# Parallel Programming Project Report
# FDTD on Microstrip Patch Antenna using CUDA

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## Project Repository

The source code and resources for this project can be found on GitHub at the following link:

`https://github.com/EricLin0123/CUDA_FDTD_antenna`

## Topic

A microstrip patch antenna is a type of radio antenna with a low profile, which can be mounted on a flat surface. It consists of a flat rectangular sheet or "patch" of metal, mounted over a larger sheet of metal called a ground plane.

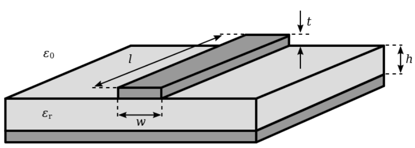
Microstrip patch antennas are popular for their low cost, ease of fabrication, and versatility. They are widely used in various applications such as mobile and satellite communications, GPS devices, and radar systems.

In this project, we aim to apply the Finite-Difference Time-Domain (FDTD) method to a microstrip patch antenna to extract its circuit traits. The FDTD method will allow us to simulate the electromagnetic wave propagation and interaction with the microstrip patch antenna, providing insights into its performance characteristics such as impedance, radiation pattern, and bandwidth.

By analyzing these traits, we can optimize the design of the microstrip patch antenna for specific applications in wireless communication systems.

### Microstrip

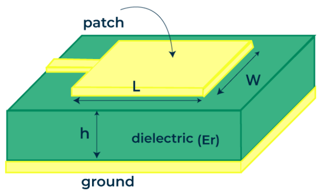
A microstrip is a type of electrical transmission line which can be fabricated using printed circuit board technology, and is used to convey microwave-frequency signals. It consists of a conducting strip separated from a ground plane by a dielectric layer.




Microstrip structure

### Microstrip patch antenna

A microstrip patch antenna is a type of patch antenna fed with a microstrip transmission line. It is widely used in wireless communication systems because of its low profile and ease of fabrication.




Microstrip patch antenna

### FDTD

FDTD stands for Finite-Difference Time-Domain. It is a numerical method for solving differential equations by approximating them with finite differences. It is a popular method for simulating electromagnetic wave propagation.

## Implementation

### Maxwell's equations

In this project, I'll replicate the FDTD simulation in the paper [1]"Application of the three-dimensional finite-difference time-domain method to the analysis of planar microstrip circuits". The simulation is done in 3D space, and the microstrip patch antenna is modeled as a planar structure.

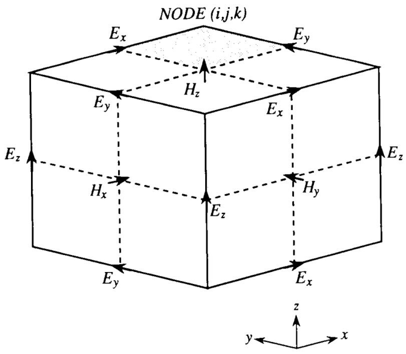
Assuming piecewise uniform, isotropic, homogenous and lossless media. The Maxwell's curl equations can be written as:

$$\mu \frac{\partial H}{\partial t} = -\nabla \times E \tag{1}$$

$$\epsilon \frac{\partial E}{\partial t} = \nabla \times H \tag{2}$$

### Finite difference equations

The FDTD method is done by discretizing the space into a grid of cells, where the electric and magnetic fields are updated at each time step based on the Maxwell's equations. The centered difference scheme is used to approximate both the time and space first-order partial derivatives. The space and the associated fields are divided into cells like the following figure (Proposed by Yee in 1966[3]):




Yee's mesh

The cantered difference approximation of (1) and (2) are:

$$
\begin{aligned}
H_{x,i,j,k}^{n+1/2} &= H_{x,i,j,k}^{n-1/2} + \frac{\Delta t}{\mu \Delta z}\left(E_{y,i,j,k}^{n} - E_{y,i,j,k-1}^{n}\right) \\
&\quad - \frac{\Delta t}{\mu \Delta y}\left(E_{z,i,j,k}^{n} - E_{z,i,j-1,k}^{n}\right) \qquad (3)
\end{aligned}
$$

$$
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\end{aligned}
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H_{z,i,j,k}^{n+1/2} &= H_{z,i,j,k}^{n-1/2} + \frac{\Delta t}{\mu \Delta y}\left(E_{x,i,j,k}^{n} - E_{x,i,j-1,k}^{n}\right) \\
&\quad - \frac{\Delta t}{\mu \Delta x}\left(E_{y,i,j,k}^{n} - E_{y,i-1,j,k}^{n}\right) \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
E_{x,i,j,k}^{n+1} &= E_{x,i,j,k}^{n} + \frac{\Delta t}{\epsilon \Delta y}\left(H_{z,i,j+1,k}^{n+1/2} - H_{z,i,j,k}^{n+1/2}\right) \\
&\quad - \frac{\Delta t}{\epsilon \Delta z}\left(H_{y,i,j,k+1}^{n+1/2} - H_{y,i,j,k}^{n+1/2}\right) \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
E_{y,i,j,k}^{n+1} &= E_{y,i,j,k}^{n} + \frac{\Delta t}{\epsilon \Delta z}\left(H_{x,i,j,k+1}^{n+1/2} - H_{x,i,j,k}^{n+1/2}\right) \\
&\quad - \frac{\Delta t}{\epsilon \Delta x}\left(H_{z,i+1,j,k}^{n+1/2} - H_{z,i,j,k}^{n+1/2}\right) \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
E_{z,i,j,k}^{n+1} &= E_{z,i,j,k}^{n} + \frac{\Delta t}{\epsilon \Delta x}\left(H_{y,i+1,j,k}^{n+1/2} - H_{y,i,j,k}^{n+1/2}\right) \\
&\quad - \frac{\Delta t}{\epsilon \Delta y}\left(H_{x,i,j+1,k}^{n+1/2} - H_{x,i,j,k}^{n+1/2}\right). \qquad (8)
\end{aligned}
$$

### stability constraint

The maximum time step that may be used is limited by the stability restriction of the finite difference equations

$$\Delta t \leq \frac{1}{v_{max}}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right)^{-1/2} \qquad (9)$$

where $v_{max}$ is the maximum velocity of EM wave in the medium.

### Time marching algorithm

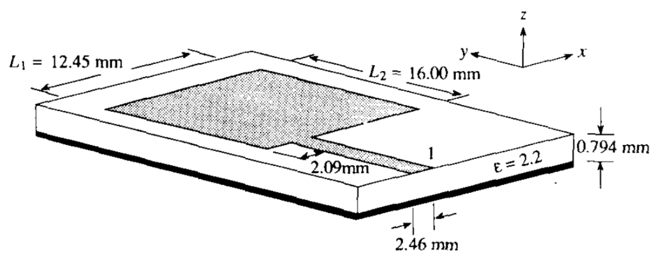
The essence of FDTD is to update the electric and magnetic fields at each time step. The time marching algorithm is as follows:

1. Initialize all fields to 0 (at $t = n = 0$)
2. Repeat the following for a desired number of time steps:
   (a) Gaussian excitation is imposed on the input port.
   (b) $H^{n+1/2}$ is updated using (3), (4), (5).
   (c) $E^{n+1}$ is updated using (6), (7), (8).
   (d) Tangential electric field is set to 0 on conductors.
   (e) Save desired field values.
   (f) $n \to n+1$.

The reason fro using Gaussian excitation is that it has a broad frequency spectrum, which is suitable for broadband analysis.(Its Fourier transform is also a Gaussian function)

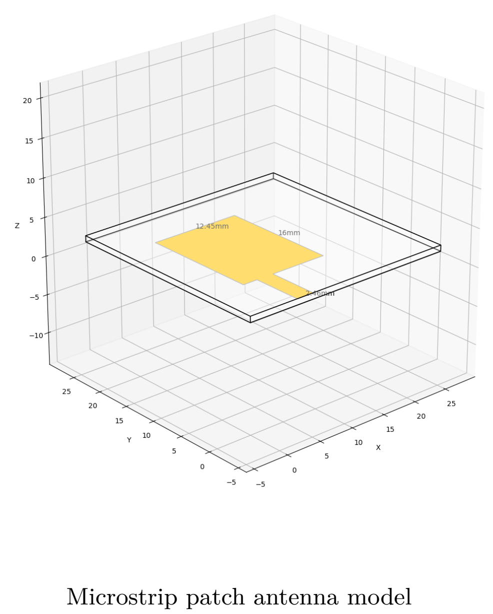
### Target structure

The main reason why I chose to replicate the paper is that it showed a good agreement between the simulation and the measurement.(which is the whole point of running FDTD) The target structure is a microstrip patch antenna with a rectangular patch and a ground plane. The patch is fed by a microstrip transmission line. The simulation will provide the impedance, radiation pattern, and bandwidth of the antenna. The exact dimensions are shown below:




Microstrip patch antenna structure

Here is a model I created using Matplotlib in Python that can later be used directly in the FDTD simulation.




Microstrip patch antenna model

**Parameters**

The 3D space is divided into cells of size

$$\Delta x = 0.389mm$$
$$\Delta y = 0.400mm$$
$$\Delta z = 0.265mm$$

The time step is determined by the stability constraint (9). The maximum velocity of EM wave in the medium is $3 \times 10^8 m/s$. The time step is chosen to be $\Delta t = 0.441ps$.

Check the stability constraint:

$$\Delta t \leq \frac{1}{3 \times 10^8} \left(\frac{1}{0.389^2} + \frac{1}{0.4^2} + \frac{1}{0.265^2}\right)^{-1/2} \times 10^3$$
$$= 6.4 \times 10^{-7} s$$

The geometry of the microstrip patch antenna is about tens of millimeters ($\frac{\lambda}{4}$), which corresponds to resonant wave length about 40 to 50 millimeters, namely 7 to 8 GHz. We should choose the appropriate Gaussian pulse width to incorporate the frequency range of interest.

A good rule of thumb is to have a pulse width ($\Delta$t) that satisfies the time-bandwidth product relationship:

$$\Delta t \times \Delta f \approx 0.441 \text{ for Gaussian pulses}$$

The appropriate pulse width:

$$\Delta t = \frac{0.441}{\Delta f} = \frac{0.441}{7.5 \times 10^9} = 58ps$$

In the original paper, The Gaussian source half width is chose to be $T = \sqrt{2}\sigma = 15ps$ which is in the same order of magnitude of our calculated result. The even shorter Gaussian pulse allow us to inspect frequencies beyond 7 GHz as the paper showed accuracy can be maintained up to 20 GHz. Time delay is $t_0 = 3T = 45ps$ to ensure the pulse starts from zero amplitude after the first time step. The simulation is run for 8000 steps as the resonant wave died out.

# Results

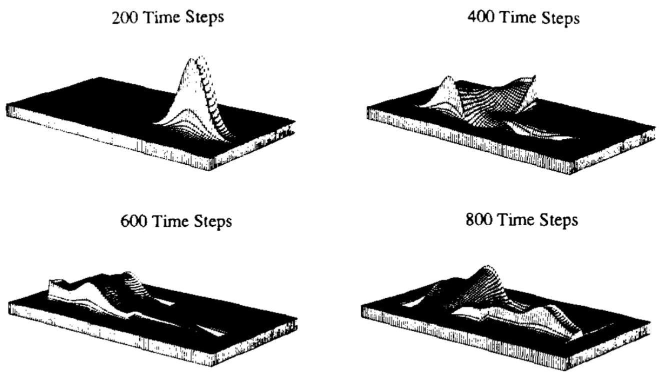
## Speedup

The sequential code is implemented in C++ ref [2]. I parallelized the program using CUDA. The simulation time for the sequential code is on average 26 seconds. And the parallelized code running on a RTX4060 GPU is on average 2.5 seconds. The speedup is about 10 times.

| Implementation | Average Time (s) | Speedup |
|---|---|---|
| Sequential (C++) | 26 | x1 |
| Parallel (CUDA) | 2.5 | x10 |

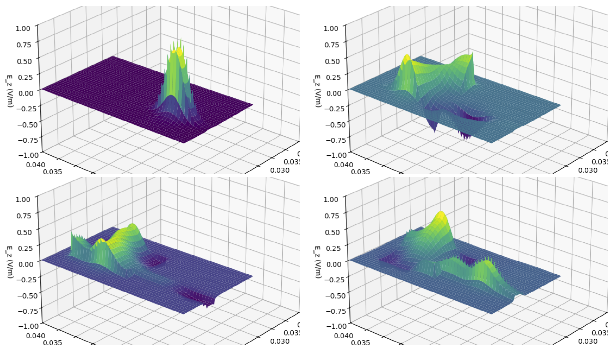
## Time domain waveform

In the original paper, it showed some time slice of the electric field waveform as following:




Time domain waveform (Paper)

Here is the corresponding waveform I obtained from my CUDA simulation.



Time domain waveform (My)

As you can see they are strongly agree with each other.

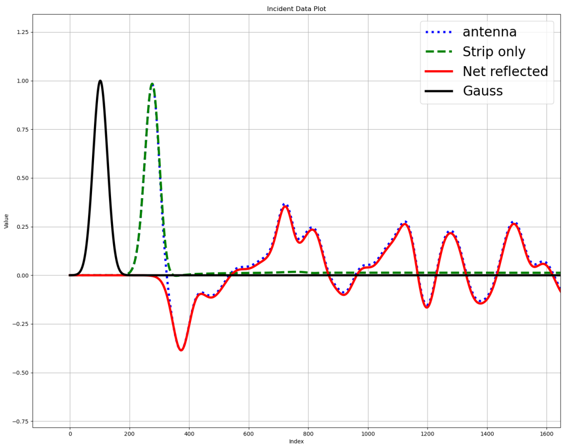
## Return loss

The main purpose of antenna simulation is to see if the device resonate at the desired frequency. Frequency domain scattering matrix can be calculated by

$$S_{jk} = \frac{\mathcal{F}\{V_j(t)\}}{\mathcal{F}\{V_k(t)\}}$$

where $S$ is the scattering matrix. To obtain the scattering parameter $S_{11}(\omega)$ (which is the Return loss), the incident and reflected waveforms must be known. The FDTD simulation calculates the sum of incident and reflected waveforms. To obtain the incident waveform, the calculation is performed using only the port 1 microstrip line, which will now be of infinite extent (i.e,. from source to far absorbing wall), and the incident waveform is recorded. This incident waveform may now be subtracted from the incident plus reflected waveform to yield the reflected waveform for port 1.

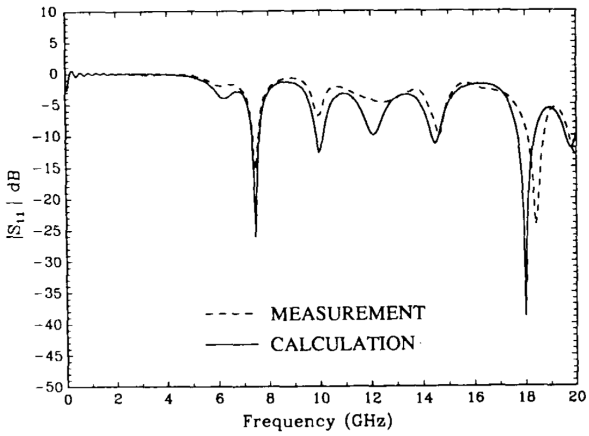
Here is the time domain electric field under the feeding line (port 1) and the reflected electric field at the same location.

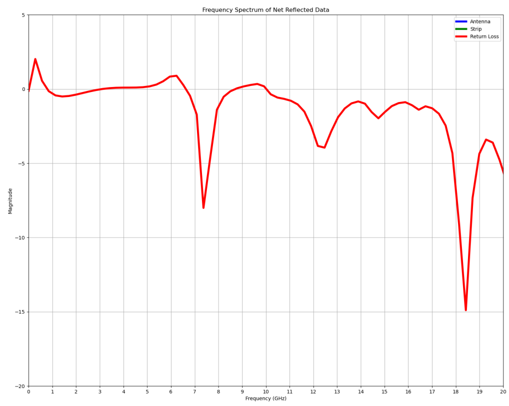



Port 1 and reflected electric field

The desired return loss should be like the following figure (Left). Note that in the original paper they used HP 8510 network analyzer calibrated to 18 GHz to perform the measurement.

On the right (red) is the return loss I obtained from my CUDA simulation. (plotted using Matplotlib and Numpy in Python) They are in good agreement with each other as the resonant frequency is around 7.5 GHz and 18.5 GHz. My Implementation is **even more accurate** than the original paper as the return loss at 18 to 19 GHz of my result is more close to the measured frequency (more close to 19 GHz, dotted line is the target).




# References

[1] D. M. Sheen, S. M. Ali, M. D. Abouzahra and J. A. Kong, "Application of the three-dimensional finite-difference time-domain method to the analysis of planar microstrip circuits," in IEEE Transactions on Microwave Theory and Techniques, vol. 38, no. 7, pp. 849-857, July 1990, doi: 10.1109/22.55775. keywords: Finite difference methods;Time domain analysis;Microstrip antennas;Microstrip components;Circuits;Antenna accessories;Impedance;Patch antennas;Frequency;Scattering parameters,

[2] Source code from Pete Bevelacqua - EE517 (https://www.antenna-theory.com/antennas/patches/patch6.php)

[3] G. Mur, "Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic-Field Equations," in IEEE Transactions on Electromagnetic Compatibility, vol. EMC-23, no. 4, pp. 377-382, Nov. 1981, doi: 10.1109/TEMC.1981.303970. keywords: Electromagnetic-field equations;time domain;finite-difference approximation;absorbing boundary conditions,

[4] D. M. Sheen, S. M. Ali, M. D. Abouzahra and J. A. Kong, "Application of the three-dimensional finite-difference time-domain method to the analysis of planar microstrip circuits," in IEEE Transactions on Microwave Theory and Techniques, vol. 38, no. 7, pp. 849-857, July 1990, doi: 10.1109/22.55775. keywords: Finite difference methods;Time domain analysis;Microstrip antennas;Microstrip components;Circuits;Antenna accessories;Impedance;Patch antennas;Frequency;Scattering parameters,