Electron Momentum Measurement

ENPH/PHYS 453

January 3, 2022

Contents

1 Learning Outcomes

1. Investigate the experiment through simulation
2. Determine the z component of the electron momentum
3. Measure the Fermi momentum of the sample material
4. Calculate the core/valence ratio for the sample material

2 Introduction

This experiment makes use of nuclear physics techniques to measure the momentum of electrons in a metal. For reasons we will discuss below, most electrons in a metal have energies that are significantly greater than thermal energies (typically about 10 eV). However, when positrons are incident on a metal, they will slow down to thermal energies. This loss of energy will be through a combination of collision and radiative losses and these positrons will most likely annihilate into gammas as

$$e^{+} + e^{-} \rightarrow 2\gamma \tag{1}$$

with this resulting in two 511 keV γ rays. Annihilation into one gamma is theoretically possible but violates conservation of momentum unless in the presence of a nucleus. Since momentum is conserved, the momenta of the produced γ rays, each with a magnitude of 511 keV/c, need to add vectorially to produce a total momentum of something on the order of a few keV/c. This can only happen if the γ rays are almost back-to-back, with an angular separation that is slightly different from 180°.

This experiment measures the angle between the two gammas (the opening angle) very precisely, and relates the distribution of opening angles to the distribution of electron momenta in metals. In this remote experiment we will "use" copper as the metal, but it could theoretically be any metal.

3 Background

Electrons in Metals

Please refer to the references for detailed theory. This write up is designed to give you a broad overview of the physics, without trying to do many derivations. *Ashcroft and Mermin* (Wikipedia article here, and the text is available in the library) is a good reference for the theory of materials.

Electron wave functions in metals differ from those in insulators because the electron wave function extends throughout the entire crystal; that is to say the wave function is delocalized. This is the same statement as the non-quantum mechanical statement that an electron is free to move from lattice site to lattice site, and is not bound to a particular atom.

It is important to note that the electrons in the samples used for this experiment can be sorted into two categories – core and valence. The core category comprises the inner, more tightly bound electrons while the valence electrons are more widely distributed and responsible for heat and electrical conduction. The potential momenta for these two types of electrons are governed by different distributions, with the difference between these distributions being usable to analyse the data from this experiment.

To calculate electron wave functions, we start with a lattice filled with atoms and their inner (core) electrons. The potential that is obtained in this way is periodic, with minima at the lattice sites and maxima in-between. The wave function that one calculates has the same periodicity with maxima at the lattice sites and minima in-between. One fills wave functions in a manner that is very similar to filling wave functions in an atom – two electrons per wave function, one of each spin, starting at the lowest energy state and filling afterwards. The fact that there are about 10^{22} valence electrons/cm^3 in a metal and that one is dealing with a macroscopic object doesn't matter - one should treat the crystal of metal the same as a single, but very large, atom. The energy of the final electron is called the Fermi Energy and its momentum is Fermi Momentum.

The fact that the electron wave functions extend throughout the crystal means that it doesn't make sense to speak of the "position" of the electron, but rather to think about an electron that is smeared out throughout the entire crystal.

Positron wave functions are calculated with the same potential. They, however, will have minima at the lattice sites and maxima at the interstitial

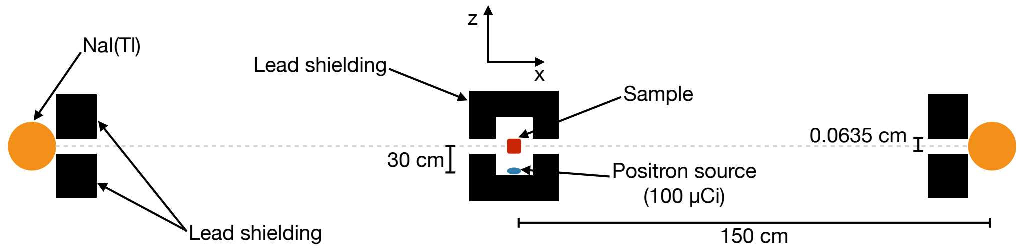

Figure 1: Schematic diagram of the experimental setup

sites. Since the rate of positron production in the source is not high, it is fair to assume that there is only one positron in the metal at a given instant, so it will de-excite until it is within thermal energies (1/40 eV) of the ground state.

The fact that the positron wave functions are maximum at the interstices means that positrons will preferentially annihilate valence electrons; this is a definite advantage in this experiment, which is going to measure the momentum distribution of the valence electrons.

Dislocations in the Lattice

When there are dislocations in the lattice, there are places where the electrostatic field is even smaller. Positrons tend to favour these locations; even though dislocations make up a very small fraction of the total volume, a large percentage of positrons will be found there. This leads to two changes in the momentum distribution:

1. there is a significantly larger contribution from valence electrons (since there are no core electrons at the dislocations) and
2. there will be a slight broadening of the distribution for valence electrons (since the electron energies will be slightly higher)

4 Equipment

The basic feature of the apparatus is shown in Figure 1. The positron source is ^{22}Na. It is contained in a lead container to prevent the gamma rays from the source itself from striking the NaI(Tl) detectors and being detected. The

1.	Positron Source	4.	Moveable source (unlabelled)
2.	Sample	5.	NaI(Tl) detector
3.	Lead source shielding	6.	Lead detector shielding

Table 1: Equipment list for Figure 1

positrons from the ^{22}Na source irradiate a sample disk of either aluminum, deformed aluminum or copper mounted right above it. Two slits in the lead container allow the gammas produced in the annihilation within the samples to escape and reach the detectors. **While this portion is not relevant this year, it is included for your information.** *Do not try to open up the lead container on your own. Ask the TA or professor to show you how to change the sample.*
There are two long cylindrical NaI(Tl) detectors at each end of the apparatus. A lead shield, with an adjustable slit width, defines the angle of the γ rays. Such geometry is necessary because the NaI(Tl) detectors need to be relatively large in order to have a reasonable efficiency to detect a gamma ray.

5 Carrying Out Your Lab

This experiment, when delivered remotely, breaks down into two distinct portions. The first is to simulate the expected results from the physical setup, the second is to analyze data taken from the actual experiment. These two pieces will be treated separately.

5.1 Simulation

The rough steps to achieve an accurate simulation of this experiment are as follows:

1. Figure out how many positrons are incident on the sample, and assume they all annihilate

2. Set the angle for the outgoing gamma rays (by randomly choosing from all possible options) to see if they would be incident on the detectors

3. Decide whether this annihilated with a valence or core electron

4. Pick out the z-momentum from the distribution for valence or core, depending on step 3
5. Decide whether this change in angle (caused by the addition of the electron momentum in step 4) assuming the gamma incident on the stationary detector is at 180° impacts on the movable detector
6. Do steps 2 to 5 for the time while the detector is at that height
7. Move the movable detector and do it again for as many steps as you wish

[A faster way, obviously, is to do the movable detector in all positions at once, just multiplying the total time by the number of steps]

Doing this will build up a distribution of “coincident” hits which would be seen in the actual experiment. This will be used later.

5.2 Analysis

Now that the simulation has been carried out, the different distributions for the core and valence electrons are known. Data will be presented from both the stationary and movable detectors, collected by your professor. The task is to fit the distributions of these two types of electrons to the data to determine both the ratio of the number of each classification and also the Fermi momentum for the sample.

Theoretical Considerations

1. Is it possible to produce anything other than two γs? How might this affect the results?
2. Can the positron annihilate with an electron while not at rest? Does this have to be considered in the final analysis?
3. What is the average distance in the metal at which the positrons will have achieved thermal energies?

Experimental Considerations

1. The signal should be traced through the entire apparatus using an oscilloscope.

2. Do the four (4) NaI(Tl) detectors show a similar response to calibration γs? (Side note: does the response look qualitatively the same for all detectors? If not, why not?)

Predictions

1. What is the Fermi momentum of the electrons in the lattice?

2. What effect with introducing defects into the lattice have?

3. What shape will the distribution of the valence electrons have? What about the core electrons?

4. How might the resolution of the detectors affect the final result?

Design

1. Why might the lead shields in front of the detector be wider or narrower? What is the optimal opening for the slits in the lead shields?

2. What impact can backgrounds have on this measurement? How can the equipment be used to provide a response only to gamma rays of the desired energy?

3. When considering the coincidence of hits from gamma rays, what should the time window be? What are the effects of having it wider or narrower?

4. How many different angle measurements should be taken? How far apart should they be spaced?

References

[1] Perrin, F. *J. Chem. Phys.*, **10**, 415 (1942)

[2] Modern Experiments in Physics, *Mellissinos*

[3] L.Widrow, *Origin of galactic and extragalactic magnetic fields*, Rev. Mod. Phys 74 (775), July 2002

[4] Queen's University Laser Safety Program, Document No. SOP-Radiation-02, Rev.1.0, Nov. 18, 2004, http://www.safety.queensu.ca/laser/laserpolicy.pdf