

1 Question 1

1.1 a

$$\begin{aligned}
\vec{\nabla} \cdot (\vec{\nabla} \times \vec{V}) &= \partial_l (\eta^{li} \epsilon_{ijk} \partial^j V^k) \\
&= \partial^i \epsilon_{ijk} \partial^j V^k = \epsilon_{ijk} \partial^i \partial^j V^k \\
&= \frac{1}{2} (\epsilon_{ijk} \partial^i \partial^j V^k + \epsilon_{jik} \partial^j \partial^i V^k) \\
&= \frac{1}{2} (\epsilon_{ijk} \partial^i \partial^j V^k - \epsilon_{ijk} \partial^j \partial^i V^k) \\
&= \frac{1}{2} (\epsilon_{ijk} \partial^i \partial^j V^k - \epsilon_{ijk} \partial^i \partial^j V^k) = 0 \quad \square
\end{aligned}$$

1.2 b

$$\begin{aligned}
\vec{\nabla} \times (\vec{\nabla} \times \vec{V}) &= \vec{\nabla} (\vec{\nabla} \cdot \vec{V}) - \vec{\nabla}^2 \vec{V} \\
\epsilon_{ijk} \partial_j (\epsilon_{klm} \partial_l v_m) &= \partial_i (\partial_j v_j) - \partial_j \partial_j v_i \\
\epsilon_{ijk} \epsilon_{klm} \partial_j \partial_l v_m &= \partial_j \partial_i v_j - \partial_j \partial_j v_i \\
(\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) \partial_j \partial_l v_m &= \partial_j \partial_i v_j - \partial_j \partial_j v_i \\
\partial_j \partial_i v_j - \partial_j \partial_j v_i &= \partial_j \partial_i v_j - \partial_j \partial_j v_i \quad \square
\end{aligned}$$

2 Question 2

for $r \neq 0$

$$\nabla^2 \left(\frac{1}{r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial r^{-1}}{\partial r} \right) = 0$$

for $r = 0$

$$\int \nabla^2 r^{-1} dr^3 = \oint \nabla r^{-1} dS$$

where $\nabla r^{-1} = \frac{-\hat{r}}{r^2}$ and $dS = \hat{r} r^2 d\Omega$

$$\oint \nabla r^{-1} \cdot dS = - \int dr = -4\pi$$

thus $r = -4\pi \delta^3(\vec{r})$

3 Question 3

$$\nabla \cdot \vec{E} = \partial_i E_i$$

$$0 = (\nabla \times E) = \epsilon_{ijk} \partial_j E_k = -\epsilon_{ijk} \partial_j \partial_k V$$

$$\nabla^2 V \iff \partial_i \partial_i V = -\partial_i E_i = \frac{-\rho}{\epsilon_0}$$

$$\nabla^2 V = \frac{-\rho}{\epsilon_0}$$

when $(\nabla \times \vec{E}) = 0$ \square

4 Question 4

Given a point charge in the corner of a cube, and we are supposed to calculate the flux of \vec{E} going through the opposite facing side as given in the problem sheet. We can construct a larger cube consisting of 8 smaller cubers, and given gauges law, the total flux of the charge is equal to the flux passing through the 24 external faces of the 8 cubes.

$$\phi = \frac{q}{24\epsilon_0}$$

5 Question 5

$$F_m = q(\vec{v} \times \vec{B})$$

$$W = \int f dr = \int F \vec{v} dt$$

$$P := \frac{dW}{dt} = F \cdot v = q(\vec{v} \times \vec{B}) \cdot v = 0$$

6 Question 6

Deriving the continuity equation starting with inhomogenous maxwell equation

$$\partial_\mu F^{\mu\nu} = \mu_0 J^\nu$$

$$\partial_\nu (\partial_\mu F^{\mu\nu}) = \mu_0 \partial_\nu J^\nu$$

but $F^{\mu\nu}$ is anti symmetric under exchange of $\mu \iff \nu$ and $\partial_\nu \partial_\mu$ is symmetric under exchange of $\mu \iff \nu$. hence it must be true that $\partial_\nu \partial_\mu F^{\mu\nu} = 0$. thus

$$\partial_\nu J^\nu = 0$$

$$\partial_\nu J^\nu = \partial_0 J^0 + \partial_i J^i = 0$$

$$\frac{1}{c} \frac{\partial(c\rho)}{\partial t} + \nabla \cdot \vec{J} = 0$$

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

7 Question 7

$$[\vec{E}] = \frac{V}{m}$$

$$[V] = kgm^2s^{-3}A^{-1}$$

$$[\vec{E}] = kgms^{-3}A^{-1}$$

$$[\vec{B}] = kgs^{-2}A^{-1}$$

where $[m] = [s] = E^{-1}$, $[kg] = E^{+1}$, and $[I] = [t^{-1}] = E^{+1}$, so

$$[\vec{E}] = (E)(E^{-1})(E^3)(E^{-1}) = E^2$$

$$[\vec{B}] = (E)(E^2)(E^{-1}) = E^2$$

8 Question 8

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$[F] = kg^{-1}m^{-2}s^4A^2$$

$$[H] = kgm^2s^{-2}A^{-2}$$

$$[\epsilon_0] = Fm^{-1} = kg^{-1}m^{-3}s^4A^2$$

$$[\mu_0] = Hm^{-1} = kgms^{-2}A^{-2}$$

$$\begin{aligned} [c] &= \left[\frac{1}{\sqrt{\epsilon_0 \mu_0}} \right] = ((kg^{-1}m^{-3}s^4A^2)(kgms^{-2}A^{-2}))^{\frac{-1}{2}} = \\ &= (m^{-2}s^2)^{\frac{-1}{2}} = ms^{-1} \end{aligned}$$