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# Independently and Identically Distributed

$\lambda_{IID}$

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Syntax

TODO

Static Semantics

TODO

Dynamic Semantic

Small-Step Semantics

$$\frac{\langle e, n, m \rangle \rightarrow \langle e', n', m' \rangle}{\langle E(e), n, m \rangle \rightarrow \langle E(e'), n', m' \rangle} \text{CONTEXT}$$

$$\frac{}{\langle \langle \text{case inl}_{\tau_1 + \tau_2} e \text{ of } e_2 \mid e_3 \rangle, n, m \rangle \rightarrow \langle e_2 e, n, m \rangle} \text{CASE-LEFT}$$

$$\frac{}{\langle (\lambda x : \tau. e) e_2, n, m \rangle \rightarrow \langle e\{e_2/x\}, n, m \rangle} \beta\text{-REDUCTION}$$

$$\frac{}{\langle \langle \text{case inr}_{\tau_1 + \tau_2} e \text{ of } e_2 \mid e_3 \rangle, n, m \rangle \rightarrow \langle e_3 e, n, m \rangle} \text{CASE-RIGHT}$$

$$\frac{}{\langle \text{let } x = e_1 \text{ in } e_2, n, m \rangle \rightarrow \langle e_2\{e_1/x\}, n, m \rangle} \text{LET}$$

$$\frac{r_1 \bar{\oplus} r_2 = r}{\langle r_1 \oplus r_2, n, m \rangle \rightarrow \langle r, n, m \rangle} \text{BOP}$$

$$\frac{}{\langle \#1 (e_1, e_2), n, m \rangle \rightarrow \langle e_1, n, m \rangle} \text{PROJ-1}$$

$$\frac{}{\langle \text{coin}, n, m \rangle \rightarrow \langle \text{hd } n, \text{tl } n, m \rangle} \text{COIN}$$

$$\frac{}{\langle \#2 (e_1, e_2), n, m \rangle \rightarrow \langle e_2, n, m \rangle} \text{PROJ-2}$$

$$\frac{}{\langle \text{rand}, n, m \rangle \rightarrow \langle \text{hd } m, n, \text{tl } m \rangle} \text{RAND}$$

$$\frac{}{\langle v_1 \text{ to } x \text{ in } e_2, n, m \rangle \rightarrow \langle e_2\{v_1/x\}, n, m \rangle} \text{TO-PC}$$

Big-Step Semantics

$$\frac{\langle e_1, n, m \rangle \Downarrow \langle \lambda x. e'_1, n', m' \rangle \quad \langle e'_1\{e_2/x\}, n', m' \rangle \Downarrow \langle v, n'', m'' \rangle}{\langle e_1 e_2, n, m \rangle \Downarrow \langle v, n'', m'' \rangle} \beta\text{-REDUCTION}$$

$$\frac{\langle e_2\{e_1/x\}, n, m \rangle \Downarrow \langle v, n', m' \rangle}{\langle \text{let } x = e_1 \text{ in } e_2, n, m \rangle \Downarrow \langle v, n', m' \rangle} \text{LET}$$

$$\frac{\langle e_1, n, m \rangle \Downarrow \langle r_1, n', m' \rangle \quad \langle e_2, n', m' \rangle \Downarrow \langle r_2, n'', m'' \rangle \quad r_1 \bar{\oplus} r_2 = r}{\langle e_1 \oplus e_2, n, m \rangle \Downarrow \langle r, n'', m'' \rangle} \text{BOP}$$

$$\frac{}{\langle \text{coin}, n, m \rangle \Downarrow \langle \text{hd } n, \text{tl } n, m \rangle} \text{COIN}$$

$$\frac{}{\langle \text{rand}, n, m \rangle \Downarrow \langle \text{hd } m, n, \text{tl } m \rangle} \text{RAND}$$

$$\frac{\langle e_2 e_1, n, m \rangle \Downarrow \langle v, n', m' \rangle}{\langle \langle \text{case inl}_{\tau_1 + \tau_2} e_1 \text{ of } e_2 \mid e_3 \rangle, n, m \rangle \Downarrow \langle v, n', m' \rangle} \text{CASE-LEFT}$$

$$\begin{array}{c}
\frac{\langle e_3 \ e_1, n, m \rangle \Downarrow \langle v, n', m' \rangle}{\langle (\text{case inr}_{\tau_1 + \tau_2} e_1 \text{ of } e_2 \mid e_3), n, m \rangle \Downarrow \langle v, n', m' \rangle} \text{CASE-LEFT} \\
\\
\frac{\langle e_1, n, m \rangle \Downarrow \langle v, n', m' \rangle}{\langle \#1 \ (e_1, e_2), n, m \rangle \Downarrow \langle v, n', m' \rangle} \text{PROJ-1} \\
\\
\frac{\langle e_2, n, m \rangle \Downarrow \langle v, n', m' \rangle}{\langle \#2 \ (e_1, e_2), n, m \rangle \Downarrow \langle v, n', m' \rangle} \text{PROJ-2} \\
\\
\frac{\langle e_2 \{ (\lambda y. e_1)() / x \}, n, m \rangle \Downarrow \langle v, n', m' \rangle}{\langle e_1 \text{ to } x \text{ in } e_2, n, m \rangle \Downarrow \langle v, n', m' \rangle} \text{TO-PC}
\end{array}$$

## Denotational Semantics

$$\begin{aligned}
\llbracket r \rrbracket &\triangleq \delta_r \\
\llbracket \lambda x : \tau. e \rrbracket &\triangleq x \mapsto \llbracket e \rrbracket \\
\llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket &\triangleq \llbracket e_2 \{ e_1 / x \} \rrbracket \\
\llbracket e_1 \ e_2 \rrbracket &\triangleq \llbracket e_1 \rrbracket \circ \llbracket e_2 \rrbracket \\
\llbracket \text{coin} \rrbracket &\triangleq \frac{1}{2}(\delta_0 + \delta_1) \\
\llbracket \text{rand} \rrbracket &\triangleq \lambda
\end{aligned}$$

## Translation to CBPV

$$\begin{aligned}
\mathcal{T} \llbracket x \rrbracket &\triangleq \text{force } x \\
\mathcal{T} \llbracket \lambda x. e \rrbracket &\triangleq \lambda x. \llbracket e \rrbracket \\
\mathcal{T} \llbracket \text{let } x = e_1 \text{ in } e_2 \rrbracket &\triangleq \text{let } x \text{ be } \mathcal{T} \llbracket e_1 \rrbracket. \mathcal{T} \llbracket e_2 \rrbracket \\
\mathcal{T} \llbracket e_1 \ e_2 \rrbracket &\triangleq (\text{thunk } \mathcal{T} \llbracket e_2 \rrbracket) \cdot \mathcal{T} \llbracket e_1 \rrbracket \\
\mathcal{T} \llbracket \text{coin} \rrbracket &\triangleq \\
\mathcal{T} \llbracket \text{rand} \rrbracket &\triangleq \\
\mathcal{T} \llbracket \text{inl}_{\tau_1 + \tau_2} e \rrbracket &\triangleq \text{produce inl thunk } \mathcal{T} \llbracket e \rrbracket \\
\mathcal{T} \llbracket \text{inr}_{\tau_1 + \tau_2} e \rrbracket &\triangleq \text{produce inr thunk } \mathcal{T} \llbracket e \rrbracket \\
\mathcal{T} \llbracket \text{case } e_1 \text{ of } e_2 \mid e_3 \rrbracket &\triangleq \\
\mathcal{T} \llbracket (e_1, e_2) \rrbracket &\triangleq \\
\mathcal{T} \llbracket \#1 \ e \rrbracket &\triangleq \\
\mathcal{T} \llbracket \#2 \ e \rrbracket &\triangleq \\
\mathcal{T} \llbracket e_1 \text{ to } x \text{ in } e_2 \rrbracket &\triangleq
\end{aligned}$$