Question 1

Under such condition, the price of the derivative will be 75 dollars. I will assume the derivative is a perpetual call or nothing option. Let's set the price of the derivation as p. At t = 0, I will short a contact of the option at price p and use the amount of p to buy p/750 shares of the Tesla. When the price of the Tesla stock hit 1000 dollars, I will sell p/750 shares of the Tesla at 1000 * p/750 = (4/3) *p. At the same time, I need to pay 100 dollars payoff for the long position party. Under no-arbitrage condition, (4/3) *p should be equal to 100 dollars. Thus, p is 75 dollars. Therefore, the price of the call or nothing digital option should be 75 dollars.

Question 2

For the range of x, let's use butterfly constraint to find:

$$-5 + 2 \times x - 10 \leq 0$$
$$x \leq 7.5$$

By using calendar spread, the price of put option with same maturity 1 year but a higher strike price will be more expensive than that of a lower strike price. Hence,

$$5 \leq x \leq 7.5$$

For the range of y, let's use butterfly constraint to find:

$$-y + 2 \times 9 - 12 \leq 0$$
$$y \geq 6$$

By using calendar spread, the value of y should be smaller or equal to 9. Hence,

$$6 \leq y \leq 9$$

Question 3

$$\sum 1 = 0.3, \sum 2 = 0.2, S_0 = 100, q = 0\%, r_f = 0\%.$$

For the 1y call option, we have:

$$d1 = \frac{\ln(1) + \frac{1}{2} \times \left(\sum 1\right)^2 \times 1}{\sum 1 \times \sqrt{1}} = 0.15.$$

$$d2 = d1 - \sum 1 \times \sqrt{1} = -0.15$$

$$N(d1) = 0.5596$$
$$N(d2) = 0.4403$$

So, for the 1y call option, the price will be:

$$C_1 = Z_0(1) \times (F_0(1) \times N(d1) - K \times N(d2)$$
$$C_1 = 1 \times (100 \times 0.5596 - 100 \times 0.4403)$$
$$C_1 = \$11.92$$

For the 2y call option, we have:

$$d1 = \frac{\ln(1) + \frac{1}{2} \times (\sum 2)^2 \times 2}{\sum 2 \times \sqrt{2}} = 0.1414$$

$$d2 = d1 - \sum 2 \times \sqrt{2} = -0.1414$$

$$N(d1) = 0.5562$$
$$N(d2) = 0.4437$$

$$C_2 = Z_0(1) \times (F_0(1) \times N(d1) - K \times N(d2)$$
$$C_2 = 1 \times (100 \times 0.5562 - 100 \times 0.4437)$$
$$C_2 = \$11.25$$

As we know, the call option with the same underlying and strike price but different maturity should have different prices. The option with longer maturity having higher time value should have a higher market price. However, in our case, the 2y call option is pricing \$11.25 which is lower than the 1y call option \$11.92. Hence, there is arbitrage opportunity.

We should short the C_1 and long C_2. By setting the C_2 price to be \$11.92.

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In [15]: import math
import numpy as np
from scipy.optimize import newton
from math import log, sqrt
from scipy.linalg import norm
def cnorm(x):
    return (1.0 + math.erf(x / math.sqrt(2.0))) / 2.0
# Define the option pricing model (Black-Scholes)
def bs_call(s, k, t, r, sigma):
    d1 = (log(s/k) + (r + 0.5 * sigma**2) * t) / (sigma * sqrt(t))
    d2 = d1 - sigma * sqrt(t)
    return s * cnorm(d1) - k * np.exp(-r * t) * cnorm(d2)

# lets set the market price be 11.92

# Define the function to find the implied volatility
def find_implied_vol(market_price, s, k, t, r):
    def func(sigma):
        return bs_call(s, k, t, r, sigma) - market_price
    return newton(func, x0=0.2) # x0 is the initial guess of volatility

# Example usage
market_price = 11.92
s = 100
k = 100
t = 2
r = 0
implied_vol = find_implied_vol(market_price, s, k, t, r)
print("Implied volatility: ", implied_vol)

Implied volatility:  0.2120686071905006
```

So, the minimum $\sum 2_{min} 0.2121$.