

TimeSeries

September 20, 2020

0.1 Use of linear Regression to predict a time serie

V1

Min. :10.05

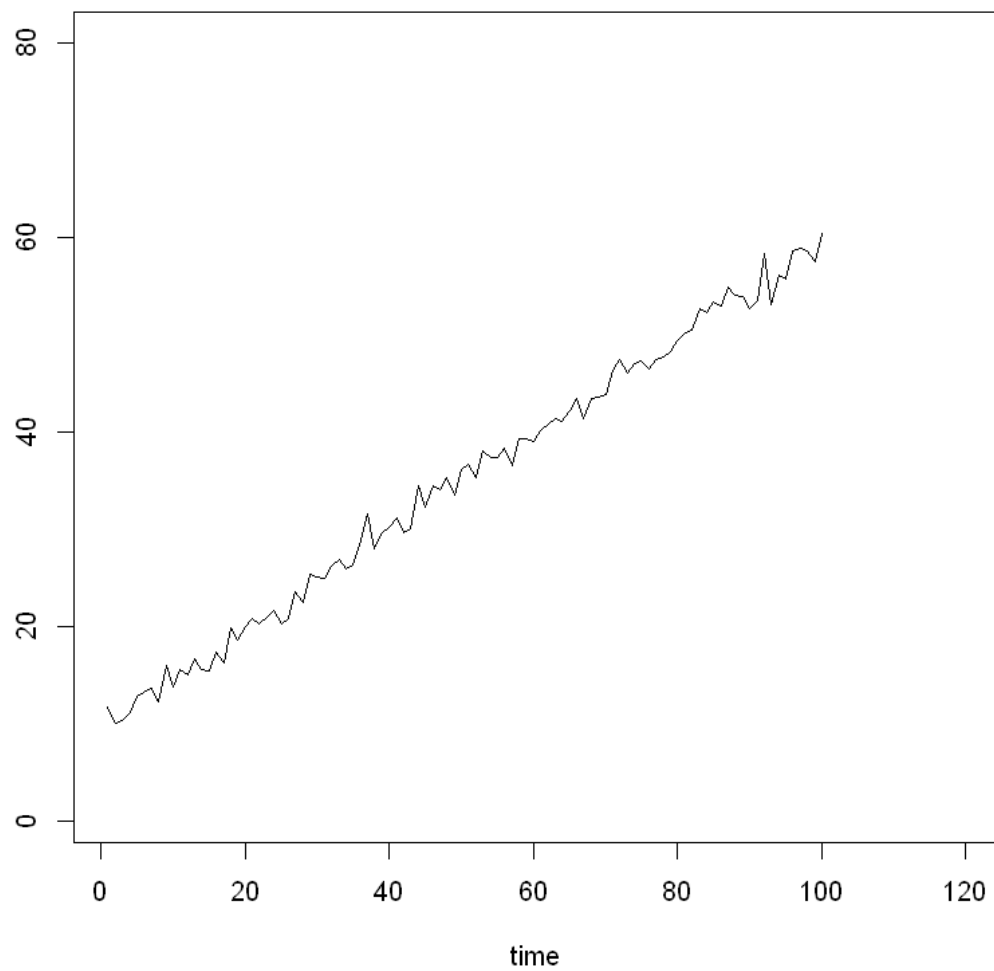
1st Qu.:21.53

Median :35.75

Mean :35.00

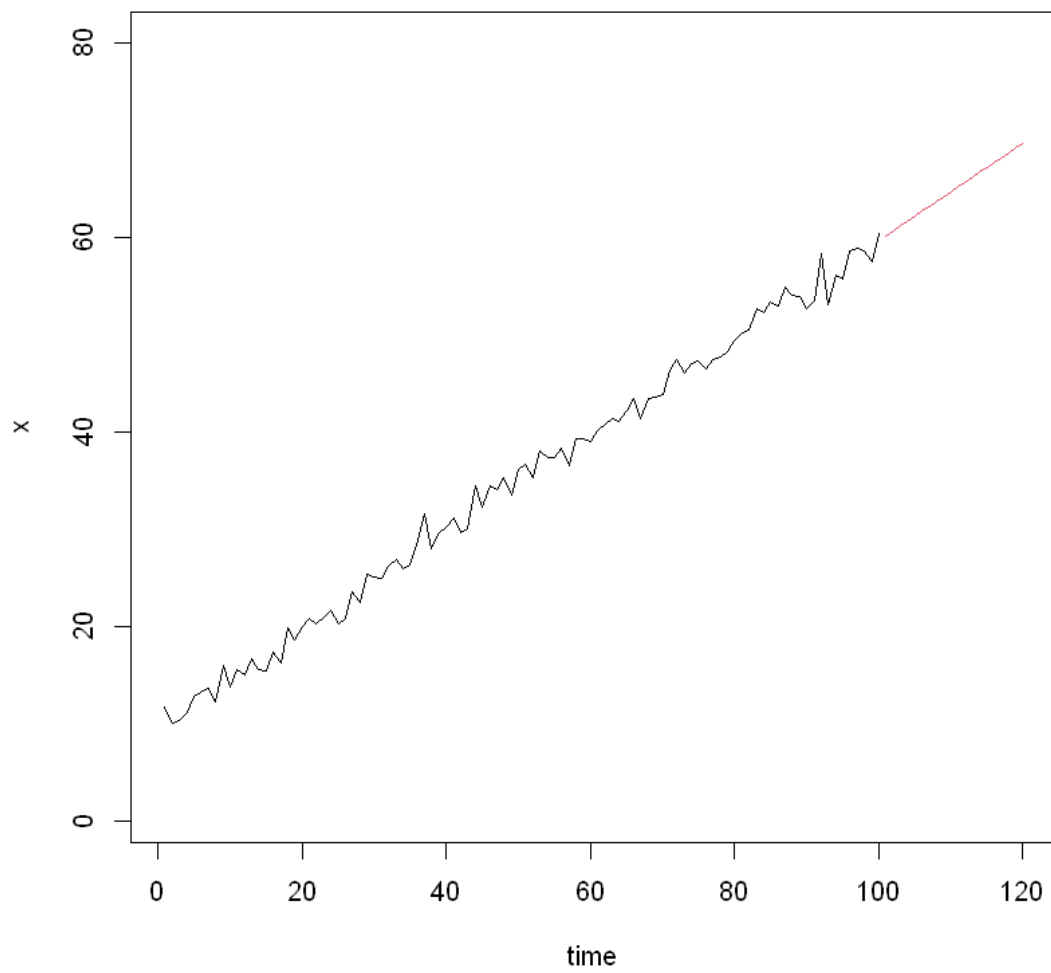
3rd Qu.:47.33

Max. :60.36



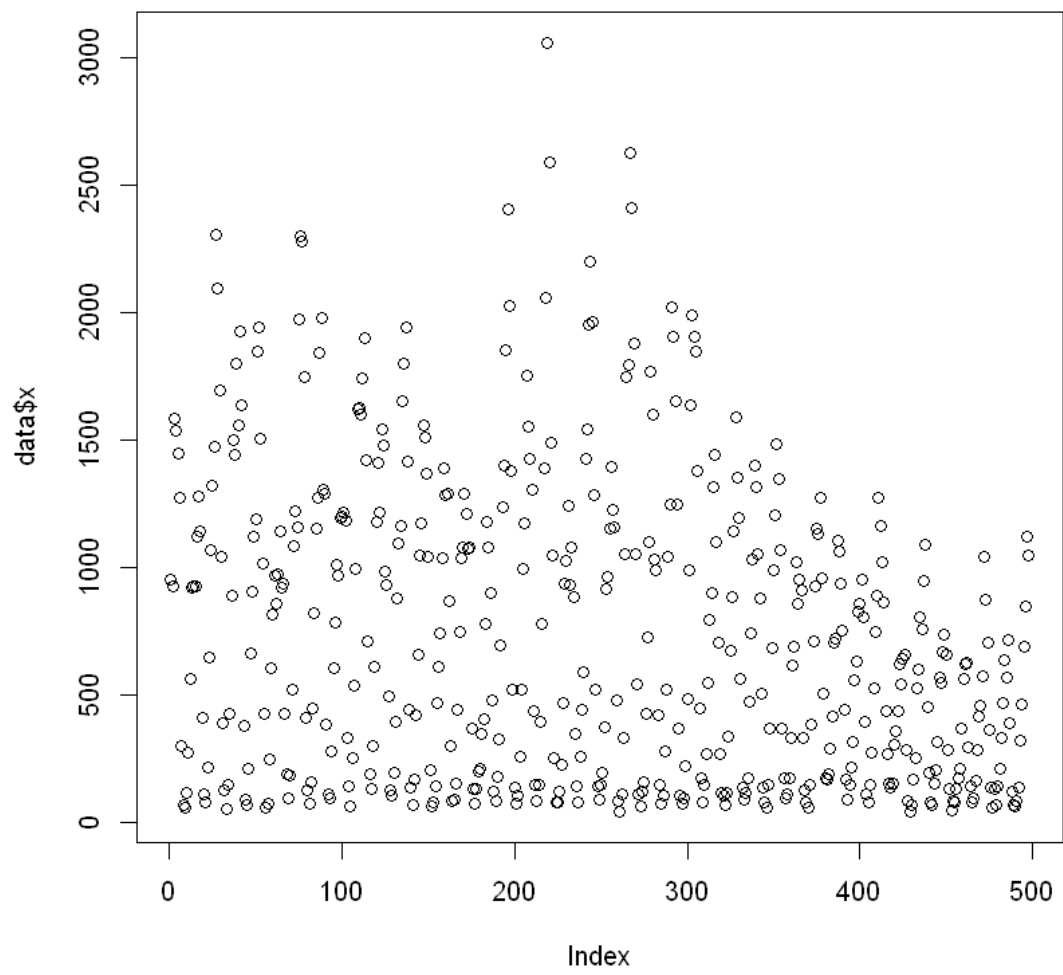
A data.frame: 6 × 1

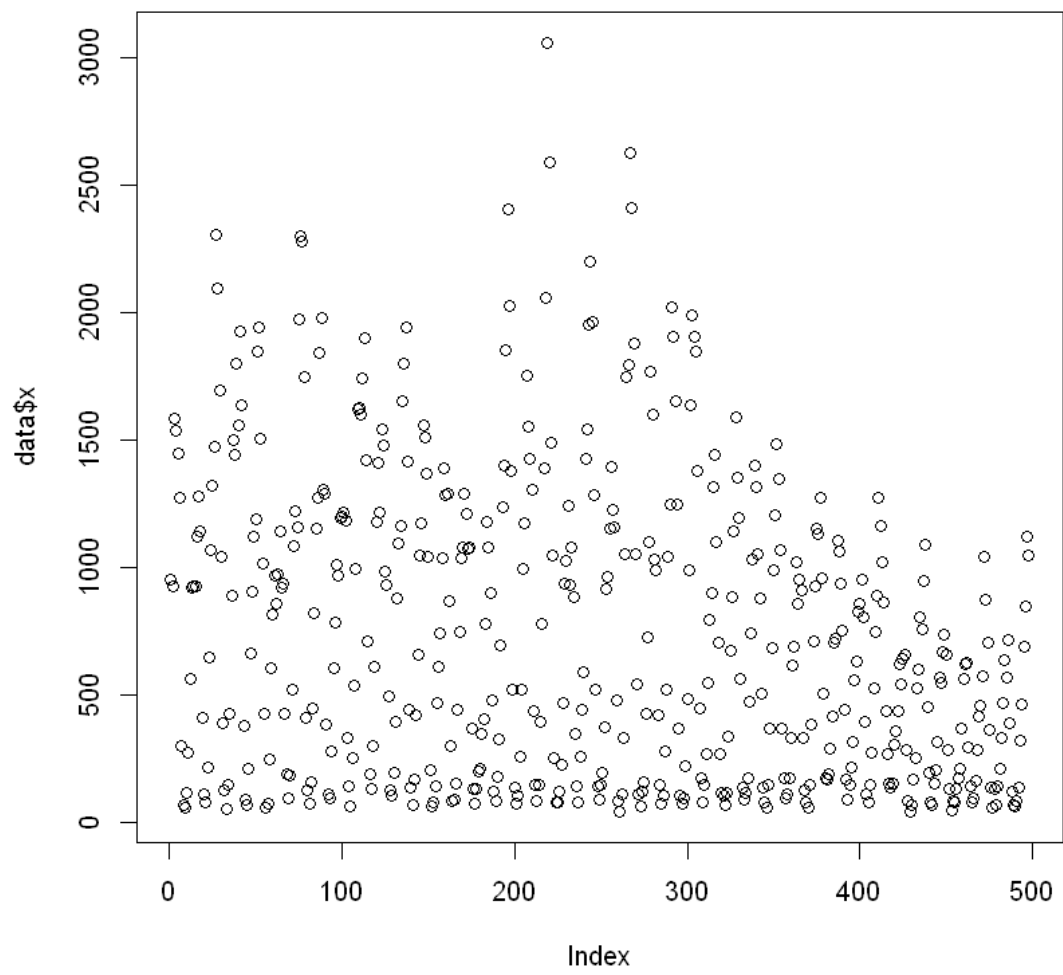
	V1 <dbl>
1	11.71048
2	10.05097
3	10.37200
4	11.24996
5	12.86221
6	13.24769

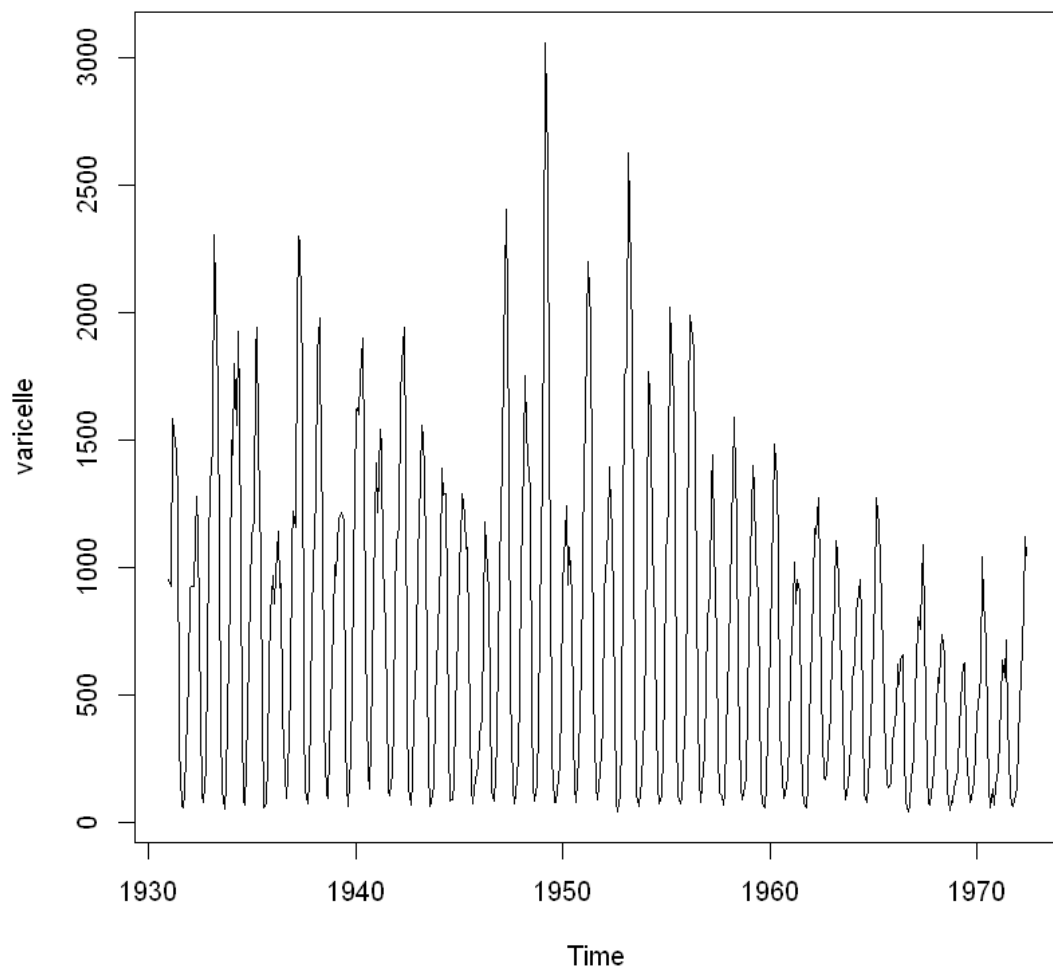


0.2 Creation of a Time Serie

Time-Series [1:144] from 1949 to 1961: 112 118 132 129 121 135 148 148 136 119
 ...







Registered S3 method overwritten by 'quantmod':

```
method      from
as.zoo.data.frame zoo
```

Error in autoplot(varicelle): objet 'varicelle' introuvable
Traceback:

```
1. autoplot(varicelle)
```

Installing package into 'C:/Users/erick/R'

(as 'lib' is unspecified)

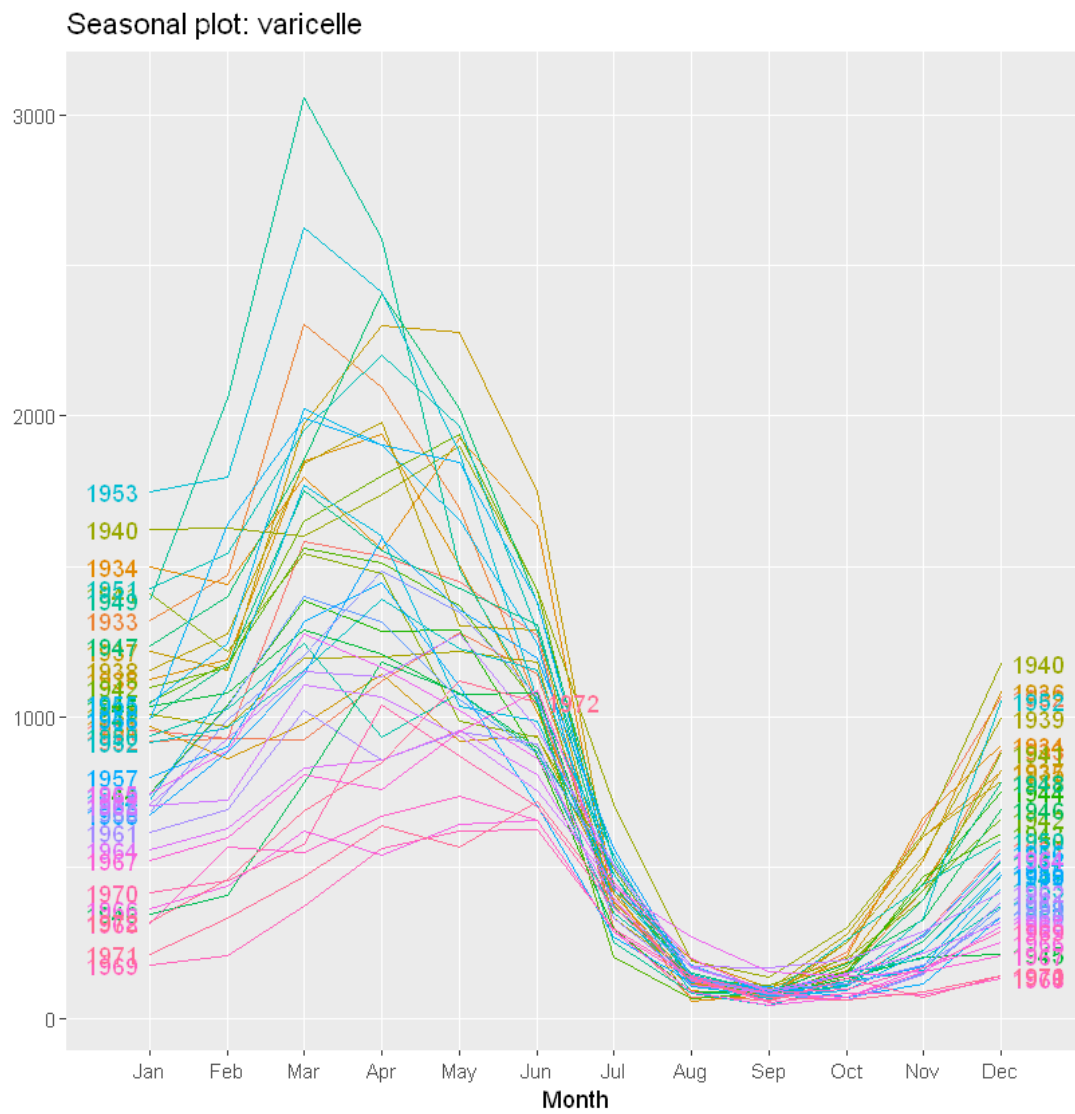
also installing the dependencies 'xts', 'TTR', 'quadprog', 'quantmod',
'fracdiff', 'lmtest', 'timeDate', 'tseries', 'urca', 'zoo'

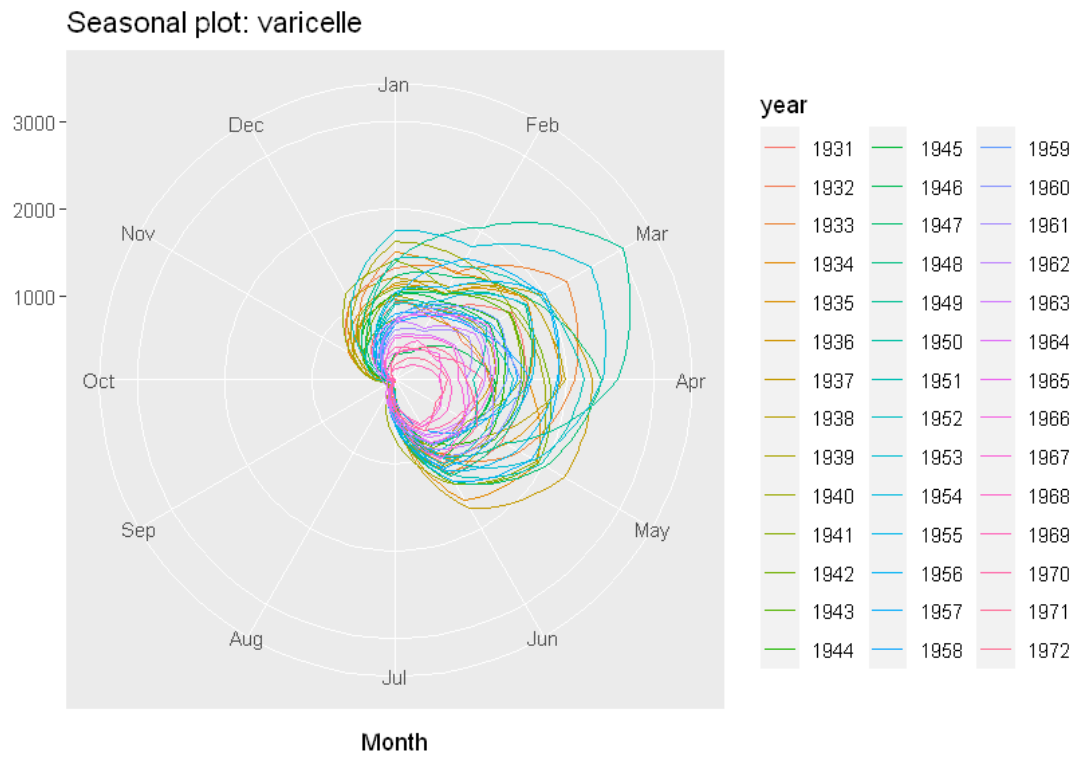
There is a binary version available but the source version is later:
binary source needs_compilation
forecast 8.12 8.13 TRUE

Binaries will be installed
package 'xts' successfully unpacked and MD5 sums checked
package 'TTR' successfully unpacked and MD5 sums checked
package 'quadprog' successfully unpacked and MD5 sums checked
package 'quantmod' successfully unpacked and MD5 sums checked
package 'fracdiff' successfully unpacked and MD5 sums checked
package 'lmtest' successfully unpacked and MD5 sums checked
package 'timeDate' successfully unpacked and MD5 sums checked
package 'tseries' successfully unpacked and MD5 sums checked
package 'urca' successfully unpacked and MD5 sums checked
package 'zoo' successfully unpacked and MD5 sums checked
package 'forecast' successfully unpacked and MD5 sums checked

The downloaded binary packages are in
C:\Users\erick\AppData\Local\Temp\Rtmp29aZBp\downloaded_packages

0.2.1 Seasonal plot



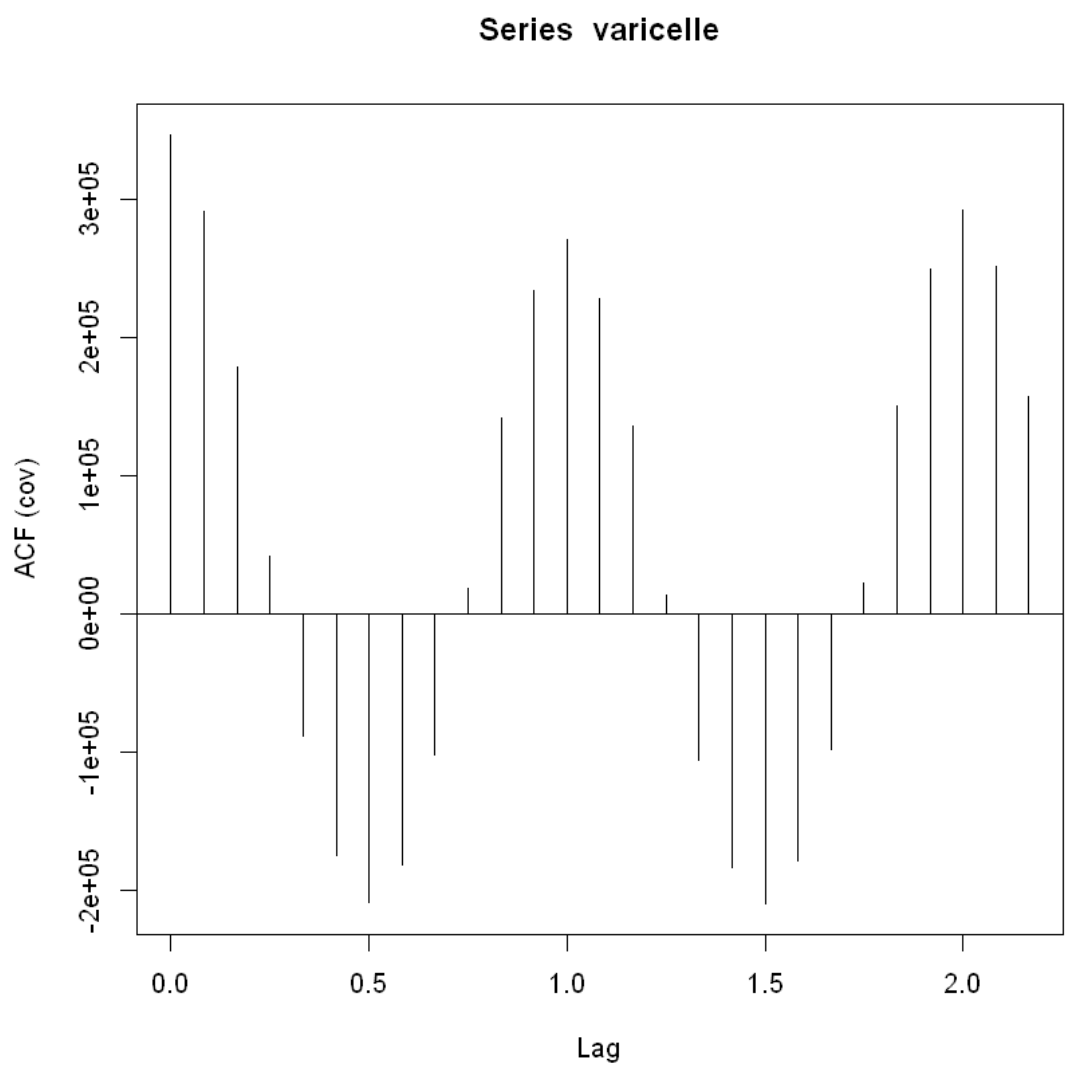


732.407630522088

347785.412979887

0.2.2 Auto Covariance - Correlogram

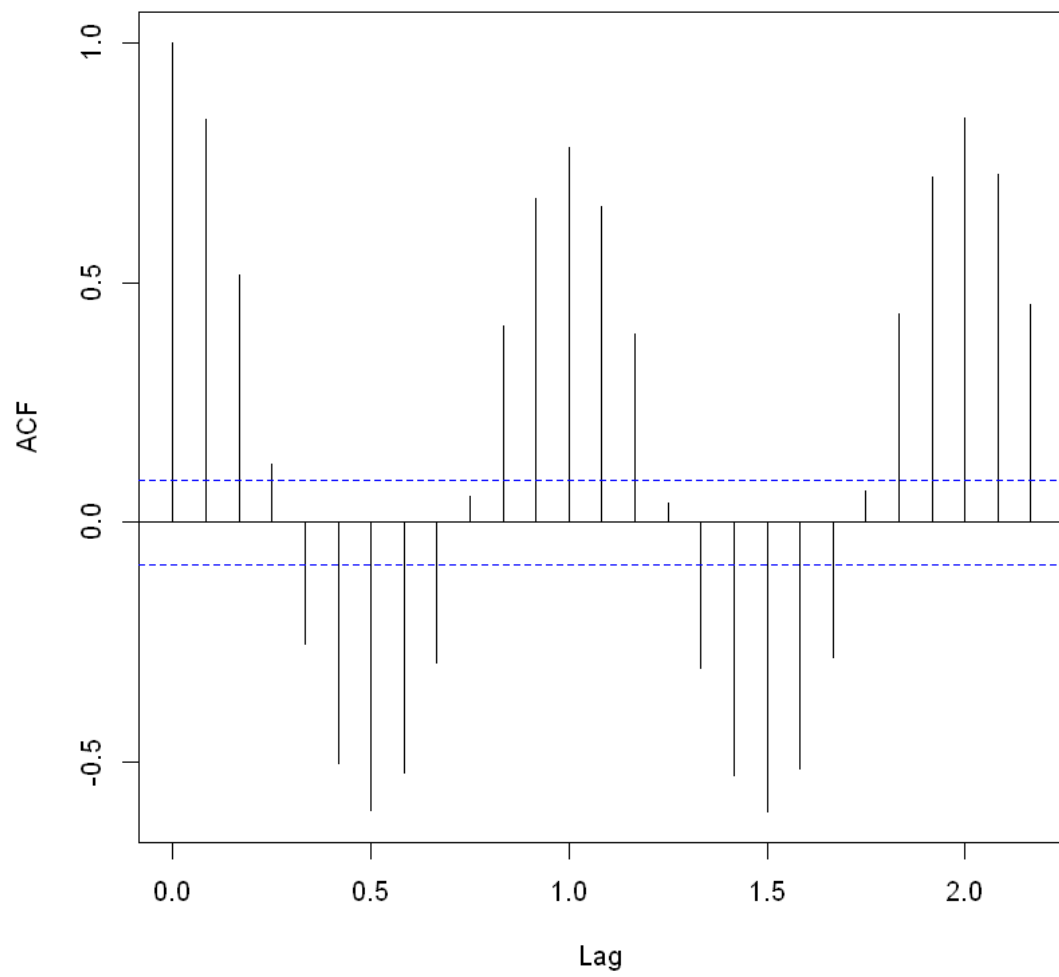
1. 347087.048696795 2. 291348.508444424 3. 179126.120673465



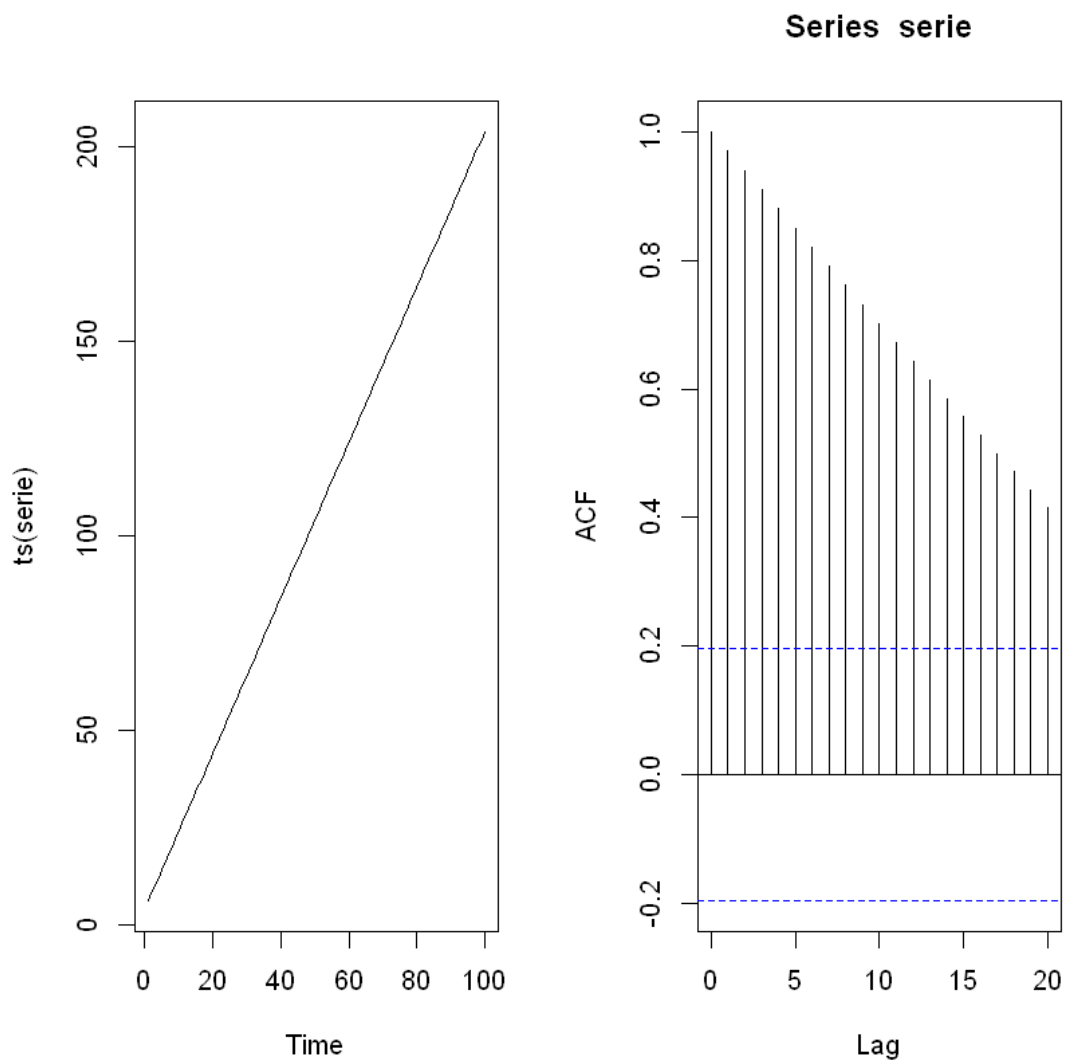
0.2.3 Auto-correlation

1. 1 2. 0.839410486615239 3. 0.516084138967525

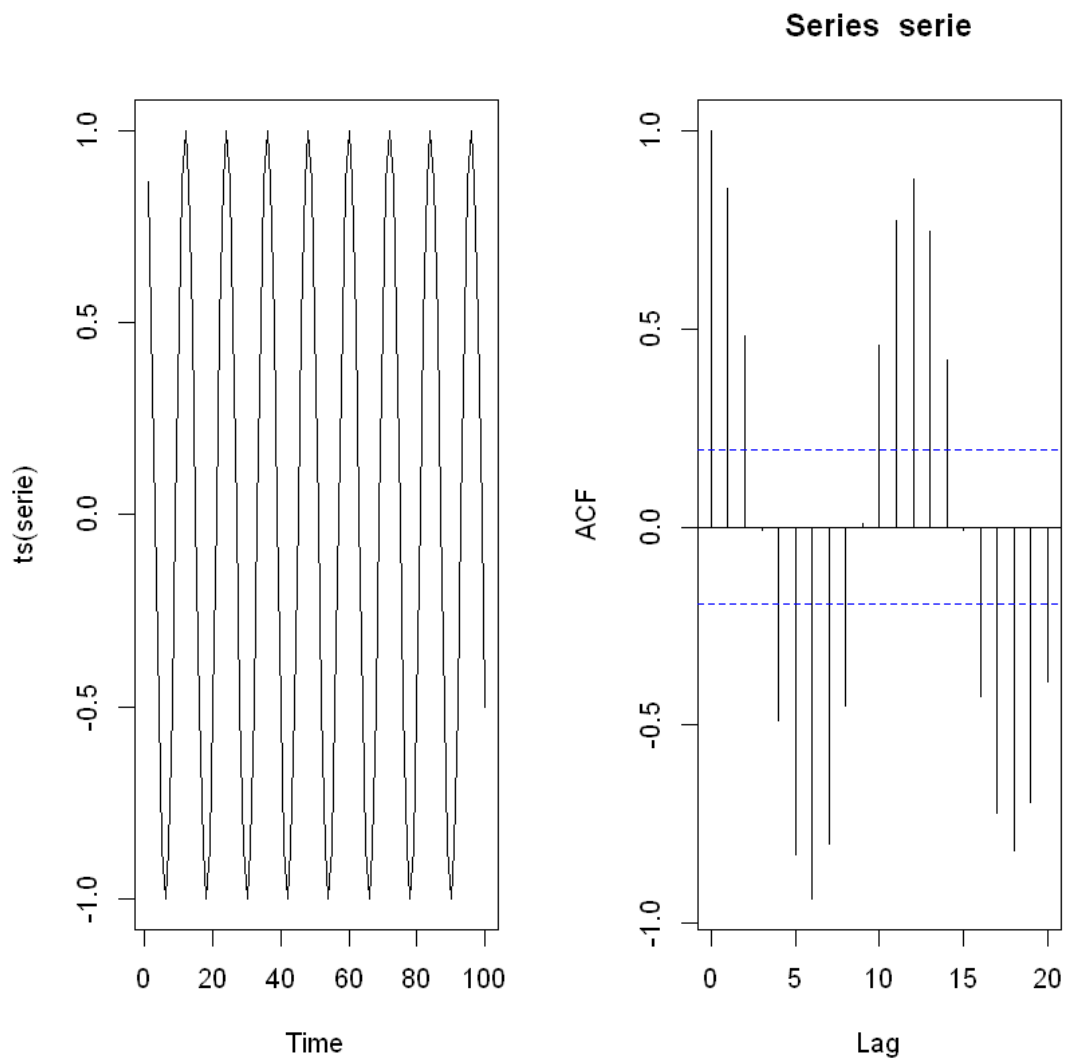
Series varicelle



0.2.4 Auto Correlation on a linear trend



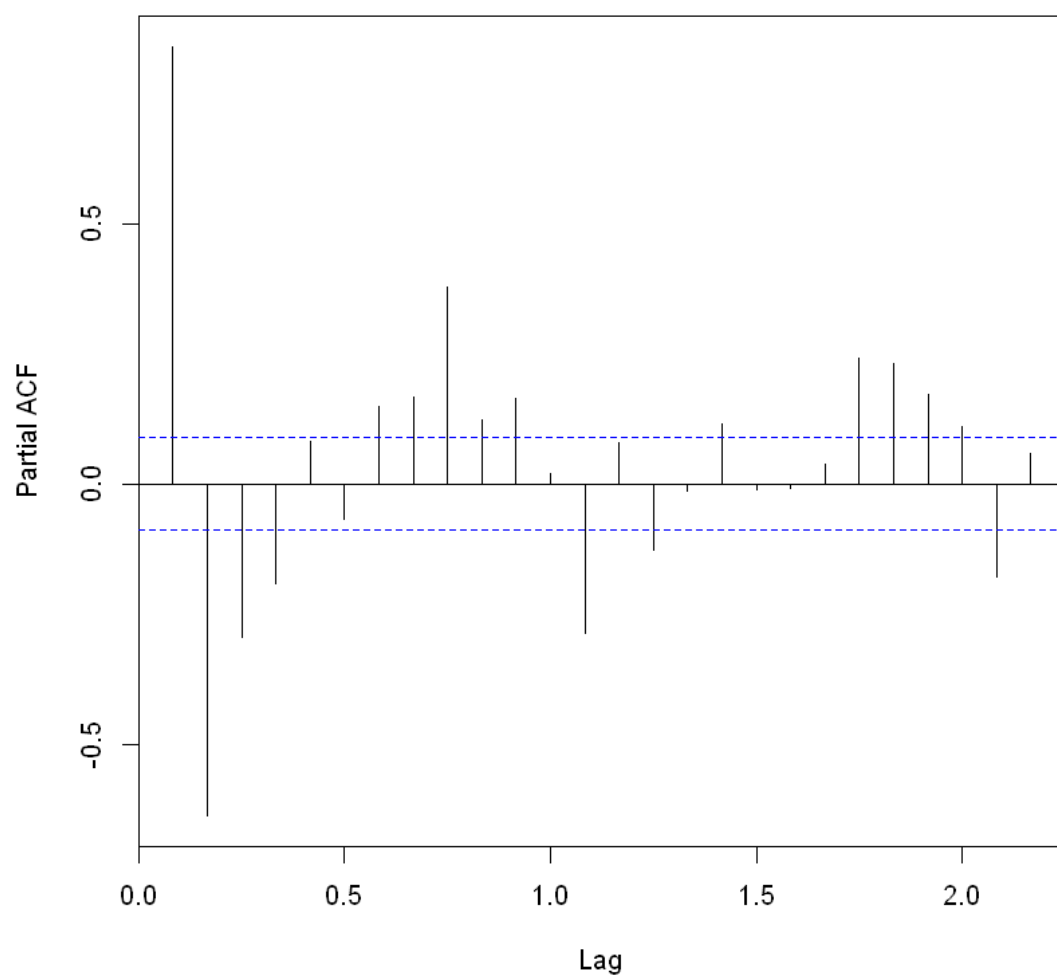
0.2.5 Auto Correlation on a periodic time serie

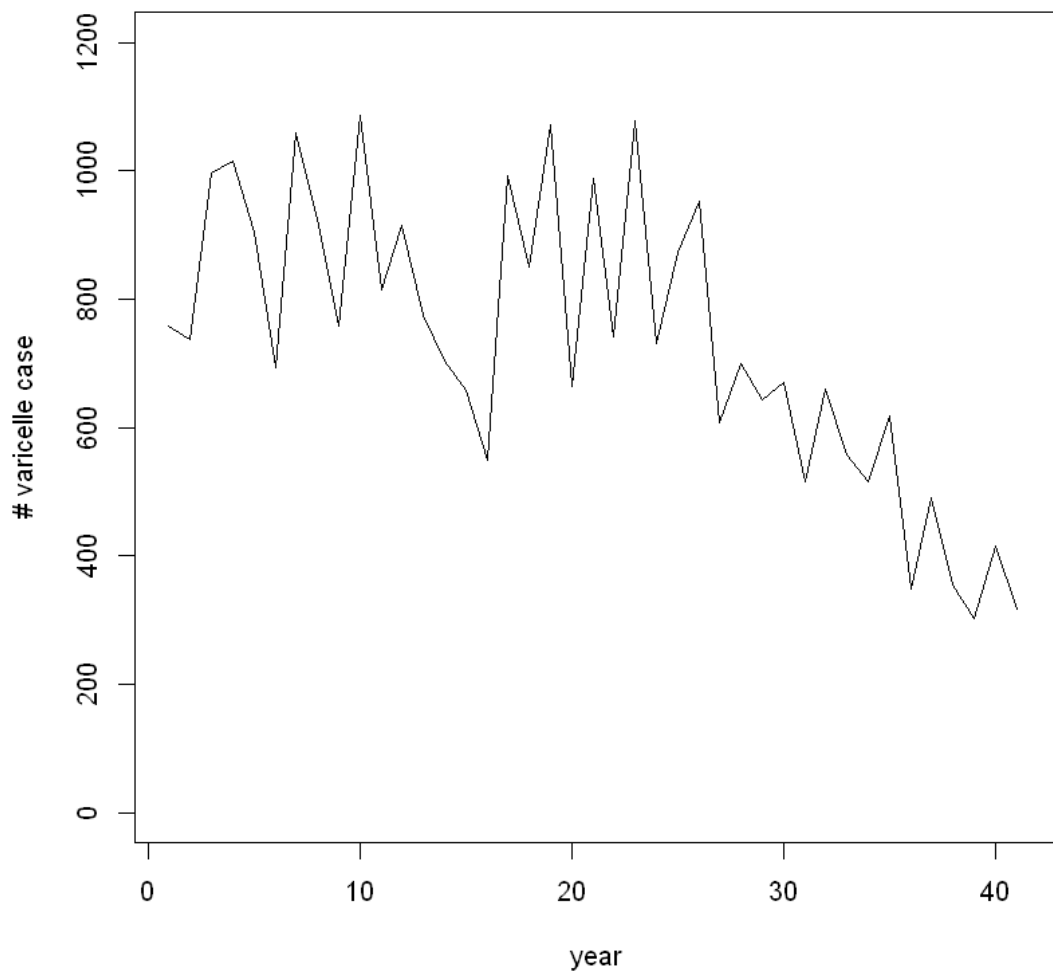


0.2.6 Partial Auto Correlation

1. 0.839410486615239 2. -0.638226763801978 3. -0.294447479813629

Series varicelle

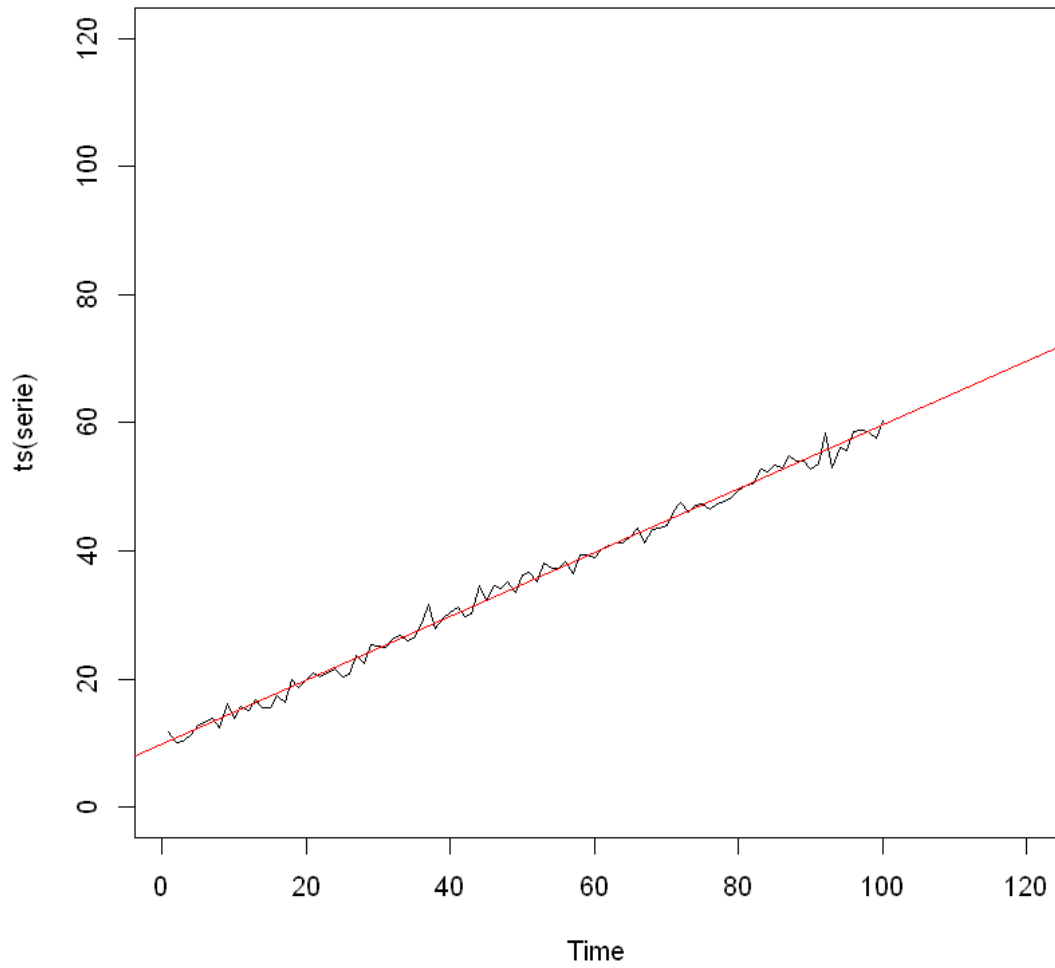




During the world war, less babies => less varicelle and after 1960 some kind of linear decrease (some kind of prevention)

If I need to do a time forecast I will remove the data before 1960

0.3 Exponential Smoothing



	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1950	936	1026	1244	935	1079	884	349	144	79	260	445	592
1951	1427	1545	1951	2200	1964	1284	523	142	93	148	198	374
1952	915	963	1154	1393	1227	1158	478	84	44	113	331	1052
1953	1747	1796	2625	2411	1877	1052	543	110	67	124	160	430
1954	726	1101	1769	1599	1035	988	424	147	76	105	281	524
1955	1044	1247	2023	1903	1653	1247	372	107	75	94	224	487
1956	989	1639	1991	1905	1846	1381	451	176	83	150	272	550
1957	798	902	1316	1443	1102	705	272	119	106	72	115	337
1958	677	885	1142	1590	1355	1198	565	136	89	115	174	477
1959	741	1034	1401	1316	1056	882	506	136	80	62	149	368
1960	683	993	1205	1485	1349	1067	369	173	95	113	175	335
1961	619	691	1022	858	953	913	332	127	82	62	147	384
1962	711	928	1152	1134	1277	961	509	173	170	193	290	415
1963	707	724	1105	1065	938	755	442	170	91	150	219	317
1964	561	631	829	857	955	808	398	111	82	147	276	528
1965	746	889	1274	1164	1024	863	436	270	156	139	156	306
1966	362	438	624	543	642	659	286	86	43	68	168	253
1967	526	601	809	759	950	1088	452	198	82	72	154	206
1968	316	569	549	671	736	659	287	132	51	85	79	133
1969	177	210	372	562	623	626	296	142	82	96	166	288
1970	416	459	576	1042	873	704	366	137	58	134	71	142
1971	211	331	471	639	569	718	391	123	72	63	86	141
1972	320	463	690	847	1121	1048						

ERROR while rich displaying an object: Error in arr_partition(a, rows, cols):
rows >= 2L is not TRUE

Traceback:

```

1. FUN(X[[i]], ...)
2. tryCatch(withCallingHandlers({
  .   if (!mime %in% names(repr::mime2repr))
  .     stop("No repr_* for mimetype ", mime, " in repr::mime2repr")
  .   rpr <- repr::mime2repr[[mime]](obj)
  .   if (is.null(rpr))
  .     return(NULL)
  .   prepare_content(is.raw(rpr), rpr)
  . }, error = error_handler), error = outer_handler)
3. tryCatchList(expr, classes, parentenv, handlers)
4. tryCatchOne(expr, names, parentenv, handlers[[1L]])
5. doTryCatch(return(expr), name, parentenv, handler)
6. withCallingHandlers({
  .   if (!mime %in% names(repr::mime2repr))
  .     stop("No repr_* for mimetype ", mime, " in repr::mime2repr")
  .   rpr <- repr::mime2repr[[mime]](obj)
  .   if (is.null(rpr))
  .     return(NULL)
  .   prepare_content(is.raw(rpr), rpr)

```

```

. }, error = error_handler)
7. repr::mime2repr[[mime]](obj)
8. repr_text.ts(obj)
9. repr_ts_generic(obj, repr_text, "%s:\n%s", ...)
10. repr_func(vec, ..., rows = nrow(vec), cols = ncol(vec), caption_override =
"Time Series")
11. repr_text.matrix(vec, ..., rows = nrow(vec), cols = ncol(vec),
.     caption_override = "Time Series")
12. ellip_limit_arr(obj, rows, cols)
13. arr_partition(a, rows, cols)
14. stopifnot(rows >= 2L, cols >= 2L)
ERROR while rich displaying an object: Error in arr_partition(a, rows, cols):
rows >= 2L is not TRUE

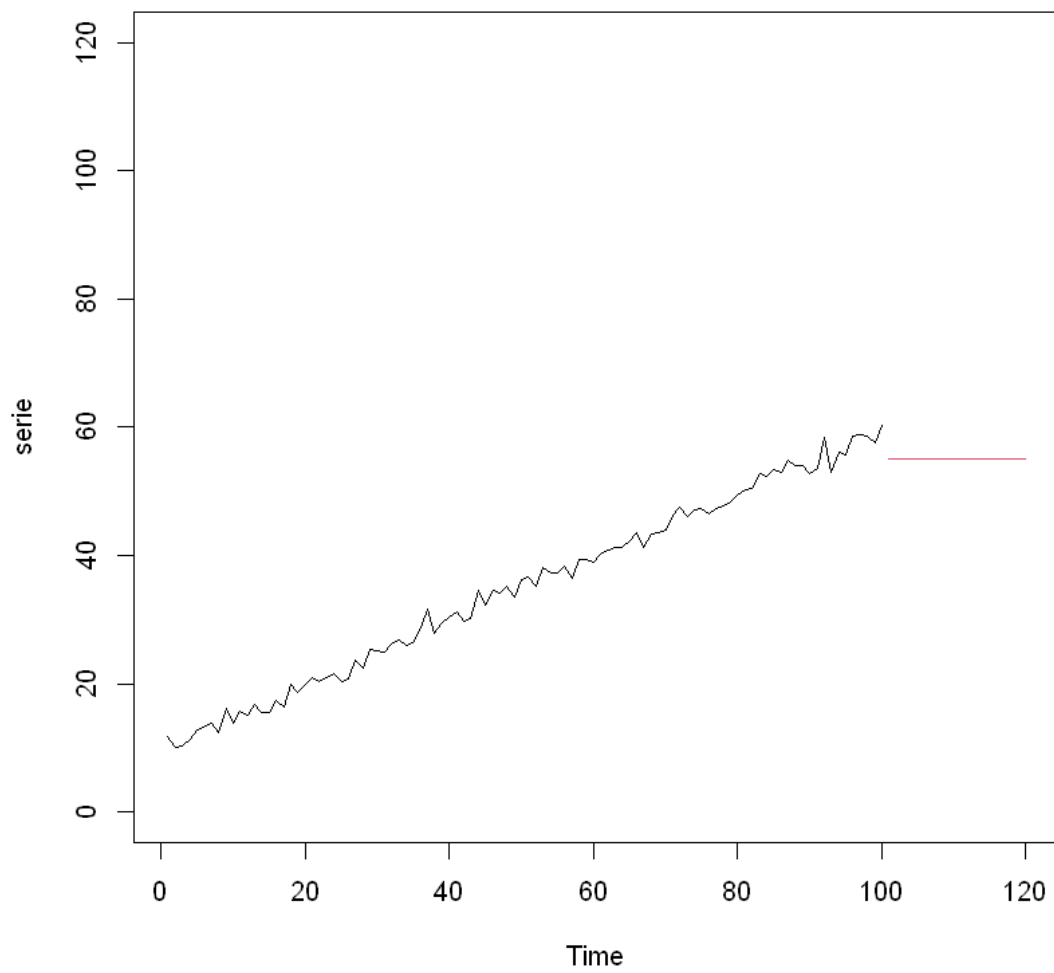
```

Traceback:

```

1. FUN(X[[i]], ...)
2. tryCatch(withCallingHandlers({
.   if (!mime %in% names(repr::mime2repr))
.     stop("No repr_* for mimetype ", mime, " in repr::mime2repr")
.   rpr <- repr::mime2repr[[mime]](obj)
.   if (is.null(rpr))
.     return(NULL)
.   prepare_content(is.raw(rpr), rpr)
. }, error = error_handler), error = outer_handler)
3. tryCatchList(expr, classes, parentenv, handlers)
4. tryCatchOne(expr, names, parentenv, handlers[[1L]])
5. doTryCatch(return(expr), name, parentenv, handler)
6. withCallingHandlers({
.   if (!mime %in% names(repr::mime2repr))
.     stop("No repr_* for mimetype ", mime, " in repr::mime2repr")
.   rpr <- repr::mime2repr[[mime]](obj)
.   if (is.null(rpr))
.     return(NULL)
.   prepare_content(is.raw(rpr), rpr)
. }, error = error_handler)
7. repr::mime2repr[[mime]](obj)
8. repr_text.ts(obj)
9. repr_ts_generic(obj, repr_text, "%s:\n%s", ...)
10. repr_func(vec, ..., rows = nrow(vec), cols = ncol(vec), caption_override =
"Time Series")
11. repr_text.matrix(vec, ..., rows = nrow(vec), cols = ncol(vec),
.     caption_override = "Time Series")
12. ellip_limit_arr(obj, rows, cols)
13. arr_partition(a, rows, cols)
14. stopifnot(rows >= 2L, cols >= 2L)

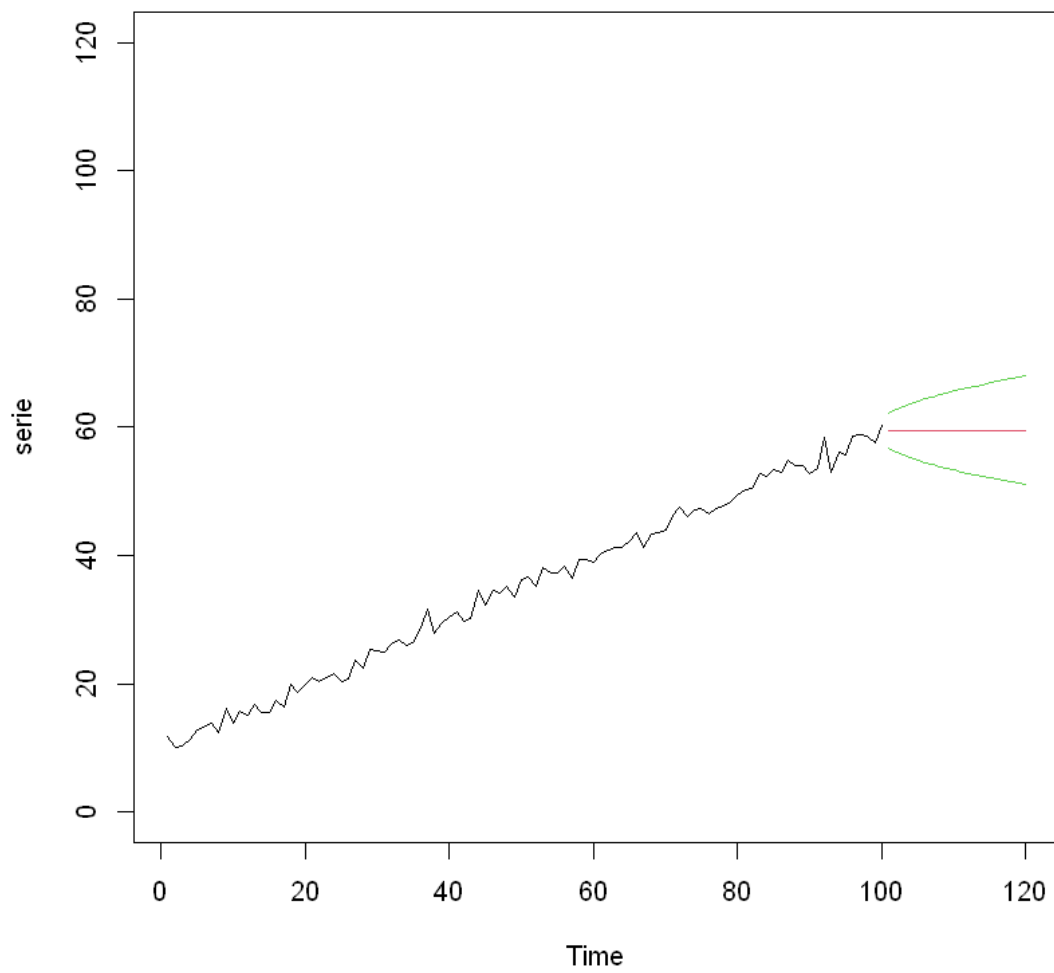
```



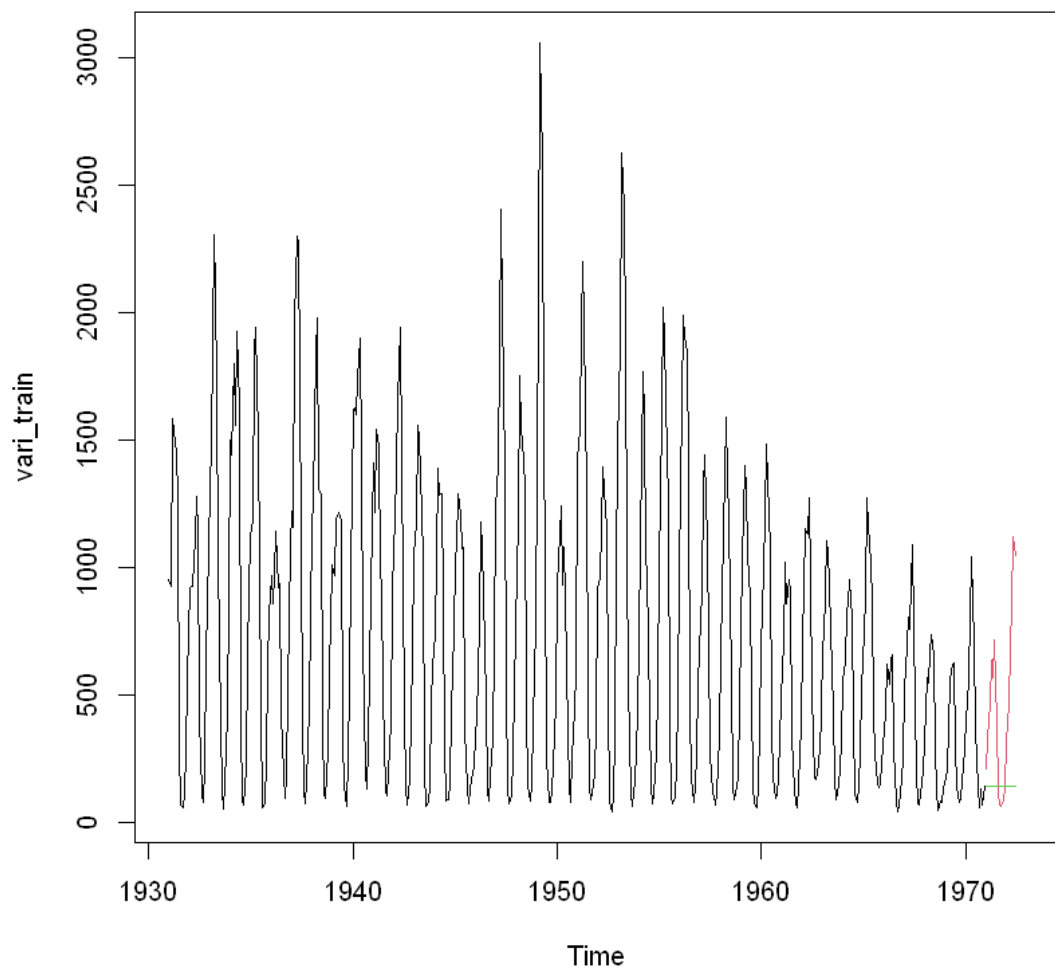
Same with auto calculation of alpha

[1] 0.658856

Same with a prediction interval

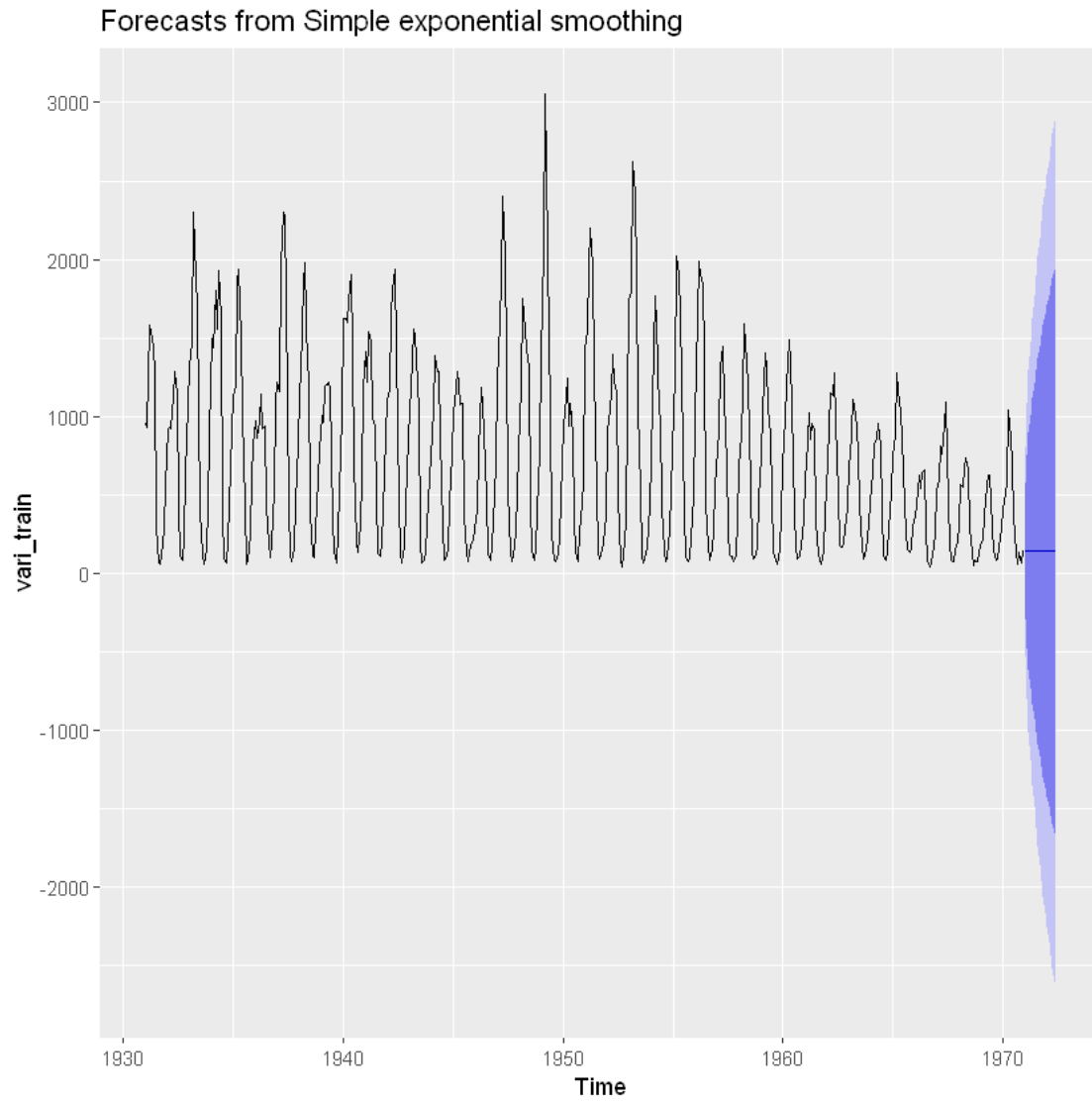


Prediction of periodical time serie with SES

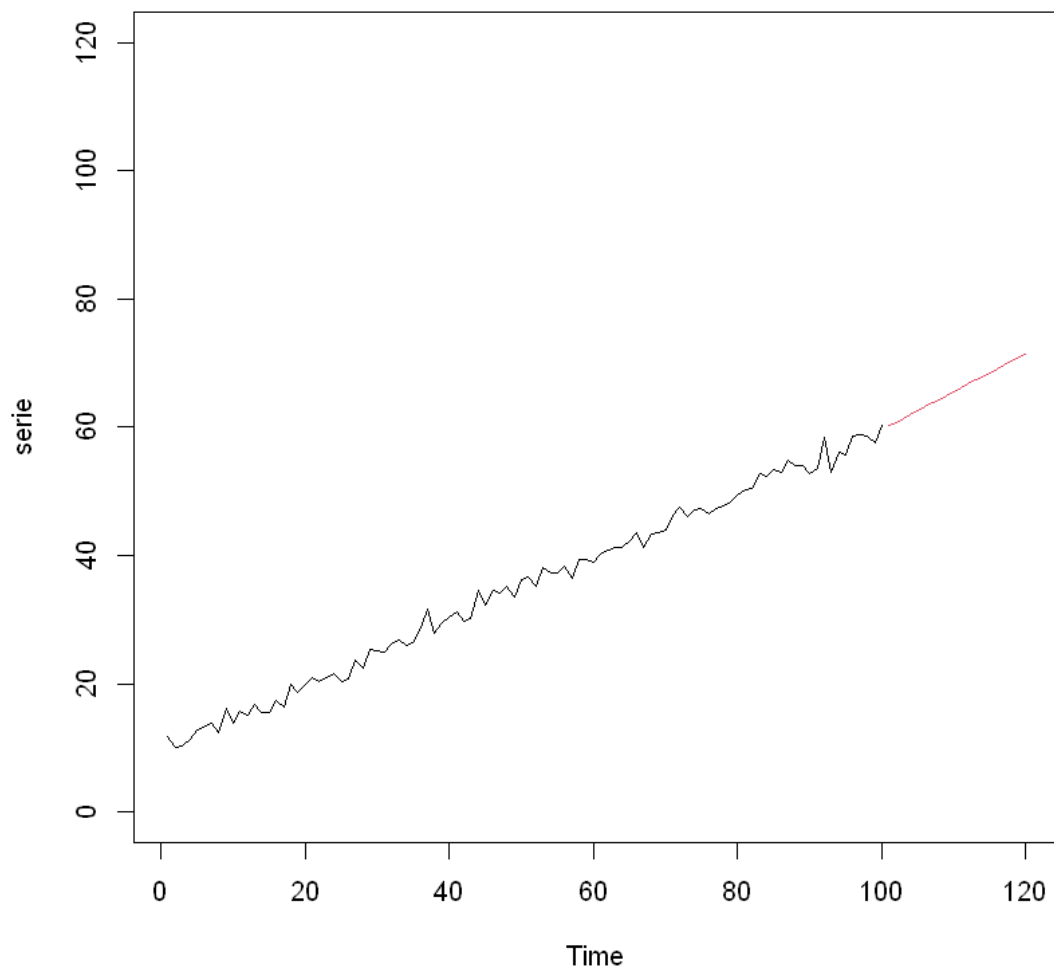


A matrix: 1×7 of type dbl

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Training set	-1.6	338.15	251.29	-24.53	61	1.04	0.51

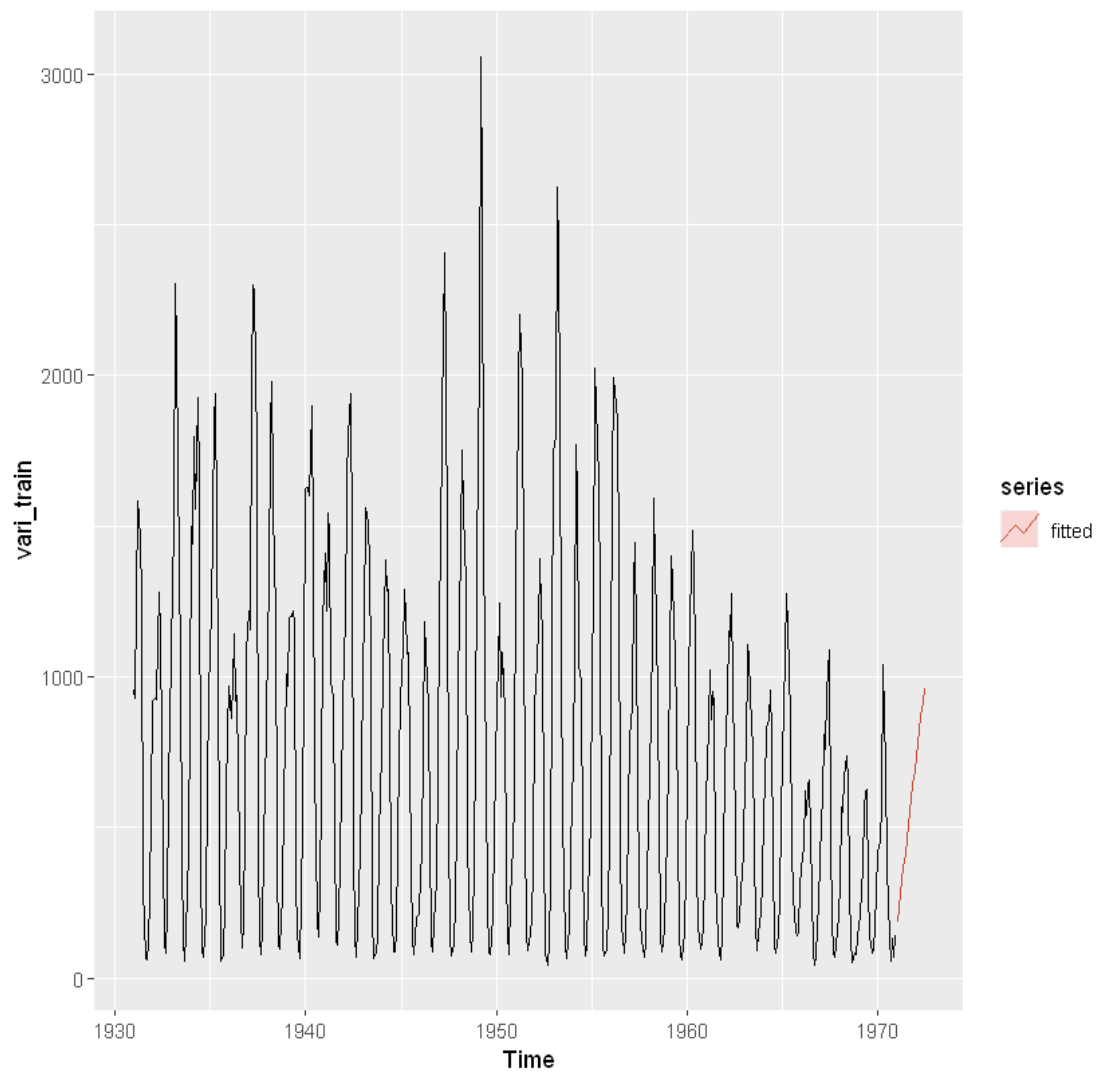


Non seasonal Holt-Winters smoothing

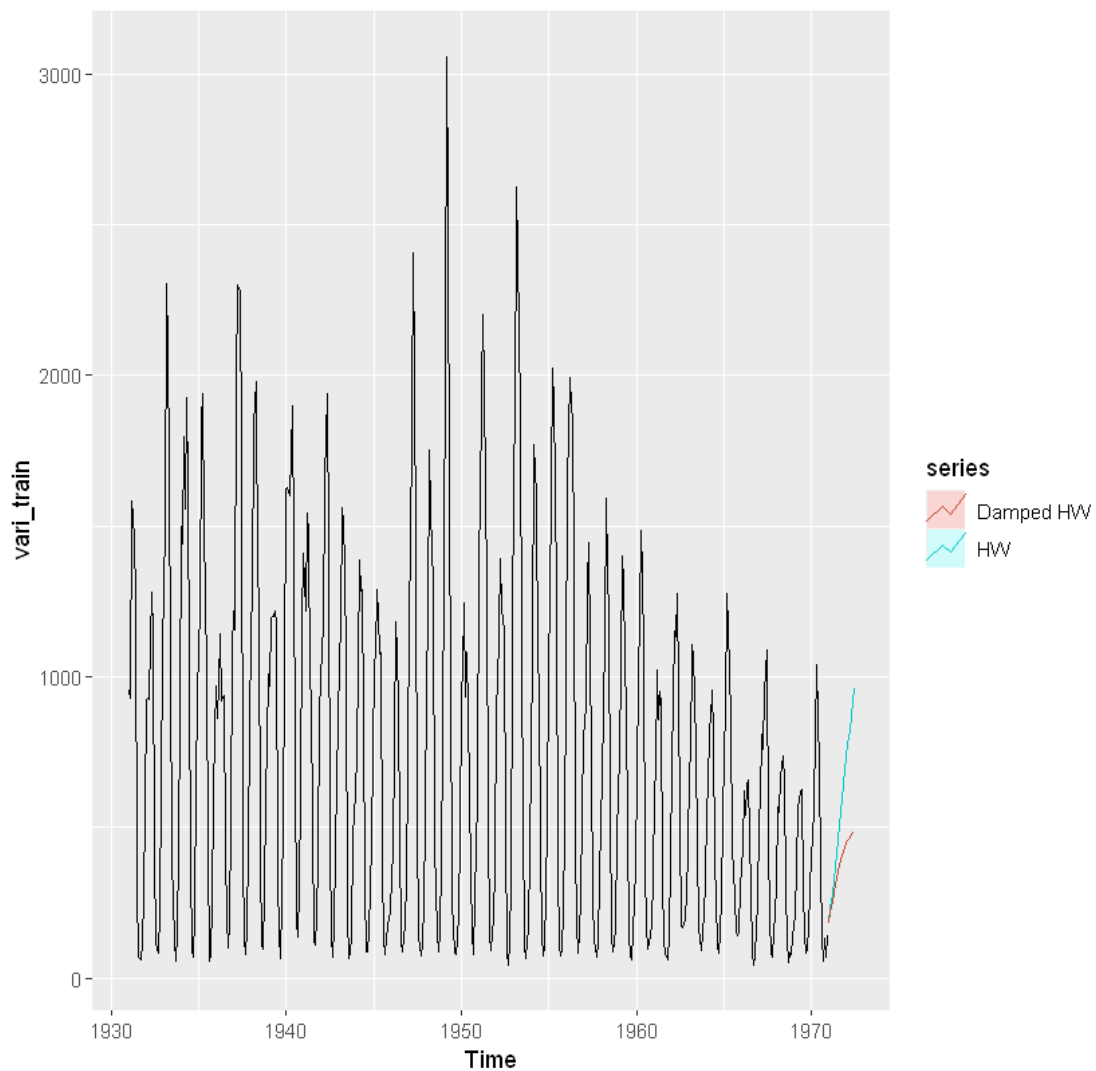


Non seasonal Holt-Winters smoothing with HOLT

A matrix: 1×7 of type dbl		ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
	Training set	0.22	330.42	259.65	54.15	89.34	1.08	0.01



Prediction with damped parameter



```
Installing package into 'C:/Users/erick/R'
(as 'lib' is unspecified)
```

```
also installing the dependencies 'fma', 'expsmooth'
```

```
package 'fma' successfully unpacked and MD5 sums checked
package 'expsmooth' successfully unpacked and MD5 sums checked
package 'fpp' successfully unpacked and MD5 sums checked
```

```
The downloaded binary packages are in
  C:\Users\erick\AppData\Local\Temp\Rtmp29aZBp\downloaded_packages
```

0.3.1 Cross validation with tsCV

Loading required package: fma

Loading required package: expsmooth

Loading required package: lmtest

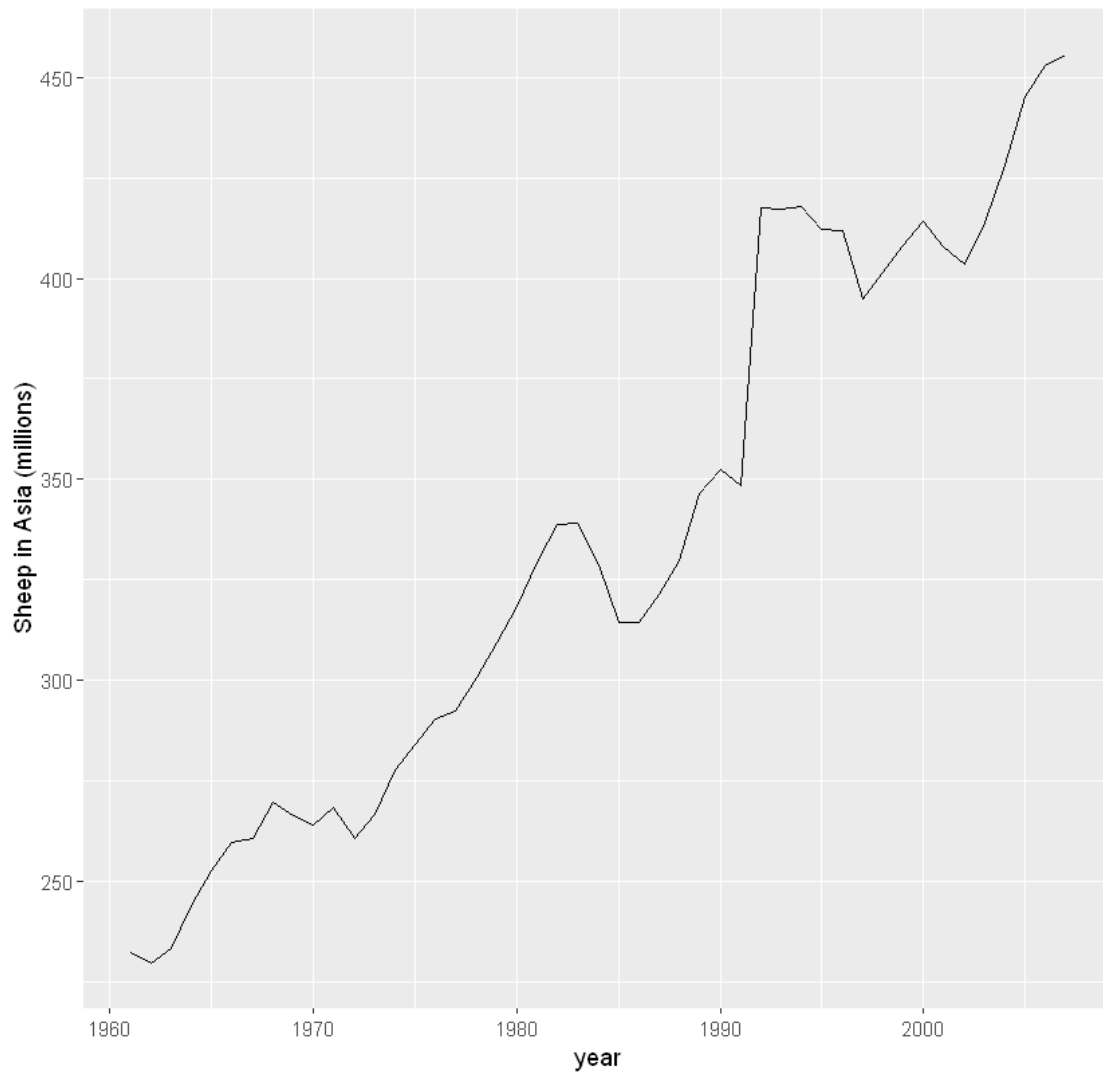
Loading required package: zoo

Attaching package: 'zoo'

The following objects are masked from 'package:base':

as.Date, as.Date.numeric

Loading required package: tseries



178.25311317288

173.364985413827

162.627434407659

At the end the smoothing with damped is the best one

Model description

Damped Holt's method

Call:

```
holt(y = livestock, h = 12, damped = TRUE)
```

Smoothing parameters:

alpha = 0.9999
beta = 3e-04
phi = 0.9798

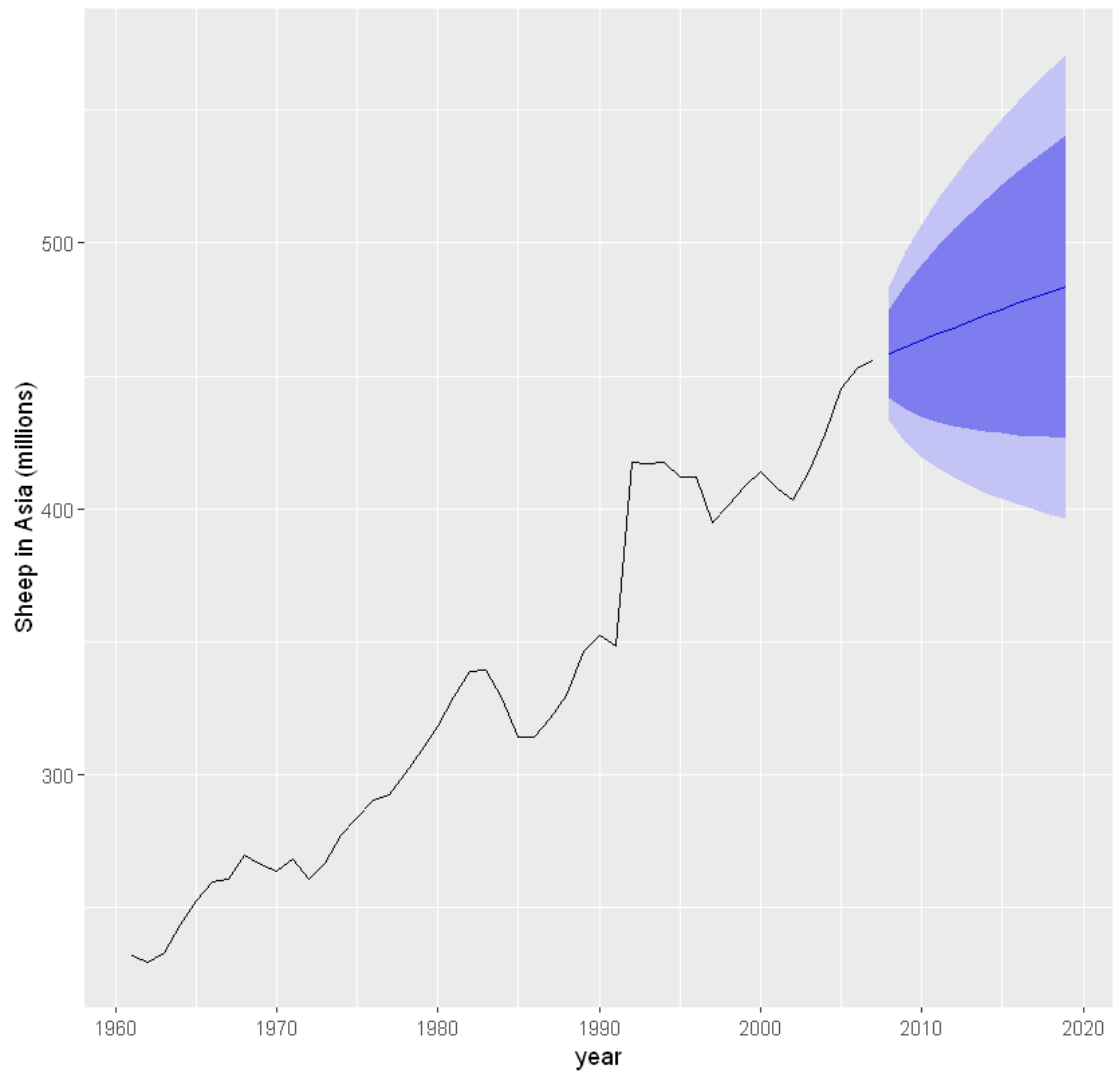
Initial states:

l = 223.35

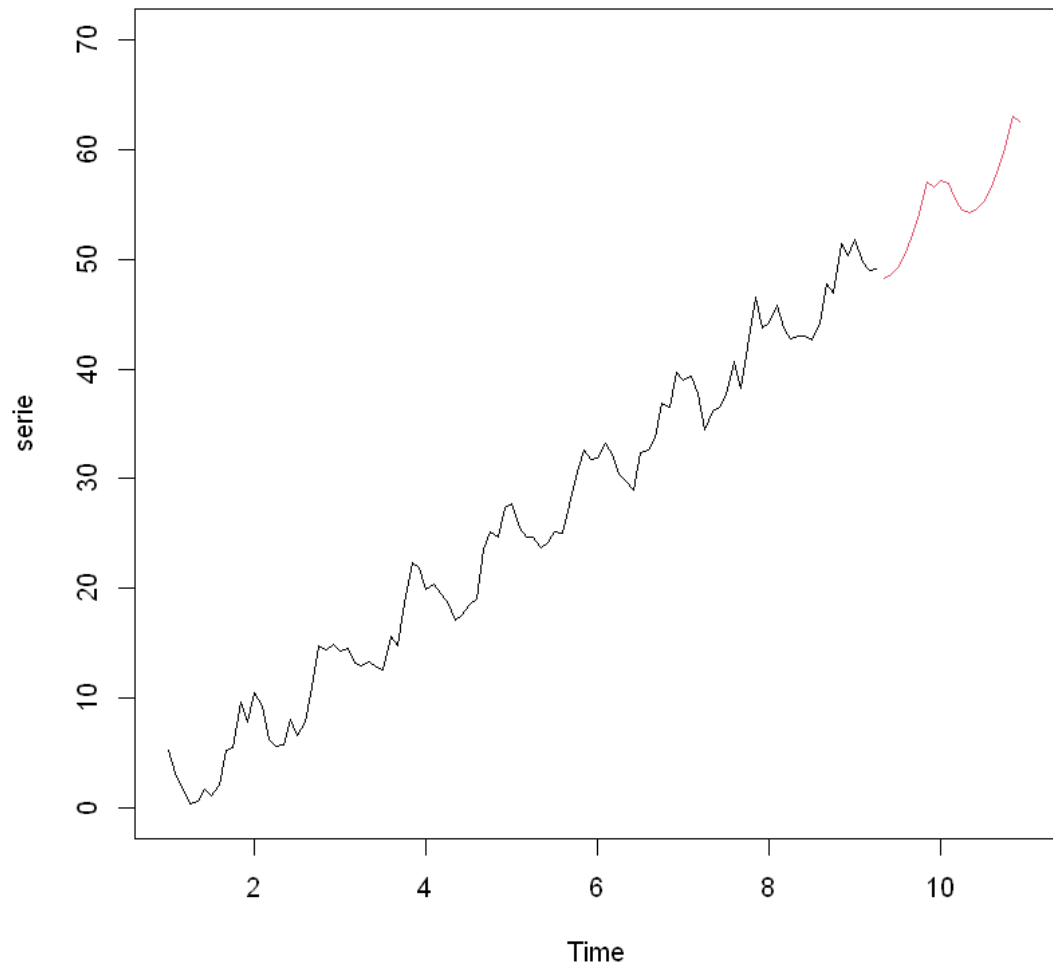
b = 6.9046

sigma: 12.8435

AIC	AICc	BIC
427.6370	429.7370	438.7379

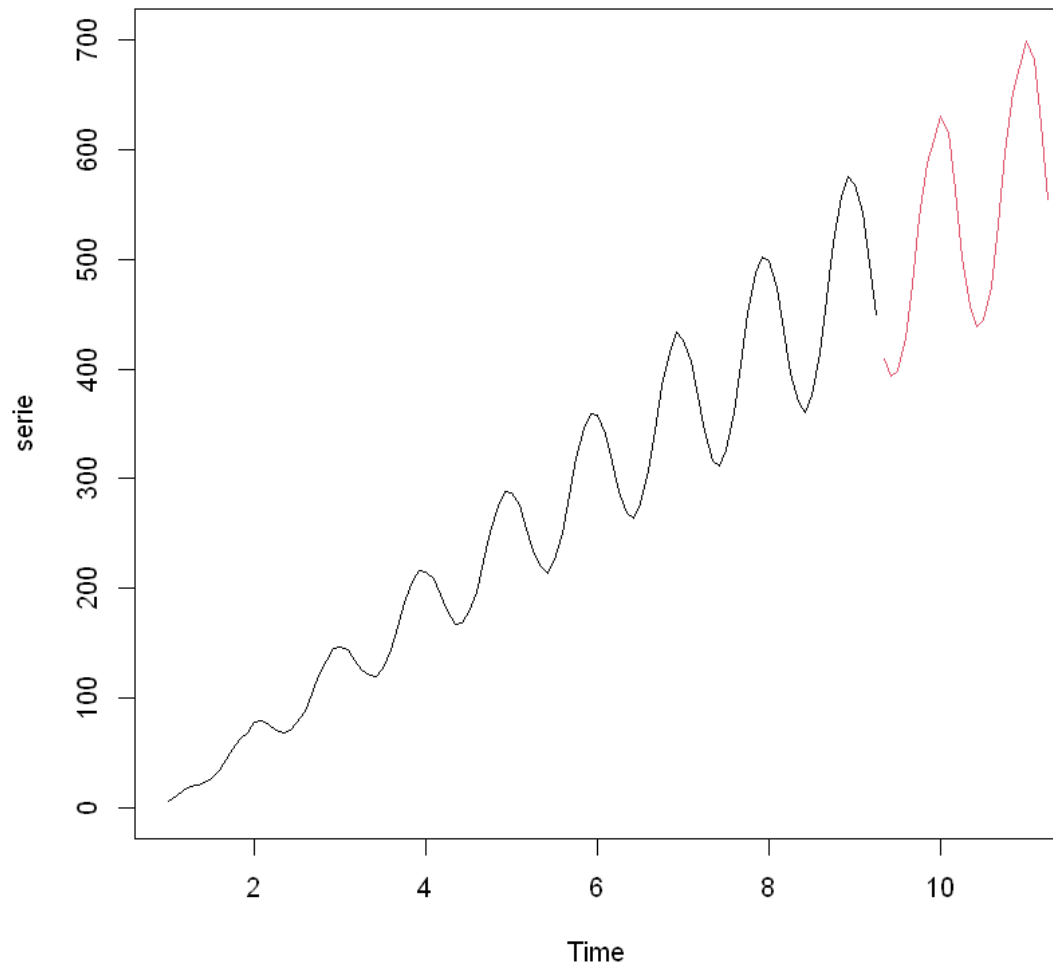


0.3.2 Additive Seasonal model pattern

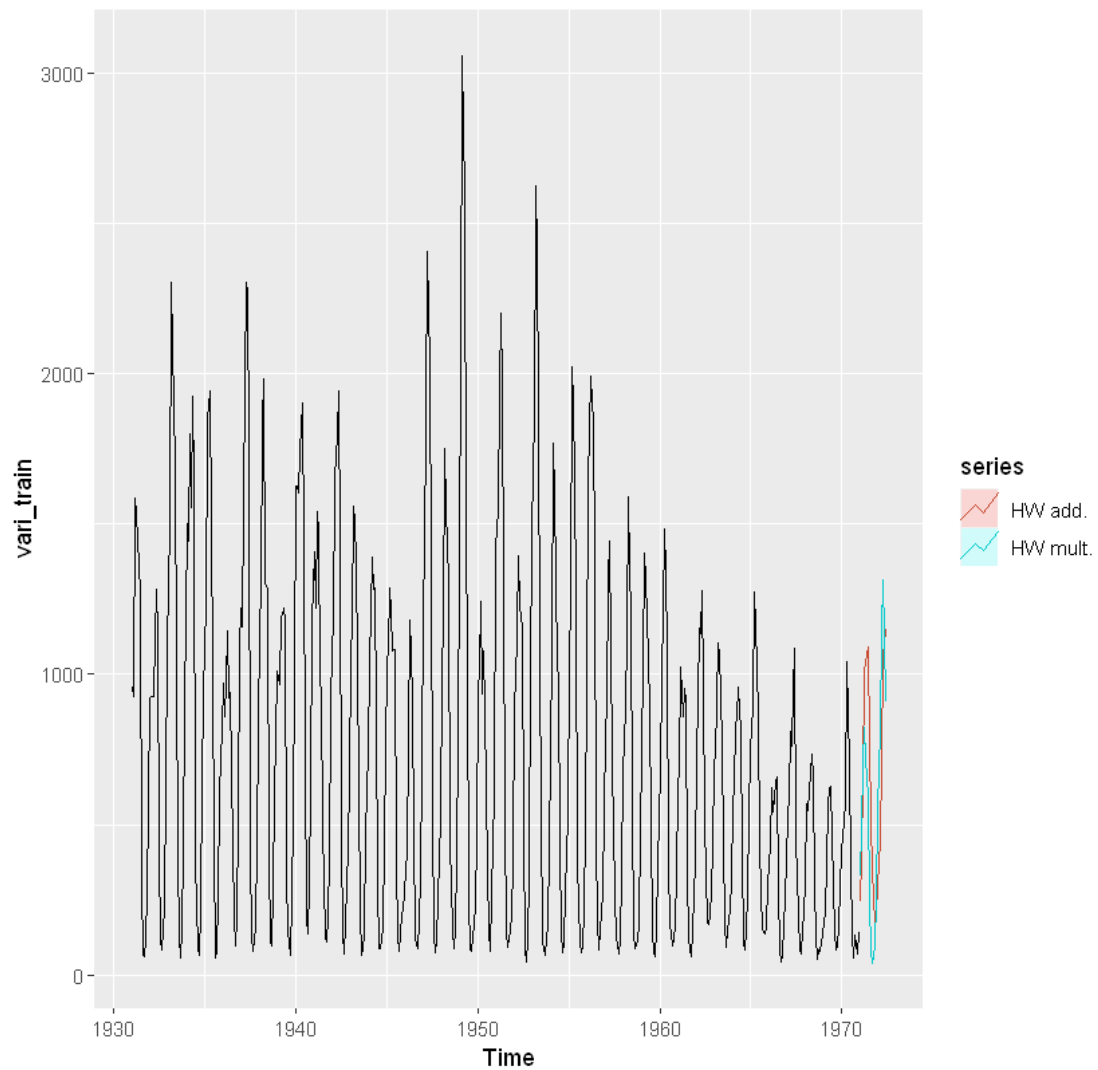


the value of the frequency in the ts function is MANDATORY

0.3.3 Multiplicative Seasonal



0.3.4 Testing additive and multiplicative effect on Varicelle Time series



No need to remove first years

[1] 238.2674

[1] 214.7901

Multiplicative model is better

0.3.5 the same with damped effect

[1] 238.2674

[1] 279.7424

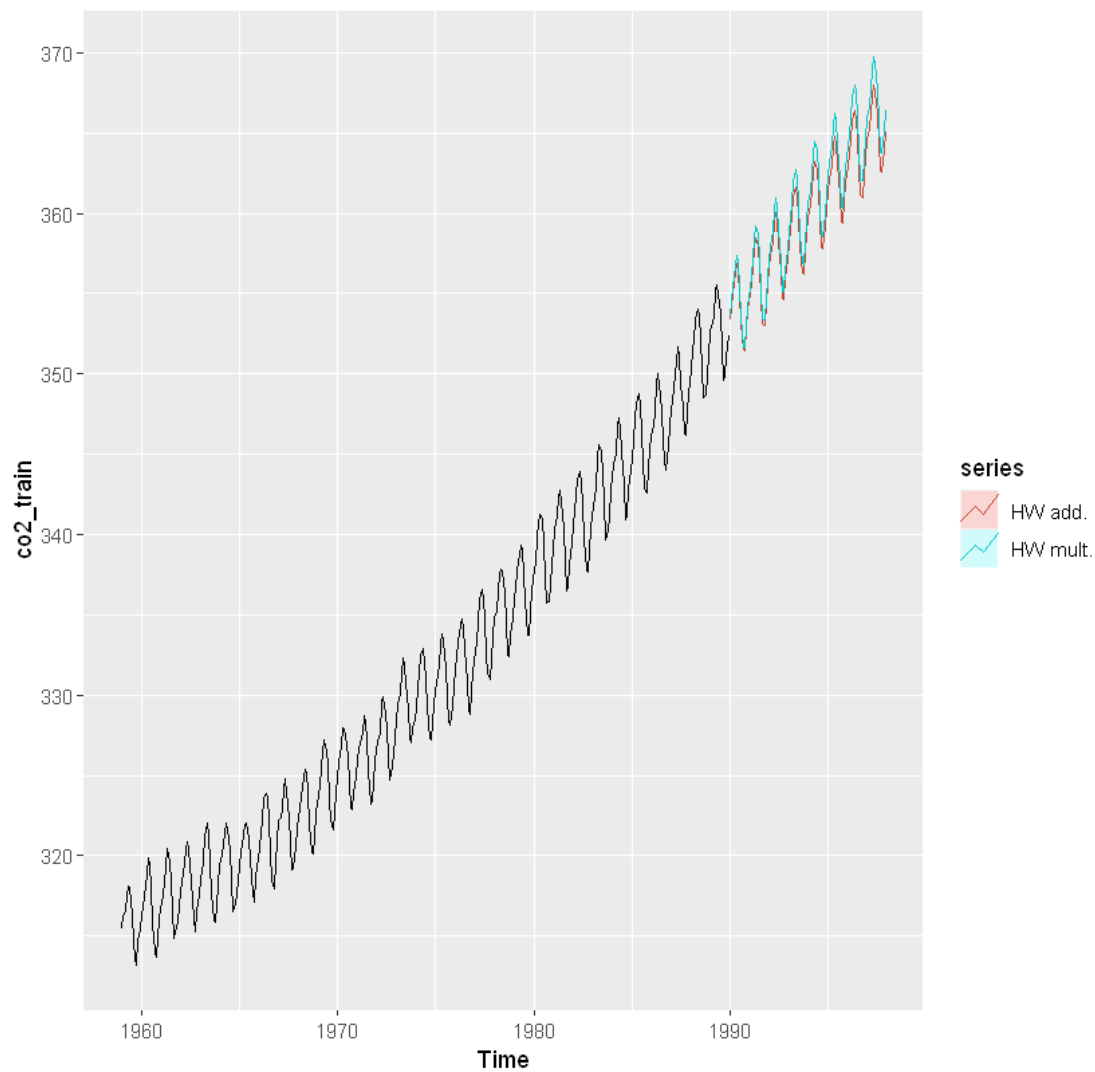
[1] 375.6358

damp effect makes it worst, so we keep the simple multiplicativ effect

0.3.6 Exemple with co2 file

Time-Series [1:468] from 1959 to 1998: 315 316 316 318 318 ...

co2 is already a built in time serie in R



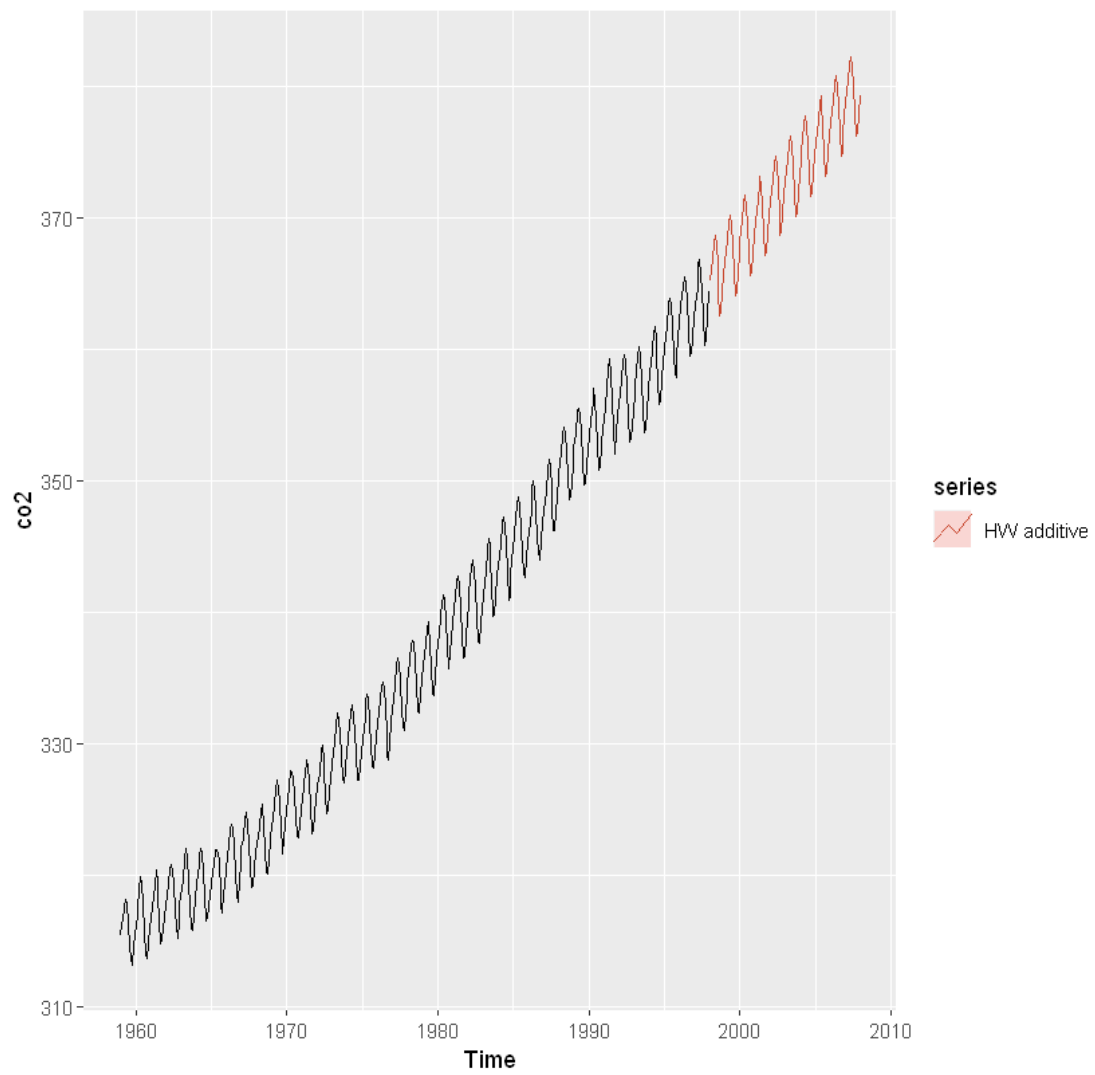
[1] 1.316165

[1] 2.199909

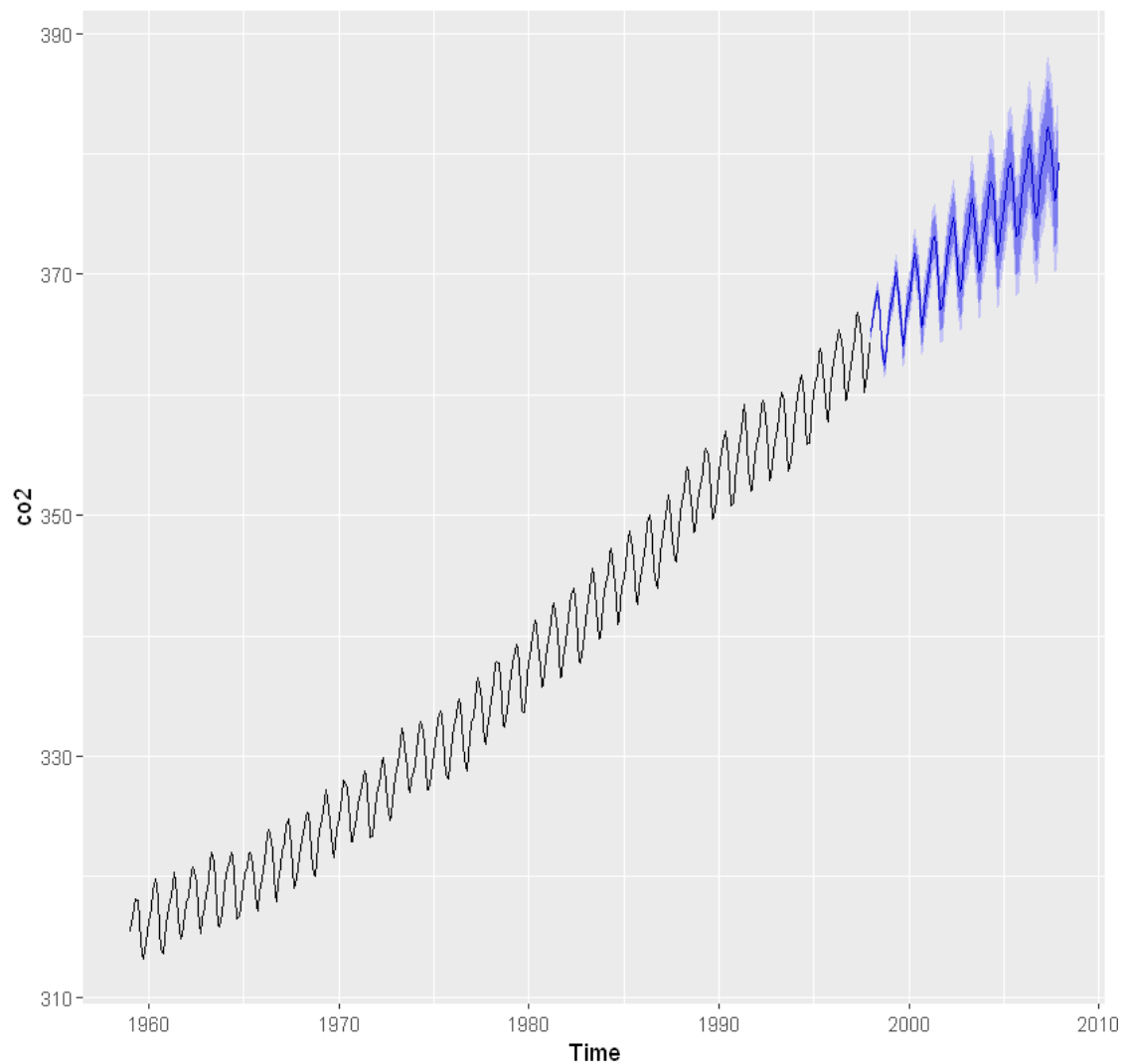
[1] 3.686467

[1] 3.541412

Best model is fit1 : additive without damped



with confident level



0.3.7 Exemple San Francisco

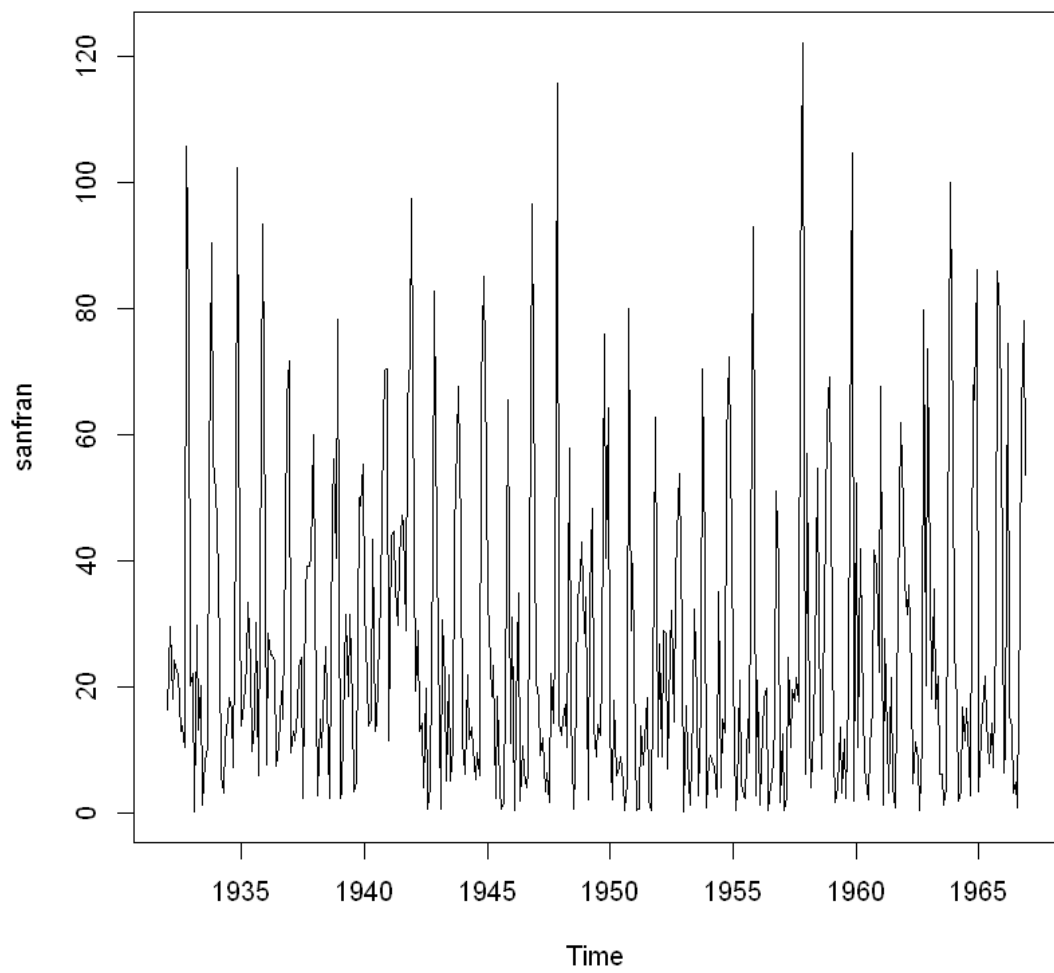
```
Error in read.table(file = "sanfran.dat", sep = " ", header = TRUE):  
↪ more columns than column names  
Traceback:
```

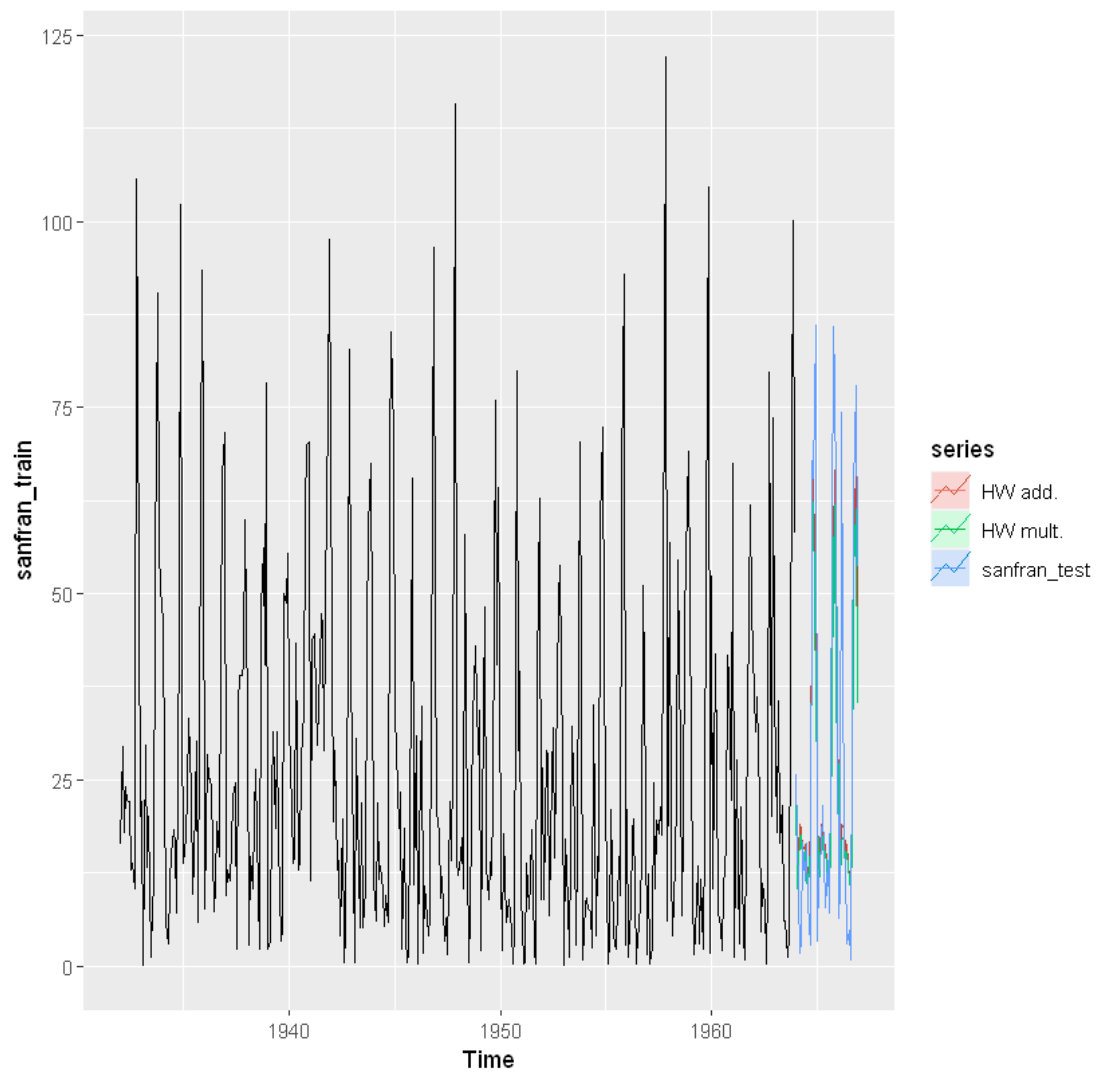
```
1. read.table(file = "sanfran.dat", sep = " ", header = TRUE)
```

```
2. stop("more columns than column names")
```

```
1. 16.26 2. 29.46 3. 18.03 4. 24.13 5. 22.35 6. 22.1 7. 12.95 8. 13.72 9. 10.41 10. 105.66 11. 79.76
```

12. 20.07 13. 22.1 14. 0 15. 29.72 16. 13.21 17. 20.07 18. 1.27 19. 8.13 20. 10.41 21. 52.83 22. 90.42
 23. 56.13 24. 50.8 25. 46.23 26. 25.91 27. 5.84 28. 3.05 29. 11.18 30. 14.99 31. 18.29 32. 16.51 33. 7.11
 34. 46.48 35. 102.36 36. 34.54 37. 13.72 38. 17.27 39. 23.62 40. 33.27 41. 23.62 42. 9.65 43. 14.22
 44. 30.23 45. 5.84 46. 33.27 47. 93.47 48. 69.09 49. 7.62 50. 28.45 51. 25.15 52. 25.15 53. 23.88 54. 7.37
 55. 10.67 56. 19.3 57. 14.73 58. 42.16 59. 65.79 60. 71.63 61. 9.4 62. 12.95 63. 11.43 64. 16 65. 22.86
 66. 24.64 67. 2.29 68. 35.05 69. 39.12 70. 39.12 71. 40.39 72. 59.94 73. 33.53 74. 2.79 75. 14.73
 76. 10.41 77. 20.32 78. 26.42 79. 9.91 80. 2.29 81. 48.01 82. 56.13 83. 40.39 84. 78.23 85. 2.29 86. 3.3
 87. 22.61 88. 31.5 89. 18.54 90. 31.5 91. 16.76 92. 3.3 93. 4.83 94. 50.04 95. 48.77 96. 55.37 97. 32.51
 98. 21.08 99. 13.72 100. 14.73 101. 43.43 102. 12.95 103. 14.22 104. 27.69 105. 34.29 106. 60.96
 107. 70.1 108. 70.36 109. 11.43 110. 43.69 111. 44.7 112. 35.56 113. 29.72 114. 40.64 115. 47.24
 116. 45.72 117. 28.96 118. 64.77 119. 72.14 120. 97.54 121. 42.42 122. 19.3 123. 28.96 124. 12.95
 125. 14.22 126. 4.06 127. 19.81 128. 0.51 129. 4.06 130. 43.18 131. 82.8 132. 41.4 133. 26.92 134. 0.51
 135. 30.48 136. 20.83 137. 5.08 138. 21.84 139. 5.08 140. 10.41 141. 44.7 142. 58.93 143. 67.56
 144. 43.94 145. 11.43 146. 6.1 147. 21.84 148. 11.68 149. 13.46 150. 10.16 151. 5.33 152. 9.4 153. 5.84
 154. 67.06 155. 85.09 156. 71.37 157. 34.04 158. 28.96 159. 18.54 160. 23.37 161. 2.29 162. 18.54
 163. 3.81 164. 0.51 165. 1.52 166. 38.35 167. 65.53 168. 10.92 169. 30.99 170. 0.25 171. 16.26 172. 34.8
 173. 1.78 174. 10.67 175. 6.6 176. 4.06 177. 6.6 178. 64.52 179. 96.52 180. 45.97 181. 20.83 182. 18.03
 183. 9.14 184. 11.94 185. 3.3 186. 6.35 187. 1.52 188. 22.1 189. 14.22 190. 27.94 191. 115.82 192. 14.22
 193. 12.19 194. 14.73 195. 17.27 196. 10.41 197. 57.91 198. 16 199. 0.51 200. 6.6 201. ... 202. 7.11
 203. 8.89 204. 4.83 205. 0.25 206. 4.83 207. 80.01 208. 28.96 209. 39.62 210. 9.91 211. 0.25 212. 0.51
 213. 13.72 214. 7.62 215. 11.43 216. 18.29 217. 2.03 218. 0.25 219. 39.12 220. 62.74 221. 8.89 222. 26.67
 223. 8.89 224. 28.96 225. 28.45 226. 6.86 227. 25.65 228. 32 229. 14.48 230. 37.59 231. 49.28 232. 53.85
 233. 27.18 234. 0 235. 17.02 236. 8.64 237. 1.27 238. 16.26 239. 32.26 240. 18.03 241. 2.79 242. 17.02
 243. 70.36 244. 40.13 245. 0.76 246. 8.38 247. 9.14 248. 7.62 249. 7.37 250. 2.54 251. 35.05 252. 4.06
 253. 14.73 254. 12.7 255. 59.18 256. 72.39 257. 42.42 258. 22.35 259. 0.25 260. 3.3 261. 21.08 262. 4.57
 263. 2.79 264. 2.29 265. 14.73 266. 9.4 267. 65.28 268. 92.96 269. 2.79 270. 21.08 271. 1.27 272. 7.62
 273. 18.03 274. 19.81 275. 0.25 276. 4.32 277. 4.83 278. 13.72 279. 51.05 280. 38.86 281. 1.52
 282. 12.45 283. 0.25 284. 2.54 285. 24.64 286. 10.41 287. 19.56 288. 17.78 289. 21.34 290. 17.53
 291. 82.55 292. 122.17 293. 6.1 294. 56.9 295. 13.21 296. 4.06 297. 13.72 298. 23.62 299. 54.61
 300. 27.94 301. 6.86 302. 18.29 303. 46.48 304. 62.23 305. 69.09 306. 48.77 307. 9.65 308. 1.52
 309. 4.06 310. 13.46 311. 3.05 312. 11.68 313. 2.29 314. 18.8 315. 55.63 316. 104.65 317. 1.78
 318. 52.32 319. 10.41 320. 41.91 321. 27.18 322. 7.11 323. 5.33 324. 2.03 325. 10.41 326. 16.26
 327. 41.66 328. 38.86 329. 22.35 330. 67.56 331. 1.27 332. 27.69 333. 12.19 334. 3.05 335. 21.34
 336. 4.06 337. 0.76 338. 17.78 339. 48.01 340. 61.98 341. 48.01 342. 36.32 343. 31.5 344. 36.07
 345. 21.84 346. 4.57 347. 11.18 348. 8.38 349. 0.25 350. 11.18 351. 79.76 352. 20.07 353. 73.66
 354. 36.83 355. 18.03 356. 35.56 357. 16.51 358. 21.59 359. 6.1 360. 6.1 361. 1.27 362. 4.06 363. 38.86
 364. 100.08 365. 58.17 366. 25.65 367. 17.53 368. 1.78 369. 3.3 370. 16.76 371. 11.43 372. 16.51
 373. 8.38 374. 2.79 375. 67.82 376. 65.53 377. 86.11 378. 3.3 379. 11.94 380. 16 381. 21.59 382. 14.48
 383. 7.87 384. 14.22 385. 7.11 386. 28.45 387. 85.85 388. 78.74 389. 62.23 390. 6.35 391. 10.16
 392. 74.42 393. 15.24 394. 13.72 395. 3.05 396. 4.83 397. 0.76 398. 34.29 399. 64.01 400. 77.98
 401. 53.59





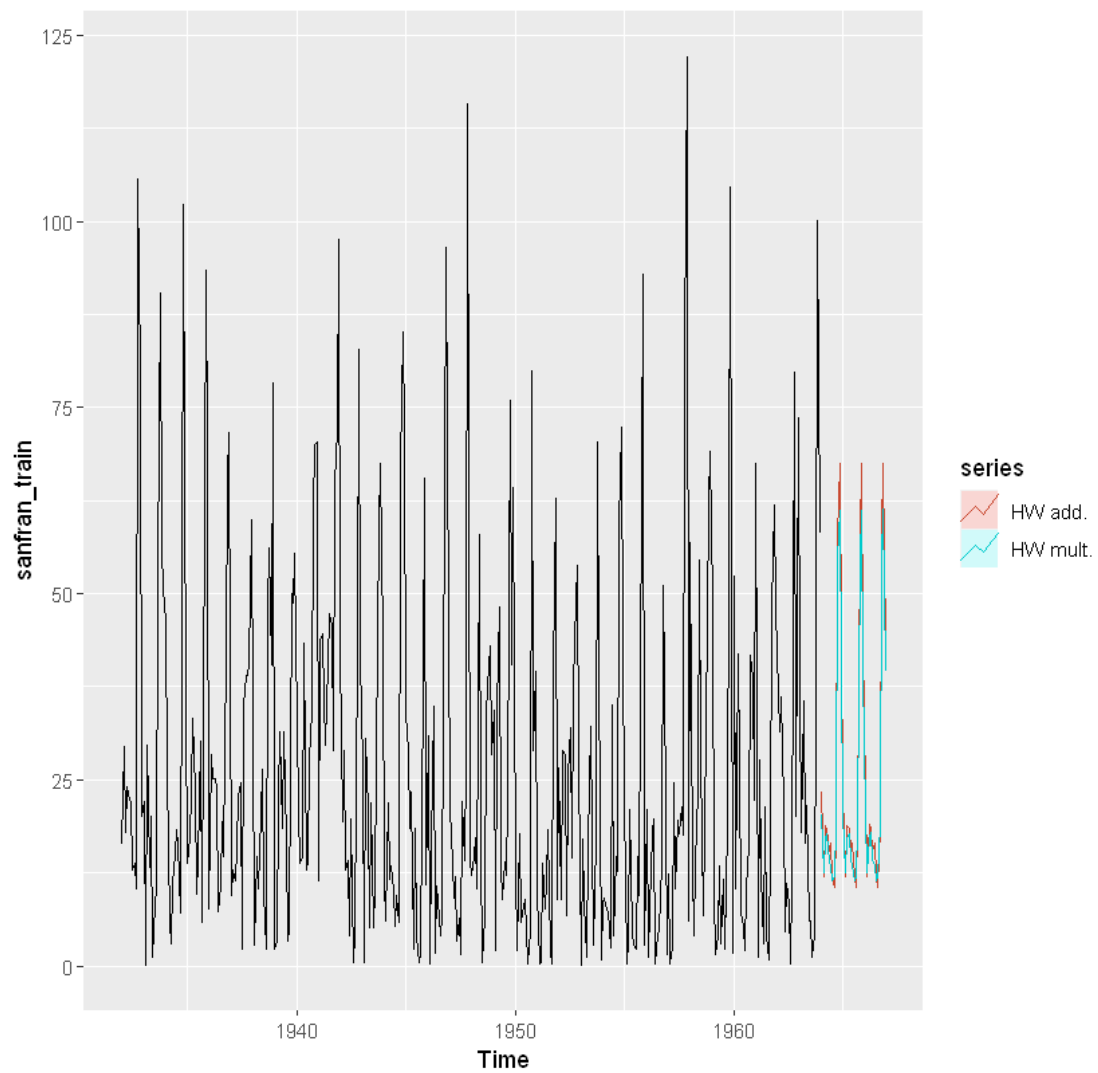
Modify data to replace 0 value by 0.1 value to be able to run multiplicativ model

0.1

253

[1] 15.87358

[1] 17.65021



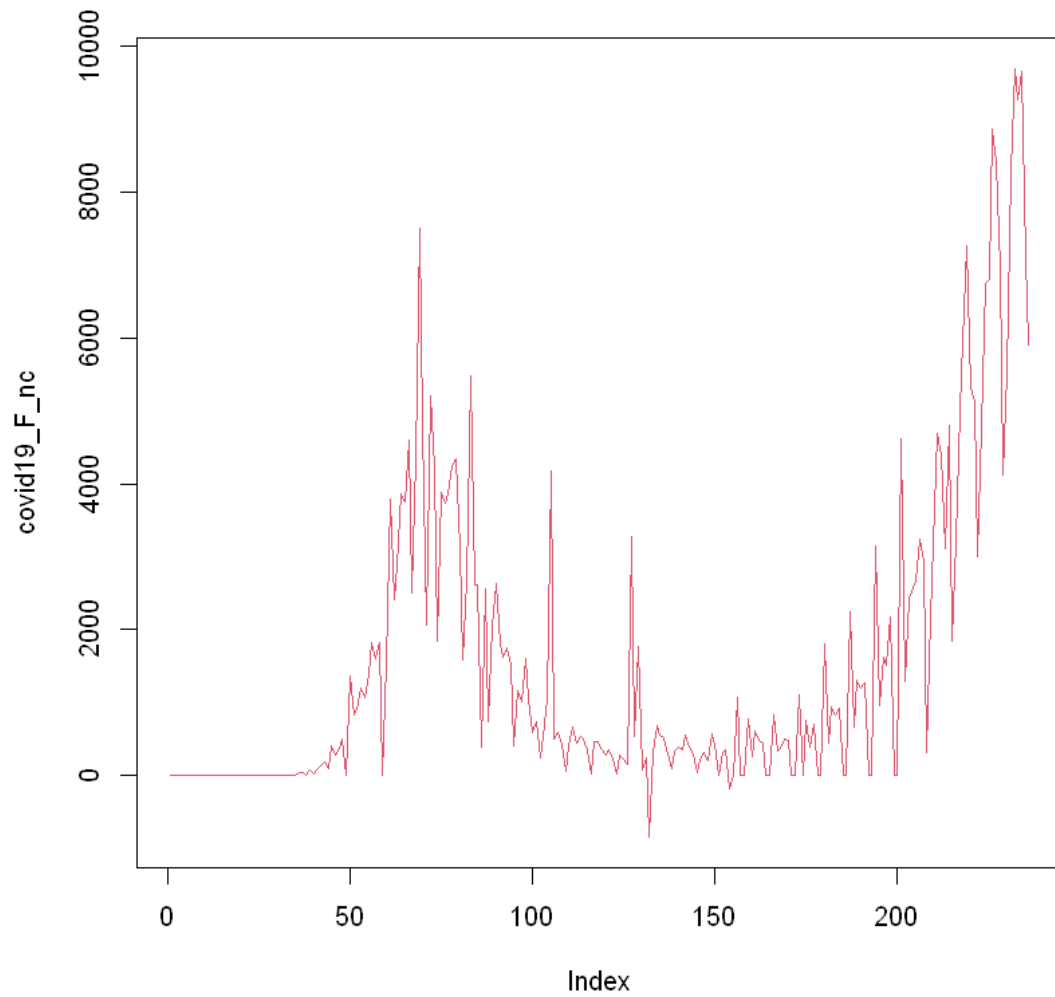
[1] 15.739

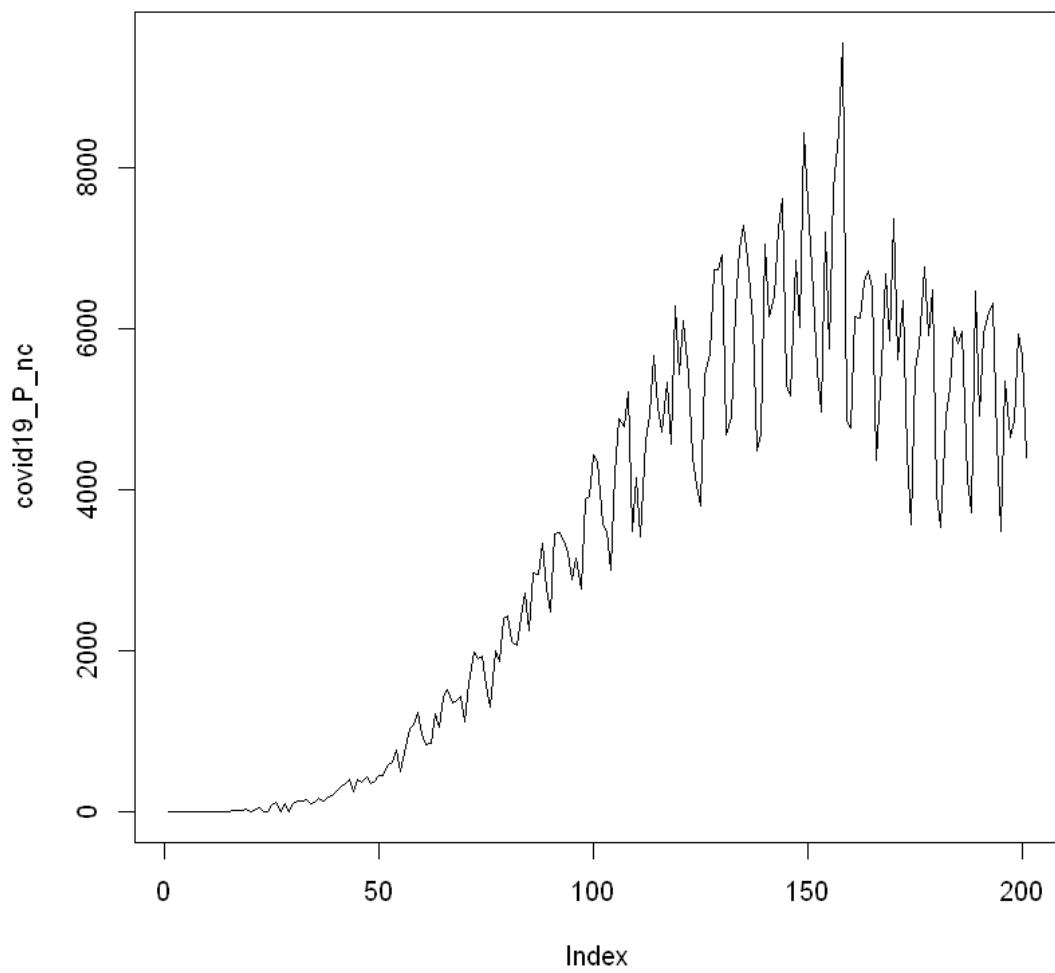
[1] 16.64443

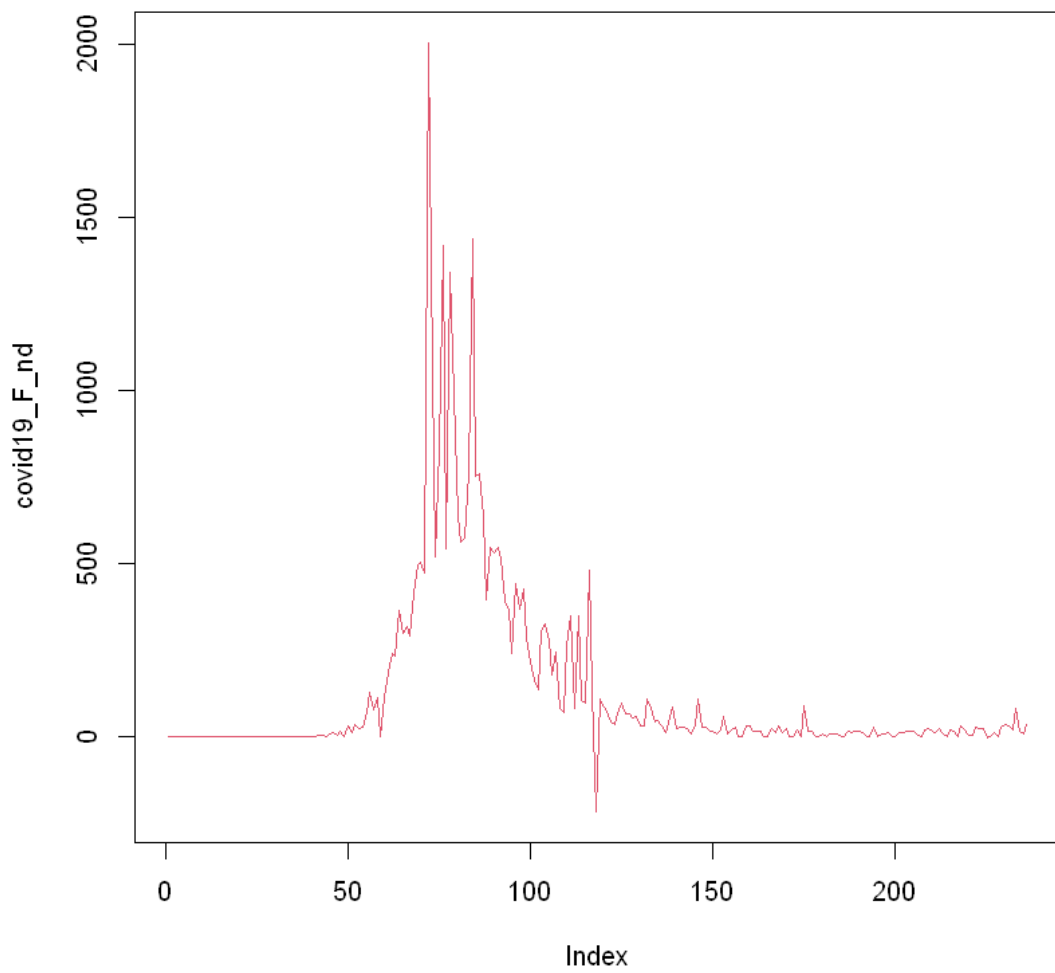
Best model is fit 3 : additiv with damped

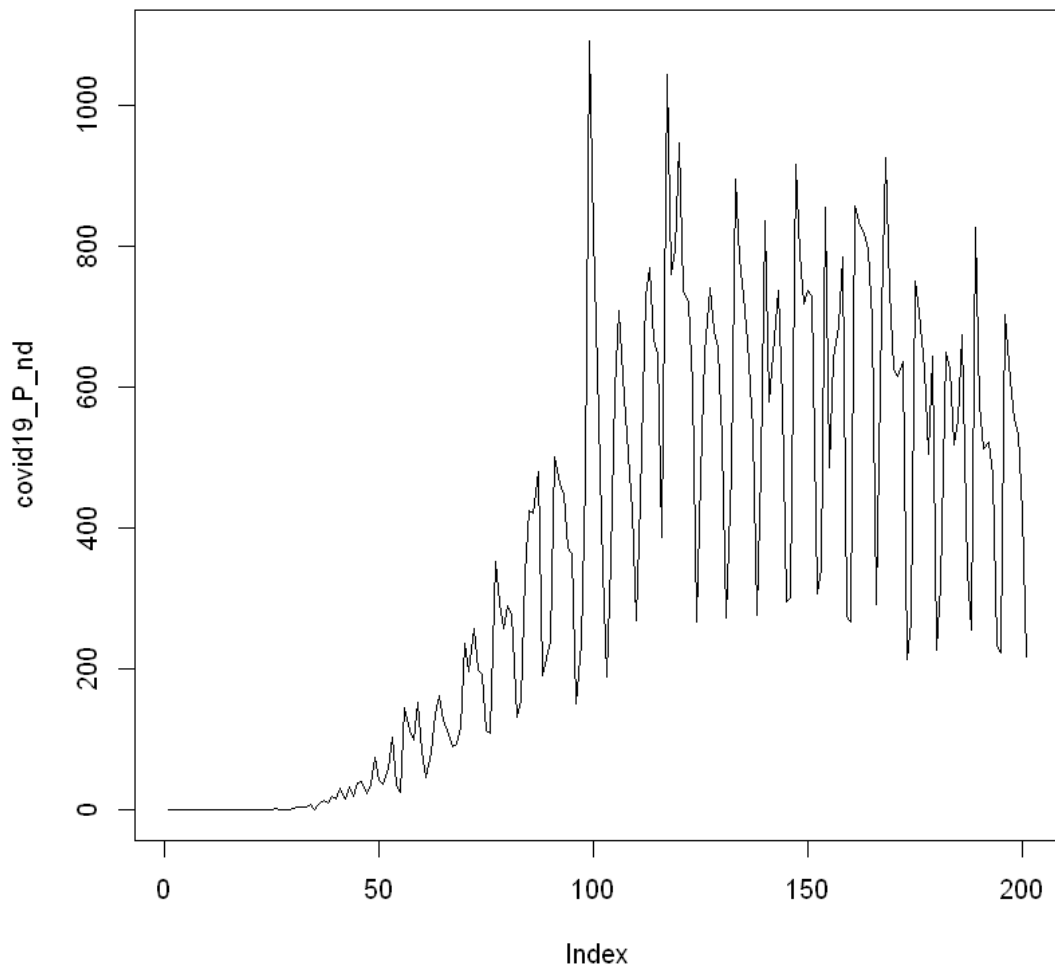
1 Day 2

1.1 auto correlation for new cases of Covid19 in France

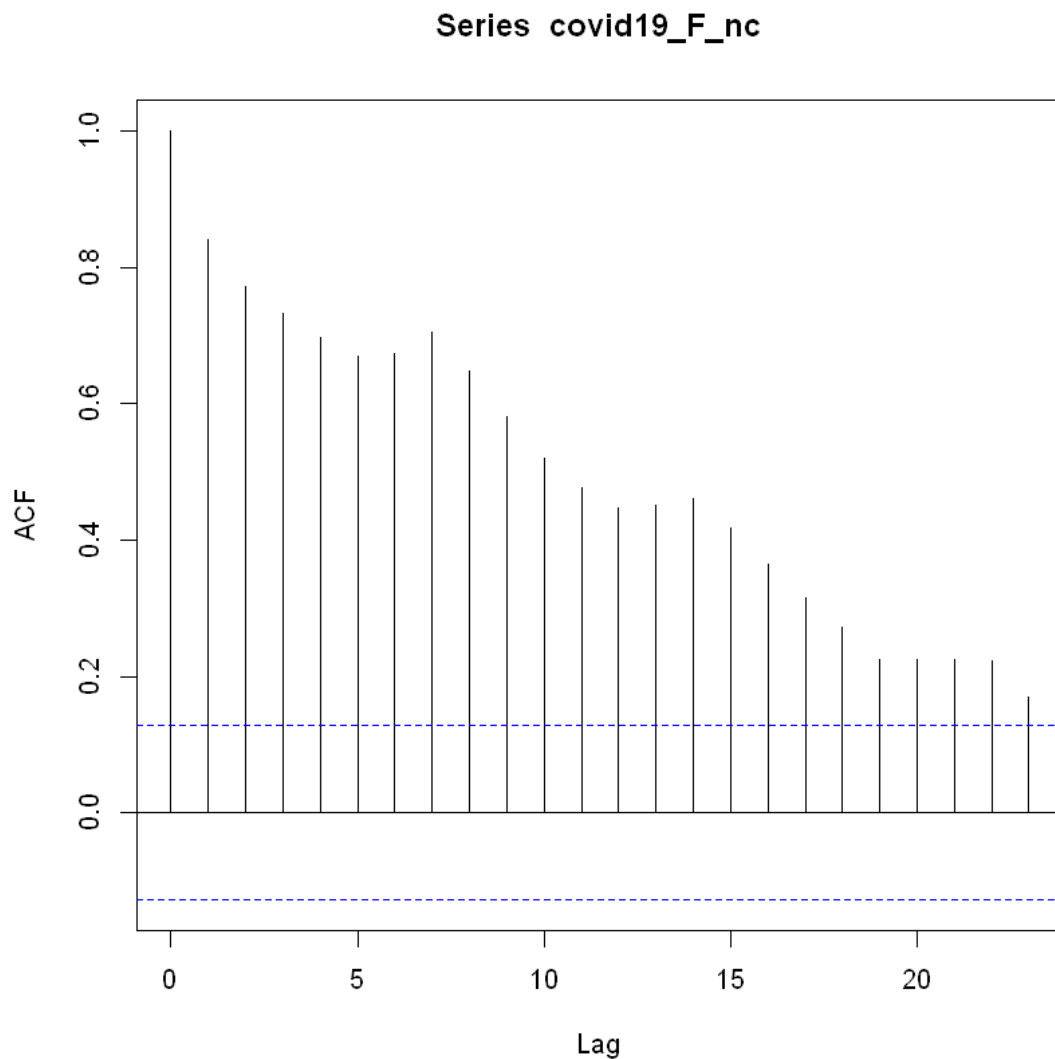








1. 1 2. 0.84001982804108 3. 0.771013894579328



1) There is a kind of linear trend

2) There is a peridodic effect (7 days)

1.2 Removing trend ans seasonal pattern - non parametric

1.2.1 Moving average - non parametric

Registered S3 method overwritten by 'quantmod':

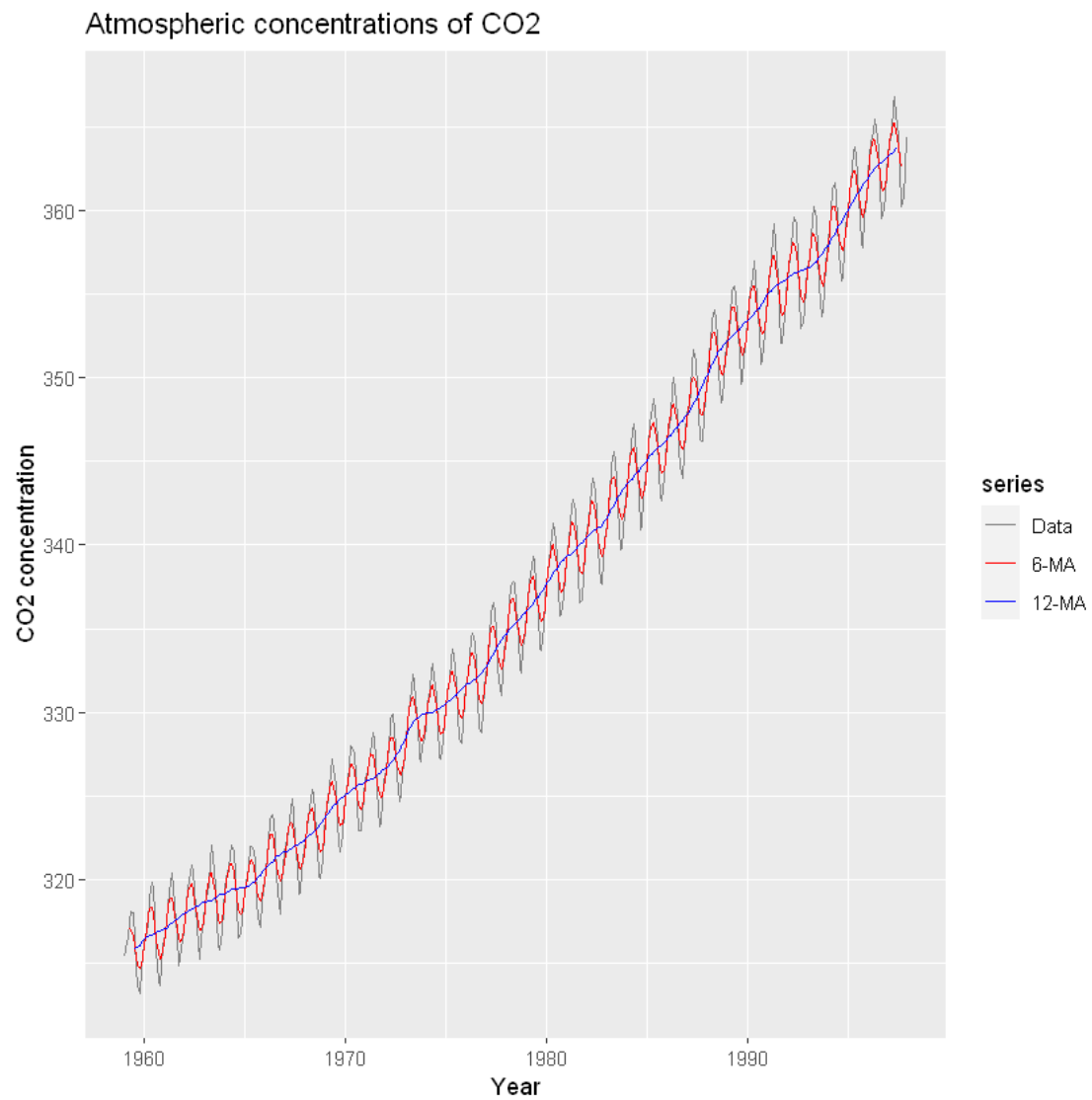
```
method      from
as.zoo.data.frame zoo
```

Warning message:

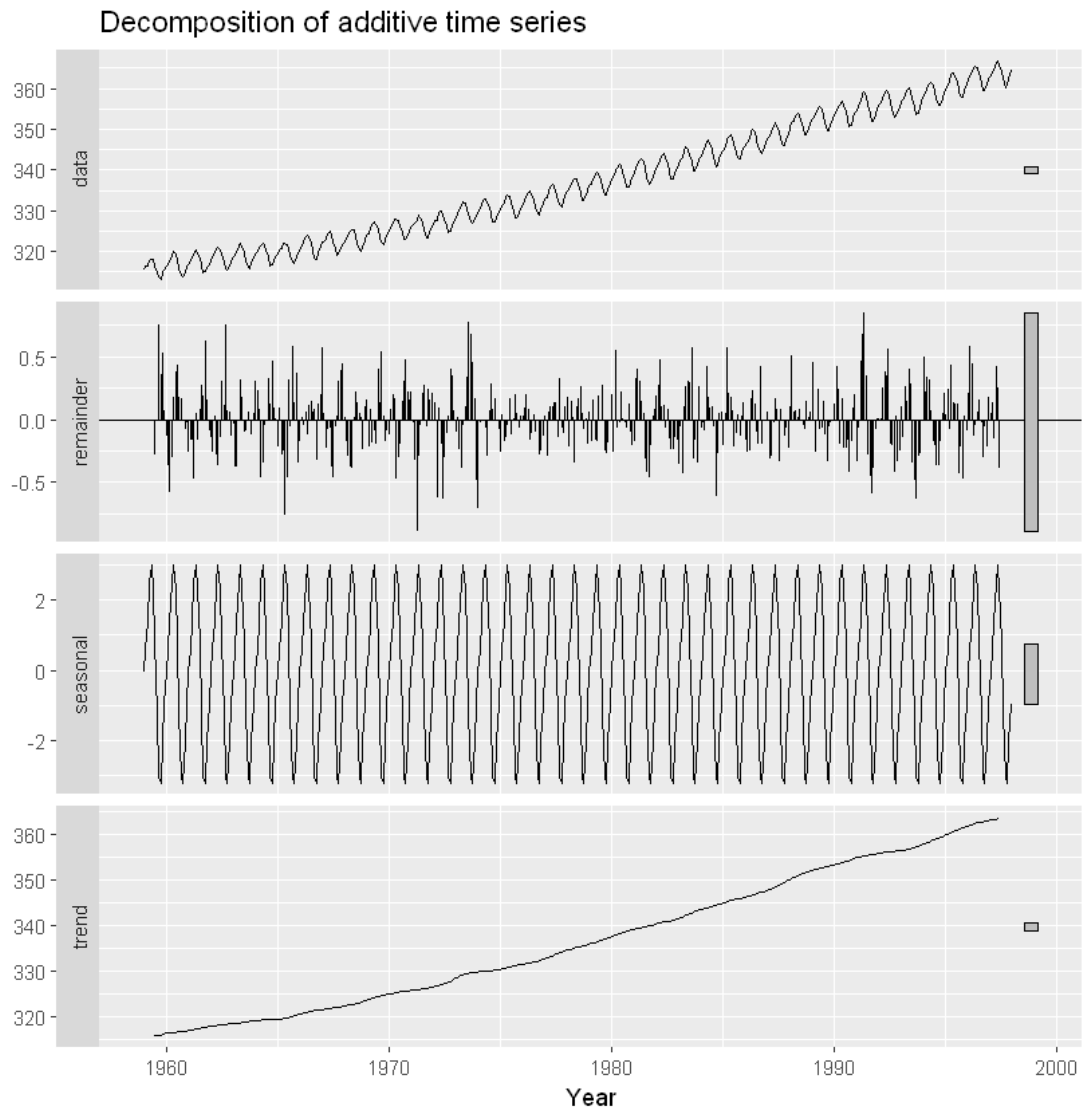
"Removed 6 row(s) containing missing values (geom_path)."

Warning message:

"Removed 12 row(s) containing missing values (geom_path)."

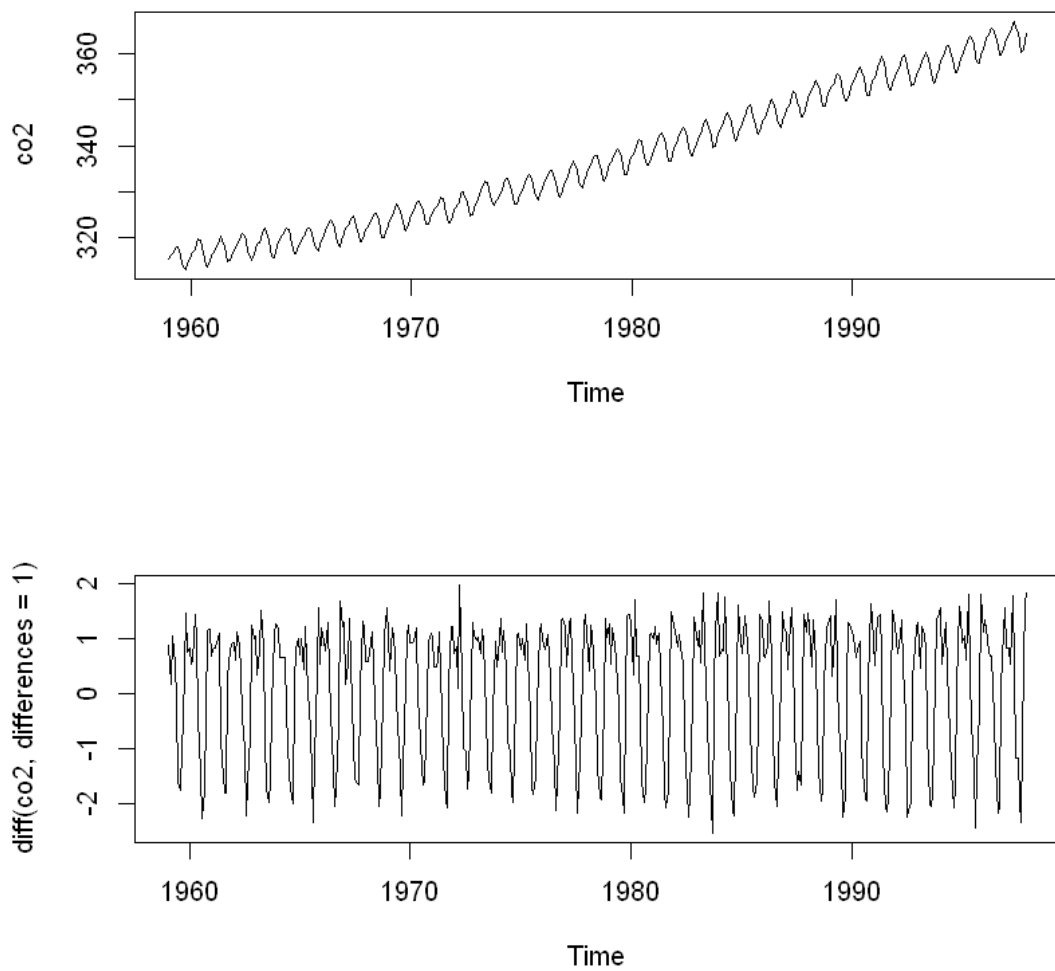


1.2.2 Decompose function - non parametric

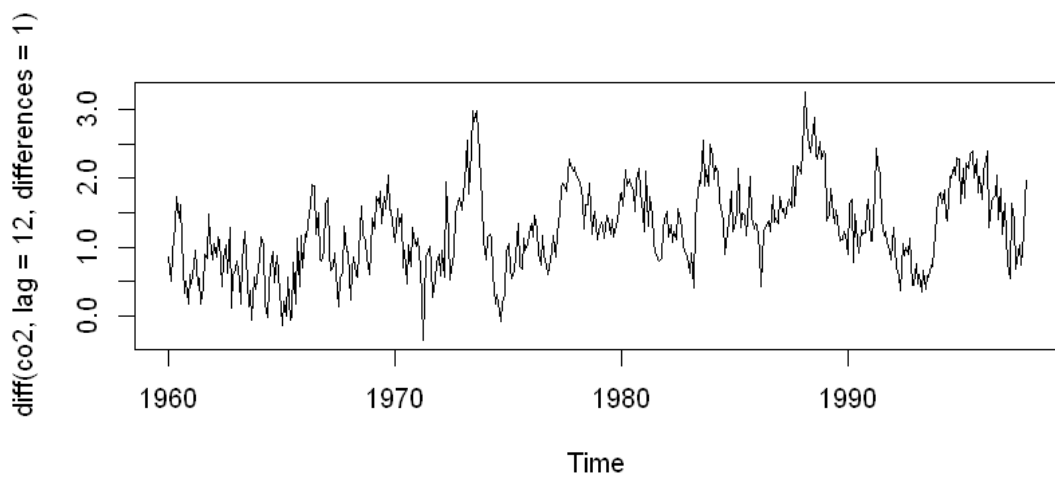
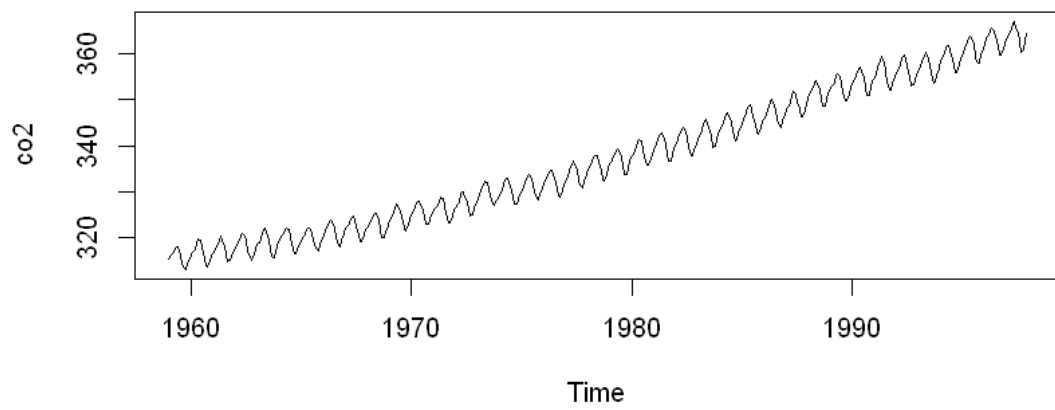


1.3 Differencing - parametric method

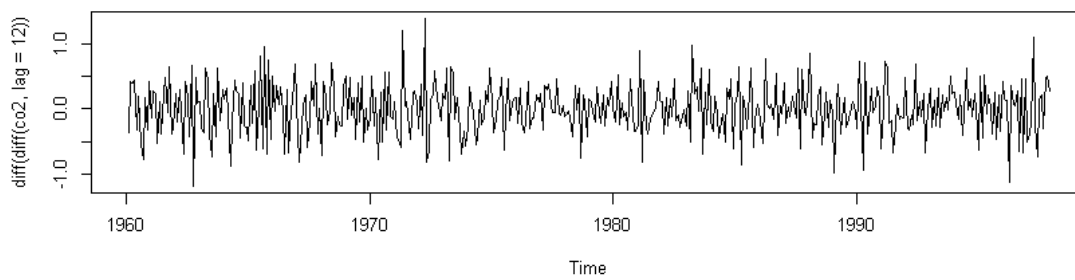
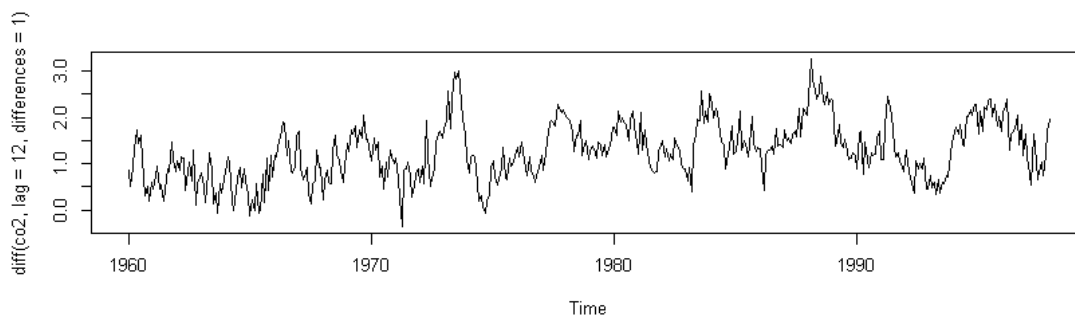
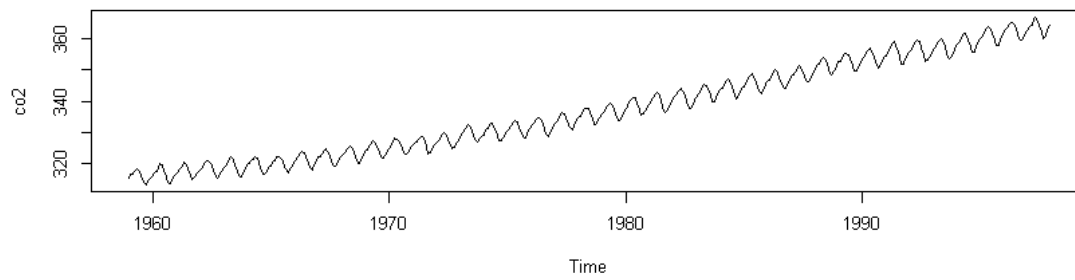
Applying diff function to co2 dataset to remove trend

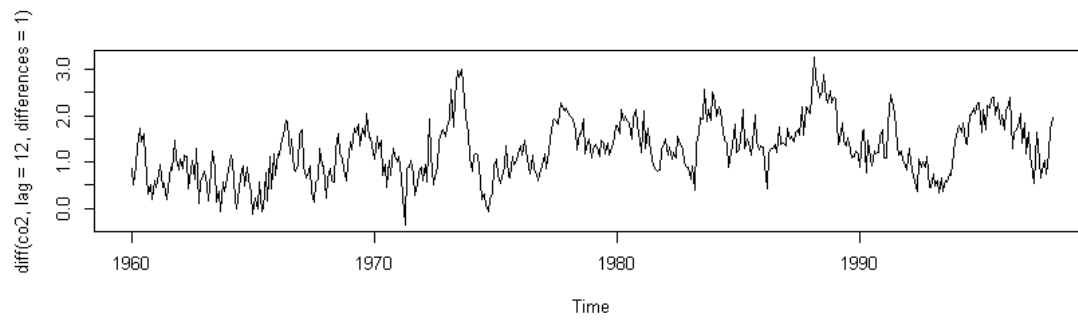


Applying diff function to co2 dataset to remove trend and seasonal



We just have the noise remaining





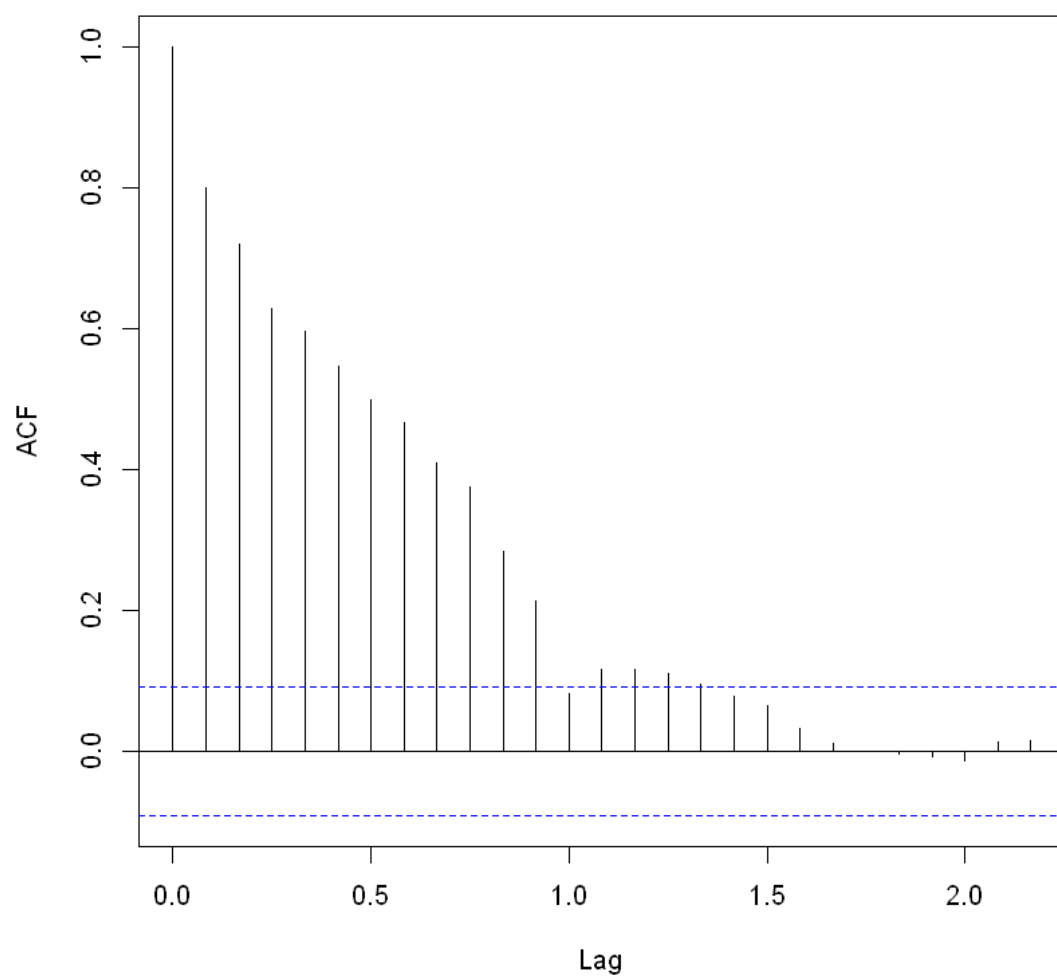
1.3.1 Testing if our residual is a white noise ?

Box-Ljung test

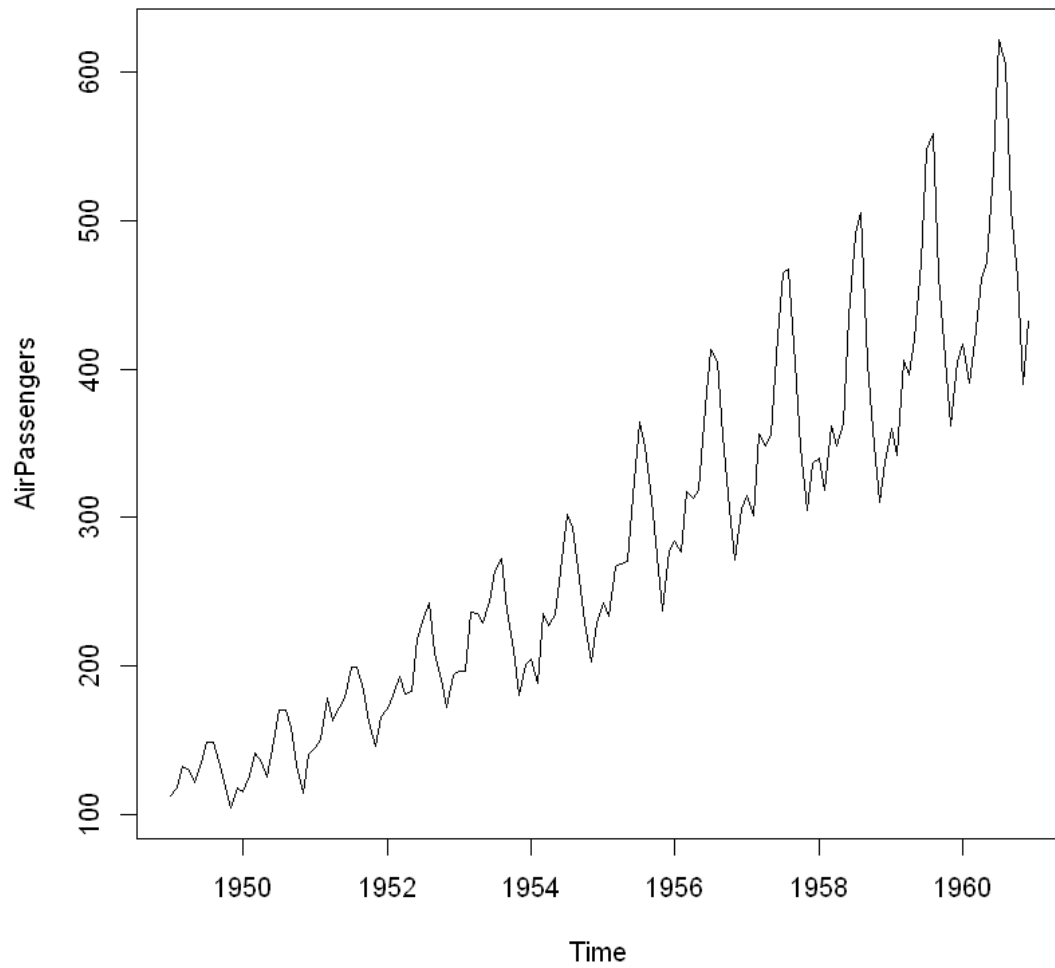
```
data: diff(co2, lag = 12, differences = 1)
X-squared = 1415.4, df = 10, p-value < 2.2e-16
```

Auto correlation of the noise

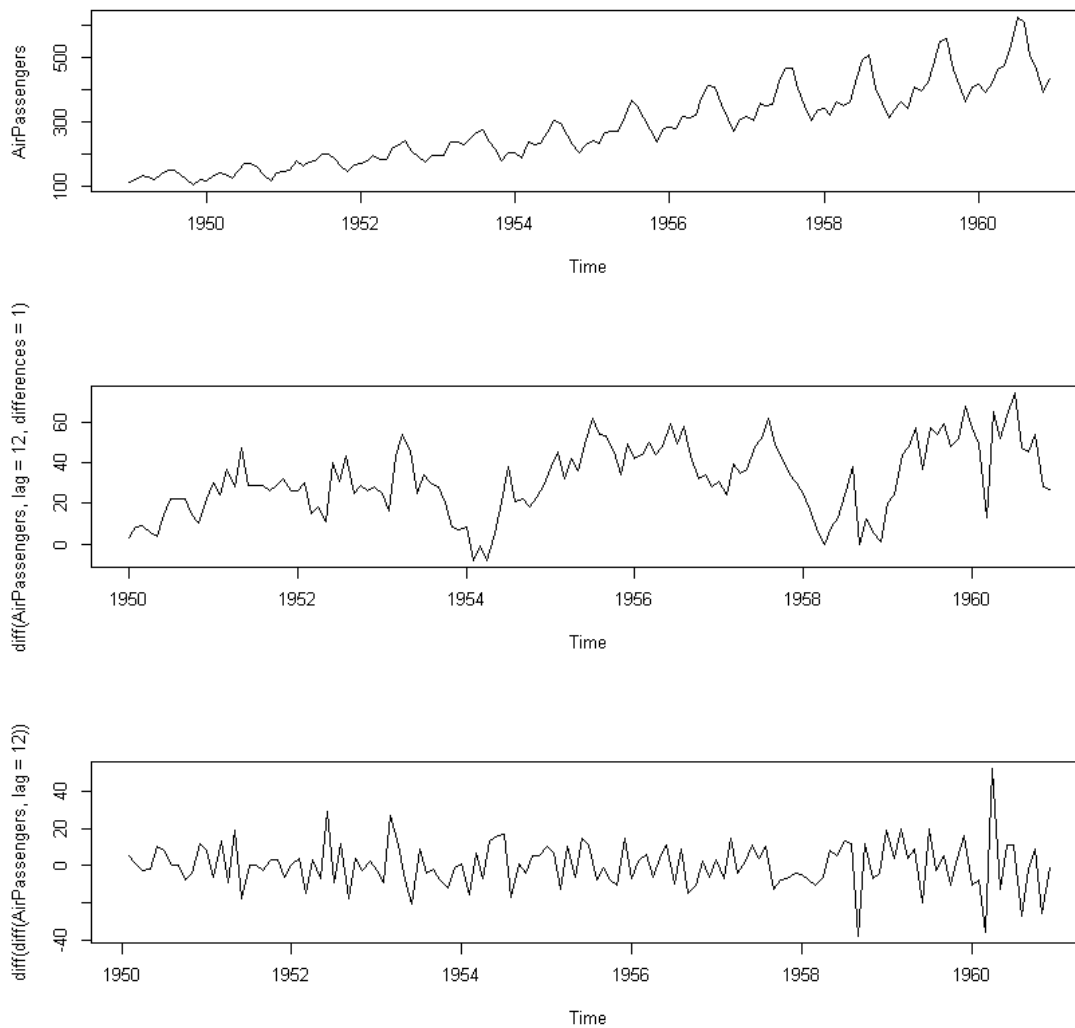
Series diff(co2, lag = 12, differences = 1)



1.3.2 Exercise on AirPassengers



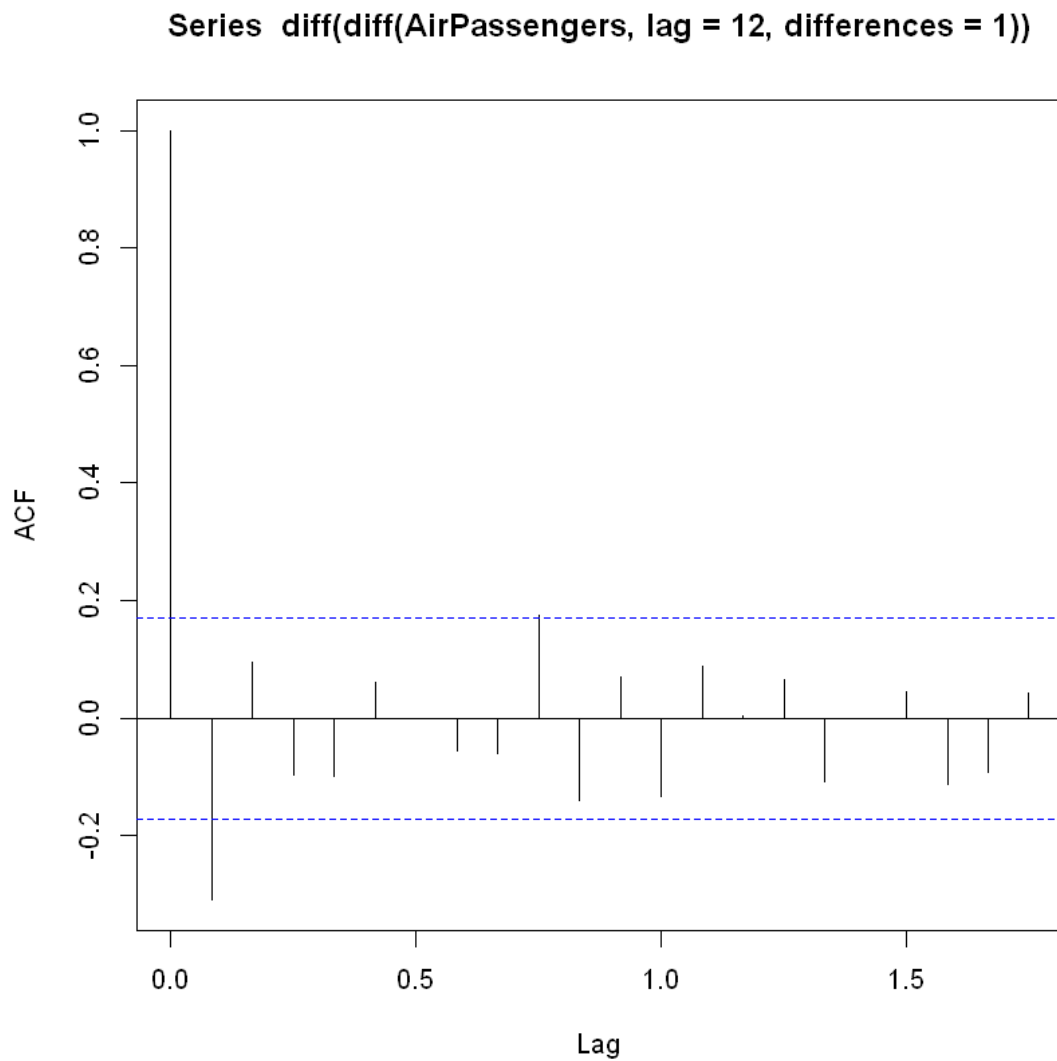
It seems to have trend and seasonnality with a muplicativ effect



Check the residual noise

Box-Ljung test

```
data: diff(diff(AirPassengers, lag = 12, differences = 1))
X-squared = 25.451, df = 10, p-value = 0.004553
```

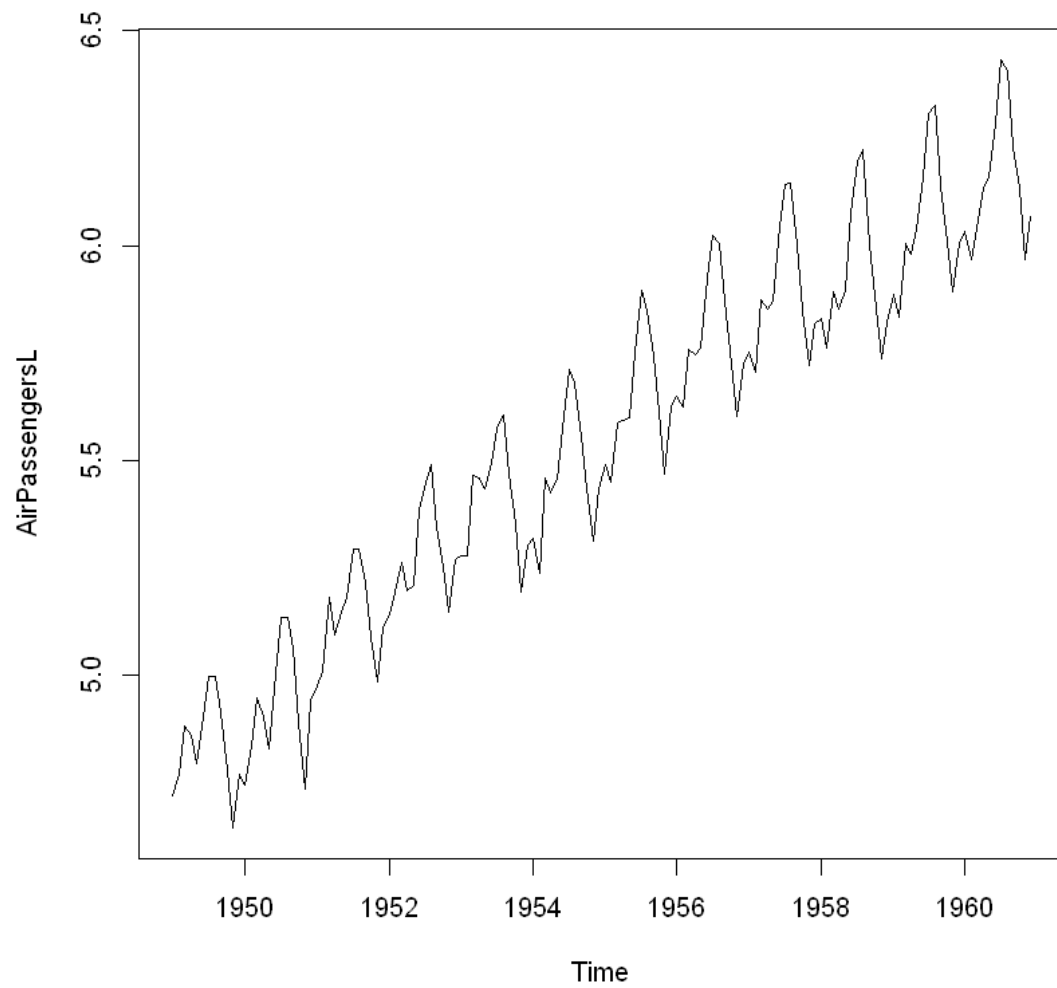


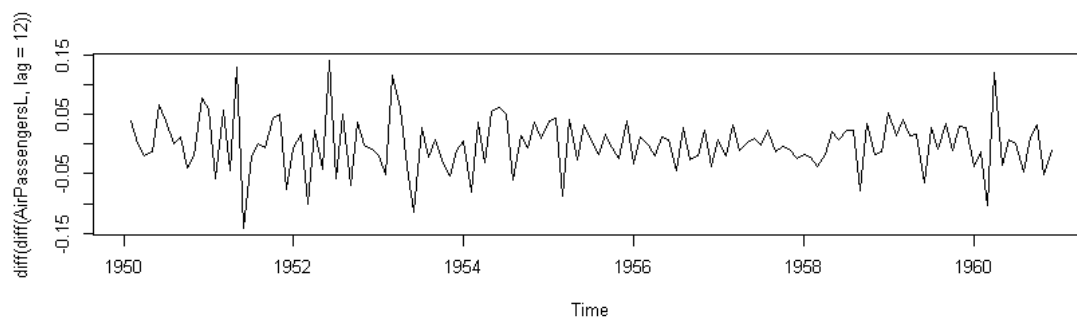
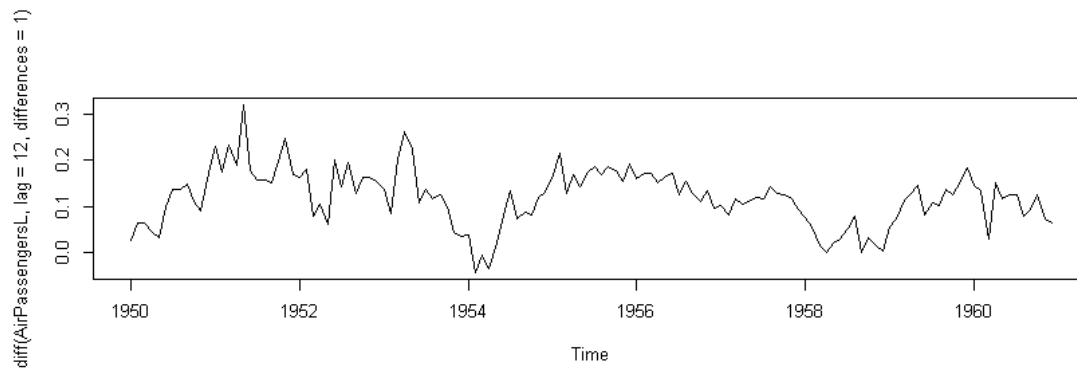
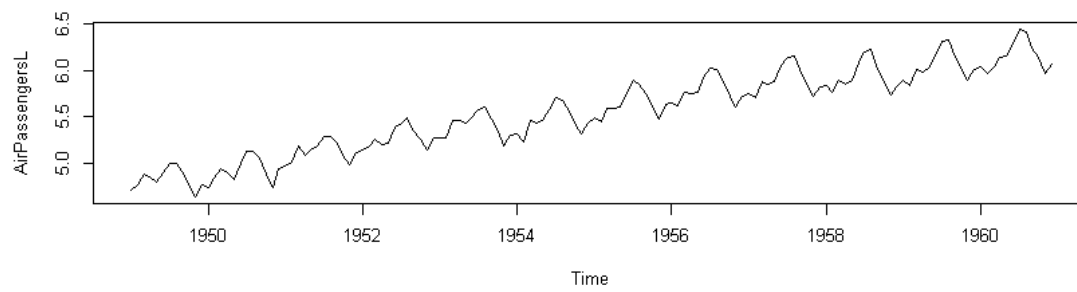
p-value is close to 0 so we reject the fact that it is a white noise

$\text{corr}(X_t, X_{t-1}) = -0.3$: a large number of passenger month n then month $n+1$ should be small

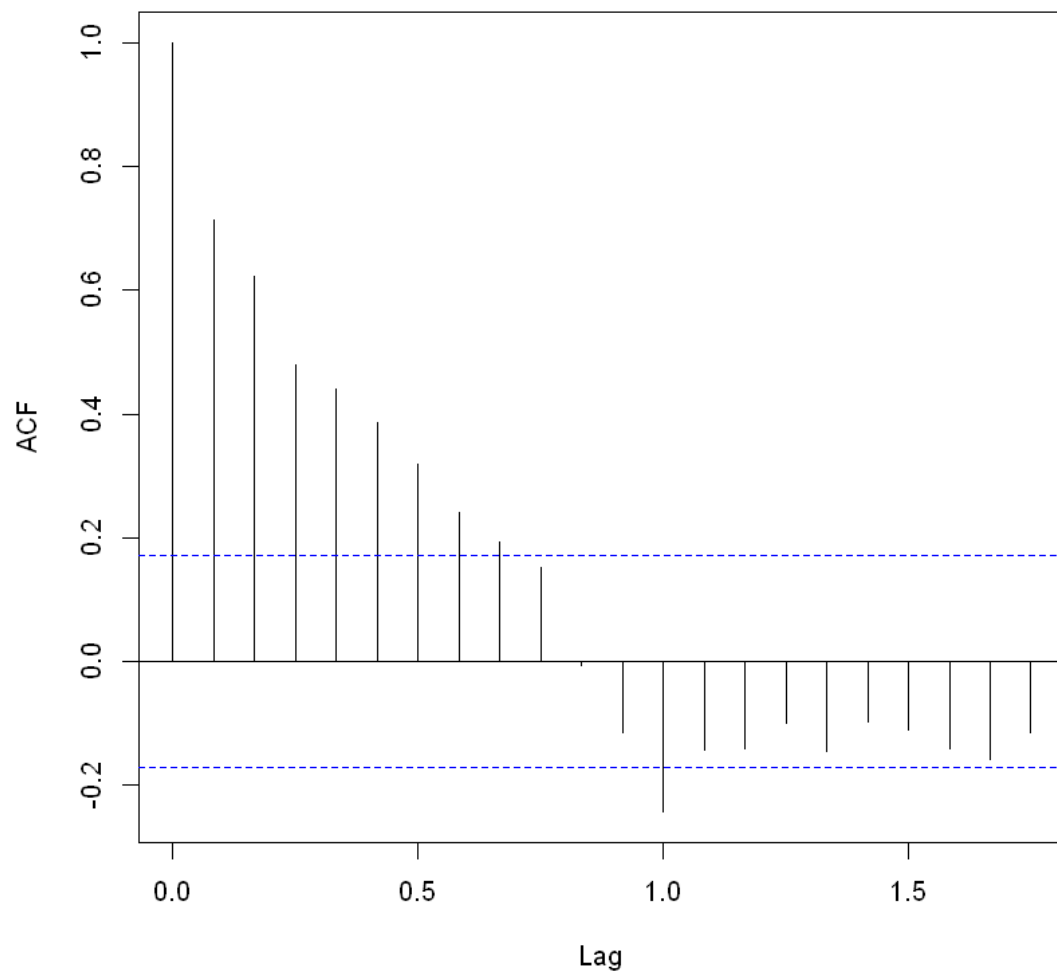
ATTENTION : here we should use **LOG** since multiplicative effect

1.3.3 Exercise AirPassengers With Log to take in account multiplicative effect

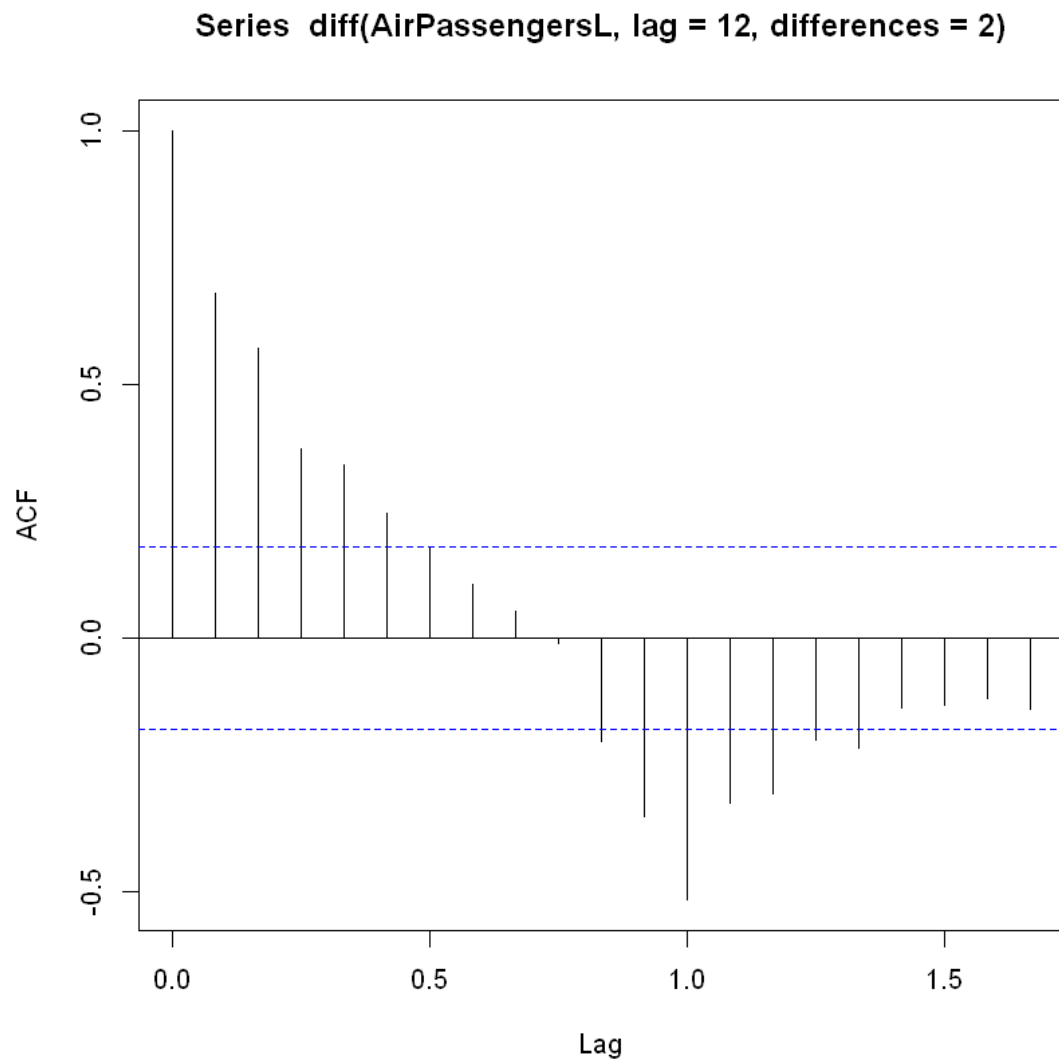




Series diff(AirPassengersL, lag = 12, differences = 1)

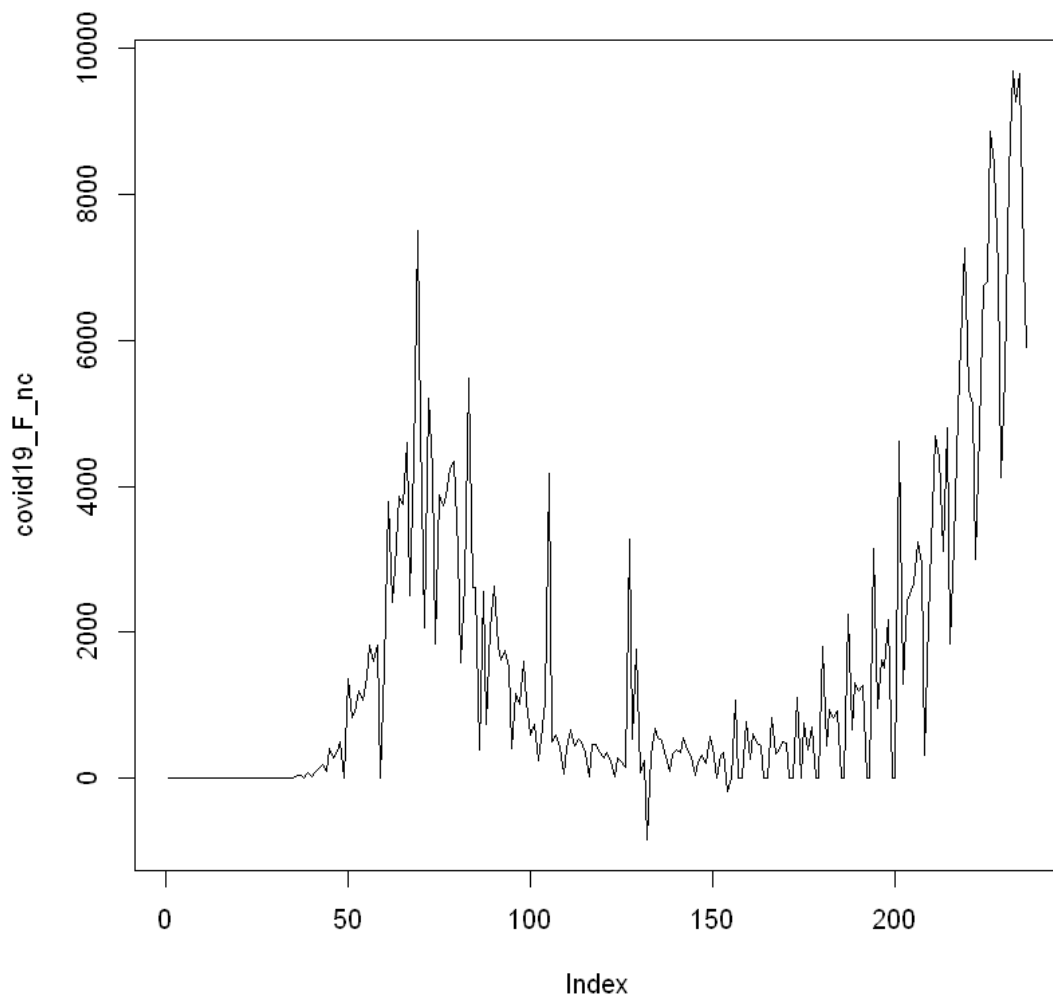


Still a linear trend. So we apply differenciatiion again



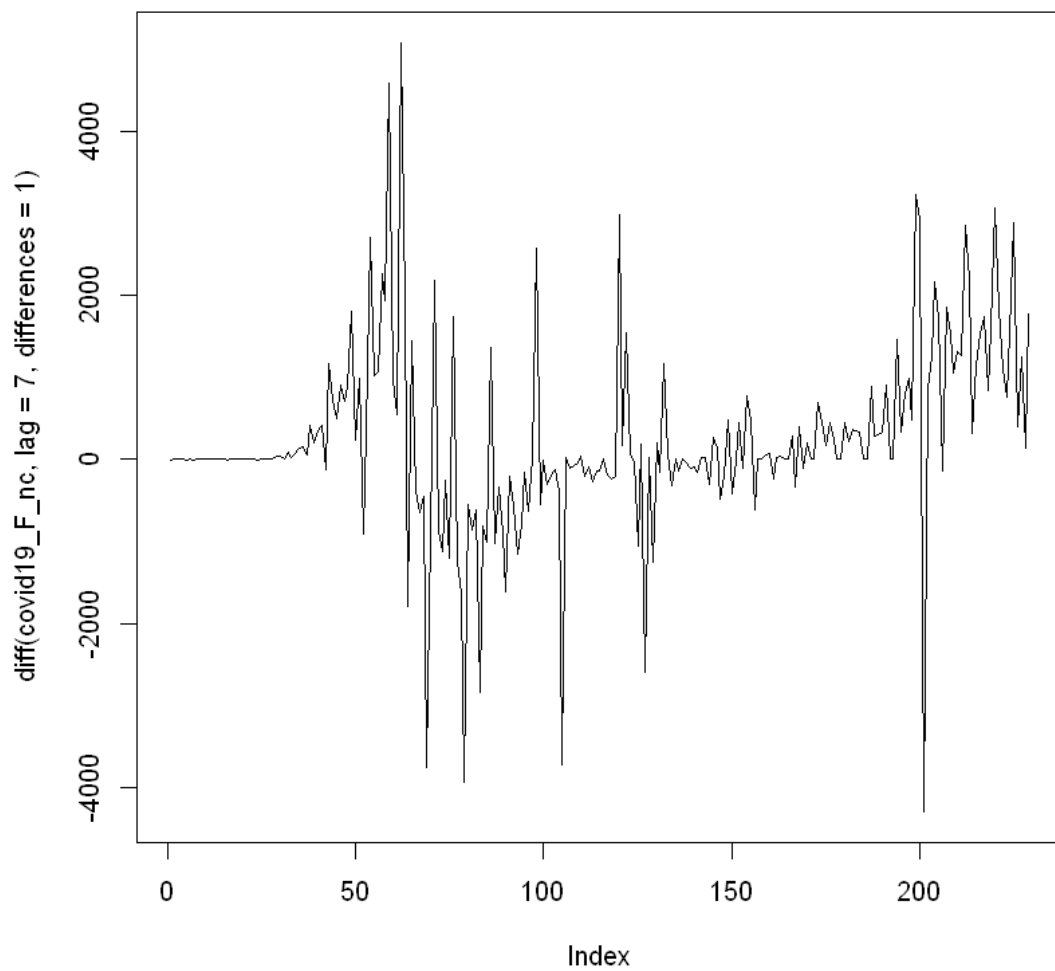
After applying differenciation a second time and still the same graph, so something else is there ... we will see later ..

1.3.4 Exercice on Covid 19

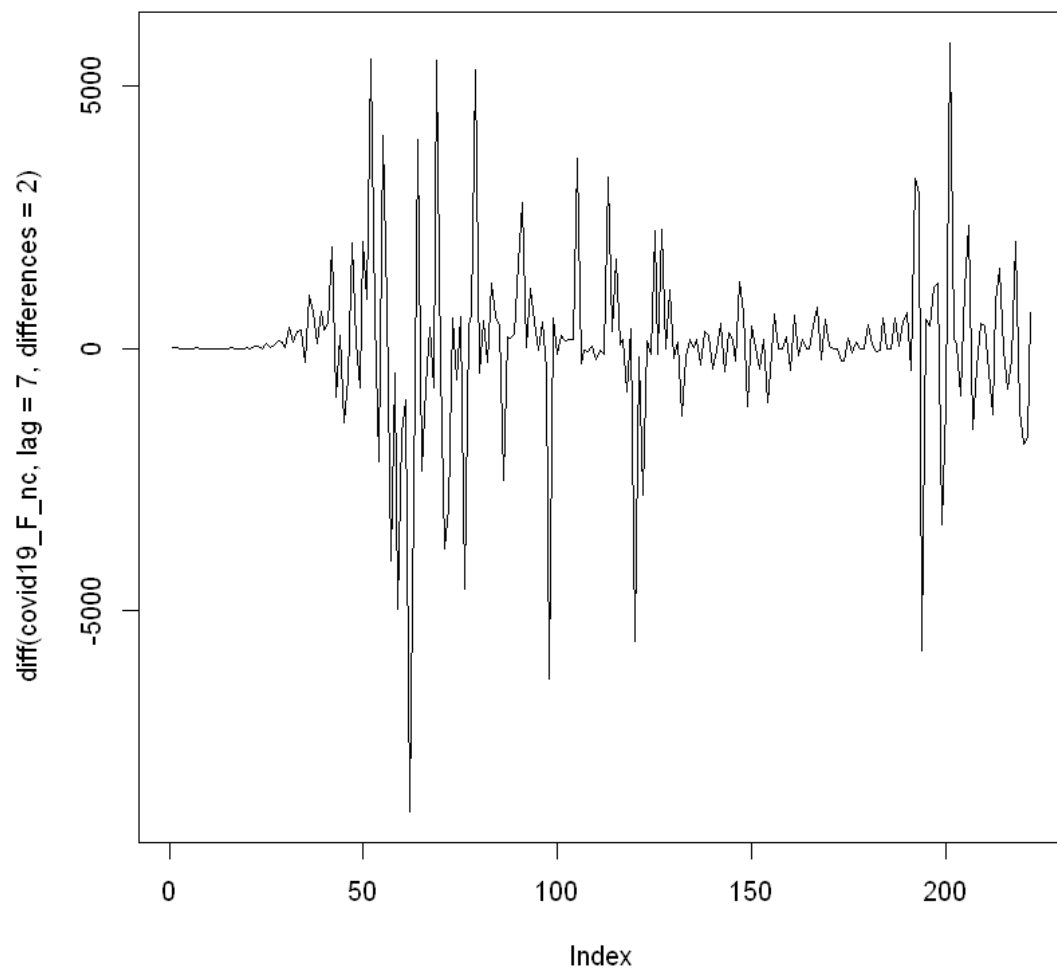


We see here a trend and a seasonality of 7 days

We will remove this with differenciatiion



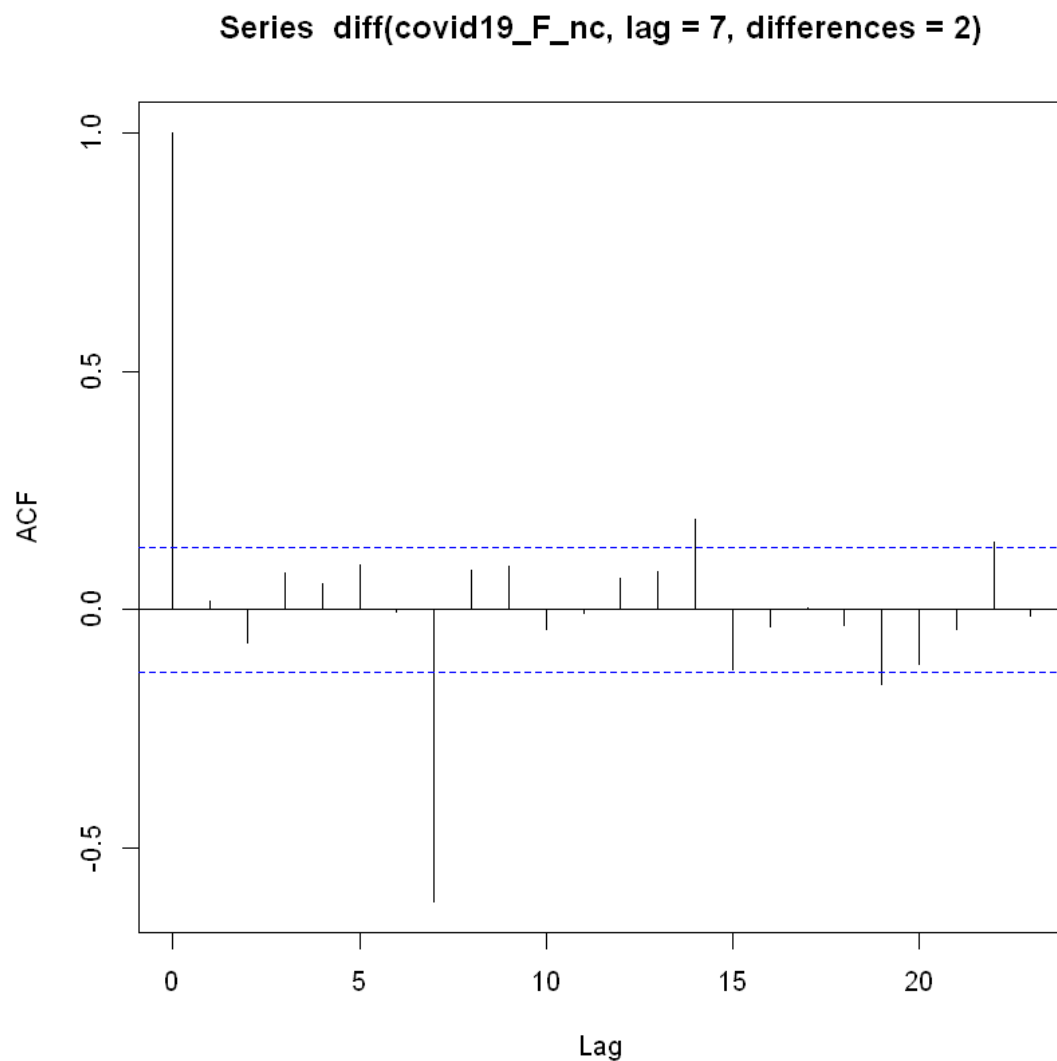
We can see that we still have a trend so we apply differentiation again



Now we can analyse the white noise

Box-Ljung test

```
data: diff(covid19_F_nc, lag = 7, differences = 2)
X-squared = 95.766, df = 10, p-value = 3.331e-16
```



We reject the fact that there is a white noise since pvalue is closed to 0

1.3.5 Exercice Google stock Price

```
-- Attaching packages -----
fpp2 2.4 --
```

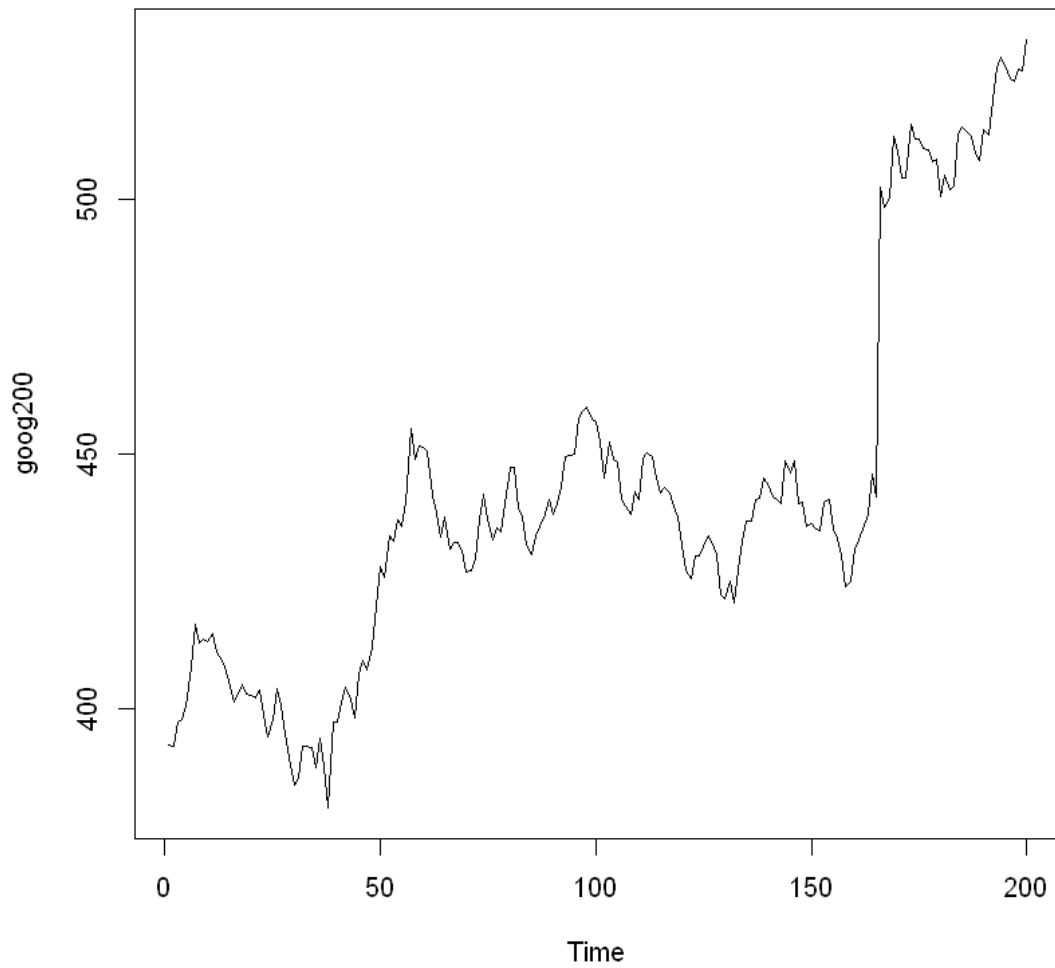
```
v fma      2.4    v expsmooth 2.3
```

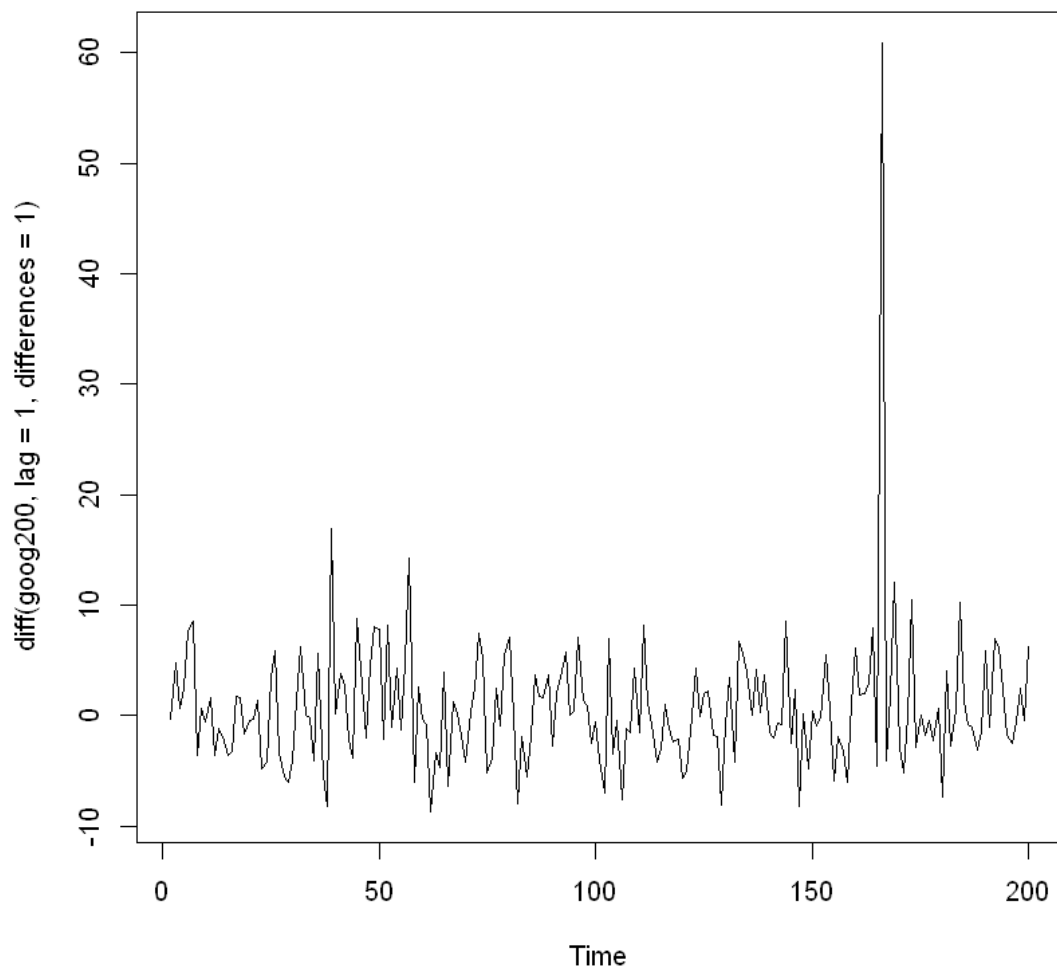
```
Installing package into 'C:/Users/erick/R'
(as 'lib' is unspecified)
```

package 'fpp2' successfully unpacked and MD5 sums checked

The downloaded binary packages are in

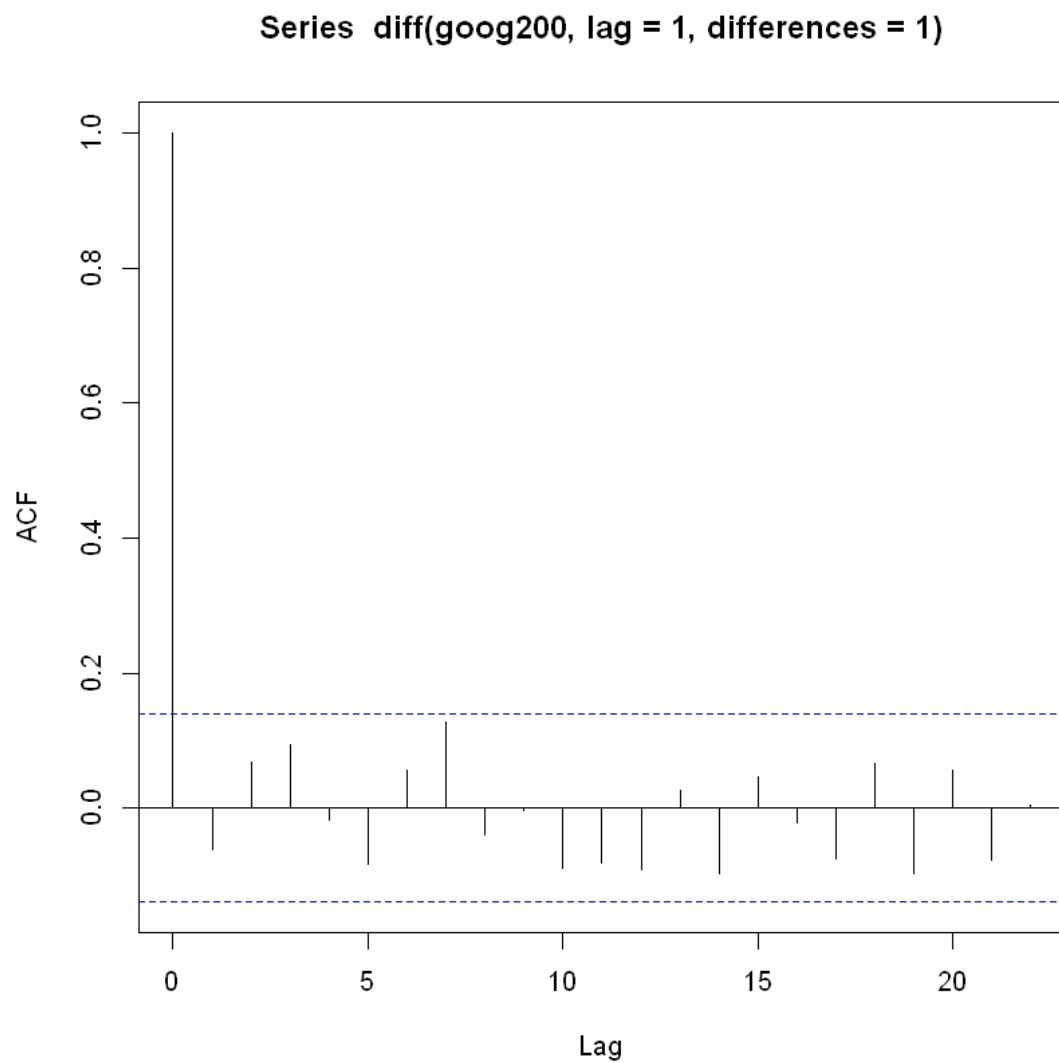
C:\Users\erick\AppData\Local\Temp\Rtmp8o1e1M\downloaded_packages





Box-Ljung test

```
data: diff(goog200, lag = 1, differences = 1)
X-squared = 11.031, df = 10, p-value = 0.3551
```

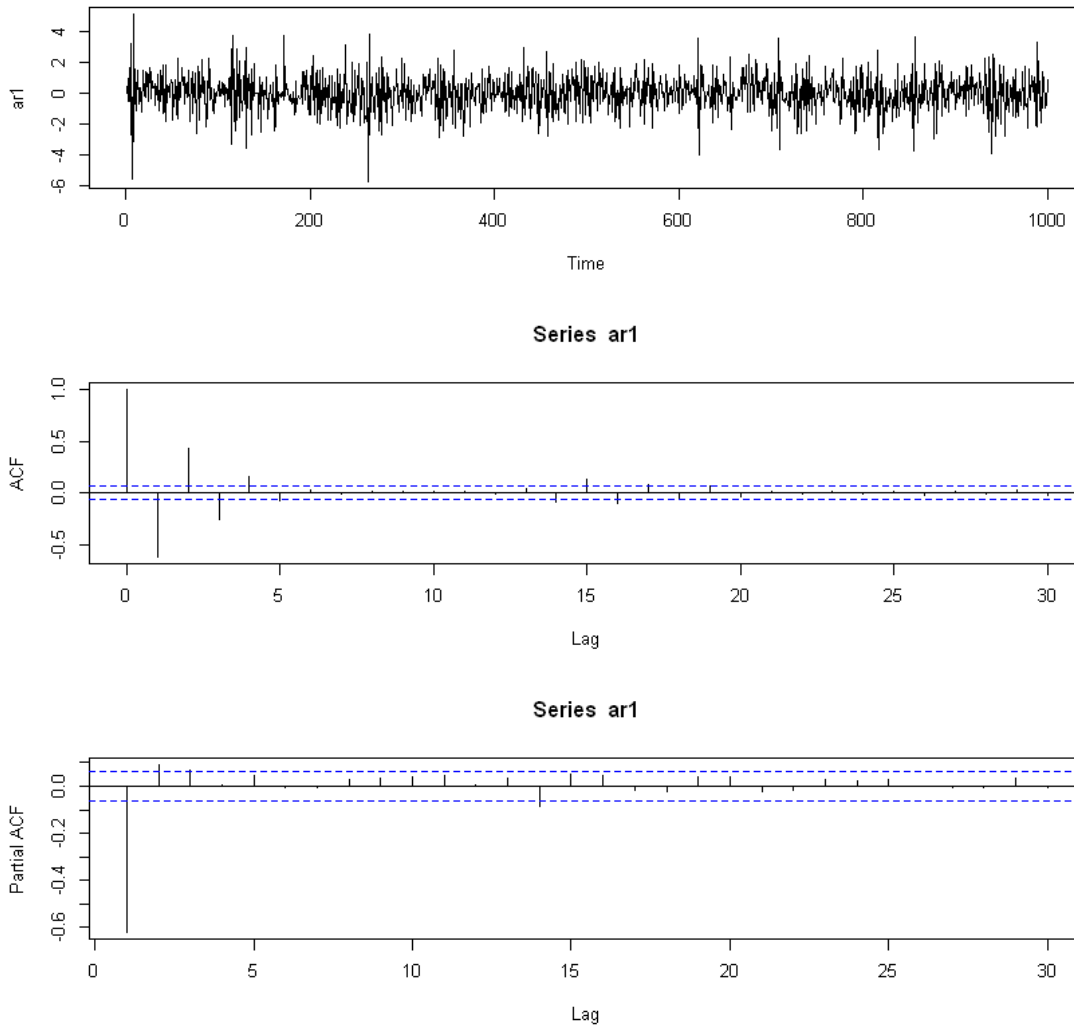


Box Test is true, we accept that we have a white noise

The correlation plot shows that there is no correlation

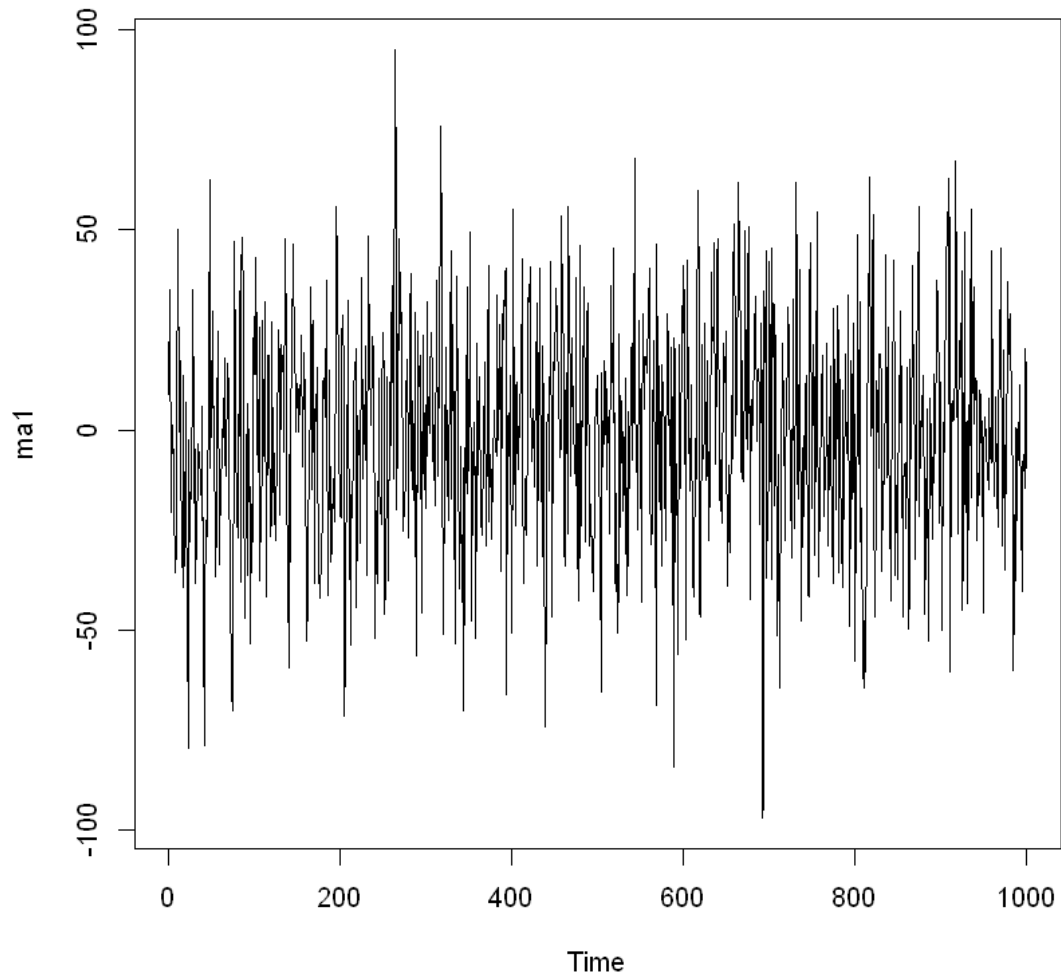
1.4 ARIMA Models

1.4.1 Simulate auto-regressive models with arima.sim function

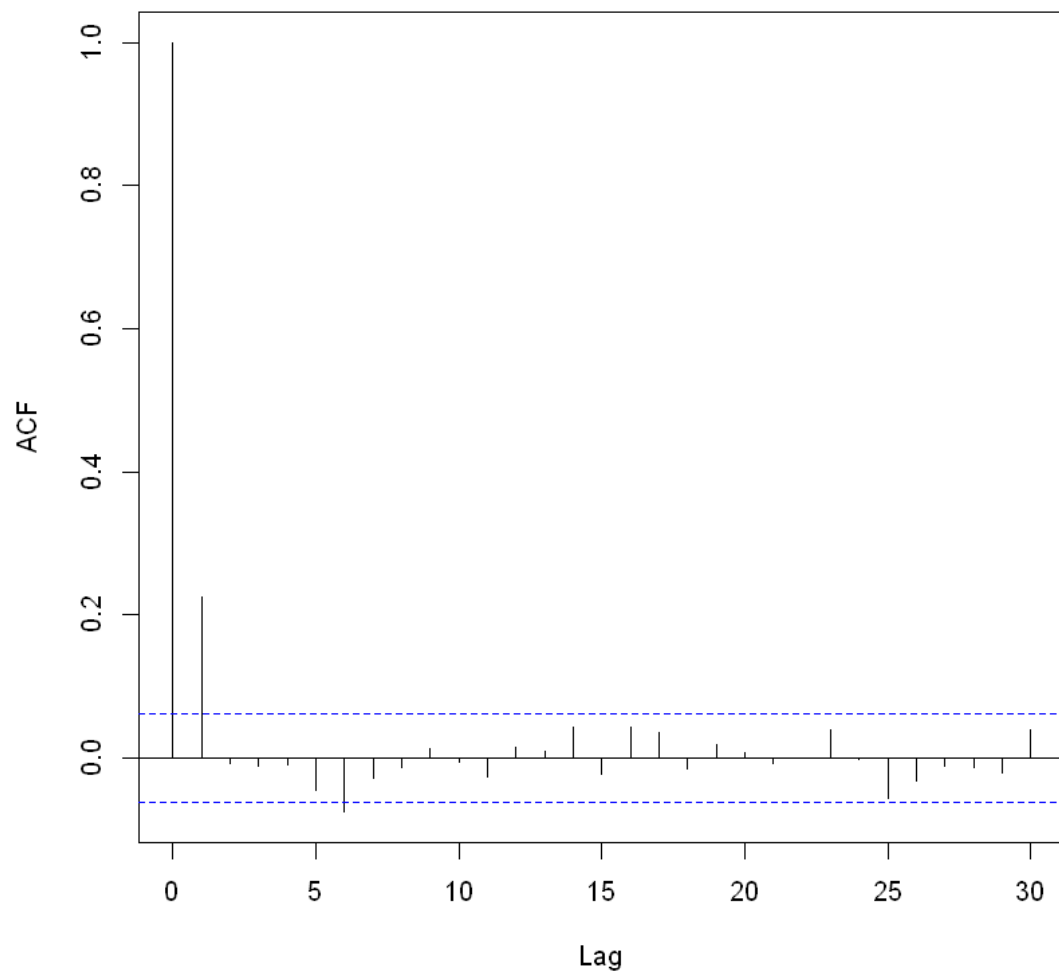


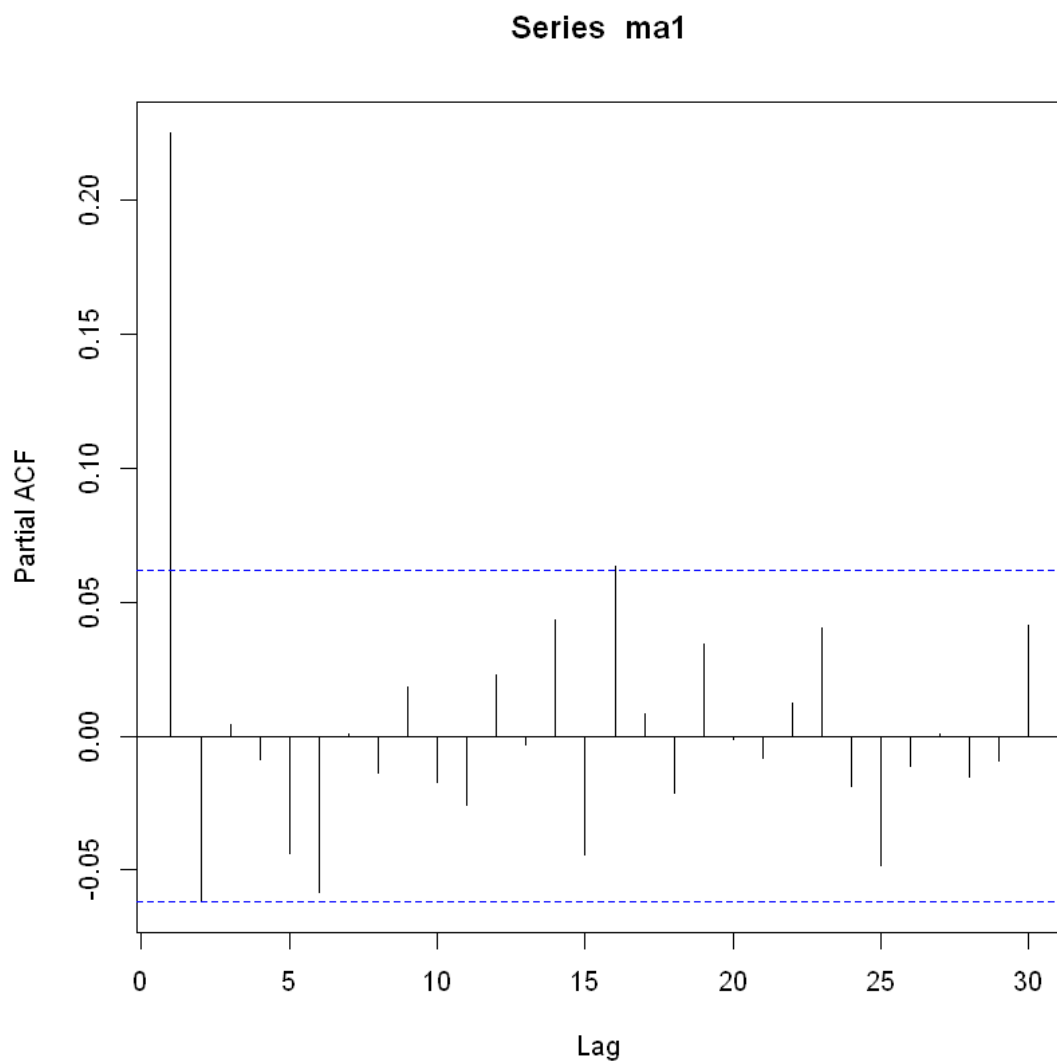
TAKE CARE : If the time series size is 1000, it's ok. With only 100 observations it is not so clear

1.4.2 Simulate moving average models with `arma.sim` function



Series ma1





1.4.3 non-seasonal ARIMA models

Series: uschange[, "Consumption"]
 ARIMA(2,0,2) with non-zero mean

Coefficients:

	ar1	ar2	ma1	ma2	mean
	1.3908	-0.5813	-1.1800	0.5584	0.7463
s.e.	0.2553	0.2078	0.2381	0.1403	0.0845

sigma² estimated as 0.3511: log likelihood=-165.14
 AIC=342.28 AICc=342.75 BIC=361.67

Looking for the smaller AIC (d=0 because no trend)

1.4.4 Quick solution, use auto.arima function to get the parameters automatically

```
Series: uschange[, "Consumption"]  
ARIMA(1,0,3)(1,0,1)[4] with non-zero mean
```

Coefficients:

	ar1	ma1	ma2	ma3	sar1	sma1	mean
	-0.3548	0.5958	0.3437	0.4111	-0.1376	0.3834	0.7460
s.e.	0.1592	0.1496	0.0960	0.0825	0.2117	0.1780	0.0886

```
sigma^2 estimated as 0.3481: log likelihood=-163.34  
AIC=342.67 AICc=343.48 BIC=368.52
```

1.4.5 More complexe but allow to understand what happens

differentiation,

```
Series: uschange[, "Consumption"]  
ARIMA(3,0,0) with non-zero mean
```

Coefficients:

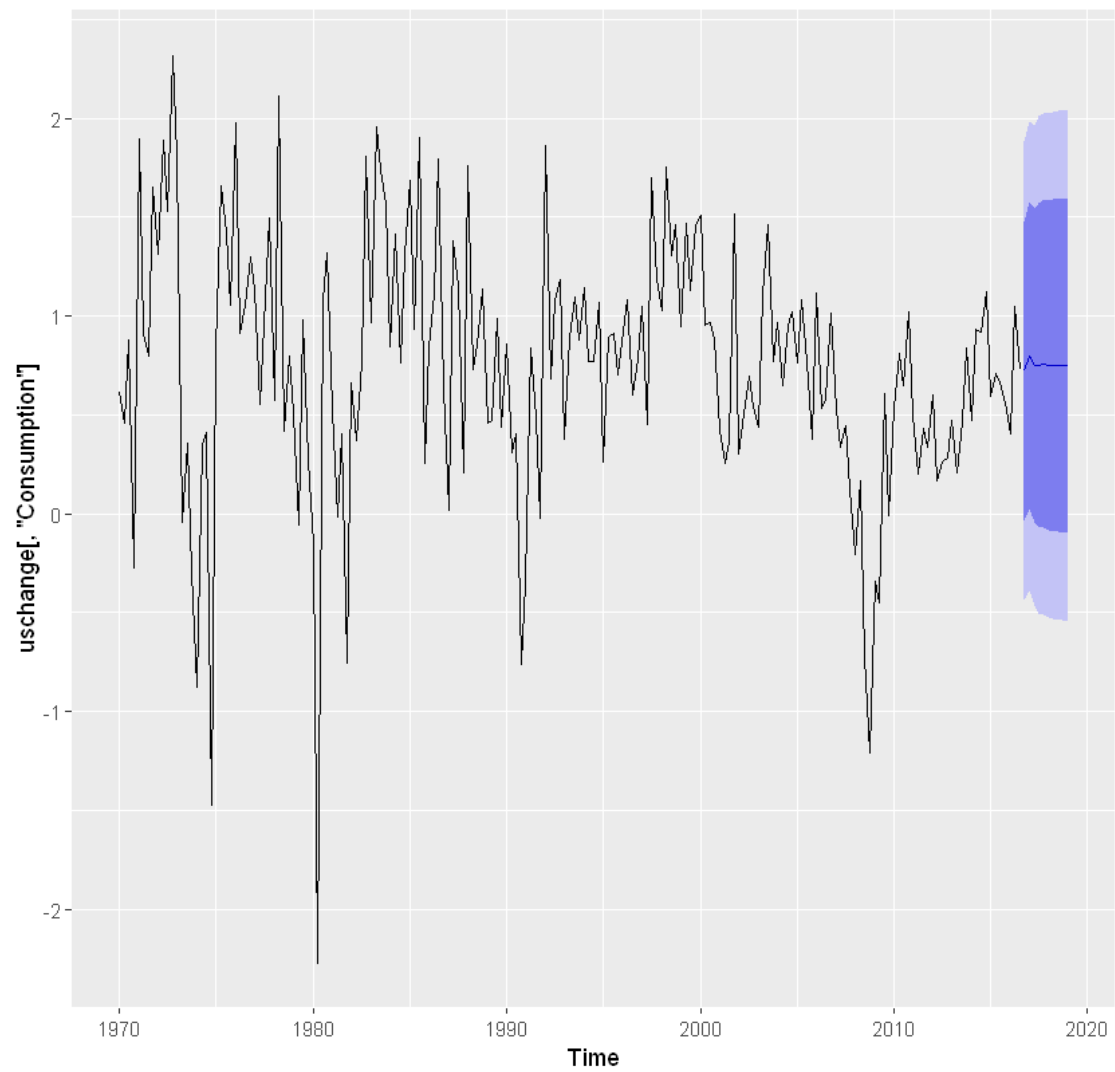
	ar1	ar2	ar3	mean
	0.2274	0.1604	0.2027	0.7449
s.e.	0.0713	0.0723	0.0712	0.1029

```
sigma^2 estimated as 0.3494: log likelihood=-165.17  
AIC=340.34 AICc=340.67 BIC=356.5
```

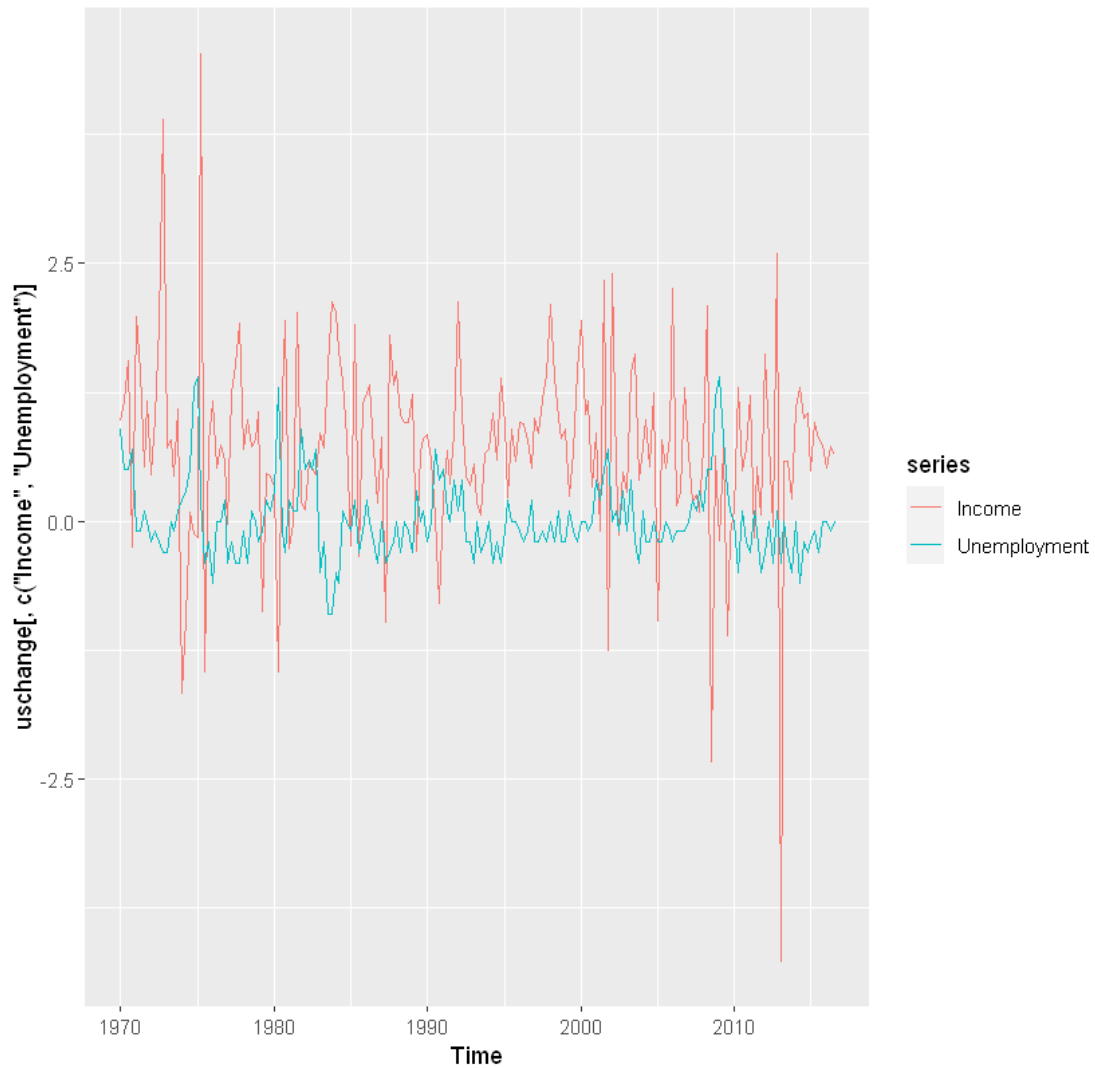
In fact we get a better result with manual AIC = 340 then with automatic AIC = 342
!!

1.4.6 Forecasting with ARMA

Forecasts from ARIMA(3,0,0) with non-zero mean



1.4.7 Exercice on us change



First method use automatique solution to find arima

```
Series: uschange[, c("Income")]  
ARIMA(0,0,0) with non-zero mean
```

Coefficients:

```
      mean  
      0.7176  
s.e.  0.0679
```

sigma² estimated as 0.8674: log likelihood=-251.54

AIC=507.08 AICc=507.15 BIC=513.55

Series: uschange[, c("Unemployment")]
ARIMA(2,0,0)(1,0,2)[4] with zero mean

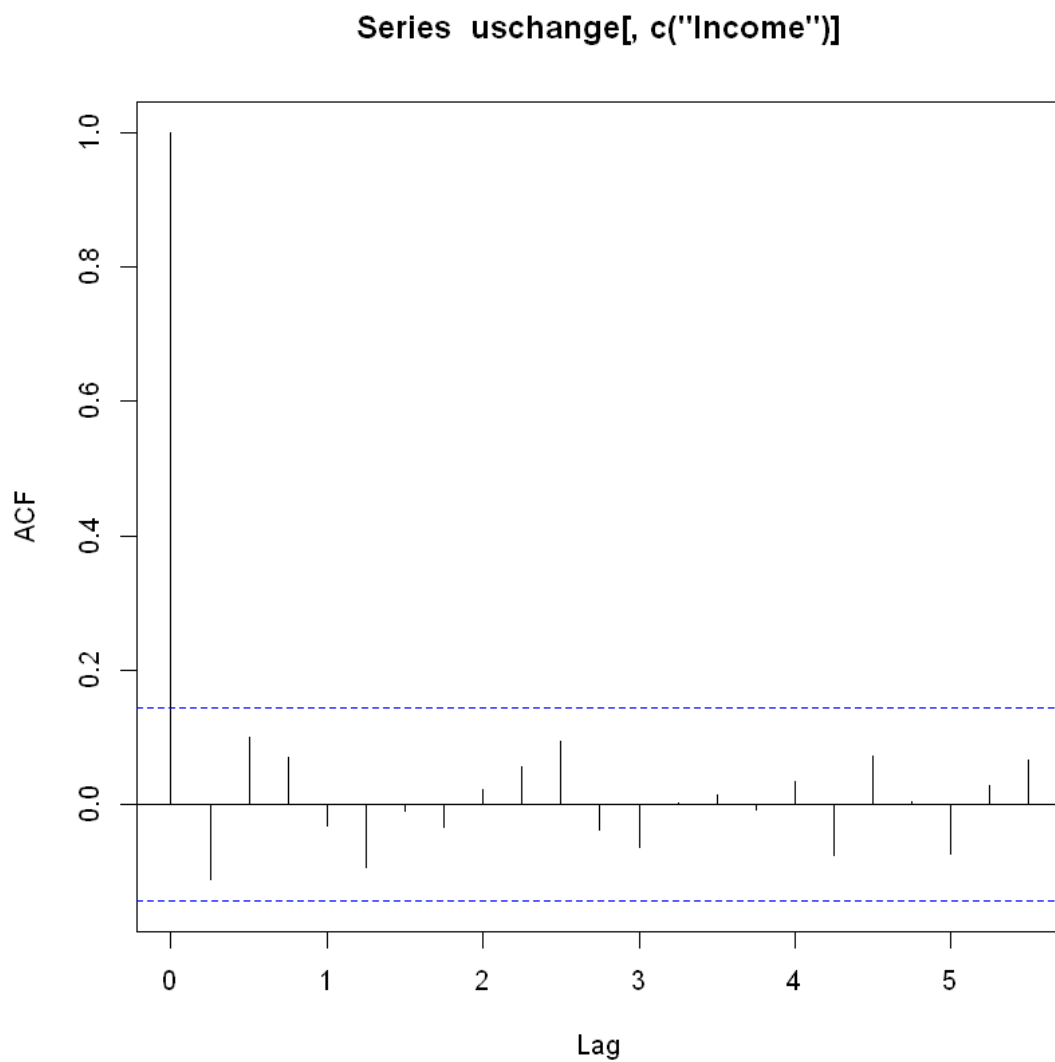
Coefficients:

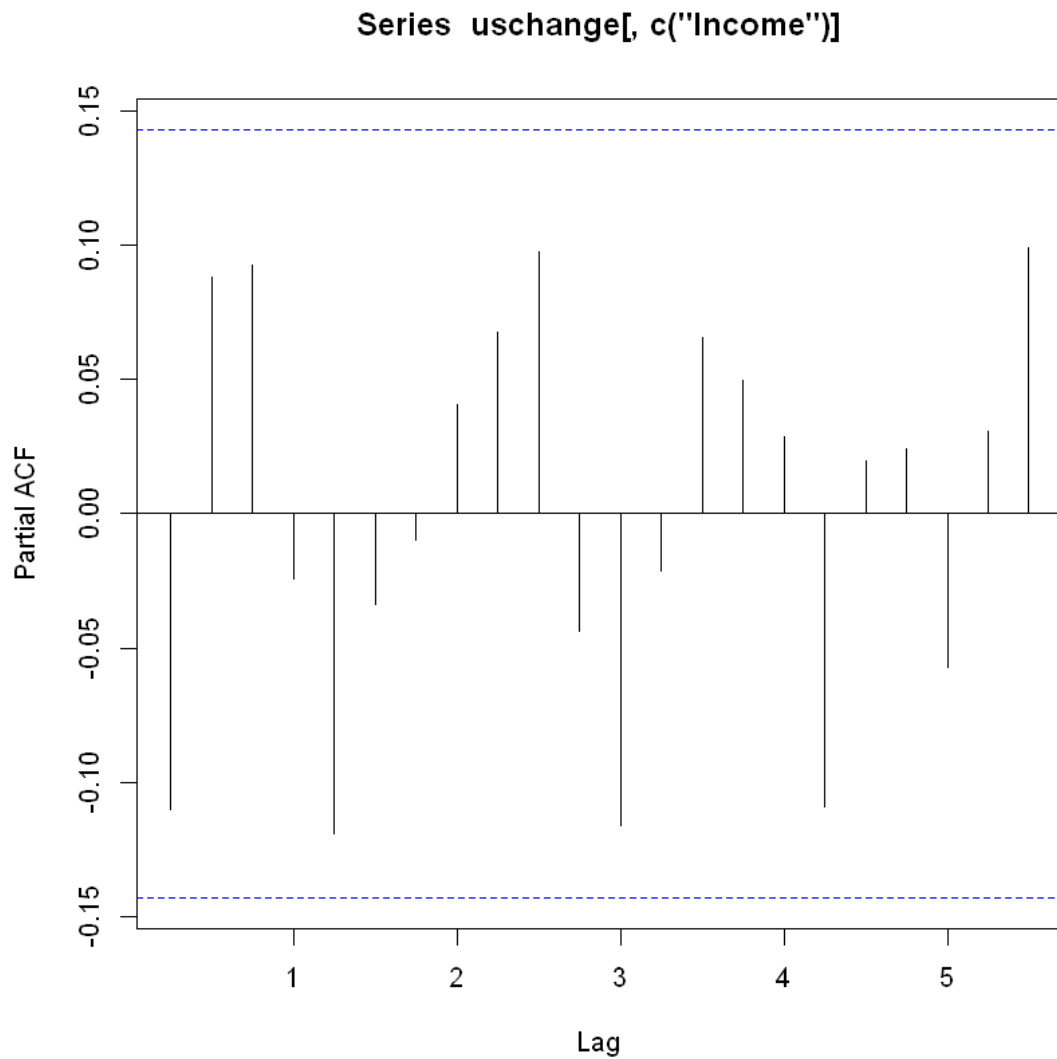
	ar1	ar2	sar1	sma1	sma2
	0.4412	0.2505	0.0152	-0.2894	-0.2290
s.e.	0.0751	0.0813	0.2844	0.2773	0.1219

sigma² estimated as 0.08953: log likelihood=-37.82
AIC=87.64 AICc=88.11 BIC=107.03

For this one ARIMA(2,0,0)

Trying with Manual methode



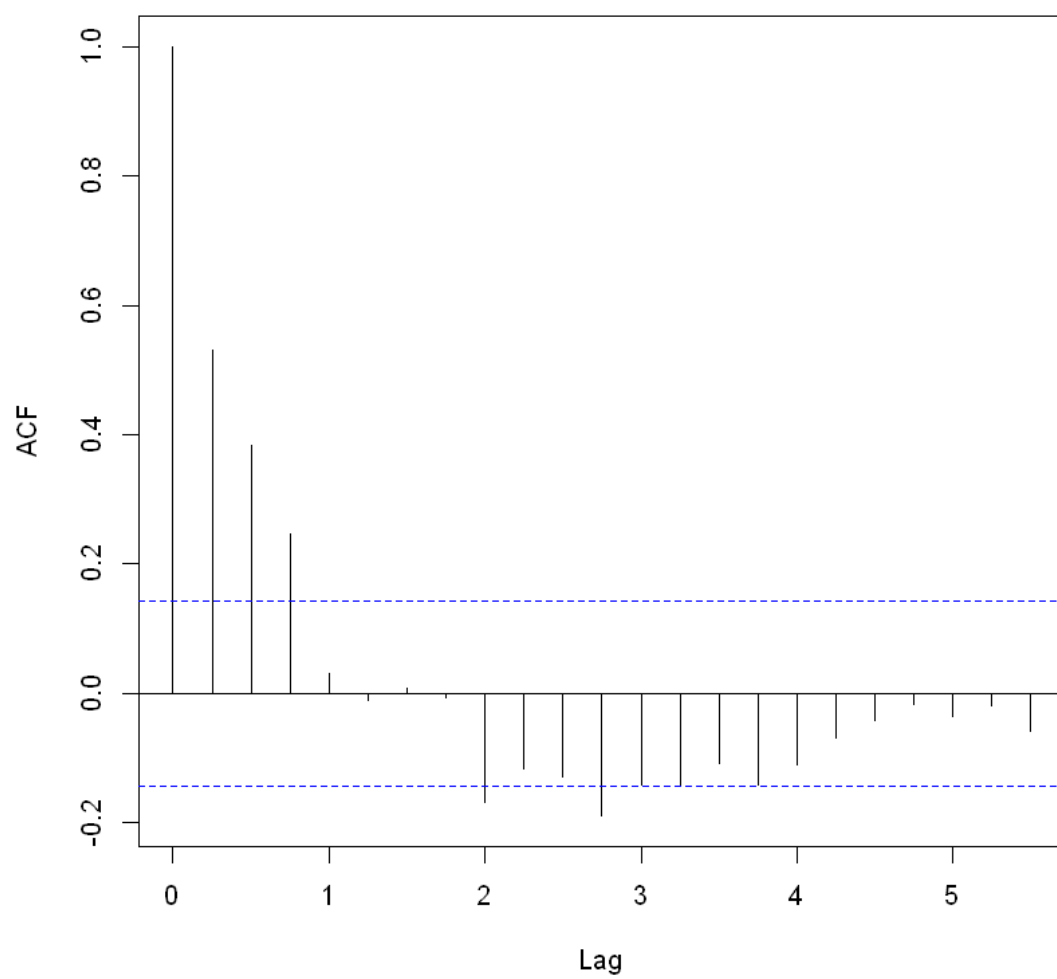


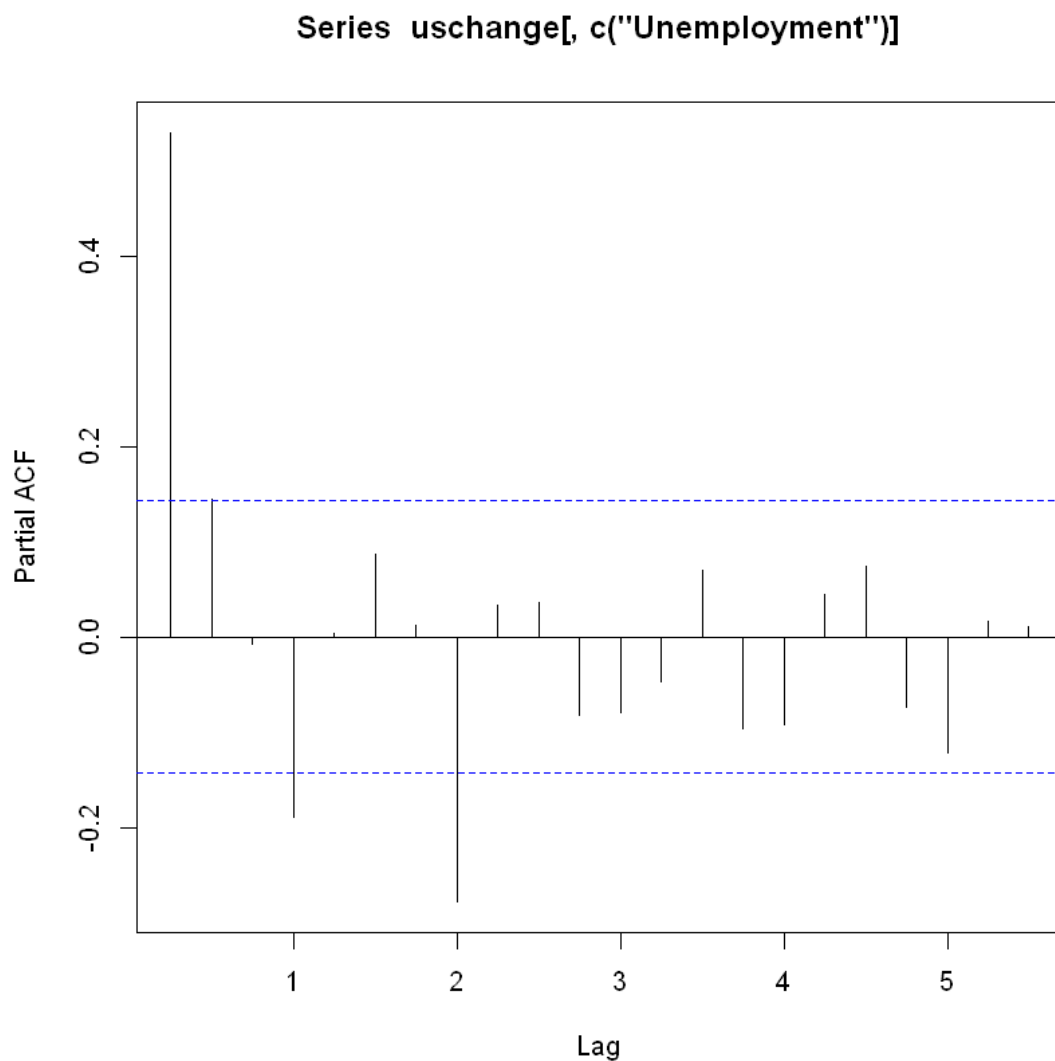
Box-Ljung test

```
data: uschange[, c("Income")]
X-squared = 9.8076, df = 10, p-value = 0.4575
```

Noise (not white because not centered) so no stochastic part for this one => best one = mean

Series uschange[, c("Unemployment")]





I would say MA3 ARMA(0,0,3) - automatic for ARMA(2,0,0), AIC = 87.64

Series: uschange[, "Unemployment"]
 ARIMA(0,0,3) with non-zero mean

Coefficients:

	ma1	ma2	ma3	mean
	0.4804	0.4845	0.3972	0.0108
s.e.	0.0694	0.0848	0.0702	0.0517

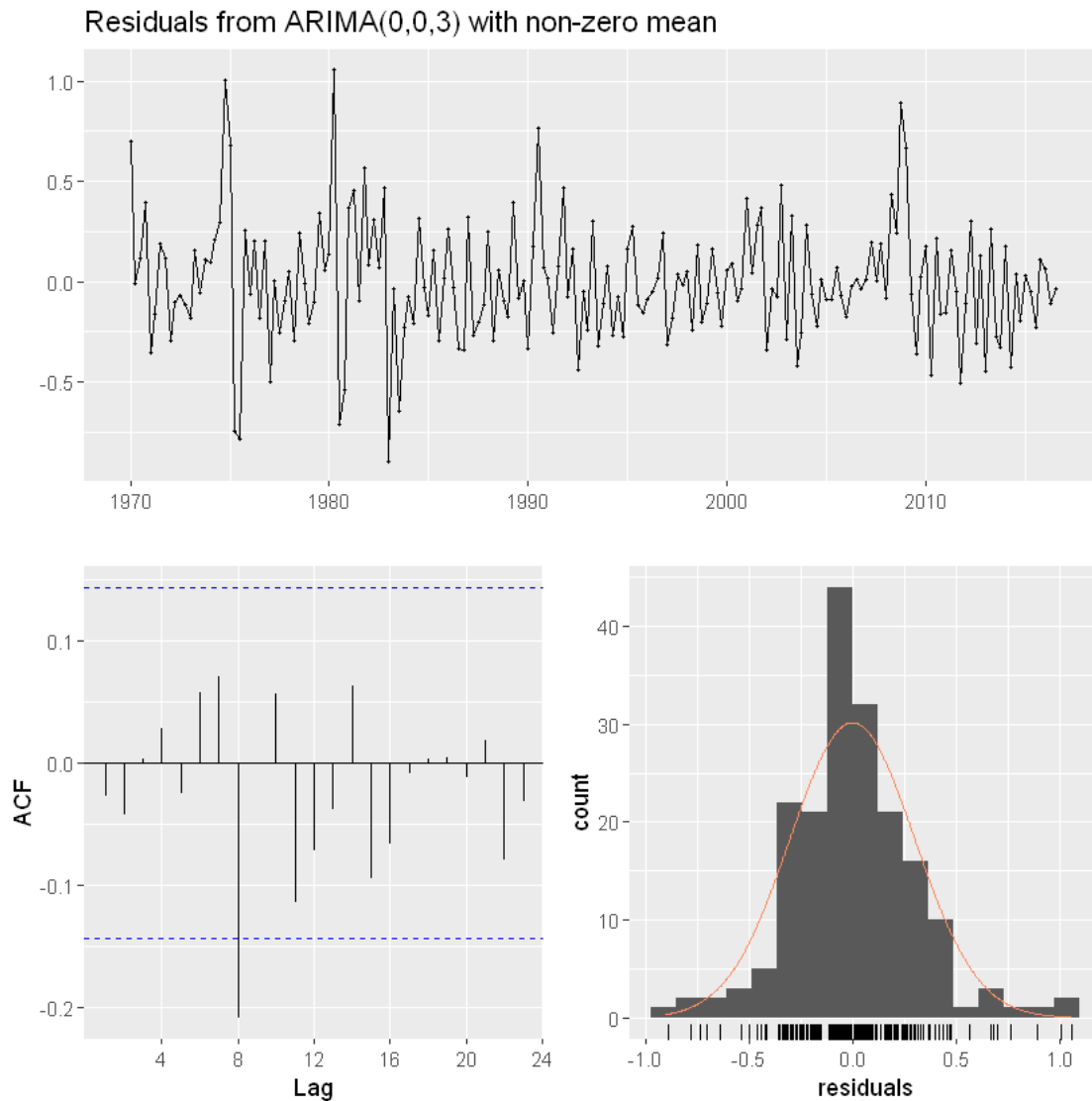
sigma² estimated as 0.09281: log likelihood=-41.47
 AIC=92.94 AICc=93.27 BIC=109.09

We check the residuals, to be sure that their are gaussian

Ljung-Box test

data: Residuals from ARIMA(0,0,3) with non-zero mean
Q* = 10.983, df = 4, p-value = 0.02675

Model df: 4. Total lags used: 8

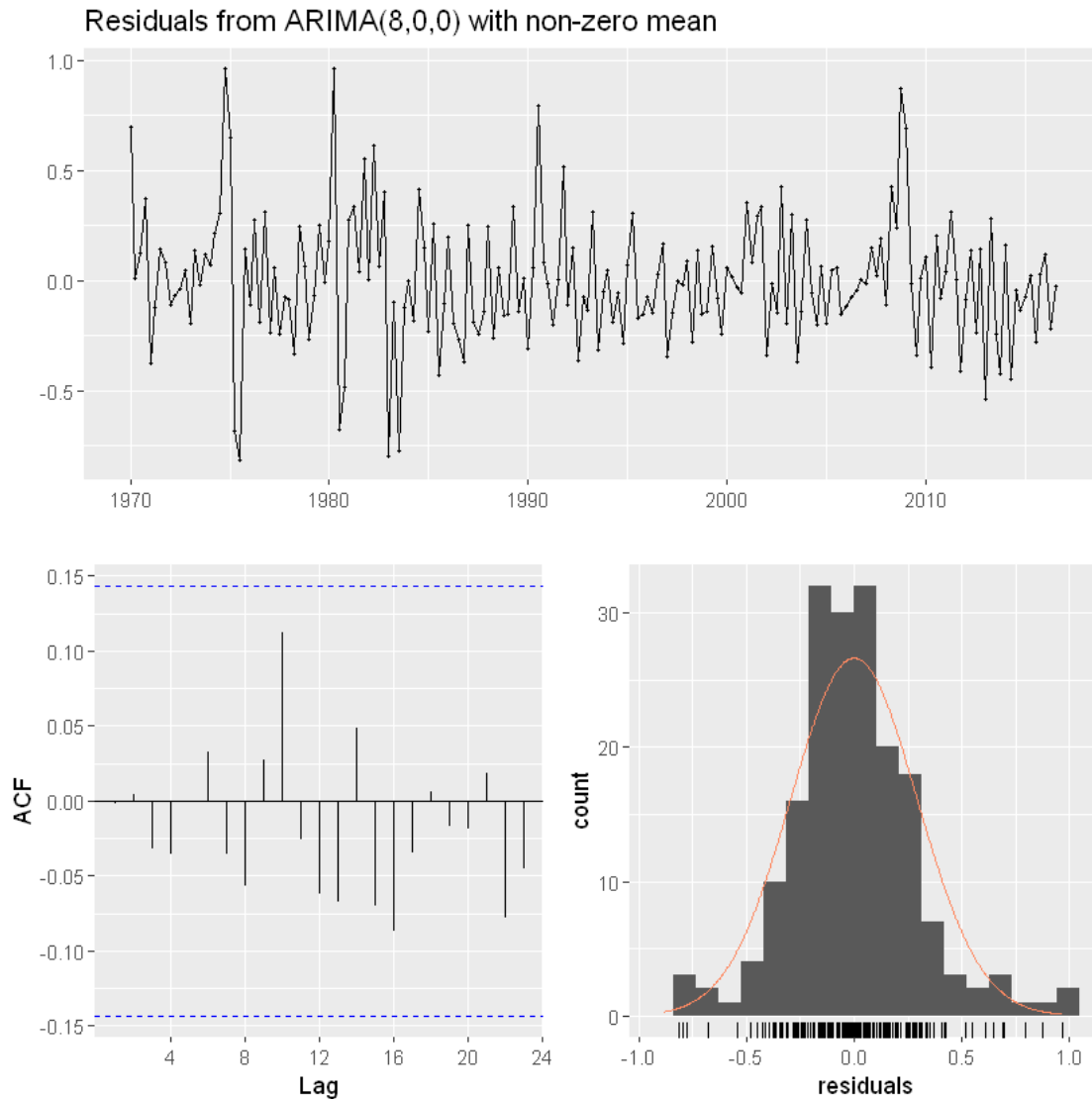


Not good, p-value < 0.05, so try another one

Ljung-Box test

data: Residuals from ARIMA(8,0,0) with non-zero mean
Q* = 5.1242, df = 3, p-value = 0.1629

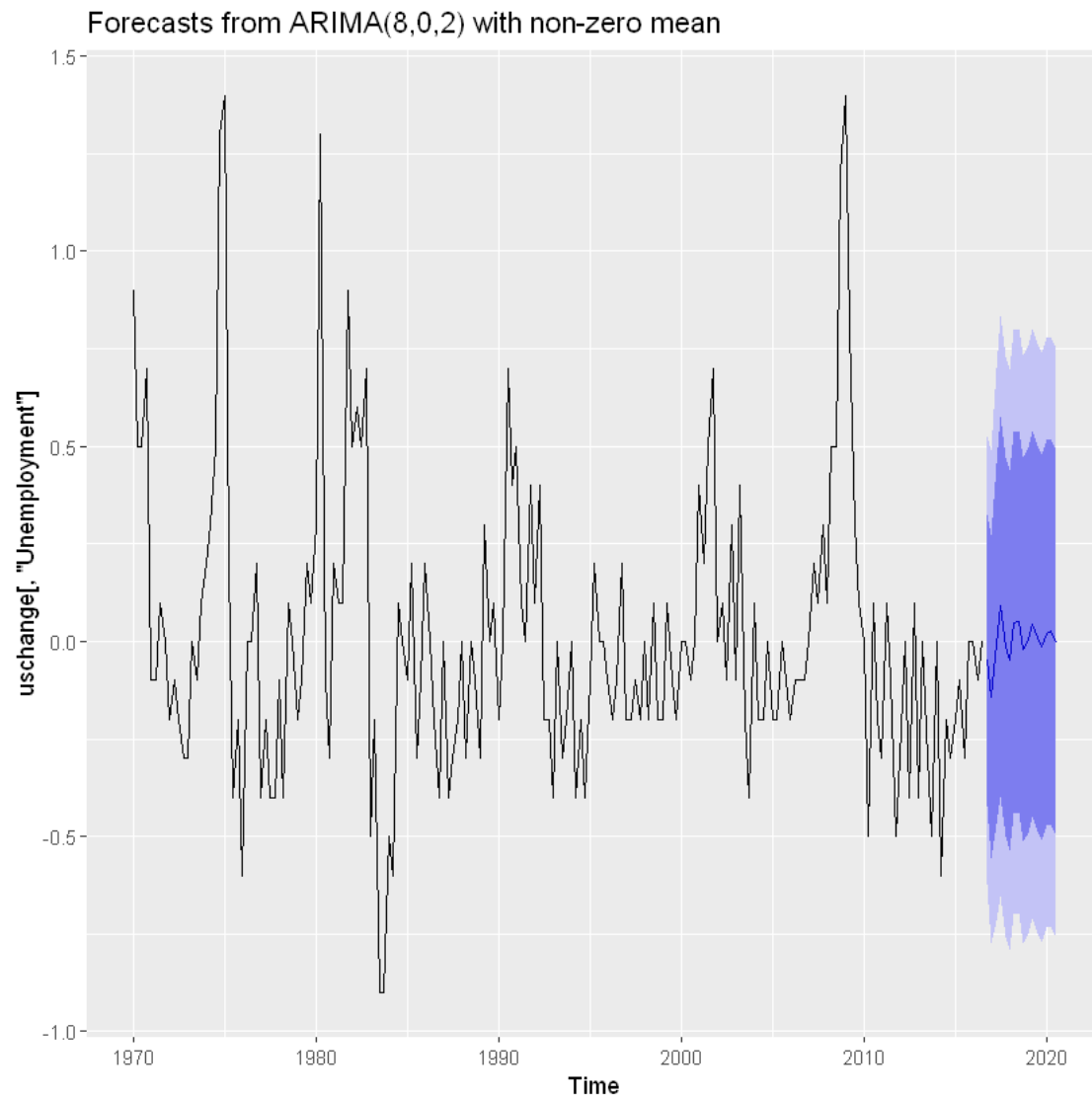
Model df: 9. Total lags used: 12



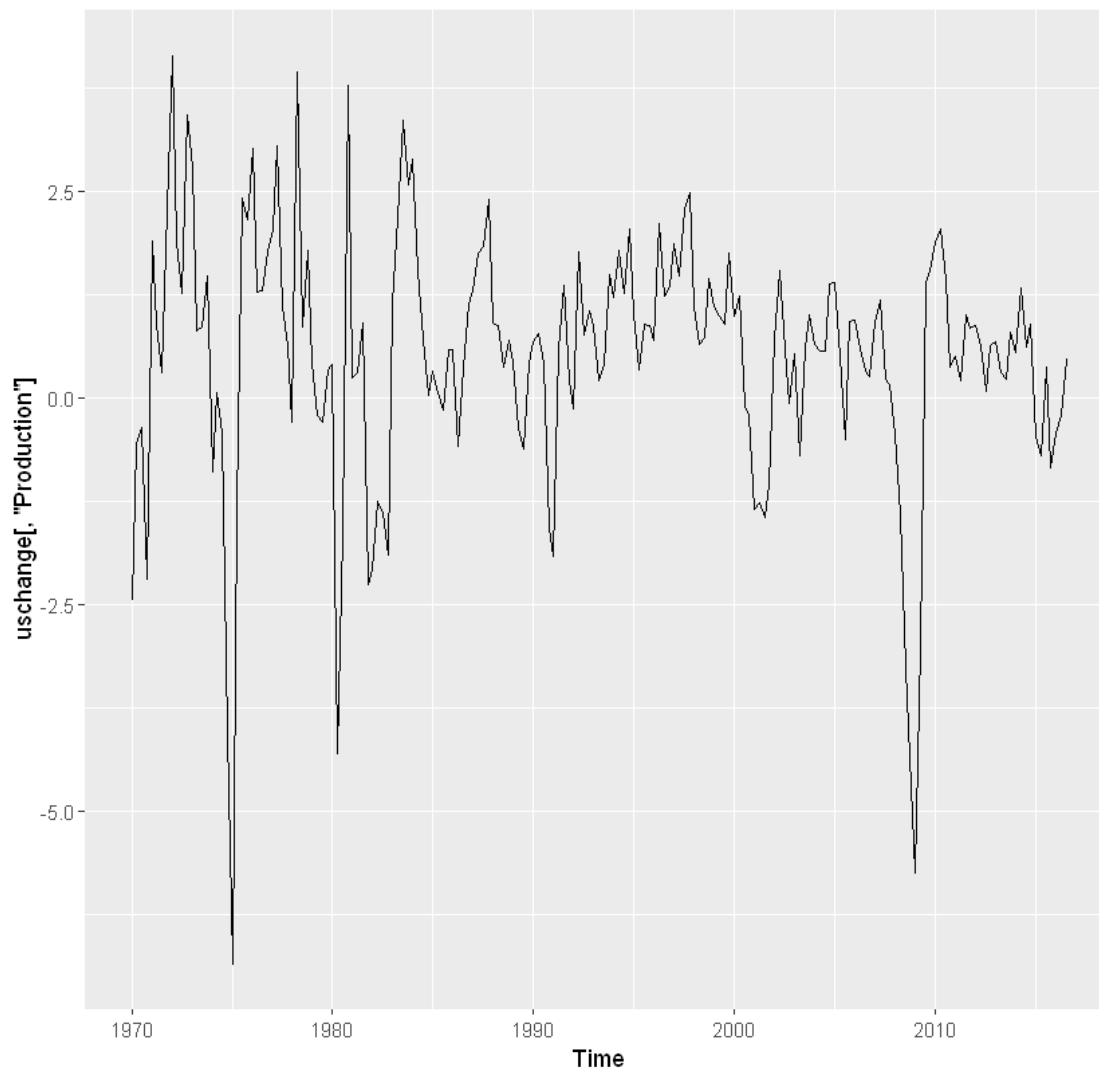
This time it is ok p-value > 0.05

We can find a better p-value with (8,0,2) - also ok for residuals - so let's forecast with ARMA(8,0,2)

Forecast with this hypothesis



1.4.8 Exercise 1 on Production data



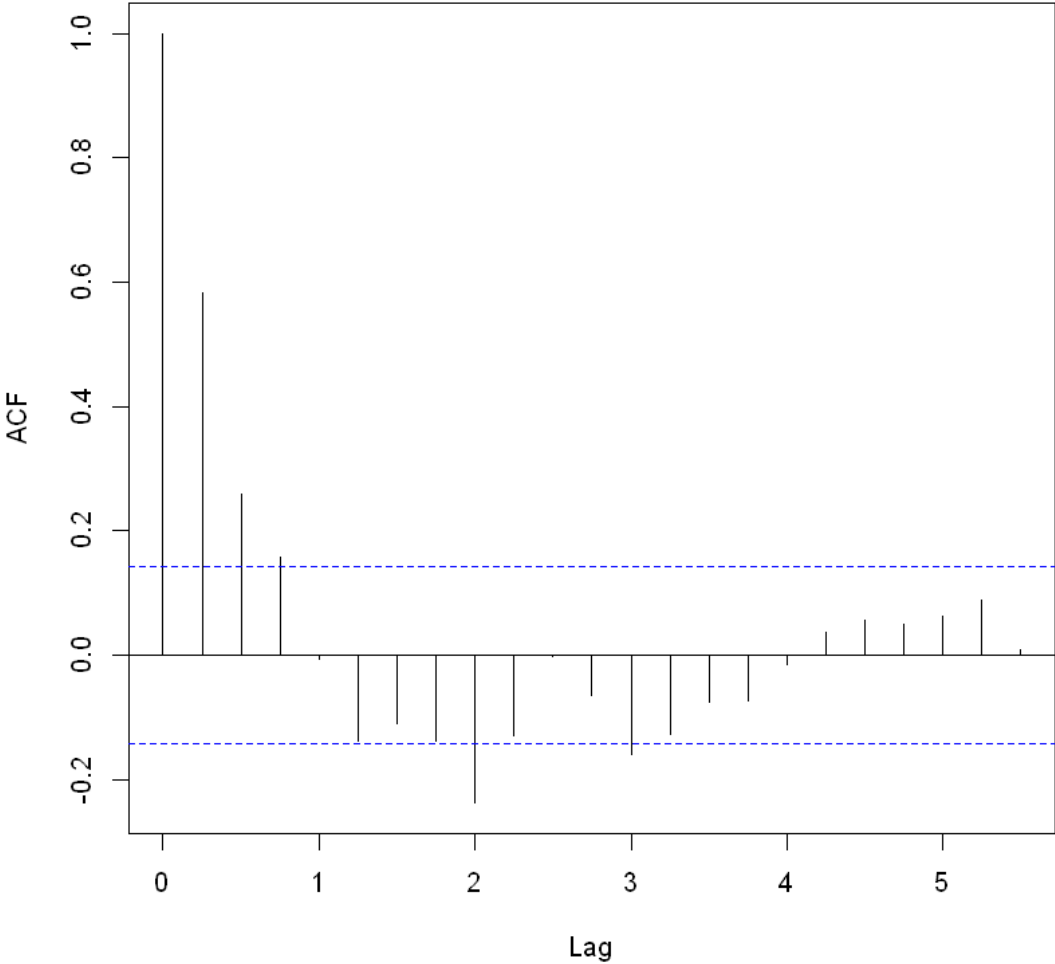
**No trend it seems, let's look if it is noise ?

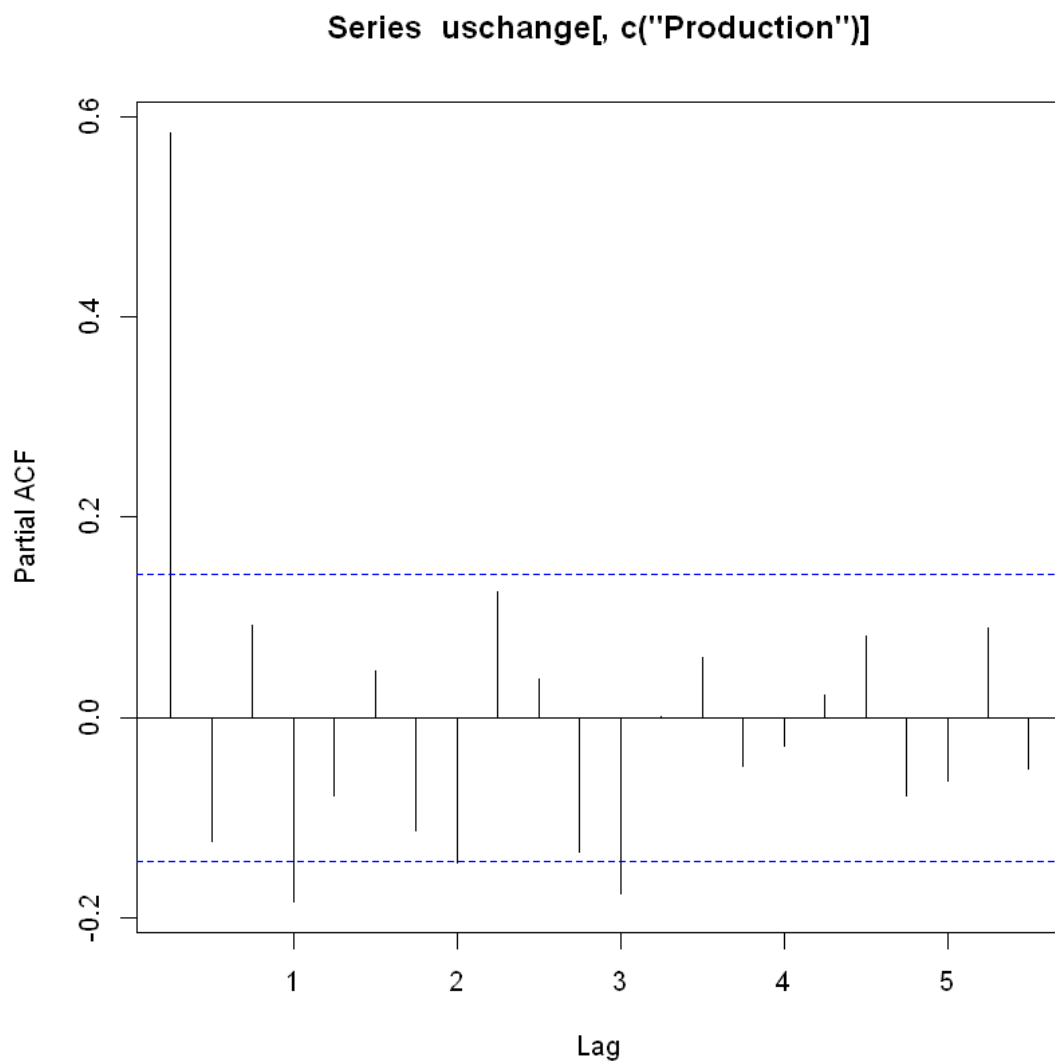
Box-Ljung test

```
data: uschange[, c("Production")]  
X-squared = 106.92, df = 10, p-value < 2.2e-16
```

White Noise is rejected

Series uschange[, c("Production")]





605.742751778692

Ljung-Box test

data: Residuals from ARIMA(0,0,12) with non-zero mean
Q* = 5.0375, df = 3, p-value = 0.1691

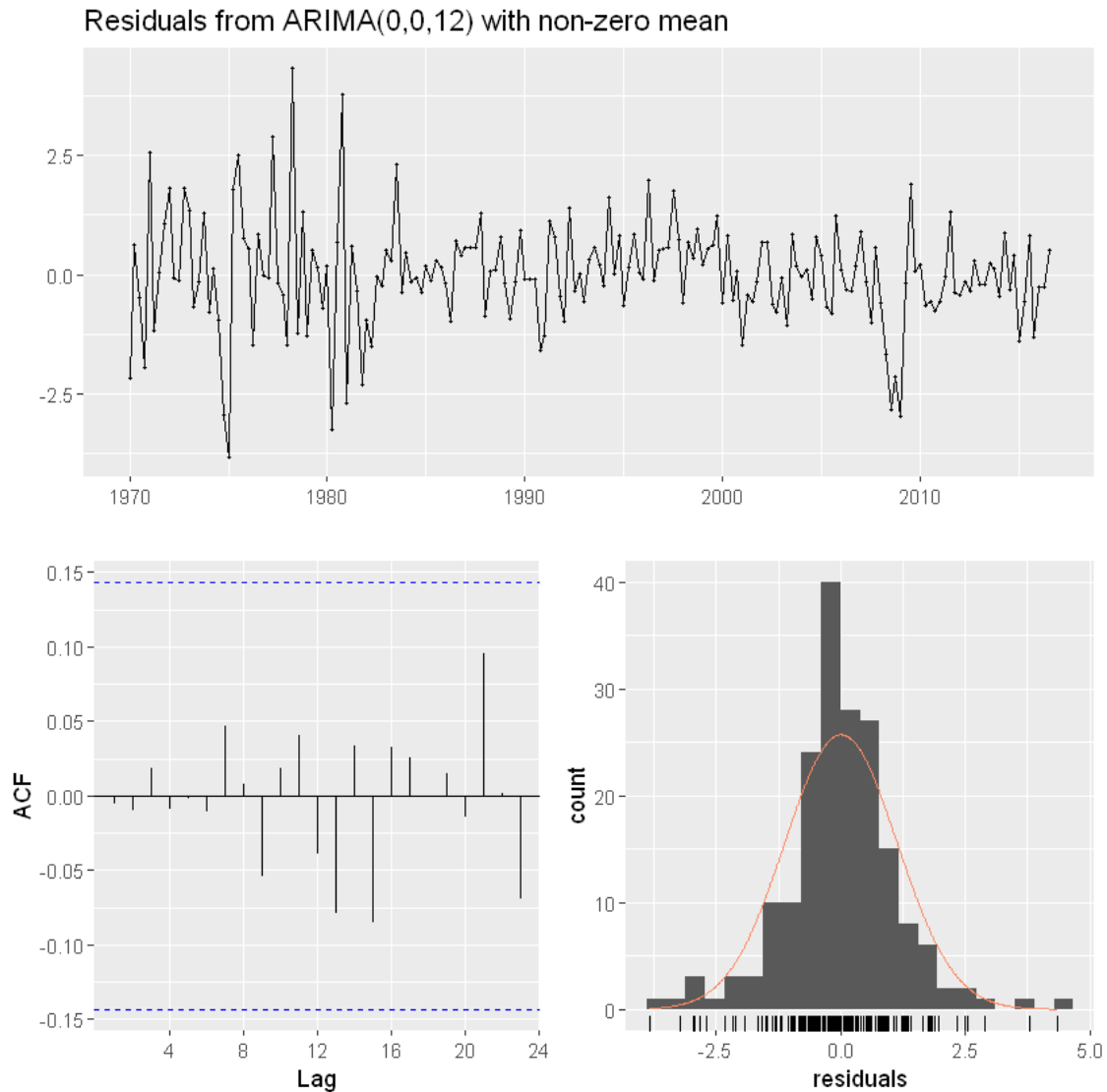
Model df: 13. Total lags used: 16

0.169075109360558

Ljung-Box test

data: Residuals from ARIMA(0,0,12) with non-zero mean
Q* = 5.0375, df = 3, p-value = 0.1691

Model df: 13. Total lags used: 16



Ljung-Box test

data: Residuals from ARIMA(0,0,0) with non-zero mean
Q* = 103.54, df = 7, p-value < 2.2e-16

Model df: 1. Total lags used: 8

```
[1] 0
[1] 0
```

Ljung-Box test

data: Residuals from ARIMA(0,0,1) with non-zero mean
Q* = 25.448, df = 6, p-value = 0.000282

Model df: 2. Total lags used: 8

```
[1] 0
[1] 1
```

Ljung-Box test

data: Residuals from ARIMA(0,0,2) with non-zero mean
Q* = 18.99, df = 5, p-value = 0.00193

Model df: 3. Total lags used: 8

```
[1] 0
[1] 2
```

Ljung-Box test

data: Residuals from ARIMA(0,0,3) with non-zero mean
Q* = 17.417, df = 4, p-value = 0.001604

Model df: 4. Total lags used: 8

```
[1] 0
[1] 3
```

Ljung-Box test

data: Residuals from ARIMA(0,0,4) with non-zero mean
Q* = 11.815, df = 3, p-value = 0.008044

Model df: 5. Total lags used: 8

```
[1] 0
[1] 4
```

Ljung-Box test

data: Residuals from ARIMA(0,0,5) with non-zero mean
Q* = 12.997, df = 3, p-value = 0.004643

Model df: 6. Total lags used: 9

[1] 0

[1] 5

Ljung-Box test

data: Residuals from ARIMA(0,0,6) with non-zero mean
Q* = 13.86, df = 3, p-value = 0.003101

Model df: 7. Total lags used: 10

[1] 0

[1] 6

Ljung-Box test

data: Residuals from ARIMA(0,0,7) with non-zero mean
Q* = 10.862, df = 3, p-value = 0.0125

Model df: 8. Total lags used: 11

[1] 0

[1] 7

Ljung-Box test

data: Residuals from ARIMA(0,0,8) with non-zero mean
Q* = 11.755, df = 3, p-value = 0.008271

Model df: 9. Total lags used: 12

[1] 0

[1] 8

Ljung-Box test

data: Residuals from ARIMA(0,0,9) with non-zero mean
Q* = 5.5555, df = 3, p-value = 0.1354

Model df: 10. Total lags used: 13

[1] 0

[1] 9

Ljung-Box test

data: Residuals from ARIMA(0,0,10) with non-zero mean
Q* = 5.5956, df = 3, p-value = 0.133

Model df: 11. Total lags used: 14

[1] 0
[1] 10

Ljung-Box test

data: Residuals from ARIMA(0,0,11) with non-zero mean
Q* = 5.9439, df = 3, p-value = 0.1144

Model df: 12. Total lags used: 15

[1] 0
[1] 11

Ljung-Box test

data: Residuals from ARIMA(0,0,12) with non-zero mean
Q* = 5.0375, df = 3, p-value = 0.1691

Model df: 13. Total lags used: 16

[1] 0
[1] 12

Ljung-Box test

data: Residuals from ARIMA(1,0,0) with non-zero mean
Q* = 24.571, df = 6, p-value = 0.0004098

Model df: 2. Total lags used: 8

[1] 1
[1] 0

Ljung-Box test

data: Residuals from ARIMA(1,0,1) with non-zero mean
Q* = 18.557, df = 5, p-value = 0.002323

Model df: 3. Total lags used: 8

[1] 1
[1] 1

Ljung-Box test

data: Residuals from ARIMA(1,0,2) with non-zero mean
Q* = 16.291, df = 4, p-value = 0.002652

Model df: 4. Total lags used: 8

[1] 1
[1] 2

Ljung-Box test

data: Residuals from ARIMA(1,0,3) with non-zero mean
Q* = 14.574, df = 3, p-value = 0.002219

Model df: 5. Total lags used: 8

[1] 1
[1] 3

Ljung-Box test

data: Residuals from ARIMA(1,0,4) with non-zero mean
Q* = 12.29, df = 3, p-value = 0.006454

Model df: 6. Total lags used: 9

[1] 1
[1] 4

Ljung-Box test

data: Residuals from ARIMA(1,0,5) with non-zero mean
Q* = 13.788, df = 3, p-value = 0.003209

Model df: 7. Total lags used: 10

[1] 1
[1] 5

Ljung-Box test

data: Residuals from ARIMA(1,0,6) with non-zero mean
Q* = 13.267, df = 3, p-value = 0.004094

Model df: 8. Total lags used: 11

[1] 1

[1] 6

Ljung-Box test

data: Residuals from ARIMA(1,0,7) with non-zero mean
Q* = 15.13, df = 3, p-value = 0.001709

Model df: 9. Total lags used: 12

[1] 1

[1] 7

Ljung-Box test

data: Residuals from ARIMA(1,0,8) with non-zero mean
Q* = 9.1263, df = 3, p-value = 0.02766

Model df: 10. Total lags used: 13

[1] 1

[1] 8

Ljung-Box test

data: Residuals from ARIMA(1,0,9) with non-zero mean
Q* = 5.675, df = 3, p-value = 0.1285

Model df: 11. Total lags used: 14

[1] 1

[1] 9

Ljung-Box test

data: Residuals from ARIMA(1,0,10) with non-zero mean
Q* = 6.5504, df = 3, p-value = 0.0877

Model df: 12. Total lags used: 15

[1] 1

[1] 10

Ljung-Box test

data: Residuals from ARIMA(1,0,11) with non-zero mean
Q* = 5.7195, df = 3, p-value = 0.1261

Model df: 13. Total lags used: 16

```
[1] 1
[1] 11
```

Ljung-Box test

data: Residuals from ARIMA(1,0,12) with non-zero mean
Q* = 4.8142, df = 3, p-value = 0.1859

Model df: 14. Total lags used: 17

```
[1] 1
[1] 12
```

Ljung-Box test

data: Residuals from ARIMA(2,0,0) with non-zero mean
Q* = 23.407, df = 5, p-value = 0.0002821

Model df: 3. Total lags used: 8

```
[1] 2
[1] 0
```

Ljung-Box test

data: Residuals from ARIMA(2,0,1) with non-zero mean
Q* = 16.498, df = 4, p-value = 0.002418

Model df: 4. Total lags used: 8

```
[1] 2
[1] 1
```

Ljung-Box test

data: Residuals from ARIMA(2,0,2) with non-zero mean
Q* = 15.747, df = 3, p-value = 0.001278

Model df: 5. Total lags used: 8

```
[1] 2
[1] 2
```

Ljung-Box test

data: Residuals from ARIMA(2,0,3) with non-zero mean
Q* = 12.505, df = 3, p-value = 0.005838

Model df: 6. Total lags used: 9

[1] 2

[1] 3

Ljung-Box test

data: Residuals from ARIMA(2,0,4) with non-zero mean
Q* = 7.5703, df = 3, p-value = 0.05578

Model df: 7. Total lags used: 10

[1] 2

[1] 4

Ljung-Box test

data: Residuals from ARIMA(2,0,5) with non-zero mean
Q* = 12.503, df = 3, p-value = 0.005844

Model df: 8. Total lags used: 11

[1] 2

[1] 5

Ljung-Box test

data: Residuals from ARIMA(2,0,6) with non-zero mean
Q* = 6.6684, df = 3, p-value = 0.08325

Model df: 9. Total lags used: 12

[1] 2

[1] 6

Ljung-Box test

data: Residuals from ARIMA(2,0,7) with non-zero mean
Q* = 8.0126, df = 3, p-value = 0.04575

Model df: 10. Total lags used: 13

[1] 2

[1] 7

Ljung-Box test

data: Residuals from ARIMA(2,0,8) with non-zero mean
Q* = 6.5042, df = 3, p-value = 0.0895

Model df: 11. Total lags used: 14

[1] 2

[1] 8

Ljung-Box test

data: Residuals from ARIMA(2,0,9) with non-zero mean
Q* = 5.5235, df = 3, p-value = 0.1372

Model df: 12. Total lags used: 15

[1] 2

[1] 9

Ljung-Box test

data: Residuals from ARIMA(2,0,10) with non-zero mean
Q* = 2.9738, df = 3, p-value = 0.3957

Model df: 13. Total lags used: 16

[1] 2

[1] 10

Ljung-Box test

data: Residuals from ARIMA(2,0,11) with non-zero mean
Q* = 3.7467, df = 3, p-value = 0.2902

Model df: 14. Total lags used: 17

[1] 2

[1] 11

Ljung-Box test

data: Residuals from ARIMA(2,0,12) with non-zero mean
Q* = 3.6874, df = 3, p-value = 0.2973

Model df: 15. Total lags used: 18

[1] 2

[1] 12

Ljung-Box test

data: Residuals from ARIMA(3,0,0) with non-zero mean
Q* = 17.218, df = 4, p-value = 0.001754

Model df: 4. Total lags used: 8

[1] 3
[1] 0

Ljung-Box test

data: Residuals from ARIMA(3,0,1) with non-zero mean
Q* = 14.365, df = 3, p-value = 0.002448

Model df: 5. Total lags used: 8

[1] 3
[1] 1

Ljung-Box test

data: Residuals from ARIMA(3,0,2) with non-zero mean
Q* = 8.7774, df = 3, p-value = 0.0324

Model df: 6. Total lags used: 9

[1] 3
[1] 2

Ljung-Box test

data: Residuals from ARIMA(3,0,3) with non-zero mean
Q* = 7.5543, df = 3, p-value = 0.05618

Model df: 7. Total lags used: 10

[1] 3
[1] 3

Ljung-Box test

data: Residuals from ARIMA(3,0,4) with non-zero mean
Q* = 6.0714, df = 3, p-value = 0.1082

Model df: 8. Total lags used: 11

[1] 3

[1] 4

Ljung-Box test

data: Residuals from ARIMA(3,0,5) with non-zero mean
Q* = 6.3775, df = 3, p-value = 0.09462

Model df: 9. Total lags used: 12

[1] 3

[1] 5

Ljung-Box test

data: Residuals from ARIMA(3,0,6) with non-zero mean
Q* = 8.2637, df = 3, p-value = 0.04087

Model df: 10. Total lags used: 13

[1] 3

[1] 6

Ljung-Box test

data: Residuals from ARIMA(3,0,7) with non-zero mean
Q* = 9.0118, df = 3, p-value = 0.02913

Model df: 11. Total lags used: 14

[1] 3

[1] 7

Ljung-Box test

data: Residuals from ARIMA(3,0,8) with non-zero mean
Q* = 2.74, df = 3, p-value = 0.4335

Model df: 12. Total lags used: 15

[1] 3

[1] 8

Ljung-Box test

data: Residuals from ARIMA(3,0,9) with non-zero mean
Q* = 3.2301, df = 3, p-value = 0.3575

Model df: 13. Total lags used: 16

```
[1] 3
[1] 9
```

Ljung-Box test

data: Residuals from ARIMA(3,0,10) with non-zero mean
Q* = 4.6774, df = 3, p-value = 0.197

Model df: 14. Total lags used: 17

```
[1] 3
[1] 10
```

Ljung-Box test

data: Residuals from ARIMA(3,0,11) with non-zero mean
Q* = 3.6784, df = 3, p-value = 0.2984

Model df: 15. Total lags used: 18

```
[1] 3
[1] 11
```

Ljung-Box test

data: Residuals from ARIMA(3,0,12) with non-zero mean
Q* = 4.48, df = 3, p-value = 0.2141

Model df: 16. Total lags used: 19

```
[1] 3
[1] 12
```

Ljung-Box test

data: Residuals from ARIMA(4,0,0) with non-zero mean
Q* = 11.11, df = 3, p-value = 0.01115

Model df: 5. Total lags used: 8

```
[1] 4
[1] 0
```

Ljung-Box test

data: Residuals from ARIMA(4,0,1) with non-zero mean
Q* = 7.7627, df = 3, p-value = 0.05118

Model df: 6. Total lags used: 9

[1] 4

[1] 1

Ljung-Box test

data: Residuals from ARIMA(4,0,2) with non-zero mean
Q* = 7.0981, df = 3, p-value = 0.06883

Model df: 7. Total lags used: 10

[1] 4

[1] 2

Ljung-Box test

data: Residuals from ARIMA(4,0,3) with non-zero mean
Q* = 6.6679, df = 3, p-value = 0.08327

Model df: 8. Total lags used: 11

[1] 4

[1] 3

Ljung-Box test

data: Residuals from ARIMA(4,0,4) with non-zero mean
Q* = 6.4779, df = 3, p-value = 0.09054

Model df: 9. Total lags used: 12

[1] 4

[1] 4

Ljung-Box test

data: Residuals from ARIMA(4,0,5) with non-zero mean
Q* = 7.9667, df = 3, p-value = 0.04671

Model df: 10. Total lags used: 13

[1] 4

[1] 5

Ljung-Box test

data: Residuals from ARIMA(4,0,6) with non-zero mean
Q* = 8.339, df = 3, p-value = 0.0395

Model df: 11. Total lags used: 14

[1] 4

[1] 6

Ljung-Box test

data: Residuals from ARIMA(4,0,7) with non-zero mean
Q* = 8.4713, df = 3, p-value = 0.03721

Model df: 12. Total lags used: 15

[1] 4

[1] 7

Ljung-Box test

data: Residuals from ARIMA(4,0,8) with non-zero mean
Q* = 3.3582, df = 3, p-value = 0.3396

Model df: 13. Total lags used: 16

[1] 4

[1] 8

Ljung-Box test

data: Residuals from ARIMA(4,0,9) with non-zero mean
Q* = 8.4571, df = 3, p-value = 0.03745

Model df: 14. Total lags used: 17

[1] 4

[1] 9

Ljung-Box test

data: Residuals from ARIMA(4,0,10) with non-zero mean
Q* = 4.1483, df = 3, p-value = 0.2459

Model df: 15. Total lags used: 18

[1] 4

[1] 10

Ljung-Box test

data: Residuals from ARIMA(4,0,11) with non-zero mean
Q* = 4.232, df = 3, p-value = 0.2375

Model df: 16. Total lags used: 19

[1] 4
[1] 11

Ljung-Box test

data: Residuals from ARIMA(4,0,12) with non-zero mean
Q* = 3.3768, df = 3, p-value = 0.3371

Model df: 17. Total lags used: 20

[1] 4
[1] 12

Ljung-Box test

data: Residuals from ARIMA(5,0,0) with non-zero mean
Q* = 10.024, df = 3, p-value = 0.01836

Model df: 6. Total lags used: 9

[1] 5
[1] 0

Ljung-Box test

data: Residuals from ARIMA(5,0,1) with non-zero mean
Q* = 12.417, df = 3, p-value = 0.006082

Model df: 7. Total lags used: 10

[1] 5
[1] 1

Ljung-Box test

data: Residuals from ARIMA(5,0,2) with non-zero mean
Q* = 6.5925, df = 3, p-value = 0.08609

Model df: 8. Total lags used: 11

[1] 5

[1] 2

Ljung-Box test

data: Residuals from ARIMA(5,0,3) with non-zero mean
Q* = 7.6837, df = 3, p-value = 0.05302

Model df: 9. Total lags used: 12

[1] 5

[1] 3

Ljung-Box test

data: Residuals from ARIMA(5,0,4) with non-zero mean
Q* = 7.1064, df = 3, p-value = 0.06858

Model df: 10. Total lags used: 13

[1] 5

[1] 4

Ljung-Box test

data: Residuals from ARIMA(5,0,5) with non-zero mean
Q* = 6.5751, df = 3, p-value = 0.08675

Model df: 11. Total lags used: 14

[1] 5

[1] 5

Ljung-Box test

data: Residuals from ARIMA(5,0,6) with non-zero mean
Q* = 6.0752, df = 3, p-value = 0.108

Model df: 12. Total lags used: 15

[1] 5

[1] 6

Ljung-Box test

data: Residuals from ARIMA(5,0,7) with non-zero mean
Q* = 7.3797, df = 3, p-value = 0.06073

Model df: 13. Total lags used: 16

```
[1] 5
[1] 7
```

Ljung-Box test

data: Residuals from ARIMA(5,0,8) with non-zero mean
Q* = 8.1658, df = 3, p-value = 0.04271

Model df: 14. Total lags used: 17

```
[1] 5
[1] 8
```

Ljung-Box test

data: Residuals from ARIMA(5,0,9) with non-zero mean
Q* = 4.0948, df = 3, p-value = 0.2514

Model df: 15. Total lags used: 18

```
[1] 5
[1] 9
```

Ljung-Box test

data: Residuals from ARIMA(5,0,10) with non-zero mean
Q* = 6.4703, df = 3, p-value = 0.09084

Model df: 16. Total lags used: 19

```
[1] 5
[1] 10
```

Ljung-Box test

data: Residuals from ARIMA(5,0,11) with non-zero mean
Q* = 4.5884, df = 3, p-value = 0.2045

Model df: 17. Total lags used: 20

```
[1] 5
[1] 11
```

Ljung-Box test

data: Residuals from ARIMA(5,0,12) with non-zero mean
Q* = 4.5601, df = 3, p-value = 0.207

Model df: 18. Total lags used: 21

[1] 5
[1] 12

Ljung-Box test

data: Residuals from ARIMA(6,0,0) with non-zero mean
Q* = 13.265, df = 3, p-value = 0.004097

Model df: 7. Total lags used: 10

[1] 6
[1] 0

Ljung-Box test

data: Residuals from ARIMA(6,0,1) with non-zero mean
Q* = 13.073, df = 3, p-value = 0.004482

Model df: 8. Total lags used: 11

[1] 6
[1] 1

Ljung-Box test

data: Residuals from ARIMA(6,0,2) with non-zero mean
Q* = 7.0648, df = 3, p-value = 0.06986

Model df: 9. Total lags used: 12

[1] 6
[1] 2

Ljung-Box test

data: Residuals from ARIMA(6,0,3) with non-zero mean
Q* = 6.4413, df = 3, p-value = 0.09201

Model df: 10. Total lags used: 13

[1] 6
[1] 3

Ljung-Box test

data: Residuals from ARIMA(6,0,4) with non-zero mean
Q* = 8.1457, df = 3, p-value = 0.0431

Model df: 11. Total lags used: 14

[1] 6
[1] 4

Ljung-Box test

data: Residuals from ARIMA(6,0,5) with non-zero mean
Q* = 6.7033, df = 3, p-value = 0.08198

Model df: 12. Total lags used: 15

[1] 6
[1] 5

Ljung-Box test

data: Residuals from ARIMA(6,0,6) with non-zero mean
Q* = 7.2419, df = 3, p-value = 0.06457

Model df: 13. Total lags used: 16

[1] 6
[1] 6

Ljung-Box test

data: Residuals from ARIMA(6,0,7) with non-zero mean
Q* = 8.2126, df = 3, p-value = 0.04182

Model df: 14. Total lags used: 17

[1] 6
[1] 7

Ljung-Box test

data: Residuals from ARIMA(6,0,8) with non-zero mean
Q* = 2.7387, df = 3, p-value = 0.4337

Model df: 15. Total lags used: 18

[1] 6
[1] 8

Ljung-Box test

data: Residuals from ARIMA(6,0,9) with non-zero mean
Q* = 2.5831, df = 3, p-value = 0.4605

Model df: 16. Total lags used: 19

[1] 6
[1] 9

Ljung-Box test

data: Residuals from ARIMA(6,0,10) with non-zero mean
Q* = 7.1386, df = 3, p-value = 0.06761

Model df: 17. Total lags used: 20

[1] 6
[1] 10

Ljung-Box test

data: Residuals from ARIMA(6,0,11) with non-zero mean
Q* = 4.7901, df = 3, p-value = 0.1878

Model df: 18. Total lags used: 21

[1] 6
[1] 11

Ljung-Box test

data: Residuals from ARIMA(6,0,12) with non-zero mean
Q* = 5.1015, df = 3, p-value = 0.1645

Model df: 19. Total lags used: 22

[1] 6
[1] 12

Ljung-Box test

data: Residuals from ARIMA(7,0,0) with non-zero mean
Q* = 10.241, df = 3, p-value = 0.01662

Model df: 8. Total lags used: 11

[1] 7

[1] 0

Ljung-Box test

data: Residuals from ARIMA(7,0,1) with non-zero mean
Q* = 14.198, df = 3, p-value = 0.002648

Model df: 9. Total lags used: 12

[1] 7

[1] 1

Ljung-Box test

data: Residuals from ARIMA(7,0,2) with non-zero mean
Q* = 4.5152, df = 3, p-value = 0.2109

Model df: 10. Total lags used: 13

[1] 7

[1] 2

Ljung-Box test

data: Residuals from ARIMA(7,0,3) with non-zero mean
Q* = 5.9854, df = 3, p-value = 0.1123

Model df: 11. Total lags used: 14

[1] 7

[1] 3

Ljung-Box test

data: Residuals from ARIMA(7,0,4) with non-zero mean
Q* = 5.9734, df = 3, p-value = 0.1129

Model df: 12. Total lags used: 15

[1] 7

[1] 4

Ljung-Box test

data: Residuals from ARIMA(7,0,5) with non-zero mean
Q* = 6.7393, df = 3, p-value = 0.08069

Model df: 13. Total lags used: 16

[1] 7
[1] 5

Ljung-Box test

data: Residuals from ARIMA(7,0,6) with non-zero mean
Q* = 6.8328, df = 3, p-value = 0.07742

Model df: 14. Total lags used: 17

[1] 7
[1] 6

Ljung-Box test

data: Residuals from ARIMA(7,0,7) with non-zero mean
Q* = 3.7399, df = 3, p-value = 0.291

Model df: 15. Total lags used: 18

[1] 7
[1] 7

Ljung-Box test

data: Residuals from ARIMA(7,0,8) with non-zero mean
Q* = 6.8122, df = 3, p-value = 0.07813

Model df: 16. Total lags used: 19

[1] 7
[1] 8

Ljung-Box test

data: Residuals from ARIMA(7,0,9) with non-zero mean
Q* = 7.5954, df = 3, p-value = 0.05516

Model df: 17. Total lags used: 20

[1] 7
[1] 9

Ljung-Box test

data: Residuals from ARIMA(7,0,10) with non-zero mean
Q* = 5.5067, df = 3, p-value = 0.1382

Model df: 18. Total lags used: 21

[1] 7
[1] 10

Ljung-Box test

data: Residuals from ARIMA(7,0,11) with non-zero mean
Q* = 6.9227, df = 3, p-value = 0.0744

Model df: 19. Total lags used: 22

[1] 7
[1] 11

Ljung-Box test

data: Residuals from ARIMA(7,0,12) with non-zero mean
Q* = 7.1331, df = 3, p-value = 0.06777

Model df: 20. Total lags used: 23

[1] 7
[1] 12

Ljung-Box test

data: Residuals from ARIMA(8,0,0) with non-zero mean
Q* = 11.364, df = 3, p-value = 0.009911

Model df: 9. Total lags used: 12

[1] 8
[1] 0

Ljung-Box test

data: Residuals from ARIMA(8,0,1) with non-zero mean
Q* = 11.785, df = 3, p-value = 0.008158

Model df: 10. Total lags used: 13

[1] 8
[1] 1

Ljung-Box test

data: Residuals from ARIMA(8,0,2) with non-zero mean
Q* = 4.5948, df = 3, p-value = 0.204

Model df: 11. Total lags used: 14

[1] 8
[1] 2

Ljung-Box test

data: Residuals from ARIMA(8,0,3) with non-zero mean
Q* = 5.6127, df = 3, p-value = 0.1321

Model df: 12. Total lags used: 15

[1] 8
[1] 3

Ljung-Box test

data: Residuals from ARIMA(8,0,4) with non-zero mean
Q* = 6.8881, df = 3, p-value = 0.07555

Model df: 13. Total lags used: 16

[1] 8
[1] 4

Ljung-Box test

data: Residuals from ARIMA(8,0,5) with non-zero mean
Q* = 7.8829, df = 3, p-value = 0.04849

Model df: 14. Total lags used: 17

[1] 8
[1] 5

Ljung-Box test

data: Residuals from ARIMA(8,0,6) with non-zero mean
Q* = 3.9171, df = 3, p-value = 0.2706

Model df: 15. Total lags used: 18

[1] 8
[1] 6

Ljung-Box test

data: Residuals from ARIMA(8,0,7) with non-zero mean
Q* = 7.5091, df = 3, p-value = 0.05732

Model df: 16. Total lags used: 19

[1] 8
[1] 7

Ljung-Box test

data: Residuals from ARIMA(8,0,8) with non-zero mean
Q* = 5.9416, df = 3, p-value = 0.1145

Model df: 17. Total lags used: 20

[1] 8
[1] 8

Ljung-Box test

data: Residuals from ARIMA(8,0,9) with non-zero mean
Q* = 5.9284, df = 3, p-value = 0.1151

Model df: 18. Total lags used: 21

[1] 8
[1] 9

Ljung-Box test

data: Residuals from ARIMA(8,0,10) with non-zero mean
Q* = 6.1729, df = 3, p-value = 0.1035

Model df: 19. Total lags used: 22

[1] 8
[1] 10

Ljung-Box test

data: Residuals from ARIMA(8,0,11) with non-zero mean
Q* = 6.6963, df = 3, p-value = 0.08223

Model df: 20. Total lags used: 23

[1] 8

[1] 11

Ljung-Box test

data: Residuals from ARIMA(8,0,12) with non-zero mean
Q* = 7.2561, df = 3, p-value = 0.06417

Model df: 21. Total lags used: 24

[1] 8

[1] 12

Ljung-Box test

data: Residuals from ARIMA(9,0,0) with non-zero mean
Q* = 9.838, df = 3, p-value = 0.01999

Model df: 10. Total lags used: 13

[1] 9

[1] 0

Ljung-Box test

data: Residuals from ARIMA(9,0,1) with non-zero mean
Q* = 9.7579, df = 3, p-value = 0.02074

Model df: 11. Total lags used: 14

[1] 9

[1] 1

Ljung-Box test

data: Residuals from ARIMA(9,0,2) with non-zero mean
Q* = 6.3999, df = 3, p-value = 0.0937

Model df: 12. Total lags used: 15

[1] 9

[1] 2

Ljung-Box test

data: Residuals from ARIMA(9,0,3) with non-zero mean
Q* = 7.0097, df = 3, p-value = 0.07159

Model df: 13. Total lags used: 16

```
[1] 9
[1] 3
```

Ljung-Box test

data: Residuals from ARIMA(9,0,4) with non-zero mean
Q* = 4.5291, df = 3, p-value = 0.2097

Model df: 14. Total lags used: 17

```
[1] 9
[1] 4
```

Ljung-Box test

data: Residuals from ARIMA(9,0,5) with non-zero mean
Q* = 4.6154, df = 3, p-value = 0.2022

Model df: 15. Total lags used: 18

```
[1] 9
[1] 5
```

Ljung-Box test

data: Residuals from ARIMA(9,0,6) with non-zero mean
Q* = 10.05, df = 3, p-value = 0.01814

Model df: 16. Total lags used: 19

```
[1] 9
[1] 6
```

Ljung-Box test

data: Residuals from ARIMA(9,0,7) with non-zero mean
Q* = 8.8128, df = 3, p-value = 0.03189

Model df: 17. Total lags used: 20

```
[1] 9
[1] 7
```

Ljung-Box test

data: Residuals from ARIMA(9,0,8) with non-zero mean
Q* = 7.4626, df = 3, p-value = 0.05853

Model df: 18. Total lags used: 21

[1] 9

[1] 8

Ljung-Box test

data: Residuals from ARIMA(9,0,9) with non-zero mean
Q* = 6.213, df = 3, p-value = 0.1017

Model df: 19. Total lags used: 22

[1] 9

[1] 9

Ljung-Box test

data: Residuals from ARIMA(9,0,10) with non-zero mean
Q* = 8.5798, df = 3, p-value = 0.03543

Model df: 20. Total lags used: 23

[1] 9

[1] 10

Ljung-Box test

data: Residuals from ARIMA(9,0,11) with non-zero mean
Q* = 8.2003, df = 3, p-value = 0.04205

Model df: 21. Total lags used: 24

[1] 9

[1] 11

Ljung-Box test

data: Residuals from ARIMA(9,0,12) with non-zero mean
Q* = 7.6228, df = 3, p-value = 0.05449

Model df: 22. Total lags used: 25

[1] 9

[1] 12

Ljung-Box test

```
data: Residuals from ARIMA(10,0,0) with non-zero mean
Q* = 9.5353, df = 3, p-value = 0.02296
```

```
Model df: 11.    Total lags used: 14
```

```
[1] 10
[1] 0
```

```
Ljung-Box test
```

```
data: Residuals from ARIMA(10,0,1) with non-zero mean
Q* = 10.137, df = 3, p-value = 0.01744
```

```
Model df: 12.    Total lags used: 15
```

```
[1] 10
[1] 1
```

```
Ljung-Box test
```

```
data: Residuals from ARIMA(10,0,2) with non-zero mean
Q* = 7.0269, df = 3, p-value = 0.07105
```

```
Model df: 13.    Total lags used: 16
```

```
[1] 10
[1] 2
```

```
Ljung-Box test
```

```
data: Residuals from ARIMA(10,0,3) with non-zero mean
Q* = 7.3174, df = 3, p-value = 0.06244
```

```
Model df: 14.    Total lags used: 17
```

```
[1] 10
[1] 3
```

```
Ljung-Box test
```

```
data: Residuals from ARIMA(10,0,4) with non-zero mean
Q* = 5.3003, df = 3, p-value = 0.1511
```

```
Model df: 15.    Total lags used: 18
```

```
[1] 10
[1] 4
```

Ljung-Box test

data: Residuals from ARIMA(10,0,5) with non-zero mean
Q* = 5.9676, df = 3, p-value = 0.1132

Model df: 16. Total lags used: 19

[1] 10
[1] 5

Ljung-Box test

data: Residuals from ARIMA(10,0,6) with non-zero mean
Q* = 5.9369, df = 3, p-value = 0.1147

Model df: 17. Total lags used: 20

[1] 10
[1] 6

Ljung-Box test

data: Residuals from ARIMA(10,0,7) with non-zero mean
Q* = 8.2575, df = 3, p-value = 0.04098

Model df: 18. Total lags used: 21

[1] 10
[1] 7

Ljung-Box test

data: Residuals from ARIMA(10,0,8) with non-zero mean
Q* = 7.0207, df = 3, p-value = 0.07124

Model df: 19. Total lags used: 22

[1] 10
[1] 8

Ljung-Box test

data: Residuals from ARIMA(10,0,9) with non-zero mean
Q* = 6.4956, df = 3, p-value = 0.08984

Model df: 20. Total lags used: 23

[1] 10

[1] 9

Ljung-Box test

data: Residuals from ARIMA(10,0,10) with non-zero mean
Q* = 7.484, df = 3, p-value = 0.05797

Model df: 21. Total lags used: 24

[1] 10

[1] 10

Ljung-Box test

data: Residuals from ARIMA(10,0,11) with non-zero mean
Q* = 7.706, df = 3, p-value = 0.0525

Model df: 22. Total lags used: 25

[1] 10

[1] 11

Ljung-Box test

data: Residuals from ARIMA(10,0,12) with non-zero mean
Q* = 7.891, df = 3, p-value = 0.04832

Model df: 23. Total lags used: 26

[1] 10

[1] 12

Ljung-Box test

data: Residuals from ARIMA(11,0,0) with non-zero mean
Q* = 6.8474, df = 3, p-value = 0.07692

Model df: 12. Total lags used: 15

[1] 11

[1] 0

Ljung-Box test

data: Residuals from ARIMA(11,0,1) with non-zero mean
Q* = 4.2138, df = 3, p-value = 0.2393

Model df: 13. Total lags used: 16


```
[1] 11
[1] 1
```

Ljung-Box test

data: Residuals from ARIMA(11,0,2) with non-zero mean
Q* = 3.6156, df = 3, p-value = 0.3061

Model df: 14. Total lags used: 17

```
[1] 11
[1] 2
```

Ljung-Box test

data: Residuals from ARIMA(11,0,3) with non-zero mean
Q* = 5.657, df = 3, p-value = 0.1295

Model df: 15. Total lags used: 18

```
[1] 11
[1] 3
```

Ljung-Box test

data: Residuals from ARIMA(11,0,4) with non-zero mean
Q* = 5.6417, df = 3, p-value = 0.1304

Model df: 16. Total lags used: 19

```
[1] 11
[1] 4
```

Ljung-Box test

data: Residuals from ARIMA(11,0,5) with non-zero mean
Q* = 8.9718, df = 3, p-value = 0.02967

Model df: 17. Total lags used: 20

```
[1] 11
[1] 5
```

Ljung-Box test

data: Residuals from ARIMA(11,0,6) with non-zero mean
Q* = 7.6644, df = 3, p-value = 0.05348

Model df: 18. Total lags used: 21

[1] 11

[1] 6

Ljung-Box test

data: Residuals from ARIMA(11,0,7) with non-zero mean
Q* = 9.099, df = 3, p-value = 0.028

Model df: 19. Total lags used: 22

[1] 11

[1] 7

Ljung-Box test

data: Residuals from ARIMA(11,0,8) with non-zero mean
Q* = 6.5633, df = 3, p-value = 0.0872

Model df: 20. Total lags used: 23

[1] 11

[1] 8

Ljung-Box test

data: Residuals from ARIMA(11,0,9) with non-zero mean
Q* = 7.5055, df = 3, p-value = 0.05742

Model df: 21. Total lags used: 24

[1] 11

[1] 9

Ljung-Box test

data: Residuals from ARIMA(11,0,10) with non-zero mean
Q* = 7.1972, df = 3, p-value = 0.06587

Model df: 22. Total lags used: 25

[1] 11

[1] 10

Ljung-Box test

data: Residuals from ARIMA(11,0,11) with non-zero mean
Q* = 7.8445, df = 3, p-value = 0.04934

Model df: 23. Total lags used: 26

[1] 11
[1] 11

Ljung-Box test

data: Residuals from ARIMA(11,0,12) with non-zero mean
Q* = 7.8095, df = 3, p-value = 0.05012

Model df: 24. Total lags used: 27

[1] 11
[1] 12

Ljung-Box test

data: Residuals from ARIMA(12,0,0) with non-zero mean
Q* = 2.8509, df = 3, p-value = 0.4152

Model df: 13. Total lags used: 16

[1] 12
[1] 0

Ljung-Box test

data: Residuals from ARIMA(12,0,1) with non-zero mean
Q* = 2.8063, df = 3, p-value = 0.4225

Model df: 14. Total lags used: 17

[1] 12
[1] 1

Ljung-Box test

data: Residuals from ARIMA(12,0,2) with non-zero mean
Q* = 4.9283, df = 3, p-value = 0.1771

Model df: 15. Total lags used: 18

[1] 12
[1] 2

Ljung-Box test

data: Residuals from ARIMA(12,0,3) with non-zero mean
Q* = 3.2, df = 3, p-value = 0.3618

Model df: 16. Total lags used: 19

[1] 12
[1] 3

Ljung-Box test

data: Residuals from ARIMA(12,0,4) with non-zero mean
Q* = 3.6926, df = 3, p-value = 0.2966

Model df: 17. Total lags used: 20

[1] 12
[1] 4

Ljung-Box test

data: Residuals from ARIMA(12,0,5) with non-zero mean
Q* = 3.8013, df = 3, p-value = 0.2837

Model df: 18. Total lags used: 21

[1] 12
[1] 5

Ljung-Box test

data: Residuals from ARIMA(12,0,6) with non-zero mean
Q* = 5.2775, df = 3, p-value = 0.1526

Model df: 19. Total lags used: 22

[1] 12
[1] 6

Ljung-Box test

data: Residuals from ARIMA(12,0,7) with non-zero mean
Q* = 6.5211, df = 3, p-value = 0.08883

Model df: 20. Total lags used: 23

[1] 12

[1] 7

Ljung-Box test

data: Residuals from ARIMA(12,0,8) with non-zero mean
Q* = 7.0018, df = 3, p-value = 0.07184

Model df: 21. Total lags used: 24

[1] 12

[1] 8

Ljung-Box test

data: Residuals from ARIMA(12,0,9) with non-zero mean
Q* = 7.3979, df = 3, p-value = 0.06024

Model df: 22. Total lags used: 25

[1] 12

[1] 9

Ljung-Box test

data: Residuals from ARIMA(12,0,10) with non-zero mean
Q* = 7.6424, df = 3, p-value = 0.05401

Model df: 23. Total lags used: 26

[1] 12

[1] 10

Ljung-Box test

data: Residuals from ARIMA(12,0,11) with non-zero mean
Q* = 7.0776, df = 3, p-value = 0.06947

Model df: 24. Total lags used: 27

[1] 12

[1] 11

Ljung-Box test

data: Residuals from ARIMA(12,0,12) with non-zero mean
Q* = 7.4559, df = 3, p-value = 0.0587

Model df: 25. Total lags used: 28

[1] 12
[1] 12

	Qtr1	Qtr2	Qtr3	Qtr4
1970	-2.45270031	-0.55152509	-0.35870786	-2.18545486
1971	1.90973412	0.90153584	0.30801942	2.29130441
1972	4.14957387	1.89062398	1.27335290	3.43689207
1973	2.79907636	0.81768862	0.86899693	1.47296187
1974	-0.88248358	0.07427919	-0.41314971	-4.06411893
1975	-6.85103912	-1.33129558	2.42435972	2.16904208
1976	3.02720471	1.27881101	1.30386487	1.77537765
1977	2.05516067	3.05838507	1.10308888	0.63346850
1978	-0.29339056	3.94815264	0.87114701	1.78447991
1979	0.42594327	-0.20491944	-0.29723637	0.33560928
1980	0.41056141	-4.30076832	-1.64181977	3.78045520
1981	0.24627687	0.30977573	0.91707444	-2.25457797
1982	-2.07131293	-1.24766384	-1.40050430	-1.90375664
1983	1.14655720	2.17942248	3.36771897	2.58168445
1984	2.89709545	1.53821324	0.72128740	0.04115557
1985	0.32353159	0.07020996	-0.14046924	0.57978813
1986	0.58132135	-0.57641778	0.37249329	1.13734778
1987	1.30758228	1.75000563	1.84366200	2.40645058
1988	0.92013121	0.87316353	0.38103668	0.70292025
1989	0.43372685	-0.36675732	-0.62142121	0.42443392
1990	0.68265169	0.77446547	0.41944800	-1.57345296
1991	-1.91422028	0.59131506	1.36255645	0.21710308
1992	-0.13365365	1.76874773	0.76167388	1.05024577
1993	0.87901471	0.21755108	0.40135891	1.49618275
1994	1.22213656	1.78250275	1.26718100	2.04370404
1995	1.02552601	0.33785685	0.90043887	0.87467273
1996	0.69285195	2.11134752	1.24418680	1.35396890
1997	1.86714700	1.48763922	2.28632066	2.48091341
1998	1.10343775	0.65122238	0.72551955	1.44421674
1999	1.10341663	0.98574261	0.90279881	1.75533234
2000	0.99682019	1.23293805	-0.10225268	-0.20388383
2001	-1.35143911	-1.25954437	-1.44101744	-1.06013675
2002	0.70916406	1.54280957	0.59478143	-0.05776556
2003	0.53922789	-0.69876172	0.60727351	1.00599126
2004	0.65792806	0.57461780	0.56330030	1.38522763
2005	1.39435718	0.50586367	-0.50305848	0.93365010
2006	0.95057853	0.59636010	0.33552773	0.25603401
2007	0.91794957	1.19594247	0.22356909	0.16424632
2008	-0.42872571	-1.41297022	-3.26349945	-4.35417741
2009	-5.75045075	-3.00372447	1.39880419	1.54400617
2010	1.88006931	2.05402479	1.42683671	0.37927209
2011	0.50174040	0.21878696	1.01113866	0.85151692
2012	0.88651817	0.62923586	0.07880166	0.63305509
2013	0.67713243	0.30744961	0.23440888	0.79208722
2014	0.54709166	1.33801074	0.62352731	0.90355427
2015	-0.46710878	-0.69702162	0.38060610	

A Time Series: 46×4

Ljung-Box test

data: Residuals from ARIMA(0,0,0) with non-zero mean
Q* = 100.44, df = 7, p-value < 2.2e-16

Model df: 1. Total lags used: 8

[1] 0

[1] 0

Ljung-Box test

data: Residuals from ARIMA(0,0,1) with non-zero mean
Q* = 24.76, df = 6, p-value = 0.0003783

Model df: 2. Total lags used: 8

[1] 0

[1] 1

Ljung-Box test

data: Residuals from ARIMA(0,0,2) with non-zero mean
Q* = 18.363, df = 5, p-value = 0.002524

Model df: 3. Total lags used: 8

[1] 0

[1] 2

Ljung-Box test

data: Residuals from ARIMA(0,0,3) with non-zero mean
Q* = 17.273, df = 4, p-value = 0.001711

Model df: 4. Total lags used: 8

[1] 0

[1] 3

Ljung-Box test

data: Residuals from ARIMA(0,0,4) with non-zero mean
Q* = 11.33, df = 3, p-value = 0.01007

Model df: 5. Total lags used: 8


```
[1] 0
[1] 4
```

Ljung-Box test

data: Residuals from ARIMA(0,0,5) with non-zero mean
Q* = 12.684, df = 3, p-value = 0.005373

Model df: 6. Total lags used: 9

```
[1] 0
[1] 5
```

Ljung-Box test

data: Residuals from ARIMA(0,0,6) with non-zero mean
Q* = 13.453, df = 3, p-value = 0.003753

Model df: 7. Total lags used: 10

```
[1] 0
[1] 6
```

Ljung-Box test

data: Residuals from ARIMA(0,0,7) with non-zero mean
Q* = 10.481, df = 3, p-value = 0.01489

Model df: 8. Total lags used: 11

```
[1] 0
[1] 7
```

Ljung-Box test

data: Residuals from ARIMA(0,0,8) with non-zero mean
Q* = 11.751, df = 3, p-value = 0.008289

Model df: 9. Total lags used: 12

```
[1] 0
[1] 8
```

Ljung-Box test

data: Residuals from ARIMA(0,0,9) with non-zero mean
Q* = 5.5776, df = 3, p-value = 0.1341

Model df: 10. Total lags used: 13

[1] 0

[1] 9

Ljung-Box test

data: Residuals from ARIMA(0,0,10) with non-zero mean
Q* = 5.6446, df = 3, p-value = 0.1302

Model df: 11. Total lags used: 14

[1] 0

[1] 10

Ljung-Box test

data: Residuals from ARIMA(0,0,11) with non-zero mean
Q* = 6.0569, df = 3, p-value = 0.1089

Model df: 12. Total lags used: 15

[1] 0

[1] 11

Ljung-Box test

data: Residuals from ARIMA(0,0,12) with non-zero mean
Q* = 5.2103, df = 3, p-value = 0.157

Model df: 13. Total lags used: 16

[1] 0

[1] 12

Ljung-Box test

data: Residuals from ARIMA(1,0,0) with non-zero mean
Q* = 24.33, df = 6, p-value = 0.0004541

Model df: 2. Total lags used: 8

[1] 1

[1] 0

Ljung-Box test

data: Residuals from ARIMA(1,0,1) with non-zero mean

Q* = 18.011, df = 5, p-value = 0.002933

Model df: 3. Total lags used: 8

[1] 1

[1] 1

Ljung-Box test

data: Residuals from ARIMA(1,0,2) with non-zero mean

Q* = 15.934, df = 4, p-value = 0.003109

Model df: 4. Total lags used: 8

[1] 1

[1] 2

Ljung-Box test

data: Residuals from ARIMA(1,0,3) with non-zero mean

Q* = 14.484, df = 3, p-value = 0.002315

Model df: 5. Total lags used: 8

[1] 1

[1] 3

Ljung-Box test

data: Residuals from ARIMA(1,0,4) with non-zero mean

Q* = 11.872, df = 3, p-value = 0.007835

Model df: 6. Total lags used: 9

[1] 1

[1] 4

Ljung-Box test

data: Residuals from ARIMA(1,0,5) with non-zero mean

Q* = 13.139, df = 3, p-value = 0.004346

Model df: 7. Total lags used: 10

[1] 1

[1] 5

Ljung-Box test

data: Residuals from ARIMA(1,0,6) with non-zero mean
Q* = 13.347, df = 3, p-value = 0.003944

Model df: 8. Total lags used: 11

[1] 1
[1] 6

Ljung-Box test

data: Residuals from ARIMA(1,0,7) with non-zero mean
Q* = 14.599, df = 3, p-value = 0.002193

Model df: 9. Total lags used: 12

[1] 1
[1] 7

Ljung-Box test

data: Residuals from ARIMA(1,0,8) with non-zero mean
Q* = 9.2235, df = 3, p-value = 0.02646

Model df: 10. Total lags used: 13

[1] 1
[1] 8

Ljung-Box test

data: Residuals from ARIMA(1,0,9) with non-zero mean
Q* = 5.7005, df = 3, p-value = 0.1271

Model df: 11. Total lags used: 14

[1] 1
[1] 9

Ljung-Box test

data: Residuals from ARIMA(1,0,10) with non-zero mean
Q* = 6.6429, df = 3, p-value = 0.08419

Model df: 12. Total lags used: 15

[1] 1
[1] 10

Ljung-Box test

data: Residuals from ARIMA(1,0,11) with non-zero mean
Q* = 5.9093, df = 3, p-value = 0.1161

Model df: 13. Total lags used: 16

[1] 1
[1] 11

Ljung-Box test

data: Residuals from ARIMA(1,0,12) with non-zero mean
Q* = 5.0196, df = 3, p-value = 0.1704

Model df: 14. Total lags used: 17

[1] 1
[1] 12

Ljung-Box test

data: Residuals from ARIMA(2,0,0) with non-zero mean
Q* = 23.1, df = 5, p-value = 0.000323

Model df: 3. Total lags used: 8

[1] 2
[1] 0

Ljung-Box test

data: Residuals from ARIMA(2,0,1) with non-zero mean
Q* = 16.218, df = 4, p-value = 0.00274

Model df: 4. Total lags used: 8

[1] 2
[1] 1

Ljung-Box test

data: Residuals from ARIMA(2,0,2) with non-zero mean
Q* = 15.5, df = 3, p-value = 0.001436

Model df: 5. Total lags used: 8

```
[1] 2
[1] 2
```

Ljung-Box test

data: Residuals from ARIMA(2,0,3) with non-zero mean
Q* = 12.504, df = 3, p-value = 0.005843

Model df: 6. Total lags used: 9

```
[1] 2
[1] 3
```

Ljung-Box test

data: Residuals from ARIMA(2,0,4) with non-zero mean
Q* = 7.2445, df = 3, p-value = 0.0645

Model df: 7. Total lags used: 10

```
[1] 2
[1] 4
```

Ljung-Box test

data: Residuals from ARIMA(2,0,5) with non-zero mean
Q* = 12.092, df = 3, p-value = 0.007073

Model df: 8. Total lags used: 11

```
[1] 2
[1] 5
```

Ljung-Box test

data: Residuals from ARIMA(2,0,6) with non-zero mean
Q* = 6.4775, df = 3, p-value = 0.09055

Model df: 9. Total lags used: 12

```
[1] 2
[1] 6
```

Ljung-Box test

data: Residuals from ARIMA(2,0,7) with non-zero mean
Q* = 8.0513, df = 3, p-value = 0.04496

Model df: 10. Total lags used: 13

[1] 2

[1] 7

Ljung-Box test

data: Residuals from ARIMA(2,0,8) with non-zero mean
Q* = 6.6682, df = 3, p-value = 0.08326

Model df: 11. Total lags used: 14

[1] 2

[1] 8

Ljung-Box test

data: Residuals from ARIMA(2,0,9) with non-zero mean
Q* = 5.5069, df = 3, p-value = 0.1382

Model df: 12. Total lags used: 15

[1] 2

[1] 9

Ljung-Box test

data: Residuals from ARIMA(2,0,10) with non-zero mean
Q* = 3.0036, df = 3, p-value = 0.3911

Model df: 13. Total lags used: 16

[1] 2

[1] 10

Ljung-Box test

data: Residuals from ARIMA(2,0,11) with non-zero mean
Q* = 3.7212, df = 3, p-value = 0.2932

Model df: 14. Total lags used: 17

[1] 2

[1] 11

Ljung-Box test

data: Residuals from ARIMA(2,0,12) with non-zero mean

Q* = 3.6729, df = 3, p-value = 0.299

Model df: 15. Total lags used: 18

[1] 2
[1] 12

Ljung-Box test

data: Residuals from ARIMA(3,0,0) with non-zero mean
Q* = 16.887, df = 4, p-value = 0.002033

Model df: 4. Total lags used: 8

[1] 3
[1] 0

Ljung-Box test

data: Residuals from ARIMA(3,0,1) with non-zero mean
Q* = 14.066, df = 3, p-value = 0.002817

Model df: 5. Total lags used: 8

[1] 3
[1] 1

Ljung-Box test

data: Residuals from ARIMA(3,0,2) with non-zero mean
Q* = 17.911, df = 3, p-value = 0.0004589

Model df: 6. Total lags used: 9

[1] 3
[1] 2

Ljung-Box test

data: Residuals from ARIMA(3,0,3) with non-zero mean
Q* = 7.636, df = 3, p-value = 0.05417

Model df: 7. Total lags used: 10

[1] 3
[1] 3

Ljung-Box test

data: Residuals from ARIMA(3,0,4) with non-zero mean
Q* = 6.0475, df = 3, p-value = 0.1093

Model df: 8. Total lags used: 11

[1] 3
[1] 4

Ljung-Box test

data: Residuals from ARIMA(3,0,5) with non-zero mean
Q* = 6.2537, df = 3, p-value = 0.0999

Model df: 9. Total lags used: 12

[1] 3
[1] 5

Ljung-Box test

data: Residuals from ARIMA(3,0,6) with non-zero mean
Q* = 8.1323, df = 3, p-value = 0.04335

Model df: 10. Total lags used: 13

[1] 3
[1] 6

Ljung-Box test

data: Residuals from ARIMA(3,0,7) with non-zero mean
Q* = 6.4525, df = 3, p-value = 0.09155

Model df: 11. Total lags used: 14

[1] 3
[1] 7

Ljung-Box test

data: Residuals from ARIMA(3,0,8) with non-zero mean
Q* = 6.0506, df = 3, p-value = 0.1092

Model df: 12. Total lags used: 15

[1] 3
[1] 8

Ljung-Box test

data: Residuals from ARIMA(3,0,9) with non-zero mean
Q* = 3.3945, df = 3, p-value = 0.3347

Model df: 13. Total lags used: 16

[1] 3

[1] 9

Ljung-Box test

data: Residuals from ARIMA(3,0,10) with non-zero mean
Q* = 4.5962, df = 3, p-value = 0.2039

Model df: 14. Total lags used: 17

[1] 3

[1] 10

Ljung-Box test

data: Residuals from ARIMA(3,0,11) with non-zero mean
Q* = 3.7232, df = 3, p-value = 0.2929

Model df: 15. Total lags used: 18

[1] 3

[1] 11

Ljung-Box test

data: Residuals from ARIMA(3,0,12) with non-zero mean
Q* = 4.463, df = 3, p-value = 0.2156

Model df: 16. Total lags used: 19

[1] 3

[1] 12

Ljung-Box test

data: Residuals from ARIMA(4,0,0) with non-zero mean
Q* = 10.834, df = 3, p-value = 0.01266

Model df: 5. Total lags used: 8

```
[1] 4
[1] 0
```

Ljung-Box test

data: Residuals from ARIMA(4,0,1) with non-zero mean
Q* = 7.6642, df = 3, p-value = 0.05349

Model df: 6. Total lags used: 9

```
[1] 4
[1] 1
```

Ljung-Box test

data: Residuals from ARIMA(4,0,2) with non-zero mean
Q* = 7.1133, df = 3, p-value = 0.06837

Model df: 7. Total lags used: 10

```
[1] 4
[1] 2
```

Ljung-Box test

data: Residuals from ARIMA(4,0,3) with non-zero mean
Q* = 6.6641, df = 3, p-value = 0.08341

Model df: 8. Total lags used: 11

```
[1] 4
[1] 3
```

Ljung-Box test

data: Residuals from ARIMA(4,0,4) with non-zero mean
Q* = 6.3285, df = 3, p-value = 0.09668

Model df: 9. Total lags used: 12

```
[1] 4
[1] 4
```

Ljung-Box test

data: Residuals from ARIMA(4,0,5) with non-zero mean
Q* = 8.057, df = 3, p-value = 0.04485

Model df: 10. Total lags used: 13

[1] 4

[1] 5

Ljung-Box test

data: Residuals from ARIMA(4,0,6) with non-zero mean
Q* = 8.3247, df = 3, p-value = 0.03976

Model df: 11. Total lags used: 14

[1] 4

[1] 6

Ljung-Box test

data: Residuals from ARIMA(4,0,7) with non-zero mean
Q* = 8.4411, df = 3, p-value = 0.03772

Model df: 12. Total lags used: 15

[1] 4

[1] 7

Ljung-Box test

data: Residuals from ARIMA(4,0,8) with non-zero mean
Q* = 7.9117, df = 3, p-value = 0.04787

Model df: 13. Total lags used: 16

[1] 4

[1] 8

Ljung-Box test

data: Residuals from ARIMA(4,0,9) with non-zero mean
Q* = 8.1259, df = 3, p-value = 0.04348

Model df: 14. Total lags used: 17

[1] 4

[1] 9

Ljung-Box test

data: Residuals from ARIMA(4,0,10) with non-zero mean

Q* = 4.0675, df = 3, p-value = 0.2543

Model df: 15. Total lags used: 18

[1] 4
[1] 10

Ljung-Box test

data: Residuals from ARIMA(4,0,11) with non-zero mean
Q* = 5.7247, df = 3, p-value = 0.1258

Model df: 16. Total lags used: 19

[1] 4
[1] 11

Ljung-Box test

data: Residuals from ARIMA(4,0,12) with non-zero mean
Q* = 4.1932, df = 3, p-value = 0.2413

Model df: 17. Total lags used: 20

[1] 4
[1] 12

Ljung-Box test

data: Residuals from ARIMA(5,0,0) with non-zero mean
Q* = 9.7775, df = 3, p-value = 0.02056

Model df: 6. Total lags used: 9

[1] 5
[1] 0

Ljung-Box test

data: Residuals from ARIMA(5,0,1) with non-zero mean
Q* = 11.961, df = 3, p-value = 0.007517

Model df: 7. Total lags used: 10

[1] 5
[1] 1

Ljung-Box test

data: Residuals from ARIMA(5,0,2) with non-zero mean
Q* = 6.5783, df = 3, p-value = 0.08663

Model df: 8. Total lags used: 11

[1] 5
[1] 2

Ljung-Box test

data: Residuals from ARIMA(5,0,3) with non-zero mean
Q* = 7.8519, df = 3, p-value = 0.04917

Model df: 9. Total lags used: 12

[1] 5
[1] 3

Ljung-Box test

data: Residuals from ARIMA(5,0,4) with non-zero mean
Q* = 6.571, df = 3, p-value = 0.0869

Model df: 10. Total lags used: 13

[1] 5
[1] 4

Ljung-Box test

data: Residuals from ARIMA(5,0,5) with non-zero mean
Q* = 6.0688, df = 3, p-value = 0.1083

Model df: 11. Total lags used: 14

[1] 5
[1] 5

Ljung-Box test

data: Residuals from ARIMA(5,0,6) with non-zero mean
Q* = 5.7829, df = 3, p-value = 0.1227

Model df: 12. Total lags used: 15

[1] 5
[1] 6

Ljung-Box test

data: Residuals from ARIMA(5,0,7) with non-zero mean
Q* = 7.1808, df = 3, p-value = 0.06635

Model df: 13. Total lags used: 16

[1] 5

[1] 7

Ljung-Box test

data: Residuals from ARIMA(5,0,8) with non-zero mean
Q* = 7.7992, df = 3, p-value = 0.05035

Model df: 14. Total lags used: 17

[1] 5

[1] 8

Ljung-Box test

data: Residuals from ARIMA(5,0,9) with non-zero mean
Q* = 3.9174, df = 3, p-value = 0.2705

Model df: 15. Total lags used: 18

[1] 5

[1] 9

Ljung-Box test

data: Residuals from ARIMA(5,0,10) with non-zero mean
Q* = 6.8944, df = 3, p-value = 0.07534

Model df: 16. Total lags used: 19

[1] 5

[1] 10

Ljung-Box test

data: Residuals from ARIMA(5,0,11) with non-zero mean
Q* = 4.0288, df = 3, p-value = 0.2584

Model df: 17. Total lags used: 20

```
[1] 5
[1] 11
```

Ljung-Box test

data: Residuals from ARIMA(5,0,12) with non-zero mean
Q* = 4.5669, df = 3, p-value = 0.2064

Model df: 18. Total lags used: 21

```
[1] 5
[1] 12
```

Ljung-Box test

data: Residuals from ARIMA(6,0,0) with non-zero mean
Q* = 12.917, df = 3, p-value = 0.004821

Model df: 7. Total lags used: 10

```
[1] 6
[1] 0
```

Ljung-Box test

data: Residuals from ARIMA(6,0,1) with non-zero mean
Q* = 12.598, df = 3, p-value = 0.005593

Model df: 8. Total lags used: 11

```
[1] 6
[1] 1
```

Ljung-Box test

data: Residuals from ARIMA(6,0,2) with non-zero mean
Q* = 6.9709, df = 3, p-value = 0.07283

Model df: 9. Total lags used: 12

```
[1] 6
[1] 2
```

Ljung-Box test

data: Residuals from ARIMA(6,0,3) with non-zero mean
Q* = 6.0319, df = 3, p-value = 0.1101

Model df: 10. Total lags used: 13

[1] 6

[1] 3

Ljung-Box test

data: Residuals from ARIMA(6,0,4) with non-zero mean
Q* = 7.7723, df = 3, p-value = 0.05096

Model df: 11. Total lags used: 14

[1] 6

[1] 4

Ljung-Box test

data: Residuals from ARIMA(6,0,5) with non-zero mean
Q* = 5.904, df = 3, p-value = 0.1164

Model df: 12. Total lags used: 15

[1] 6

[1] 5

Ljung-Box test

data: Residuals from ARIMA(6,0,6) with non-zero mean
Q* = 7.7628, df = 3, p-value = 0.05118

Model df: 13. Total lags used: 16

[1] 6

[1] 6

Ljung-Box test

data: Residuals from ARIMA(6,0,7) with non-zero mean
Q* = 7.9925, df = 3, p-value = 0.04617

Model df: 14. Total lags used: 17

[1] 6

[1] 7

Ljung-Box test

data: Residuals from ARIMA(6,0,8) with non-zero mean

Q* = 1.8284, df = 3, p-value = 0.6088

Model df: 15. Total lags used: 18

[1] 6

[1] 8

Ljung-Box test

data: Residuals from ARIMA(6,0,9) with non-zero mean

Q* = 2.8666, df = 3, p-value = 0.4127

Model df: 16. Total lags used: 19

[1] 6

[1] 9

Ljung-Box test

data: Residuals from ARIMA(6,0,10) with non-zero mean

Q* = 7.5625, df = 3, p-value = 0.05597

Model df: 17. Total lags used: 20

[1] 6

[1] 10

Ljung-Box test

data: Residuals from ARIMA(6,0,11) with non-zero mean

Q* = 4.7131, df = 3, p-value = 0.1941

Model df: 18. Total lags used: 21

[1] 6

[1] 11

Ljung-Box test

data: Residuals from ARIMA(6,0,12) with non-zero mean

Q* = 4.8498, df = 3, p-value = 0.1831

Model df: 19. Total lags used: 22

[1] 6

[1] 12

Ljung-Box test

data: Residuals from ARIMA(7,0,0) with non-zero mean
Q* = 9.5486, df = 3, p-value = 0.02282

Model df: 8. Total lags used: 11

[1] 7
[1] 0

Ljung-Box test

data: Residuals from ARIMA(7,0,1) with non-zero mean
Q* = 12.252, df = 3, p-value = 0.006569

Model df: 9. Total lags used: 12

[1] 7
[1] 1

Ljung-Box test

data: Residuals from ARIMA(7,0,2) with non-zero mean
Q* = 4.2339, df = 3, p-value = 0.2373

Model df: 10. Total lags used: 13

[1] 7
[1] 2

Ljung-Box test

data: Residuals from ARIMA(7,0,3) with non-zero mean
Q* = 5.7494, df = 3, p-value = 0.1245

Model df: 11. Total lags used: 14

[1] 7
[1] 3

Ljung-Box test

data: Residuals from ARIMA(7,0,4) with non-zero mean
Q* = 5.7004, df = 3, p-value = 0.1271

Model df: 12. Total lags used: 15

[1] 7
[1] 4

Ljung-Box test

data: Residuals from ARIMA(7,0,5) with non-zero mean
Q* = 5.5019, df = 3, p-value = 0.1385

Model df: 13. Total lags used: 16

[1] 7

[1] 5

Ljung-Box test

data: Residuals from ARIMA(7,0,6) with non-zero mean
Q* = 5.384, df = 3, p-value = 0.1457

Model df: 14. Total lags used: 17

[1] 7

[1] 6

Ljung-Box test

data: Residuals from ARIMA(7,0,7) with non-zero mean
Q* = 8.8009, df = 3, p-value = 0.03206

Model df: 15. Total lags used: 18

[1] 7

[1] 7

Ljung-Box test

data: Residuals from ARIMA(7,0,8) with non-zero mean
Q* = 6.7058, df = 3, p-value = 0.08189

Model df: 16. Total lags used: 19

[1] 7

[1] 8

Ljung-Box test

data: Residuals from ARIMA(7,0,9) with non-zero mean
Q* = 8.5312, df = 3, p-value = 0.03622

Model df: 17. Total lags used: 20

```
[1] 7
[1] 9
```

Ljung-Box test

data: Residuals from ARIMA(7,0,10) with non-zero mean
Q* = 7.8513, df = 3, p-value = 0.04919

Model df: 18. Total lags used: 21

```
[1] 7
[1] 10
```

Ljung-Box test

data: Residuals from ARIMA(7,0,11) with non-zero mean
Q* = 4.5045, df = 3, p-value = 0.2119

Model df: 19. Total lags used: 22

```
[1] 7
[1] 11
```

Ljung-Box test

data: Residuals from ARIMA(7,0,12) with non-zero mean
Q* = 6.9788, df = 3, p-value = 0.07258

Model df: 20. Total lags used: 23

```
[1] 7
[1] 12
```

Ljung-Box test

data: Residuals from ARIMA(8,0,0) with non-zero mean
Q* = 10.692, df = 3, p-value = 0.01351

Model df: 9. Total lags used: 12

```
[1] 8
[1] 0
```

Ljung-Box test

data: Residuals from ARIMA(8,0,1) with non-zero mean
Q* = 11.396, df = 3, p-value = 0.009768

Model df: 10. Total lags used: 13

[1] 8

[1] 1

Ljung-Box test

data: Residuals from ARIMA(8,0,2) with non-zero mean
Q* = 4.4688, df = 3, p-value = 0.2151

Model df: 11. Total lags used: 14

[1] 8

[1] 2

Ljung-Box test

data: Residuals from ARIMA(8,0,3) with non-zero mean
Q* = 5.6325, df = 3, p-value = 0.1309

Model df: 12. Total lags used: 15

[1] 8

[1] 3

Ljung-Box test

data: Residuals from ARIMA(8,0,4) with non-zero mean
Q* = 6.7948, df = 3, p-value = 0.07873

Model df: 13. Total lags used: 16

[1] 8

[1] 4

Ljung-Box test

data: Residuals from ARIMA(8,0,5) with non-zero mean
Q* = 7.7366, df = 3, p-value = 0.05178

Model df: 14. Total lags used: 17

[1] 8

[1] 5

Ljung-Box test

data: Residuals from ARIMA(8,0,6) with non-zero mean

Q* = 7.8032, df = 3, p-value = 0.05026

Model df: 15. Total lags used: 18

[1] 8

[1] 6

Ljung-Box test

data: Residuals from ARIMA(8,0,7) with non-zero mean

Q* = 6.2029, df = 3, p-value = 0.1021

Model df: 16. Total lags used: 19

[1] 8

[1] 7

Ljung-Box test

data: Residuals from ARIMA(8,0,8) with non-zero mean

Q* = 5.6357, df = 3, p-value = 0.1307

Model df: 17. Total lags used: 20

[1] 8

[1] 8

Ljung-Box test

data: Residuals from ARIMA(8,0,9) with non-zero mean

Q* = 5.7522, df = 3, p-value = 0.1243

Model df: 18. Total lags used: 21

[1] 8

[1] 9

Ljung-Box test

data: Residuals from ARIMA(8,0,10) with non-zero mean

Q* = 6.0236, df = 3, p-value = 0.1105

Model df: 19. Total lags used: 22

[1] 8

[1] 10

Ljung-Box test

data: Residuals from ARIMA(8,0,11) with non-zero mean
Q* = 6.4409, df = 3, p-value = 0.09202

Model df: 20. Total lags used: 23

[1] 8
[1] 11

Ljung-Box test

data: Residuals from ARIMA(8,0,12) with non-zero mean
Q* = 7.2332, df = 3, p-value = 0.06482

Model df: 21. Total lags used: 24

[1] 8
[1] 12

Ljung-Box test

data: Residuals from ARIMA(9,0,0) with non-zero mean
Q* = 9.6361, df = 3, p-value = 0.02193

Model df: 10. Total lags used: 13

[1] 9
[1] 0

Ljung-Box test

data: Residuals from ARIMA(9,0,1) with non-zero mean
Q* = 9.5763, df = 3, p-value = 0.02253

Model df: 11. Total lags used: 14

[1] 9
[1] 1

Ljung-Box test

data: Residuals from ARIMA(9,0,2) with non-zero mean
Q* = 6.2767, df = 3, p-value = 0.0989

Model df: 12. Total lags used: 15

[1] 9
[1] 2

Ljung-Box test

data: Residuals from ARIMA(9,0,3) with non-zero mean
Q* = 6.9525, df = 3, p-value = 0.07343

Model df: 13. Total lags used: 16

[1] 9

[1] 3

Ljung-Box test

data: Residuals from ARIMA(9,0,4) with non-zero mean
Q* = 7.6059, df = 3, p-value = 0.0549

Model df: 14. Total lags used: 17

[1] 9

[1] 4

Ljung-Box test

data: Residuals from ARIMA(9,0,5) with non-zero mean
Q* = 7.426, df = 3, p-value = 0.05949

Model df: 15. Total lags used: 18

[1] 9

[1] 5

Ljung-Box test

data: Residuals from ARIMA(9,0,6) with non-zero mean
Q* = 8.4429, df = 3, p-value = 0.03769

Model df: 16. Total lags used: 19

[1] 9

[1] 6

Ljung-Box test

data: Residuals from ARIMA(9,0,7) with non-zero mean
Q* = 8.9644, df = 3, p-value = 0.02977

Model df: 17. Total lags used: 20

```
[1] 9
[1] 7
```

Ljung-Box test

data: Residuals from ARIMA(9,0,8) with non-zero mean
Q* = 6.4546, df = 3, p-value = 0.09147

Model df: 18. Total lags used: 21

```
[1] 9
[1] 8
```

Ljung-Box test

data: Residuals from ARIMA(9,0,9) with non-zero mean
Q* = 6.4424, df = 3, p-value = 0.09196

Model df: 19. Total lags used: 22

```
[1] 9
[1] 9
```

Ljung-Box test

data: Residuals from ARIMA(9,0,10) with non-zero mean
Q* = 9.5214, df = 3, p-value = 0.0231

Model df: 20. Total lags used: 23

```
[1] 9
[1] 10
```

Ljung-Box test

data: Residuals from ARIMA(9,0,11) with non-zero mean
Q* = 8.0008, df = 3, p-value = 0.04599

Model df: 21. Total lags used: 24

```
[1] 9
[1] 11
```

Ljung-Box test

data: Residuals from ARIMA(9,0,12) with non-zero mean
Q* = 7.8222, df = 3, p-value = 0.04983

Model df: 22. Total lags used: 25

[1] 9
[1] 12

Ljung-Box test

data: Residuals from ARIMA(10,0,0) with non-zero mean
Q* = 9.366, df = 3, p-value = 0.0248

Model df: 11. Total lags used: 14

[1] 10
[1] 0

Ljung-Box test

data: Residuals from ARIMA(10,0,1) with non-zero mean
Q* = 10.023, df = 3, p-value = 0.01837

Model df: 12. Total lags used: 15

[1] 10
[1] 1

Ljung-Box test

data: Residuals from ARIMA(10,0,2) with non-zero mean
Q* = 6.9788, df = 3, p-value = 0.07258

Model df: 13. Total lags used: 16

[1] 10
[1] 2

Ljung-Box test

data: Residuals from ARIMA(10,0,3) with non-zero mean
Q* = 6.973, df = 3, p-value = 0.07276

Model df: 14. Total lags used: 17

[1] 10
[1] 3

Ljung-Box test

data: Residuals from ARIMA(10,0,4) with non-zero mean

Q* = 7.3372, df = 3, p-value = 0.06189

Model df: 15. Total lags used: 18

[1] 10

[1] 4

Ljung-Box test

data: Residuals from ARIMA(10,0,5) with non-zero mean

Q* = 7.7591, df = 3, p-value = 0.05126

Model df: 16. Total lags used: 19

[1] 10

[1] 5

Ljung-Box test

data: Residuals from ARIMA(10,0,6) with non-zero mean

Q* = 9.4753, df = 3, p-value = 0.0236

Model df: 17. Total lags used: 20

[1] 10

[1] 6

Ljung-Box test

data: Residuals from ARIMA(10,0,7) with non-zero mean

Q* = 6.4455, df = 3, p-value = 0.09184

Model df: 18. Total lags used: 21

[1] 10

[1] 7

Ljung-Box test

data: Residuals from ARIMA(10,0,8) with non-zero mean

Q* = 6.4682, df = 3, p-value = 0.09093

Model df: 19. Total lags used: 22

[1] 10

[1] 8

Ljung-Box test

data: Residuals from ARIMA(10,0,9) with non-zero mean
Q* = 6.5276, df = 3, p-value = 0.08858

Model df: 20. Total lags used: 23

[1] 10
[1] 9

Ljung-Box test

data: Residuals from ARIMA(10,0,10) with non-zero mean
Q* = 7.2716, df = 3, p-value = 0.06373

Model df: 21. Total lags used: 24

[1] 10
[1] 10

Ljung-Box test

data: Residuals from ARIMA(10,0,11) with non-zero mean
Q* = 7.7163, df = 3, p-value = 0.05225

Model df: 22. Total lags used: 25

[1] 10
[1] 11

Ljung-Box test

data: Residuals from ARIMA(10,0,12) with non-zero mean
Q* = 7.9577, df = 3, p-value = 0.04689

Model df: 23. Total lags used: 26

[1] 10
[1] 12

Ljung-Box test

data: Residuals from ARIMA(11,0,0) with non-zero mean
Q* = 6.8659, df = 3, p-value = 0.0763

Model df: 12. Total lags used: 15

[1] 11
[1] 0

Ljung-Box test

data: Residuals from ARIMA(11,0,1) with non-zero mean
Q* = 4.1808, df = 3, p-value = 0.2426

Model df: 13. Total lags used: 16

[1] 11

[1] 1

Ljung-Box test

data: Residuals from ARIMA(11,0,2) with non-zero mean
Q* = 3.7127, df = 3, p-value = 0.2942

Model df: 14. Total lags used: 17

[1] 11

[1] 2

Ljung-Box test

data: Residuals from ARIMA(11,0,3) with non-zero mean
Q* = 5.4814, df = 3, p-value = 0.1398

Model df: 15. Total lags used: 18

[1] 11

[1] 3

Ljung-Box test

data: Residuals from ARIMA(11,0,4) with non-zero mean
Q* = 8.555, df = 3, p-value = 0.03583

Model df: 16. Total lags used: 19

[1] 11

[1] 4

Ljung-Box test

data: Residuals from ARIMA(11,0,5) with non-zero mean
Q* = 8.6775, df = 3, p-value = 0.0339

Model df: 17. Total lags used: 20

```
[1] 11
[1] 5
```

Ljung-Box test

data: Residuals from ARIMA(11,0,6) with non-zero mean
Q* = 7.489, df = 3, p-value = 0.05784

Model df: 18. Total lags used: 21

```
[1] 11
[1] 6
```

Ljung-Box test

data: Residuals from ARIMA(11,0,7) with non-zero mean
Q* = 9.1081, df = 3, p-value = 0.02789

Model df: 19. Total lags used: 22

```
[1] 11
[1] 7
```

Ljung-Box test

data: Residuals from ARIMA(11,0,8) with non-zero mean
Q* = 6.9375, df = 3, p-value = 0.07392

Model df: 20. Total lags used: 23

```
[1] 11
[1] 8
```

Ljung-Box test

data: Residuals from ARIMA(11,0,9) with non-zero mean
Q* = 7.3688, df = 3, p-value = 0.06103

Model df: 21. Total lags used: 24

```
[1] 11
[1] 9
```

Ljung-Box test

data: Residuals from ARIMA(11,0,10) with non-zero mean
Q* = 7.6552, df = 3, p-value = 0.0537

Model df: 22. Total lags used: 25

[1] 11

[1] 10

Ljung-Box test

data: Residuals from ARIMA(11,0,11) with non-zero mean
Q* = 7.6936, df = 3, p-value = 0.05279

Model df: 23. Total lags used: 26

[1] 11

[1] 11

Ljung-Box test

data: Residuals from ARIMA(11,0,12) with non-zero mean
Q* = 7.6684, df = 3, p-value = 0.05339

Model df: 24. Total lags used: 27

[1] 11

[1] 12

Ljung-Box test

data: Residuals from ARIMA(12,0,0) with non-zero mean
Q* = 2.8841, df = 3, p-value = 0.4098

Model df: 13. Total lags used: 16

[1] 12

[1] 0

Ljung-Box test

data: Residuals from ARIMA(12,0,1) with non-zero mean
Q* = 2.8625, df = 3, p-value = 0.4133

Model df: 14. Total lags used: 17

[1] 12

[1] 1

Ljung-Box test

data: Residuals from ARIMA(12,0,2) with non-zero mean

Q* = 4.7897, df = 3, p-value = 0.1879

Model df: 15. Total lags used: 18

[1] 12

[1] 2

Ljung-Box test

data: Residuals from ARIMA(12,0,3) with non-zero mean

Q* = 3.0764, df = 3, p-value = 0.38

Model df: 16. Total lags used: 19

[1] 12

[1] 3

Ljung-Box test

data: Residuals from ARIMA(12,0,4) with non-zero mean

Q* = 3.8084, df = 3, p-value = 0.2829

Model df: 17. Total lags used: 20

[1] 12

[1] 4

Ljung-Box test

data: Residuals from ARIMA(12,0,5) with non-zero mean

Q* = 7.487, df = 3, p-value = 0.05789

Model df: 18. Total lags used: 21

[1] 12

[1] 5

Ljung-Box test

data: Residuals from ARIMA(12,0,6) with non-zero mean

Q* = 5.6993, df = 3, p-value = 0.1272

Model df: 19. Total lags used: 22

[1] 12

[1] 6

Ljung-Box test

```
data: Residuals from ARIMA(12,0,7) with non-zero mean
Q* = 7.2965, df = 3, p-value = 0.06302
```

```
Model df: 20.    Total lags used: 23
```

```
[1] 12
[1] 7
```

Ljung-Box test

```
data: Residuals from ARIMA(12,0,8) with non-zero mean
Q* = 6.8322, df = 3, p-value = 0.07744
```

```
Model df: 21.    Total lags used: 24
```

```
[1] 12
[1] 8
```

Ljung-Box test

```
data: Residuals from ARIMA(12,0,9) with non-zero mean
Q* = 7.5014, df = 3, p-value = 0.05752
```

```
Model df: 22.    Total lags used: 25
```

```
[1] 12
[1] 9
```

```
Error in optim(init[mask], armafn, method = optim.method, hessian = _
↪TRUE, : non-finite finite-difference value [13]
Traceback:
```

```
1. Arima(prod_train, order = c(i, 0, j))

2. suppressWarnings(tmp <- stats::arima(x = x, order = order, seasonal = _
↪seasonal,
.    include.mean = include.mean, method = method, ...))

3. withCallingHandlers(expr, warning = function(w) if (inherits(w,
.    classes)) tryInvokeRestart("muffleWarning"))

4. stats::arima(x = x, order = order, seasonal = seasonal, include.mean _
↪= include.mean,
.    method = method, ...)
```

```

5. optim(init[mask], armafn, method = optim.method, hessian = TRUE,
.      control = optim.control, trans = as.logical(transform.pars))

586.257014293105

31

4.23076923076923

3

Series: prod_train
ARIMA(4,0,2) with non-zero mean

Coefficients:
          ar1          ar2          ar3          ar4          ma1          ma2          mean
      0.4844  -0.9006   0.7108  -0.2733   0.2332   0.9898   0.5134
s.e.  0.0723   0.0657   0.0607   0.0739   0.0166   0.1257   0.1889

sigma^2 estimated as 1.325:  log likelihood=-284.61
AIC=585.23   AICc=586.06   BIC=610.91

44

3.38461538461538

4

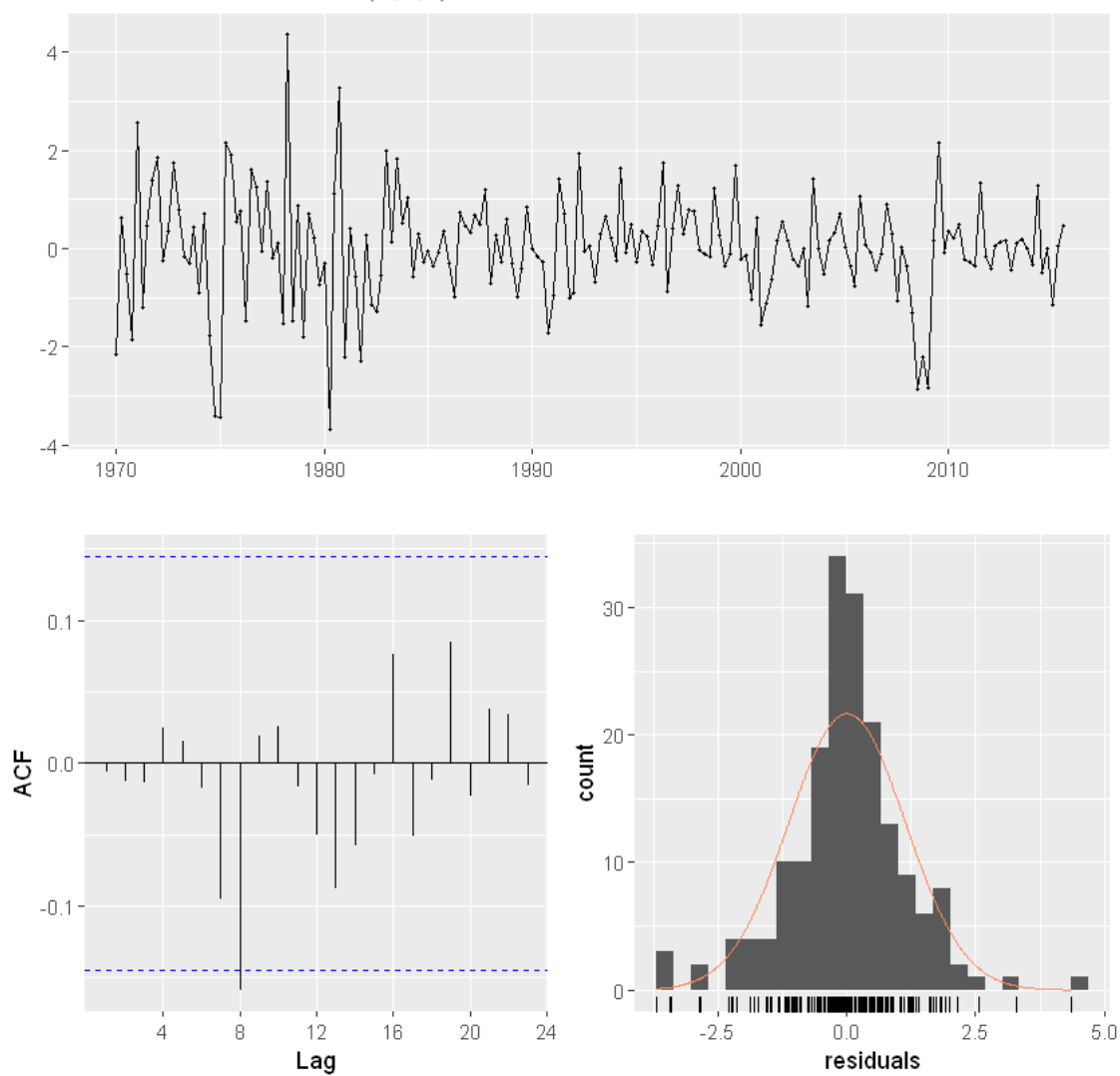
Ljung-Box test

data:  Residuals from ARIMA(4,0,2) with non-zero mean
Q* = 7.1133, df = 3, p-value = 0.06837

Model df: 7.    Total lags used: 10

```

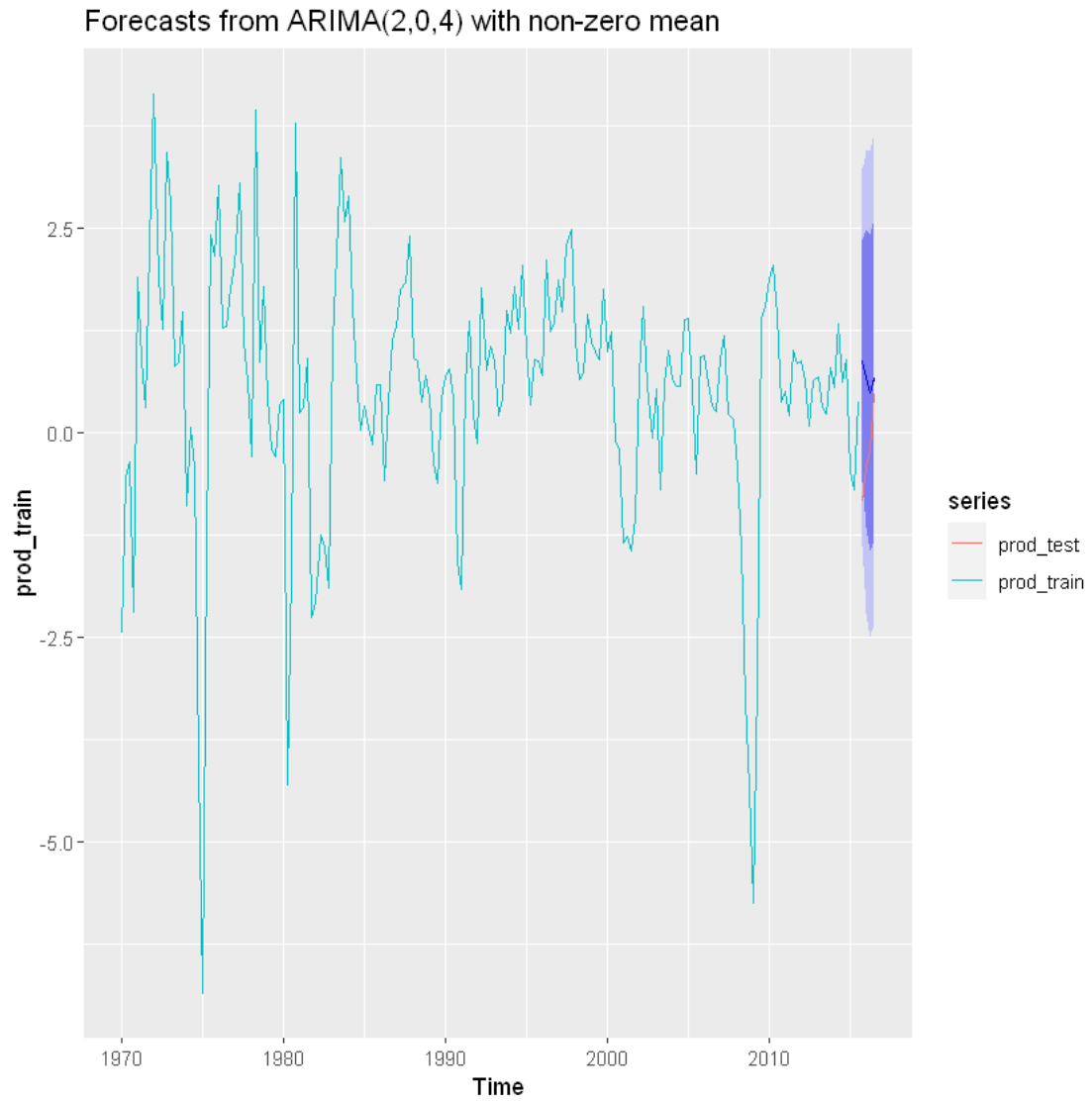
Residuals from ARIMA(4,0,2) with non-zero mean



0.068374297397761

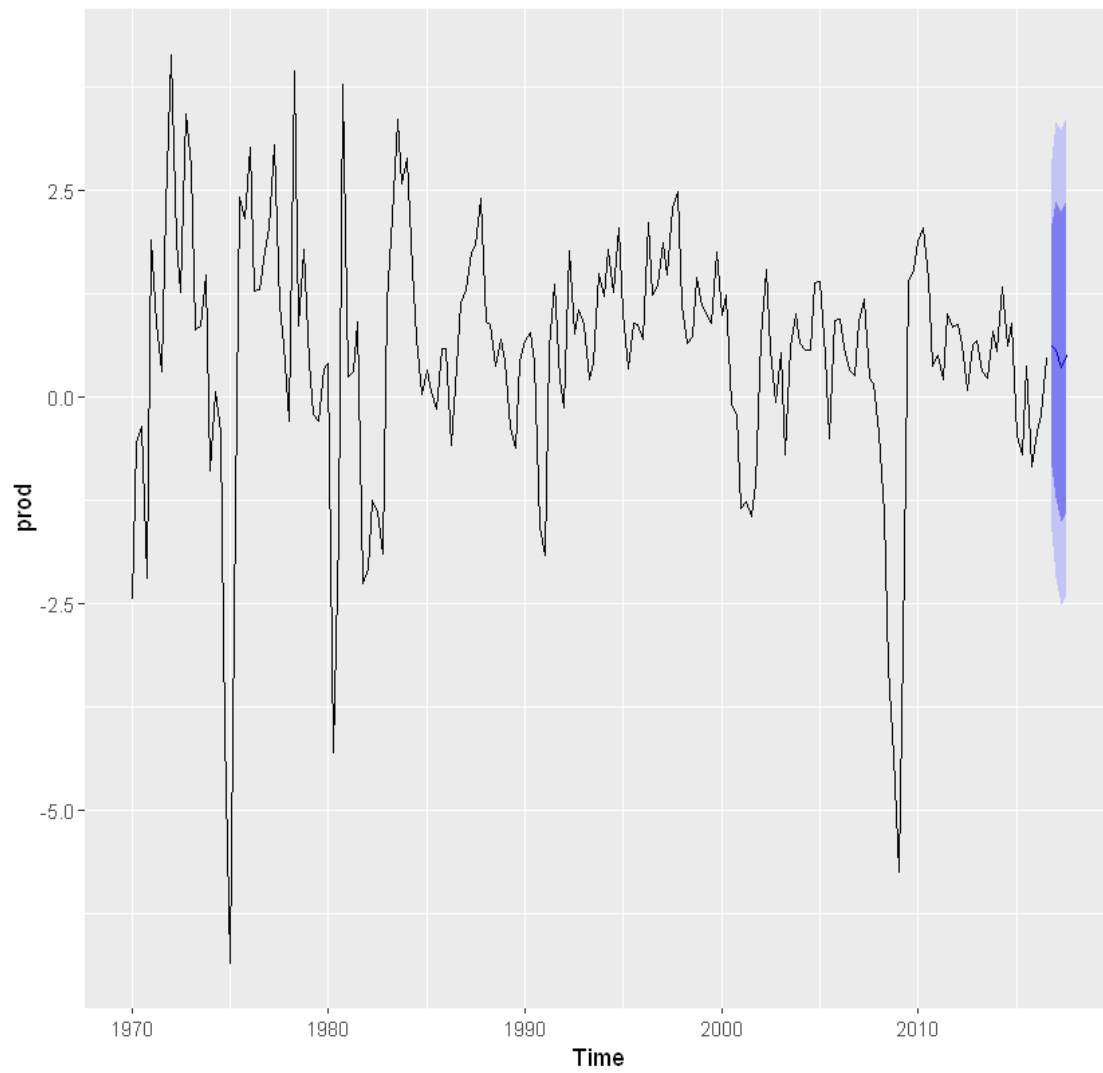
1.16855199109931

1.17456497199915

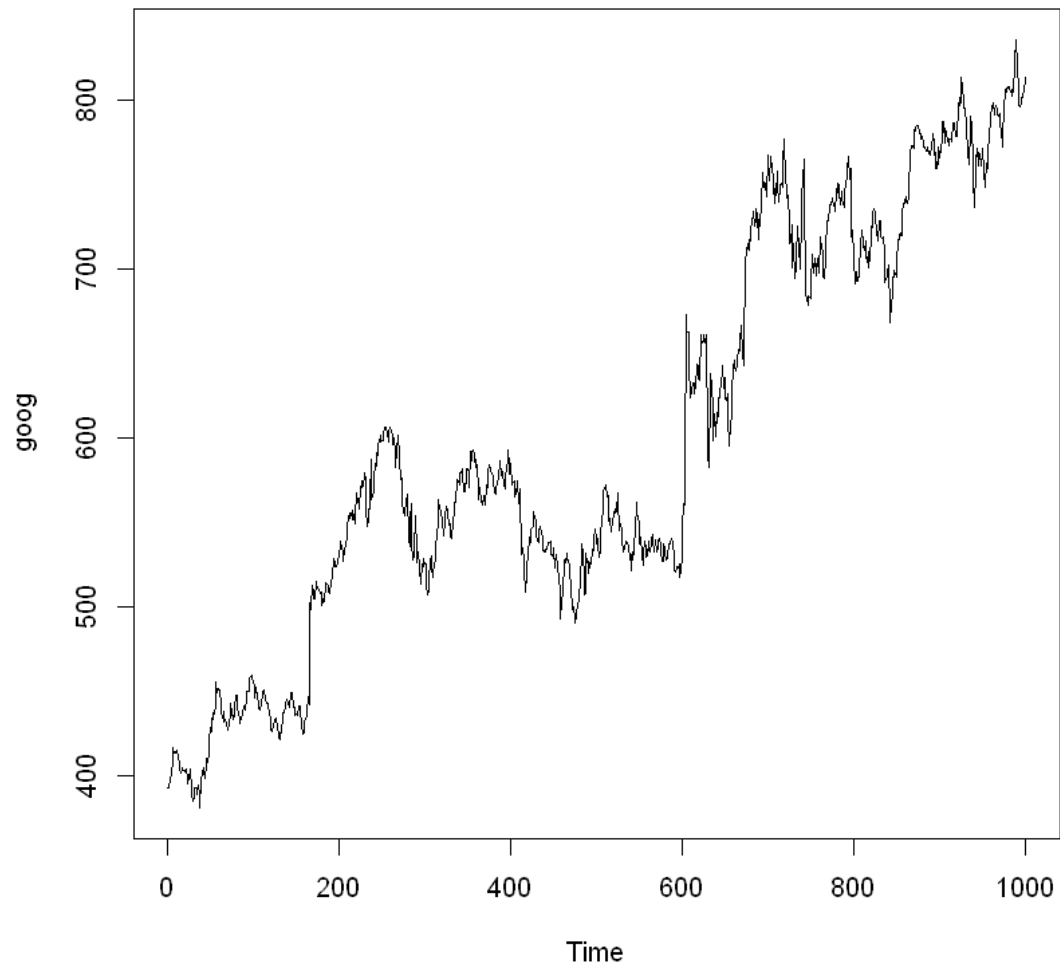


The best model is ARMA(2,0,4). We use it to plot 2017

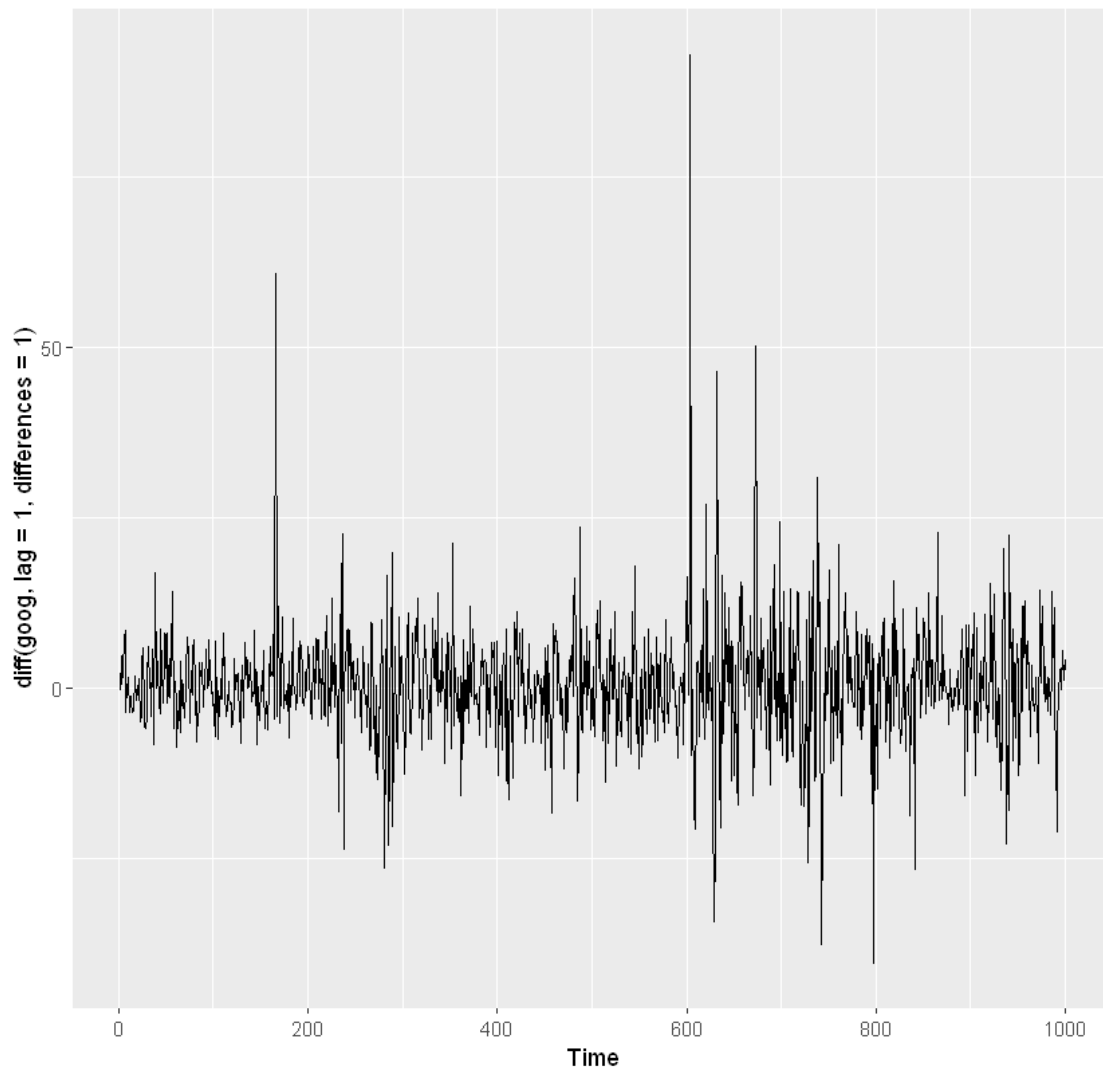
Forecasts from ARIMA(2,0,4) with non-zero mean



1.4.9 Exercise closing stock price of Google



First we remove the trend



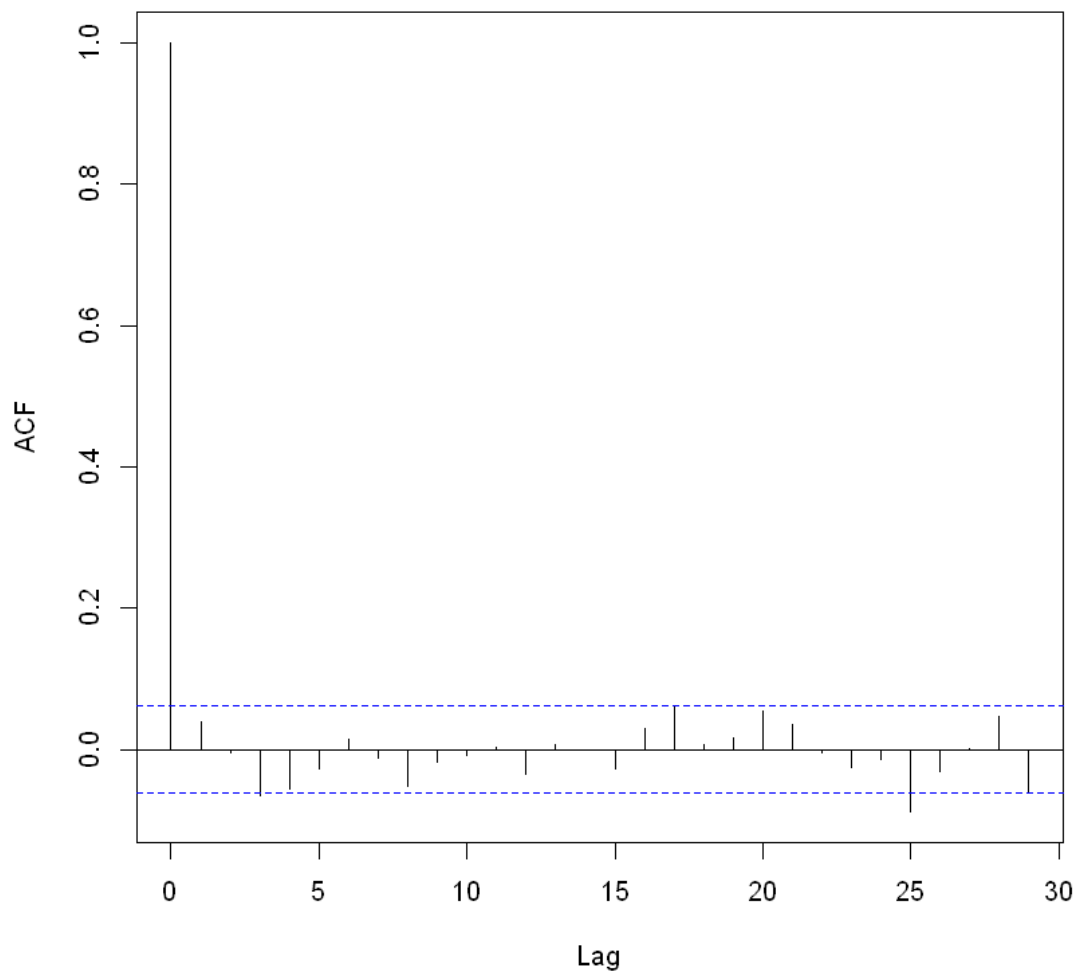
No there is no more trend, we can check if we have more than a white noise ?

Box-Ljung test

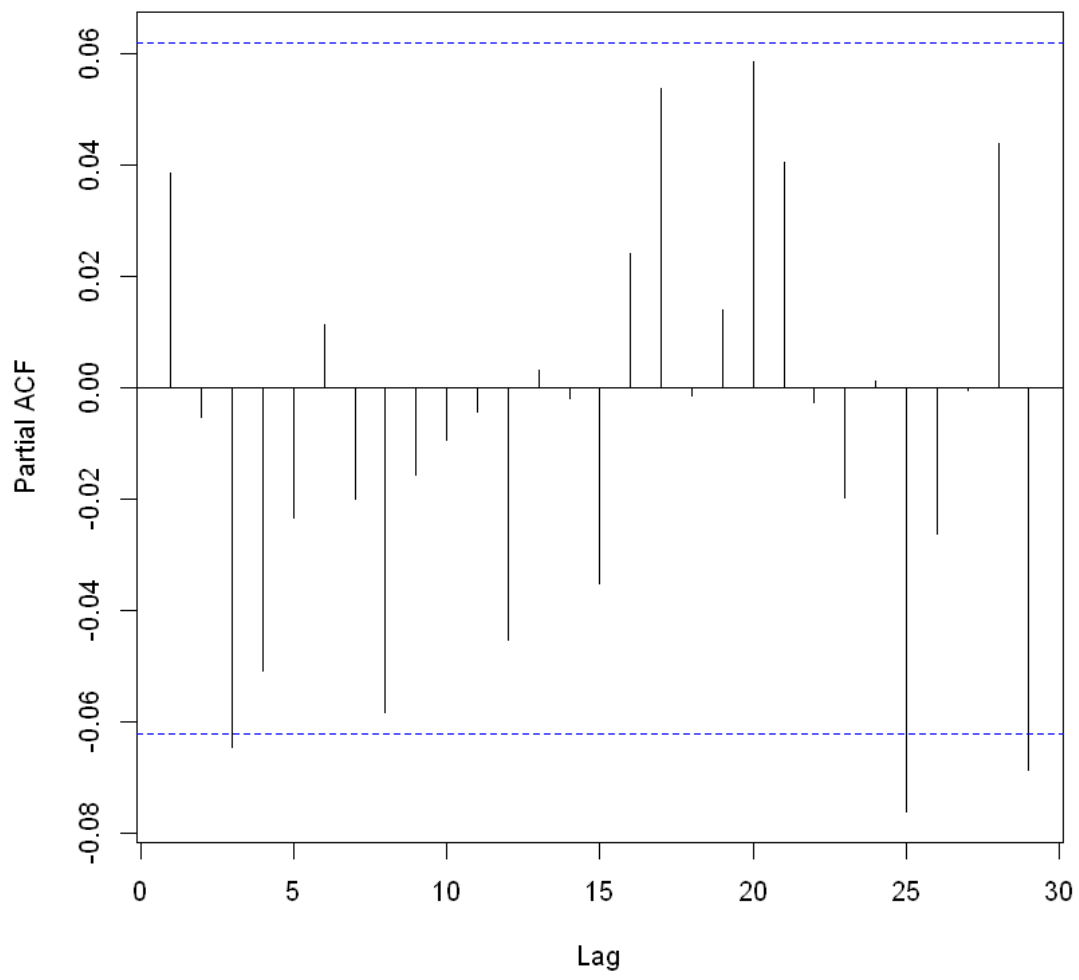
```
data: goog_diff  
X-squared = 13.123, df = 10, p-value = 0.2169
```

pvalue is > 0.05 , we reject the fact that this is white noise. Confirm with autocorrelation

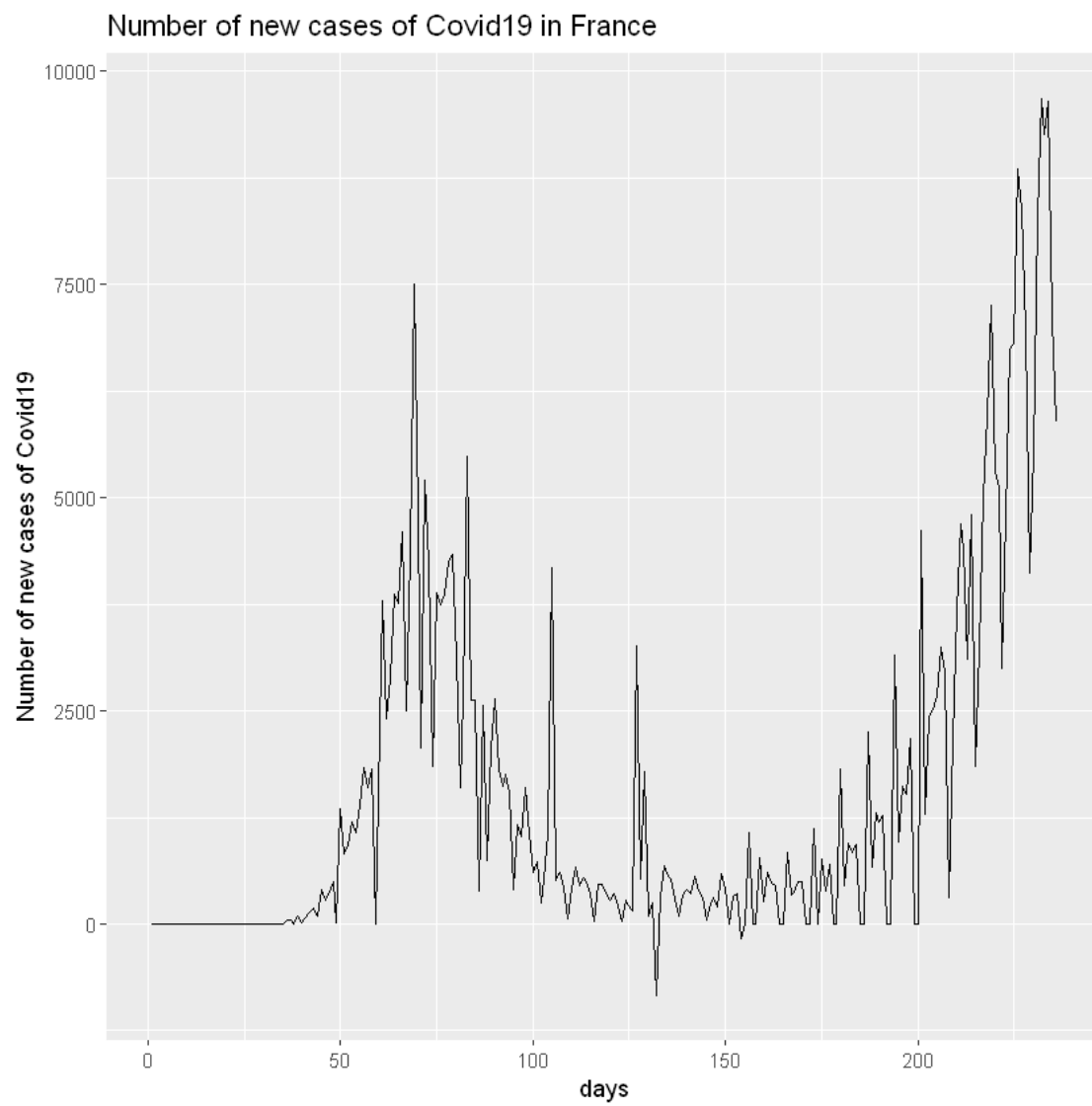
Series goog_diff



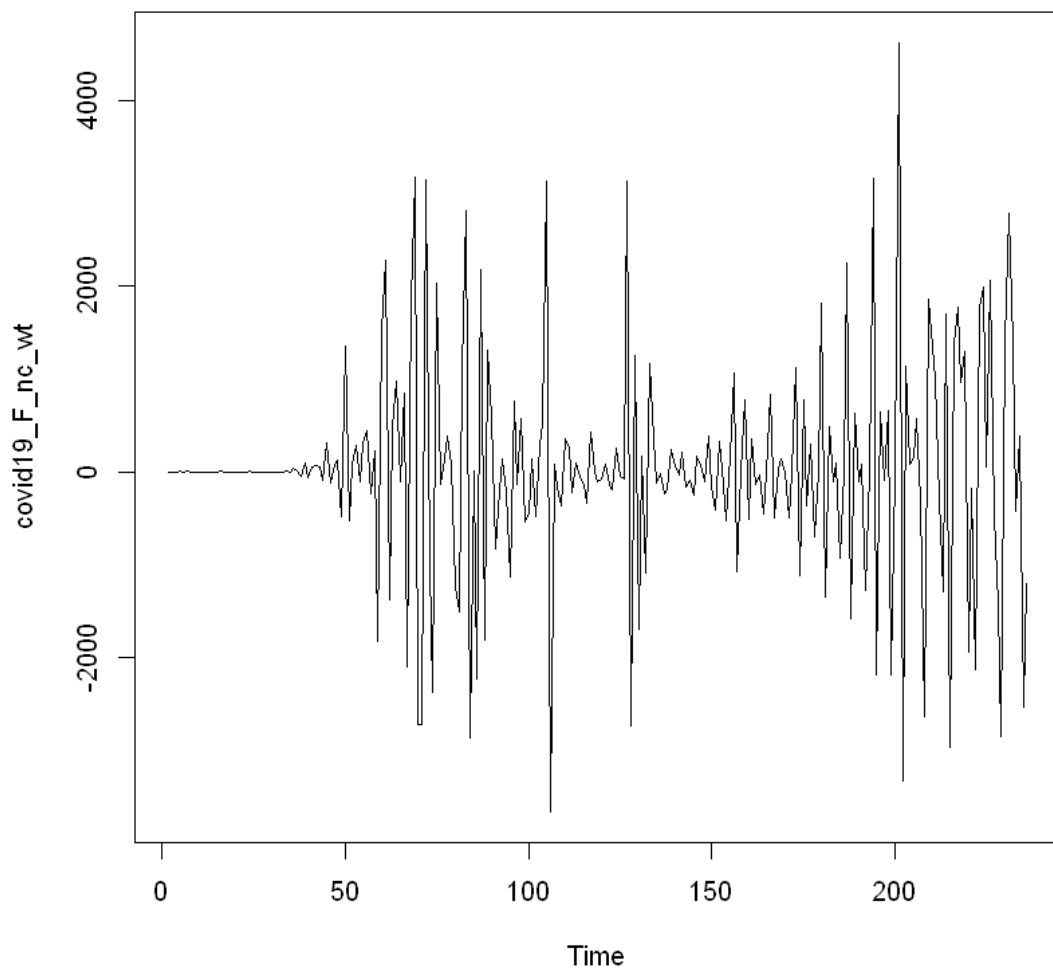
Series goog_diff



1.4.10 Exercice with Covid Data



Remove the trend



Test if it is white noise ?

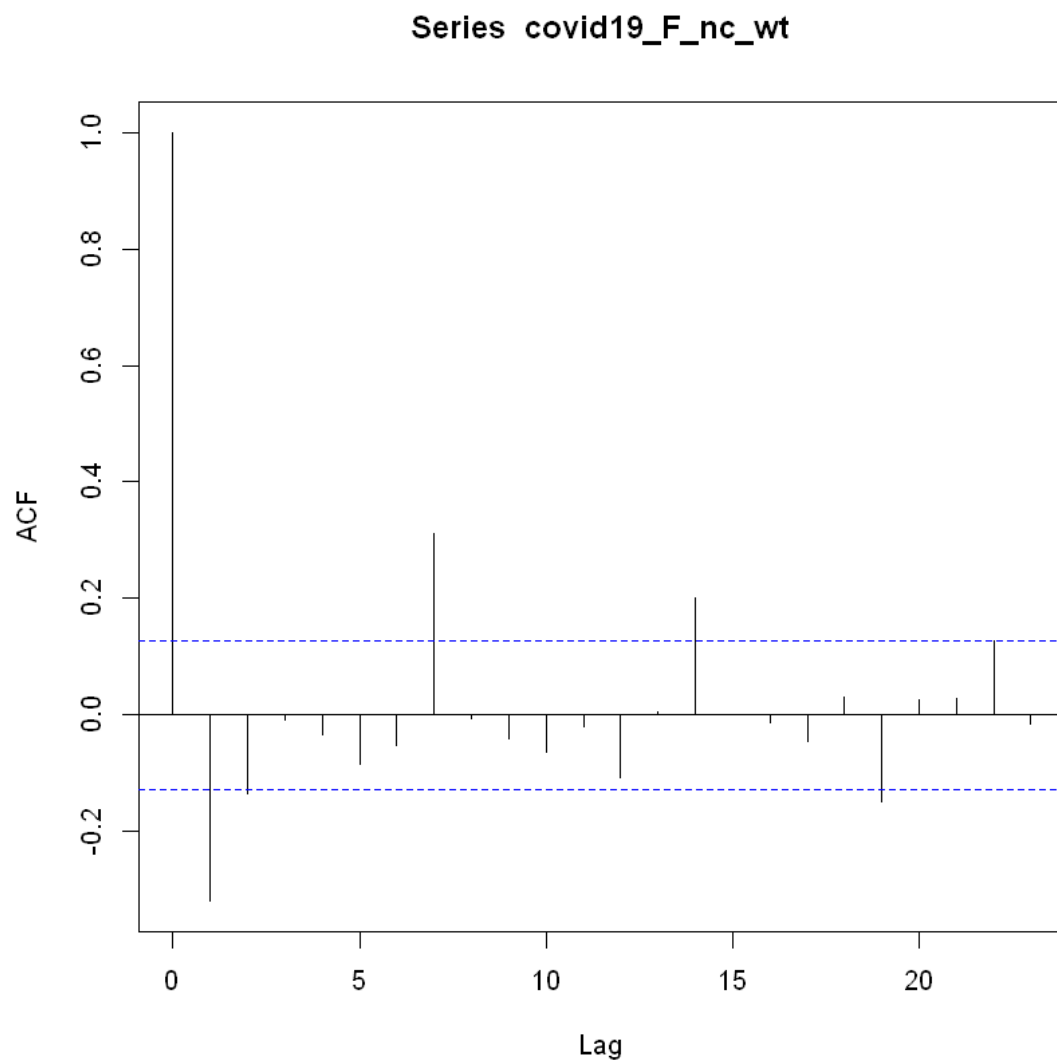
Box-Ljung test

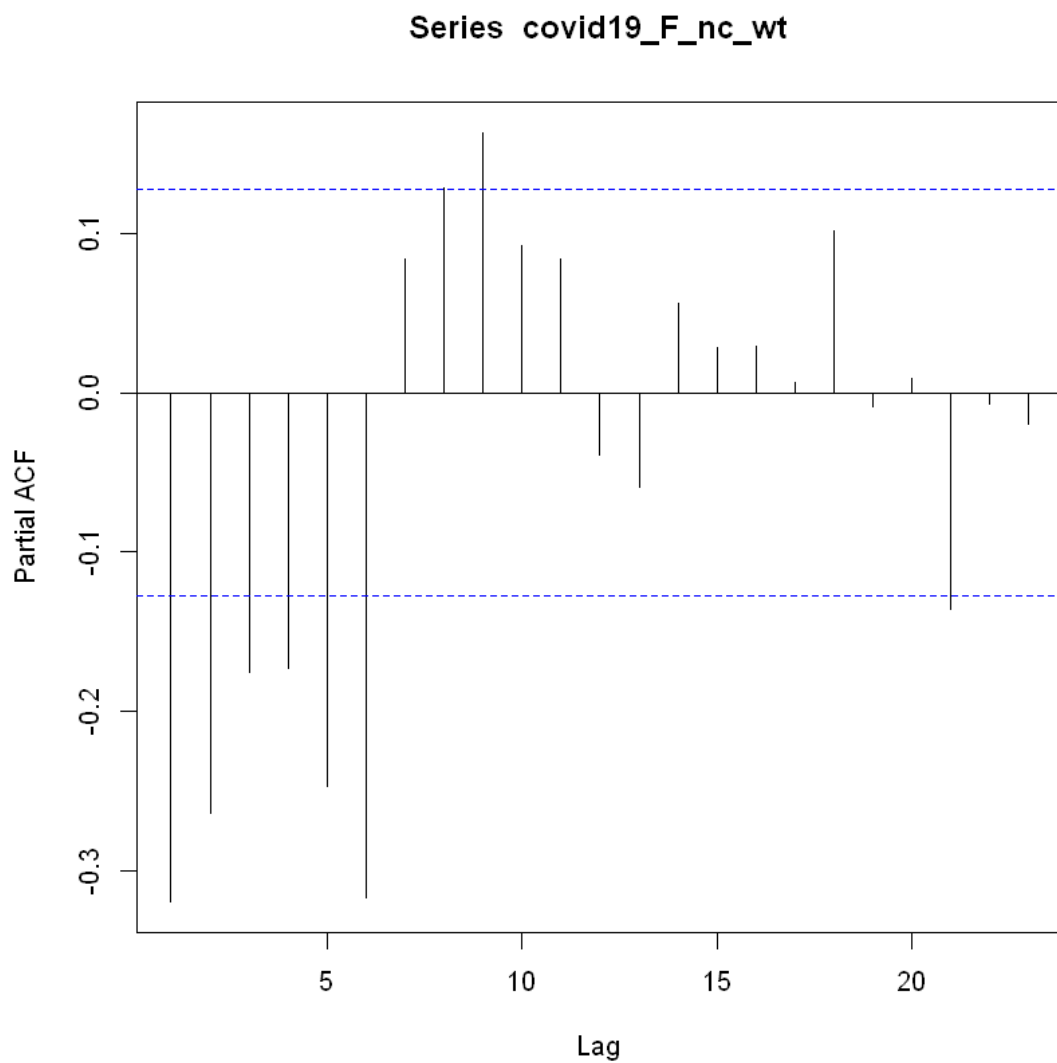
data: covid19_F_nc_wt
X-squared = 56.421, df = 10, p-value = 1.712e-08

Box-Ljung test

data: covid19_F_nd_wt
X-squared = 113.52, df = 10, p-value < 2.2e-16

Not white noise : look for ARMA function





Ljung-Box test

data: Residuals from ARIMA(0,0,0) with non-zero mean
Q* = 56.421, df = 9, p-value = 6.522e-09

Model df: 1. Total lags used: 10

[1] 0

[1] 0

Ljung-Box test

data: Residuals from ARIMA(0,0,1) with non-zero mean
Q* = 75.269, df = 8, p-value = 4.357e-13

Model df: 2. Total lags used: 10

[1] 0
[1] 1

Ljung-Box test

data: Residuals from ARIMA(0,0,2) with non-zero mean
Q* = 48.93, df = 7, p-value = 2.341e-08

Model df: 3. Total lags used: 10

[1] 0
[1] 2

Ljung-Box test

data: Residuals from ARIMA(1,0,0) with non-zero mean
Q* = 65.911, df = 8, p-value = 3.186e-11

Model df: 2. Total lags used: 10

[1] 1
[1] 0

Ljung-Box test

data: Residuals from ARIMA(1,0,1) with non-zero mean
Q* = 52.496, df = 7, p-value = 4.66e-09

Model df: 3. Total lags used: 10

[1] 1
[1] 1

Ljung-Box test

data: Residuals from ARIMA(1,0,2) with non-zero mean
Q* = 51.041, df = 6, p-value = 2.906e-09

Model df: 4. Total lags used: 10

[1] 1
[1] 2

Ljung-Box test

data: Residuals from ARIMA(2,0,0) with non-zero mean
Q* = 75.15, df = 7, p-value = 1.337e-13

Model df: 3. Total lags used: 10

[1] 2
[1] 0

Ljung-Box test

data: Residuals from ARIMA(2,0,1) with non-zero mean
Q* = 50.513, df = 6, p-value = 3.71e-09

Model df: 4. Total lags used: 10

[1] 2
[1] 1

Ljung-Box test

data: Residuals from ARIMA(2,0,2) with non-zero mean
Q* = 22.523, df = 5, p-value = 0.0004164

Model df: 5. Total lags used: 10

[1] 2
[1] 2

Ljung-Box test

data: Residuals from ARIMA(3,0,0) with non-zero mean
Q* = 74.364, df = 6, p-value = 5.185e-14

Model df: 4. Total lags used: 10

[1] 3
[1] 0

Ljung-Box test

data: Residuals from ARIMA(3,0,1) with non-zero mean
Q* = 46.237, df = 5, p-value = 8.126e-09

Model df: 5. Total lags used: 10

[1] 3

[1] 1

Ljung-Box test

data: Residuals from ARIMA(3,0,2) with non-zero mean
Q* = 43.893, df = 4, p-value = 6.754e-09

Model df: 6. Total lags used: 10

[1] 3

[1] 2

Ljung-Box test

data: Residuals from ARIMA(4,0,0) with non-zero mean
Q* = 61.052, df = 5, p-value = 7.365e-12

Model df: 5. Total lags used: 10

[1] 4

[1] 0

Ljung-Box test

data: Residuals from ARIMA(4,0,1) with non-zero mean
Q* = 42.159, df = 4, p-value = 1.546e-08

Model df: 6. Total lags used: 10

[1] 4

[1] 1

Ljung-Box test

data: Residuals from ARIMA(4,0,2) with non-zero mean
Q* = 16.671, df = 3, p-value = 0.0008258

Model df: 7. Total lags used: 10

[1] 4

[1] 2

Ljung-Box test

data: Residuals from ARIMA(5,0,0) with non-zero mean
Q* = 39.791, df = 4, p-value = 4.781e-08

Model df: 6. Total lags used: 10

```
[1] 5
[1] 0
```

Ljung-Box test

data: Residuals from ARIMA(5,0,1) with non-zero mean
Q* = 36.358, df = 3, p-value = 6.29e-08

Model df: 7. Total lags used: 10

```
[1] 5
[1] 1
```

```
Error in stats::arima(x = x, order = order, seasonal = seasonal, include.
↪mean = include.mean, : non-stationary AR part from CSS
```

Traceback:

```
1. Arima(covid19_F_nc_wt, order = c(i, 0, j))

2. suppressWarnings(tmp <- stats::arima(x = x, order = order, seasonal = ↪
↪seasonal,
.   include.mean = include.mean, method = method, ...))

3. withCallingHandlers(expr, warning = function(w) if (inherits(w,
.   classes)) tryInvokeRestart("muffleWarning"))

4. stats::arima(x = x, order = order, seasonal = seasonal, include.mean ↪
↪= include.mean,
.   method = method, ...)

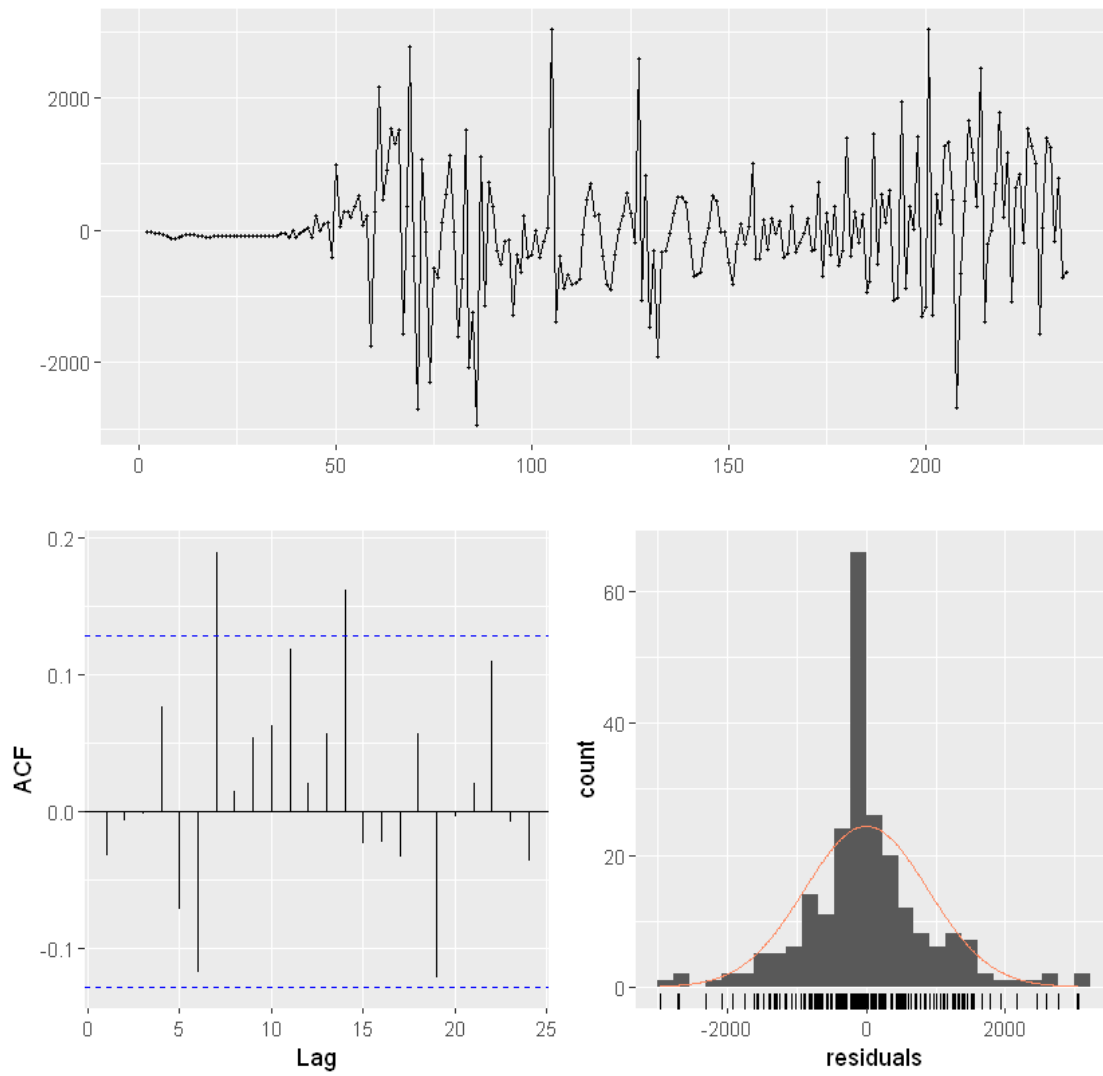
5. stop("non-stationary AR part from CSS")
```

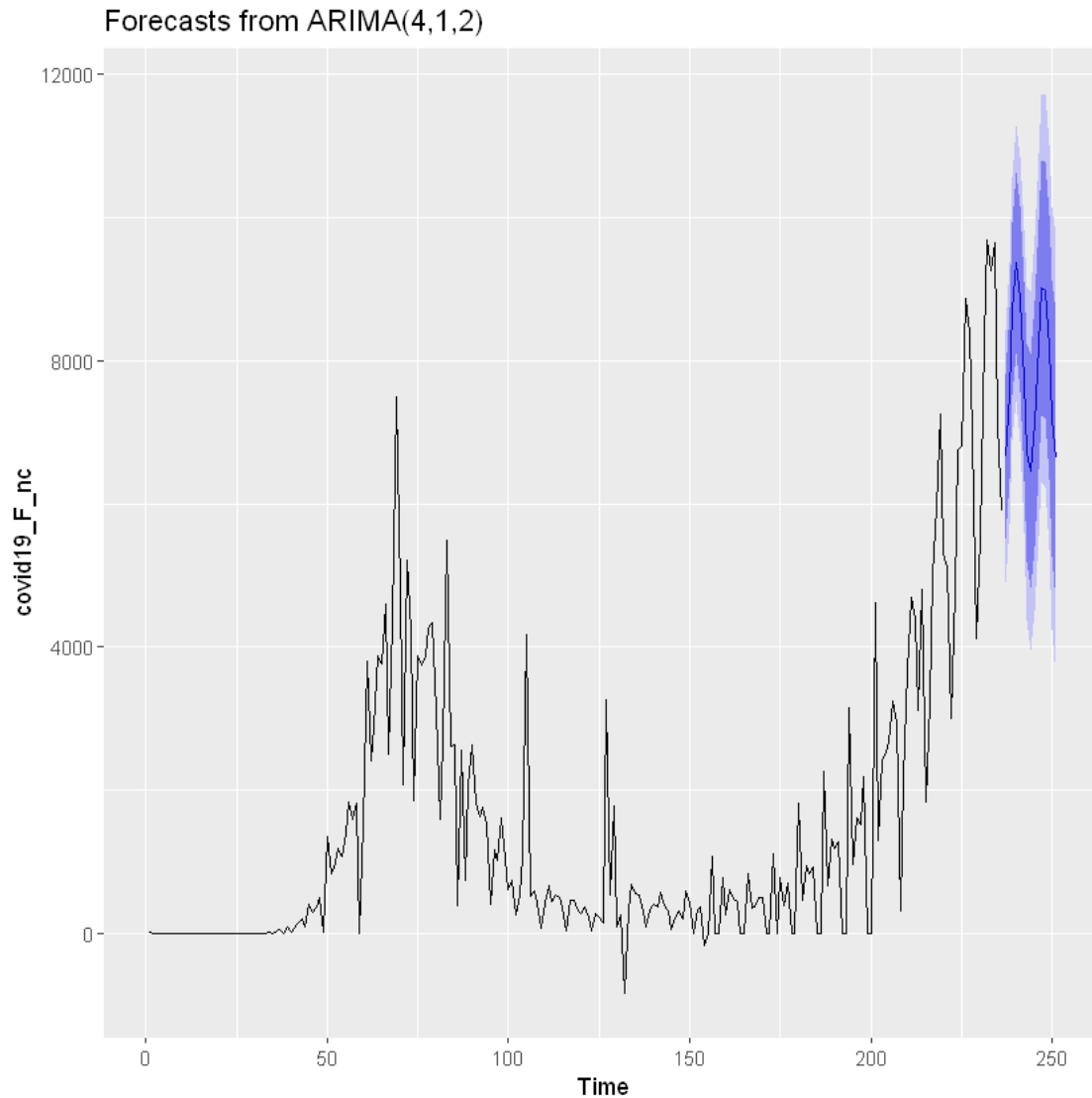
Ljung-Box test

data: Residuals from ARIMA(4,0,2) with non-zero mean
Q* = 16.671, df = 3, p-value = 0.0008258

Model df: 7. Total lags used: 10

Residuals from ARIMA(4,0,2) with non-zero mean



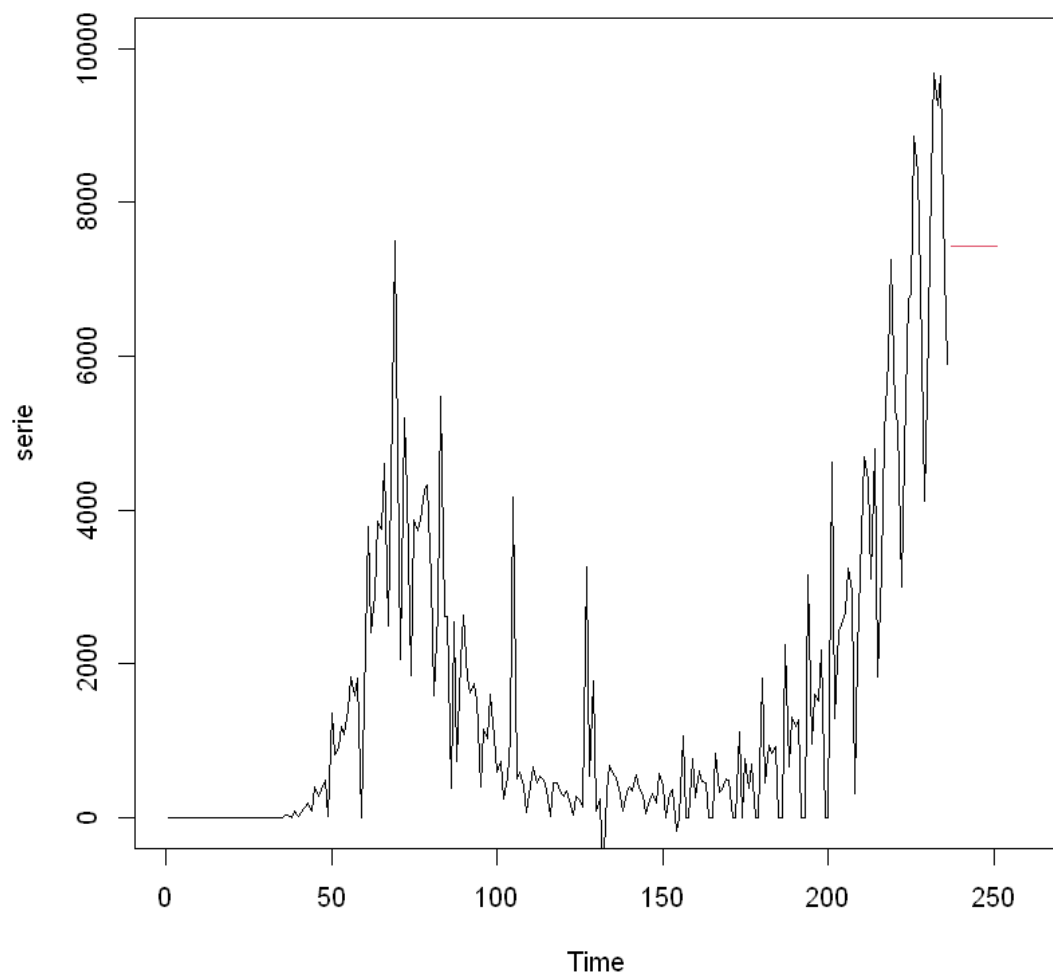


A Time Series:

1. 6657.19573342794 2. 7540.37804763402 3. 8717.75499634906 4. 9366.80986900823
 5. 8859.60746020283 6. 7717.02002410394 7. 6708.58029208465 8. 6454.05516867398
 9. 7115.24943292307 10. 8216.61817596264 11. 9007.77525932098 12. 8985.06655127728
 13. 8196.59869234538 14. 7192.15650569113 15. 6643.06764731932

with ARMA(4,1,2) : 6657.195733427947540.378047634028717.754996349069366.809869008238859.6

See the result with exponential smmothing



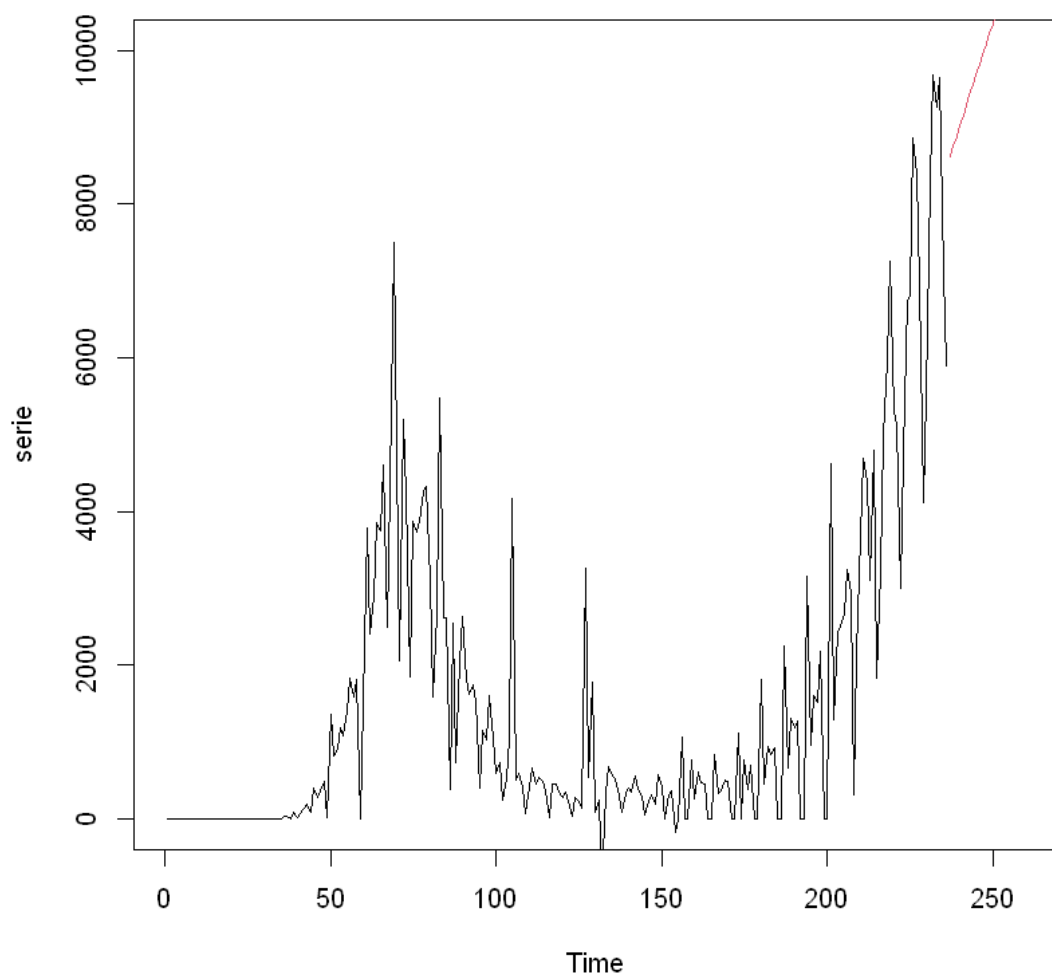
```

Time Series:
Start = 237
End = 251
Frequency = 1
fit
[1,] 7428.15
[2,] 7428.15
[3,] 7428.15
[4,] 7428.15
[5,] 7428.15
[6,] 7428.15
[7,] 7428.15
[8,] 7428.15

```

```
[9,] 7428.15  
[10,] 7428.15  
[11,] 7428.15  
[12,] 7428.15  
[13,] 7428.15  
[14,] 7428.15  
[15,] 7428.15
```

prediction : 7428



Time Series:
Start = 237
End = 251
Frequency = 1

```

fit
[1,] 8623.026
[2,] 8755.993
[3,] 8888.961
[4,] 9021.928
[5,] 9154.895
[6,] 9287.863
[7,] 9420.830
[8,] 9553.798
[9,] 9686.765
[10,] 9819.732
[11,] 9952.700
[12,] 10085.667
[13,] 10218.634
[14,] 10351.602
[15,] 10484.569

```

```

fit
[1,] 8623.026 [2,] 8755.993 [3,] 8888.961 [4,] 9021.928 [5,] 9154.895 [6,] 9287.863 [7,] 9420.830 [8,]
9553.798 [9,] 9686.765 [10,] 9819.732 [11,] 9952.700 [12,] 10085.667 [13,] 10218.634 [14,] 10351.602
[15,] 10484.569

```