Speculators in Housing Markets: A Search Model Approach

Jincheng Huang and Xuequan Peng

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Abstract

This paper aims to provide a theoretical foundation for the empirical observation in the housing market that a small number of speculators, or "flippers" contributed to a large portion of the increase in house prices and transactions during the 2003-2006 housing boom. We extend the search framework of Duffie, Garleanu and Pederson (2007) by introducing an additional type of agents who act like house flippers, in the sense that they purchase from sellers with low valuation of their houses and sell quickly to buyers with high valuation. As they spend most of their time on the market as speculative buyers, they reduce search frictions for sellers but increase search frictions for buyers. We show that under a reasonable set of parameter values, a small increase in the measure of flippers can lead to a reduction in housing vacancy and the average time of house posting on the market, as well as a significant increase in average prices. Moreover, these flippers are able to buy at below-average prices and sell at above-average prices, effectively generating a house price spread within the market.

1 Introduction

The US housing market experienced a significant boom during 2003-2006, and its eventual downturn triggered one of the greatest financial crises in US history. A large number of discussions since then have focused on the exact drivers of the housing boom. There has been a growing literature showing that the boom was to a large extent fueled by "flippers" in the housing market, i.e. speculators who purchased and sold houses within relatively short time periods to benefit from rising home prices. Using survey data, Piazzesi and Schneider (2009) point out that the boom progressed along with the number of optimistic buyers who believed house prices would rise further. They suggest that these optimistic buyers, although small in size relative to other types of buyers, accounted for a large share of transaction flow in the housing market and drove up average transaction prices. In addition, Mian and Sufi (2018) find using individual credit history data that a small group of housing speculators, while accounting for only less than 1% of total mortgage borrowers during the boom, contributed to almost 80% of the relative price growth in zip codes where access to private mortgages were easier.

Considering the relative size of house flippers and the high transaction cost of housing, it is highly unlikely that the inflow of flippers have any significant impact on the housing stock. In fact, some estimates (e.g. Haughwout et al. (2012)) indicate that housing vacancies were consistently above equilibrium and continued to grow during the housing boom. These findings are somewhat at odds with our common economic intuition, that is a rapid increase in prices should be the result of supply shortage. In the case of the last housing boom, it seems that the main driving force of house price appreciation was the amount of flipping in the transaction flow, rather than the stock.

This paper aims to provide a theoretical foundation for the empirical findings above. In particular, we would like to develop a model where a small group of housing speculators, or "flippers", are able to gain from trades with regular home buyers and sellers. To do so, we use a search and matching framework in the spirit of Duffie, Garleanu and Pederson (2007) where agents differ in their valuation of housing and trade in a decentralized market with search frictions. We extend the model by introducing a new type of agents whose valuation of housing are between the buyers' and the sellers' and are invariant over time. This new type of agents work like housing flippers in the sense that they buy from sellers with low valuation of their houses and quickly sell to buyers with high valuation. As they sell more quickly than they can buy, they spend most of their time as buyers, thereby crowding the buy side of the market while making the sell side more frictional.

We identify the role of house flippers in housing valuation using steady-state comparative statics. The results generated by our framework are threefold. First, an increase in house flippers reduces vacancy and the average amount of time houses spend on the market. Second, flippers divert transactions between regular agents towards themselves and drive up equilibrium prices. Second, they buy at below-average prices and sell at above-average prices.

Our work is inspired by a number of empirical housing studies that focus on the role of short-term flippers or speculators in amplifying price and volume dynamics, most notably Piazzesi and Schneider (2009), DeFusco, Nathanson and Zwick (2018) and Mian and Sufi (2018). These studies have all highlighted the increase of short-term buyers, i.e. flippers during the housing boom. We take the fact as given in this paper and provide an explanation for why a rise in short-term buyers would lead to higher house prices and transaction volumes. On the theoretical side, we contribute to the literature pioneered by Wheaton (1990) that uses search models to understand the housing market. We extend the model by Duffie, Garleanu and Pederson (2005, 2007) by (1) modifying assumptions about agents' valuation process to make it more applicable to housing market, and (2) adding more heterogeneity in preferences to account for the short-term buying and selling strategies by speculators. We show that these extensions enable us to explain a rich set of phenomena in the housing market through the lens of short-term traders and their speculative activities, a topic which has not been extensively studied by previous housing search literature.

The paper proceeds as the follows. We first describe our model and characterize the steady state equilibrium in Section 2. Then, in Section 3, we show numerical examples that demonstrate the key results of our model. In the last section, we conclude and discuss the future directions we plan to explore.

2 Model

Our model builds on the search framework by Duffie, Garleanu and Pederson (2005, 2007). Our primary goal is to modify and extend the model so that it matches how the housing stock is traded by buyers and sellers in a decentralized market. The first modification is our assumption of sellers' preference: unlike the typical investors in the OTC market whose preference might change before a transaction happen, a seller in our model does not change her preference of the house before transaction happens. In other words, we interpret the preference shock of a homeowner as a moving shock, which, upon arrival, would permanently lower the homeowner's valuation of the house she currently owns. As a result, the *unhappy* homeowner is better off selling the house to an agent who has higher valuation of it, and wait for the opportunity to buy another house in the future. Secondly, we introduce speculative agents, whose valuation of housing is time-invariant and is always in between a *happy* homeowner's and an *unhappy* homeowner's. This structure of preferences, together with search frictions, allows for the speculator to extract profit by trading with both parties.

We focus on the steady-state equilibrium to help us gain useful insights and show a set of equations that characterize equilibrium total surpluses, prices, and the distribution of each agent type.

2.1 Environment and Preferences

Time is continuous, and agents are infinitely-lived and risk neutral. They have access to a risk-free interest rate of r, which is also their time discount rate. The model focuses on a single housing market with a fixed stock of homes, which we normalize to have measure $s \in (0,1)$. At any point in time, there are five types of agents in the economy: happy homeowners (ho), unhappy homeowners, i.e. home sellers (lo), home buyers (hn), speculative buyers (mn) and speculative sellers (mo). The first letter in the types stands for valuation of housing assets: h means high valuation, m means median valuation, etc. The second letter in the types stand for ownership status: o means owner and n means non-owner. We assume that agents preferences are public information. We normalize the sum of happy homeowners, home sellers and home buyers to be 1 in the steady state, and the sum of speculative buyers and speculative sellers (we call them speculators or flippers in general) to be $m \in (0,1)$. Since agents are risk neutral, it is without loss of generality to assume that each agent can own at most 1 unit of housing service at any point in time.

A happy homeowner (ho) receives a flow utility of v_H from the housing service that her house provides. A home seller (lo) receives a lower flow utility of v_L, where the drop in flow utility comes from the liquidity cost of holding a house she no longer wishes to own. A speculative seller receives $v_M < v_H$, where the difference in flow utility represents the opportunity cost of investing in the housing market, as well as the additional tax burden of owning non-owner-occupant housing. We focus on the case where $v_H > v_M > v_L$.

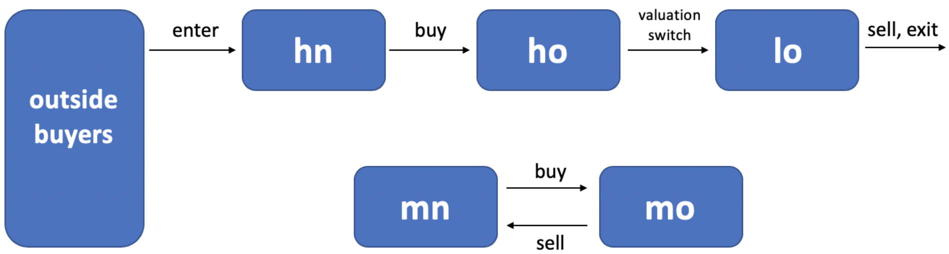
Home buyers and sellers find each other randomly in a decentralized market with search frictions. At any point of time, home buyers (hn) enter the housing market at rate e_h and search for potential sellers. We show later that trade always happens if a home buyer is matched with a regular home seller (lo) or a speculative seller (mo). After transaction, a home buyer becomes a happy homeowner (ho) and is inactive, meaning that she would never trade with other agents at this stage. A happy homeowner (ho) receives a *moving shock* at rate δ, whereupon the owner receives a much lower flow utility from the house she owns and thus becomes a seller (lo). A seller searches for potential buyers in the market, and we show later that trade always happens if she matches with a regular home buyer (hn) or a speculative buyer (mn).

Speculative home buyers (mn) stay in the housing market to gain from trade with regular home buyers and sellers. While in the housing market, a speculative buyer has to suffer a flow disutility of k. We can interpret it as an opportunity cost or liquidity cost of staying in the market. After trade, a speculative buyer becomes a speculative seller (mo) and searches for potential home buyers (hn). We can show that speculators always buy from lo and sell to hn. After selling the house, a speculative seller exits the market.

Figure 1 shows the evolution of agent types in the housing economy that we described above. Our assumption about the inflow and outflow of agents is similar in spirit to Vayanos and Wang (2006) in that we think of home buyers and speculators as coming from outside the market. This allows us to simplify the calculations later by dropping continuation values from low-type owners' valuation functions.

Buyers (hn or mn) and sellers (lo or mo) meet bilaterally at any point of time, and the bilateral meeting rate is λ. We restrict the number of houses that an agent can own to be either 0 or 1. Therefore, the total measure of housing stock should be equal to the total number of owners at all time, i.e. $\mu_{mo}(t) + \mu_{lo}(t) + \mu_{ho}(t) = s \, \forall t$. Under this condition, we can define *housing vacancy* as

Figure 1: Flow Diagram for Agent Types



the number of houses held by sellers, i.e. $\mu_{lo}(t)+\mu_{mo}(t)$.

2.2 Distribution of Agents

To solve for the distribution of agents in this model, we first use the key insight that there are 3 types of trades that generate positive gains: those between regular home buyers (hn) and home sellers (lo), regular home buyers (hn) and speculative sellers (mo), and speculative buyers (mn) and regular home sellers (lo). The rates of change for the measures of agents at any time t are therefore

$$\dot{\mu}_{hn}(t) = e_h(t) - \lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda\mu_{hn}(t)\mu_{mo}(t) \tag{1}$$

$$\dot{\mu}_{ho}(t) = \lambda\mu_{hn}(t)\mu_{mo}(t) + \lambda\mu_{hn}(t)\mu_{lo}(t) - \delta\mu_{ho}(t) \tag{2}$$

$$\dot{\mu}_{lo}(t) = \delta\mu_{ho}(t) - \lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda\mu_{lo}(t)\mu_{mn}(t) \tag{3}$$

$$\dot{\mu}_{mn}(t) = \lambda\mu_{hn}(t)\mu_{mo}(t) - \lambda\mu_{lo}(t)\mu_{mn}(t) \tag{4}$$

$$\dot{\mu}_{mo}(t) = \lambda\mu_{lo}(t)\mu_{mn}(t) - \lambda\mu_{hn}(t)\mu_{mo}(t) \tag{5}$$

The intuition is straightforward. For example, equation 1 says the rate of change in the number of home buyers is equal to the inflow $e_h(t)$, minus the measure of buyers who are successfully matched with a regular home seller (lo) or speculative seller (mo). Again, we show later that trades always happen when such matches are formed. Due to the law of large numbers, the intensity of match between home buyers (hn) and home sellers (lo) is $\lambda\mu_{hn}(t)\mu_{lo}(t)$. The interpretation of the other equations is similar and is thus omitted.

2.3 Value Functions

We now characterize the dynamic problem of each type of agents. We denote the price at which a regular home seller (lo) and buyer (hn) trade by p, the price between hn and speculative seller mo

as q_1, and the price between lo and speculative buyer mn as q_2.

In equilibrium, the value from being a buyer in the market comes from the expected gains from trade with a regular home seller (lo) or speculative seller (mo), plus capital gains if the economy is not in a stationary equilibrium. When a trade occurs, the buyer enjoys utility gain $V_{ho}(t) - V_{hn}(t)$ and pays a price denoted in utils $p(t)$ or $q_1(t)$ to the counterparty. Therefore we can write the Hamilton-Jacobi-Bellman (HJB) equation of a regular home buyer's value function as

$$rV_{hn}(t) = \lambda\left[\mu_{lo}(t)\left((V_{ho}(t) - V_{hn}(t)) - p(t)\right) + \mu_{mo}(t)\left((V_{ho}(t) - V_{hn}(t)) - q_1(t)\right)\right] + \dot{V}_{hn}(t) \quad (6)$$

Similarly, we can specify the HJB for other types of agent as follows.

$$rV_{ho}(t) = v_H + \delta\left(V_{lo}(t) - V_{ho}(t)\right) + \dot{V}_{ho}(t) \quad (7)$$

$$rV_{lo}(t) = v_L + \lambda\mu_{hn}(t)\left(p(t) - (V_{lo}(t) - 0)\right) + \lambda\mu_{mn}(t)\left(q_2(t) - (V_{lo}(t) - 0)\right) + \dot{V}_{lo}(t) \quad (8)$$

$$rV_{mn}(t) = -k + \lambda\mu_{lo}(t)\left((V_{mo}(t) - V_{mn}(t)) - q_2(t)\right) + \dot{V}_{mn}(t) \quad (9)$$

$$rV_{mo}(t) = v_M + \lambda\mu_{hn}(t)\left(q_1(t) - (V_{mo}(t) - V_{mn}(t))\right) + \dot{V}_{mo}(t) \quad (10)$$

A happy owner ho enjoys flow utility of v_H, plus the expected change in value from moving shock. A seller enjoys flow utility of v_L and the expected gain from trading with the two types of buyers. A speculative buyer pays a flow cost k to be in the housing market and tries to buy from a regular home seller lo. A speculative seller mo enjoys flow utility of v_M from housing service. We normalize the value upon exiting the housing market to 0.

2.4 Nash Bargaining and Prices

The prices are determined through Nash bargaining with complete information. There are three possible transaction types in the economy, which yield the following prices. We denote θ as the bargaining power of a buyer (hn), and η as the bargaining power of a speculator (mn and mo). We let $\triangle V_h = V_{ho} - V_{hn},\ \triangle V_m = V_{mo} - V_{mn},\ \triangle V_l = V_{lo}$, which corresponds to the reservation values of each type. Therefore we have the following results.

- Price of transactions between buyer hn and seller lo

$$\begin{aligned}
&\max_p\,(\triangle V_h - p)^{\theta}\,(p - \triangle V_l)^{1-\theta} \\
&p = (1-\theta)\triangle V_h + \theta\triangle V_l \\
&\quad = \triangle V_l + (1-\theta)(\triangle V_h - \triangle V_l)
\end{aligned} \quad (11)$$

- Price of transactions between buyer *hn* and speculative seller *mo*

$$\max_{q_1} (q_1 - \triangle V_m)^{\eta} (\triangle V_h - q_1)^{1-\eta}$$

$$\begin{aligned} q_1 &= (1-\eta) \triangle V_m + \eta \triangle V_h \\ &= \triangle V_m + \eta (\triangle V_h - \triangle V_m) \end{aligned} \tag{12}$$

- Price of transactions between speculative buyer *mn* and seller *lo*

$$\max_{q_2} (\triangle V_m - q_2)^{\eta} (q_2 - \triangle V_l)^{1-\eta}$$

$$\begin{aligned} q_2 &= (1-\eta) \triangle V_m + \eta \triangle V_l \\ &= \triangle V_l + (1-\eta)(\triangle V_m - \triangle V_l) \end{aligned} \tag{13}$$

There are two ways to understand the prices. One way is through the first expression for each price, which can be interpreted as the weighted averages of buyer and seller's reservation values. The second way is through the second expressions, which are seller's reservation values plus a fraction of the total gains from trade. After solving for the total gains from trade$\{\triangle V_h - \triangle V_m, \triangle V_m - \triangle V_l, \triangle V_h - \triangle V_l\}$, we can use them to derive $\triangle V_h$, $\triangle V_m$ and $\triangle V_l$ and thus the equilibrium prices.

Proposition 1. *Under the Nash bargaining protocol, there are three types of trade in the housing market: those between a regular home buyer* (hn) *and a home seller* (lo)*, those between a regular home buyer* (hn) *and a speculative home seller* (mo) *and those between a speculative home buyer* (mn) *and a regular home seller* (lo)*.*

We provide the proof in Appendix A. The main idea is that, any type of trade only occurs if both sides enjoy positive gains from trade. In the proof we show that the only types of trade that generate positive gains for both sides are the three types mentioned in the proposition.

2.5 Steady-State Equilibrium

The steady-state equilibrium can be characterized by (i) a set of decision rules $\{I_{\sigma,\sigma'}\}$, where $\sigma \in \{ho, hn, lo, ln, mo, mn\}$; $I_{\sigma} = 1$ if type σ would transact with type σ', and $I_{\sigma} = 0$ otherwise, and (ii) the distribution of types $\{\mu_{\sigma}(t)\}$, and (iii) equilibrium prices p, q_1 and q_2. Given prices and the distribution of types, and decision rules are optimal for each type of agents, and given the decision rules, the measures should satisfy the set of equations 1-5 above. Finally, given the decision rules and distribution of types, prices should follow equations 11-13 above. To characterize such an equilibrium, we first use the aforementioned insight that trades only happen between regular home buyers (hn) and home sellers (lo), regular home buyers (hn) and speculative sellers (mo), and speculative buyers (mn) and regular home sellers (lo). Then, we use the steady-state distribution to solve for prices and total surpluses. Finally we verify that the prices and total surpluses are indeed consistent with the equilibrium decision rules.

2.5.1 Steady State Distribution

In steady state, the measures of each type of agents should be constant. This means that for type σ agents, the rate of change in its measure $\dot{\mu}_\sigma = 0$. Substituting the condition into equations 1-5, we have the following:

$$
\begin{aligned}
0 &= e_h - \mu_{hn}\mu_{lo}\lambda - \mu_{hn}\mu_{mo}\lambda && (14)\\
0 &= \mu_{hn}\mu_{mo}\lambda + \mu_{hn}\mu_{lo}\lambda - \mu_{ho}\delta \\
0 &= \mu_{ho}\delta - \mu_{hn}\mu_{lo}\lambda - \mu_{lo}\mu_{mn}\lambda && (15)\\
0 &= \mu_{hn}\mu_{mo}\lambda - \mu_{lo}\mu_{mn}\lambda && (16)\\
0 &= \mu_{lo}\mu_{mn}\lambda - \mu_{hn}\mu_{mo}\lambda
\end{aligned}
$$

From the conditions in section 2.1, we also know the measures of each type also have to satisfy:

$$\mu_{hn} + \mu_{ho} + \mu_{lo} = 1 \tag{17}$$

$$\mu_{mn} + \mu_{mo} = m \tag{18}$$

$$\mu_{ho} + \mu_{lo} + \mu_{mo} = s \tag{19}$$

The distribution of agent types in steady state is thus characterized by equations 14-19. The inflow rates for regular home buyers, namely e_h, is a free parameter chosen to satisfy the steady state conditions. The following proposition claims the existence and uniqueness of the steady state.

Proposition 2. *There exists a unique solution to equations 14-19. The solution specifies the unique steady state distribution of agents of each type.*

In Appendix B, we show the proof of existence and uniqueness of steady state distribution.

2.5.2 Equilibrium Surplus

Having described equilibrium distribution of different types of agents, we now proceed to characterize the equilibrium total surpluses. For a transaction between *hn* and *lo*, the total surplus is $\triangle V_h - \triangle V_l$, which is the sum of gains from trade for both parties. Similarly, the total surplus for a transaction between *hn* and *mo* is $\triangle V_h - \triangle V_m$, total surplus between *mn* and *lo* is $\triangle V_m - \triangle V_l$. Appendix C shows the details of the calculation. The equilibrium total surpluses $\{\triangle V_h - \triangle V_l, \triangle V_h - \triangle V_m, \triangle V_m - \triangle V_l\}$ jointly satisfy equations 20 - 22.

- Total gains from trade $\triangle V_h - \triangle V_l$ between *hn* and *lo*

$$\begin{aligned}
&(\triangle V_h - \triangle V_l)\left[r + \lambda\left(\theta\mu_{lo} + (1-\theta)\frac{r}{r+\delta}\mu_{hn}\right)\right] \\
=&\frac{r(v_H - v_L)}{r+\delta} - \frac{r\lambda\mu_{mn}(1-\eta)}{r+\delta}(\triangle V_m - \triangle V_l) \\
&-\lambda(1-\eta)\mu_{mo}(\triangle V_h - \triangle V_m) \qquad (20)
\end{aligned}$$

- Total gains from trade $\triangle V_h - \triangle V_m$ between *hn* and *mo*

$$\begin{aligned}
&(\triangle V_h - \triangle V_m)[r + \lambda((1-\eta)\mu_{mo} + \eta\mu_{hn})] \\
=&\left[v_H - v_M - k - \frac{\delta(v_H - v_L)}{r+\delta}\right] - \lambda\left[\mu_{lo}\theta - \frac{\delta\mu_{hn}(1-\theta)}{r+\delta}\right](\triangle V_h - \triangle V_l) \\
&-\lambda\left[\eta\mu_{lo} - \frac{\delta(1-\eta)\mu_{mn}}{r+\delta}\right](\triangle V_m - \triangle V_l) \qquad (21)
\end{aligned}$$

- Total gains from trade $\triangle V_m - \triangle V_l$ between *mn* and *lo*

$$\begin{aligned}
&(\triangle V_m - \triangle V_l)[r + \lambda(\eta\mu_{lo} + (1-\eta)\mu_{mn})] \\
=&(v_M - v_L + k) + \lambda\eta\mu_{hn}(\triangle V_h - \triangle V_m) \\
&-\lambda(1-\theta)\mu_{hn}(\triangle V_h - \triangle V_l) \qquad (22)
\end{aligned}$$

20 - 22 form a system of 3 equations and 3 unknowns: $\triangle V_h - \triangle V_l, \triangle V_h - \triangle V_m$ and $\triangle V_m - \triangle V_l$, therefore there should be a unique solution to the equilibrium surpluses.

2.5.3 Equilibrium Prices

Using equilibrium total surpluses, we can solve for individual gains from trade given the values of bargaining parameter θ and η. Using the Nash bargaining solutions 11-13 and reservation values (shown in Appendix C), we can show that equilibrium prices can be characterized by equations 23 - 25.

- The price at which seller *lo* and buyer *hn* transact:

$$\begin{aligned}
p =& \frac{v_H}{r} - \frac{1}{r}\left[\frac{\delta(v_H - v_L)}{r+\delta} - \left(\frac{\delta\lambda\mu_{hn}(1-\theta)}{r+\delta} - \lambda\mu_{lo}\theta - r\theta\right)(\triangle V_h - \triangle V_l)\right. \\
&\left. + \lambda\mu_{mo}(1-\eta)(\triangle V_h - \triangle V_m) - \frac{\delta\lambda\mu_{mn}(1-\eta)}{r+\delta}(\triangle V_m - \triangle V_l)\right] \qquad (23)
\end{aligned}$$

- The price at which speculator *mo* sells property to buyer *hn*:

$$q_1 = \frac{v_M + k}{r} - \frac{1}{r}\left[\lambda \mu_{lo} \eta \left(\triangle V_m - \triangle V_l\right) - \lambda \mu_{hn} \eta \left(\triangle V_h - \triangle V_m\right) - \eta r \left(\triangle V_h - \triangle V_m\right)\right] \quad (24)$$

- The price at which speculator *mn* buys property from seller *lo*:

$$q_2 = \frac{v_M + k}{r} - \frac{1}{r}\left[\lambda \mu_{lo} \eta \left(\triangle V_m - \triangle V_l\right) - \lambda \mu_{hn} \eta \left(\triangle V_h - \triangle V_m\right) + \eta r \left(\triangle V_m - \triangle V_l\right)\right] \quad (25)$$

The three prices are the net present values of housing services ($\frac{v_H}{r}$ for *ho*, and $\frac{v_M + k}{r}$ for *mo*) minus illiquidity discounts. Note that the expressions are not closed-form solutions, as they involve the equilibrium surplus values $\triangle V_h - \triangle V_l, \triangle V_h - \triangle V_m$ and $\triangle V_m - \triangle V_l$, which come from the system of equations 20 - 22. We will derive the closed-form solutions under a special case discussed below.

According to equations 24 and 25, the speculators flip house prices by the amount

$$q_1 - q_2 = \eta \left(\triangle V_h - \triangle V_l\right) \quad (26)$$

which depends on both the speculators' bargaining power and the total surplus of housing transaction between regular home buyers and sellers. This result provides us with a useful insight that speculators earn higher profits per trade from a housing market where the gains from trade between regular home buyers and sellers are high.

3 Numerical Example

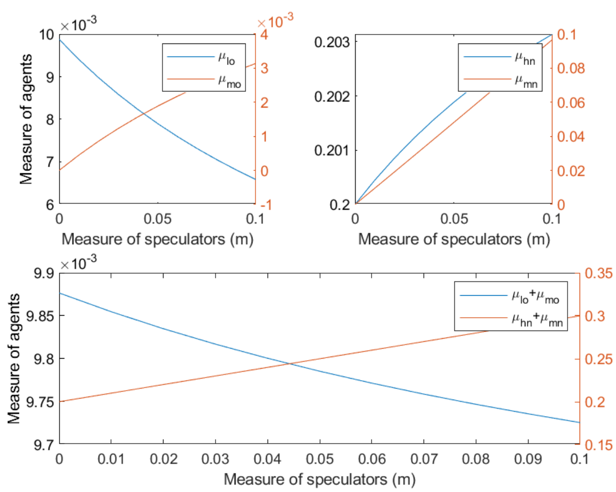
While we have shown the set of equations characterizing the equilibrium, we need to point out that the complexity of the analytical solutions makes it difficult for us to get useful insights into the model's predictions. We make up for the lack of analytical results by providing numerical examples to illustrate the effects of house flipping by housing speculators on the market equilibrium[1]. In particular, we compare steady state equilibria with different measures m of housing speculators. We let the maximal measure of speculators be 0.1, which is 10% of total regular home buyers and sellers, so as to be consistent with the idea that speculators are a relative small group in the housing market. We set the annual interest rate r at 5%, and the supply of housing s to 0.85 to match with the share of houses on the market from the American Housing Survey. The flow utility of happy

[1]For now, we refrain from explaining the exact reasons for the numerical results, as more work is needed to solve the equilibrium analytically and identify the channels.

homeowners v_H is normalized to 1, and we assume that v_M and v_L are 0.3 and 0.1 respectively. The flow fixed cost of speculation $k = 0.05$, namely 5% of happy home owners' flow utility. We let $\delta = 0.1$, which implies that agents spend about 90% of the time as happy home owners. We set the bargaining power between home buyers hn and home sellers lo to be 0.5, and set η equal to 1 or 0.5 for illustrative purpose. Finally, we set the bilateral meeting rate $\lambda = 40$, meaning that on average every agent in the market meets with 20 other agents every year.

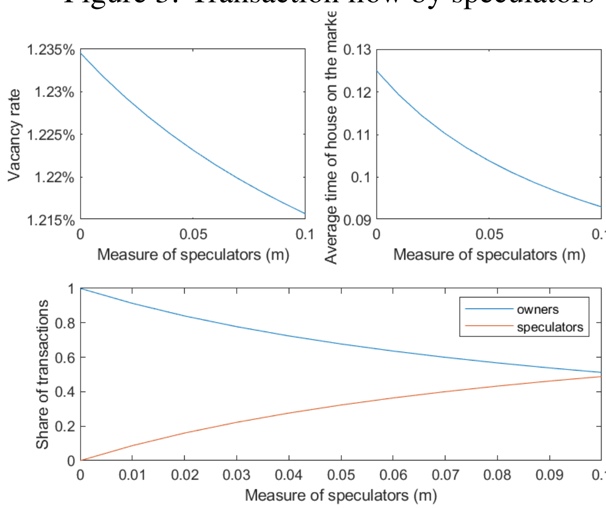
The first natural question to ask is, when m increases, how does search patterns change? Figure 2 below shows the change in distribution of each type of buyers and sellers as speculators increase in the market. The top left panel shows the measures of regular sellers (lo) in blue and speculative sellers (mo) in red, while the top right panel shows the measures of regular buyers (hn) in blue and speculative buyers (mn) in red. We can see that as the measure of speculators increases in the housing market, buyers tend to match more frequently with speculative sellers and less frequently with regular sellers, while sellers tend to match more with both speculative buyers and regular buyers. In addition, by comparing the rate of increase of speculative sellers vs. speculative buyers, we can see that they spend the majority of their time as speculative buyers. Therefore by occupying the buy side, speculators enables regular sellers to sell and exit the market more quickly, while making it more difficult for regular buyers to buy. The bottom panel shows the change in overall measures of buyers in red, and sellers in blue. Clearly, as overall demand increases while overall supply decreases, search friction goes up for buyers, but goes down for sellers. Both channels tend to increase a buyer's reservation value.

Figure 2: Distribution of each type



We can also show that speculators makes the housing market more liquid overall. The top left panel of Figure 3 shows the total share of housing vacancy on the market, defined as $\frac{\mu_{lo}+\mu_{mo}}{s}$. As the measure of speculators increases to 10% the size of regular agents, vacancy rate decreases by about 0.02%. As more speculators are in the market, lo sellers find buyers more frequently. On the other hand, since speculators sell their houses whenever they meet a higher-type buyer *hn*, they lower the average time a vacant house spends on the market. The top right panel of Figure 3 shows this result, defined by 1 over the weighted average of sellers' meeting intensity with buyers. As speculators grow from 0% to 10% of the size of regular agents, the average time of a house on the market decreases from about 0.13 years to 0.09 years. On the other hand, speculators tend to crowd out transactions between regular buyers and sellers. As the bottom panel shows, an increase in speculators increases the share of transaction flow between them and regular buyers *hn* and sellers lo[2], and also reduces the transaction flow between *hn* and lo.

Figure 3: Transaction flow by speculators



The findings above have two different implications to the housing market. First, by increasing the overall search friction for buyers and reducing the search friction for sellers, speculators raise the reservation values of buyers as well as sellers. From the interpretation of the Nash bargaining price coming from equation 11, we know that prices will increase as a result. We call it the "liquidity effect". Second, by increasing trades with speculators and decreasing those with regular agents, speculators also make prices more heavily influenced by speculators' reservation value. We call it the "crowding-out effect". To the extent that speculators normally have lower reservation values than regular buyers and higher reservation values than regular sellers (as we will see later), an increasing presence of speculators in the market will reduce the reservation value of regular buyers and sellers. These opposite directions of the two effects tell us that the overall change in the reservation values of buyers and sellers depends on whether the liquidity effect or the crowing-out effect dominates.

[2] defined as $\frac{\mu_{mo}\mu_{hn}\lambda+\mu_{mn}\mu_{lo}\lambda}{\mu_{mo}\mu_{hn}\lambda+\mu_{mn}\mu_{lo}\lambda+\mu_{hn}\mu_{lo}\lambda}$

With the two potential channels in mind, we can now examine the overall effect of speculation on equilibrium prices.

3.1 $\eta = 1$

By setting $\eta = 1$, we assume that speculators post take-it-or-leave-it offers to regular buyers and sellers, and extract all the gains from trade with them. It is easier to derive analytical solutions in this case, and we show in Appendix D that equilibrium prices are the following:

$$p = \frac{v_H}{r} - \frac{\delta}{r}\frac{v_H - v_L}{r+\delta} + \left[\frac{\delta\lambda}{r(r+\delta)}\mu_{hn}(1-\theta) - \theta\left(\frac{\lambda}{r}\mu_{lo}+1\right)\right]\frac{r(v_H - v_L)}{r(r+\delta)+\lambda\left[\theta(r+\delta)\mu_{lo}+(1-\theta)r\mu_{hn}\right]}$$
$$q_1 = \frac{v_H}{r} - \frac{\delta}{r}\frac{v_H - v_L}{r+\delta} + \left[\frac{\delta}{r}\frac{\lambda}{r+\delta}\mu_{hn}(1-\theta) - \frac{\lambda}{r}\theta\mu_{lo}\right]\frac{r(v_H - v_L)}{r(r+\delta)+\lambda\left[\theta(r+\delta)\mu_{lo}+(1-\theta)r\mu_{hn}\right]}$$
$$q_2 = \frac{v_L}{r} + \frac{\lambda}{r}(1-\theta)\mu_{hn}\frac{r(v_H - v_L)}{r(r+\delta)+\lambda\left[\theta(r+\delta)\mu_{lo}+(1-\theta)r\mu_{hn}\right]}$$

The fractions at the end of each expression are the gains from trade between a regular buyer and a regular seller $(\triangle V_h - \triangle V_l)$. As μ_{lo} decreases with m while μ_{hn} increases with m, the change in the denominator is ambiguous. However, it can be shown that given the parameters we assume, the denominator decreases with m and thus total gains from trade between hn and lo increases. We also tried several other parameters for r and δ, and it seems that the result is not qualitatively sensitive to changes in parameter. Table 1 column 6 shows that indeed, $\triangle V_h - \triangle V_l$ increases with m. On the other hand, the terms multiplying the total gains from trade between hn and lo increases unambiguously with m. Therefore through the analytical expressions we can see that prices should increase with the measure of speculators.

Setting $\eta = 1$ makes it easy for us to interpret the liquidity effect of speculation. Since speculators extract all the gains from trade with them, variations in speculators' willingness to pay does not affect the reservation values of regular buyers and sellers. As mentioned above, the presence of speculators lead to more search frictions for buyers and less search frictions for sellers, thereby increasing the reservation values for both regular buyers and sellers. It is indeed confirmed by Table 1, which shows that the reservation values of both buyers and sellers, namely $\triangle V_h$ and $\triangle V_l$, increase with m.

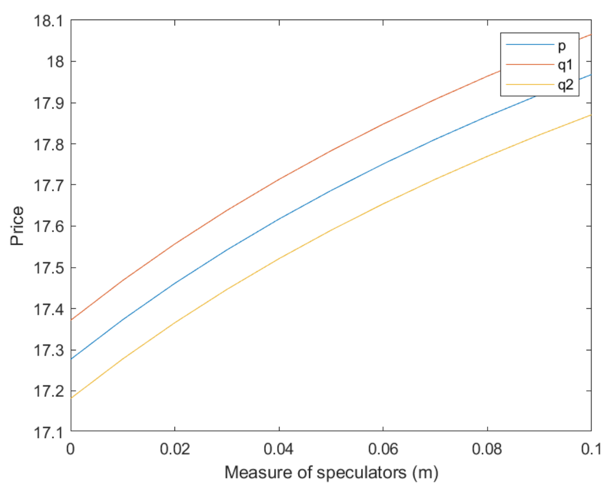
As both the reservation value of sellers and the gains from trade increase with speculators, we can infer from the interpretation of Nash bargaining results that prices should therefore increase. Figure 4 shows the prices at which regular homer buyers purchase from regular sellers (p), speculative buyers purchase from regular sellers (q_2) and speculative sellers sell to regular buyers (q_1) in each steady state equilibrium with different measures of speculators. Indeed, as the measure of speculators rises from 0% to 10% of the size of regular agents, p rises by 4% in steady state,

Table 1: Reservation values and gains from trade

m	$\triangle V_h$	$\triangle V_m$	$\triangle V_l$	$\triangle V_h - \triangle V_m$	$\triangle V_h - \triangle V_l$	$\triangle V_m - \triangle V_l$
0	17.3713	-	17.1816	-	0.1898	-
0.01	17.4681	17.3929	17.2776	0.0752	0.1905	0.1153
0.02	17.5568	17.4814	17.3655	0.0754	0.1913	0.1159
0.03	17.6383	17.5628	17.4464	0.0756	0.1919	0.1164
0.04	17.7136	17.6379	17.5210	0.0757	0.1925	0.1168
0.05	17.7832	17.7074	17.5901	0.0758	0.1931	0.1172
0.06	17.8478	17.7719	17.6543	0.0760	0.1936	0.1176
0.07	17.9080	17.8319	17.7139	0.0761	0.1941	0.1180
0.08	17.9641	17.8879	17.7696	0.0762	0.1945	0.1183
0.09	18.0166	17.9403	17.8217	0.0763	0.1949	0.1186
0.10	18.0658	17.9894	17.8705	0.0764	0.1953	0.1189

confirming our analysis above. A second important observation is that speculators buy at a price lower than p, while sells at a price higher than p, effectively generating a spread in house prices although houses are homogeneous. The stark assumption in this example that $\eta = 1$ means that speculators charge at high type's reservation value while buy at low type's reservation value, and as high types generally have higher reservation values than low types, speculators are able to earn a premium. We can show in the next example that this result doesn't change qualitatively as we change speculators' bargaining power η.

Figure 4: Flipped housing prices



One more useful insight from the figure is that the spread $q_1 - q_2$ that speculators earn is increasing with the measure of speculators. This is consistent with equation 26, which shows that the spread is strictly increasing with $\triangle V_h - \triangle V_l$, the gains from trade between regular home sellers and buyers.

3.2 $\eta = 0.5$

The previous example shows the liquidity effect of speculation. In this example, we examine the crowding-out effect by allowing for speculators' reservation values to affect regular buyers and sellers' reservation values. Although it is difficult to separately show the crowding-out effect, we can still identify it by comparing the equilibrium prices with the case where $\eta = 1$, in which the crowding-out effect is absent. As the rates of change in each type of agent's measure do not vary with η, the distribution of agent types and thus the demand and supply of housing are not different from the previous case. However, a change in speculators' bargaining power η changes the allocation of the gains from trade and should thus affect each type of agents' reservation value. Intuitively, lowering speculators' bargaining power should enable regular agents to extract more gains from trade with speculators, increasing the option value to buy and sell and lowering the reservation values.

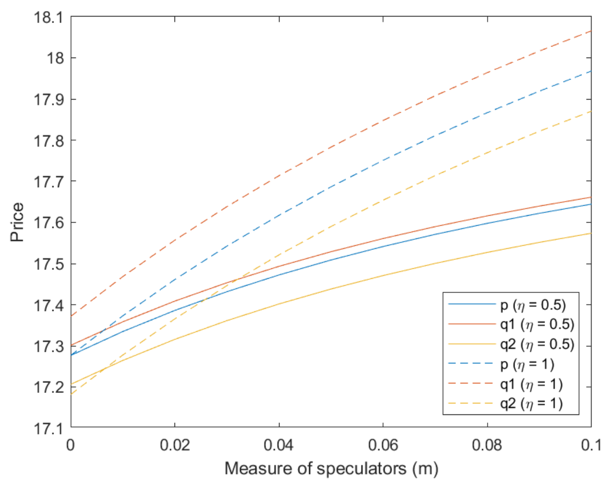
This claim can be confirmed by Table 2, which shows the reservation values for each type. We can see that although the reservation values increase with m, the rates at which they increase are noticeably lower than the case where $\eta = 1$ (Table 1). Given the same measure of m, each type's reservation value is lower than the corresponding values in Table 1, highlighting the crowding-out effect. Another important observation is that in this case, total gains from trade with regular sellers decrease with m, opposite to the case before. An intuitive way to explain this finding is that the decreasing search frictions for sellers as a result of higher m partly mitigate the crowding-out effect for regular low type agents, thereby alleviating the downward impact on $\triangle V_l$. On the other hand, the increasing search frictions for buyers would strengthen the crowding-out effect for buyers, leading to a larger downward impact on $\triangle V_h$.

Table 2: Reservation values and gains from trade

m	$\triangle V_h$	$\triangle V_m$	$\triangle V_l$	$\triangle V_h - \triangle V_m$	$\triangle V_h - \triangle V_l$	$\triangle V_m - \triangle V_l$
0	17.3713	-	17.1816	-	0.1898	-
0.01	17.4286	17.2881	17.2408	0.1405	0.1878	0.0473
0.02	17.4791	17.3384	17.2930	0.1407	0.1860	0.0454
0.03	17.5238	17.3830	17.3395	0.1408	0.1843	0.0436
0.04	17.5637	17.4228	17.3809	0.1409	0.1828	0.0419
0.05	17.5994	17.4584	17.4180	0.1410	0.1813	0.0403
0.06	17.6314	17.4903	17.4515	0.1411	0.1799	0.0389
0.07	17.6604	17.5192	17.4817	0.1411	0.1787	0.0375
0.08	17.6866	17.5454	17.5091	0.1412	0.1775	0.0363
0.09	17.7104	17.5691	17.5341	0.1412	0.1763	0.0351
0.10	17.7321	17.5908	17.5568	0.1413	0.1753	0.0340

Figure 5 shows the prices at each steady state equilibrium with different measures of speculators, as well as comparison with the previous case. The solid lines show the prices under the case where $\eta = 0.5$, and the dashed lines correspond to the case where $\eta = 1$. By comparing the solid and dashed lines with the same color, we can clearly see that the crowding-out effect of speculation, represented by the difference in the dashed and solid lines, becomes more prominent as the measure of speculators increase. The overall effect of speculation on house prices is still positive since the liquidity effect dominates the crowding-out effect, but the increase in transaction prices between regular types goes down to around 2% as speculators rise to 10% of the size of regular agents.

Figure 5: Flipped housing prices



The two examples above helps us to identify two key channels through which speculation affects house prices, namely the liquidity effect and the crowding-out effect. Using reasonable parameters, the model is able to generate patterns in housing market liquidity and prices that are in line with our economic intuition and empirical observations.

4 Conclusion and Future work

In this paper, we extend from the framework of Duffie, Garleanu and Pederson (2005, 2007) to study the effect of house flipping activities on the housing market. This conceptually simple model allows us to capture key empirical observations from previous work about housing speculation. In particular, by comparing steady state equilibria with different measures of speculators, we show that there are two key channels through which housing speculators affect the market: the liquidity effect and the crowding-out effect. By making the market more frictional for buyers and less frictional for sellers, speculators increase the reservation values for both regular sellers and buyers. Secondly, by competing with regular agents, speculators affect their counterparties' reservation values through their own willingness to pay and sell.

Our next step is to use the current model to more clearly quantify the different channels through which the entrant of speculators affect equilibrium prices and volumes. To do this, we need to quantify key variables affecting the equilibrium (e.g. market tightness, illiquidity discount) through a fully calibrated model. We could also use the calibrated model to study the welfare effects of speculative activities in the housing market.

We can also try to generate more meaningful price dynamics and transaction decisions by accounting for more idiosyncrasy in preferences. For example, we can allow regular buyers to have idiosyncratic taste of housing asset. This can take the form of a property-buyer specific match quality in the spirit of Grenesove and Han (2012) and Anenberg and Bayer (2018). In this way we can allow for the identity of marginal buyers and sellers in the market to change with the level of speculative activities and generate more variations in housing dynamics.

Lastly, we also plan to bring in a large-scale individual level housing transaction data to help us enrich the micro foundation about the speculative activities in housing market. Some of the questions we are interested in asking are, for example, (i) what segment of housing market do speculators enter and extract the most profit, (ii) how do speculative activities affect housing transactions by regular home buyers and sellers, and (iii) under what market conditions do speculators generate more positive vs. negative externality to other market participants?

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Appendix

A Proof of Proposition 1

To prove that trades exist between two types, notice that the necessary and sufficient condition is for both sides to have positive gains from trade. Using the results from Nash Bargaining, we know that each side gets a fraction of the total gains from trade. Therefore it suffices to show whether the total gains from trade, if it were to happen, are positive. For trades between *hn* and *lo*, total gains from trade is $\triangle V_h - \triangle V_l$. For trades between *hn* and *mo*, total gains from trade is $\triangle V_h - \triangle V_m$. For trades between *mn* and *lo*, total gains from trade is $\triangle V_m - \triangle V_l$. These 3 types of trades are supposed to happen in equilibrium. On the other hand, suppose there were trades between *ho* and *mn*, then total gains from trade would be $\triangle V_m - \triangle V_h$. Similarly, if trades were to happen between *mo* and *mn*, total gains from trade would be $\triangle V_m - \triangle V_m = 0$.

Therefore, proving which types of trades occur in the equilibrium requires us to show which ones of the previous gains from trade are positive. Showing this information analytically is not currently possible due to the lack of closed-form solutions for the gains from trade. However, we showed from the numerical example in Figure **??** that under reasonable parameters, we have $\triangle V_h > \triangle V_m > \triangle V_l$. This numerical result shows that trades are only possible between *hn* and *mo*, *mn* and *lo*, as well as *hn* and *lo*.

B Derive equilibrium distribution of types

Imposing the steady state condition, then equation 16 implies $\frac{\mu_{hn}}{\mu_{lo}} = \frac{\mu_{mn}}{\mu_{mo}} := w$, and therefore

$$\mu_{hn} = w\mu_{lo} \tag{27}$$

In addition, from 17-19, we have

$$(w+1)\mu_{lo} + \mu_{ho} = 1 \tag{28}$$

$$\mu_{mn} = \frac{mw}{1+w} \tag{29}$$

$$\mu_{mo} = \frac{m}{1+w} \tag{30}$$

$$\mu_{ho} + \mu_{lo} + \frac{m}{1+w} = s \tag{31}$$

Using equation 15 and the equations we derived above, we can obtain

$$Q(\mu_{lo}) \equiv w\lambda\mu_{lo}^2 + \left[\frac{w}{1+w}m\lambda + (1+w)\delta\right]\mu_{lo} - \delta = 0 \tag{32}$$

Given w we can solve for the value of μ_{lo} from equation 32. Since $Q(0) < 0$ and $Q(1) = w\lambda + \frac{w}{1+w}m\lambda + w\delta > 0$, we know there must be a solution in the interval $(0,1)$. The other root is negative and should thus be neglected. Therefore, given w there is a unique solution for $\mu_{lo}(w)$ coming from the system of equations 14-19.

To show there exists a unique solution for w, plug the solution for $\mu_{lo}(w)$, which is an expression of w, back into equations 28 and 31. We then have a system of 2 equations with 2 unknowns: μ_{ho} and w:

$$(w+1)\mu_{lo}(w) + \mu_{ho} = 1$$

$$\mu_{ho} + \mu_{lo}(w) + \frac{m}{1+w} = s$$

We should therefore have a unique solution for w.

Given the unique solution for μ_{lo} and w, we can then back out the whole distribution of types using 27-31.

C Derive equations for total surpluses

Rewrite the value functions and prices as a function of parameter values and distribution of different types

- To get $V_{ho} - V_{lo}$ by $7-8$

$$(V_{ho} - V_{lo})(r+\delta) = v_H - v_L - \lambda\mu_{hn}(1-\theta)(\triangle V_h - \triangle V_l) - \lambda\mu_{mn}(1-\eta)(\triangle V_m - \triangle V_l) \tag{33}$$

- To get $\triangle V_h$ by $7-6$ and 11, 12 and 33

$$r\triangle V_h = v_H - \delta\left(V_{ho}-V_{lo}\right) - \lambda\theta\mu_{lo}\left(\triangle V_h - \triangle V_l\right) - \lambda\left(1-\eta\right)\mu_{mo}\left(\triangle V_h - \triangle V_m\right) \tag{34}$$

- To get $\triangle V_m$ by $10-9$ and 12 and 13

$$r\triangle V_m = v_M + k + \lambda\eta\mu_{hn}\left(\triangle V_h - \triangle V_m\right) - \lambda\eta\mu_{lo}\left(\triangle V_m - \triangle V_l\right) \tag{35}$$

- To get $\triangle V_l$ by 8 and 11 and 13

$$r\triangle V_l = v_L + \lambda\left(1-\theta\right)\mu_{hn}\left(\triangle V_h - \triangle V_l\right) + \lambda\left(1-\eta\right)\mu_{mn}\left(\triangle V_m - \triangle V_l\right) \tag{36}$$

- To get $\triangle V_h - \triangle V_m$ by $34-35$, and substitute in 33 for $\left(V_{ho}-V_{lo}\right)$

$$\begin{aligned}
&\left(\triangle V_h - \triangle V_m\right)\left[r+\lambda\left(\left(1-\eta\right)\mu_{mo}+\eta\mu_{hn}\right)\right] \\
=&\left[v_H - v_M - k - \frac{\delta\left(v_H - v_L\right)}{r+\delta}\right] - \lambda\left[\mu_{lo}\theta - \frac{\delta\mu_{hn}\left(1-\theta\right)}{r+\delta}\right]\left(\triangle V_h - \triangle V_l\right) \\
&-\lambda\left(\mu_{lo}\eta - \frac{\delta\mu_{mn}\left(1-\eta\right)}{r+\delta}\right)\left(\triangle V_m - \triangle V_l\right)
\end{aligned}$$

$$\Longrightarrow \left(\triangle V_h - \triangle V_m\right)D_1 = A_1 + B_1\left(\triangle V_h - \triangle V_l\right) + C_1\left(\triangle V_m - \triangle V_l\right) \tag{37}$$

- To get $\triangle V_m - \triangle V_l$ by $35-36$

$$\begin{aligned}
&\left(\triangle V_m - \triangle V_l\right)\left[r+\lambda\left(\eta\mu_{lo}+\left(1-\eta\right)\mu_{mn}\right)\right] \\
&=\left(v_M - v_L + k\right) + \lambda\eta\mu_{hn}\left(\triangle V_h - \triangle V_m\right) \\
&-\lambda\left(1-\theta\right)\mu_{hn}\left(\triangle V_h - \triangle V_l\right)
\end{aligned}$$

$$\Longrightarrow \left(\triangle V_m - \triangle V_l\right)D_2 = A_2 + B_2\left(\triangle V_h - \triangle V_m\right) + C_2\left(\triangle V_h - \triangle V_l\right) \tag{38}$$

- To get $\triangle V_h - \triangle V_l$ by $34-36$, and substitute in 33

$$
\begin{aligned}
&(\triangle V_h - \triangle V_l)\left[r + \lambda\left(\theta\mu_{lo} + (1-\theta)\frac{r}{r+\delta}\mu_{hn}\right)\right] \\
=&\frac{r(v_H - v_L)}{r+\delta} - \lambda\frac{r\mu_{mn}(1-\eta)}{r+\delta}(\triangle V_m - \triangle V_l) \\
&-\lambda(1-\eta)\mu_{mo}(\triangle V_h - \triangle V_m)
\end{aligned}
$$

$$
\Longrightarrow\ (\triangle V_h - \triangle V_l)D_3 = A_3 + B_3(\triangle V_m - \triangle V_l) + C_3(\triangle V_h - \triangle V_m) \tag{39}
$$

Collecting terms, this is a system of three equations with three unknowns. From equations 37 – 39, we can pin down the unique equilibrium surpluses $\{\triangle V_h - \triangle V_m, \triangle V_m - \triangle V_l, \triangle V_h - \triangle V_l\}$.

$$
\begin{aligned}
(\triangle V_h - \triangle V_m)D_1 &= A_1 + B_1(\triangle V_h - \triangle V_l) + C_1(\triangle V_m - \triangle V_l) \quad (a.8) \\
(\triangle V_m - \triangle V_l)D_2 &= A_2 + B_2(\triangle V_h - \triangle V_m) + C_2(\triangle V_h - \triangle V_l) \quad (a.9) \\
(\triangle V_h - \triangle V_l)D_3 &= A_3 + B_3(\triangle V_m - \triangle V_l) + C_3(\triangle V_h - \triangle V_m) \quad (a.10)
\end{aligned}
$$

After solving for the equilibrium surpluses $\{\triangle V_h - \triangle V_m, \triangle V_m - \triangle V_l, \triangle V_h - \triangle V_l\}$, we can use them to back out equilibrium continuation values $\triangle V_h, \triangle V_m, \triangle V_l$.

D Prices when $\eta = 1$

From 37-39 we have the following set of equations:

$$
\begin{aligned}
(\triangle V_h - \triangle V_m)(r + \lambda\mu_{hn}) &= \left[v_H - v_M - k - \frac{\delta(v_H - v_L)}{r+\delta}\right] - \lambda\left[\mu_{lo}\theta - \frac{\delta\mu_{hn}(1-\theta)}{r+\delta}\right](\triangle \\
(\triangle V_m - \triangle V_l)(r + \lambda\mu_{lo}) &= (v_M - v_L + k) + \lambda\mu_{hn}(\triangle V_h - \triangle V_m) - \lambda(1-\theta)\mu_{hn}(\triangle V_h - \\
(\triangle V_h - \triangle V_l)\left[r + \lambda\left(\theta\mu_{lo} + (1-\theta)\frac{r}{r+\delta}\mu_{hn}\right)\right] &= \frac{r(v_H - v_L)}{r+\delta}
\end{aligned}
$$

and therefore by solving this system of equations we have

$$
\triangle V_h - \triangle V_l = \frac{r(v_H - v_L)}{r(r+\delta) + \lambda[\theta(r+\delta)\mu_{lo} + (1-\theta)r\mu_{hn}]}
$$

From the Nash Bargaining solutions 11-12 as well as equations 34-33 we have

$$
\begin{aligned}
p &= \triangle V_h - \theta\left(\triangle V_h - \triangle V_l\right) \\
&= \frac{v_H}{r} - \frac{\delta}{r}\left(V_{ho} - V_{lo}\right) - \frac{\lambda}{r}\theta\mu_{lo}\left(\triangle V_h - \triangle V_l\right) - \theta\left(\triangle V_h - \triangle V_l\right) \\
&= \frac{v_H}{r} - \frac{\delta}{r}\left[\frac{v_H - v_L}{r+\delta} - \frac{\lambda}{r+\delta}\mu_{hn}\left(1-\theta\right)\left(\triangle V_h - \triangle V_l\right)\right] - \theta\left(\frac{\lambda}{r}\mu_{lo} + 1\right)\left(\triangle V_h - \triangle V_l\right) \\
&= \frac{v_H}{r} - \frac{\delta}{r}\frac{v_H - v_L}{r+\delta} + \left[\frac{\delta\lambda}{r(r+\delta)}\mu_{hn}\left(1-\theta\right) - \theta\left(\frac{\lambda}{r}\mu_{lo} + 1\right)\right]\frac{r\left(v_H - v_L\right)}{r(r+\delta) + \lambda\left[\theta(r+\delta)\mu_{lo} + \left(1-\theta\right)r\mu_{hn}\right]}
\end{aligned}
$$

and

$$
\begin{aligned}
q_2 &= \triangle V_l \\
&= \frac{v_L}{r} + \frac{\lambda}{r}\left(1-\theta\right)\mu_{hn}\left(\triangle V_h - \triangle V_l\right) \\
&= \frac{v_L}{r} + \frac{\lambda}{r}\left(1-\theta\right)\mu_{hn}\frac{r\left(v_H - v_L\right)}{r(r+\delta) + \lambda\left[\theta(r+\delta)\mu_{lo} + \left(1-\theta\right)r\mu_{hn}\right]}
\end{aligned}
$$

and

$$
\begin{aligned}
q_1 &= \triangle V_h \\
&= \frac{v_H}{r} - \frac{\delta}{r}\left(V_{ho} - V_{lo}\right) - \frac{\lambda}{r}\theta\mu_{lo}\left(\triangle V_h - \triangle V_l\right) \\
&= \frac{v_H}{r} - \frac{\delta}{r}\left[\frac{v_H - v_L}{r+\delta} - \frac{\lambda}{r+\delta}\mu_{hn}\left(1-\theta\right)\left(\triangle V_h - \triangle V_l\right)\right] - \frac{\lambda}{r}\theta\mu_{lo}\left(\triangle V_h - \triangle V_l\right) \\
&= \frac{v_H}{r} - \frac{\delta}{r}\frac{v_H - v_L}{r+\delta} + \left[\frac{\delta}{r}\frac{\lambda}{r+\delta}\mu_{hn}\left(1-\theta\right) - \frac{\lambda}{r}\theta\mu_{lo}\right]\frac{r\left(v_H - v_L\right)}{r(r+\delta) + \lambda\left[\theta(r+\delta)\mu_{lo} + \left(1-\theta\right)r\mu_{hn}\right]}
\end{aligned}
$$