Parallel Graph Coloring in Shared Memory

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Outline

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2. Project Description
3. Implementation
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    b. Collision Computation
4. Evaluation
    a. Data Movement
    b. Distance 1
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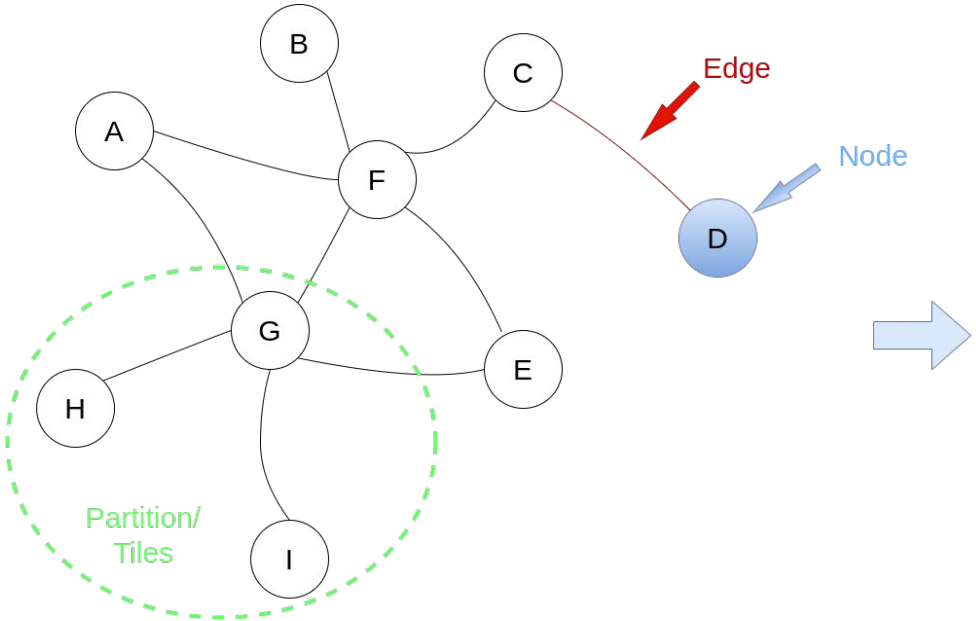
Terminology



	A	B	C	D	E	F	G	H	I
A	1	0	0	0	0	1	1	0	0
B	0	1	0	0	0	1	0	0	0
C	0	0	1	1	0	1	0	0	0
D	0	0	1	1	0	0	0	0	0
E	0	0	0	0	1	1	1	0	0
F	1	1	1	0	1	1	1	0	0
G	1	0	0	0	1	1	1	1	1
H	0	0	0	0	0	0	1	1	0
I	0	0	0	0	0	0	1	0	1

Project Description

- Max & Total collisions for Distance 1 & Distance 2 coloring
- Different hash functions for coloring
- Performance & Quality evaluation
- Comparison with cusparseDcsrcolor() and old ASC project

Tiling

- No need to access elements in different partitions
- Permutation not needed
- Only row pointer important
- Sequential

➜ Simply create partitions with constraint on maximum size

How size of partition is calculated may differ

Row_Ptr

0	5	9	13	16	20	...

Adjacent_Difference

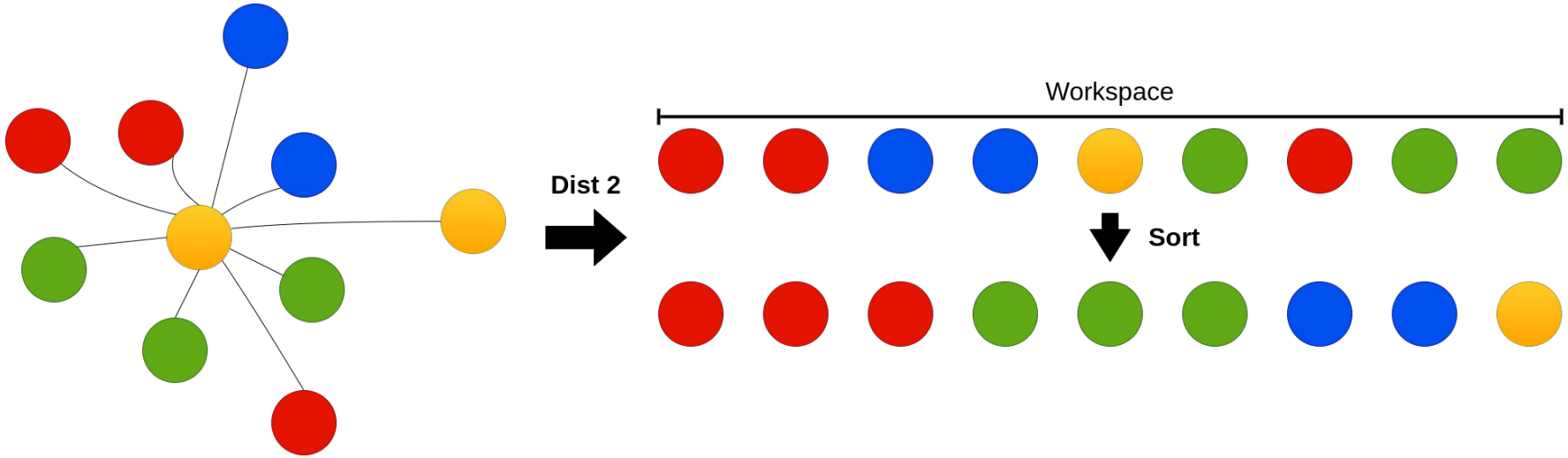
0	5	4	4	3	4	...

```
tile_start ← 0
tile_cols ← 0
i ← 1
while i < len(Row_Ptr) do
    tile_rows ← i − tile_start
    tile_cols ← tile_cols + Adjacent_Difference[i]

    tile_size ← foo(tile_rows, tile_cols)

    if tile_size > shared_mem_size then
        set_tile_start(i);
        tile_cols ← 0
    end if
    i ← i + 1
end while
```

Collision Counting

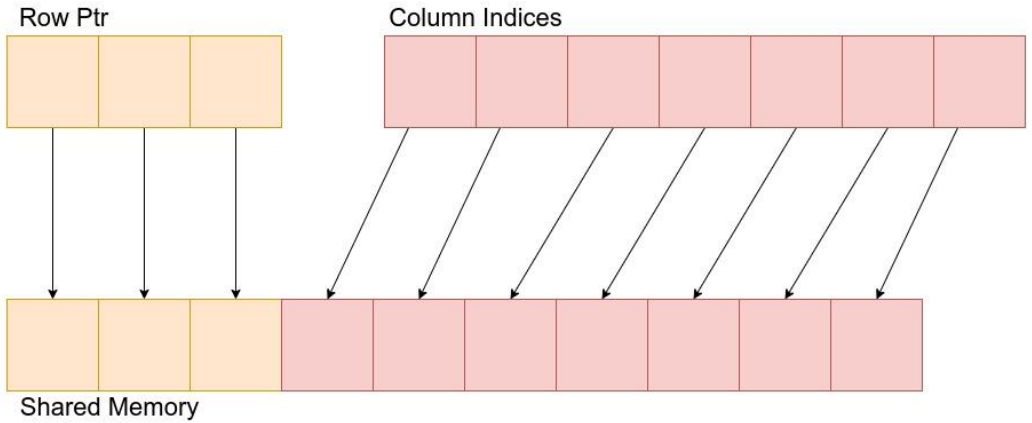


Distance 1:
1 Collision

Distance 2:
3 Collisions -> highest number of appearances

Shared Memory Load

- Coalesced load of both Row Ptr and Column Indices
- Possibly unaligned
- Barrier sync. after shared memory load



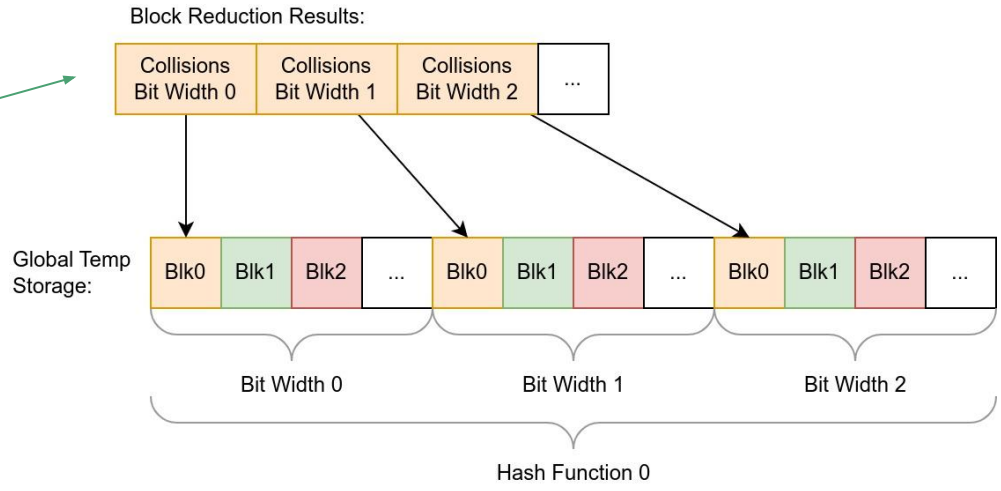
➜ No overlap of load and compute inside a single thread block

Distance 1 Kernel

Algorithm 1 Distance 1 Kernel

```
PartitionToSharedMemory(tile)
for hash_functions do
    ComputeLocalCollisions(tile)
    for bit_widths do
        BlockReduceTotal(local_collisions)
        BlockReduceMax(local_max_colisions)
        if threadID == 0 then
            global_block_total ← block_total
            global_block_max ← block_max
        end if
    end for
end for
threadfence()
if Thread == 0 then
    CountTickets()
end if
if last_block then
    ReduceAllBlocks()
end if
```

*2 * hash_functions * bit_widths* Counters
➔ 300 in our case



Hardware

RTX 2080 TI

Memory Size:	11 GB
Memory Bandwidth:	616 GB/s
Max Shared Memory per SM:	64 KB
Max Register Memory per SM:	256 KB
Max Threads per SM:	1024
Streaming Multiprocessors (SM):	68
Compute Capability	7.5

Evaluation

- We use bandwidth as metric
  - ➔ Reference to HW peak
  - ➔ Comparable
- Speedup in comparisons

Janna/Cube_Coup_dt0	
Rows:	2 164 760 (8 MiB)
NonZero Entries:	127 206 144 (485 MiB)
Avg Node degree:	59
Max Node degree:	68

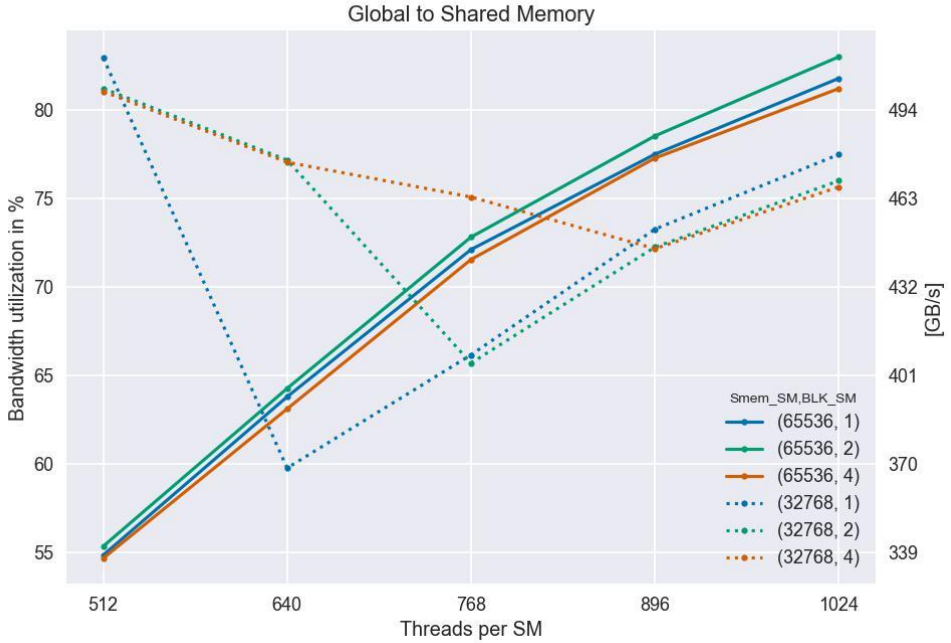
- Performance might vary with other matrices

Global to Shared Memory

Launch Bounds Configuration:

- Blocks per SM is **desired lower limit** (can be more)
- We adjust threads per block so that desired blocks fit on SM

- More Threads are better
- 2 Blocks per SM seems best

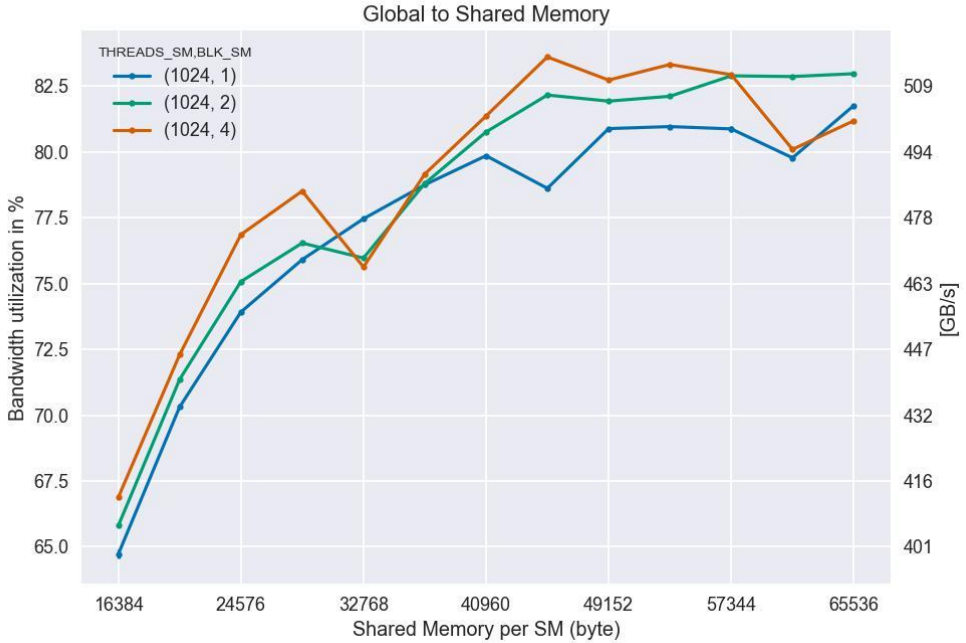


Global to Shared

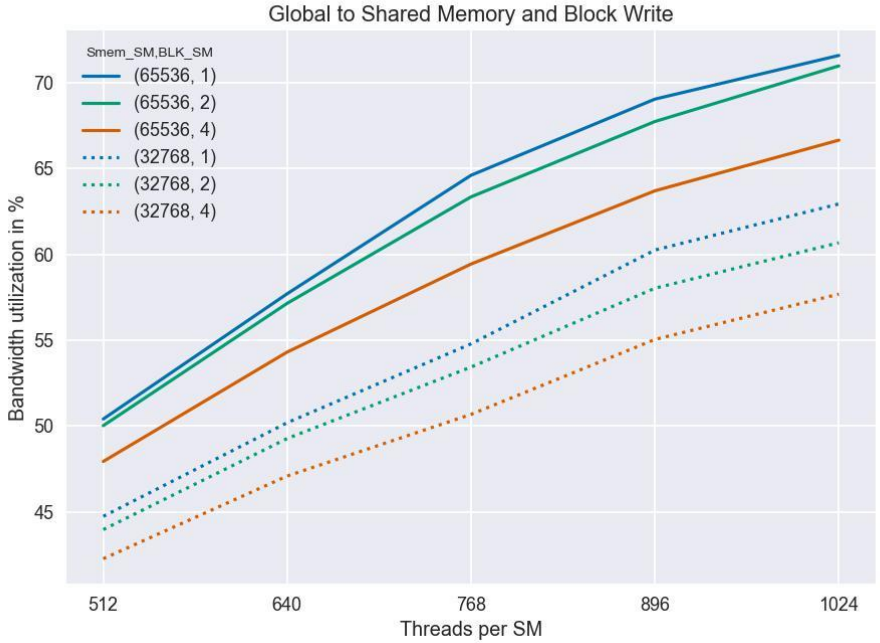
Partition Size:

- Partitions below 48 KiB loses performance
- Above 48 KiB no big change
- Distance 2 coloring
  ➡ only half shared memory for data

Less Shared Memory
⬇
More Partitions
⬇
More unaligned Memory Accesses

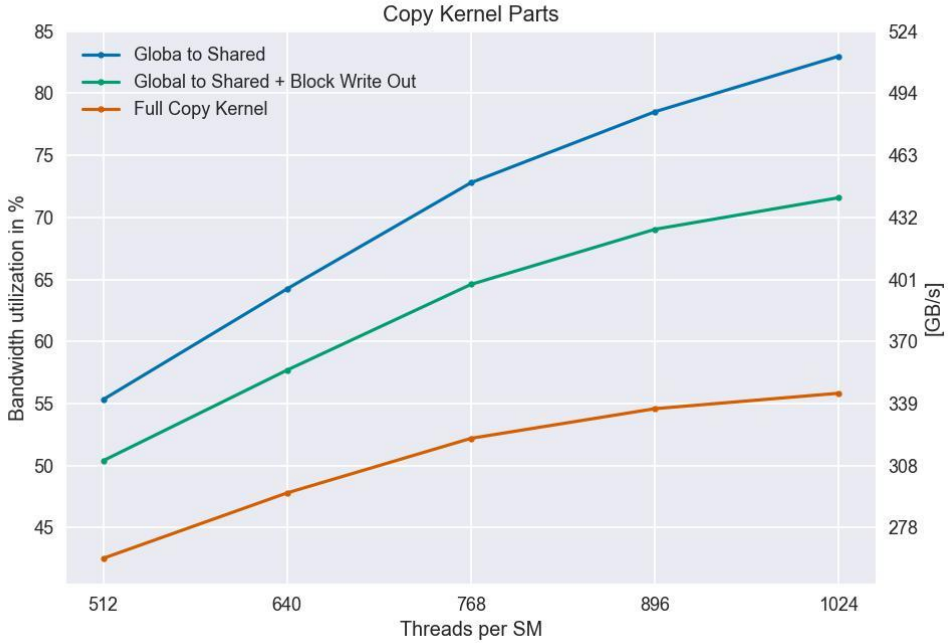


Global Load and Write of Block Results



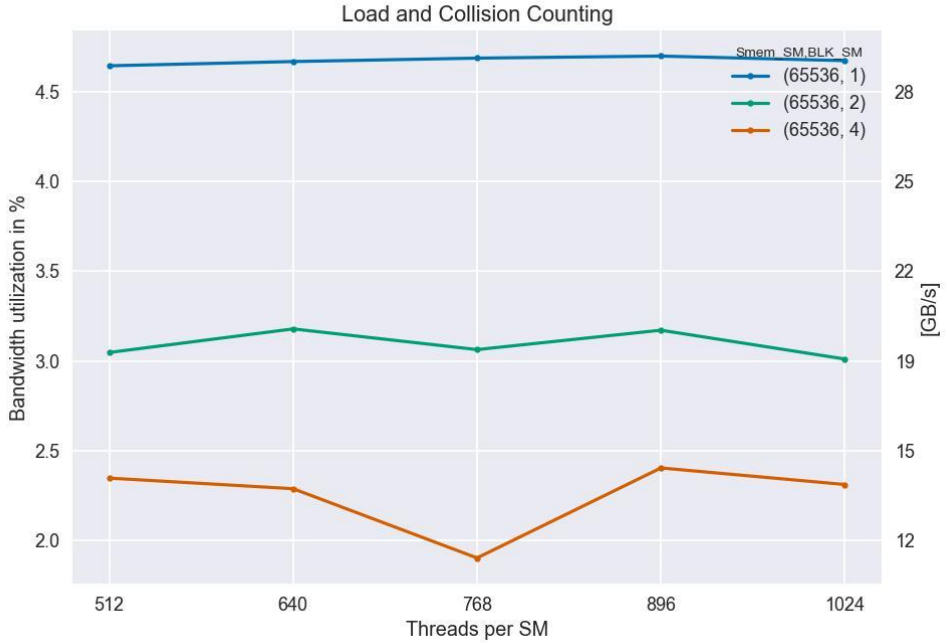
Copy Kernel Parts

We can achieve 56% of peak BW with the current global data movement



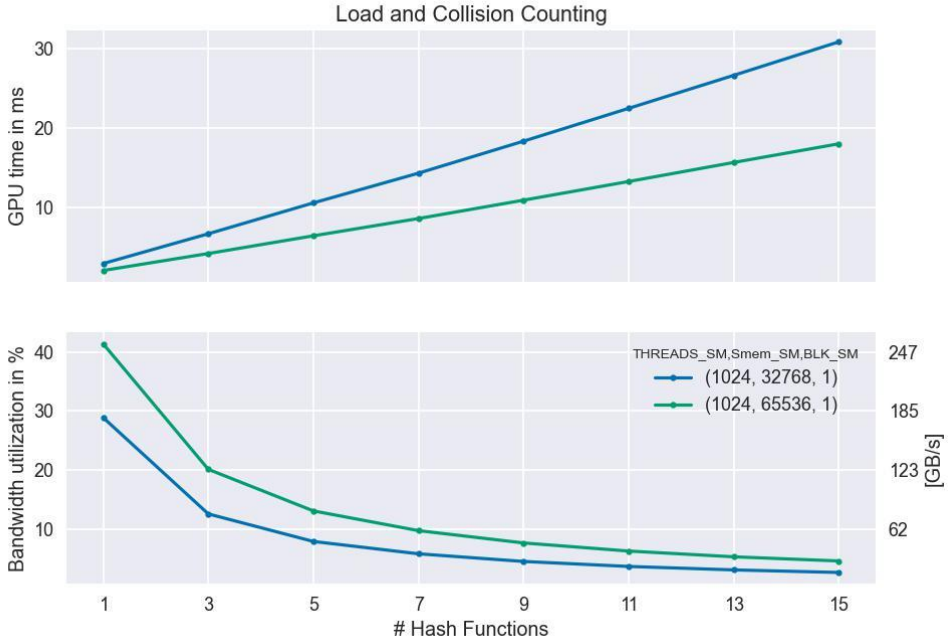
Load and Collision Counting

- Hashing significantly reduces Throughput



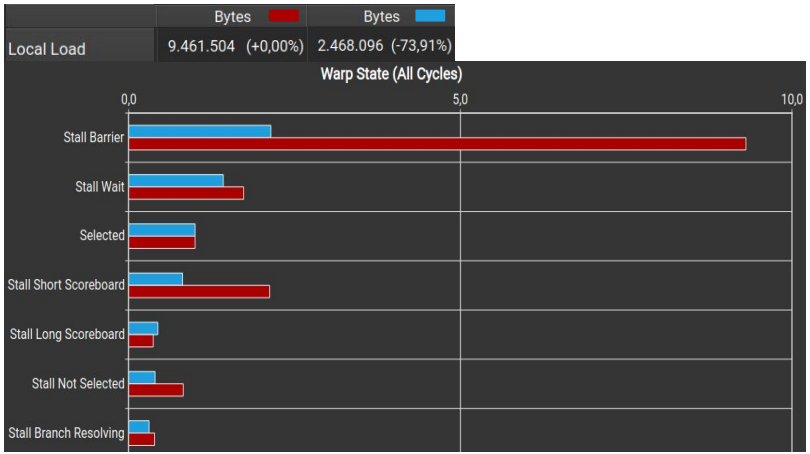
Load and Collision Counting

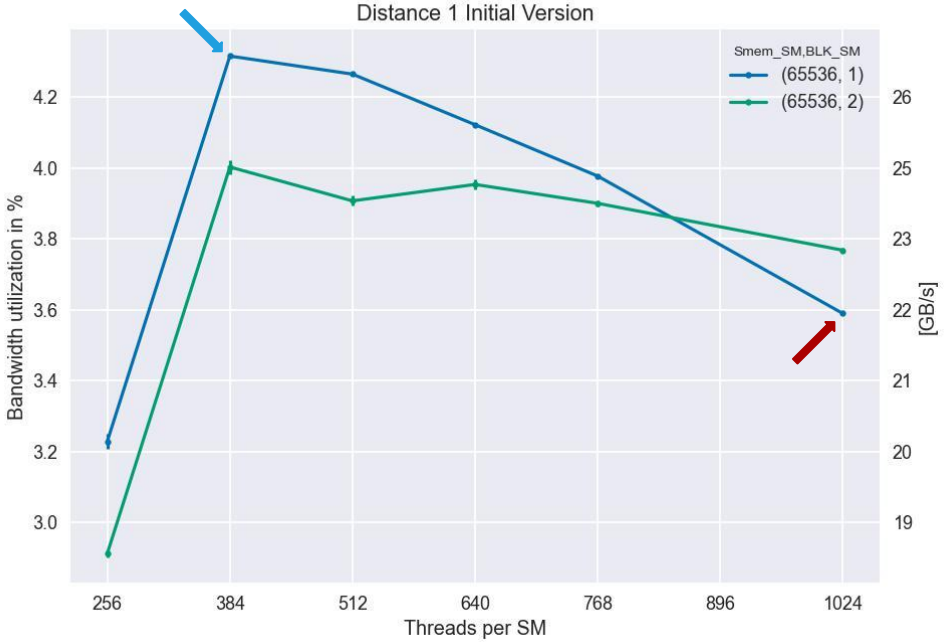
- We are compute-bound
- For 15 hash functions we spend only 4% of the time for loading data to shared memory



Distance 1 Initial Version

- Not enough parallelism for matrices with high node degree
- Only a bit register spilling with many threads





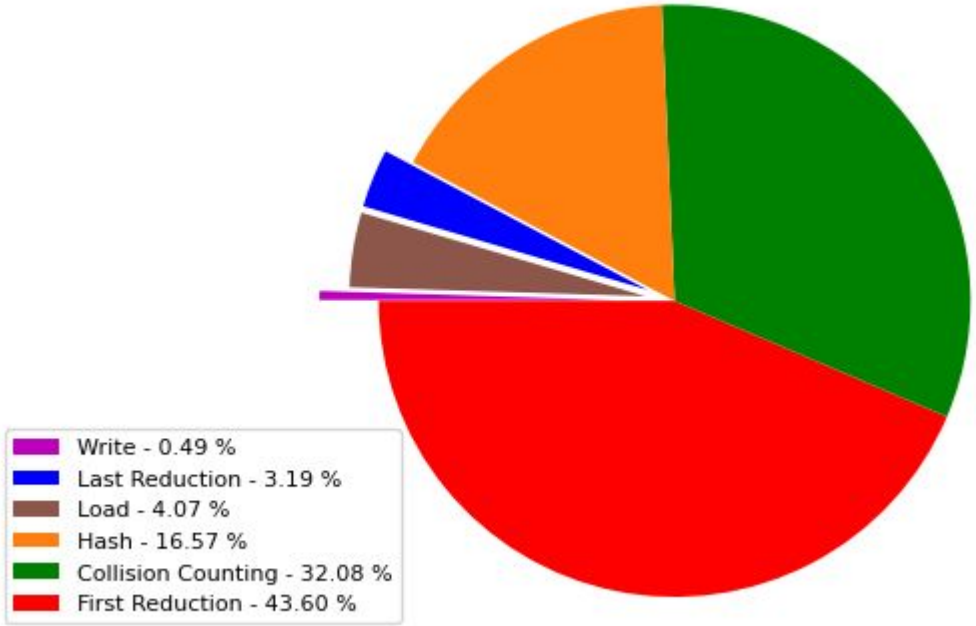
64 KiB, 1 Block, 1024 Threads per SM 64 KiB, 1 Block, 384 Threads per SM

Relative Execution Time

Main contributors:

- Block reduction
- Collision computation

➜ Focus of improvements

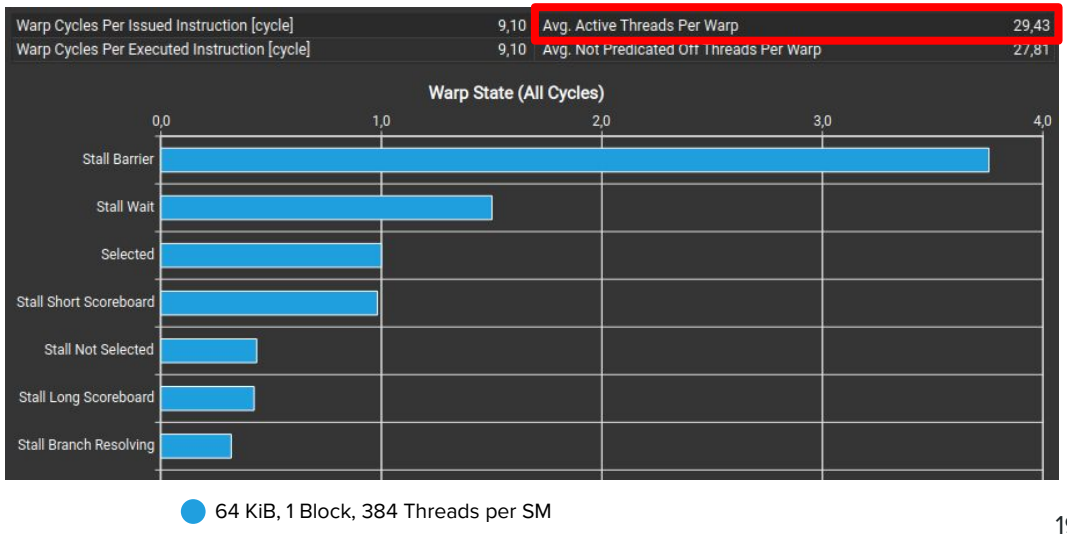


64 KiB, 1 Block, 384 Threads per SM

Constraints on Performance

Possible problems during computation:

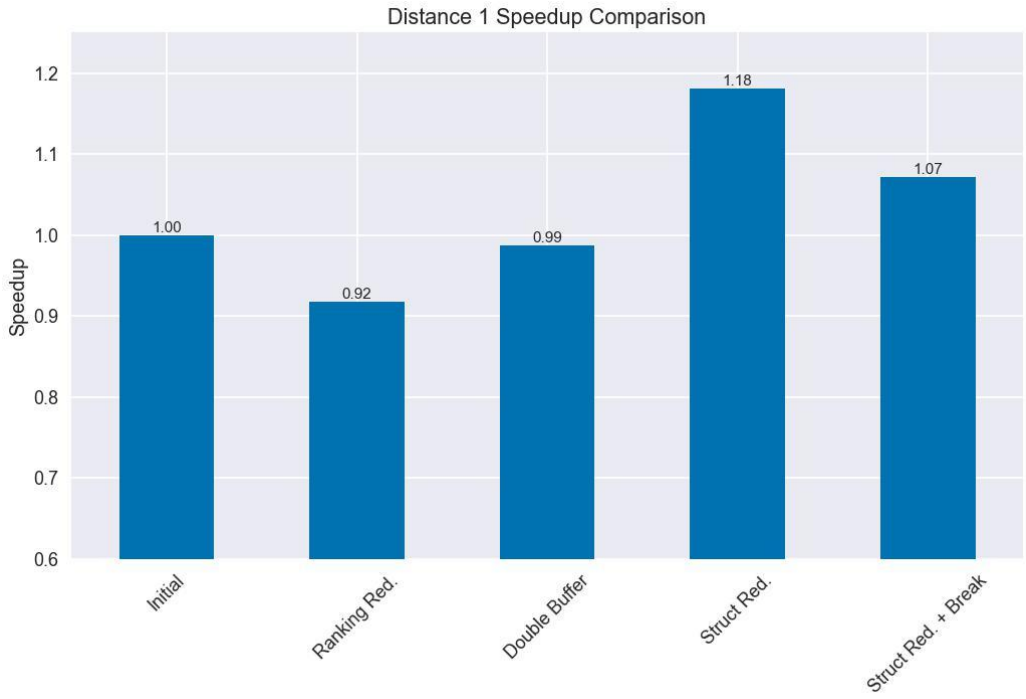
- Unnecessary computations (break loop, unrolling)
- Bank conflicts (read before hash)
- Too many synchronizations
- Other reduce algorithms
- Branch divergence



Speedup Comparison

- Warp Reduction (default) better than Ranking Reduction Algorithm (cub)
- Struct reduction decreases number of reduction calls
  ➡ less synchronization
- Breaking loop causes branch divergence

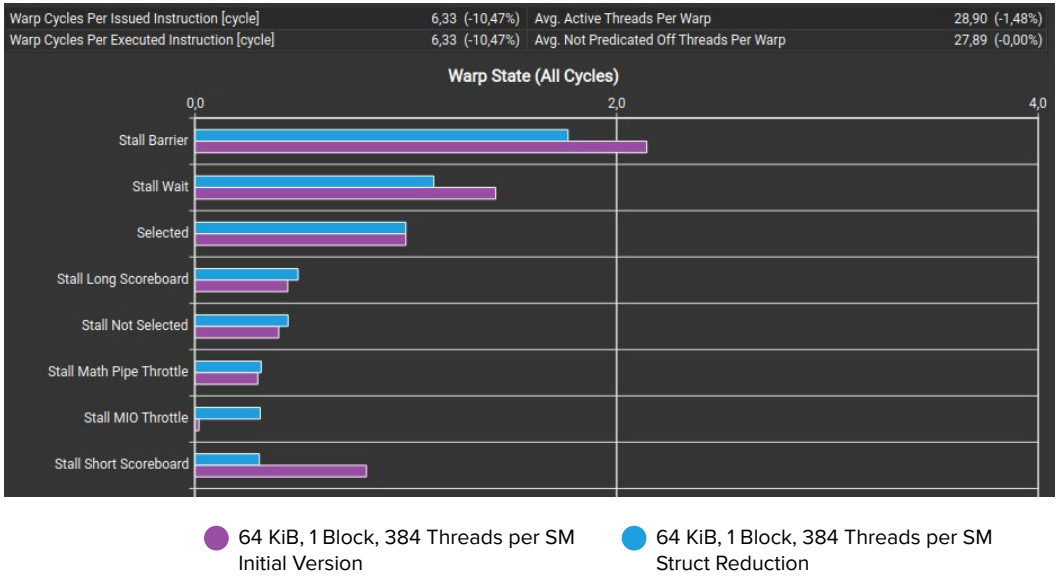
64 KiB, 1 Block, 384 Threads per SM



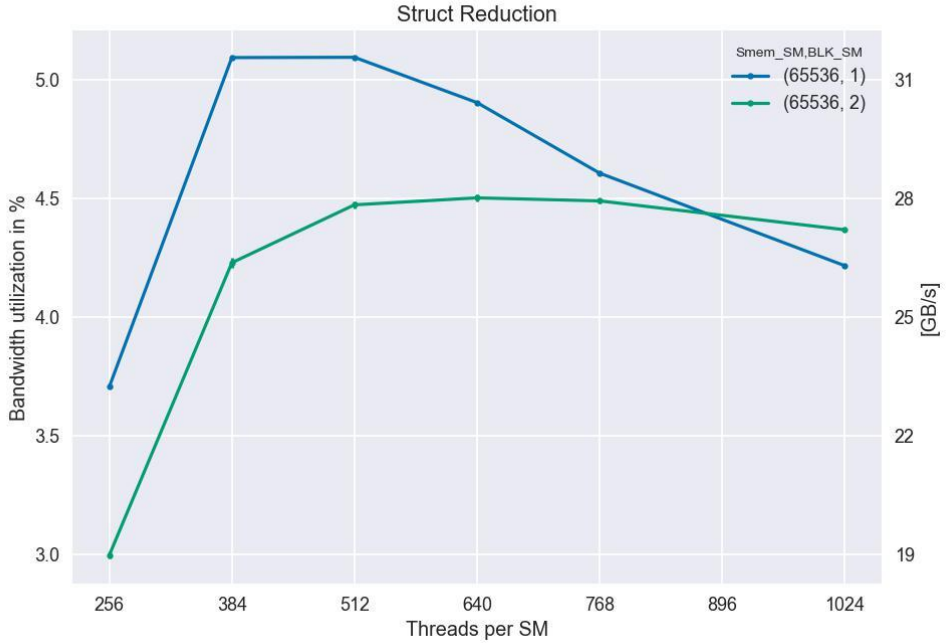
Distance 1 with Struct Reduction

- More work in single reduction
- More shared memory in single reduction (less reuse)

➜ Less stalls caused by barrier



Best version launch config

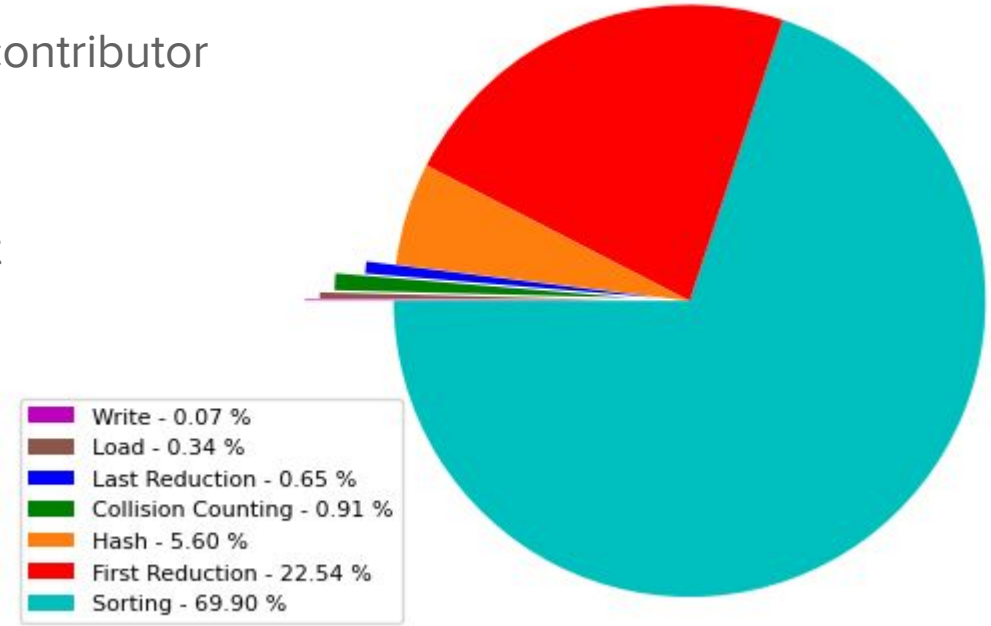


Relative Execution Time Distance 2

- Workspace sorting main contributor

  ➜ main focus

- Reduction also big impact



64 KiB, 2 Blocks, 384 Threads per SM

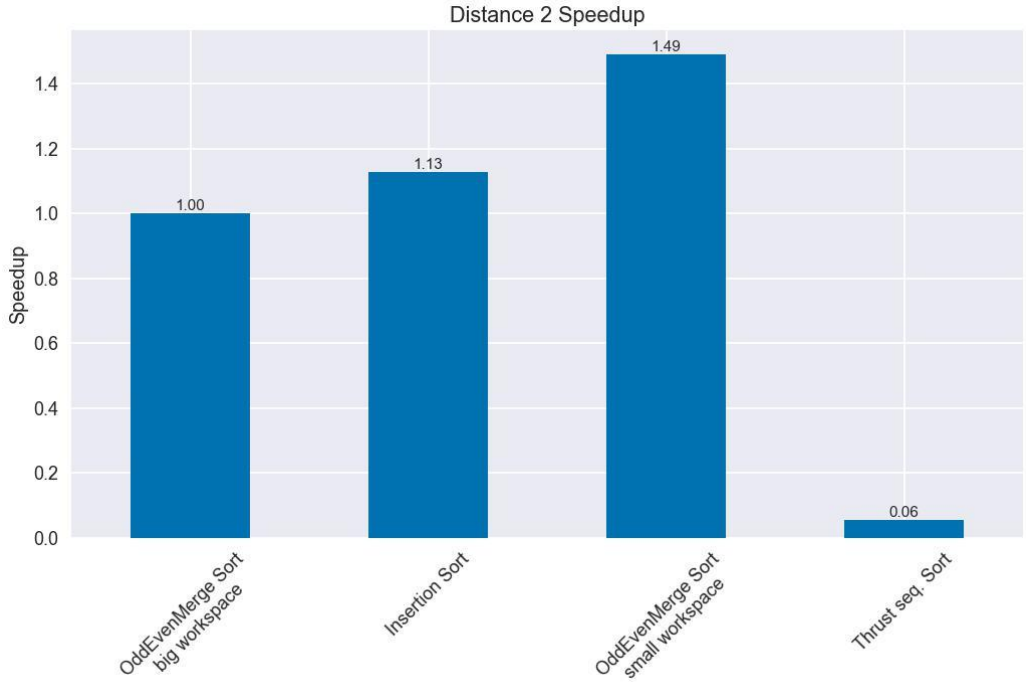
Distance 2 Speedup

64 KiB, 1 Blocks, 384 Threads per SM

- Big Workspace:
  - ➜ round up to multiple of 32 + 1

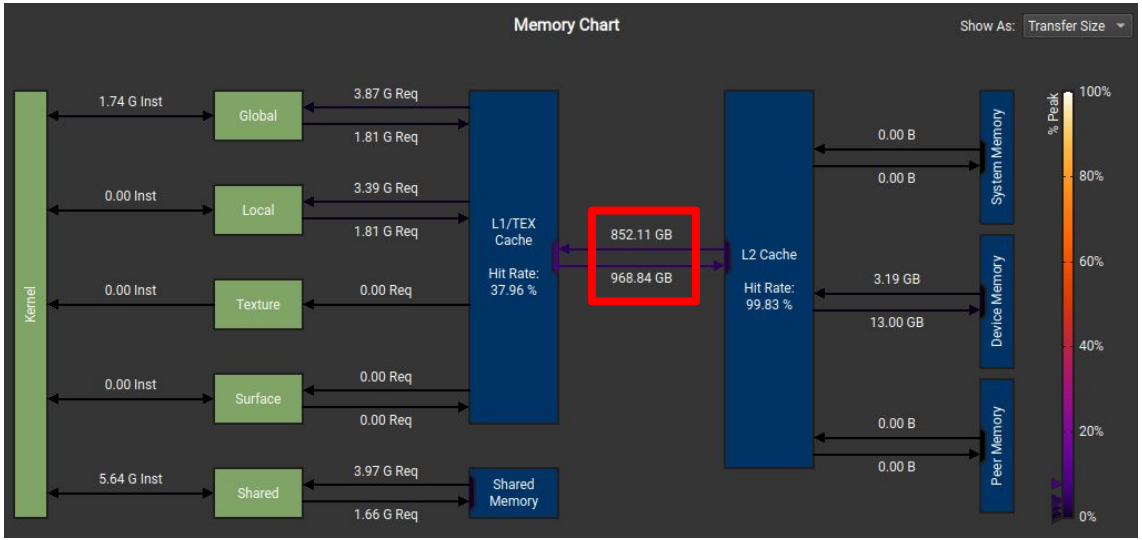
  <u>But:</u> makes partition smaller
- Sorting Network still useful

  ➜ Depending on matrix



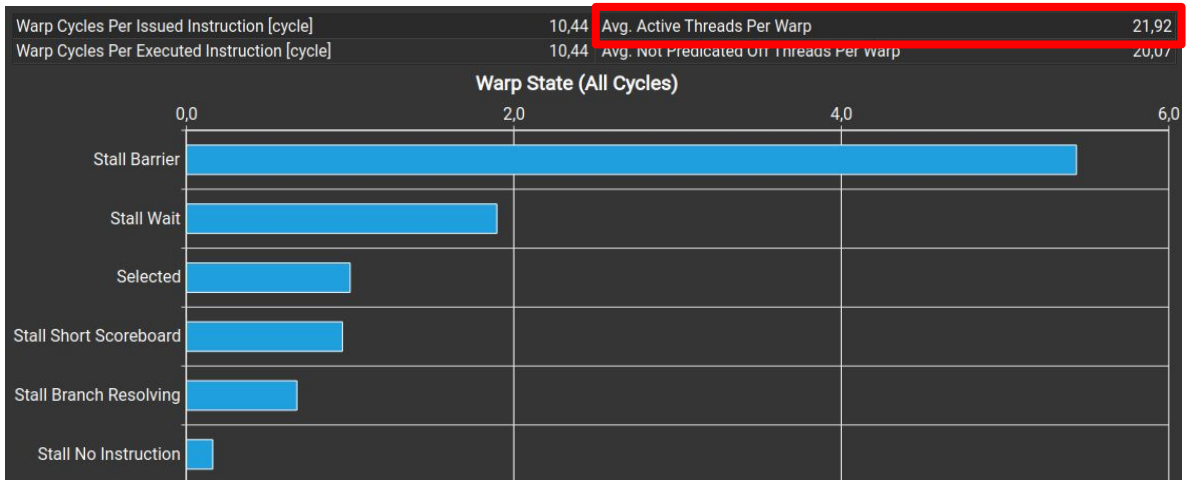
Thrust seq Sort

➜ global memory is used for temporary sorting storage



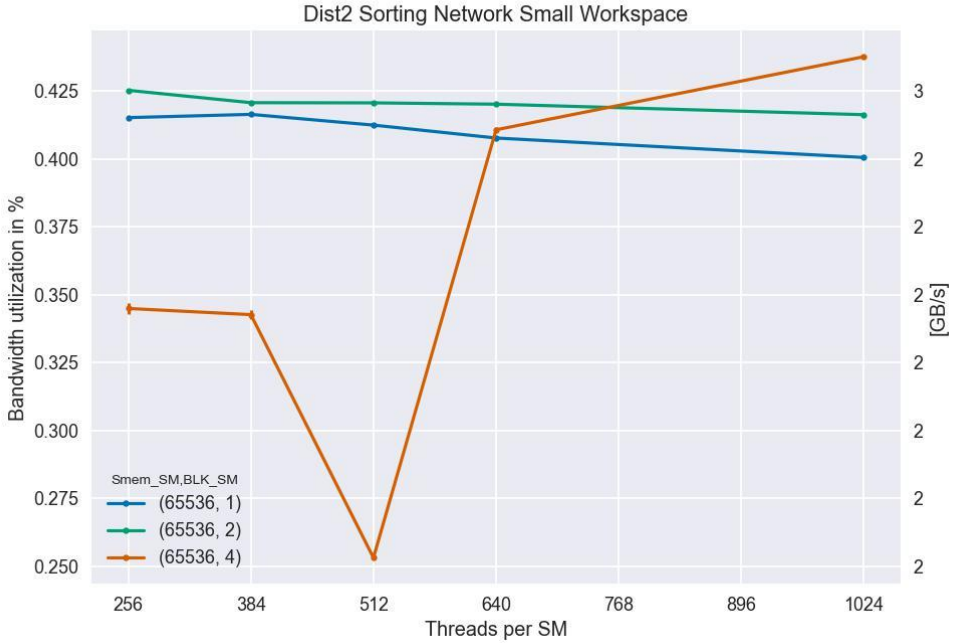
Profile Distance 2 Optimized

- Barrier still main stall reason
- Branch divergence

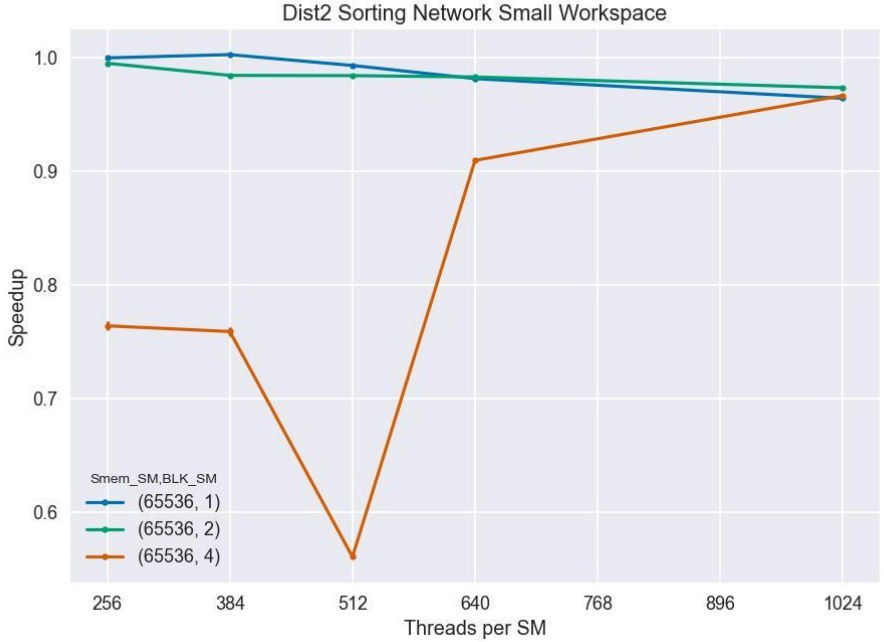


64 KiB, 1 Block, 384 Threads per SM

D2 launch config



D2 launch config

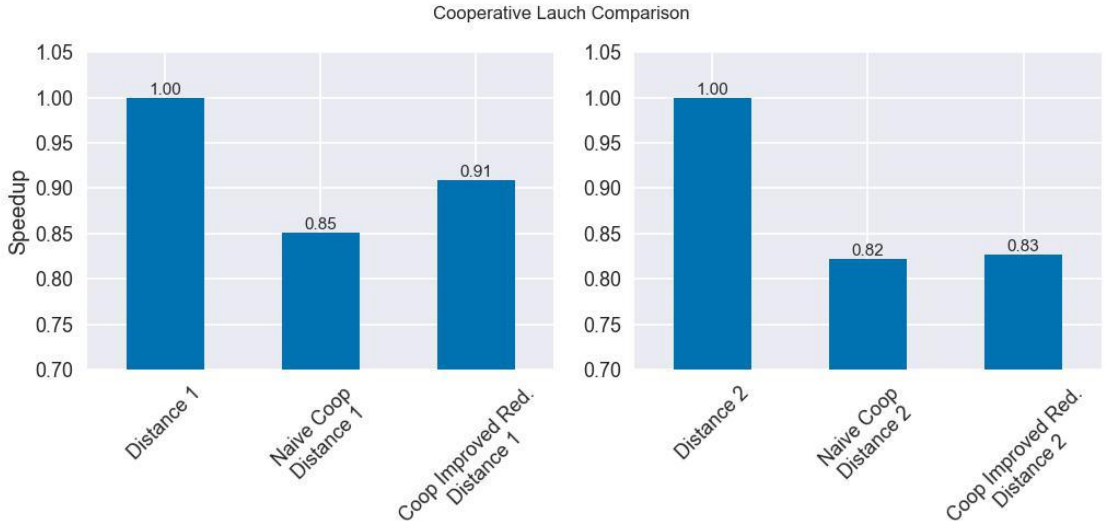


Cooperative Launch Comparison

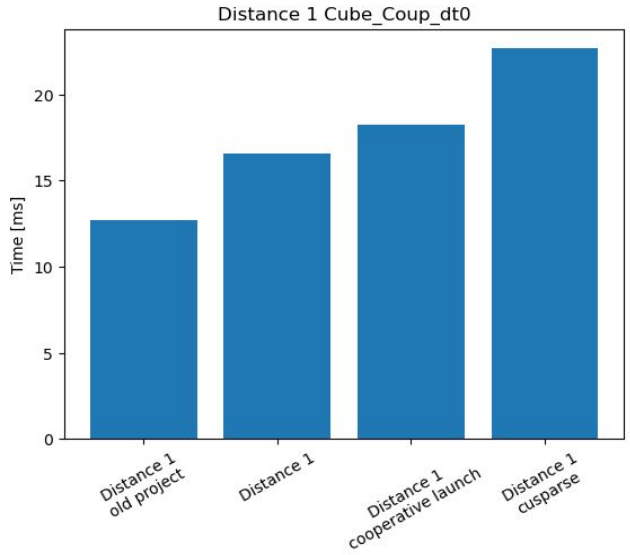
- Reduction of multiple partitions in single block can save global stores
  ➜ Improvements for Dist 1

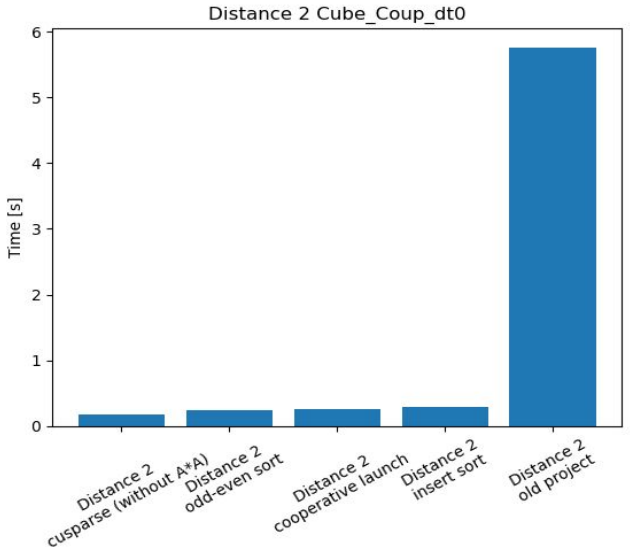
➜ Overall performance depending on matrix

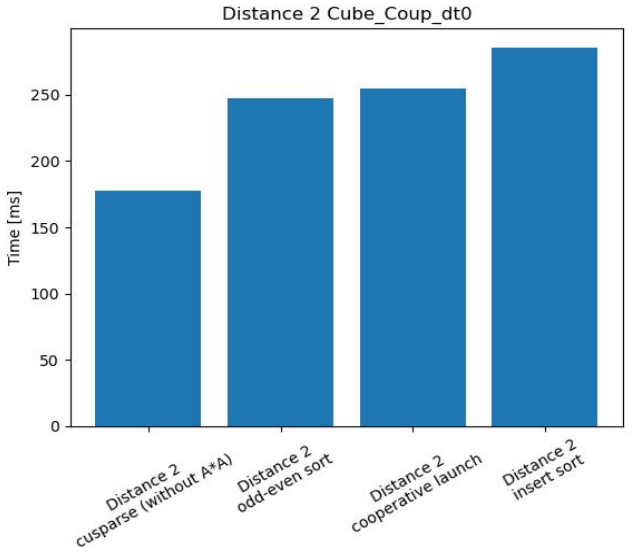
64 KiB, 1 Blocks, 384 Threads per SM



Best Version comparison

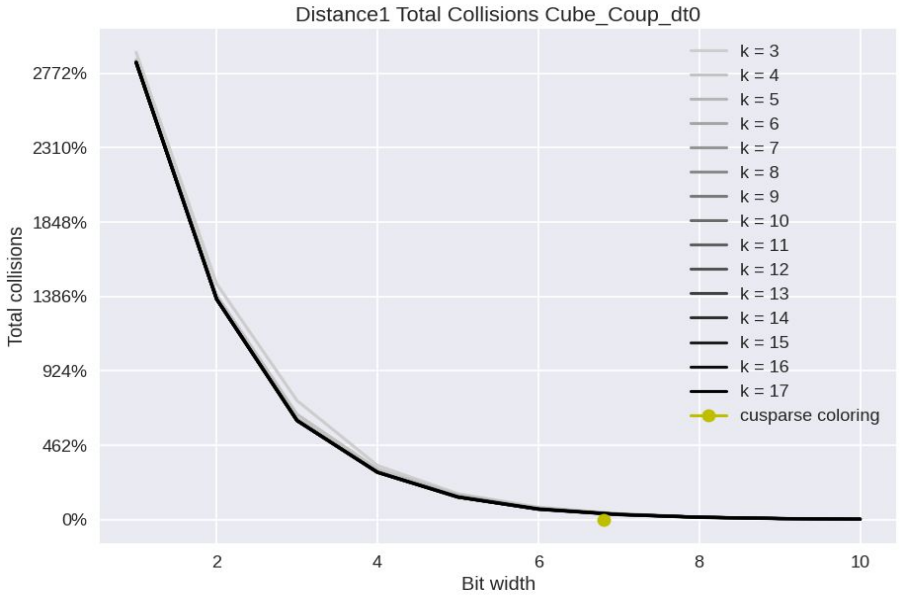


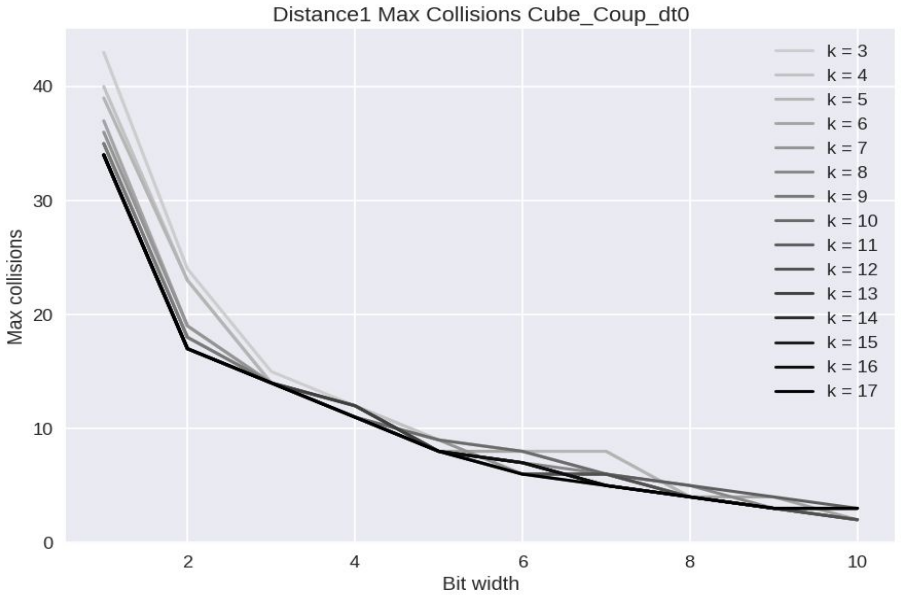




64 KiB, 1 Blocks, 384 Threads per SM

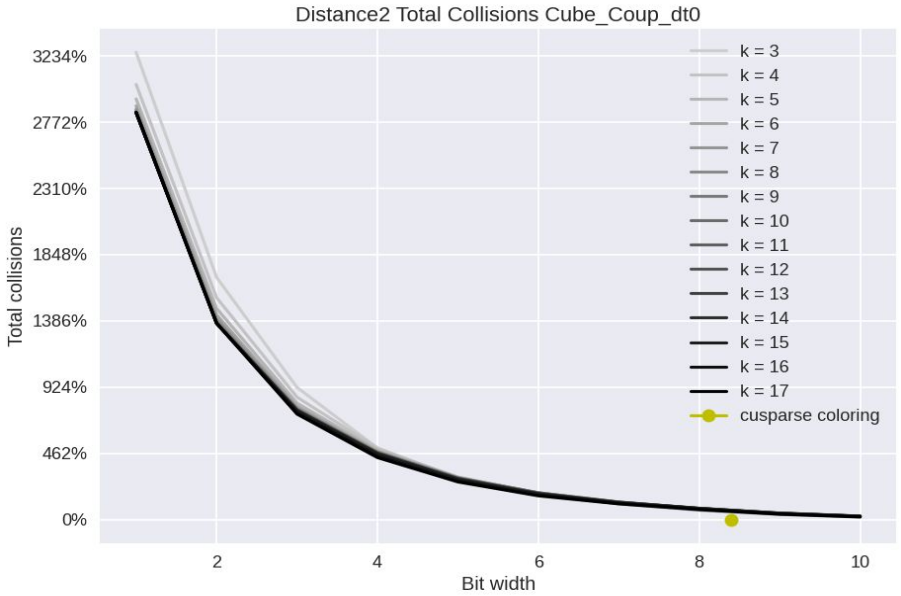
Coloring Quality Distance 1

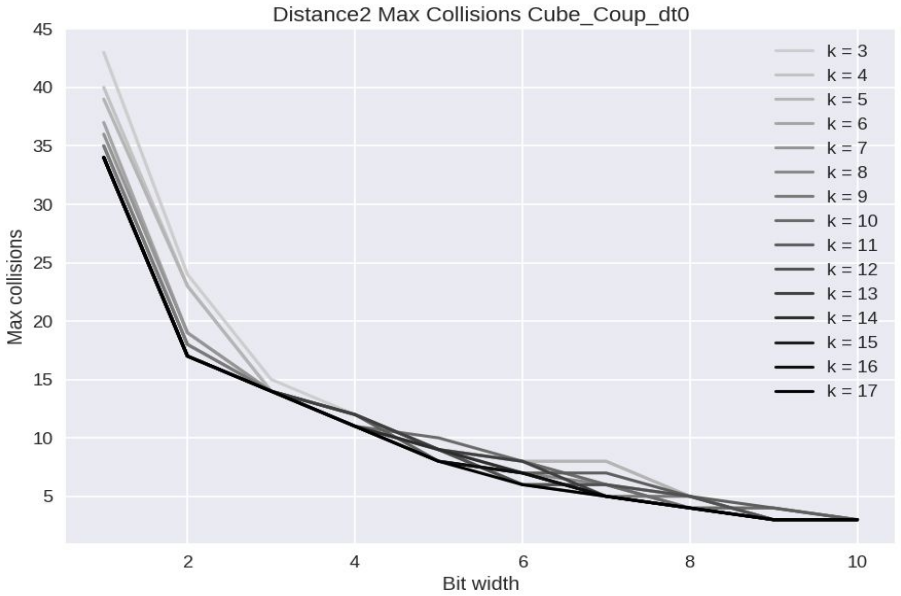




- old project: 32768 colors (k = 5)

Coloring Quality Distance 2





- old project: 65536 colors (k = 5)

Summary

- Collision computation can happen completely in shared memory
- Contrary to our expectations: compute bound
- Time spent primarily in collision computation and reduction
- Shared Memory size can limit performance
- Bank conflicts not that problematic for sorting

Further Improvements

- Trade Computation for Communication in global Memory
  ➜ Currently Global memory unused most of the time
- Think about using a whole Warp for a single Node
  ➜ Too few rows in partition for graphs with high node degree
  ➜ Might also help with branch divergence in very irregular graphs
- Use of better Hardware
  ➜ More Shared Memory and more SMs

Thank you!

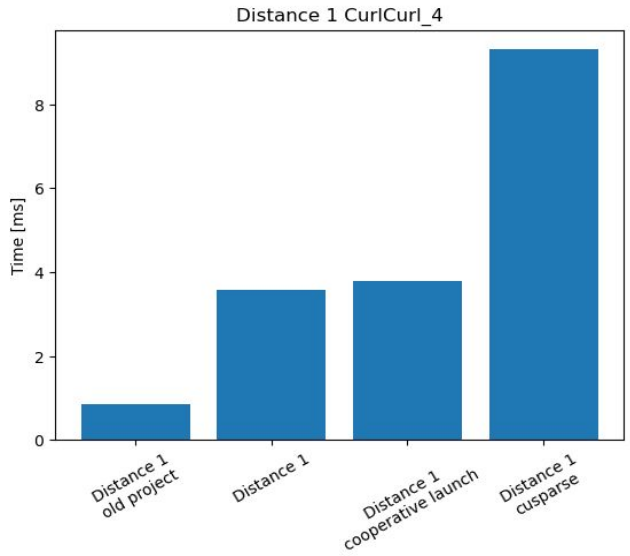
Remark on upcoming plots

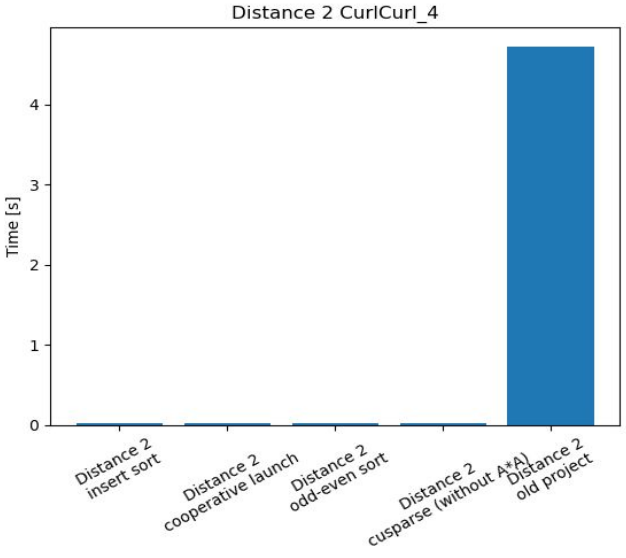
- all following plots created with best performing setup

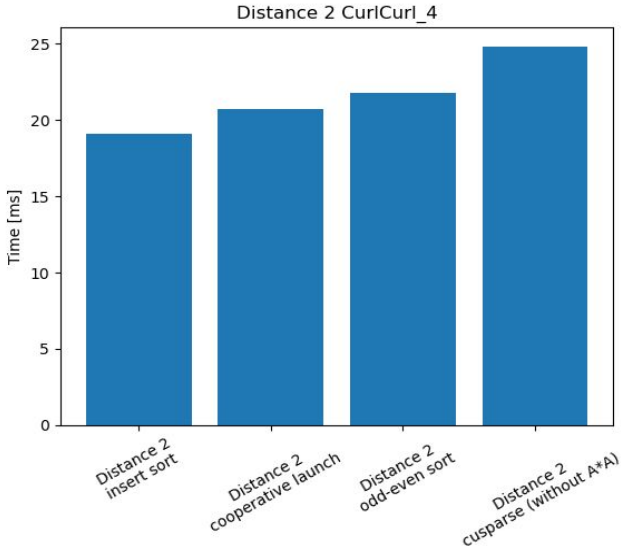
  ➔ 1 Thread Block per SM & 384 Threads per Block

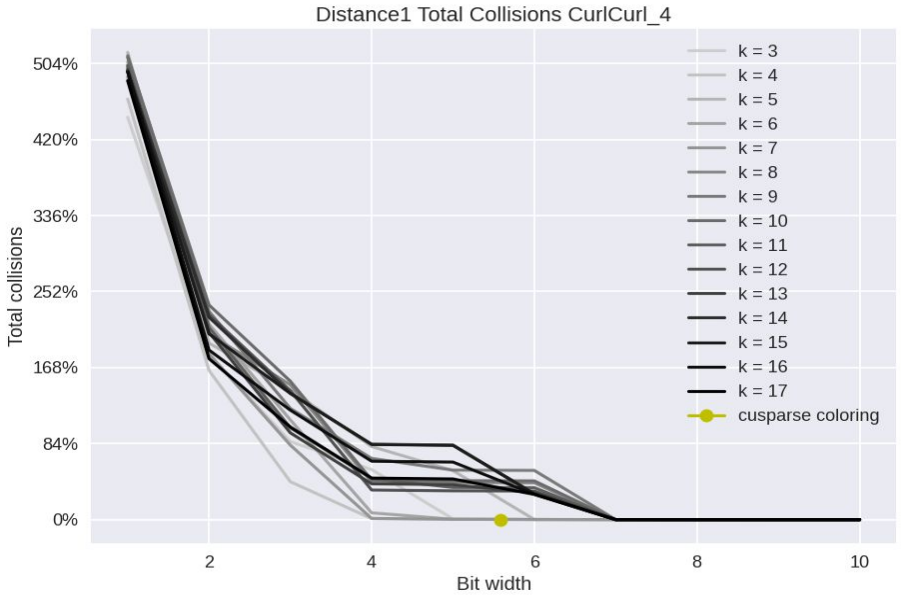
  ➔ For each Matrix a comparison of execution time and coloring quality for distance 1 & 2 with cusparseDcsrcolor() and the old project is made

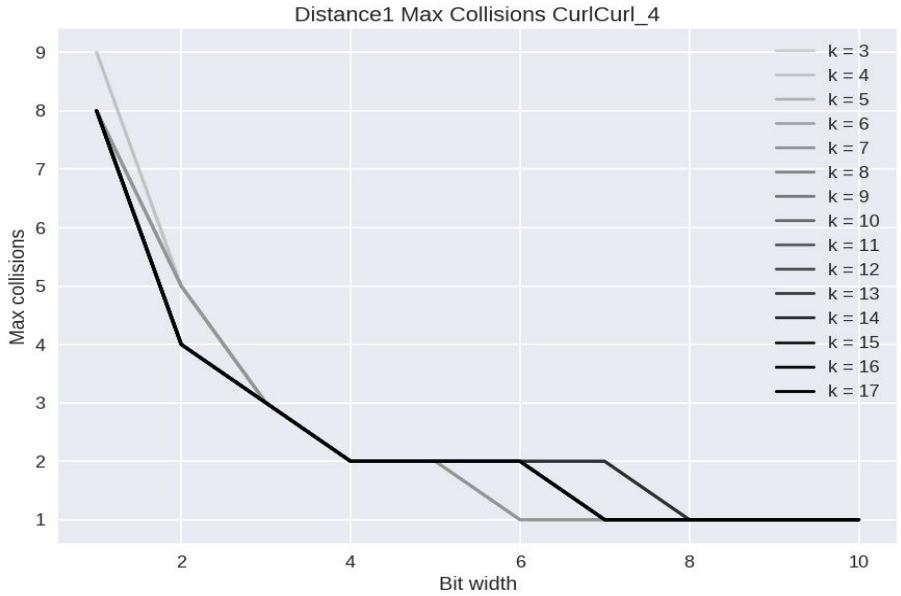
- As the for the cusparse distance 2 coloring required matrix multiplication was performed on the CPU we did not include this in our measurements


Distance 1 CurlCurl_4
Time [ms]
0
2
4
6
8
Distance 1 old project
Distance 1
Distance 1 cooperative launch
Distance 1 cusparse

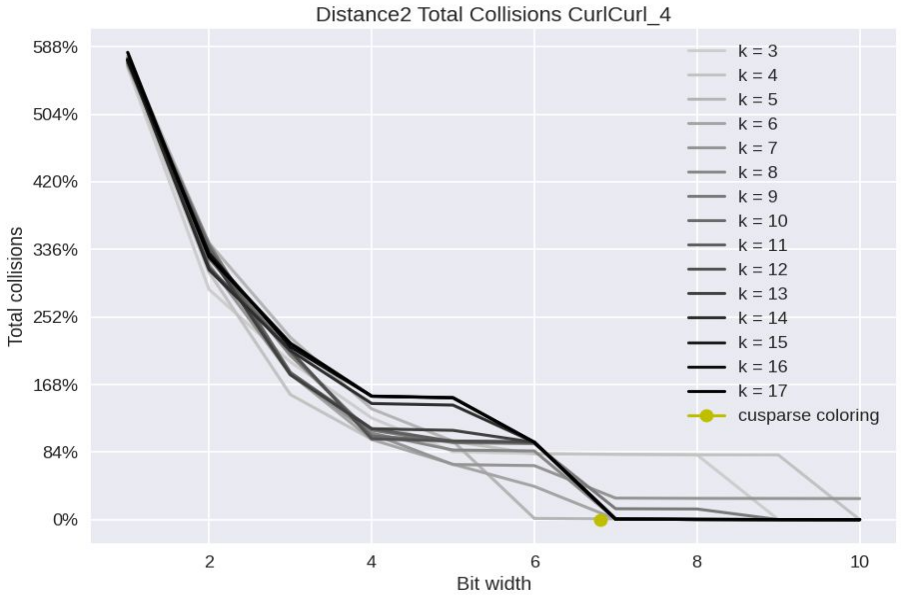

Distance 2 CurlCurl_4
Time [s]
0
1
2
3
4
Distance 2 insert sort
Distance 2 cooperative launch
Distance 2 odd-even sort
Distance 2 cusparse (without A*A)
Distance 2 old project

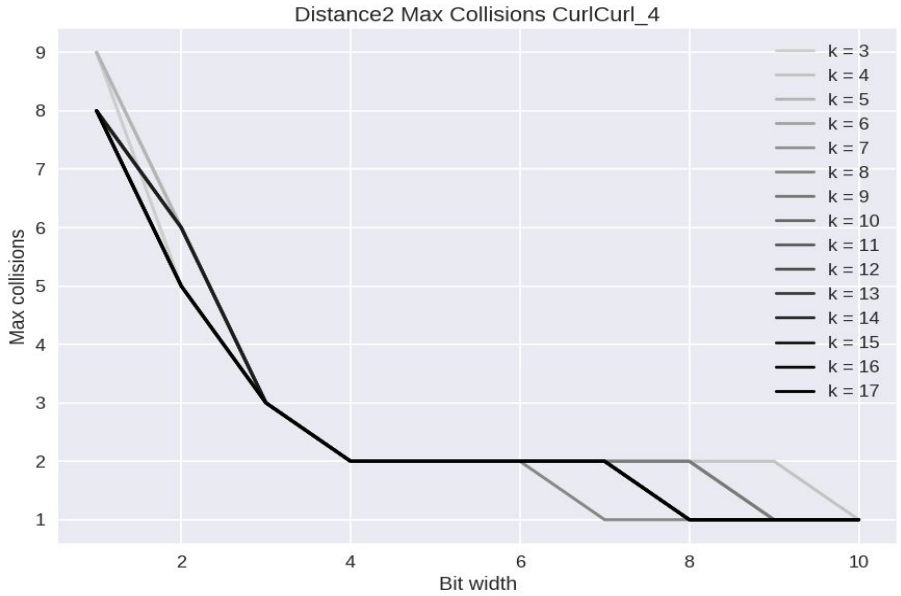

Distance 2 CurlCurl_4
Time [ms]
0
5
10
15
20
25
Distance 2 insert sort
Distance 2 cooperative launch
Distance 2 odd-even sort
Distance 2 cusparse (without A*A)





- old project: 65536 colors (k = 3)





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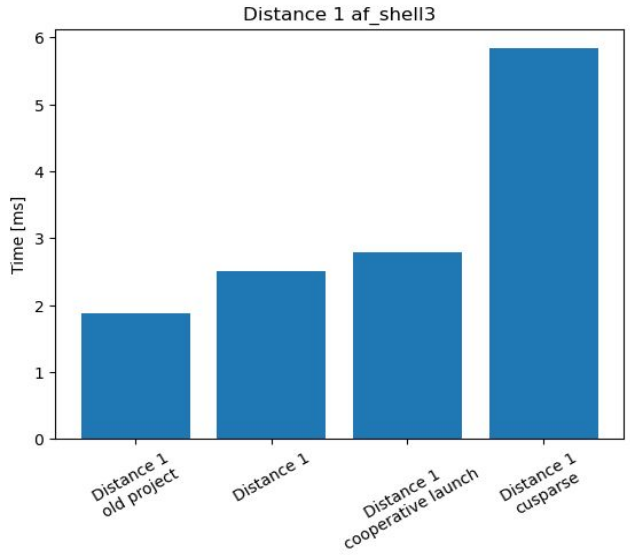
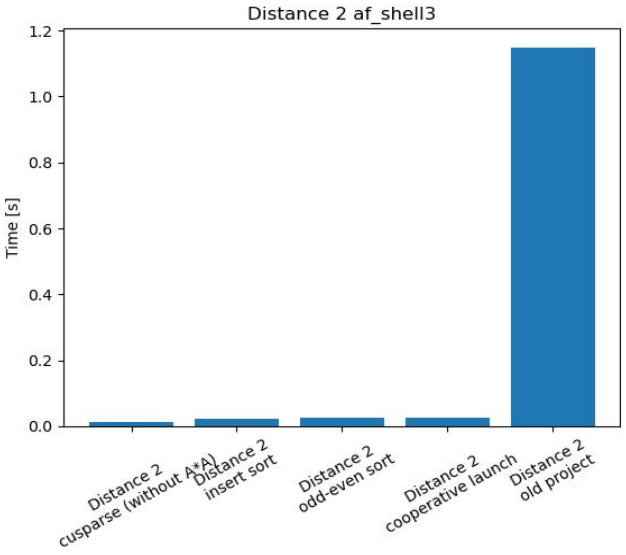
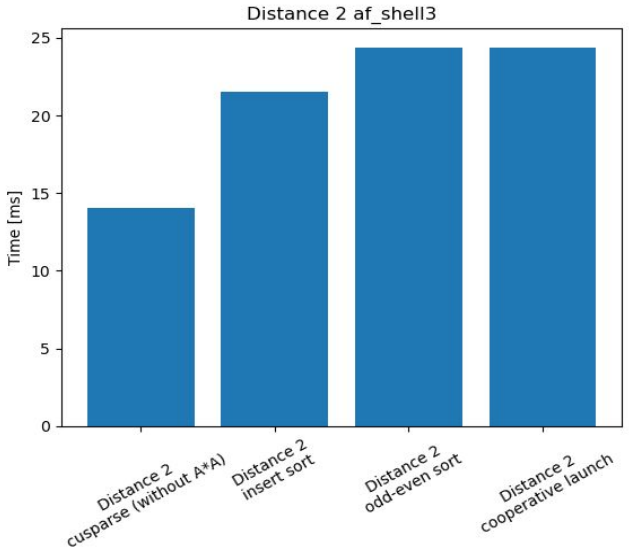
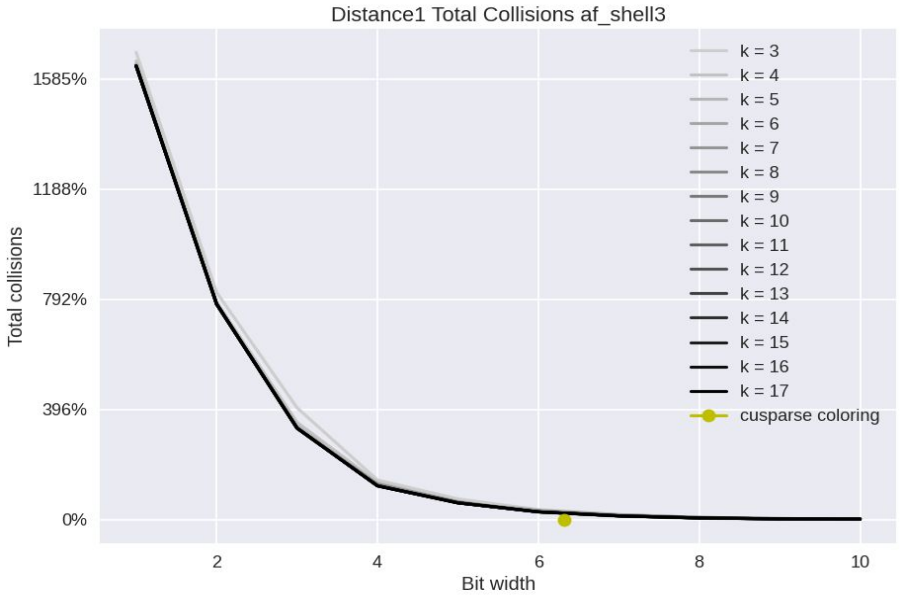
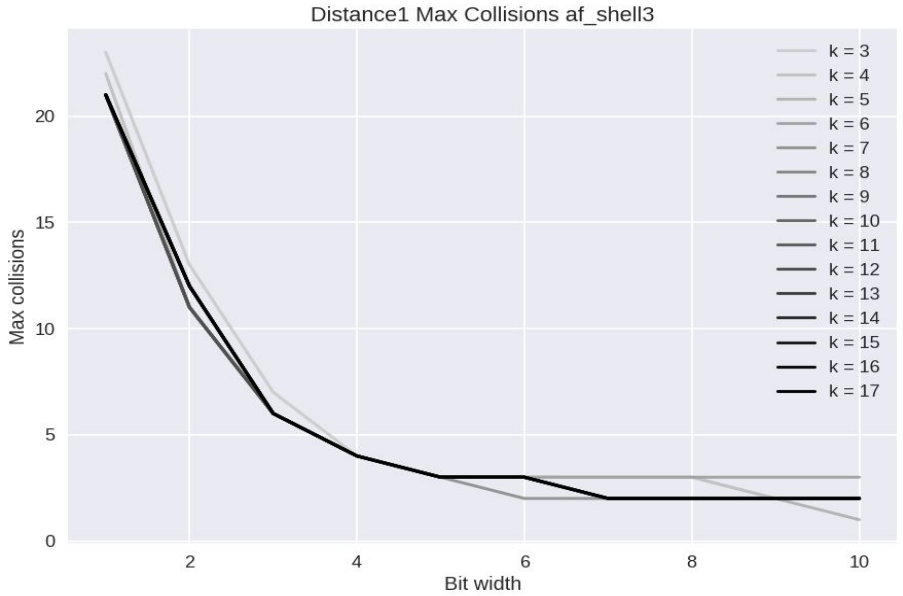

Distance 1 af_shell3
Time [ms]
0
1
2
3
4
5
6
Distance 1 old project
Distance 1
Distance 1 cooperative launch
Distance 1 cusparse

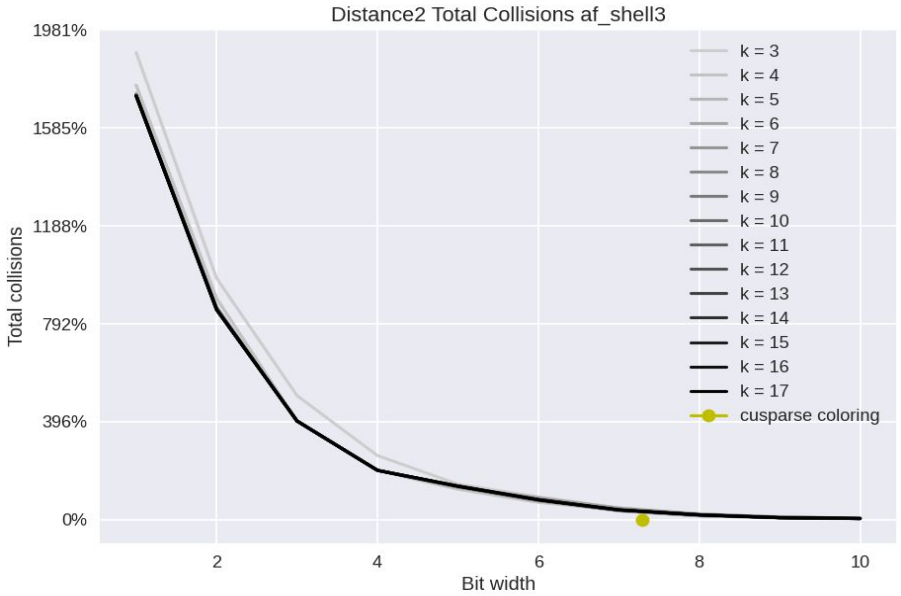

Distance 2 af_shell3
Time [s]
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 old project

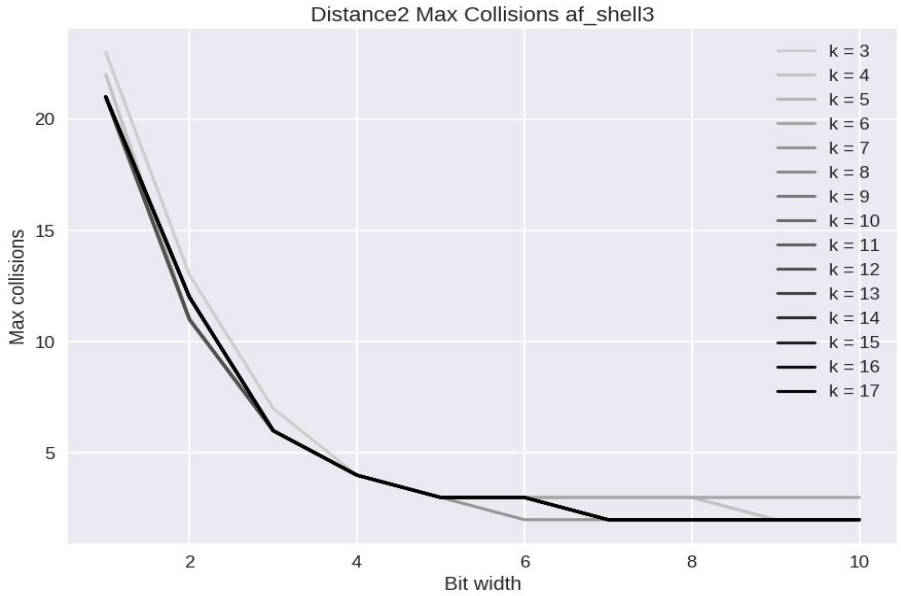

Distance 2 af_shell3
Time [ms]
0
5
10
15
20
25
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch



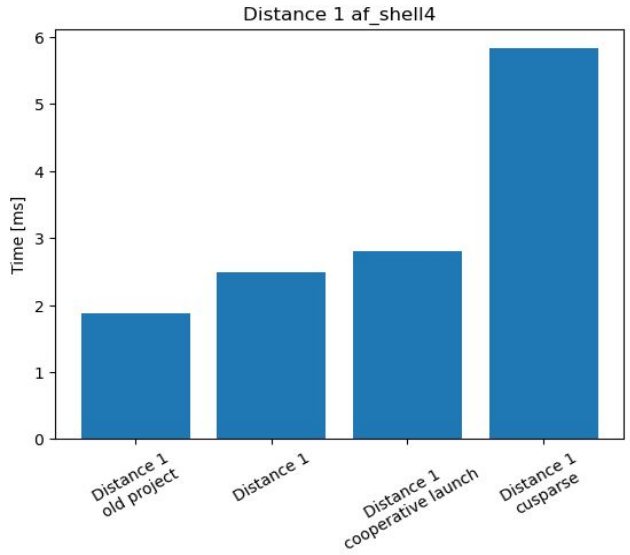


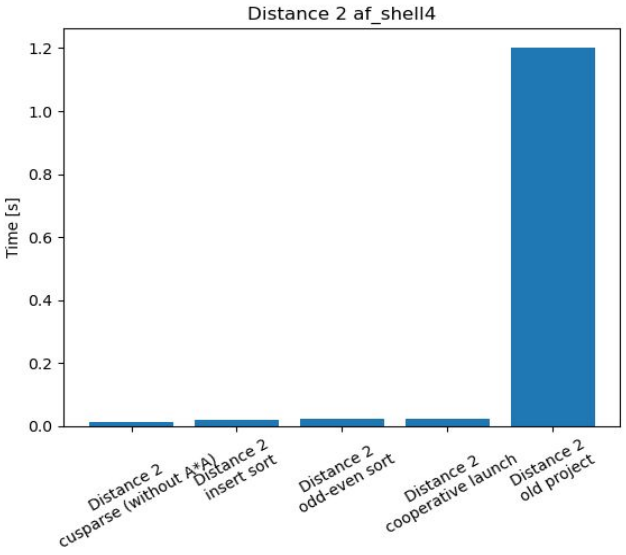
- old project: 4096 colors (k = 3)

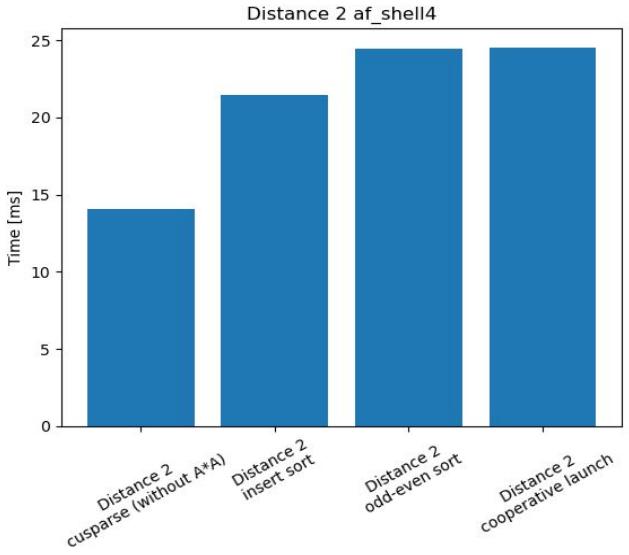


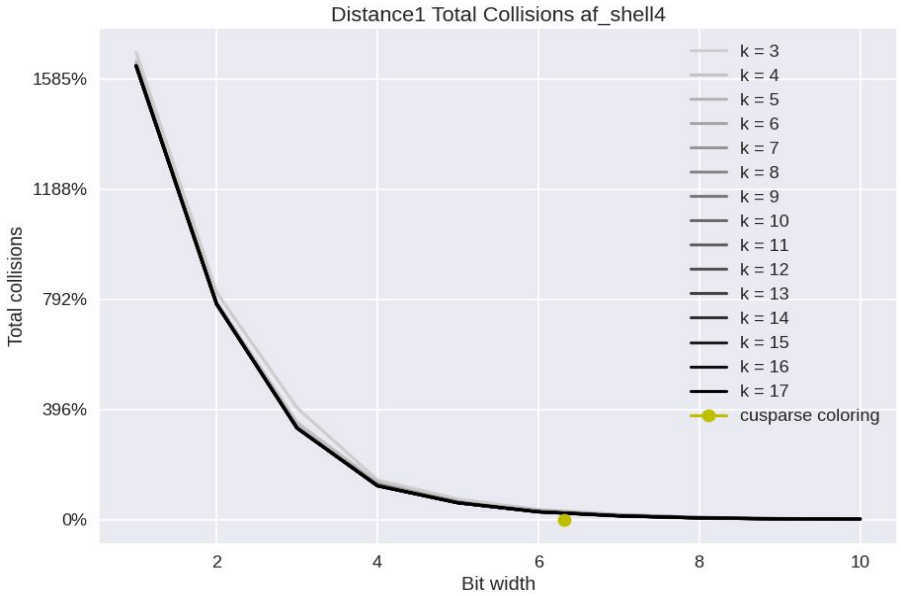


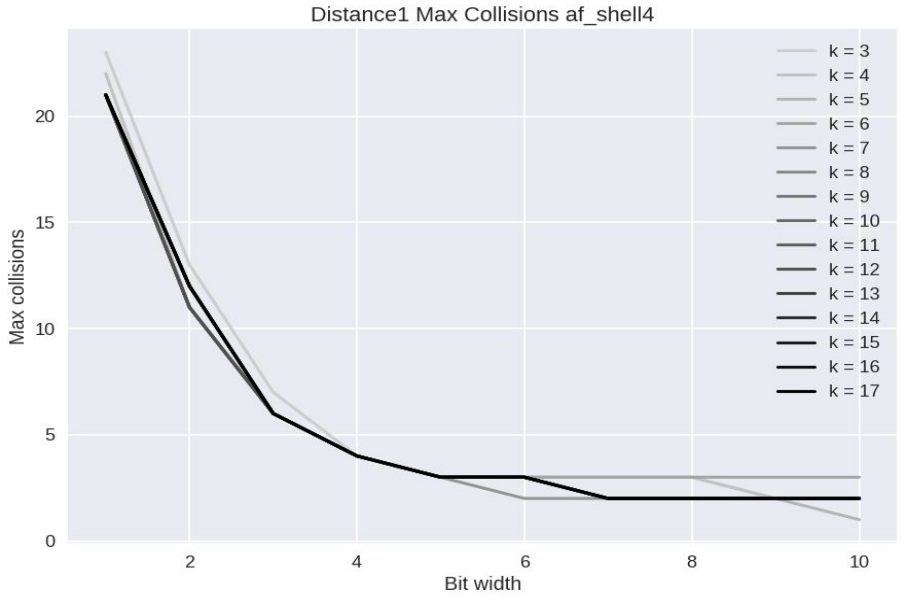
- old project: 8192 colors (k = 4)


Distance 1 af_shell4
Time [ms]
0
1
2
3
4
5
6
Distance 1 old project
Distance 1
Distance 1 cooperative launch
Distance 1 cusparse

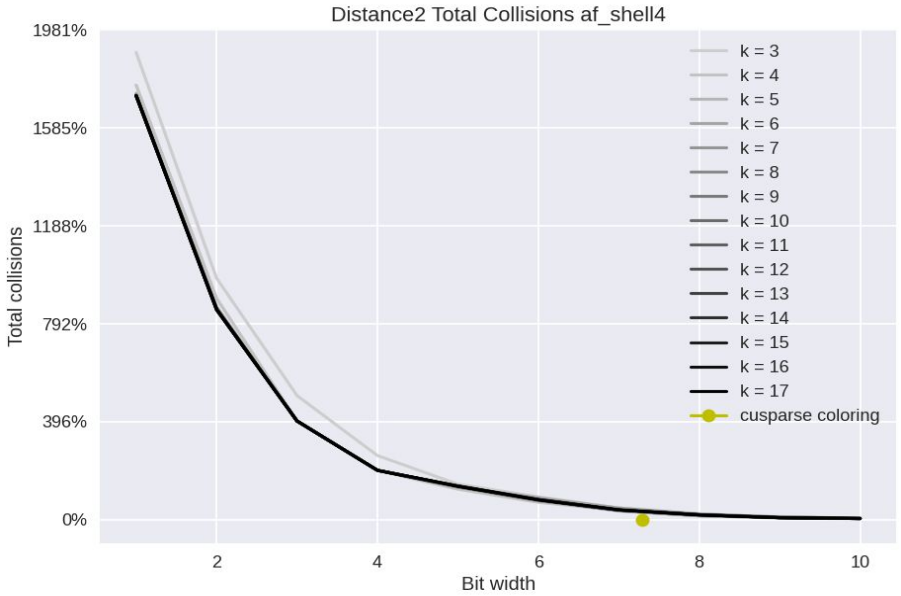

Distance 2 af_shell4
Time [s]
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 old project

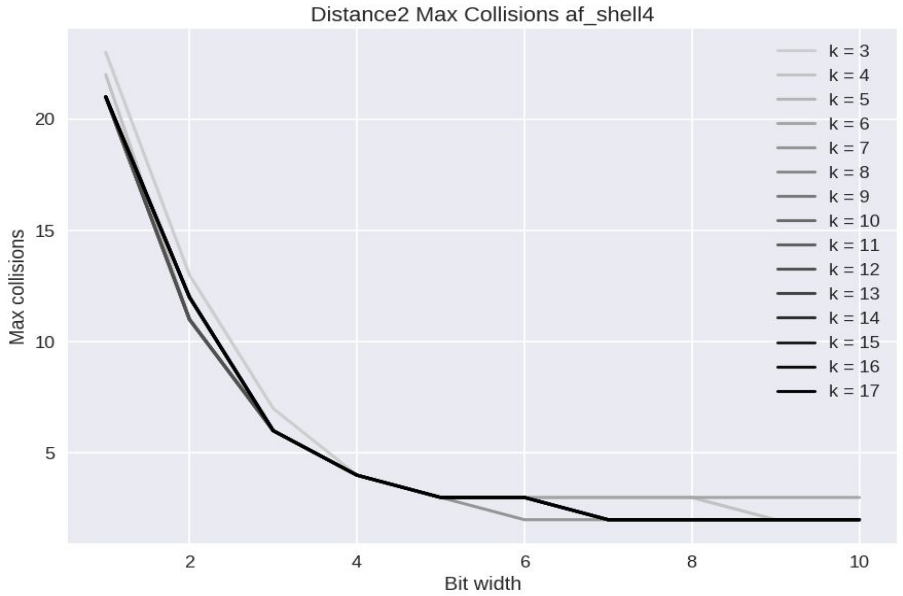

Distance 2 af_shell4
Time [ms]
0
5
10
15
20
25
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch



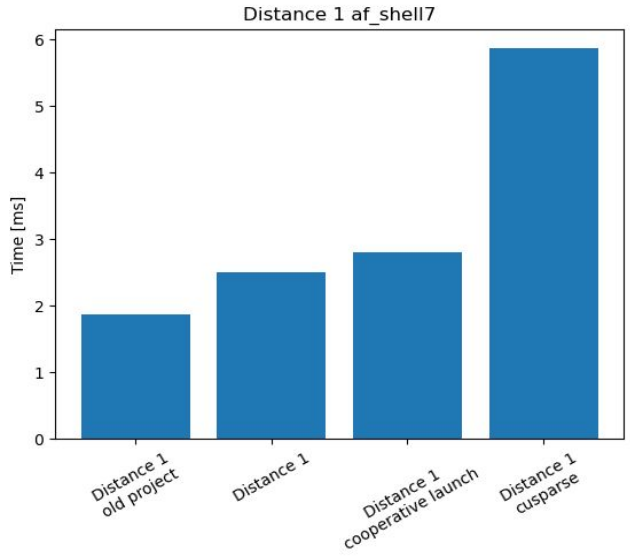


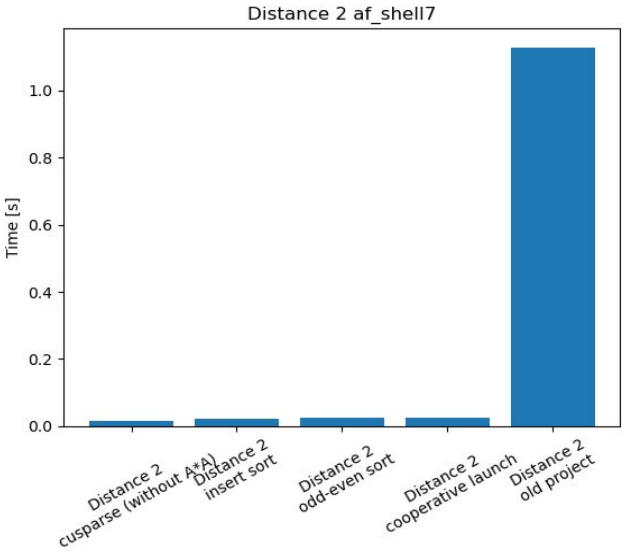
- old project: 4096 colors (k = 3)

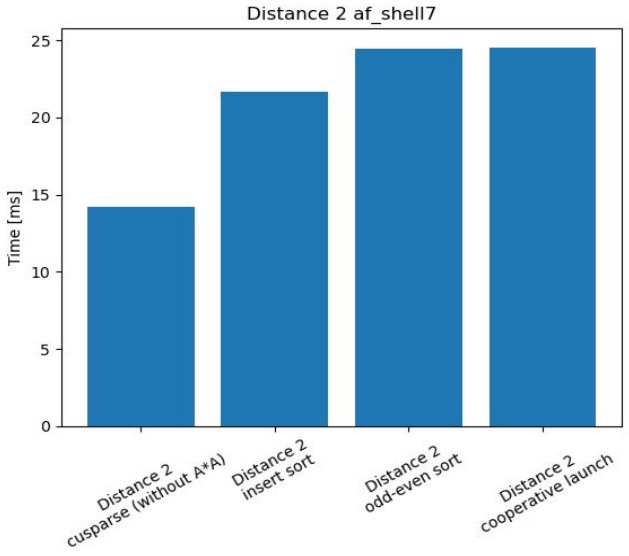


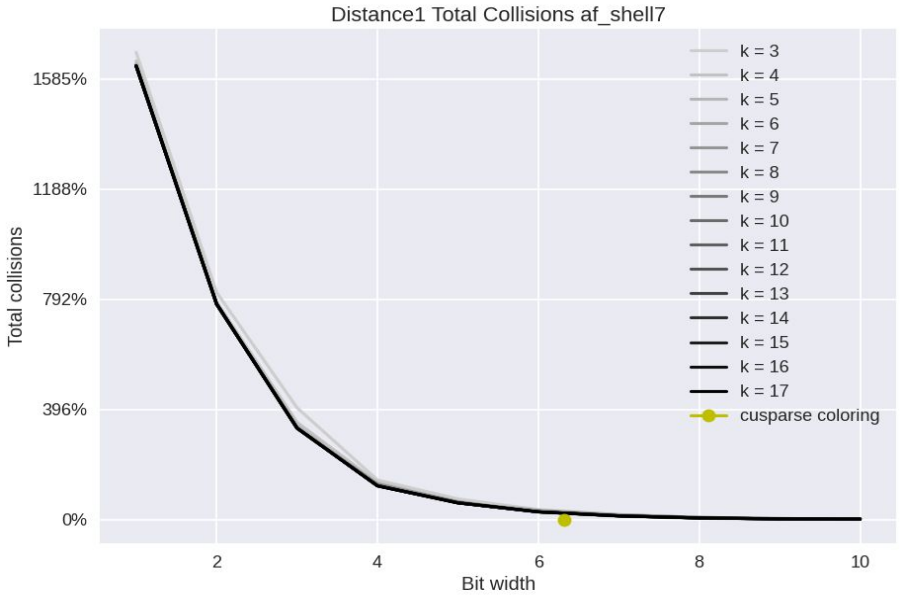


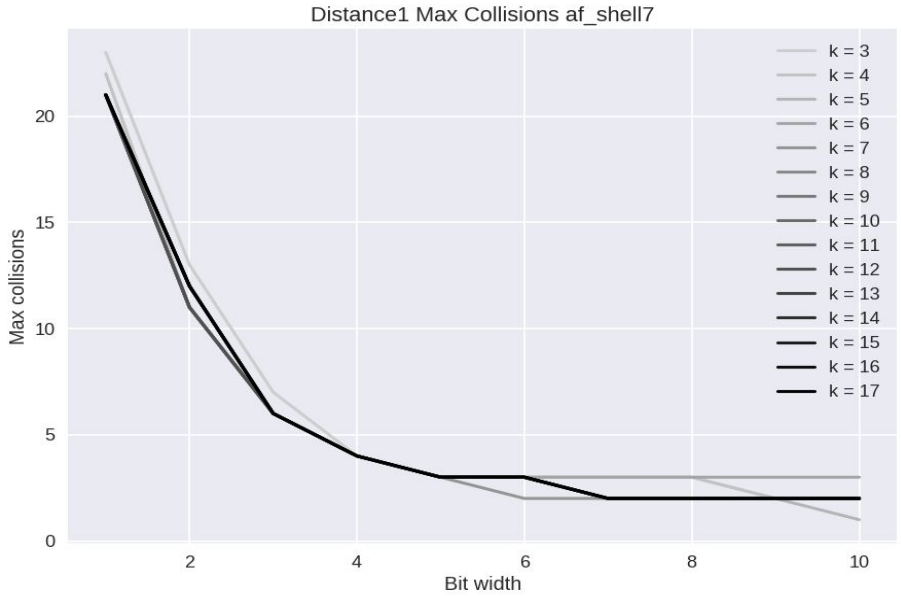
- old project: 8192 colors (k = 4)


Distance 1 af_shell7
Time [ms]
0
1
2
3
4
5
6
Distance 1 old project
Distance 1
Distance 1 cooperative launch
Distance 1 cusparse

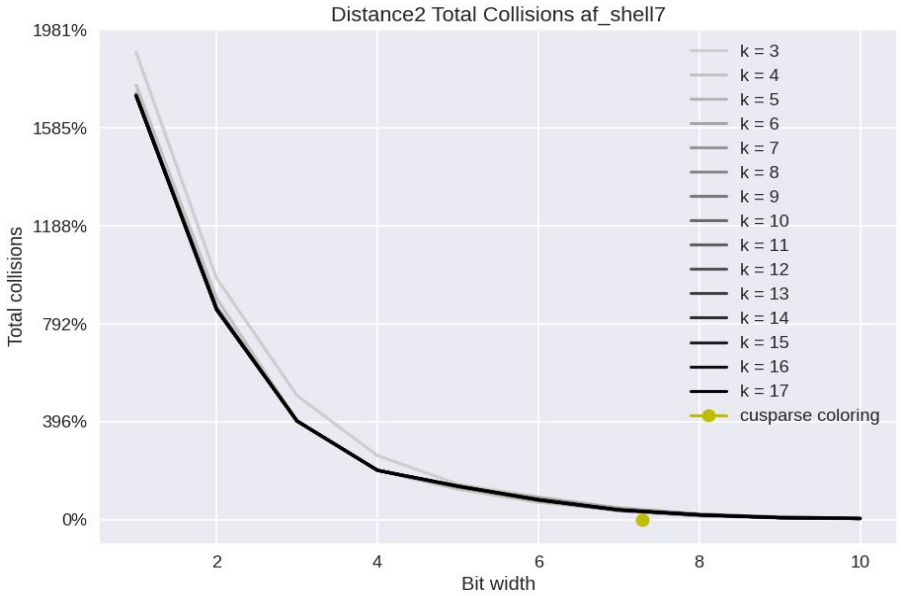

Distance 2 af_shell7
Time [s]
0.0
0.2
0.4
0.6
0.8
1.0
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 old project

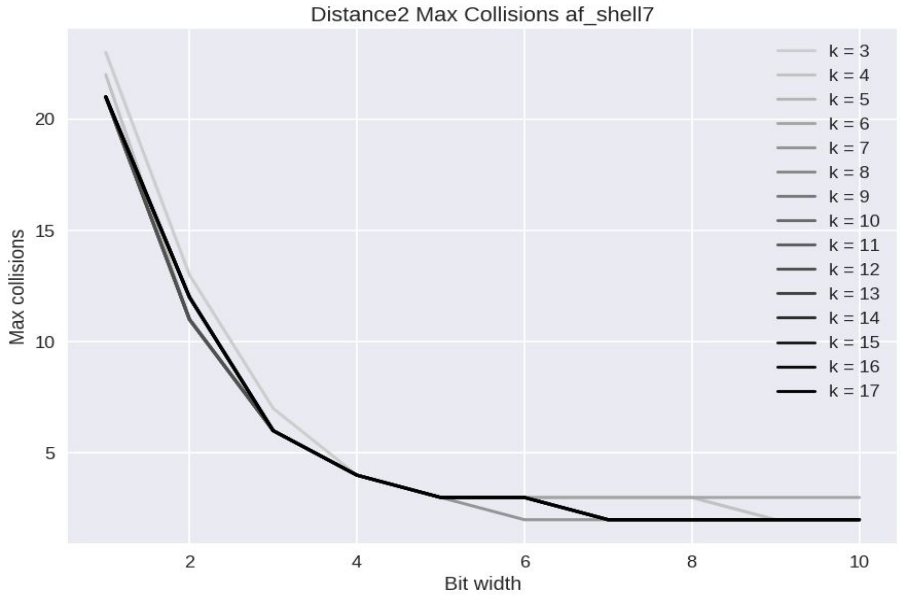

Distance 2 af_shell7
Time [ms]
0
5
10
15
20
25
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch



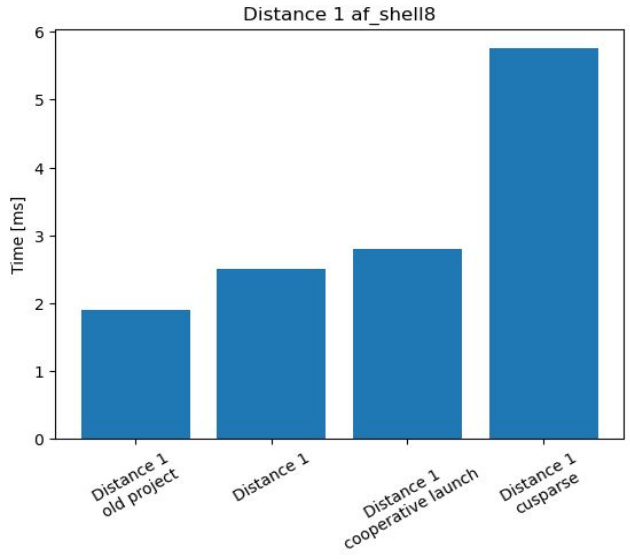


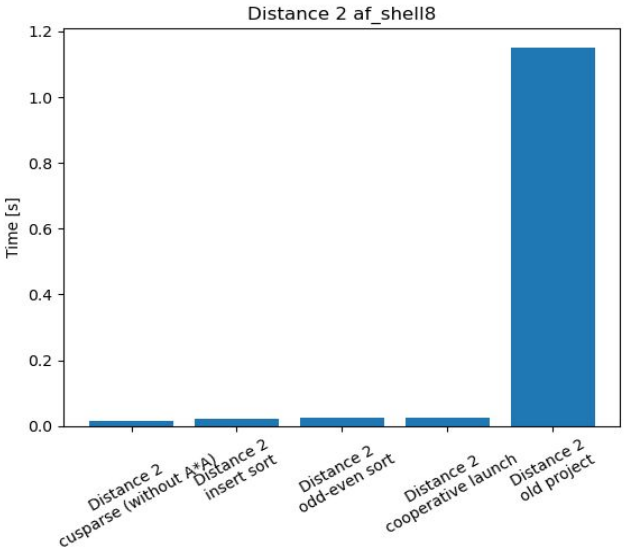
- old project: 4096 colors (k = 3)

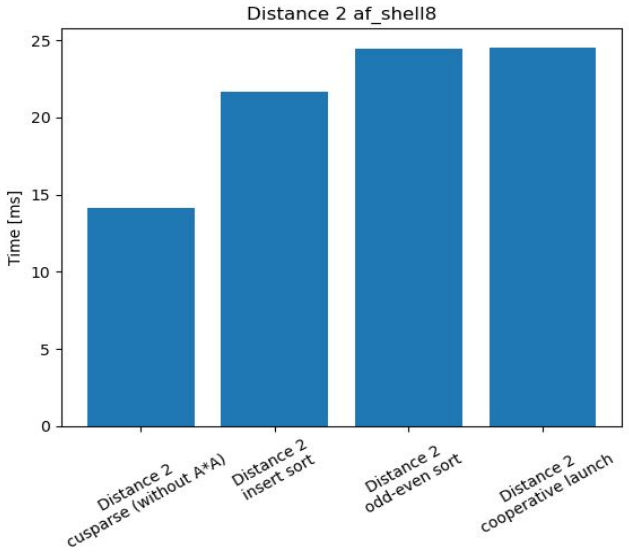


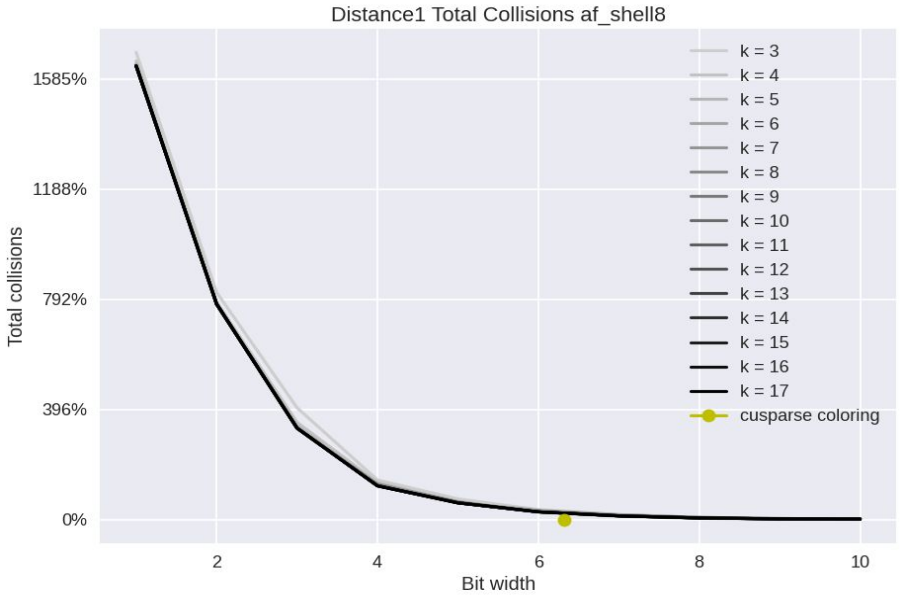


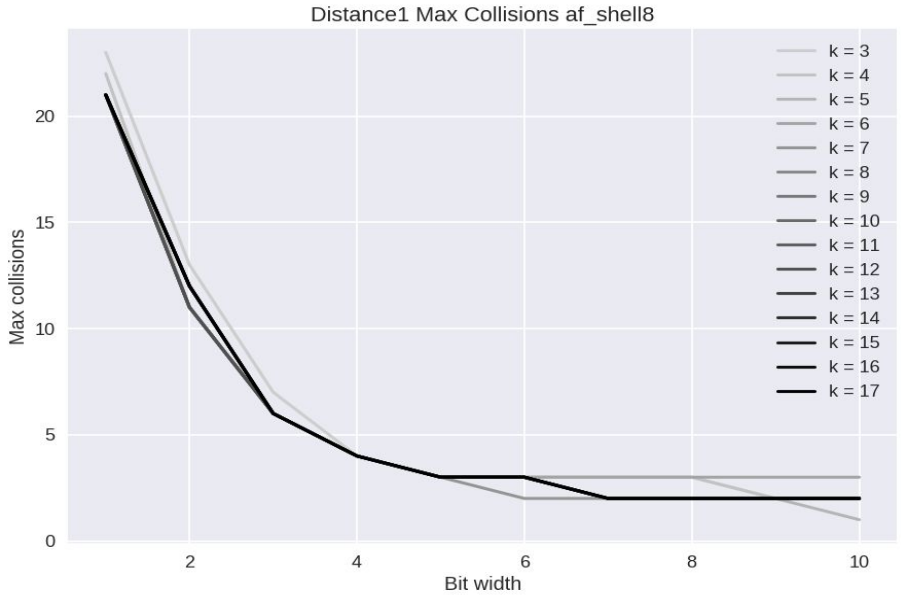
- old project: 8192 colors (k = 4)


Distance 1 af_shell8
Time [ms]
0
1
2
3
4
5
6
Distance 1 old project
Distance 1
Distance 1 cooperative launch
Distance 1 cusparse

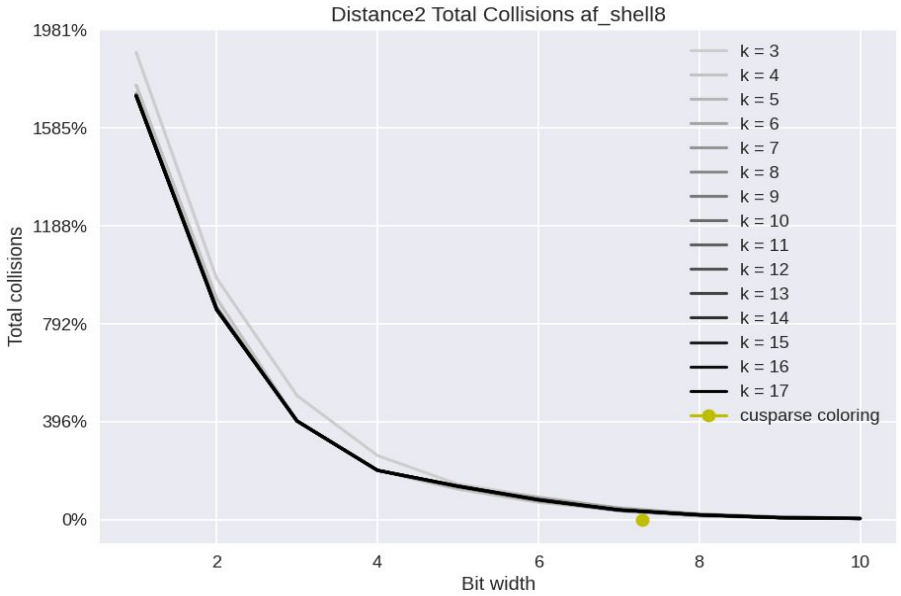

Distance 2 af_shell8
Time [s]
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 old project

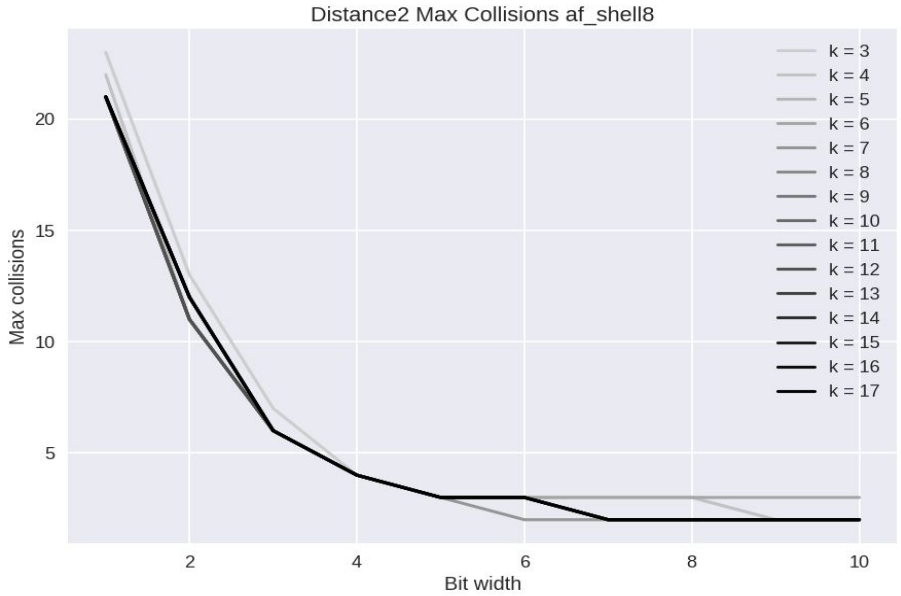

Distance 2 af_shell8
Time [ms]
0
5
10
15
20
25
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch



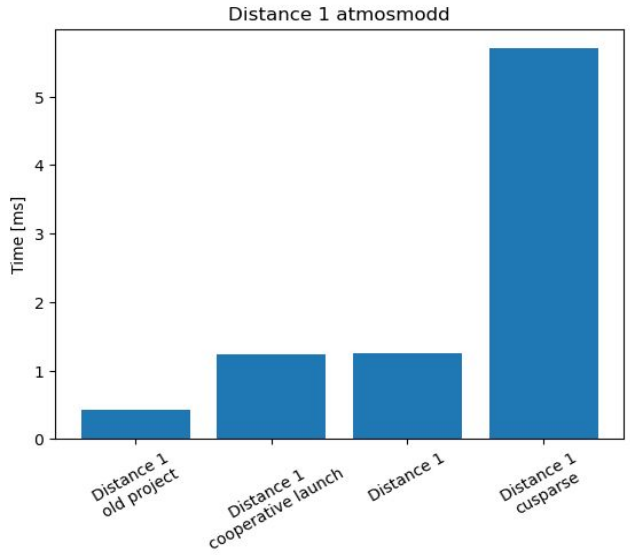


- old project: 4096 colors (k = 3)

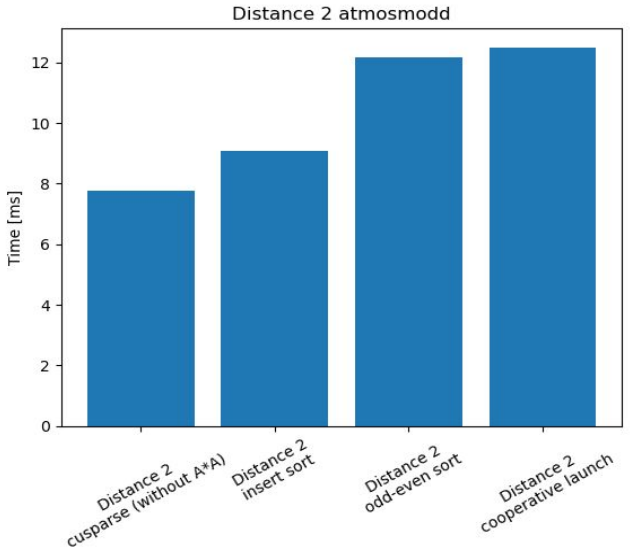


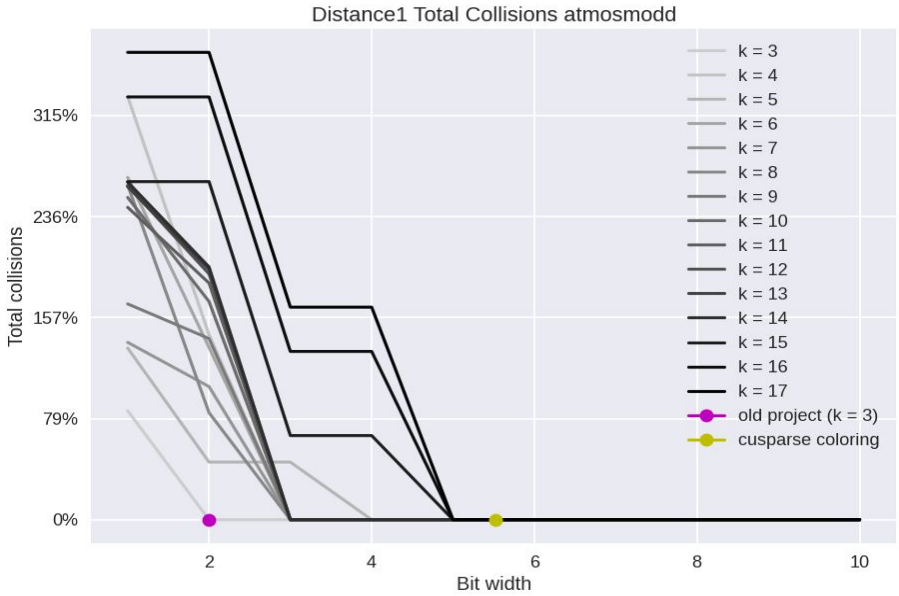


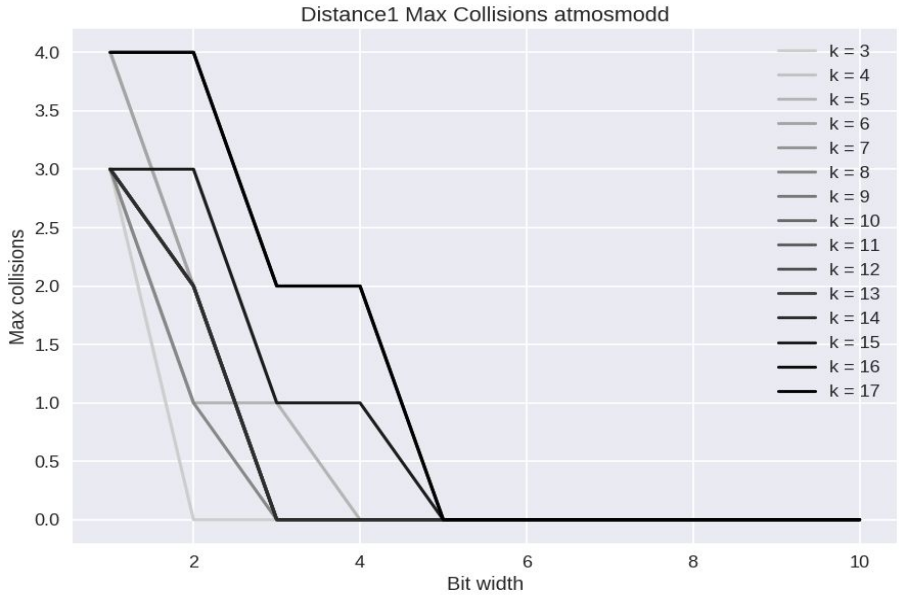
- old project: 8192 colors (k = 4)

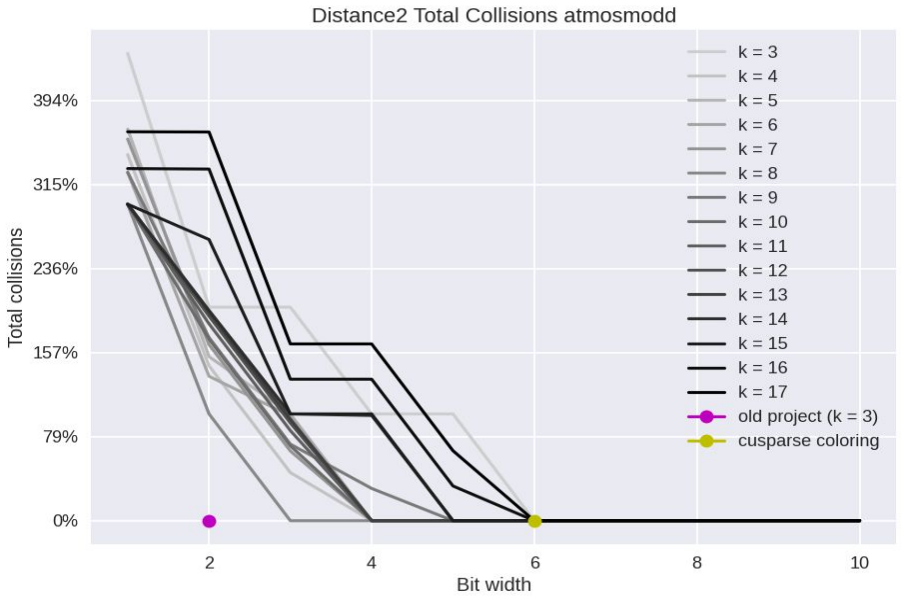

Distance 1 atmosmodd
Time [ms]
0
1
2
3
4
5
Distance 1 old project
Distance 1 cooperative launch
Distance 1
Distance 1 cusparse

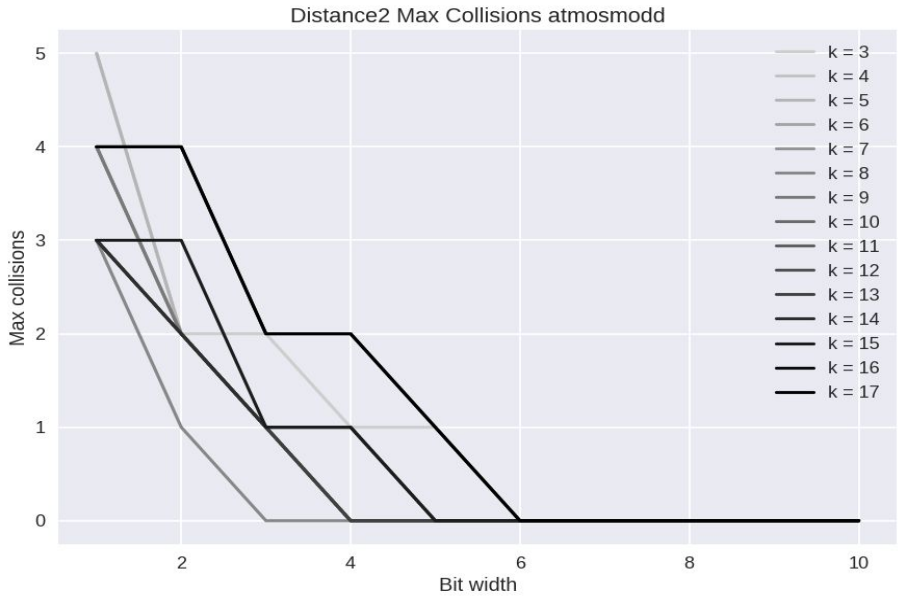

Distance 2 atmosmodd
Time [s]
0.0
0.5
1.0
1.5
2.0
2.5
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 old project

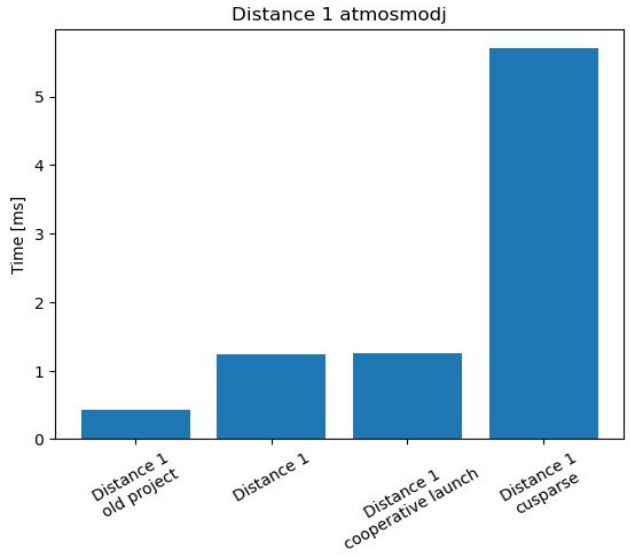

Distance 2 atmosmodd
Time [ms]
0
2
4
6
8
10
12
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch

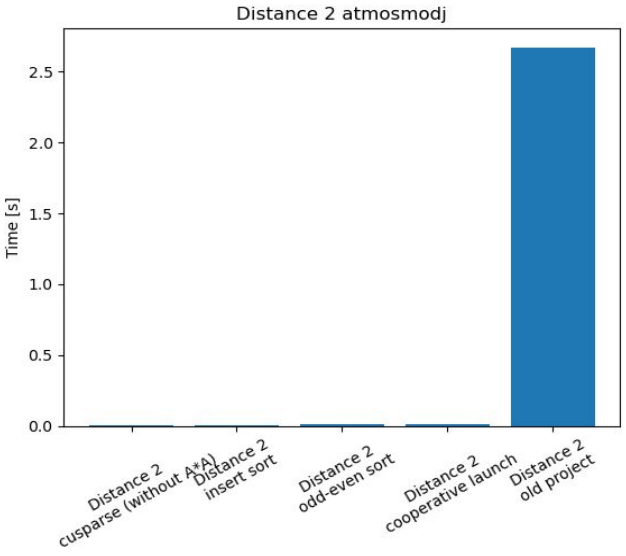

Distance1 Total Collisions atmosmodd
Total collisions
315%
236%
157%
79%
0%
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17
old project (k = 3)
cusparse coloring

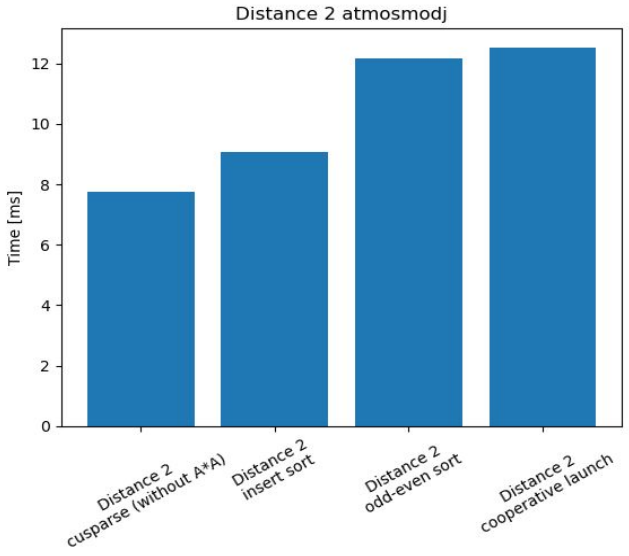

Distance1 Max Collisions atmosmodd
Max collisions
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17

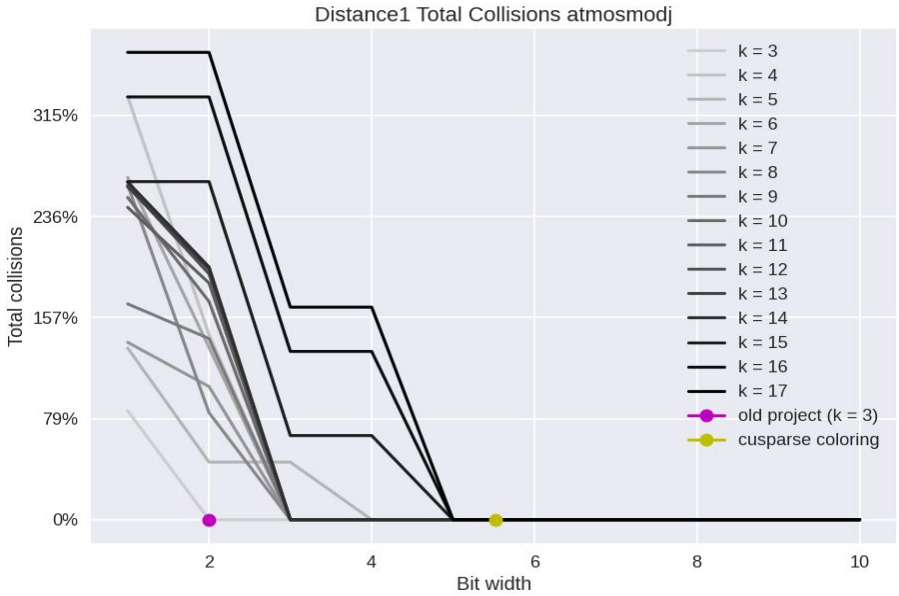

Distance2 Total Collisions atmosmodd
Total collisions
394%
315%
236%
157%
79%
0%
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17
old project (k = 3)
cusparse coloring

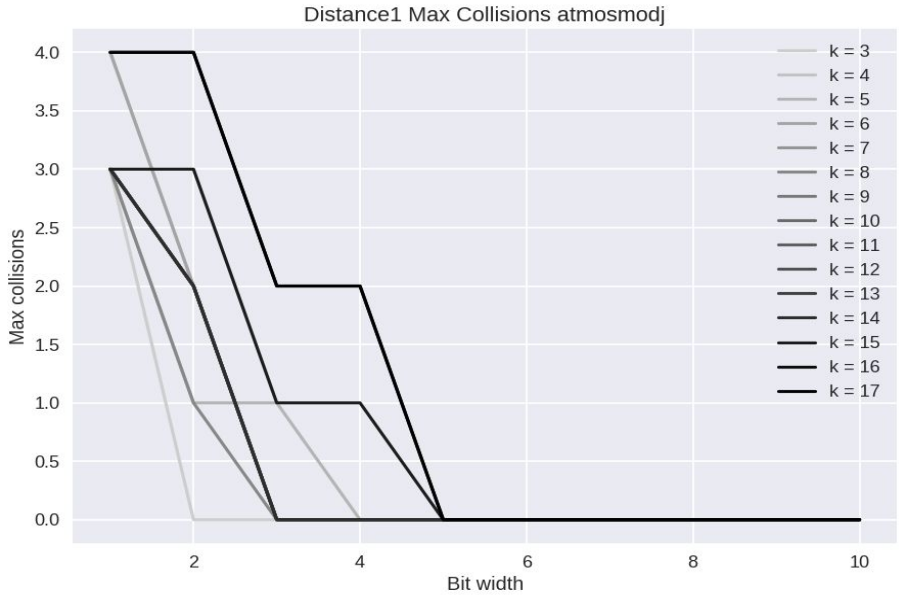

Distance2 Max Collisions atmosmodd
Max collisions
5
4
3
2
1
0
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17

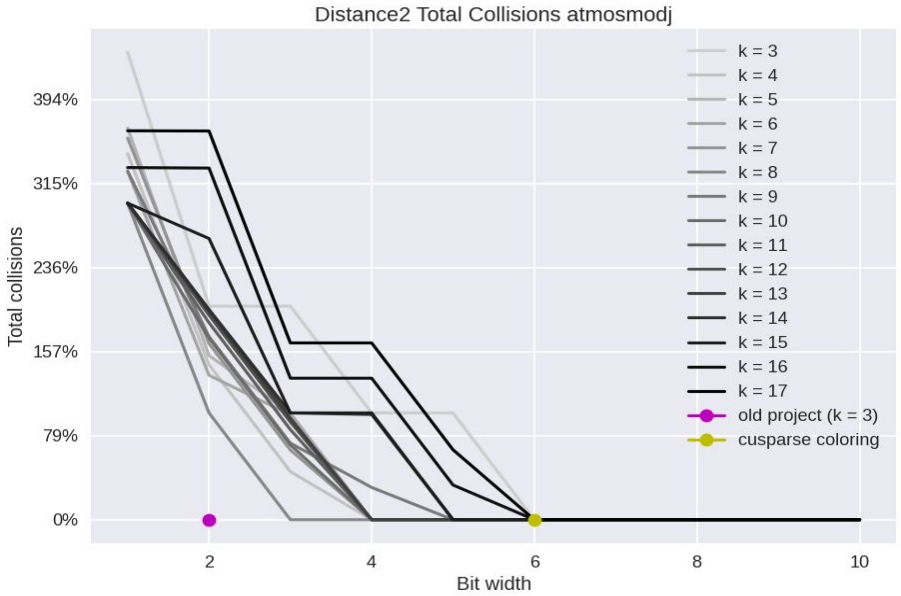

Distance 1 atmosmodj
Time [ms]
0
1
2
3
4
5
Distance 1 old project
Distance 1
Distance 1 cooperative launch
Distance 1 cusparse

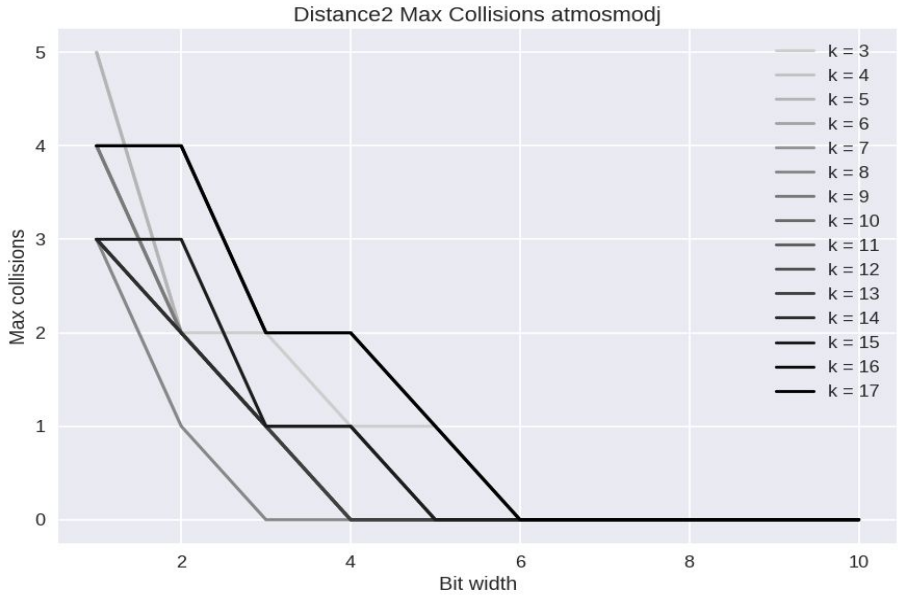

Distance 2 atmosmodj
Time [s]
0.0
0.5
1.0
1.5
2.0
2.5
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 old project

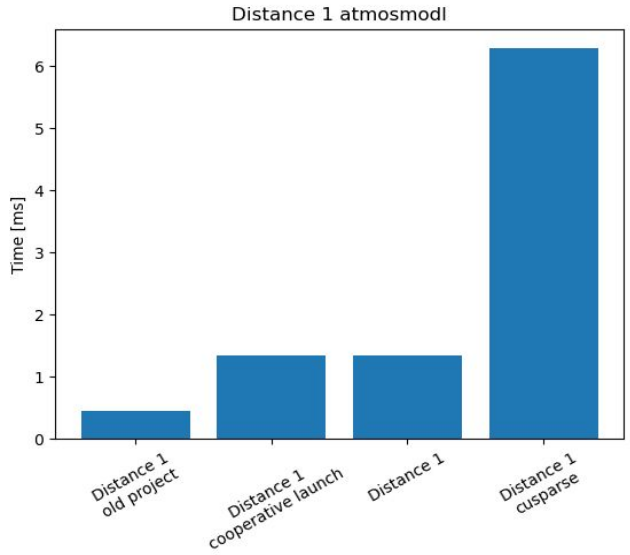

Distance 2 atmosmodj
Time [ms]
0
2
4
6
8
10
12
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch

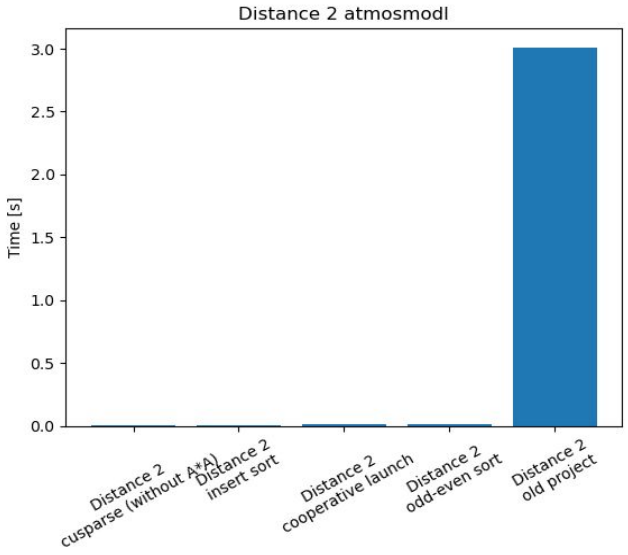

Distance1 Total Collisions atmosmodj
Total collisions
315%
236%
157%
79%
0%
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17
old project (k = 3)
cusparse coloring

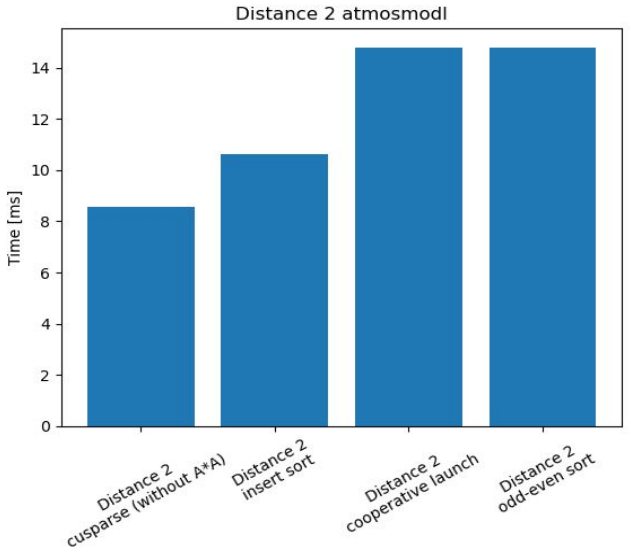

Distance1 Max Collisions atmosmodj
Max collisions
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17

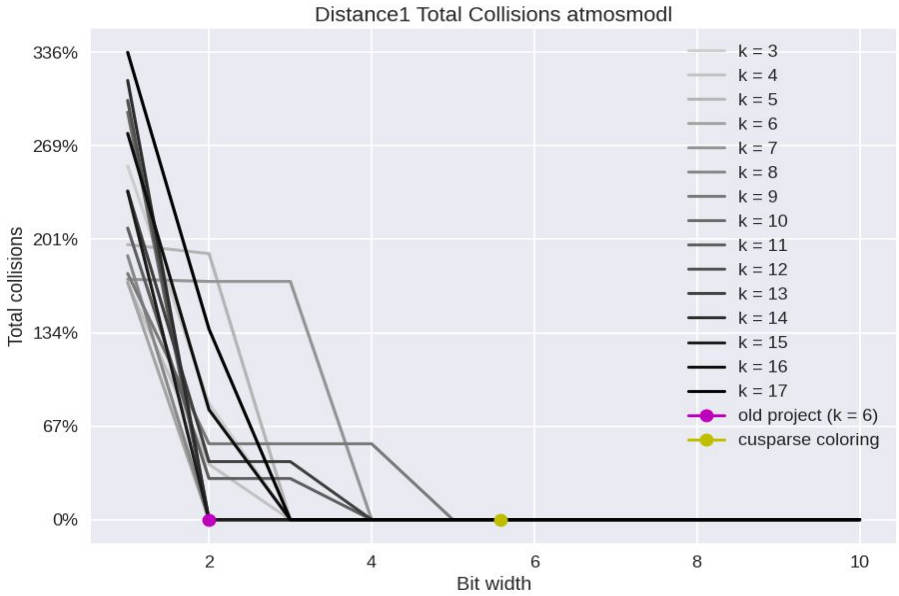

Distance2 Total Collisions atmosmodj
Total collisions
394%
315%
236%
157%
79%
0%
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17
old project (k = 3)
cusparse coloring

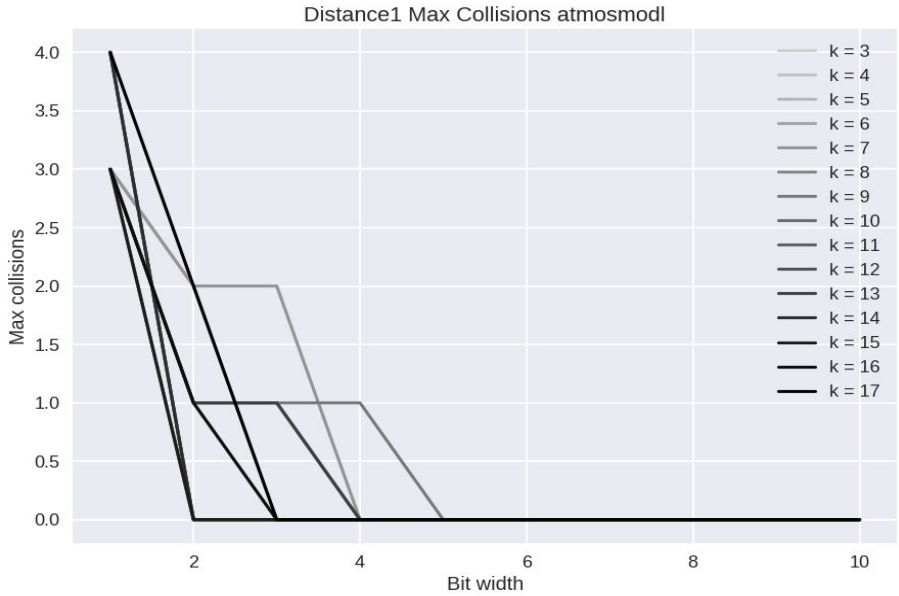

Distance2 Max Collisions atmosmodj
Max collisions
5
4
3
2
1
0
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17

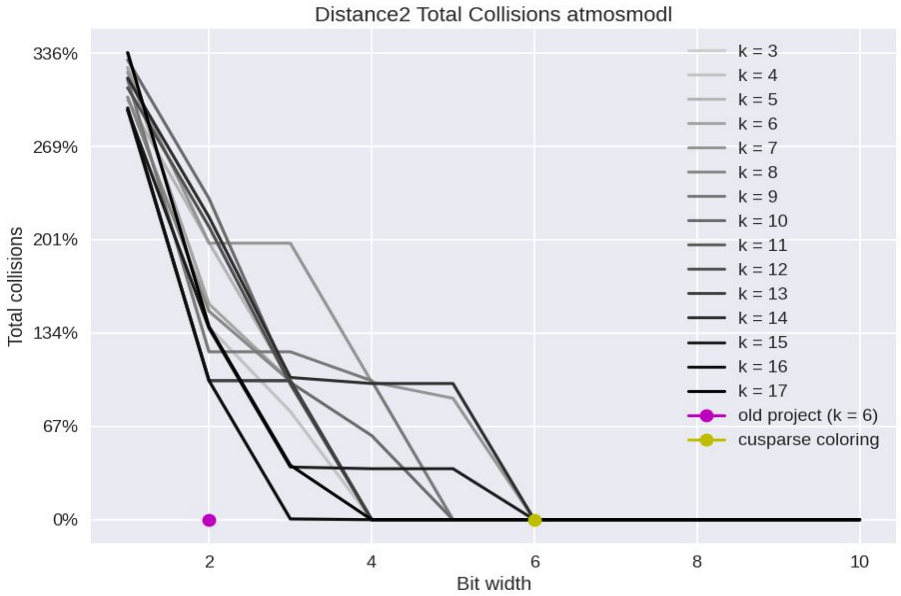

Distance 1 atmosmodl
Time [ms]
0
1
2
3
4
5
6
Distance 1 old project
Distance 1 cooperative launch
Distance 1
Distance 1 cusparse

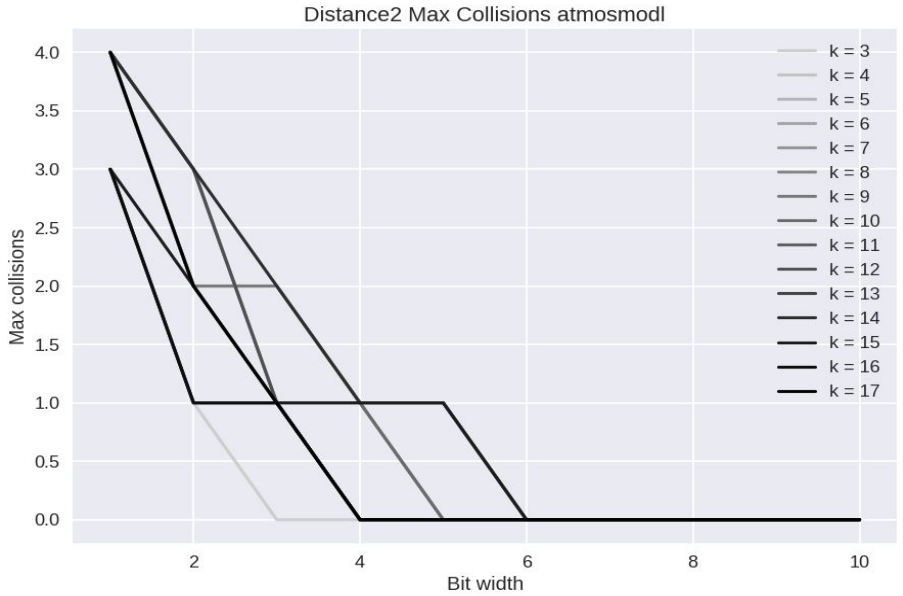

Distance 2 atmosmodl
Time [s]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 cooperative launch
Distance 2 odd-even sort
Distance 2 old project

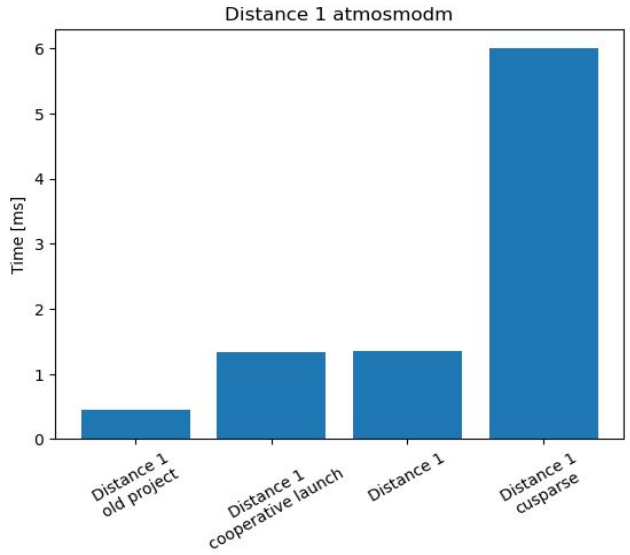

Distance 2 atmosmodl
Time [ms]
0
2
4
6
8
10
12
14
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 cooperative launch
Distance 2 odd-even sort

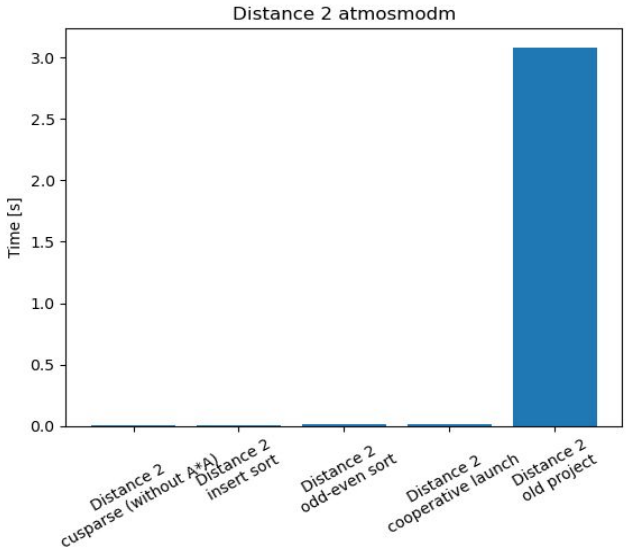

Distance1 Total Collisions atmosmodl
Total collisions
336%
269%
201%
134%
67%
0%
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17
old project (k = 6)
cusparse coloring

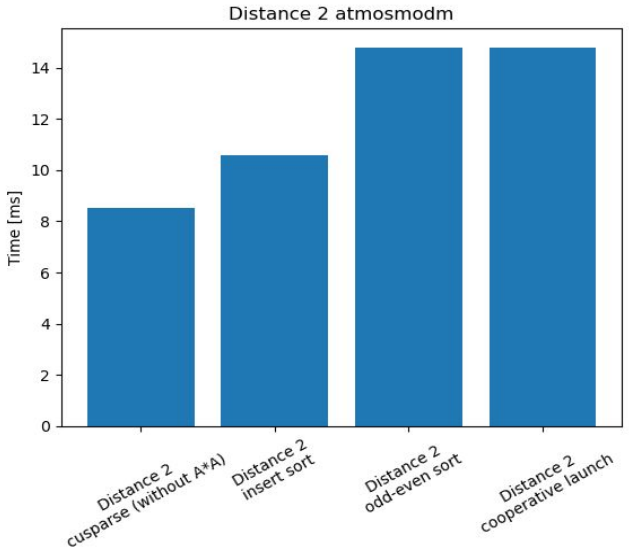

Distance1 Max Collisions atmosmodl
Max collisions
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17

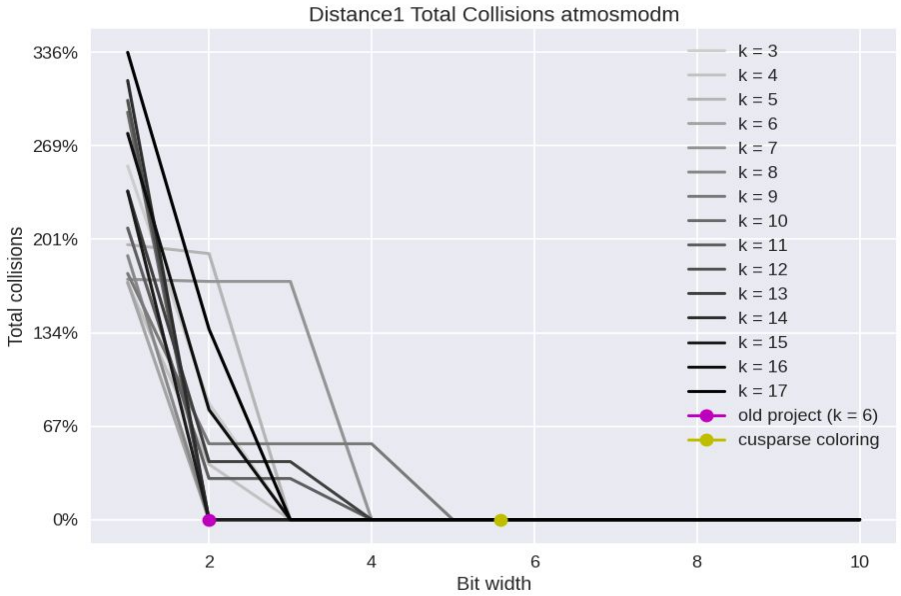

Distance2 Total Collisions atmosmodl
Total collisions
336%
269%
201%
134%
67%
0%
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17
old project (k = 6)
cusparse coloring

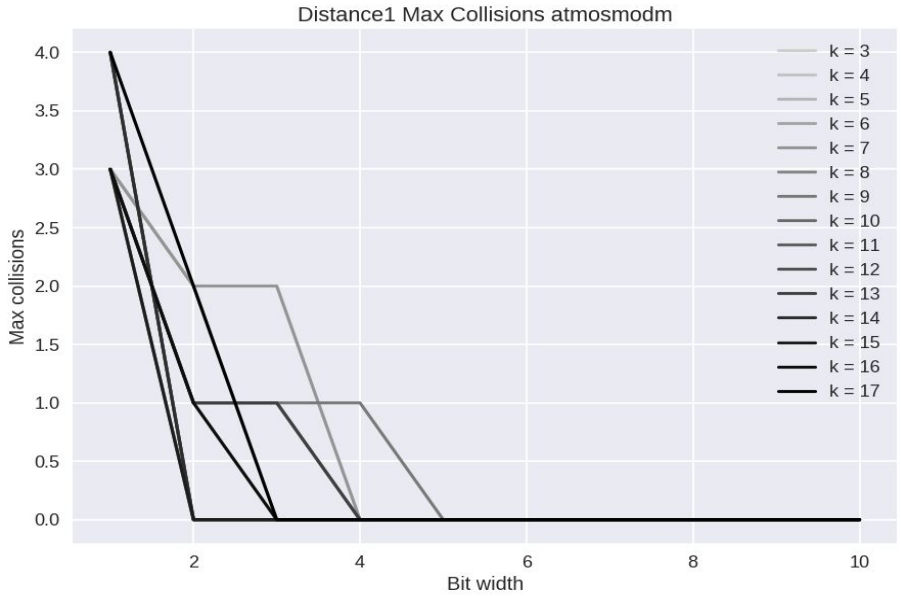

Distance2 Max Collisions atmosmodl
Max collisions
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17

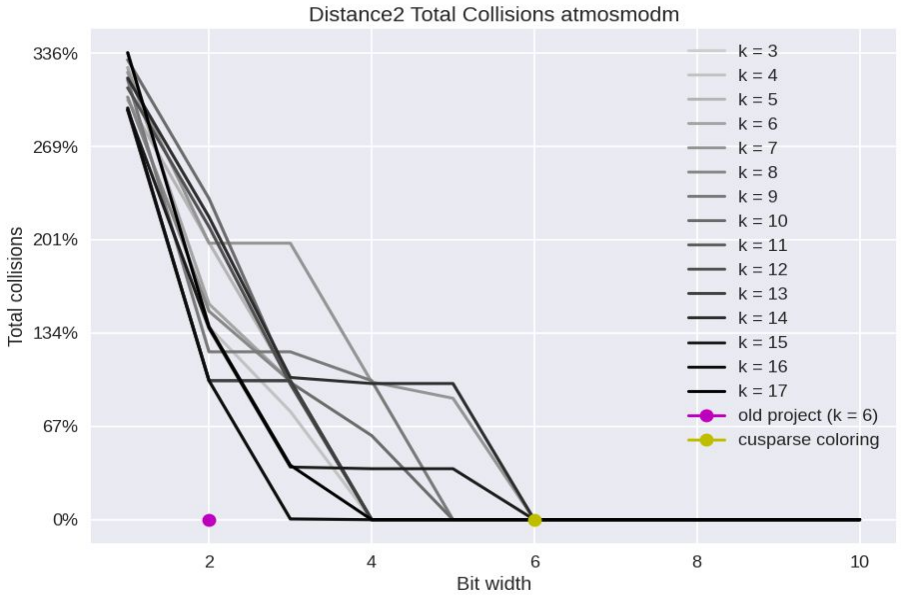

Distance 1 atmosmodm
Time [ms]
0
1
2
3
4
5
6
Distance 1 old project
Distance 1 cooperative launch
Distance 1
Distance 1 cusparse

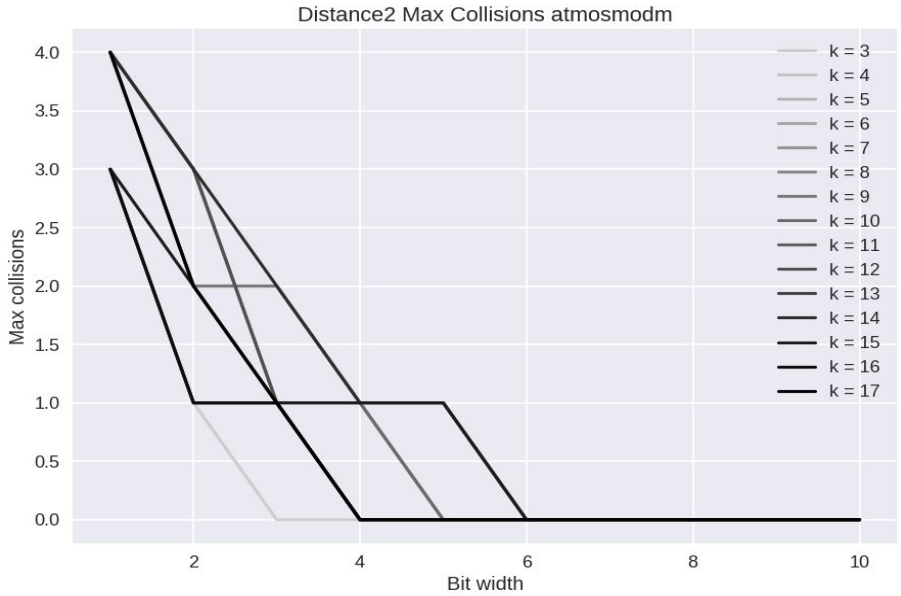

Distance 2 atmosmodm
Time [s]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 old project

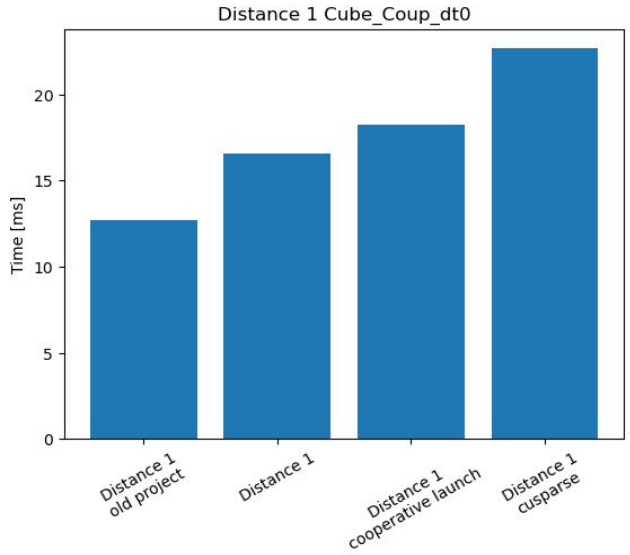

Distance 2 atmosmodm
Time [ms]
0
2
4
6
8
10
12
14
Distance 2 cusparse (without A*A)
Distance 2 insert sort
Distance 2 odd-even sort
Distance 2 cooperative launch

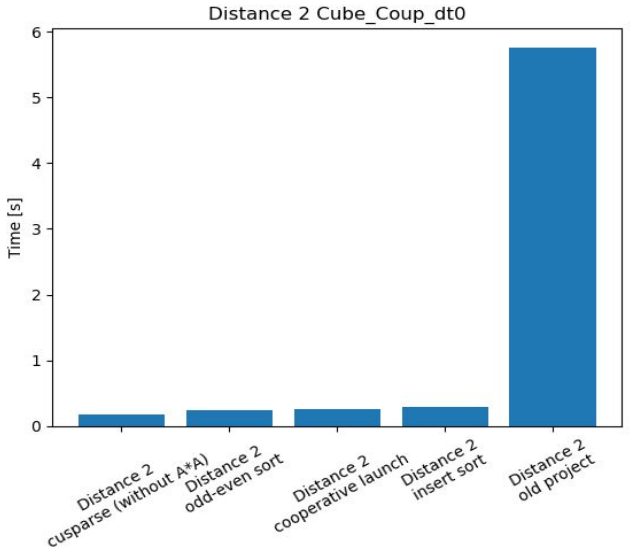

Distance1 Total Collisions atmosmodm
Total collisions
336%
269%
201%
134%
67%
0%
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17
old project (k = 6)
cusparse coloring

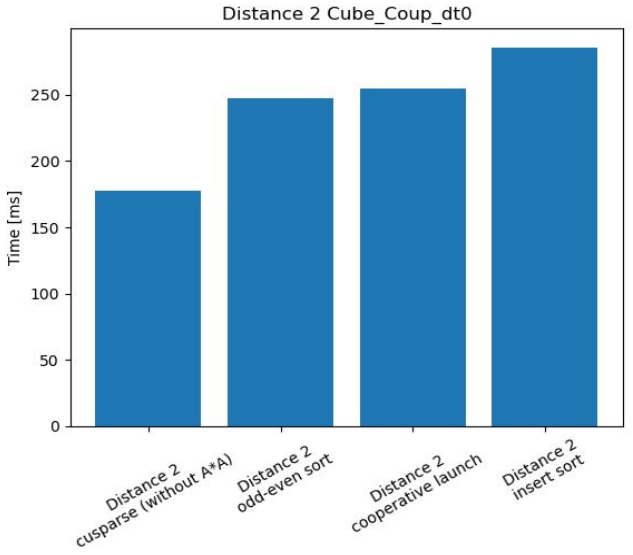

Distance1 Max Collisions atmosmodm
Max collisions
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17

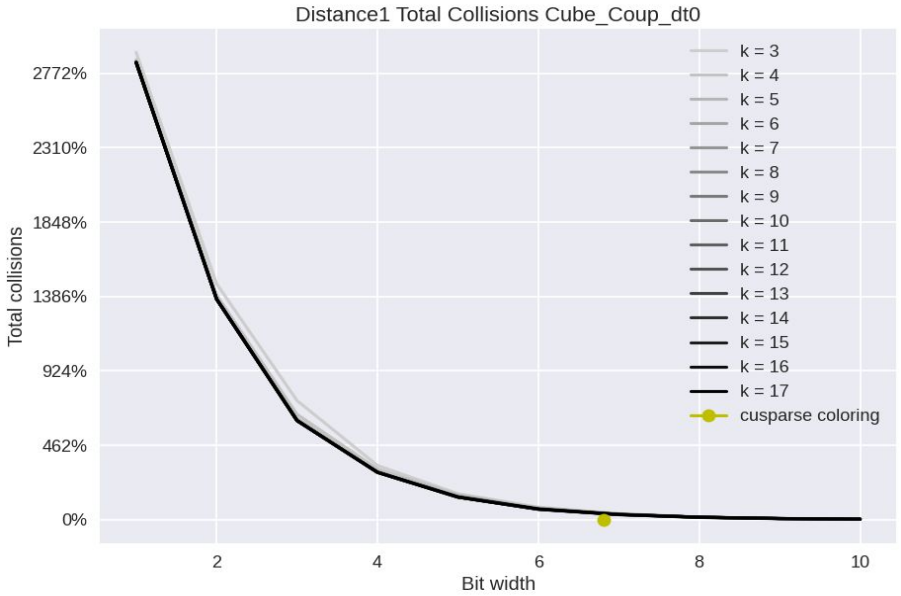

Distance2 Total Collisions atmosmodm
Total collisions
336%
269%
201%
134%
67%
0%
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17
old project (k = 6)
cusparse coloring

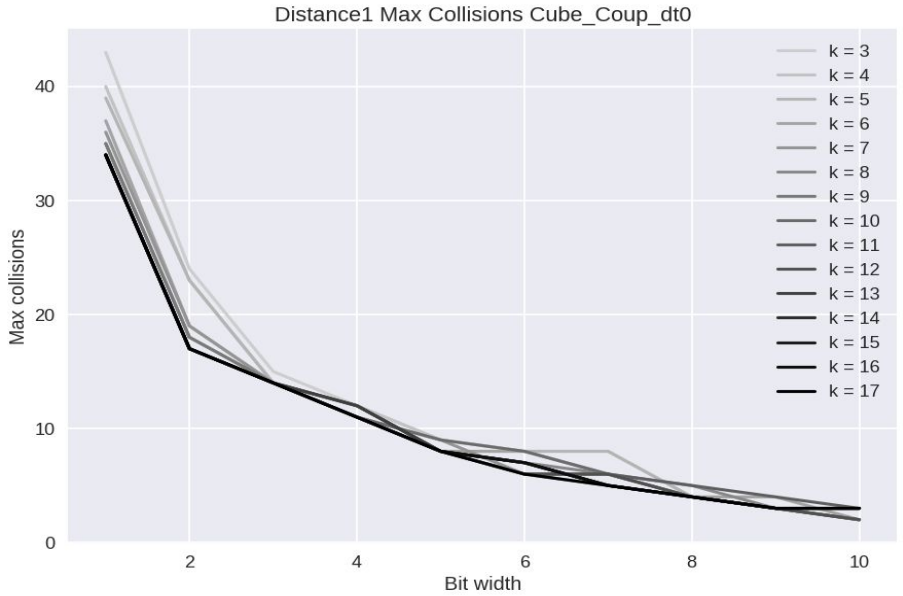

Distance2 Max Collisions atmosmodm
Max collisions
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2
4
6
8
10
Bit width
k = 3
k = 4
k = 5
k = 6
k = 7
k = 8
k = 9
k = 10
k = 11
k = 12
k = 13
k = 14
k = 15
k = 16
k = 17


Distance 1 Cube_Coup_dt0
Time [ms]
0
5
10
15
20
Distance 1 old project
Distance 1
Distance 1 cooperative launch
Distance 1 cusparse

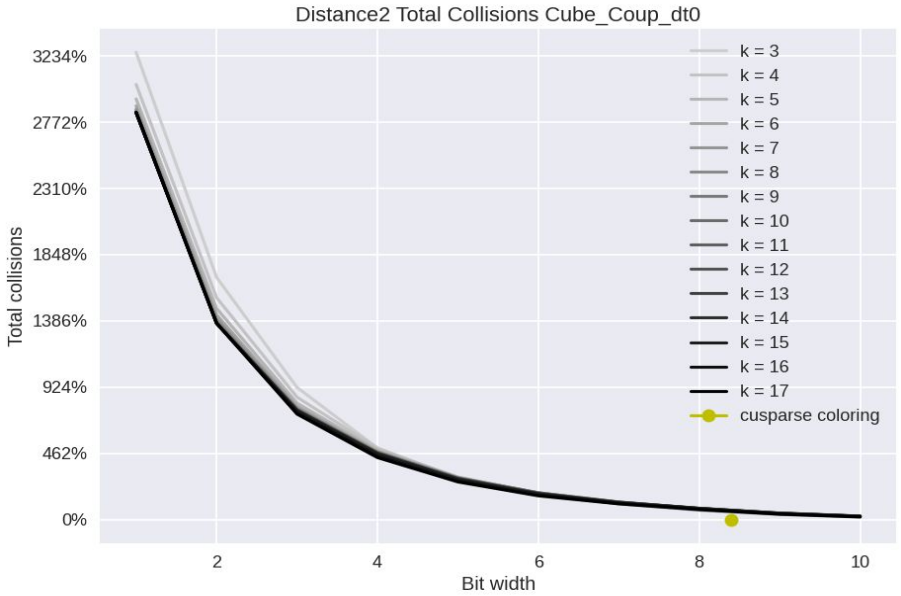

Distance 2 Cube_Coup_dt0
Time [s]
0
1
2
3
4
5
6
Distance 2 cusparse (without A*A)
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 insert sort
Distance 2 old project

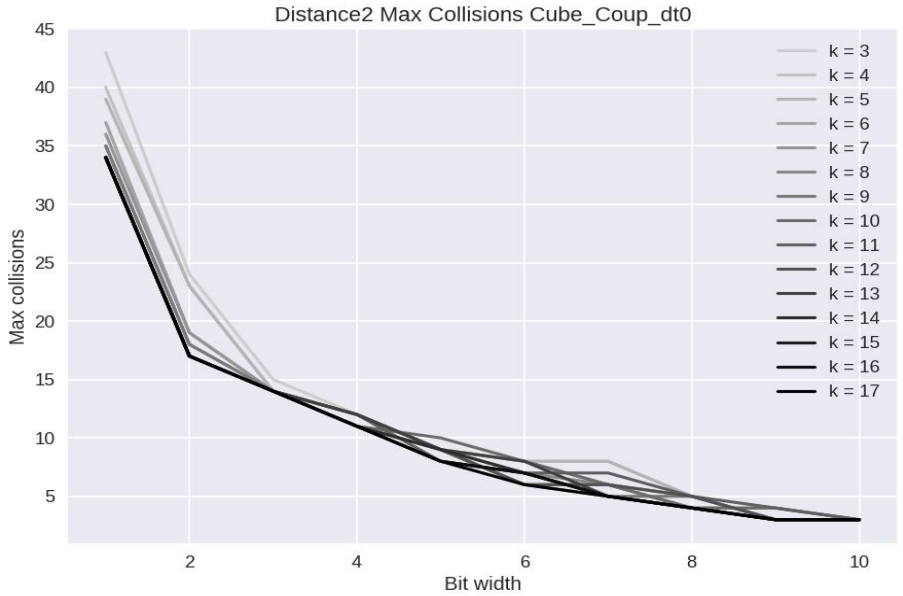

Distance 2 Cube_Coup_dt0
Time [ms]
0
50
100
150
200
250
Distance 2 cusparse (without A*A)
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 insert sort



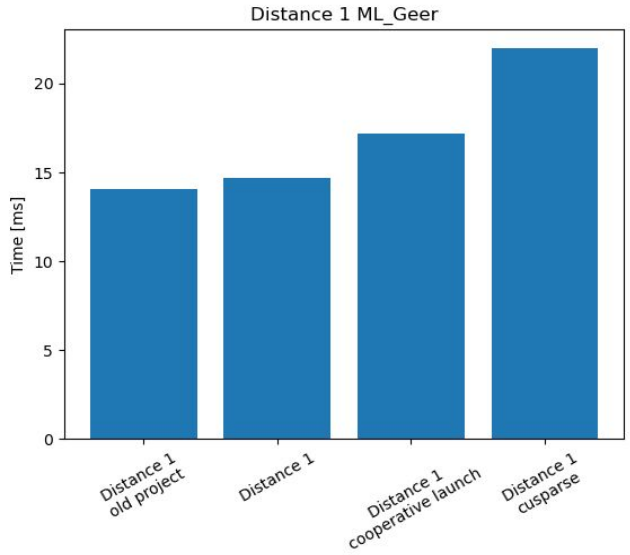


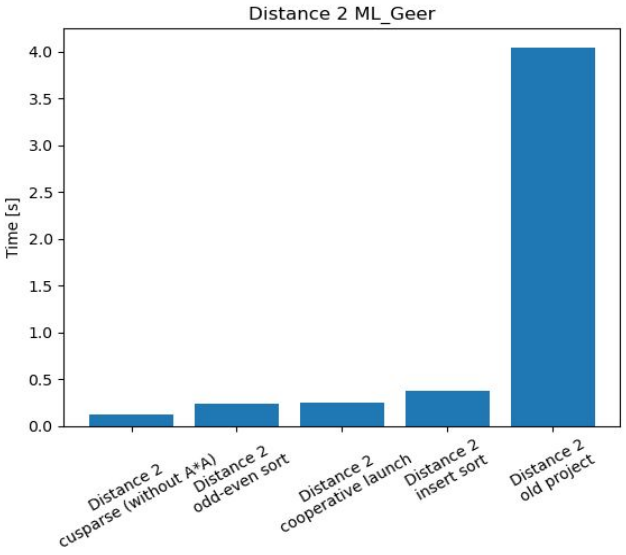
- old project: 32768 colors (k = 5)

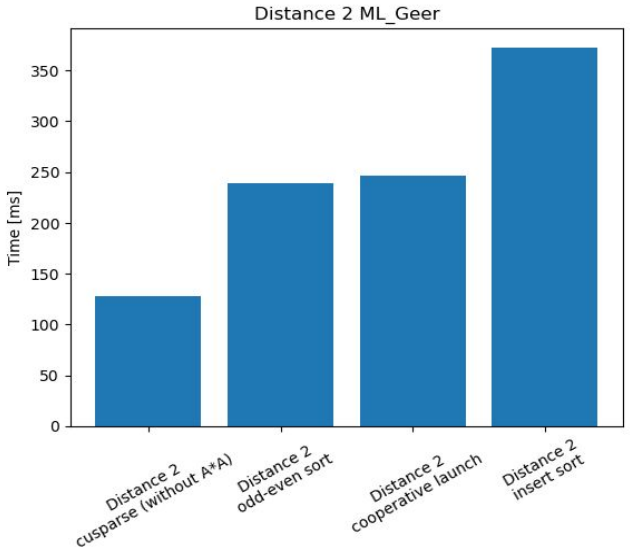


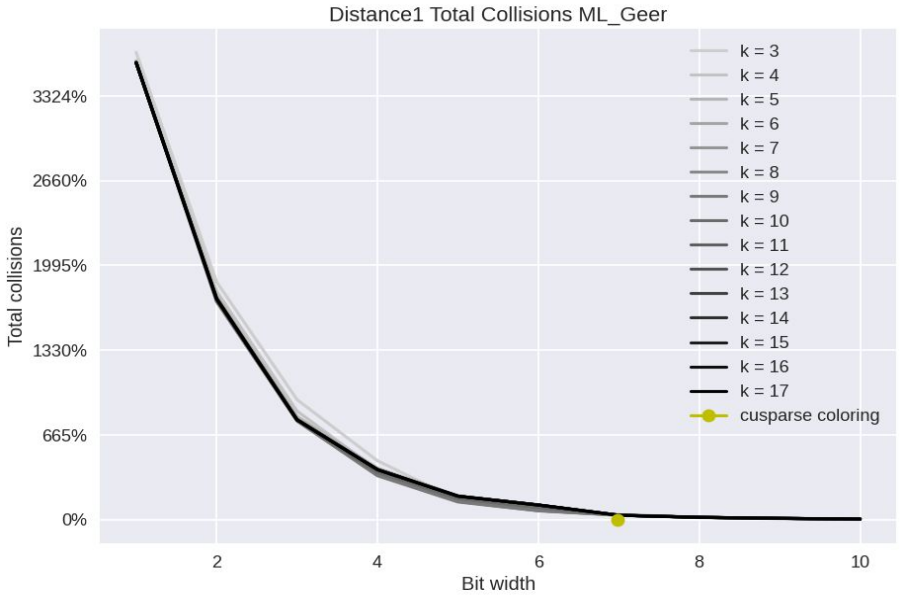


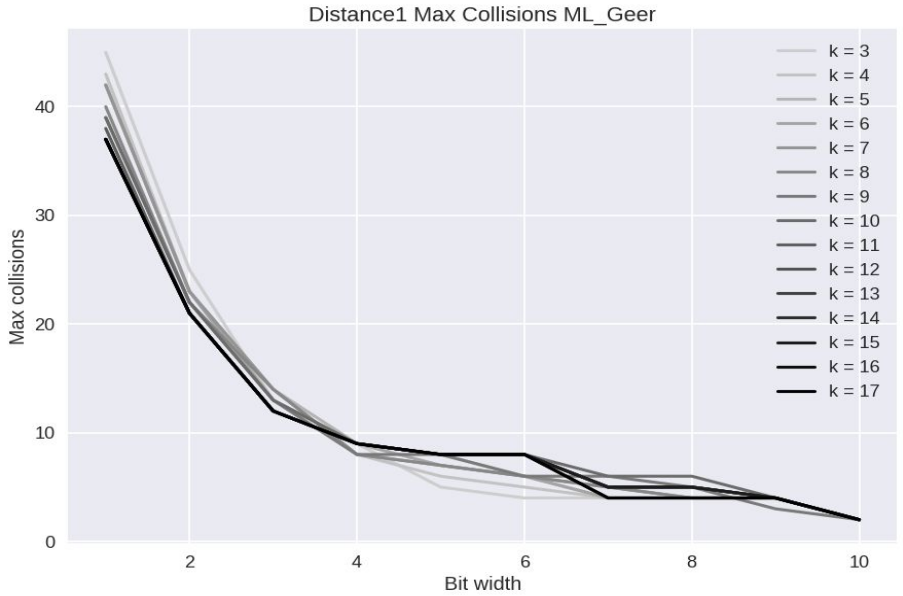
- old project: 65536 colors (k = 5)


Distance 1 ML_Geer
Time [ms]
0
5
10
15
20
Distance 1 old project
Distance 1
Distance 1 cooperative launch
Distance 1 cusparse

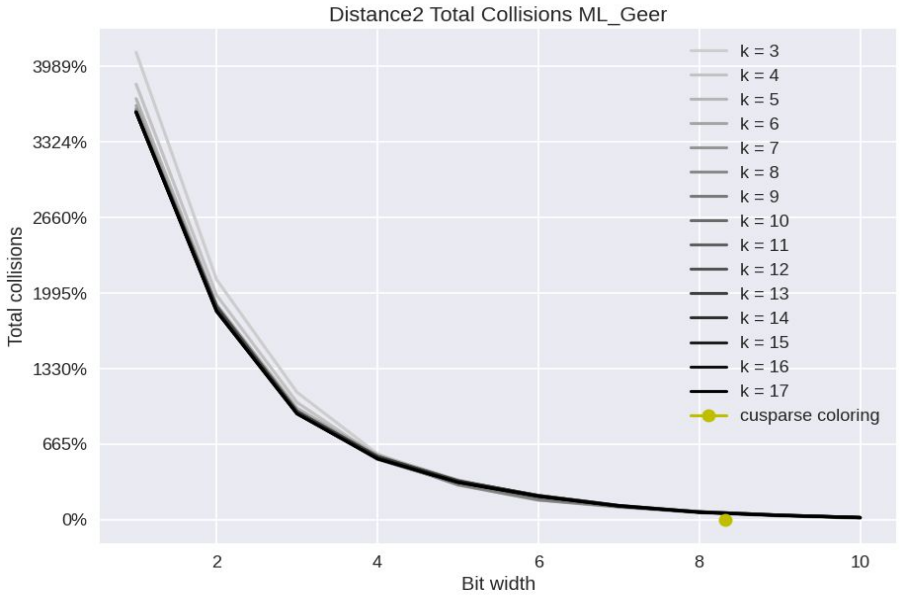

Distance 2 ML_Geer
Time [s]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Distance 2 cusparse (without A*A)
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 insert sort
Distance 2 old project

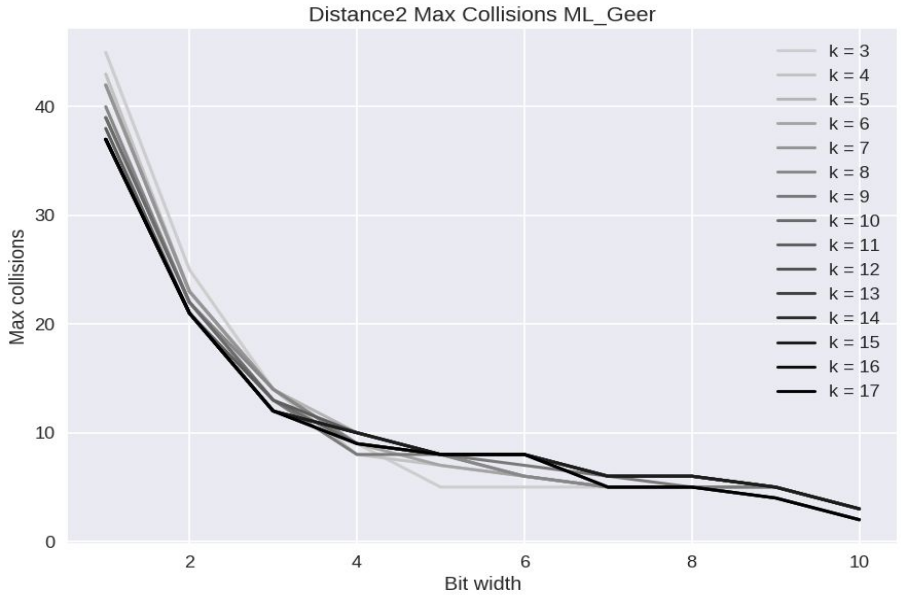

Distance 2 ML_Geer
Time [ms]
0
50
100
150
200
250
300
350
Distance 2 cusparse (without A*A)
Distance 2 odd-even sort
Distance 2 cooperative launch
Distance 2 insert sort





- old project: 8192 colors (k = 3)





- old project: 16384 colors (k = 3)