Precautionary Mismatch

(Preliminary Work in Progress)

Jincheng (Eric) Huang and Xincheng Qiu

University of Pennsylvania

ESPE 2021 Conference

June 19, 2021

Introduction

- How does wealth affect the labor market's allocation of the "right" worker to the "right" job?

Why this question?

- Misallocation of labor typically reduces total factor productivity
- Wealth as a major source of self-insurance matters for workers' job search decisions

Introduction

- How does wealth affect the labor market's allocation of the "right" worker to the "right" job?

Need a theory of

- labor market sorting with different workers and different jobs
- frictions that hinder perfect assignment so that mismatch arises
- wealth interacted with job search behavior

Introduction

- How does wealth affect the labor market's allocation of the “right” worker to the “right” job?

<u>Need a theory of</u>

- labor market sorting
- mismatch
- interaction with wealth

<u>Propose a framework with</u>

- two-sided heterogeneity
- search friction
- incomplete markets

Introduction

- How does wealth affect the labor market's allocation of the "right" worker to the "right" job?

Need a theory of

- labor market sorting
- mismatch
- interaction with wealth

Propose a framework with

- two-sided heterogeneity
- search friction
- incomplete markets

At its core is

- Becker
- Diamond-Mortensen-Pissarides
- Bewley-Huggett-Aiyagari

Introduction

- How does wealth affect the labor market's allocation of the "right" worker to the "right" job?

Need a theory of

- labor market sorting
- mismatch
- interaction with wealth

Propose a framework with

- two-sided heterogeneity
- search friction
- incomplete markets

At its core is

- Becker
- Diamond-Mortensen-Pissarides
- Bewley-Huggett-Aiyagari

- We cast the model in continuous time so that computation and analysis become feasible

Mechanism

- Trade-off between right assignment vs. speed of forming a match

Mechanism

- Trade-off between right assignment vs. speed of forming a match
    - asset holdings affect such trade-off

Mechanism

- Trade-off between right assignment vs. speed of forming a match
  - asset holdings affect such trade-off: wealth $\uparrow \Rightarrow$ MU$_c \downarrow \Rightarrow$
  - workers: more willing to take risk $\Rightarrow$ wait for better matches

Mechanism

- Trade-off between right assignment vs. speed of forming a match
  - asset holdings affect such trade-off: wealth $\uparrow \Rightarrow$ $MU_c \downarrow \Rightarrow$
  - workers: more willing to take risk $\Rightarrow$ wait for better matches
  - firms: higher wage compensation needed $\Rightarrow$ only profitable for better matches

Mechanism

- Trade-off between right assignment vs. speed of forming a match
  - asset holdings affect such trade-off: wealth $\uparrow \Rightarrow$ MU$_c \downarrow \Rightarrow$
  - workers: more willing to take risk $\Rightarrow$ wait for better matches
  - firms: higher wage compensation needed $\Rightarrow$ only profitable for better matches
  - in addition to the standard precautionary saving motive, there is *precautionary mismatch* motive

Mechanism

- Trade-off between right assignment vs. speed of forming a match
  - asset holdings affect such trade-off: wealth $\uparrow \Rightarrow$ $\text{MU}_c \downarrow \Rightarrow$
  - workers: more willing to take risk $\Rightarrow$ wait for better matches
  - firms: higher wage compensation needed $\Rightarrow$ only profitable for better matches
  - in addition to the standard precautionary saving motive, there is *precautionary mismatch* motive
- Implications that are supported by data
  - sorting is stronger among wealthy workers
  - job finding rate decreases with wealth
  - wage increases with wealth

Mechanism

- Trade-off between right assignment vs. speed of forming a match
  - asset holdings affect such trade-off: wealth $\uparrow \Rightarrow$ $\text{MU}_c \downarrow \Rightarrow$
  - workers: more willing to take risk $\Rightarrow$ wait for better matches
  - firms: higher wage compensation needed $\Rightarrow$ only profitable for better matches
  - in addition to the standard precautionary saving motive, there is *precautionary mismatch* motive
- Implications that are supported by data
  - sorting is stronger among wealthy workers
  - job finding rate decreases with wealth
  - wage increases with wealth
- Wealth determines the amount of precautionary mismatch and thus aggregate productivity

Model

Details

Environment: Preferences and Technology

- Continuous time, infinite horizon
- Continuum of workers and jobs/firms
- Workers are risk averse with flow utility $u(c)$ and discount rate ρ $(u' > 0, u'' < 0)$
- Firms are risk neutral with discount rate r
- Workers $x \in \mathbb{X}$ and firms $y \in \mathbb{Y}$ are heterogeneous
- Production function of a matched pair $f(x, y) : \mathbb{X} \times \mathbb{Y} \to \mathbb{R}_+$
- Home production b

Environment: Market Structure

- There is only one risk-free asset that agents can lend and borrow at r
- Workers face an exogenous borrowing constraint $\underline{a}$
- Labor markets are frictional: Meeting happens randomly by CRS $M(u, v)$
- Meetings $\neq$ Successful matches
- Jobs are destroyed exogenously with a Poisson rate σ
- Wage is determined by Nash bargaining with worker bargaining power η

Model Summary

This Paper

Incomplete Market Models
Bewley-Huggett-Aiyagari

Search & Matching Models
Diamond-Mortensen-Pissarides

Assignment Models
Becker

Model Summary

This Paper

Two Limiting Economies:

1. If agents are homogeneous in production type, i.e., $\mathbb{X}$ and $\mathbb{Y}$ are singletons

Incompleate Market Models
Bewley-Huggett-Aiyagari

Search & Matching Models
Diamond-Mortensen-Pissarides

Assignment Models
Becker

Krusell, Mukoyama, Sahin
(2010 Restud)

Model Summary

This Paper

Two Limiting Economies:

1. If agents are homogeneous in production type, i.e., $\mathbb{X}$ and $\mathbb{Y}$ are singletons
2. If workers are risk neutral, i.e., $u(c) = c$

Incomplete Market Models
Bewley-Huggett-Aiyagari

Search & Matching Models
Diamond-Mortensen-Pissarides

Assignment Models
Becker

Shimer and Smith
(2000 Econometrica)

Main Results

Main Results

- We focus on the model in its steady state for now
- Cross-sectional implications about the interactions between wealth, job search and wages
- Supportive evidence using linked NLSY79 + O*NET data
- For future iterations, we plan to examine dynamic implications & their empirical evidence

Proposition 1: Precautionary Mismatch

Theorem

The matching set $\mathbb{M}(a) := \{(x, y) : \Phi(a, x, y) = 1\}$ *is wider for lower-asset workers.*

That is, for $a < a'$*, we have* $\mathbb{M}(a) \supset \mathbb{M}(a')$*.*

Proof

Proposition 1: Precautionary Mismatch

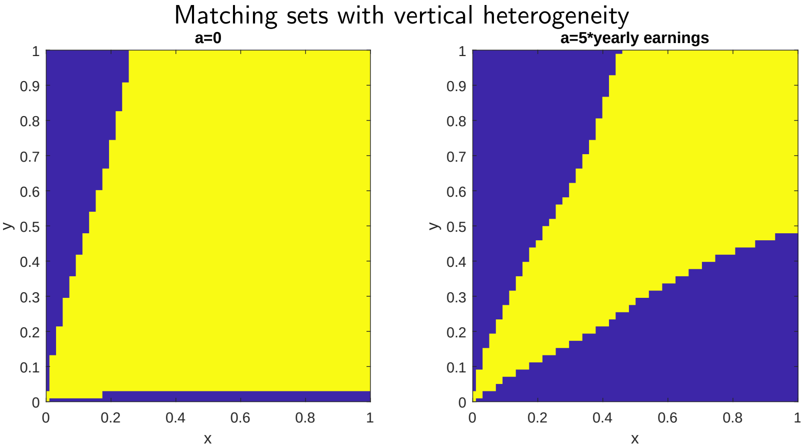
Theorem

The matching set $\mathbb{M}(a) := \{(x, y) : \Phi(a, x, y) = 1\}$ *is wider for lower-asset workers.*
That is, for $a < a'$, *we have* $\mathbb{M}(a) \supset \mathbb{M}(a')$.

Proof

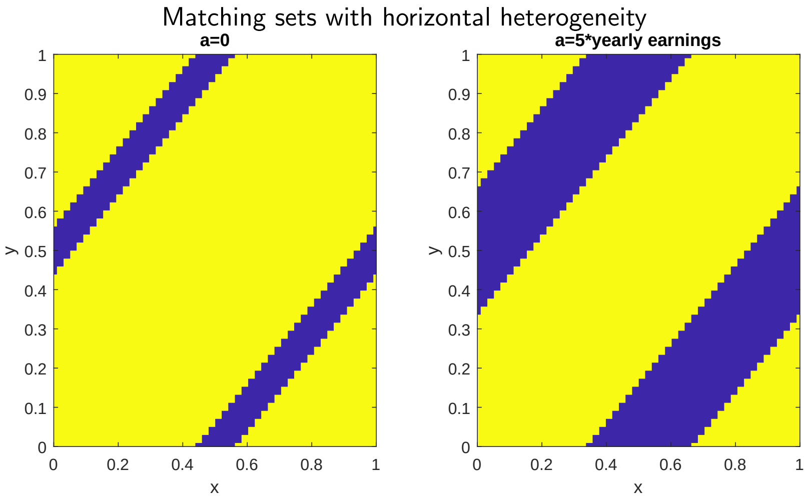
- This result does not rely on particular sorting pattern (production function)
- It is instructive to visualize
  1. a case of vertical heterogeneity that exhibits positive assortative matching Production
  2. a case of horizontal heterogeneity where agents are symmetric but not identical Production

Proposition 1: Precautionary Mismatch



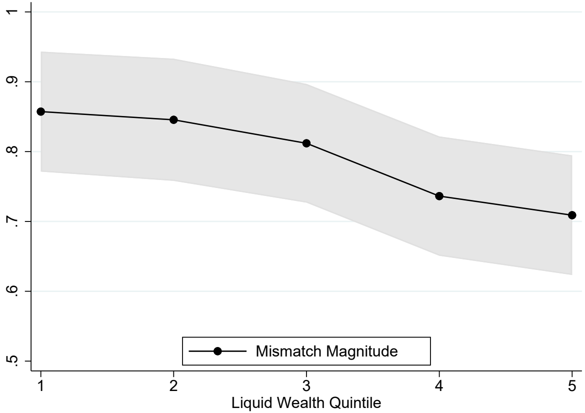
Quadratic

Proposition 1: Precautionary Mismatch



Quadratic

Supportive Evidence: Wealth and Mismatch



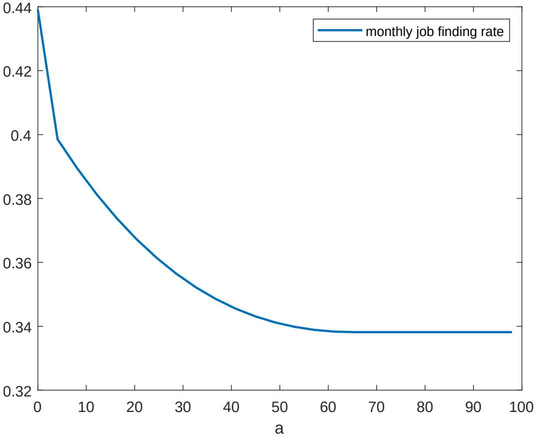
Construct worker's skills x from NLSY79 and occupation's requirements y from O*NET.
Define mismatch as $d = |x - y|$. Details

Proposition 2: Wealth-Poor Workers Find Jobs Faster

Corollary

Job finding rate $\pi_{ue}(a, x)$ is decreasing in wealth a.

Proof



Vertical

Supportive Evidence: Wealth and Job Finding Rate

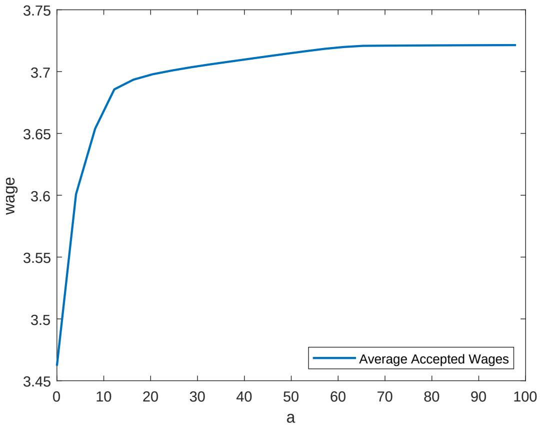
	(1)	(2)	(3)
$\log(a + \sqrt{1 + a^2})$	-0.005	-0.007**	-0.010***
	(0.003)	(0.003)	(0.004)
Demographic	No	Yes	Yes
Family background	No	No	Yes
Obs	5368	5368	4264

Standard errors in parentheses, * p<0.10, ** p<0.05, *** p<0.01

Proposition 3: Wealth and Wages

Proposition 3

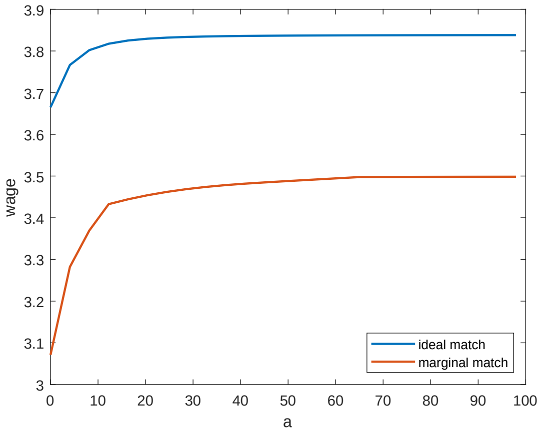
Average accepted wage $\overline{\omega}(a,x) = \int \omega(a,x,y)\Phi(a,x,y)\frac{d_v y}{v} d(y)$ *is increasing in wealth* a.



Vertical

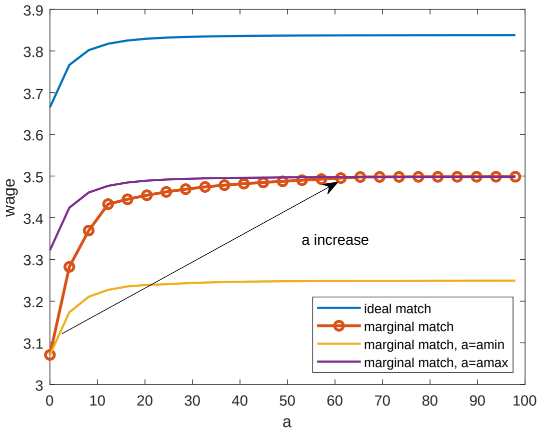
Proposition 3: Wealth and Wages

- Two reasons why average wages increase with wealth
  1. Nash bargaining: wealthier workers bargain for higher wages on each job
  2. Mismatch: wealthier workers accept smaller mismatch & get higher productivity



Proposition 3: Wealth and Wages

- Two reasons why average wages increase with wealth
  1. Nash bargaining: wealthier workers bargain for higher wages on each job
  2. Mismatch: wealthier workers accept smaller mismatch & get higher productivity



Supportive Evidence: Wealth and Wages

	(1)	(2)	(3)
$\log(a + \sqrt{1 + a^2})$	0.024***	0.015**	0.014**
	(0.007)	(0.006)	(0.007)
Demographic	No	Yes	Yes
Family background	No	No	Yes
Obs	3189	3189	2515

Standard errors in parentheses, * p<0.10, ** p<0.05, *** p<0.01

Supportive Evidence: Wealth and Wages

Proposition 4: Optimal Consumption Growth

- Euler equation for employed worker's consumption Proof

$$\frac{\dot{c}^e}{c^e} = \frac{1}{\gamma}\left\{r - \rho + \omega_a + \sigma\left[\frac{u'(c^u)}{u'(c^e)} - 1\right]\right\}$$

where arguments of $\omega(a, x, y)$, $c^u(a, x, y)$, $c^e(a, x, y)$ are suppressed for brevity.

Proposition 4: Optimal Consumption Growth

- Euler equation for employed worker's consumption Proof

$$\frac{\dot{c}^e}{c^e} = \frac{1}{\gamma}\left\{r - \rho + \omega_a + \sigma\left[\frac{u'\left(c^u\right)}{u'\left(c^e\right)} - 1\right]\right\}$$

where arguments of $\omega(a,x,y)$, $c^u(a,x,y)$, $c^e(a,x,y)$ are suppressed for brevity.

- Euler equation for unemployed worker's consumption Proof

$$\frac{\dot{c}^u}{c^u} = \frac{1}{\gamma}\left\{r - \rho - p\left(\theta\right)\int_{B(a,x)} \frac{d_v\left(y\right)}{v}\left[1 - \frac{u'\left(c^e\right)}{u'\left(c^u\right)}\right] \mathrm{d}y\right\}$$

where $B\left(a,x\right) := \left\{y : \Phi\left(a,x,y\right) = 1\right\}$ is the acceptance set of worker $\left(a,x\right)$.

Precautionary Saving

Takeaways

- Propositions 1-3 says wealth $\uparrow \Rightarrow$ mismatch $\downarrow$ search durations $\uparrow$ & wages $\uparrow$
- Proposition 4 says acceptance strategy & wages $\Rightarrow$ savings policy $\Rightarrow$ wealth accumulation
- The interactions lead to an endogenous joint distribution of wealth & wages

Takeaways

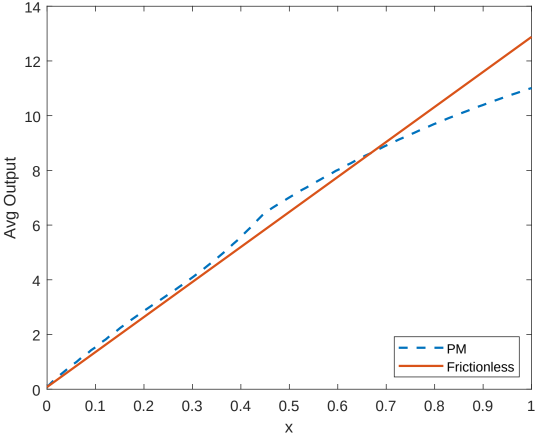
- The interactions lead to an endogenous joint distribution of wealth & wages

Experiments

How Does Wealth Matter for Labor Productivity?

- Productivity in the precautionary mismatch (PM) economy vs. a frictionless economy details

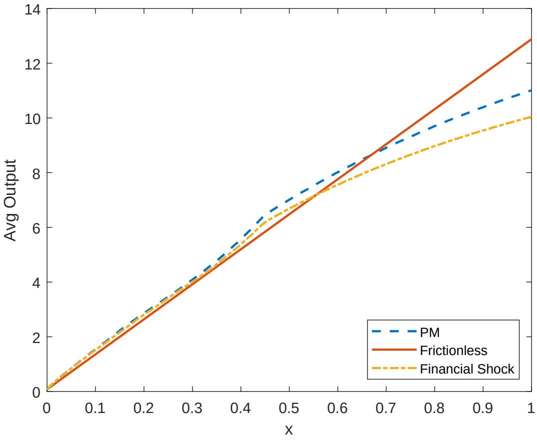
- Assume vertical heterogeneity to see how mismatch affects different types of workers



How Does Wealth Matter for Labor Productivity?

- Now hit the PM economy with a financial shock that erases 50% of wealth from all workers

details



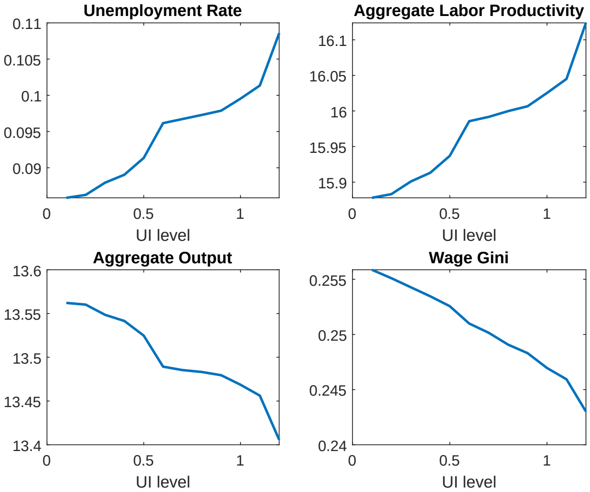
The Effect of Unemployment Benefits

- Suppose UI is financed by a lump-sum tax T. Government budget balance:

$$b \iint d_u(a, x)\, \mathrm{d}a \mathrm{d}x = T$$

- How does UI impact output, productivity and inequality?

The Effect of Unemployment Benefits



Conclusions

- Develop a framework to study how wealth affects labor market sorting
- The model generates several empirically sound relationships
  - Poor workers experience more mismatch
  - Job finding rate decreases with wealth
  - Wages increase with wealth
- Endogenous joint distribution of wealth and wages
- Study its aggregate, distributional and policy implications
  - A severe wealth shock negatively impacts labor market efficiency, primarily due to mismatch of high-skilled workers
  - UI discounrages employment but also improves labor market efficiency and decreases wage inequality

Appendix

HJB Equation: Derivation – 1/2

- Consider the discrete time problem with period length of Δ

$$W(a,x,y) = \max_c u(c)\Delta + \frac{1}{1+\rho\Delta}\Big\{ \underbrace{\Delta\sigma U(a',x)}_{\text{exogenous separation}} +$$
$$+ (1-\Delta\sigma)\max\left[W(a',x,y), \underbrace{U(a',x)}_{\text{endogeneous separation}}\right]\Big\}$$
$$\text{s.t.} \quad a' = a + (ra + \omega(a,x,y) - c)\Delta$$

- Consider acceptable matches $W(a,x,y) \geq U(a,x)$. As $\Delta \to 0$, $a' \to a$, continuity will preserve that $W(a',x,y) \geq U(a',x)$.

- We can take the max operator off for small Δ

$$W(a,x,y) = u(c^e)\Delta + \frac{1}{1+\rho\Delta}\{\Delta\sigma U(a + (ra + \omega - c^e)\Delta, x)$$
$$+ (1-\Delta\sigma) W(a + (ra + \omega - c^e)\Delta, x, y)$$

HJB Equation: Derivation – 2/2

- Multiply both sides by $(1+\rho\Delta)$, subtract W, and then divide them by Δ

$$\begin{aligned}\rho W(a,x,y) = u(c^e)(1+\rho\Delta) + \frac{1}{\Delta}\left[W(a+(ra+\omega-c^e)\Delta,x,y) - W(a,x,y)\right] \\ + \sigma\left[U(a+(ra+\omega-c^e)\Delta,x) - W(a+(ra+\omega-c^e)\Delta,x,y)\right]\end{aligned}$$

- Take the limit $\Delta \to 0$,

$$\rho W(a,x,y) = u(c^e) + \underbrace{(ra+\omega-c^e)}_{\dot{a}} W_a(a,x,y) + \sigma\left[U(a,x) - W(a,x,y)\right]$$

- Other value functions can be derived similarly

Back

Nash Bargaining: Proof – 1/3

- Consider the discrete time problem with period length of Δ

- The tilde-value of a worker with temporarily deviating flow wage w is

$$\tilde{W}(w,a,x,y;\Delta) = \max_c u(c)\Delta + \frac{1}{1+\rho\Delta}\left\{(1-\Delta\sigma)W(a',x,y) + \Delta\sigma U(a',x)\right\}$$
$$\text{s.t.} \quad a' = a + (ra + w - c)\Delta$$

- Envelop condition

$$\tilde{W}_w(w,a,x,y;\Delta) = \frac{1}{1+\rho\Delta}\{(1-\Delta\sigma)W_a(a+(ra+w-\tilde{c}^e)\Delta,x,y)\Delta$$
$$+\Delta\sigma U_a(a+(ra+w-\tilde{c}^e)\Delta,x)\Delta\}$$

where $\tilde{c}^e(w,a,x,y)$ is the optimal consumption policy.

Nash Bargaining: Proof – 2/3

- The discrete-time version tilde-value for a producing job is

$$\tilde{J}(w,a,x,y) = f(x,y)\,\Delta - w\Delta + \frac{1}{1+r\Delta}\left[(1-\Delta\sigma)\,J(a',x,y) + \Delta\sigma V(y)\right]$$

where $a' = a + (ra + w - \tilde{c}^e(w,a,x,y))\,\Delta$ is taken as given for firms

- Take derivative of $\tilde{J}$ w.r.t. w

$$\tilde{J}_w(w,a,x,y) = -\Delta + \frac{1}{1+r\Delta}(1-\Delta\sigma)\,J_a(a + (ra + w - \tilde{c}^e(w,a,x,y))\,\Delta, x, y) \times (1 - \tilde{c}^e_w(w,a,x,y))\,\Delta$$

Nash Bargaining: Proof – 3/3

- Under Nash bargaining, the wage policy is determined by

$$\omega(a,x,y) = \arg\max_{w} \left[\tilde{W}(w,a,x,y) - U(a,x)\right]^{\eta} \left[\tilde{J}(w,a,x,y) - V(y)\right]^{1-\eta}$$

- The first order condition for the bargaining problem is

$$\eta \frac{\tilde{J}(w,a,x,y) - V(y)}{-\tilde{J}_w(w,a,x,y)} = (1-\eta) \frac{\tilde{W}(w,a,x,y) - U(a,x)}{\tilde{W}_w(w,a,x,y)}$$

- Taking limit $\Delta \to 0$, we can show that

$$\eta \frac{J(a,x,y) - V(y)}{1 - J_a(a,x,y)} = (1-\eta) \frac{W(a,x,y) - U(a,x)}{W_a(a,x,y)}$$

where we use result from Lemma.

Back

Nash Bargaining: Lemma – 1/2

Lemma

$$\lim_{\Delta\to 0} \frac{-\tilde{J}_w(w,a,x,y;\Delta)}{\tilde{W}_w(w,a,x,y;\Delta)} = \frac{1-J_a(a,x,y)}{W_a(a,x,y)}.$$

Proof.

Plug in the expressions for $\tilde{J}_w$ and $\tilde{W}_w$ derived before,

$$\begin{aligned}
\frac{-\tilde{J}_w}{\tilde{W}_w} &= \frac{\Delta - \frac{1}{1+r\Delta}(1-\Delta\sigma) J_a(a+(ra+w-\tilde{c}^e(w,a,x,y))\Delta,x,y)(1-\tilde{c}^e_w(w,a,x,y))\Delta}{\frac{1}{1+\rho\Delta}\{(1-\Delta\sigma) W_a(a+(ra+w-c)\Delta,x,y)\Delta+\Delta\sigma U_a(a+(ra+w-c)\Delta,x)\Delta\}} \\
&= \frac{(1+r\Delta)-(1-\Delta\sigma) J_a(a+(ra+w-\tilde{c}^e(w,a,x,y))\Delta,x,y)}{\{(1-\Delta\sigma) W_a(a+(ra+w-c)\Delta,x,y)+\Delta\sigma U_a(a+(ra+w-c)\Delta,x)\}} \times \frac{1+\rho\Delta}{1+r\Delta} \\
&\to \frac{1-J_a(a,x,y)}{W_a(a,x,y)}
\end{aligned}$$

as $\Delta \to 0$, where $\lim_{\Delta\to 0} \tilde{c}^e_w(w,a,x,y;\Delta) = 0$ is proved in Lemma. □

Nash Bargaining: Lemma – 2/2

Lemma

$$\lim_{\Delta \to 0} \tilde{c}^e_w \left(w, a, x, y; \Delta\right) = 0.$$

Proof.

The first order condition to employed worker's problem is

$$u'\left(\tilde{c}^e\right) = \frac{1}{1+\rho\Delta} \left\{ \left(1 - \Delta\sigma\right) W_a \left(a + \left(ra + w - \tilde{c}^e\right)\Delta, x, y\right) + \Delta\sigma U_a \left(a + \left(ra + w - \tilde{c}^e\right)\Delta, x\right) \right\}.$$

Taking limit $\Delta \to 0$, we have

$$\lim_{\Delta \to 0} u'\left(\tilde{c}^e\left(w, a, x, y; \Delta\right)\right) = W_a\left(a, x, y\right).$$

Under mild technical conditions,

$$\lim_{\Delta \to 0} \frac{\partial \tilde{c}^e}{\partial w}\left(w, a, x, y; \Delta\right) = \frac{\partial}{\partial w} \lim_{\Delta \to 0} \tilde{c}^e\left(w, a, x, y; \Delta\right) = \frac{\partial}{\partial w} u'^{(-1)}\left(W_a\left(a, x, y\right)\right) = 0.$$

Wage

- Rewrite the Nash solution as

$$\eta \frac{rJ(a,x,y) + (\rho - r) J(a,x,y) - \rho V(y)}{1 - J_a(a,x,y)} = (1-\eta) \frac{\rho W(a,x,y) - \rho U(a,x)}{W_a(a,x,y)}$$

- Plug in the HJB equation of rJ and ρW:

$$\eta \frac{f(x,y) - \omega - \sigma [J(a,x,y) - V(y)] + (ra + \omega - c^e) J_a(a,x,y) + (\rho - r) J(a,x,y) - \rho V(y)}{1 - J_a(a,x,y)}$$
$$= (1-\eta) \frac{u(c^e) - \sigma [W(a,x,y) - U(a,x)] + (ra + \omega - c^e) W_a(a,x,y) - \rho U(a,x)}{W_a(a,x,y)}$$

- Collecting terms

$$\begin{aligned} \omega(a,x,y) = &\eta \frac{f(x,y) + (ra - c^e) J_a(a,x,y) + (\rho - r) J(a,x,y) - \rho V(y)}{1 - J_a(a,x,y)} \\ &- (1-\eta) \frac{u(c^e) + (ra - c^e) W_a(a,x,y) - \rho U(a,x)}{W_a(a,x,y)} \end{aligned}$$

Back

Formal Equilibrium Definition

Definition

A stationary equilibrium consists of

- a set of value functions $\{W(a,x,y), U(a,x), J(a,x,y), V(y)\}$,
- a set of consumption policy functions $\{c^e(a,x,y), c^u(a,x)\}$,
- acceptance policy $\Phi(a,x,y)$ and wage policy $\omega(a,x,y)$,
- an invariant distribution of $d_m(a,x,y)$ and $d_u(a,x)$, and market tightness θ,
- an interest rate r

such that:

1. The value functions and policy functions solve worker and firm's optimization problem;
2. Wage setting and matching acceptance decision satisfy Nash bargaining;
3. The stationary distributions satisfy the Kolmogorov Forward equations;
4. Market tightness adjusts so that free entry gives zero economic profits to vacant firms;
5. Interest rate clears the asset market. Back

Algorithm

1. Guess θ and $d_v(y_k)/v$
2. Guess bargaining solution for each pair $w(a_i, x_j, y_k)$
3. Solve the workers' and firms' problem using the implicit method as in Achdou et al. (2020)
4. Calculate stationary distribution of workers: Discretize the Kolmogorov Forward equation

$$
\begin{aligned}
0 &= -\frac{s_{i,F}^{jk,W+} d_{i,}^{jk,W} - s_{i-1,F}^{jk,W+} d_{i-1}^{jk,W}}{\Delta_a} - \frac{s_{i+1,B}^{jk,W-} d_{i+1}^{jk,W} - s_{i,B}^{jk,W+} d_{i}^{jk,W}}{\Delta_a} - \delta d_i^{jk,W} + p(\theta) d_v(k) \mathbf{1}_i^{jk} d_i^{j,U} \\
0 &= -\frac{s_{i,F}^{j,U+} d_i^{j,U} - s_{i-1,F}^{j,U+} d_{i-1}^{j,u}}{\Delta_a} - \frac{s_{i+1,B}^{j,U-} d_{i+1}^{j,U} - s_{i,B}^{j,U-} d_i^{j,u}}{\Delta_a} - p(\theta) \sum_k d_v(k) \mathbf{1}_i^{jk} d_i^{j,U} + \delta \sum_k d_i^{jk,W}
\end{aligned}
$$

5. Update wage schedule using the wage function from Nash bargaining
6. Update θ and d_v
7. Stop if convergence criteria are met, otherwise go back to 3.

Back

Type Normalization

- Suppose $\tilde{F}(\tilde{x})$ and $\tilde{G}(\tilde{y})$ are CDF of worker and job types, with production function $\tilde{f}(\tilde{x}, \tilde{y})$
- Redefine a type according to its rank, i.e., $x := \tilde{F}(\tilde{x})$ and $y := \tilde{G}(\tilde{y})$
- The distribution of the rank-based type is uniform:

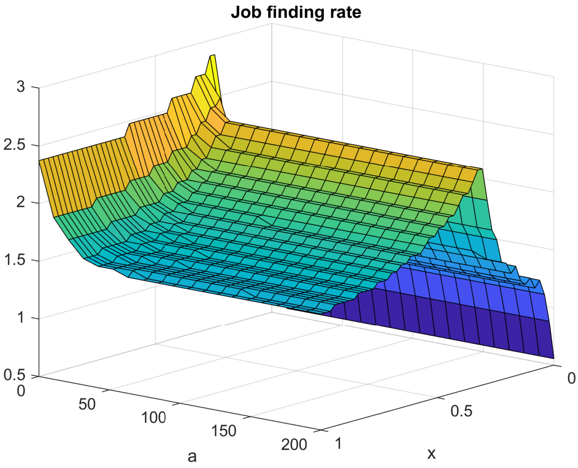
$$\mathbb{P}(x \leq t) = \mathbb{P}\left(\tilde{F}(\tilde{x}) \leq t\right) = \mathbb{P}\left(\tilde{x} \leq s, \text{ for some } s \in \tilde{F}^{-1}(t)\right) = t$$

- Rewrite the production function accordingly

$$f(x, y) := \tilde{f}\left(\tilde{F}^{-1}(x), G^{-1}(y)\right)$$

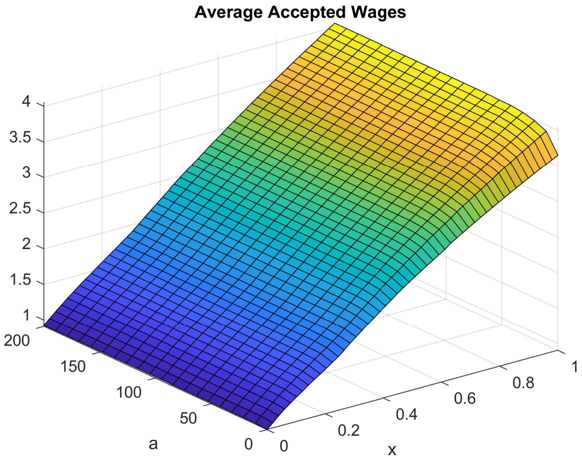
Back

Job Finding Rates, Vertical Heterogeneity



Back

Average Accepted Wages, Vertical Heterogeneity



back

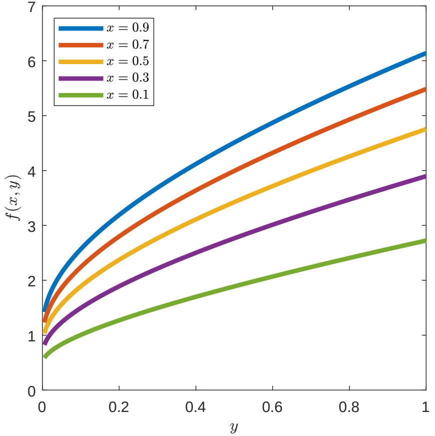
Supportive Evidence for Labor Market Outcomes

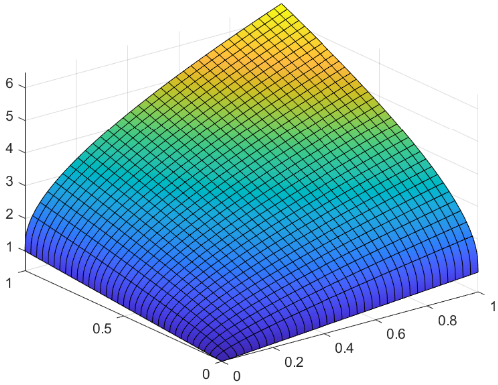
Supportive evidence from the literature

- Card et al. (2007): lump sum transfer of 2 months of salary $\downarrow$ job finding rate by 8-12%
- Chetty (2008): elasticity of job finding rate w.r.t. UI decreases with liquid wealth
- Herkennhoff et al (WP): individuals who can replace 10% more of their earnings with credit take 0.53 weeks longer to find a job, and increases replacement rates by 1.34%

Back

Production Function with Vertical Heterogeneity

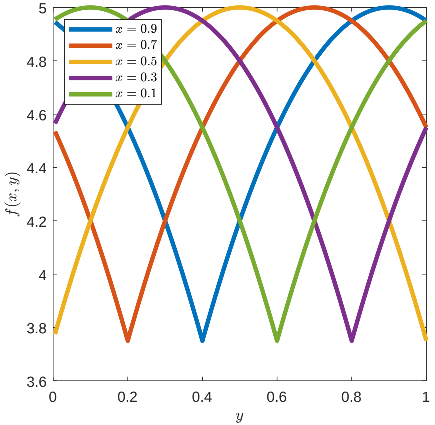


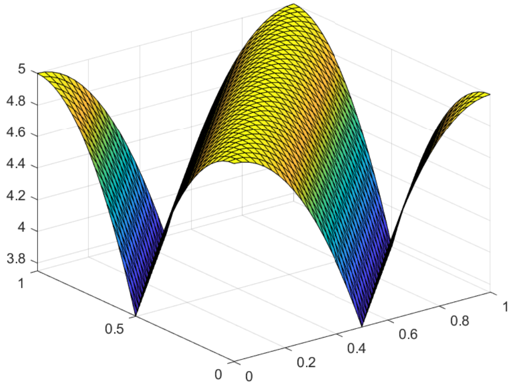


$$f(x, y) = 1 + \psi \left(x^{\xi} + y^{\xi} \right)^{1/\xi}$$

Calibration Proposition

Production Function with Horizontal Heterogeneity





$$f\,(x, y) = a - b \min\left(|x - y|, |1 + x - y|, |1 + y - x|\right)^2$$

Calibration Proposition

Proof: Precautionary Mismatch

- Lemma: $c^e(a,x,y) > c^u(a,x)$

- Consider a marginally acceptable match (a,x,y). Nash bargaining implies

$$\Delta\left(a;x,y\right) := W\left(a,x,y\right) - U\left(a,x\right) = 0$$

- Take the derivative with respect to a

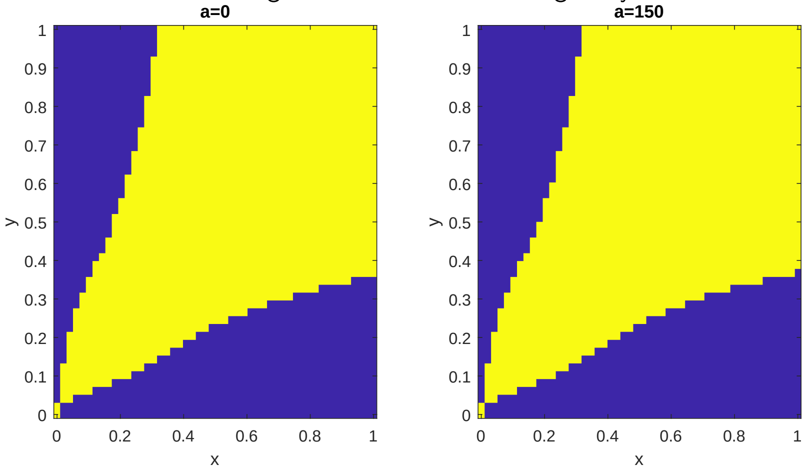
$$\Delta_a = W_a - U_a = u'\left(c^e\right) - u'\left(c^u\right) < 0.$$

- Therefore, we will have $\hat{S}\left(a',x,y\right) < 0$ for $a' > a$ and $\hat{S}\left(a'',x,y\right) > 0$ for $a'' < a$

Back

Precautionary Mismatch: Quadratic Utility

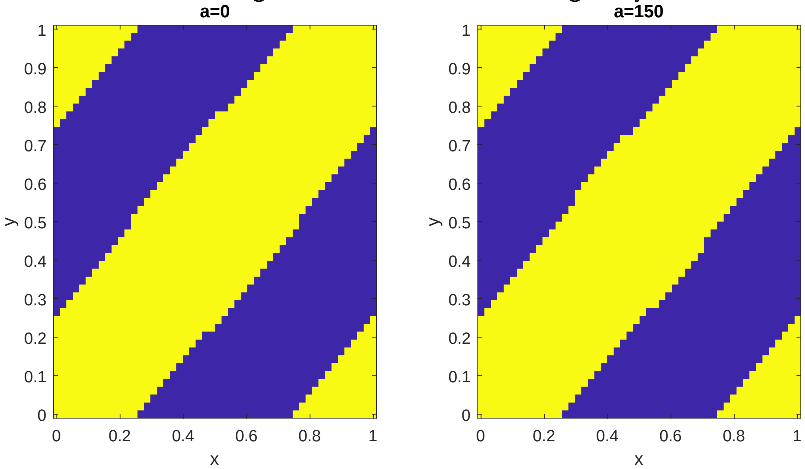
Matching sets with vertical heterogeneity



Back

Precautionary Mismatch: Quadratic Utility

Matching sets with horizontal heterogeneity



Back

Skill Mismatch Measures

- Measure workers' and occupations' heterogeneity (Lise and Postel-Vinay 2020)
- NLSY97 to construct workers' skill measures:
  - ASVAB scores for each subject, mental health score, emotional and behavioral score, BMI, and criminal activity
  - take the first 2 principal components with the exclusion restrictions
    1. ASVAB mathematics knowledge score only reflects *cognitive* skills
    2. ASVAB automotive & shop information score only reflects *manual* skills
- O*NET to measure occupations' skill requirements
  - take the first 2 principal components with the exclusion restrictions
    1. mathematics score only reflects *cognitive* skill requirements
    2. mechanical knowledge score only reflects *manual* skill requirements
- Define mismatch as

$$d_k = |x_k - y_k|, k = \{\text{cog, man}\}$$

Back

Proof: Job finding rate is decreasing in wealth

- Consider $a > a'$
- From Theorem "precautionary mismatch", we know that $\Phi(a, x, y) \leq \Phi(a', x, y)$
- The job finding rate of the worker of type x with wealth

$$\begin{aligned}
\pi_{ue}(a, x) &= p(\theta) \int \frac{d_v(y)}{v} \Phi(a, x, y) \mathrm{d}y \\
&\leq p(\theta) \int \frac{d_v(y)}{v} \Phi(a', x, y) \mathrm{d}y \\
&= \pi_{ue}(a', x)
\end{aligned}$$

Back

Proof: Euler equation for employerd worker's consumption

- Total differentiating $W_a(a,x,y)$, we have

$$\mathrm{d}W_a(a,x,y) = W_{aa}(a,x,y)\,\mathrm{d}t$$

- Apply the Envelope theorem to employed value $W(a,x,y)$ with respect to a,

$$\rho W_a(a,x,y) = \sigma\left[U_a(a,x) - W_a(a,x,y)\right] + \dot{a}W_{aa}(a,x,y) + \left[r + \omega_a(a,x,y)\right]W_a(a,x,y)\,.$$

- Note that $W_a(a,x,y) = u'(c^e(a,x,y))$ and $U_a(a,x) = u'(c^u(a,x))$ by FOCs

$$u''(c^e)\,\mathrm{d}c^e = (\rho - r - \omega_a)\,u'(c^e)\,\mathrm{d}t - \sigma\left[u'(c^u) - u'(c^e)\right]\mathrm{d}t$$

- Rearrange

$$\underbrace{-\frac{u''(c^e)\,c^e}{u'(c^e)}}_{\text{relative risk aversion}} \cdot \underbrace{\frac{\mathrm{d}c^e/\mathrm{d}t}{c^e}}_{\text{consumption growth}} = r - \rho + \omega_a + \sigma\left[\frac{u'(c^u)}{u'(c^e)} - 1\right]$$

Back

Proof: Euler equation for unemployerd worker's consumption

- Total differentiating $U_a(a, x)$, we have

$$\mathrm{d}U_a(a, x) = U_{aa}(a, x)[ra + b - c^u]\,\mathrm{d}t$$

- Apply the Envelope theorem to unemployed value $U(a, x)$ with respect to a

$$\rho U_a(a, x) = p(\theta) \int \frac{d_v(y)}{v} [W_a(a, x, y) - U_a(a, x)]^+ \,\mathrm{d}y + \dot{a} U_{aa}(a, x) + rU_a(a, x)$$

- Plugging in FOCs

$$u''(c^u)\,\mathrm{d}c^u = (\rho - r)\, u'(c^u)\,\mathrm{d}t - p(\theta) \int_{B(a,x)} \frac{d_v(y)}{v} [u'(c^e) - u'(c^u)]\,\mathrm{d}y\mathrm{d}t$$

- Rearrange

$$-\frac{u''(c^u)\, c}{u'(c^u)} \cdot \frac{\mathrm{d}c^u/\mathrm{d}t}{c} = r - \rho + p(\theta) \int_{B(a,x)} \frac{d_v(y)}{v} \left[\frac{u'(c^e)}{u'(c^u)} - 1\right] \mathrm{d}y$$

Back

Precautionary Saving

- If $u''' > 0$, i.e., u' is convex, then

$$\frac{\dot{c}^e}{c^e} = \frac{1}{\gamma}\left\{r - \rho + \omega_a + \frac{1}{u'(c^e)}\left[\sigma u'(c^u) + (1-\sigma)u'(c^e)\right] - 1\right\}$$
$$> \frac{1}{\gamma}\left\{r - \rho + \omega_a + \frac{u'(\sigma c^u + (1-\sigma)c^e)}{u'(c^e)} - 1\right\}$$

- And

$$\frac{\dot{c}^u}{c^u} = \frac{1}{\gamma}\left\{r - \rho - 1 + \frac{1}{u'(c^u)}\left[p(\theta)\int_{B(a,x)} \frac{d_v(y)}{v} u'(c^e)\,\mathrm{d}y + (1-\pi_{ue})u'(c^u)\right]\right\}$$
$$> \frac{1}{\gamma}\left\{r - \rho + \frac{u'(\mathbb{E}c)}{u'(c^u)} - 1\right\}$$

Back

Hamilton-Jacobi-Bellman Equations: Workers

- Employed worker of type x with wealth a working at job of type y:

$$\begin{aligned}\rho W(a,x,y) = \max_c u(c) + \sigma\left[U(a,x) - W(a,x,y)\right] + \dot{a} W_a(a,x,y) \\ \text{s.t. } \dot{a} = ra + \omega(a,x,y) - c \\ a \geq \underline{a}\end{aligned}$$

Derivation

- Unemployed worker of type x with wealth a:

$$\begin{aligned}\rho U(a,x) = \max_c u(c) + p(\theta)\int \frac{d_v(y)}{v}\left[W(a,x,y) - U(a,x)\right]^+ \mathrm{d}y + \dot{a} U_a(a,x) \\ \text{s.t. } \dot{a} = ra + b - c \\ a \geq \underline{a}\end{aligned}$$

where $[\bullet]^+ := \max\{\bullet, 0\}$.

Hamilton-Jacobi-Bellman Equations: Jobs

- Producing job of type y, with employee of type x with wealth a:

$$rJ(a,x,y) = f(x,y) - \omega(a,x,y) + \sigma[V(y) - J(a,x,y)] + \dot{a}^e J_a(a,x,y)$$

where $\dot{a}^e := ra + \omega(a,x,y) - c^e(a,x,y)$ is the optimal saving policy of the employee.

- Vacant job of type y:

$$rV(y) = q(\theta)\iint \frac{d_u(a,x)}{u}[J(a,x,y) - V(y)]^+ \,\mathrm{d}a\mathrm{d}x$$

- Free entry condition:

$$\kappa = \int V(y)\,dG(y)$$

Wage Setting: Nash Bargaining

- Wage policy is determined by

$$\omega\left(a,x,y\right)=\arg\max_{w}\left[\tilde{W}\left(w,a,x,y\right)-U\left(a,x\right)\right]^{\eta}\left[\tilde{J}\left(w,a,x,y\right)-V\left(y\right)\right]^{1-\eta}$$

- The Nash solution can be characterized by Proof

$$\eta\frac{J\left(a,x,y\right)-V\left(y\right)}{1-J_a\left(a,x,y\right)}=\left(1-\eta\right)\frac{W\left(a,x,y\right)-U\left(a,x\right)}{W_a\left(a,x,y\right)}$$

- Intuition: workers & firms split *adjusted* surplus ($\rightarrow$ *standard* surplus as $u(c)\rightarrow$ linear)

$$\hat{S}\left(a,x,y\right):=\frac{1}{W_a\left(a,x,y\right)}\left[W\left(a,x,y\right)-U\left(a,x\right)\right]+\frac{1}{1-J_a\left(a,x,y\right)}\left[J\left(a,x,y\right)-V\left(y\right)\right]$$

- Matching and separation is privately (bilaterally) efficient

$$\Phi(a,x,y)=\mathbb{1}\{W\left(a,x,y\right)\geq U\left(a,x\right)\}=\mathbb{1}\{J\left(a,x,y\right)\geq V\left(y\right)\}=\mathbb{1}\{\hat{S}\left(a,x,y\right)\geq 0\}$$

- Delivers an analytical formula for wage that facilitates computation Wage

Steady State: Kolmogorov Forward Equations

- Inflow-outflow balancing equation for employed workers

$$0 = \underbrace{-\frac{\partial}{\partial a}\left[\dot{a}^{e}(a,x,y)\, d_m(a,x,y)\right]}_{\text{asset evolution}} \underbrace{-\sigma d_m(a,x,y)}_{\text{separation}} \underbrace{+ d_u(a,x)\, p(\theta)\, \frac{d_v(y)}{v}\, \Phi(a,x,y)}_{\text{job finding}}$$

- Inflow-outflow balancing equation for unemployed workers

$$0 = \underbrace{-\frac{\partial}{\partial a}\left[\dot{a}^{u}(a,x)\, d_u(a,x)\right]}_{\text{asset evolution}} \underbrace{-\int p(\theta)\, \frac{d_v(y)}{v}\, \Phi(a,x,y)\, d_u(a,x)\, \mathrm{d}y}_{\text{job finding}} \underbrace{+\sigma \int d_m(a,x,y)\, \mathrm{d}y}_{\text{separation}}$$

General Equilibrium

- Interest rate r clears the asset market

$$\iiint a d_m(a,x,y)\,\mathrm{d}a\mathrm{d}x\mathrm{d}y + \iint a d_u(a,x)\,\mathrm{d}a\mathrm{d}x$$
$$= \iiint J(a,x,y)\,d_m(a,x,y)\,\mathrm{d}a\mathrm{d}x\mathrm{d}y + \int V(y)\,d_v(y)\,\mathrm{d}y$$

- Formal equilibrium definition Equilibrium

Back to Model

Model Comparison: Aiyagari vs. KMS vs. Precautionary Mismatch

- We compare the joint wage and wealth distributions of three models
  1. Precautionary Mismatch (PM) Model
  2. Krusell, Mukoyama and Sahin (2010) (KMS)
  3. Aiyagari w/ 2-state income process (employed & unemployed)
- For fair comparison, we assume horizontal heterogeneity in PM
- KMS and Aiyagari are calibrated to have the same average wages and U2E rates as in PM
- We compare steady state joint wage-wealth distributions from the three models

Calibration: Parameterization

- Flow utility function exhibits constant relative risk aversion (CRRA):

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad , \quad \gamma > 0.$$

- Meeting function is assumed to be Cobb-Douglas:

$$M(u, v) = \chi u^{\alpha} v^{1-\alpha}.$$

- Worker and job types are normalized to be uniformly distributed WLOG Normalization

- Borrowing constraint is set to $\underline{a} = 0$

Calibration

Table: External Calibration

Parameter	Symbol	Value
discount rate	ρ	0.01
relative risk aversion	γ	2
separation rate	σ	0.1
bargaining power	η	0.5
meeting elasticity	α	0.5

Table: Internal Calibration

Parameter	Symbol	Targeted Moment
matching efficiency	χ	job finding rate
unemployment benefit	b	replacement rate
entry cost	κ	market tightness
production function	ξ, ψ	wage Gini
		$\text{corr}(a, w)$

How Does Wealth Matter for Labor Productivity?

- In a frictionless economy, $\exists$ a rule $\mu : \mathcal{A} \times \mathbb{X} \to \mathbb{Y}$ so that output is maximized

$$Y^{OPT} = \int_{\mathbb{X}} \int_{\mathcal{A}} f(x, \mu(a, x)) \mathrm{d}a \, \mathrm{d}x$$

- In the model's precautionary mismatch (PM) equilibrium, output is

$$Y^{PM} = \int_{\mathbb{Y}} \int_{\mathbb{X}} \int_{\mathcal{A}} f(x, y) d_m(a, x, y) \mathrm{d}a \, \mathrm{d}x \, \mathrm{d}y$$

- With a unit measure of workers, aggregate productivity for the frictionless economy is Y^{OPT} and

$$Y^{PM} / \int_{\mathbb{Y}} \int_{\mathbb{X}} \int_{\mathcal{A}} d_m(a, x, y) \mathrm{d}a \, \mathrm{d}x \, \mathrm{d}y$$

for the PM equilibrium

back

How Does Wealth Matter for Labor Productivity?

- Upon impact of the wealth shock, the distributions of employed and unemployed workers with previous wealth a become

$$\hat{d}_m(a, x, y) = d_m\left(\frac{a}{2}, x, y\right) \text{ and } \hat{d}_u(a, x) = d_u\left(\frac{a}{2}, x\right)$$

- Their job strategy becomes

$$\hat{\Phi}(a, x, y) = \Phi\left(\frac{a}{2}, x, y\right)$$

- So the measure of workers x employed at job y immediately after the shock is

$$\hat{d}(x, y) = \int_{\mathcal{A}} \hat{d}_u(a, x) p(\theta) \frac{d_v(y)}{v} \hat{\Phi}(a, x, y) + (1 - \sigma)\hat{d}_m(a, x, y) \mathrm{d}a$$

- Total output after the financial shock

$$Y^{FIN} = \int_0^1 \int_0^1 f(x, y)\hat{d}(x, y) \mathrm{d}x \, \mathrm{d}y$$

back