^{jk} - \delta J_i^{jk} + s_i^{jk,W} J_{a,i}^{jk}$$

Update the value function by

$$\frac{J^{l+1} - J^l}{\Delta} = f^{jk} - w_i^{jk} + \mathbf{A}_{1,e} J^{l+1} - (\rho + \delta) J^{l+1} \Rightarrow J^{l+1} = \left[\left(\frac{1}{\Delta} + \rho + \delta \right) - \mathbf{A}_{1,e} \right]^{-1} \left(f^{jk} - w_i^{jk} + \frac{1}{\Delta} J^l \right)$$

5. Update wage schedule

6. update θ and dv, go back to 1.

7. Derive other densities and update guess in step 1.

Detailed algorithm is shown in the appendix.

5 Theoretical Results

5.1 Two Limiting Economies

Our model provides a unified framework of incomplete market and frictional sorting. It is a generalization that nests Krusell, Mukoyama and Şahin (2010) and Shimer and Smith (2000).

If worker's preference is risk neutral, i.e., if the flow utility function u is linear in consumption c, then our model becomes the frictional sorting model a la Shimer and Smith (2000). Aternatively, if workers have access to a full set of Arrow securities (i.e., complete market), the model becomes Shimer and Smith (2000). In either case, asset level will not affect optimal decision and becomes irrelvant to decision making.

If $\mathbb{X}$ and $\mathbb{Y}$ are singletons, then we have homogenous workers (in terms of the skills; workers are still heterogenous with respect to wealth) and firms. There is no sorting so to speak. The model becomes a standard Bewley (1977); Huggett (1993); Aiyagari (1994) type incomplete market model with Diamond-Mortensen-Pissarides search frictions. This has been explored by Krusell, Mukoyama and Şahin (2010).

5.2 Precautionary Mismatch

Theorem 1. ***Precautionary Mismatch.*** *The matching set is wider for lower-asset holders. Fix worker type x and job type y. If a is the marginal wealth level such that the adjusted match surplus is exactly* *zero, then wealthier workers reject the match while pooer workers accept the match. Formally, fix arbitrary x and y, if $\hat{S}(a,x,y)=0$, then $\hat{S}(a',x,y)<0$ for any $a'>a$, and $\hat{S}(a'',x,y)>0$ for any $a''<a$.*

Proof. From the discussion before, we know that Nash bargaining implies the following relationship for the adjusted match surplus could be written as

$$\frac{W(a,x,y)-U(a,x)}{W_a(a,x,y)}=\eta\hat{S}(a,x,y).$$

Worker optimization gives rise to the first order condition such that $W_a(a,x,y)=\mathrm{u}'(c^e(a,x,y))>0$. Therefore, whether a match is formed or not, i.e., whether $\hat{S}(a,x,y)>0$ is equivalent to whether $W(a,x,y)-U(a,x)>0$.

Consider a such that $W(a,x,y)-U(a,x)=0$, i.e., a marginally acceptable match. Define $\Delta(a;x,y):=W(a,x,y)-U(a,x)$. Differentiate both sides with respect to wealth a:

$$\Delta_a=W_a-U_a=\mathrm{u}'(c^e)-\mathrm{u}'(c^u),$$

where the arguments are suppressed for simplicity. It is obvious that for acceptable matches, $c^e>c^u$. Since the flow utility exhibits the usual concavity property $\mathrm{u}''<0$, it must be that $\Delta_a=\mathrm{u}'(c^e)-\mathrm{u}'(c^u)<0$.

Therefore, for any $a'>a$ we will have $\hat{S}(a',x,y)<0$ and for any $a''<a$ we will have $\hat{S}(a'',x,y)>0$. □

6 Quantitative Results

6.1 Parameterization

We adopt standard functional form assumptions to faciliate numerical analysis. We assume the flow utility function exhibits constant relative risk aversion (CRRA):

$$u(c)=\frac{c^{1-\gamma}}{1-\gamma}\quad,\quad\gamma>0.$$

The meeting function is assumed to take the Cobb-Douglas form:

$$M(u,v)=\chi u^{\alpha}v^{1-\alpha}.$$

Without loss of generality, worker and job types are normalized to be uniformly distributed. To see its generality, suppose the $\tilde{F}(\tilde{x})$ and $\tilde{G}(\tilde{y})$ are the cumulative density functions of the distribution of worker and job types, respectively, with a production function $\tilde{f}(\tilde{x}, \tilde{y})$. We could redefine a type according to its rank, i.e., $x := \tilde{F}(\tilde{x})$ and $y := \tilde{G}(\tilde{y})$, and rewrite the production function accordingly $f(x, y) := \tilde{f}\left(\tilde{F}^{-1}(x), G^{-1}(y)\right)$. The distribution of the rank-based type is thus uniform, as the CDF of any random variable is uniformly distributed between 0 and 1.[1] We specify a production function that induces positive assortative matching (PAM):

$$f(x, y) = f_0 + f_1 \left(x^{\xi} + y^{\xi}\right)^{1/\xi}. \tag{15}$$

Empirical evidence in Hagedorn, Law and Manovskii (2017) supports PAM as a description of data.

6.2 Calibration

We assume the borrowing constraint is $\underline{a} = 0$ such that workers cannot have negative net worth.

We consider 200 worker types and 200 occupations in the numerical exercise. We assume that in the stationary equilibrium, the vacancy posting cost is such that there is the same number of jobs as workers.

We take standard parameter values in the literature.

Note that although Hosios condition is imposed, it does guarantee efficiency here.

6.3 Equilibrium Outcomes

The stationary equilibrium features a set of matching sets between workers and firms, as well as endogenous joint distributions of wealth, worker skills and firm productivity. Here we examine a few equilibrium outcomes of interest.

Figure 4 shows matching sets, represented by yellow areas, for workers at the borrowing constraint ($a = 0$) and those far from it ($a = 150$). Both matching sets show that with a production function allowing for complementarity between works and firms, there is positive sorting. However, the degree of sorting varies significantly for workers of different wealth levels. In particular, wealth-poor workers accept a wider range of jobs, as can be seen from the lower bounds of the matching sets. Therefore, given a skill level, wealth-poor workers tend to find jobs more quickly but work at firms producing lower outputs. This is a result of precautionary saving motive as

[1] To see this, denote the transformed cumulative distribution functions as F and G such that $x \sim F$ and $y \sim G$. Consider an arbitrary $t \in [0, 1]$. We have

$$F(t) = \mathbb{P}(x \leq t) = \mathbb{P}\left(\tilde{F}(\tilde{x}) \leq t\right) = \mathbb{P}\left(\tilde{x} \leq s, \text{ for some } s \in \tilde{F}^{-1}(t)\right) = t.$$

Therefore $x \sim \mathcal{U}[0, 1]$. Similarly, $y \sim \mathcal{U}[0, 1]$.

Table 7: Calibration

Parameter	Value	Source
	External Calibration	
discount rate	$\rho = 0.015$	annual discount factor 0.955
interest rate	$r = 0.01$	annual interest rate 4%
relative risk aversion	$\gamma = 2$	common parameterization
bargaining power	$\eta = 0.5$	symmetric bargaining
matching elasticity	$\alpha = 0.5$	common parameterization
separation rate	$\sigma = 0.06$	monthly job losing rate 0.02
production function	PAM	Hagedorn, Law and Manovskii (2017)
	Internal Calibration	
unemployment insurance	$b = 0.4$	UI/wage 40
matching efficiency	$\chi = 0.9$	monthly job finding rate 0.3

Notes: The table summarizes the calibrated parameters and their sources.

workers close to borrowing constraints are more willing to find a job in order to move away from the constraint.

Workers' job acceptance strategy can in turn lead to heterogeneous job finding rates, as shown by Figure 5. For any given worker skill, job finding rate decreases with wealth as worker becomes more patient. Given wealth level, job finding rates tend to be hump-shaped, as workers with skills at the very low end are accepted by very few firms and those at the very top only accept offers from firms with high productivity.

We can also examine how different job acceptance strategies affect average wages and products of workers who otherwise have the same skills. Figure 6 shows the average product (top panel) and wages (bottom panel) of matched workers by wealth and skill level, where wealth-skill combinations that do not occur in the equilibrium are omitted. We can see that average product increase both in worker skills and wealth levels. First, average product increase in worker skills due to higher match-specific productivities and positive sorting. Second, average product also increase in wealth due to the effect of wealth on sorting (as shown by Figure 4). This directly leads to a wage function, shown by the bottom panel, which is also increasing in both worker skills and wealth. This result echos the findings from a number of empirical papers.

Figure 4: Matching Sets

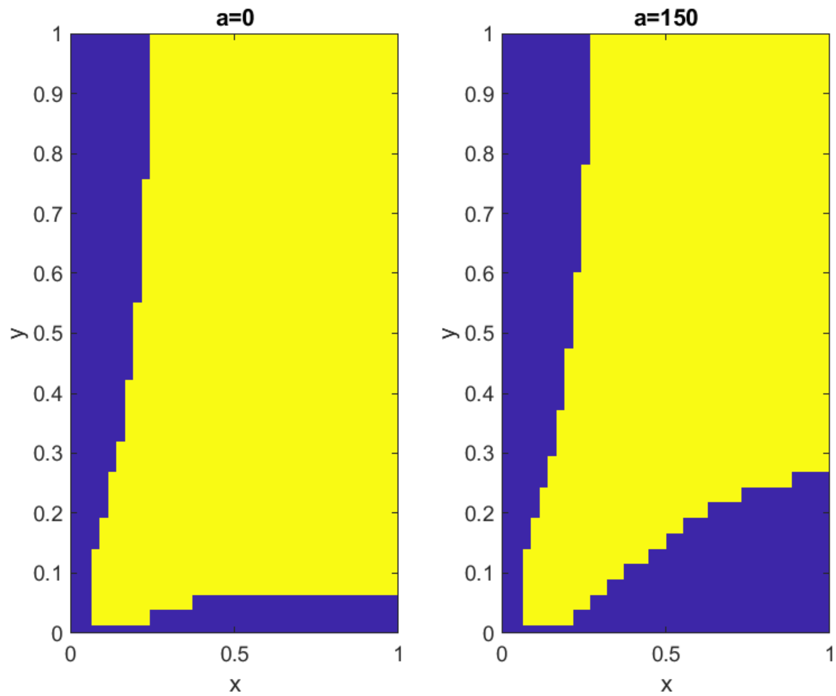


Figure 5: Job Finding Rates

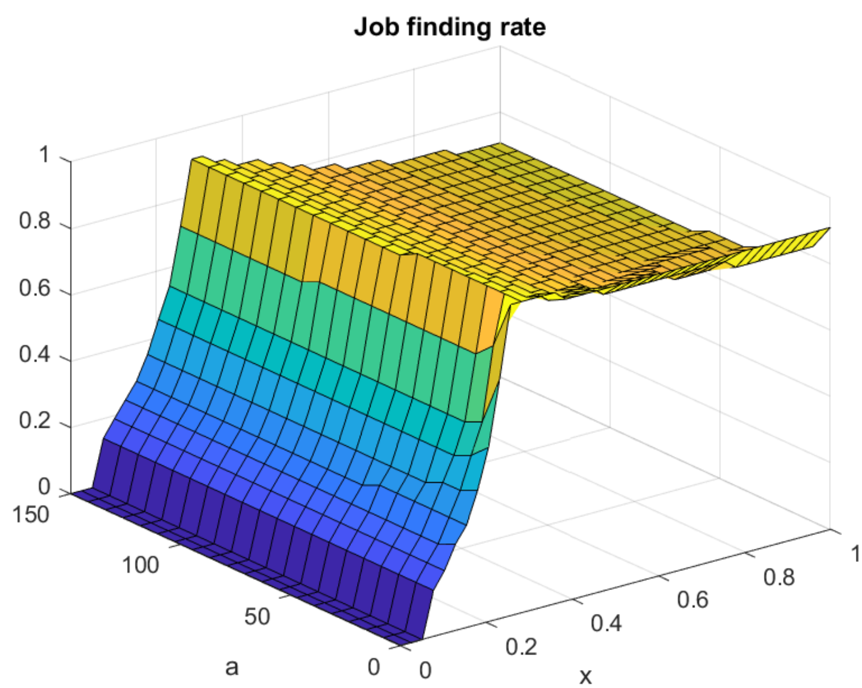
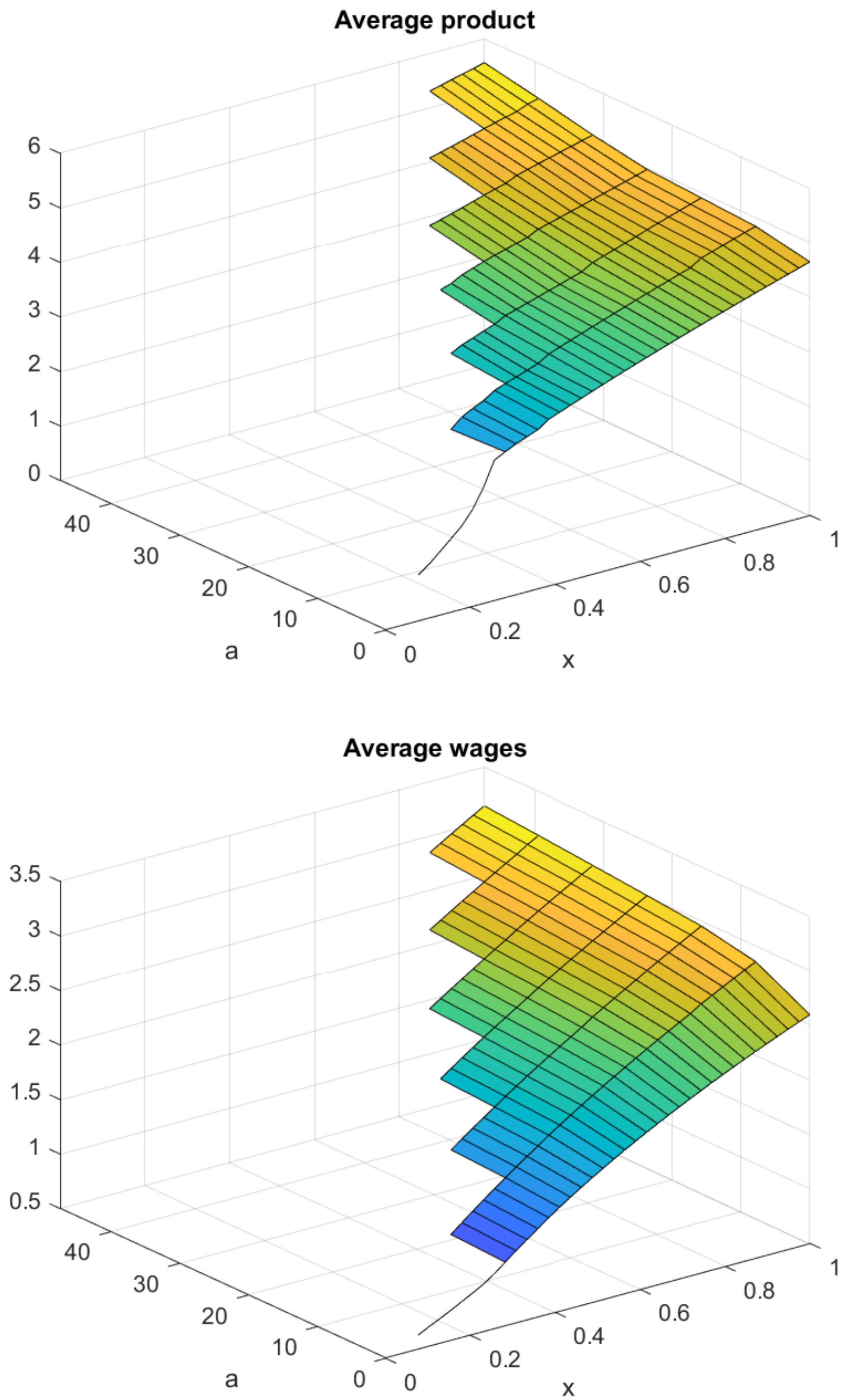


Figure 6: Average Wages



7 Conclusion

This paper examines the causes of labor market misallocation through the lens of search frictions under incomplete market, and examines the implications on productivity. We have shown empirically that labor misallocation can be measured in the form of skill mismatch, which is high when workers have limited ability to smooth consumption. Shortage of liquid wealth induces workers to find jobs more quickly at the cost of a lower match-specific productivity. Our major modelling contribution is to introduce two-sided skill heterogeneity into an incomplete market model with search. Due to precautionary saving motive, workers close to borrowing constraints accept a wider range of vacancies in order to smooth consumption. This increases their job finding rate, but on the other hand lowers their match-specific productivities and wages. Therefore in an economy where more workers are poor, labor productivity will be lower due to more skill mismatch.

The findings from this paper has several important implications for macro labor research. First, we show that wealth can affect wages through job-finding strategy. Second, on a more aggregate level, we show that higher job finding rate and thus more employment might come at the cost of lower output per worker, as precautionary saving motive induces higher job-finding rate but more skill mismatch. Therefore our research provides yet another evidence that inequality matters for macro (see the discussion by Ahn et al. (2018)). In addition, our findings also highlight the importance of taking into account allocative efficiency in the design of optimal unemployment insurance policy. The aggregate emplonent rate masks important heterogeneity in match quality which matters for aggregate productivity. Ignoring the effect of unemployment insurance on allocative efficiency would lead to an over-emphasis on job finding and result in unemployment insurance that is too low compared with an optimal level.

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A Mathematical Appendix

A.1 Nash Bargaining

To derive the wage setting, we start with the discrete time problem with period length of Δ. The value for employed worker of type x with asset a that works at a job of type y for an arbitrarily deviating flow wage w (recognizing that in the following period the wage will go back to the equilibrium bargained wage) satisfies

$$\tilde{W}(w,a,x,y) = \max_{c} u\left(c\right)\Delta + \frac{1}{1+\rho\Delta}\left\{\left(1-\Delta\sigma\right)W\left(a',x,y\right) + \Delta\sigma U\left(a',x\right)\right\}$$
$$\text{s.t.} \quad a' = a + \left(ra + w - c\right)\Delta.$$

Denote the optimal consumption policy by $\tilde{c}^{e}\left(w,a,x,y\right)$. The value function could be written as

$$\tilde{W}(w,a,x,y) = u\left(\tilde{c}^{e}\right)\Delta + \frac{1}{1+\rho\Delta}\left\{\left(1-\Delta\sigma\right)W(a+\left(ra+w-\tilde{c}^{e}\right)\Delta,x,y) + \Delta\sigma U\left(a+\left(ra+w-\tilde{c}^{e}\right)\Delta,x\right)\right\}.$$

Multiply both sides by $\left(1+\rho\Delta\right)$, subtract $\tilde{W}$, and then divide them by Δ,

$$\rho\tilde{W}(w,a,x,y) = u\left(\tilde{c}^{e}\right)\left(1+\rho\Delta\right) + \frac{1}{\Delta}\left[W(a+\left(ra+w-\tilde{c}^{e}\right)\Delta,x,y) - \tilde{W}(w,a,x,y)\right]$$
$$+\,\sigma\left[U\left(a+\left(ra+w-\tilde{c}^{e}\right)\Delta,x\right) - W(a+\left(ra+w-\tilde{c}^{e}\right)\Delta,x,y)\right].$$

Similarly, the value for such a producing job is

$$\tilde{J}(w,a,x,y) = f\left(x,y\right)\Delta - w\Delta + \frac{1}{1+\rho\Delta}\left[\left(1-\Delta\sigma\right)J\left(a',x,y\right) + \Delta\sigma V\left(y\right)\right],$$

where $a' = a + \left(ra + w - \tilde{c}^{e}\left(w,a,x,y\right)\right)\Delta$ is taken as given from the firm's point of view. Using a similar procedure (i.e., multiply both sides by $1+\rho\Delta$, subtract $\tilde{J}$, and then divide them by Δ), we obtain

$$\rho\tilde{J}(w,a,x,y) = f\left(x,y\right)\left(1+\rho\Delta\right) - w\left(1+\rho\Delta\right) + \frac{1}{\Delta}\left[J\left(a',x,y\right) - \tilde{J}(w,a,x,y)\right] + \sigma\left[V\left(y\right) - J\left(a',x,y\right)\right].$$

Under Nash bargaining, the wage policy is determined by

$$\omega\left(a,x,y\right) = \arg\max_{w}\left[\tilde{W}(w,a,x,y) - U\left(a,x\right)\right]^{\eta}\left[\tilde{J}\left(w,a,x,y\right) - V\left(y\right)\right]^{1-\eta}.$$

The first order condition for the bargaining problem is

$$\eta\left(\tilde{J}\left(w,a,x,y\right)-V\left(y\right)\right)\tilde{W}_{w}(w,a,x,y)+\left(1-\eta\right)\left(\tilde{W}(w,a,x,y)-U\left(a,x\right)\right)\tilde{J}_{w}\left(w,a,x,y\right)=0.$$

From the Envelop theorem, we have

$$\tilde{W}_{w}(w,a,x,y)=\frac{1}{1+\rho\Delta}\left\{\left(1-\Delta\sigma\right)W_{a}(a+\left(ra+w-c\right)\Delta,x,y)\Delta+\Delta\sigma U_{a}\left(a+\left(ra+w-c\right)\Delta,x\right)\Delta\right\},$$

and

$$\tilde{J}_{w}(w,a,x,y)=-\Delta+\frac{1}{1+\rho\Delta}\left(1-\Delta\sigma\right)J_{a}(a+\left(ra+w-\tilde{c}^{e}\left(w,a,x,y\right)\right)\Delta,x,y)\left(1-\tilde{c}_{w}^{e}\left(w,a,x,y\right)\right)\Delta.$$

Insert the expressions for $\tilde{W}$, $\tilde{J}$, $\tilde{W}_{w}$, and $\tilde{J}_{w}$ into the Bargaining FOC, divide both sides by Δ and take the limit $\Delta\to 0$,

$$\begin{aligned}&\eta\left\{f\left(x,y\right)-w+\left(ra+w-\tilde{c}^{e}\right)J_{a}(a,x,y)+\sigma\left[V\left(y\right)-J(a,x,y)\right]-\rho V\left(y\right)\right\}W_{a}(a,x,y)\\&=\left(1-\eta\right)\left(u\left(\tilde{c}^{e}\right)+\left(ra+w-\tilde{c}^{e}\right)W_{a}(a,x,y)+\sigma\left[U\left(a,x\right)-W(a,x,y)\right]-\rho U\left(a,x\right)\right)\left\{1-J_{a}(a,x,y)\right\},\end{aligned}$$

where we have used the result that $\lim_{\Delta\to 0}\tilde{c}_{w}^{e}\left(w,a,x,y;\Delta\right)=0$. This is true because the optimal consumption policy is characterized by its first order condition

$$u'\left(\tilde{c}^{e}\right)=\frac{1}{1+\rho\Delta}\left\{\left(1-\Delta\sigma\right)W_{a}(a+\left(ra+w-\tilde{c}^{e}\right)\Delta,x,y)+\Delta\sigma U_{a}\left(a+\left(ra+w-\tilde{c}^{e}\right)\Delta,x\right)\right\}.$$

Notice that as $\Delta\to 0$, the limiting FOC becomes $\lim_{\Delta\to 0}u'\left(\tilde{c}^{e}\right)=W_{a}(a,x,y)$. Under mild technical conditions,

$$\lim_{\Delta\to 0}\frac{\partial\tilde{c}^{e}}{\partial w}\left(w,a,x,y;\Delta\right)=\frac{\partial}{\partial w}\lim_{\Delta\to 0}\tilde{c}^{e}\left(w,a,x,y;\Delta\right)=\frac{\partial}{\partial w}u'^{(-1)}\left(W_{a}(a,x,y)\right)=0.$$

It is helpful to recognize that as $\Delta\to 0$,

$$\frac{\tilde{J}_{w}(w,a,x,y)}{\tilde{W}_{w}(w,a,x,y)}\to\frac{J_{a}(a,x,y)-1}{W_{a}(a,x,y)}.$$

This could be easily seen if one plugs in the expressions for $\tilde{J}_{w}$ and $\tilde{W}_{w}$ derived from the Envelop theorem. Notice that the bargaining FOC can now be written as

$$\eta\left[J\left(a,x,y\right)-V\left(y\right)\right]W_{a}(a,x,y)=\left(1-\eta\right)\left[W(a,x,y)-U\left(a,x\right)\right]\left(1-J_{a}(a,x,y)\right),$$

which simplifies the bargaining FOC to

$$\begin{aligned}&\eta\left\{f(x,y)-w+(ra+w-\tilde{c}^{e})\,J_{a}(a,x,y)-\rho V(y)\right\}W_{a}(a,x,y)\\=\;&(1-\eta)\left(u(\tilde{c}^{e})+(ra+w-\tilde{c}^{e})\,W_{a}(a,x,y)-\rho U(a,x)\right)(1-J_{a}(a,x,y)).\end{aligned}$$

The bargained wage can be solved to:

$$\begin{aligned}w=\;&\eta\frac{\left\{f(x,y)+(ra-\tilde{c}^{e})\,J_{a}(a,x,y)-\rho V(y)\right\}}{1-J_{a}(a,x,y)}\\&-(1-\eta)\,\frac{u(\tilde{c}^{e})+(ra-\tilde{c}^{e})\,W_{a}(a,x,y)-\rho U(a,x)}{W_{a}}.\end{aligned}$$

B Algorithmic Appendix

B.1 HJB Equations

Rewrite $W(a,x,y)$ as the employed value, and $U(a,x)$ as the unemployed value. The HJB equations are $\rho W(w,a,x,y)->c_{w}$

$$\begin{aligned}\rho W(a,x,y)&=\max_{c}u(c)+\delta\left[U(a,x)-W(a,x,y)\right]+(ra+w(a,x,y)-c)\,W_{a}(a,x,y)\\\rho U(a,x)&=\max_{c}u(c)+p(\theta)\sum_{k}\frac{d_{v}(k)}{v}\left[W(a,x,y_{k})-U(a,x)\right]^{+}+(ra+b(x)-c)\,U_{a}(a,x)\end{aligned}$$

with the first order conditions $u'(c)=W_{a}(a,x,y)$ and $u'(c)=U_{a}(a,x)$ respectively. The FD approximation to the HJB equations are

$$\rho W\left(a_{i},x_{j},y_{k}\right)=u\left(c_{i,j,k}\right)+\delta\left[U\left(a_{i},x_{j}\right)-W\left(a_{i},x_{j},y_{k}\right)\right]+\left(ra_{i}+w\left(a_{i},x_{j},y_{k}\right)-c_{i,j,k}\right)W_{a,}\left(a_{i},x_{j},y_{k}\right)\tag{16}$$

$$\rho U\left(a_{i},x_{j}\right)=u\left(c_{i,j}\right)+p(\theta)\sum_{k}\frac{d_{v}(k)}{v}\left[W\left(a_{i},x_{k},y_{k}\right)-U\left(a_{i},x_{j}\right)\right]^{+}+\left(ra_{i}+b\left(x_{j}\right)-c\right)U_{a}\left(a_{i},x_{k}\right)\tag{17}$$

B.2 Upwind Scheme

To compute the HJB equations, we need to approximate the derivatives of value functions numerically. Here we follow Achdou et al. (2020) and use the upwind scheme. The idea is to basically use the forward difference approximation whenever savings policy is positive, and backward difference whenever savings is negative.

Define the forward difference and backward difference as

$$W_{a,F}\left(a_i,x_j,y_k\right)=\frac{W\left(a_{i+1},x_j,y_k\right)-W\left(a_i,x_j,y_k\right)}{\Delta_a}$$

$$W_{a,B}\left(a_i,x_j,y_k\right)=\frac{W\left(a_i,x_j,y_k\right)-W\left(a_{i-1},x_j,y_k\right)}{\Delta_a}$$

$$\bar{W}_a\left(a_i,x_j,y_k\right)=u'\left(ra_i+w\left(a_i,x_j,y_k\right)\right)$$

We use the "upwind scheme". From the first order condition we can get $c=\left(u'\right)^{-1}W_a\left(a,x,y\right)$. Define

$$s^W_{i,j,k,F}=ra_i+w\left(a_i,x_j,y_k\right)-\left(u'\right)^{-1}\left(W_{a,F}\left(a_i,x_j,y_k\right)\right)$$

$$s^W_{i,j,k,B}=ra_i+w\left(a_i,x_j,y_k\right)-\left(u'\right)^{-1}\left(W_{a,B}\left(a_i,x_j,y_k\right)\right)$$

and approximate the derivative as follows

$$W_a\left(a_i,x_j,y_k\right)=W_{a,B}\left(a_i,x_j,y_k\right)\mathbf{1}_{\left\{s^W_{i,j,k,B}<0\right\}}+W_{a,F}\left(a_i,x_j,y_k\right)\mathbf{1}_{\left\{s^W_{i,j,k,F}>0\right\}}+\bar{W}_a\left(a_i,x_j,y_k\right)\mathbf{1}_{\left\{s^W_{i,j,k,F}<0<s^W_{i,j,k,B}\right\}}. \tag{18}$$

Since W is concave in a, we have $s^W_{i,j,k,F}<s^W_{i,j,k,B}$, then at some point i we have $s^W_{i,j,k,F}<0<s^W_{i,j,k,B}$, in which case we set savings to 0. Plugging the expression (18) into the discretized HJB equation (16), then the HJB equation can be written as

$$\rho W_i^{jk}=u\left(c_i^{jk}\right)+\delta\left[U_i^j-W_i^{jk}\right]+\underbrace{\frac{W_{i+1}^{jk}-W_i^{jk}}{\Delta_a}}_{W_{a,F}}s_{i,F}^{jk,W+}+\underbrace{\frac{W_i^{jk}-W_{i-1}^{jk}}{\Delta_a}}_{W_{a,B}}s_{i,B}^{jk,W-}$$

In matrix notation

$$\rho W_i^{jk}=u\left(c_i^{jk}\right)+\delta\left[U_i^j-W_i^{jk}\right]+\frac{1}{\Delta_a}\left[\begin{array}{ccc}-s_{i,B}^{jk,W-}, & s_{i,B}^{jk,W-}-s_{i,F}^{jk,W+}, & s_{i,F}^{jk,W+}\end{array}\right]\left[\begin{array}{c}W_{i-1}^{jk}\\ W_i^{jk}\\ W_{i+1}^{jk}\end{array}\right] \tag{19}$$

Similarly define

$$\rho U_i^j=u\left(c_i^j\right)+p\left(\theta\right)\sum_k\frac{d_v\left(k\right)}{v}\left[W_i^{jk}-U_i^j\right]^+ +\frac{1}{\Delta_a}\left[\begin{array}{ccc}-s_{i,B}^{j,U-}, & s_{i,B}^{j,U-}-s_{i,F}^{j,U+}, & s_{i,F}^{j,U+}\end{array}\right]\left[\begin{array}{c}U_{i-1}^j\\ U_i^j\\ U_{i+1}^j\end{array}\right] \tag{20}$$

B.3 Implicit method

Let $\mathbf{W}$ denote the vector that stacks all value functions together. The implicit method updates the value functions in the following way:

$$\frac{1}{\Delta}\left(\mathbf{W}^{n+1}-\mathbf{W}^{n}\right)+\rho\mathbf{W}^{n+1}=\tilde{\mathbf{u}}\left(\mathbf{W}^{n}\right)+\mathbf{A}\left(\mathbf{W}^{n}\right)\mathbf{W}^{n+1}$$

which gives

$$\left(\left(\rho+\frac{1}{\Delta}\right)\mathbf{I}-\mathbf{A}\left(\mathbf{W}^{n}\right)\right)\mathbf{W}^{n+1}=\tilde{\mathbf{u}}\left(\mathbf{W}^{n}\right)+\frac{1}{\Delta}\mathbf{W}^{n}$$

$$\Rightarrow \mathbf{W}^{n+1}=\left(\left(\rho+\frac{1}{\Delta}\right)\mathbf{I}-\mathbf{A}\left(\mathbf{W}^{n}\right)\right)^{-1}\left(\tilde{\mathbf{u}}\left(\mathbf{W}^{n}\right)+\frac{1}{\Delta}\mathbf{W}^{n}\right)$$

Stack the value $\mathbf{W}$ where we first loop over assets $a_1,...,a_{Na}$, then over worker skills $x_1,..,x_{N_x}$, and then finally over firm type $y_1,...,y_{N_y}$ in the outer loop.

The matrix $\mathbf{A}\left(\mathbf{W}^{n}\right)$ has three components: one with respect to asset accumulation (the last terms of equations (19) and (20)), another with respect to job separation $\delta\left[U_i^j-W_i^{jk}\right]$, and the last one with respect to job matching $p\left(\theta\right)\sum_k\frac{d_v(k)}{v}\left[W_i^{jk}-U_i^j\right]^+$, which we denote as $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$ respectively, then $\mathbf{A}\left(\mathbf{W}^{n}\right)=\mathbf{A}_1+\mathbf{A}_2+\mathbf{A}_3$ such that

$$\mathbf{A}_1=\begin{bmatrix}\mathbf{A}_{1e} & 0\\ 0 & \mathbf{A}_{1u}\end{bmatrix}$$

$$\mathbf{A}_{1e}=\begin{bmatrix}\beta_1^{11,W} & \gamma_1^{11,W} & 0 & \cdots & 0\\ \alpha_2^{11,W} & \beta_2^{11,W} & \gamma_2^{11,W} & 0 & \cdots\\ 0 & \alpha_3^{11,W} & \beta_3^{11,W} & \gamma_3^{11,W} & 0\\ \vdots & \ddots & \ddots & \ddots & \ddots\\ 0 & \ddots & \ddots & \alpha_{Na}^{NxNy,W} & \beta_{Na}^{N_xN_y,W}\end{bmatrix},\mathbf{A}_{1u}=\begin{bmatrix}\beta_1^{1,U} & \gamma_1^{1,U} & 0 & \cdots & 0\\ \alpha_2^{1,U} & \beta_2^{1,U} & \gamma_2^{1,U} & 0 & 0\\ 0 & \alpha_3^{1,U} & \beta_3^{1,U} & \gamma_3^{1,U} & 0\\ \ddots & \ddots & \ddots & \ddots & \ddots\\ 0 & \cdots & 0 & \beta_{Na}^{N_x,U} & \gamma_{N_a}^{N_x,U}\end{bmatrix}$$

where

$$\alpha_i^{jk,W}=\frac{-s_{i,B}^{jk,W-}}{\Delta_a}$$

$$\beta_i^{jk,W}=\frac{s_{i,B}^{jk,W-}-s_{i,F}^{jk,W+}}{\Delta_a}$$

$$\gamma_i^{jk,W}=\frac{s_{i,F}^{jk,W+}}{\Delta_a}$$

and analogously for the unemployed coefficients.

$$
\mathbf{A}_2 = \begin{bmatrix}
-\delta & 0 & \cdots & 0 & & & & & & & & & 0 & \delta & 0 & \cdots & 0 \\
0 & -\delta & 0 & 0 & & & & & & & & & \vdots & 0 & \delta & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots & & & & & & & & & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & -\delta & & & & & & & & & \vdots & 0 & \cdots & 0 & \delta \\
\vdots & \ddots & \ddots & \ddots & \ddots & & & & & & & & \vdots & \delta & 0 & \cdots & 0 \\
\vdots & & & & & \ddots & & & & & & & \vdots & 0 & \delta & 0 & 0 \\
\vdots & & & & & & \ddots & & & & & & \vdots & \vdots & \ddots & \ddots & \vdots \\
\vdots & & & & & & & \ddots & & & & & \vdots & 0 & \cdots & 0 & \delta \\
\vdots & & & & & & & & & \ddots & & & \vdots & \delta & 0 & \cdots & 0 \\
\vdots & & & & & & & & & & \ddots & & \vdots & 0 & \delta & 0 & 0 \\
\vdots & & & & & & & & & & & \ddots & 0 & \vdots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & -\delta & 0 & \cdots & 0 & \delta \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0
\end{bmatrix}
$$

where each diagonal submatrix corresponds to a loop over asset states $a_1, ..., a_{N_a}$ and worker skills $x_1, ..., x_{N_x}$. The bottom part is a matrix of $N_1 \times N_2$ zeros where $N_1 = N_a \times N_x$ and $N_2 = N_a \times N_x \times (N_y + 1)$.

$$
\mathbf{A}_3 = p(\theta) \times \begin{bmatrix}
0 & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & 0 \\
\mathbf{A}_{31} & \mathbf{A}_{32} & \cdots & \mathbf{A}_{3N_y} & \mathbf{A}_{3N_y+1}
\end{bmatrix}
$$

where

$$
\mathbf{A}_{3k} = \begin{bmatrix}
d_v^k \mathbb{1}_1^{1k} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & d_v^k \mathbb{1}_2^{1k} & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & d_v^k \mathbb{1}_{N_a}^{1k} & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & 0 & d_v^k \mathbb{1}_{N_a}^{N_x k}
\end{bmatrix}
$$

$$\mathbb{1}_i^{jk} = \begin{cases} 1 & \text{if } U\left(a_i, x_j, y_k\right) > W\left(a_i, x_j\right) \\ 0 & \text{otherwise} \end{cases}$$

and

$$\mathbf{A}_{3N_y+1} = \begin{bmatrix} -\sum_k d_v^k \mathbb{1}_1^{1k} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & -\sum_k d_v^k \mathbb{1}_2^{1k} & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\sum_k d_v^k \mathbb{1}_{N_a}^{1k} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots\cdots & \cdots\cdots & \cdots\cdots & 0 & -\sum_k d_v^k \mathbb{1}_{N_a}^{N_x k} \end{bmatrix}$$

and the top part is a matrix of $N_1 \times N_2$ zeros where $N_1 = N_a \times N_x \times N_y$ and $N_2 = N_a \times N_x \times \left(N_y + 1\right)$.

Alternatively, we loop over j and k

$$\rho W_i^{jk} = u\left(c_i^{jk}\right) + \delta\left[U_i^j - W_i^{jk}\right] + \frac{1}{\Delta_a}\left[\begin{array}{ccc} -s_{i,B}^{jk,W-}, & s_{i,B}^{jk,W-} - s_{i,F}^{jk,W+}, & s_{i,F}^{jk,W+} \end{array}\right]\begin{bmatrix} W_{i-1}^{jk} \\ W_i^{jk} \\ W_{i+1}^{jk} \end{bmatrix}$$

$$\mathbf{W}_{jk}^{n+1} = \left(\left(\rho + \frac{1}{\Delta}\right)\mathbf{I} - \mathbf{A}\left(\mathbf{W}_{jk}^n\right)\right)^{-1}\left(\tilde{\mathbf{u}}\left(\mathbf{W}^n\right)_{jk}^W + \frac{1}{\Delta}\mathbf{W}_{jk}^n\right)$$

and then loop over j

$$\mathbf{U}_j^{n+1} = \left(\left(\rho + \frac{1}{\Delta}\right)\mathbf{I} - \mathbf{A}\left(\mathbf{U}_j^n\right)\right)^{-1}\left(\tilde{\mathbf{u}}\left(\mathbf{W}^n\right)_j^U + \frac{1}{\Delta}\mathbf{U}_j^n\right)$$

and then stack

$$\mathbf{W}^{n+1} = \begin{pmatrix} \mathbf{W}_{11}^{n+1} \\ \mathbf{W}_{12}^{n+1} \\ \vdots \\ \mathbf{W}_{N_x N_y}^{n+1} \\ \mathbf{U}_1^{n+1} \\ \mathbf{U}_{12}^{n+1} \\ \vdots \\ \mathbf{U}_{N_x N_y}^{n+1} \end{pmatrix}$$

This case is easy to code because $\mathbf{A}$ is standard (although $\tilde{\mathbf{u}}$ is new, but this is straightforward).

However, the loop may slowdown the algorithm.

B.4 Stationary Density

Recall the Kolmogorov Forward (KF) equations for density:

$$
\begin{aligned}
0 &= -\frac{\partial}{\partial a}\left[s_e\left(a,x,y\right)d_m\left(a,x,y\right)\right] - \delta d_m\left(a,x,y\right) + p\left(\theta\right)\frac{d_v\left(y\right)}{v}\Phi\left(a,x,y\right)d_u\left(a,x\right) \\
0 &= -\frac{\partial}{\partial a}\left[s_u\left(a,x\right)d_u\left(a,x\right)\right] - \int p\left(\theta\right)\frac{d_v\left(y\right)}{v}\Phi\left(a,x,y\right)d_u\left(a,x\right)dy + \int \delta d_m\left(a,x,y\right)dy
\end{aligned}
$$

together with the condition that density integrates to 1:

$$
1 = \int_{\underline{a}}^{\infty} d_m\left(a,x,y\right)dadxdy + \int_{\underline{a}}^{\infty} d_u\left(a,x\right)dadx
$$

as well as

$$
\begin{aligned}
d_x &= \int_{\underline{a}}^{\infty} d_m\left(a,x,y\right)dady + \int_{\underline{a}}^{\infty} d_u\left(a,x\right)da \\
d_y &= \int_{\underline{a}}^{\infty} d_m\left(a,x,y\right)dadx + d_v\left(y\right)
\end{aligned}
$$

which can be discretized as

$$
\begin{aligned}
0 &= -\frac{\partial}{\partial a}\left[s_i^{jk,W}d_i^{jk,W}\right] - \delta d_i^{jk,W} + p\left(\theta\right)\frac{d_v^k}{v}\mathbb{1}_i^{jk}d_i^{j,U} \\
0 &= -\frac{\partial}{\partial a}\left[s_i^{j,U}d_i^{j,U}\right] - p\left(\theta\right)\sum_{k=1}^{N_y}\frac{d_v^k}{v}\mathbb{1}_i^{jk}d_i^{j,U} + \delta\sum_{k=1}^{N_y}d_i^{jk,W}
\end{aligned}
$$

and

$$
\begin{aligned}
1 &= \sum_{i=1}^{N_a}\sum_{j=1}^{N_x}\sum_{k=1}^{N_y}d_i^{jk,W}\Delta_a\Delta_x\Delta_y + \sum_{i=1}^{N_a}\sum_{j=1}^{N_x}d_i^{j,U}\Delta_a\Delta_x \qquad (21)\\
d_x^j &= \sum_{i=1}^{N_a}\sum_{k=1}^{N_y}d_i^{jk,W}\Delta_a\Delta_y + \sum_{i=1}^{N_a}d_i^{j,U}\Delta_a \\
d_y^k &= \sum_{i=1}^{N_a}\sum_{j=1}^{N_x}d_i^{jk,W}\Delta_a\Delta_x + d_{vy}^k
\end{aligned}
$$

B.5 Upwind Scheme

For the derivatives, we again use the forward scheme

$$0 = -\frac{s^{jk,W+}_{i,F} d^{jk,W}_i - d^{jk,W}_{i-1} s^{jk,W+}_{i-1,F}}{\Delta_a} - \frac{d^{jk,W}_{i+1} s^{jk,W-}_{i+1,B} - d^{jk,W}_i s^{jk,W-}_{i,B}}{\Delta_a} - \delta d^{jk,W}_i + p\left(\theta\right) \frac{d^k_v}{v} \mathbb{1}^{jk}_i d^{j,U}_i$$

$$0 = -\frac{s^{j,U+}_{i,F} d^{j,U}_i - d^{j,U}_{i-1} s^{j,U+}_{i-1,F}}{\Delta_a} - \frac{d^{j,U}_{i+1} s^{j,U-}_{i+1,B} - d^{j,U}_i s^{j,U-}_{i,B}}{\Delta_a} - p\left(\theta\right) \sum_{k=1}^{N_y} \frac{d^k_v}{v} \mathbb{1}^{jk}_i d^{j,U}_i + \delta \sum_{k=1}^{N_y} d^{jk,W}_i$$

Collecting terms, we have

$$0 = d^{jk,W}_{i-1} \alpha^{jk,W}_{i-1} + d^{jk,W}_i \beta^{jk,W}_i + d^{jk,W}_{i+1} \gamma^{jk,W}_{i+1} + p\left(\theta\right) \frac{d^k_v}{v} \mathbb{1}^{jk}_i d^{j,U}_i$$

$$0 = d^{j,U}_{i-1} \alpha^{j,U}_{i-1} + d^{j,U}_i \beta^{j,U}_i + d^{j,U}_{i+1} \gamma^{j,U}_{i+1} + \delta \sum_{k=1}^{N_y} d^{jk,W}_i$$

where

$$\begin{cases} \alpha^{jk,W}_{i-1} & = \frac{s^{jk,W+}_{i-1,F}}{\Delta_a} \\ \beta^{jk,W}_i & = \frac{s^{jk,W-}_{i,B} - s^{jk,W+}_{i,F}}{\Delta_a} - \delta \\ \gamma^{jk,W}_i & = -\frac{s^{jk,W-}_{i+1}}{\Delta_a} \end{cases} \begin{cases} \alpha^{j,U}_{i-1} & = \frac{s^{j,U+}_{i-1,F}}{\Delta_a} \\ \beta^{j,U}_i & = \frac{s^{j,U-}_{i,B} - s^{j,U+}_{i,F}}{\Delta_a} - p\left(\theta\right) \sum_{k=1}^{N_y} \frac{d^k_v}{v} \mathbb{1}^{jk}_i \\ \gamma^{j,U}_i & = -\frac{s^{j,U-}_{i+1}}{\Delta_a} \end{cases}$$

Let $\mathbf{d}$ be the stacked vector of densities (arranged in the same order as $\mathbf{W}$), then the KF equations expressed using the upwind scheme can be written as

$$\mathbf{A}^T \mathbf{d} = 0 \tag{22}$$

where $\mathbf{A}^T$ is the same matrix that was defined in Section **??**.

To solve the problem of equation (22) subject to the constraints (21), we can do the following. Fix (1) either $d^{jk,W}_i$ or $d^{j,U}_i$ to be 0.1 (or any other non-zero number) for arbitrary (i, j, k); (2) then solve the system for some $\tilde{d}$ and then to renormalize $d^{jk,W}_i = \tilde{d}^{jk,W}_i / \left(\sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \sum_{k=1}^{N_y} \tilde{d}^{jk,W}_i \Delta_a \Delta_x \Delta_y + \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \tilde{d}^{j,U}_i\right.$ and $d^{j,U}_i = \tilde{d}^{j,U}_i / \left(\sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \sum_{k=1}^{N_y} \tilde{d}^{jk,W}_i \Delta_a \Delta_x \Delta_y + \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \tilde{d}^{j,U}_i \Delta_a \Delta_x\right)$.

C Data Appendix

C.1 Construction of Worker and Firm Types

This section describes the method to construct multi-dimensional worker skills and job skill requirements, used by Lise and Postel-Vinay (2020).

We create 2-dimensional worker skill bundles and job skill requirement bundles using a data set combining NLSY79 job history and O*NET, following Lise and Postel-Vinay (2020).

For jobs, we

- match weekly NLSY79 job record to O*NET data which contains measures of a variety of job skill descriptors
- take the first 2 principal components of these measures in the panel
- recombine the 2 principal components so that they satisfy the following exclusion restrictions: (1) the *mathematics* measure only reflects cognitive skill requirements; (2) the *mechanical knowledge* scores only reflects manual skill requirements
- normalize the skill requirements so that each component lies in $[0, 1]$

For workers, we

- use all 10 components of individual ASVAB test scores and a measure of health (BMI)
- take the first 2 principal components of these measures
- recombine them so that (1) the ASVAB *mathematics knowledge* score only reflects cognitive skills; (2) the ASVAB *automotive and shop information* score only reflects manual skills
- normalize the skill measures so that each component lies in $[0, 1]$

In the analysis above, we only use the first component, i.e. cognitive skill/skill requirement.