Independently and Identically Distributed

λ_{IID}

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Syntax

TODO

Static Semantics

TODO

Dynamic Semantic

Small-Step Semantics

$$\frac{\langle e,\, n,\, m\rangle \to \langle e',\, n',\, m'\rangle}{\langle E(e),\, n,\, m\rangle \to \langle E(e'),\, n',\, m'\rangle}\ \textsc{Context}$$

$$\frac{}{\langle(\mathbf{case}\ \mathbf{inl}_{\tau_1+\tau_2} e\ \mathbf{of}\ e_2 \,|\, e_3),\, n,\, m\rangle \to \langle e_2\, e,\, n,\, m\rangle}\ \textsc{Case-Left}$$

$$\frac{}{\langle(\lambda x:\tau.\, e)\, e_2,\, n,\, m\rangle \to \langle e\{e_2/x\},\, n,\, m\rangle}\ \beta\textsc{-Reduction}$$

$$\frac{}{\langle(\mathbf{case}\ \mathbf{inr}_{\tau_1+\tau_2} e\ \mathbf{of}\ e_2 \,|\, e_3),\, n,\, m\rangle \to \langle e_3\, e,\, n,\, m\rangle}\ \textsc{Case-Right}$$

$$\frac{}{\langle\mathbf{let}\ x = e_1\ \mathbf{in}\ e_2,\, n,\, m\rangle \to \langle e_2\{e_1/x\},\, n,\, m\rangle}\ \textsc{Let}$$

$$\frac{}{\langle\mathbf{\#1}\ (e_1, e_2),\, n,\, m\rangle \to \langle e_1,\, n,\, m\rangle}\ \textsc{Proj-1}$$

$$\frac{r_1 \bar{\oplus} r_2 = r}{\langle r_1 \oplus r_2,\, n,\, m\rangle \to \langle r,\, n,\, m\rangle}\ \textsc{Bop}$$

$$\frac{}{\langle\mathbf{\#2}\ (e_1, e_2),\, n,\, m\rangle \to \langle e_2,\, n,\, m\rangle}\ \textsc{Proj-2}$$

$$\frac{}{\langle\mathbf{coin},\, n,\, m\rangle \to \langle\mathbf{hd}\ n,\, \mathbf{tl}\ n,\, m\rangle}\ \textsc{Coin}$$

$$\frac{}{\langle\mathbf{rand},\, n,\, m\rangle \to \langle\mathbf{hd}\ m,\, n,\, \mathbf{tl}\ m\rangle}\ \textsc{Rand}$$

$$\frac{}{\langle v_1\ \mathbf{to}\ x\ \mathbf{in}\ e_2, n, m\rangle \to \langle e_2\{v_1/x\}, n, m\rangle}\ \textsc{To-pc}$$

Big-Step Semantics

$$\frac{\langle e_1, n, m\rangle \Downarrow \langle \lambda x.\, e_1', n', m'\rangle \qquad \langle e_1'\{e_2/x\}, n', m'\rangle \Downarrow \langle v, n'', m''\rangle}{\langle e_1\, e_2,\, n,\, m\rangle \Downarrow \langle v,\, n'',\, m''\rangle}\ \beta\textsc{-Reduction}$$

$$\frac{\langle e_2\{e_1/x\}, n, m\rangle \Downarrow \langle v, n', m'\rangle}{\langle\mathbf{let}\ x = e_1\ \mathbf{in}\ e_2,\, n,\, m\rangle \Downarrow \langle v,\, n',\, m'\rangle}\ \textsc{Let}$$

$$\frac{\langle e_1, n, m\rangle \Downarrow \langle r_1, n', m'\rangle \qquad \langle e_2, n', m'\rangle \Downarrow \langle r_2, n'', m''\rangle \qquad r_1 \bar{\oplus} r_2 = r}{\langle e_1 \oplus e_2,\, n,\, m\rangle \Downarrow \langle r,\, n'',\, m''\rangle}\ \textsc{Bop}$$

$$\frac{}{\langle\mathbf{coin},\, n,\, m\rangle \Downarrow \langle\mathbf{hd}\ n,\, \mathbf{tl}\ n,\, m\rangle}\ \textsc{Coin}$$

$$\frac{}{\langle\mathbf{rand},\, n,\, m\rangle \Downarrow \langle\mathbf{hd}\ m,\, n,\, \mathbf{tl}\ m\rangle}\ \textsc{Rand}$$

$$\frac{\langle e_2\, e_1, n, m\rangle \Downarrow \langle v, n', m'\rangle}{\langle(\mathbf{case}\ \mathbf{inl}_{\tau_1+\tau_2} e_1\ \mathbf{of}\ e_2 \,|\, e_3),\, n,\, m\rangle \Downarrow \langle v,\, n',\, m'\rangle}\ \textsc{Case-Left}$$

$$\frac{\langle e_3\, e_1, n, m\rangle \Downarrow \langle v, n', m'\rangle}{\langle(\textbf{case inr}_{\tau_1+\tau_2} e_1 \textbf{ of } e_2 \mid e_3),\, n,\, m\rangle \Downarrow \langle v,\, n',\, m'\rangle}\ \textsc{Case-Left}$$

$$\frac{\langle e_1, n, m\rangle \Downarrow \langle v, n', m'\rangle}{\langle \textbf{\#1}\ (e_1, e_2),\, n,\, m\rangle \Downarrow \langle v, n', m'\rangle}\ \textsc{Proj-1}$$

$$\frac{\langle e_2, n, m\rangle \Downarrow \langle v, n', m'\rangle}{\langle \textbf{\#2}\ (e_1, e_2),\, n,\, m\rangle \Downarrow \langle v, n', m'\rangle}\ \textsc{Proj-2}$$

$$\frac{\langle e_2\{(\lambda y.\, e_1)()/x\}, n, m\rangle \Downarrow \langle v, n', m'\rangle}{\langle e_1 \textbf{ to } x \textbf{ in } e_2, n, m\rangle \Downarrow \langle v, n', m'\rangle}\ \textsc{To-pc}$$

Denotational Semantics

$$\begin{aligned}
[\![r]\!] &\triangleq \delta_r \\
[\![\lambda x : \tau.\, e]\!] &\triangleq x \mapsto [\![e]\!] \\
[\![\textbf{let } x = e_1 \textbf{ in } e_2]\!] &\triangleq [\![e_2\{e_1/x\}]\!] \\
[\![e_1\ e_2]\!] &\triangleq [\![e_1]\!] \circ [\![e_2]\!] \\
[\![\textbf{coin}]\!] &\triangleq \frac{1}{2}(\delta_0 + \delta_1) \\
[\![\textbf{rand}]\!] &\triangleq \lambda
\end{aligned}$$

Translation to CBPV

$$\begin{aligned}
\mathcal{T}[\![x]\!] &\triangleq \textbf{force } x \\
\mathcal{T}[\![\lambda x.\, e]\!] &\triangleq \lambda x.\, [\![e]\!] \\
\mathcal{T}[\![\textbf{let} x = e_1 \textbf{ in } e_2]\!] &\triangleq \textbf{let } x \textbf{ be } \mathcal{T}[\![e_1]\!].\, \mathcal{T}[\![e_2]\!] \\
\mathcal{T}[\![e_1\, e_2]\!] &\triangleq (\textbf{thunk}\mathcal{T}[\![e_2]\!])\text{`}\, \mathcal{T}[\![e_1]\!] \\
\mathcal{T}[\![\textbf{coin}]\!] &\triangleq \textbf{coin} \\
\mathcal{T}[\![\textbf{rand}]\!] &\triangleq \textbf{rand} \\
\mathcal{T}[\![\textbf{inl}_{\tau_1+\tau_2} e]\!] &\triangleq \textbf{produce inl thunk } \mathcal{T}[\![e]\!] \\
\mathcal{T}[\![\textbf{inr}_{\tau_1+\tau_2} e]\!] &\triangleq \textbf{produce inr thunk } \mathcal{T}[\![e]\!] \\
\mathcal{T}[\![\textbf{case } e_1 \textbf{of } e_2 | e_3]\!] &\triangleq \mathcal{T}[\![e_1]\!] \textbf{ to } z.\ \textbf{pm } z \textbf{ as } \{\textbf{inl } x.\mathcal{T}[\![e_2]\!],\ \textbf{inr } x.\mathcal{T}[\![e_3]\!]\} \\
\mathcal{T}[\![(e_1, e_2)]\!] &\triangleq \\
\mathcal{T}[\![\textbf{\#1 } e]\!] &\triangleq \\
\mathcal{T}[\![\textbf{\#2 } e]\!] &\triangleq \\
\mathcal{T}[\![e_1 \textbf{ to } x \textbf{ in } e2]\!] &\triangleq
\end{aligned}$$