# 1 Question 1

## 1.1 a

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{V}) = \partial_l(\eta^{li}\epsilon_{ijk}\partial^j V^k)$$
$$= \partial^i\epsilon_{ijk}\partial^j V^k = \epsilon_{ijk}\partial^i\partial^j V^k$$
$$= \frac{1}{2}(\epsilon_{ijk}\partial^i\partial^j V^k + \epsilon_{jik}\partial^j\partial^i V^k)$$
$$= \frac{1}{2}(\epsilon_{ijk}\partial^i\partial^j V^k - \epsilon_{ijk}\partial^j\partial^i V^k)$$
$$= \frac{1}{2}(\epsilon_{ijk}\partial^i\partial^j V^k - \epsilon_{ijk}\partial^i\partial^j V^k) = 0 \quad \square$$

## 1.2 b

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{V}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{V}) - \vec{\nabla}^2\vec{V}$$
$$\epsilon_{ijk}\partial_j(\epsilon_{klm}\partial_l v_m) = \partial_i(\partial_j v_j) - \partial_j\partial_j v_i$$
$$\epsilon_{ijk}\epsilon_{klm}\partial_j\partial_l v_m = \partial_j\partial_i v_j - \partial_j\partial_j v_i$$
$$(\delta_{il}\delta jm - \delta im\delta jl)\partial_j\partial_l v_m = \partial_j\partial_i v_j - \partial_j\partial_j v_i$$
$$\partial_j\partial_i v_j - \partial_j\partial_j v_i = \partial_j\partial_i v_j - \partial_j\partial_j v_i \quad \square$$

# 2 Question 2

for $r \neq 0$

$$\nabla^2\left(\frac{1}{r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial r^{-1}}{\partial r}\right) = 0$$

for $r = 0$

$$\int \nabla^2 r^{-1} dr^3 = \oint \nabla r^{-1} dS$$

where $\nabla r^{-1} = \frac{-\hat{r}}{r^2}$ and $dS = \hat{r}r^2 d\Omega$

$$\oint \nabla r^{-1}\cdot dS = -\int dr - 4\pi$$

thus $r = -4\pi\delta^3(\vec{r})$

## 3 Question 3

$$\nabla \cdot \vec{E} = \partial_i E_i$$

$$0 = (\nabla \times E) = \epsilon_{ijk}\partial_j E_k = -\epsilon_{ijk}\partial_j \partial_k V$$

$$\nabla^2 V \iff \partial_i \partial_i V = -\partial_i E_i = \frac{-\rho}{\epsilon_0}$$

$$\nabla^2 V = \frac{-\rho}{\epsilon_0}$$

when $(\nabla \times \vec{E}) = 0 \qquad \square$

## 4 Question 4

Given a point charge in the corner of a cube, and we are supposed to calculate the flux of $\vec{E}$ going through the opposite facing side as given in the problem sheet. We can construct a larger cube consisting of 8 smaller cubers, and given gauses law, the total flux of the charge is equal to the flux passing through the 24 external faces of the 8 cubes.

$$\phi = \frac{q}{24\epsilon_0}$$

## 5 Question 5

$$F_m = q(\vec{v} \times \vec{B})$$

$$W = \int f dr = \int F \vec{v} dt$$

$$P := \frac{dW}{dt} = F \cdot v = q(\vec{v} \times \vec{B}) \cdot v = 0$$

## 6 Question 6

Deriving the continuity equation starting with inhomogenous maxwell equation

$$\partial_\mu F^{\mu\nu} = \mu_0 J^\nu$$

$$\partial_\nu(\partial_\mu F^{\mu\nu}) = \mu_0 \partial_\nu J^\nu$$

but $F^{\mu\nu}$ is anti symmetric under exchange of $\mu \iff \nu$ and $\partial_\nu \partial_\mu$ is symmetric under exchange of $\mu \iff \nu$. hence it must be true that $\partial_\nu \partial_\mu F^{\mu\nu} = 0$. thus

$$\partial_\nu J^\nu = 0$$

$$\partial_\nu J^\nu = \partial_0 J^0 + \partial_i J^i = 0$$

$$\frac{1}{c}\frac{\partial(c\rho)}{\partial t} + \nabla \cdot \vec{J} = 0$$

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

# 7 Question 7

$$[\vec{E}] = \frac{V}{m}$$

$$[V] = kgm^2 s{-}3A^{-1}$$

$$[\vec{E}] = kgms^{-3}A^{-1}$$

$$[\vec{B}] = kgs^{-2}A^{-1}$$

where $[m] = [s] = E^{-1}$, $[kg] = E^{+1}$, and $[I] = [t^{-1}] = E^{+1}$, so

$$[\vec{E}] = (E)(E^{-1})(E^3)(E^{-1}) = E^2$$

$$[\vec{B}] = (E)(E^2)(E^{-1}) = E^2$$

# 8 Question 8

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$[F] = kg^{-1}m^{-2}s^4A^2$$

$$[H] = kgm^2s^{-2}A^{-2}$$

$$[\epsilon_0] = Fm^{-1} = kg^{-1}m^{-3}s^4A^2$$

$$[\mu_0] = Hm^{-1} = kgms^{-2}A^{-2}$$

$$[c] = [\frac{1}{\sqrt{\epsilon_0\mu_0}}] = ((kg^{-1}m^{-3}s^4A^2)(kgms^{-2}A^{-2}))^{\frac{-1}{2}} =$$

$$= (m^{-2}s^2)^{\frac{-1}{2}} = ms^{-1}$$