Course Name

Assignment

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Problem1. Prove that $P = NP$.
Solution. In progress.

Remark. It's known as one of "The Seven Difficulties of Millennials".

Problem2. State the differential privacy.
Solution. A randomized algorithm $\mathcal{M}$ with domain $\mathbb{N}^{|\mathcal{X}|}$ is (ϵ, δ)-differential private if for all $\mathcal{S} \in \text{Range}(\mathcal{M})$ and for all $x, y \in \mathbb{N}^{|\mathcal{X}|}$ such that $||x - y||_1 \leq 1$:

$$\Pr[\mathcal{M}(x) \in \mathcal{S}] \leq \exp(\epsilon) \Pr[\mathcal{M}(y) \in \mathcal{S}] + \delta \tag{1}$$

Remark. In some cases, the δ is 0.

Problem3. Describe the Dijkstra algorithm.
Solution. Algorithm 1 presents the Dijkstra algorithm in pseudo code format. As it will take a graph G (weighted or unweighted) and query pair (s, t) where s represents the source and t is terminal as input. And compute the shortest distance $d(s, t)$. In Line 1, we firstly create the minimum heap $\mathcal{Q}$ which stores each vertex v and its distance to s: $d(v, s)$ as values ($\mathcal{Q}$ is maintained by $d(v, s)$), distance array dis and Boole array vis to record whether each vertex has been visited (*i.e.* its shortest distance has been computed). Then EXTRACTHEAPTOP selects the vertex u who has the shortest distance to s at current stage (Line 3). Obviously, at the first run of the while loop, u will be s. After extraction of u, it will be marked as visited in vis. Finally in Line 4, the algorithm will update u's neighbors' distances and insert the neighbors in $\mathcal{Q}$ whose distances are updated.

UPDATENEIGHBORS. We go a little bit further in Line 4. Note v as one of u's neighbors. The procedure will firstly compare $dis[v]$ with $dis[u] + w(u, v)$. If the former is bigger, then the $dis[v]$ will be updated as $dis[u] + w(u, v)$ and v with its updated distance will be appended to the heap $\mathcal{Q}$.

Remark. At the beginning of CREATEAUXILIARYDATASTRUCTURES, all elements in dis will be set to infinity and for entries in vis will be $false$. But finally we will set $dis[s]$ to 0 and then s will be inserted into $\mathcal{Q}$.

Algorithm 1 Dijkstra algorithm

Require: Data graph G and query vertex pair (s, t).
Ensure: The shortest distance $d(s, t)$ between source s and terminal t.

1: $\mathcal{Q}, dis, vis \leftarrow$ CREATEAUXILIARYDATASTRUCTURES(G, s) ▷ $\mathcal{Q}$ is a min heap
2: **while** $\mathcal{Q}$ is **not** empty **do**
3: $u \leftarrow$ EXTRACTHEAPTOP($\mathcal{Q}$)
4: UPDATENEIGHBORS(u, G, dis, vis, $\mathcal{Q}$) ▷ update the distance of u's neighbors
5: **end while**
6: $d(s, t) \leftarrow dis[t]$
7: **return** $d(s, t)$

Problem4. Any interesting problems.

Solution.

THEOREM 1. This is a theorem.

LEMMA 1. This is a lemma.

DEFINITION 1 (**A DEFINITION**). This is a definition.

EXAMPLE 1. This is an example.

Proof. This is a proof.

Remark.