MDP & Q-Learning & SARSA

Shan-Hung Wu & DataLab
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In this lab, we will introduce the conception of Markov Decision Process(MDP) and two solution algorithms, and then we will introduce the Q-Learning and SARSA algorithm, finally we will use the Q-learning algorithm to train an agent to play "Flappy Bird" game.

Markov Decision Process(MDP)

A Markov decision process (MDP) is a random process, i.e. a sequence of random states S1, S2, ... with the Markov property. It provides a mathematical framework for modeling decision making in situations where outcomes are partly random and partly under the control of a decision maker.
There are two algorithms to solve the MDP problem: **value iteration** and **policy iteration**.

Value iteration

The algorithm is like below:

Input: MDP $(\mathbb{S}, \mathbb{A}, \mathrm{P}, R, \gamma, H \to \infty)$
Output: $\pi^*(s)$'s for all s's

For each state s, initialize $V^*(s) \leftarrow 0$;
repeat
  foreach s **do**
    $V^*(s) \leftarrow \max_a \sum_{s'} \mathrm{P}(s'|s;a)[R(s,a,s') + \gamma V^*(s')]$;
  end
until $V^*(s)$*'s converge*;
foreach s **do**
  $\pi^*(s) \leftarrow \arg\max_a \sum_{s'} \mathrm{P}(s'|s;a)[R(s,a,s') + \gamma V^*(s')]$;
end

Next, we will show an example code of Gridworld to see how value iteration works.

please install gym.

commands: **pip install gym==0.21.0**

In [2]:
```
import numpy as np
import sys
from gym.envs.toy_text import discrete
```

In [3]:
```
# four actions in the game
UP = 0
RIGHT = 1
DOWN = 2
LEFT = 3
```

In [4]:
```
class GridworldEnv(discrete.DiscreteEnv):
    """
    Grid World environment from Sutton's Reinforcement Learning book chapter 4.
    You are an agent on an MxN grid and your goal is to reach the terminal
    state at the top left or the bottom right corner.
    For example, a 4x4 grid looks as follows:
    T  o  o  o
    o  x  o  o
    o  o  o  o
    o  o  o  T
    x is your position and T are the two terminal states.
    You can take actions in each direction (UP=0, RIGHT=1, DOWN=2, LEFT=3).
    Actions going off the edge leave you in your current state.
    You receive a reward of -1 at each step until you reach a terminal state.
    """

    metadata = {'render.modes': ['human', 'ansi']}

    def __init__(self, shape=[4, 4]):
        if not isinstance(shape, (list, tuple)) or not len(shape) == 2:
            raise ValueError('shape argument must be a list/tuple of length 2')

        self.shape = shape

        nS = np.prod(shape)
        nA = 4

        MAX_Y = shape[0]
        MAX_X = shape[1]

        P = {}
        grid = np.arange(nS).reshape(shape)
        it = np.nditer(grid, flags=['multi_index'])

        while not it.finished:
            s = it.iterindex
            y, x = it.multi_index

            P[s] = {a: [] for a in range(nA)}

            is_done = lambda s: s == 0 or s == (nS - 1)
            reward = 0.0 if is_done(s) else -1.0
```

```python
            # We're stuck in a terminal state
            if is_done(s):
                P[s][UP] = [(1.0, s, reward, True)]
                P[s][RIGHT] = [(1.0, s, reward, True)]
                P[s][DOWN] = [(1.0, s, reward, True)]
                P[s][LEFT] = [(1.0, s, reward, True)]
            # Not a terminal state
            else:
                ns_up = s if y == 0 else s - MAX_X
                ns_right = s if x == (MAX_X - 1) else s + 1
                ns_down = s if y == (MAX_Y - 1) else s + MAX_X
                ns_left = s if x == 0 else s - 1
                P[s][UP] = [(1.0, ns_up, reward, is_done(ns_up))]
                P[s][RIGHT] = [(1.0, ns_right, reward, is_done(ns_right))]
                P[s][DOWN] = [(1.0, ns_down, reward, is_done(ns_down))]
                P[s][LEFT] = [(1.0, ns_left, reward, is_done(ns_left))]

            it.iternext()

        # Initial state distribution is uniform
        isd = np.ones(nS) / nS

        # We expose the model of the environment for educational purposes
        # This should not be used in any model-free learning algorithm
        self.P = P

        super(GridworldEnv, self).__init__(nS, nA, P, isd)

    def render(self, mode='human', close=False):
        if close:
            return

        outfile = StringIO() if mode == 'ansi' else sys.stdout

        grid = np.arange(self.nS).reshape(self.shape)
        it = np.nditer(grid, flags=['multi_index'])
        while not it.finished:
            s = it.iterindex
            y, x = it.multi_index

            if self.s == s:
                output = " x "
            elif s == 0 or s == self.nS - 1:
                output = " T "
            else:
                output = " o "

            if x == 0:
                output = output.lstrip()
            if x == self.shape[1] - 1:
                output = output.rstrip()

            outfile.write(output)

            if x == self.shape[1] - 1:
                outfile.write("\n")

            it.iternext()
```

In [5]:
```python
env = GridworldEnv()
```

In [6]:
```python
def value_iteration(env, theta=0.0001, discount_factor=1.0):
    """
    Value Iteration Algorithm.

    Args:
        env: OpenAI env. env.P represents the transition probabilities of the environment.
            env.P[s][a] is a list of transition tuples (prob, next_state, reward, done).
            env.nS is a number of states in the environment.
            env.nA is a number of actions in the environment.
        theta: We stop evaluation once our value function change is less than theta for all states.
        discount_factor: Gamma discount factor.

    Returns:
        A tuple (policy, V) of the optimal policy and the optimal value function.
    """


    def one_step_lookahead(state, V):
        """
        Given an state, calculate the new value function V(s) based on the value iteration algorithm
        Args:
            state: represents each state in the Gridworld, an integer
            V: the current value function of the states(V(s)), the lengh is env.nS

        Returns:
            a new V(s)
        """
        A = np.zeros(env.nA)
        for a in range(env.nA):
            for prob, next_state, reward, done in env.P[state][a]:
                A[a] += prob * (reward + discount_factor * V[next_state])
        return A

    V = np.zeros(env.nS)
    while True:
        delta = 0

        for s in range(env.nS):
            # Do a one-step lookahead to find the best action
            A = one_step_lookahead(s, V)
            best_action_value = np.max(A)
            # Calculate delta across all states seen so far
            delta = max(delta, np.abs(best_action_value - V[s]))
            # Update the value function
            V[s] = best_action_value
            # Check if we can stop
        if delta < theta:
            break

    # Create a deterministic policy using the optimal value function
    policy = np.zeros([env.nS, env.nA])
    for s in range(env.nS):
        # One step lookahead to find the best action for this state
```

```
            A = one_step_lookahead(s, V)
            best_action = np.argmax(A)
            # Always take the best action
            policy[s, best_action] = 1.0

    return policy, V


policy, v = value_iteration(env)

print("Policy Probability Distribution:")
print(policy)
print("")

print("Reshaped Grid Policy (0=up, 1=right, 2=down, 3=left):")
print(np.reshape(np.argmax(policy, axis=1), env.shape))
print("")

print("Value Function:")
print(v)
print("")

print("Reshaped Grid Value Function:")
print(v.reshape(env.shape))
print("")
```

```
Policy Probability Distribution:
[[1. 0. 0. 0.]
 [0. 0. 0. 1.]
 [0. 0. 0. 1.]
 [0. 0. 1. 0.]
 [1. 0. 0. 0.]
 [1. 0. 0. 0.]
 [1. 0. 0. 0.]
 [0. 0. 1. 0.]
 [1. 0. 0. 0.]
 [1. 0. 0. 0.]
 [0. 1. 0. 0.]
 [0. 0. 1. 0.]
 [1. 0. 0. 0.]
 [0. 1. 0. 0.]
 [0. 1. 0. 0.]
 [1. 0. 0. 0.]]

Reshaped Grid Policy (0=up, 1=right, 2=down, 3=left):
[[0 3 3 2]
 [0 0 0 2]
 [0 0 1 2]
 [0 1 1 0]]

Value Function:
[ 0. -1. -2. -3. -1. -2. -3. -2. -2. -3. -2. -1. -3. -2. -1.  0.]

Reshaped Grid Value Function:
[[ 0. -1. -2. -3.]
 [-1. -2. -3. -2.]
 [-2. -3. -2. -1.]
 [-3. -2. -1.  0.]]
```

Policy iteration

The algorithm is like below:

Input: MDP $(\mathbb{S}, \mathbb{A}, \mathrm{P}, R, \gamma, H \to \infty)$
Output: $\pi(s)$'s for all s's

For each state s, initialize $\pi(s)$ randomly;
repeat
  For each state s, initialize $V_\pi(s) \leftarrow 0$;
  repeat Policy evaluation
    foreach s **do**
      $V_\pi(s) \leftarrow \sum_{s'} \mathrm{P}(s'|s;\pi(s))[R(s,\pi(s),s') + \gamma V_\pi(s')]$;
    end
  until $V_\pi(s)$'s *converge*;
  foreach s **do** Policy improvement
    $\pi(s) \leftarrow \arg\max_a \sum_{s'} \mathrm{P}(s'|s;a)[R(s,a,s') + \gamma V_\pi(s')]$;
  end
until $\pi(s)$'s *converge*;

Next, we will show an example code of Gridworld to see how value iteration works.

In [7]:
```
import numpy as np
import sys
from gym.envs.toy_text import discrete
```

In [8]:
```
env = GridworldEnv()
```

In [9]:
```
def policy_eval(policy, env, discount_factor=1.0, theta=0.00001):
    """
    Evaluate a policy given an environment and a full description of the environment's dynamics.

    Args:
        policy: [S, A] shaped matrix representing the policy.
        env: OpenAI env. env.P represents the transition probabilities of the environment.
            env.P[s][a] is a list of transition tuples (prob, next_state, reward, done).
```

```python
            env.nS is a number of states in the environment.
            env.nA is a number of actions in the environment.
        theta: We stop evaluation once our value function change is less than theta for all states.
        discount_factor: Gamma discount factor.

    Returns:
        Vector of length env.nS representing the value function.
    """
    # Start with a random (all 0) value function
    V = np.zeros(env.nS)
    while True:
        delta = 0
        # For each state, perform a "full backup"
        for s in range(env.nS):
            v = 0
            # Look at the possible next actions
            for a, action_prob in enumerate(policy[s]):
                # For each action, look at the possible next states...
                for  prob, next_state, reward, done in env.P[s][a]:
                    # Calculate the expected value
                    v += action_prob * prob * (reward + discount_factor * V[next_state])
            # How much our value function changed (across any states)
            delta = max(delta, np.abs(v - V[s]))
            V[s] = v
        # Stop evaluating once our value function change is below a threshold
        if delta < theta:
            break

    return np.array(V)
```

In [10]:

```python
def policy_improvement(env, policy_eval_fn=policy_eval, discount_factor=1.0):
    """
    Policy Improvement Algorithm. Iteratively evaluates and improves a policy
    until an optimal policy is found.

    Args:
        env: The OpenAI environment.
        policy_eval_fn: Policy Evaluation function that takes 3 arguments:
            policy, env, discount_factor.
        discount_factor: gamma discount factor.

    Returns:
        A tuple (policy, V).
        policy is the optimal policy, a matrix of shape [S, A] where each state s
        contains a valid probability distribution over actions.
        V is the value function for the optimal policy.

    """

    def one_step_lookahead(state, V):
        """
        Helper function to calculate the value for all action in a given state.

        Args:
            state: The state to consider (int)
            V: The value to use as an estimator, Vector of length env.nS

        Returns:
            A vector of length env.nA containing the expected value of each action.
        """
        A = np.zeros(env.nA)
        for a in range(env.nA):
            for prob, next_state, reward, done in env.P[state][a]:
                A[a] += prob * (reward + discount_factor * V[next_state])
        return A

    # Start with a random policy
    policy = np.ones([env.nS, env.nA]) / env.nA

    while True:
        # Evaluate the current policy
        V = policy_eval_fn(policy, env, discount_factor)

        # Will be set to false if we make any changes to the policy
        policy_stable = True

        # For each state...
        for s in range(env.nS):
            # The best action we would take under the current policy
            chosen_a = np.argmax(policy[s])

            # Find the best action by one-step lookahead
            # Ties are resolved arbitarily
            action_values = one_step_lookahead(s, V)
            best_a = np.argmax(action_values)

            # Greedily update the policy
            if chosen_a != best_a:
                policy_stable = False
            policy[s] = np.eye(env.nA)[best_a]

        # If the policy is stable we've found an optimal policy. Return it
        if policy_stable:
            return policy, V
```

In [11]:

```python
policy, v = policy_improvement(env)
print("Policy Probability Distribution:")
print(policy)
print("")

print("Reshaped Grid Policy (0=up, 1=right, 2=down, 3=left):")
print(np.reshape(np.argmax(policy, axis=1), env.shape))
print("")

print("Value Function:")
print(v)
print("")

print("Reshaped Grid Value Function:")
print(v.reshape(env.shape))
print("")
```

```
Policy Probability Distribution:
[[1. 0. 0. 0.]
 [0. 0. 0. 1.]
 [0. 0. 0. 1.]
 [0. 0. 1. 0.]
 [1. 0. 0. 0.]
 [1. 0. 0. 0.]
 [1. 0. 0. 0.]
 [0. 0. 1. 0.]
 [1. 0. 0. 0.]
 [1. 0. 0. 0.]
 [0. 1. 0. 0.]
 [0. 0. 1. 0.]
 [1. 0. 0. 0.]
 [0. 1. 0. 0.]
 [0. 1. 0. 0.]
 [1. 0. 0. 0.]]

Reshaped Grid Policy (0=up, 1=right, 2=down, 3=left):
[[0 3 3 2]
 [0 0 0 2]
 [0 0 1 2]
 [0 1 1 0]]

Value Function:
[ 0. -1. -2. -3. -1. -2. -3. -2. -2. -3. -2. -1. -3. -2. -1.  0.]

Reshaped Grid Value Function:
[[ 0. -1. -2. -3.]
 [-1. -2. -3. -2.]
 [-2. -3. -2. -1.]
 [-3. -2. -1.  0.]]
```

Value iteration VS. Policy iteration

Difference:

1. Initialization
   $v(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$

2. Policy Evaluation
   Repeat
   $\quad\Delta \leftarrow 0$
   $\quad$For each $s \in \mathcal{S}$:
   $\quad\quad temp \leftarrow v(s)$
   $\quad\quad v(s) \leftarrow \sum_{s'} p(s'|s,\pi(s))\left[r(s,\pi(s),s') + \gamma v(s')\right]$
   $\quad\quad \Delta \leftarrow \max(\Delta, |temp - v(s)|)$
   until $\Delta < \theta$ (a small positive number)

3. Policy Improvement
   $policy\text{-}stable \leftarrow true$
   For each $s \in \mathcal{S}$:
   $\quad temp \leftarrow \pi(s)$
   $\quad \pi(s) \leftarrow \arg\max_a \sum_{s'} p(s'|s,a)\left[r(s,a,s') + \gamma v(s')\right]$
   $\quad$If $temp \neq \pi(s)$, then $policy\text{-}stable \leftarrow false$
   If $policy\text{-}stable$, then stop and return v and π; else go to 2

Figure 4.3: Policy iteration (using iterative policy evaluation) for v_*. This algorithm has a subtle bug, in that it may never terminate if the policy continually switches between two or more policies that are equally good. The bug can be fixed by adding additional flags, but it makes the pseudocode so ugly that it is not worth it. :-)

finding optimal value function

Initialize array v arbitrarily (e.g., $v(s) = 0$ for all $s \in \mathcal{S}^+$)

Repeat
$\quad\Delta \leftarrow 0$
$\quad$For each $s \in \mathcal{S}$:
$\quad\quad temp \leftarrow v(s)$
$\quad\quad v(s) \leftarrow \max_a \sum_{s'} p(s'|s,a)[r(s,a,s') + \gamma v(s')]$
$\quad\quad \Delta \leftarrow \max(\Delta, |temp - v(s)|)$
until $\Delta < \theta$ (a small positive number)

Output a deterministic policy, π, such that
$\quad \boxed{\pi(s) = \arg\max_a \sum_{s'} p(s'|s,a)\left[r(s,a,s') + \gamma v(s')\right]}$

Figure 4.5: Value iteration.

one policy update (extract policy from the optimal value function

This image comes from a answer from stackoverflow

Q-Learning

Q-Learning is an off-policy, model-free RL algorithm.

Q-learning: An off-policy TD control algorithm

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(terminal\text{-}state, \cdot) = 0$
Repeat (for each episode):
$\quad$Initialize S
$\quad$Repeat (for each step of episode):
$\quad\quad$Choose A from S using policy derived from Q (e.g., ϵ-greedy)
$\quad\quad$Take action A, observe R, S'
$\quad\quad Q(S,A) \leftarrow Q(S,A) + \alpha\left[R + \gamma \max_a Q(S',a) - Q(S,A)\right]$
$\quad\quad S \leftarrow S'$
$\quad$until S is terminal

This image comes from Reinforcement Learning: An Introduction

SARSA

SARSA is an on-policy, model-free RL algorithm.

Sarsa: An on-policy TD control algorithm

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Choose A from S using policy derived from Q (e.g., ϵ-greedy)
    Repeat (for each step of episode):
        Take action A, observe R, S'
        Choose A' from S' using policy derived from Q (e.g., ϵ-greedy)
        $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma Q(S',A') - Q(S,A)\big]$
        $S \leftarrow S'$; $A \leftarrow A'$;
    until S is terminal

This image comes from Reinforcement Learning: An Introduction

Q-Learning VS. SARSA

Difference:

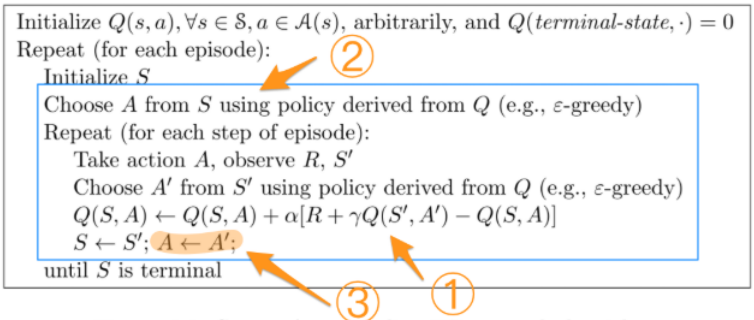

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Repeat (for each step of episode):
        Take action A, observe R, S'
        Choose A' from S' using policy derived from Q (e.g., ε-greedy)
        $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma Q(S',A') - Q(S,A)]$
        $S \leftarrow S'$; $A \leftarrow A'$;
    until S is terminal

Figure 6.9: Sarsa: An on-policy TD control algorithm.

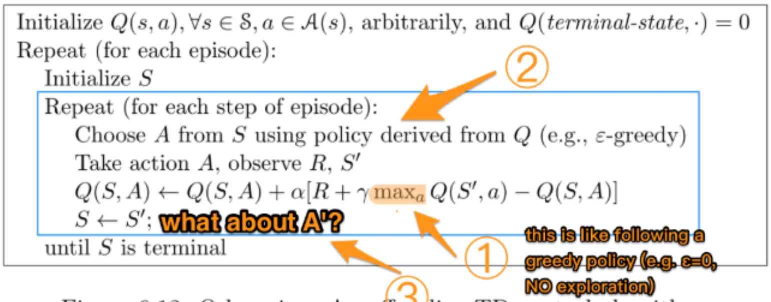
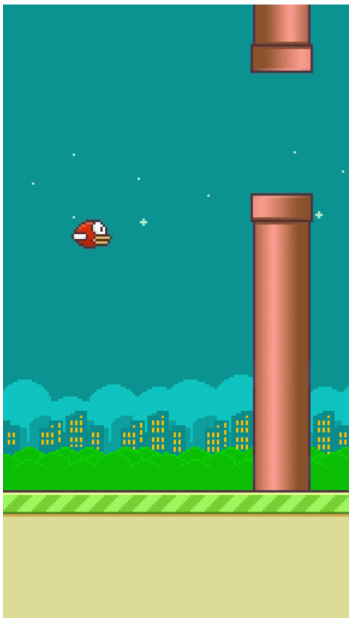

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma \max_a Q(S',a) - Q(S,A)]$
        $S \leftarrow S'$;
    until S is terminal

Figure 6.12: Q-learning: An off-policy TD control algorithm.

This image comes from a answer from stackoverflow

Flappy Bird Game

Flappybird is a side-scrolling game where the agent must successfully nagivate through gaps between pipes.
Next, we wiil train an agent to play "Flappy Bird" game using Q-learning algorithm.



First, we should install PyGame Learning Environment(PLE) which provides the environment to train an agent.

1. Clone the repo

command: git clone https://github.com/ntasfi/PyGame-Learning-Environment

```
$ git clone https://github.com/ntasfi/PyGame-Learning-Environment
Cloning into 'PyGame-Learning-Environment'...
remote: Enumerating objects: 1118, done.
remote: Total 1118 (delta 0), reused 0 (delta 0), pack-reused 1118
Receiving objects: 100% (1118/1118), 8.06 MiB | 800.00 KiB/s, done.
Resolving deltas: 100% (592/592), done.
```

2. Install PLE(in the PyGame-Learning-Environment folder)

command: **pip install -e .**(Please don't ignore this period)

```
$ pip install -e .
Obtaining file:///E:/DL/Lab/RL/PyGame-Learning-Environment
Requirement already satisfied: numpy in c:\users\vincent\anaconda3\lib\site-pack
ages (from ple==0.0.1) (1.16.4)
Requirement already satisfied: Pillow in c:\users\vincent\anaconda3\lib\site-pac
kages (from ple==0.0.1) (6.1.0)
Installing collected packages: ple
  Found existing installation: ple 0.0.1
    Uninstalling ple-0.0.1:
      Successfully uninstalled ple-0.0.1
  Running setup.py develop for ple
Successfully installed ple
```

3. Install pygame (1.9.6)

command: **pip install pygame**

Now, we can train our agent to play the game.

It is not necessary to create the code file in the in the PyGame-Learning-Environment folder, you could create the code file wherever you want.

Code

In [3]:

```
from ple.games.flappybird import FlappyBird
from ple import PLE
import matplotlib.pyplot as plt
import os
import numpy as np

%matplotlib inline
os.environ["SDL_VIDEODRIVER"] = "dummy"  # this line disable pop-out window
game = FlappyBird()
env = PLE(game, fps=30, display_screen=False)  # environment interface to game
env.reset_game()
```

```
libpng warning: iCCP: known incorrect sRGB profile
libpng warning: iCCP: known incorrect sRGB profile
libpng warning: iCCP: known incorrect sRGB profile
libpng warning: iCCP: known incorrect sRGB profile
libpng warning: iCCP: known incorrect sRGB profile
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libpng warning: iCCP: known incorrect sRGB profile
libpng warning: iCCP: known incorrect sRGB profile
libpng warning: iCCP: known incorrect sRGB profile
libpng warning: iCCP: known incorrect sRGB profile
```

In [4]:

```
# return a dictionary whose key is action description and value is action index
print(game.actions)
# return a list of action index (include None)
print(env.getActionSet())
```

```
{'up': 119}
[119, None]
```

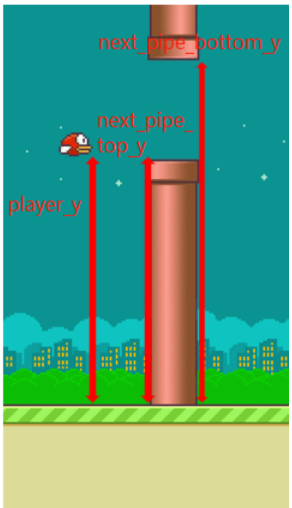
In [5]:

```
# a dictionary describe state
'''
    player y position.
    players velocity.
    next pipe distance to player
    next pipe top y position
```

```
    next pipe bottom y position
    next next pipe distance to player
    next next pipe top y position
    next next pipe bottom y position
'''
game.getGameState()
```

Out[5]:


```
{'player_y': 256,
 'player_vel': 0,
 'next_pipe_dist_to_player': 309.0,
 'next_pipe_top_y': 144,
 'next_pipe_bottom_y': 244,
 'next_next_pipe_dist_to_player': 453.0,
 'next_next_pipe_top_y': 160,
 'next_next_pipe_bottom_y': 260}
```

In [6]:

```
import math
import copy
from collections import defaultdict
MIN_EXPLORING_RATE = 0.01
MIN_LEARNING_RATE = 0.5


class Agent:

    def __init__(self,
                 bucket_range_per_feature,
                 num_action,
                 t=0,
                 discount_factor=0.99):
        self.update_parameters(t)  # init explore rate and learning rate
        self.q_table = defaultdict(lambda: np.zeros(num_action))
        self.discount_factor = discount_factor
        self.num_action = num_action

        # how to discretize each feature in a state
        # the higher each value, less time to train but with worser performance
        # e.g. if range = 2, feature with value 1 is equal to feature with value 0 bacause int(1/2) = int(0/2)
        self.bucket_range_per_feature = bucket_range_per_feature

    def select_action(self, state):
        # epsilon-greedy
        state_idx = self.get_state_idx(state)
        if np.random.rand() < self.exploring_rate:
            action = np.random.choice(num_action)  # Select a random action
        else:
            action = np.argmax(
                self.q_table[state_idx])  # Select the action with the highest q
        return action

    def update_policy(self, state, action, reward, state_prime):
        state_idx = self.get_state_idx(state)
        state_prime_idx = self.get_state_idx(state_prime)
        # Update Q_value using Q-learning update rule
        best_q = np.max(self.q_table[state_prime_idx])
        self.q_table[state_idx][action] += self.learning_rate * (
            reward + self.discount_factor * best_q - self.q_table[state_idx][action])

    def get_state_idx(self, state):
        # instead of using absolute position of pipe, use relative position
        state = copy.deepcopy(state)
        state['next_next_pipe_bottom_y'] -= state['player_y']
        state['next_next_pipe_top_y'] -= state['player_y']
        state['next_pipe_bottom_y'] -= state['player_y']
        state['next_pipe_top_y'] -= state['player_y']

        # sort to make list converted from dict ordered in alphabet order
        state_key = [k for k, v in sorted(state.items())]

        # do bucketing to decrease state space to speed up training
        state_idx = []
        for key in state_key:
            state_idx.append(
                int(state[key] / self.bucket_range_per_feature[key]))
        return tuple(state_idx)

    def update_parameters(self, episode):
        self.exploring_rate = max(MIN_EXPLORING_RATE,
                                  min(0.5, 0.99**((episode) / 30)))
        self.learning_rate = max(MIN_LEARNING_RATE, min(0.5, 0.99
                                                        ** ((episode) / 30)))

    def shutdown_explore(self):
        # make action selection greedy
        self.exploring_rate = 0
```

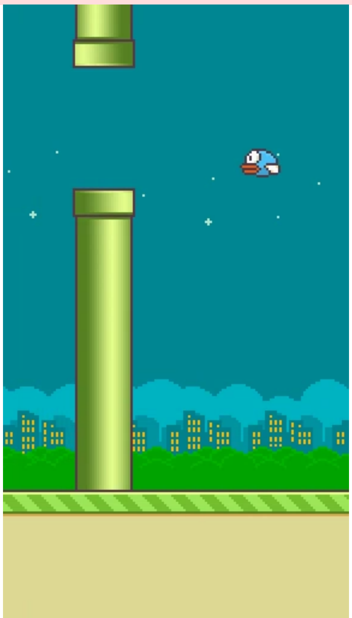
In [7]:
```
num_action = len(env.getActionSet())
bucket_range_per_feature = {
  'next_next_pipe_bottom_y': 40,
  'next_next_pipe_dist_to_player': 512,
  'next_next_pipe_top_y': 40,
  'next_pipe_bottom_y': 20,
  'next_pipe_dist_to_player': 20,
  'next_pipe_top_y': 20,
  'player_vel': 4,
  'player_y': 16
}
# init agent
agent = Agent(bucket_range_per_feature, num_action)
```

In [8]:
```
import moviepy.editor as mpy


def make_anim(images, fps=60, true_image=False):
    duration = len(images) / fps

    def make_frame(t):
        try:
            x = images[int(len(images) / duration * t)]
        except:
            x = images[-1]

        if true_image:
            return x.astype(np.uint8)
        else:
            return ((x + 1) / 2 * 255).astype(np.uint8)

    clip = mpy.VideoClip(make_frame, duration=duration)
    clip.fps = fps
    return clip
```

In [9]:
```
from IPython.display import Image, display

reward_per_epoch = []
lifetime_per_epoch = []
exploring_rates = []
learning_rates = []
print_every_episode = 500
show_gif_every_episode = 5000
NUM_EPISODE = 40000
for episode in range(0, NUM_EPISODE):

    # Reset the environment
    env.reset_game()

    # record frame
    frames = [env.getScreenRGB()]

    # for every 500 episodes, shutdown exploration to see performance of greedy action
    if episode % print_every_episode == 0:
        agent.shutdown_explore()

    # the initial state
    state = game.getGameState()
    # cumulate reward for this episode
    cum_reward = 0
    t = 0

    while not env.game_over():

        # select an action
        action = agent.select_action(state)

        # execute the action and get reward
        # reward = +1 when pass a pipe, -5 when die
        reward = env.act(env.getActionSet()[action])

        frames.append(env.getScreenRGB())

        # cumulate reward
        cum_reward += reward

        # observe the result
        state_prime = game.getGameState()  # get next state

        # update agent
        agent.update_policy(state, action, reward, state_prime)

        # Setting up for the next iteration
        state = state_prime
        t += 1

    # update exploring_rate and learning_rate
    agent.update_parameters(episode)

    if episode % print_every_episode == 0:
        print("Episode {} finished after {} time steps, cumulated reward: {}, exploring rate: {}, learning rate: {}".format(
            episode,
            t,
            cum_reward,
            agent.exploring_rate,
            agent.learning_rate
        ))
        reward_per_epoch.append(cum_reward)
        exploring_rates.append(agent.exploring_rate)
        learning_rates.append(agent.learning_rate)
        lifetime_per_epoch.append(t)

    # for every 5000 episode, record an animation
    if episode % show_gif_every_episode == 0:
        print("len frames:", len(frames))
        clip = make_anim(frames, fps=60, true_image=True).rotate(-90)
        display(clip.ipython_display(fps=60, autoplay=1, loop=1))
```

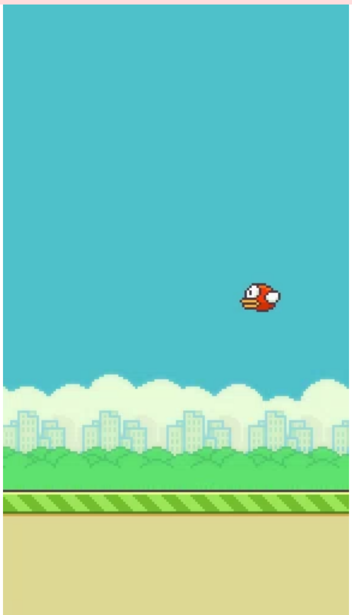
```
Episode 0 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.5, learning rate: 0.5
len frames: 63
Moviepy - Building video __temp__.mp4.
Moviepy - Writing video __temp__.mp4
```

```
Moviepy - Done !
Moviepy - video ready __temp__.mp4
```



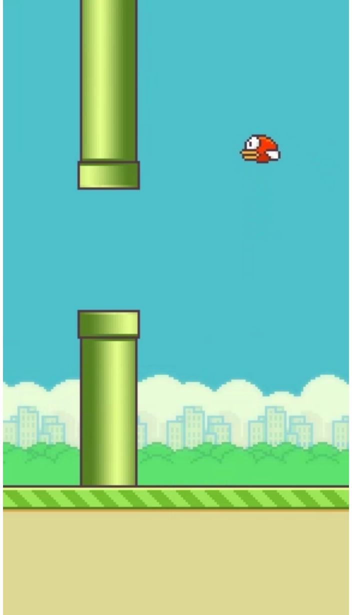
```
Episode 500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.5, learning rate: 0.5
Episode 1000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.5, learning rate: 0.5
Episode 1500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.5, learning rate: 0.5
Episode 2000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.5, learning rate: 0.5
Episode 2500 finished after 59 time steps, cumulated reward: -5.0, exploring rate: 0.43277903725889943, learning rate: 0.5
Episode 3000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.3660323412732292, learning rate: 0.5
Episode 3500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.30957986252419073, learning rate: 0.5
Episode 4000 finished after 67 time steps, cumulated reward: -4.0, exploring rate: 0.26183394327157605, learning rate: 0.5
Episode 4500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.22145178723886091, learning rate: 0.5
Episode 5000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.18729769509073985, learning rate: 0.5
len frames: 63
Moviepy - Building video __temp__.mp4.
Moviepy - Writing video __temp__.mp4
```

```
Moviepy - Done !
Moviepy - video ready __temp__.mp4
```



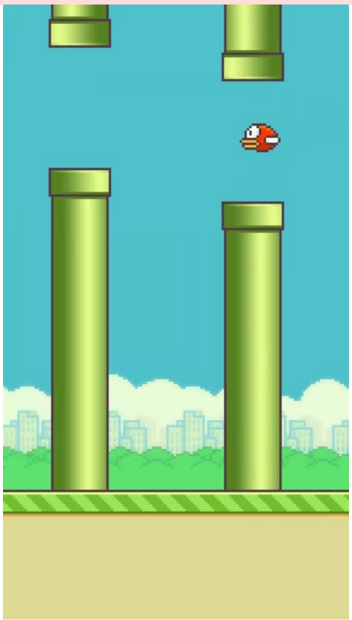
```
Episode 5500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.15841112426184903, learning rate: 0.5
Episode 6000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.13397967485796172, learning rate: 0.5
Episode 6500 finished after 52 time steps, cumulated reward: -5.0, exploring rate: 0.11331624189077398, learning rate: 0.5
Episode 7000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.09583969128049684, learning rate: 0.5
Episode 7500 finished after 98 time steps, cumulated reward: -4.0, exploring rate: 0.08105851616218128, learning rate: 0.5
Episode 8000 finished after 56 time steps, cumulated reward: -5.0, exploring rate: 0.0685570138491429, learning rate: 0.5
Episode 8500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.05798359469728905, learning rate: 0.5
Episode 9000 finished after 65 time steps, cumulated reward: -5.0, exploring rate: 0.04904089407128572, learning rate: 0.5
Episode 9500 finished after 72 time steps, cumulated reward: -4.0, exploring rate: 0.04147740932356356, learning rate: 0.5
Episode 10000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.03508042658630376, learning rate: 0.5
len frames: 63
Moviepy - Building video __temp__.mp4.
Moviepy - Writing video __temp__.mp4
```

```
Moviepy - Done !
Moviepy - video ready __temp__.mp4
```



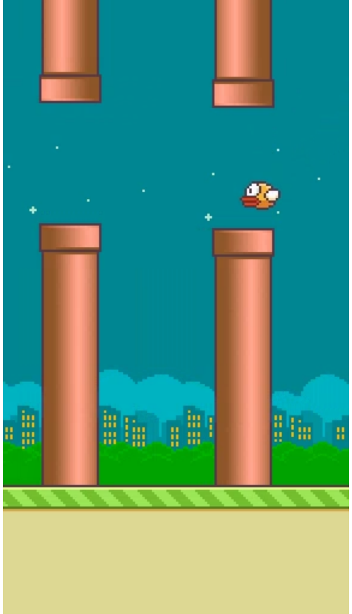
```
Episode 10500 finished after 61 time steps, cumulated reward: -5.0, exploring rate: 0.029670038450977102, learning rate: 0.5
Episode 11000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.02509408428990297, learning rate: 0.5
Episode 11500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.021223870922486707, learning rate: 0.5
Episode 12000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.017950553275045137, learning rate: 0.5
Episode 12500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.015182073244652034, learning rate: 0.5
Episode 13000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.012840570676248398, learning rate: 0.5
Episode 13500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.010860193639877882, learning rate: 0.5
Episode 14000 finished after 44 time steps, cumulated reward: -5.0, exploring rate: 0.01, learning rate: 0.5
Episode 14500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.01, learning rate: 0.5
Episode 15000 finished after 133 time steps, cumulated reward: -3.0, exploring rate: 0.01, learning rate: 0.5
len frames: 134
Moviepy - Building video __temp__.mp4.
Moviepy - Writing video __temp__.mp4
```

```
Moviepy - Done !
Moviepy - video ready __temp__.mp4
```

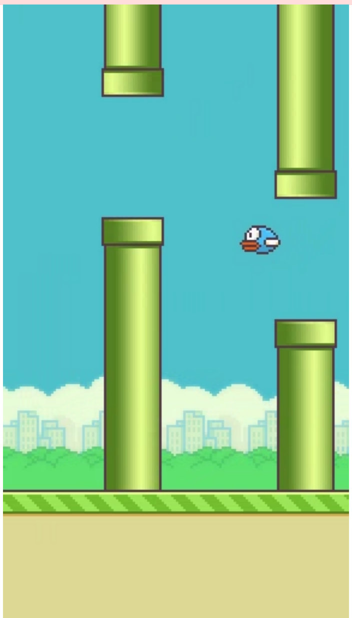


```
Episode 15500 finished after 221 time steps, cumulated reward: 0.0, exploring rate: 0.01, learning rate: 0.5
Episode 16000 finished after 134 time steps, cumulated reward: -3.0, exploring rate: 0.01, learning rate: 0.5
Episode 16500 finished after 627 time steps, cumulated reward: 10.0, exploring rate: 0.01, learning rate: 0.5
Episode 17000 finished after 134 time steps, cumulated reward: -3.0, exploring rate: 0.01, learning rate: 0.5
Episode 17500 finished after 401 time steps, cumulated reward: 4.0, exploring rate: 0.01, learning rate: 0.5
Episode 18000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.01, learning rate: 0.5
Episode 18500 finished after 211 time steps, cumulated reward: -1.0, exploring rate: 0.01, learning rate: 0.5
Episode 19000 finished after 247 time steps, cumulated reward: 0.0, exploring rate: 0.01, learning rate: 0.5
Episode 19500 finished after 586 time steps, cumulated reward: 9.0, exploring rate: 0.01, learning rate: 0.5
Episode 20000 finished after 663 time steps, cumulated reward: 11.0, exploring rate: 0.01, learning rate: 0.5
len frames: 664
Moviepy - Building video __temp__.mp4.
Moviepy - Writing video __temp__.mp4
```

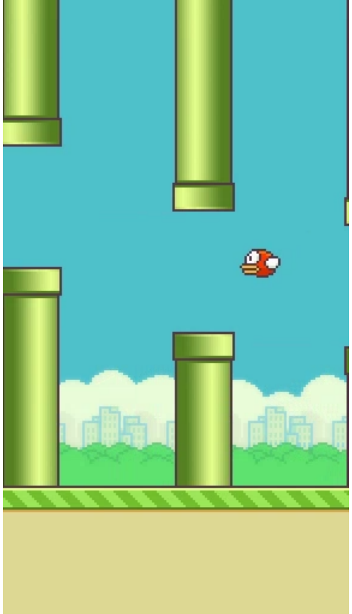
```
Moviepy - Done !
Moviepy - video ready __temp__.mp4
```



```
Episode 20500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.01, learning rate: 0.5
Episode 21000 finished after 776 time steps, cumulated reward: 14.0, exploring rate: 0.01, learning rate: 0.5
Episode 21500 finished after 1531 time steps, cumulated reward: 34.0, exploring rate: 0.01, learning rate: 0.5
Episode 22000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.01, learning rate: 0.5
Episode 22500 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.01, learning rate: 0.5
Episode 23000 finished after 134 time steps, cumulated reward: -3.0, exploring rate: 0.01, learning rate: 0.5
Episode 23500 finished after 175 time steps, cumulated reward: -2.0, exploring rate: 0.01, learning rate: 0.5
Episode 24000 finished after 62 time steps, cumulated reward: -5.0, exploring rate: 0.01, learning rate: 0.5
Episode 24500 finished after 1531 time steps, cumulated reward: 34.0, exploring rate: 0.01, learning rate: 0.5
Episode 25000 finished after 211 time steps, cumulated reward: -1.0, exploring rate: 0.01, learning rate: 0.5
len frames: 212
Moviepy - Building video __temp__.mp4.
Moviepy - Writing video __temp__.mp4

Moviepy - Done !
Moviepy - video ready __temp__.mp4
```

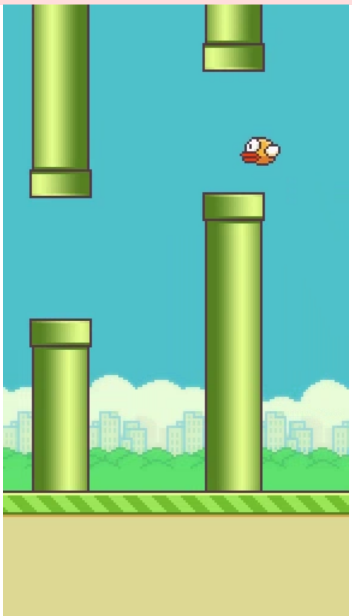


```
Episode 25500 finished after 134 time steps, cumulated reward: -3.0, exploring rate: 0.01, learning rate: 0.5
Episode 26000 finished after 175 time steps, cumulated reward: -2.0, exploring rate: 0.01, learning rate: 0.5
Episode 26500 finished after 187 time steps, cumulated reward: -1.0, exploring rate: 0.01, learning rate: 0.5
Episode 27000 finished after 812 time steps, cumulated reward: 15.0, exploring rate: 0.01, learning rate: 0.5
Episode 27500 finished after 134 time steps, cumulated reward: -3.0, exploring rate: 0.01, learning rate: 0.5
Episode 28000 finished after 636 time steps, cumulated reward: 11.0, exploring rate: 0.01, learning rate: 0.5
Episode 28500 finished after 925 time steps, cumulated reward: 18.0, exploring rate: 0.01, learning rate: 0.5
Episode 29000 finished after 2322 time steps, cumulated reward: 55.0, exploring rate: 0.01, learning rate: 0.5
Episode 29500 finished after 586 time steps, cumulated reward: 9.0, exploring rate: 0.01, learning rate: 0.5
Episode 30000 finished after 1483 time steps, cumulated reward: 33.0, exploring rate: 0.01, learning rate: 0.5
len frames: 1484
Moviepy - Building video __temp__.mp4.
Moviepy - Writing video __temp__.mp4

Moviepy - Done !
Moviepy - video ready __temp__.mp4
```



```
Episode 30500 finished after 554 time steps, cumulated reward: 9.0, exploring rate: 0.01, learning rate: 0.5
Episode 31000 finished after 2923 time steps, cumulated reward: 71.0, exploring rate: 0.01, learning rate: 0.5
Episode 31500 finished after 214 time steps, cumulated reward: -1.0, exploring rate: 0.01, learning rate: 0.5
Episode 32000 finished after 5409 time steps, cumulated reward: 137.0, exploring rate: 0.01, learning rate: 0.5
Episode 32500 finished after 1496 time steps, cumulated reward: 34.0, exploring rate: 0.01, learning rate: 0.5
Episode 33000 finished after 2223 time steps, cumulated reward: 53.0, exploring rate: 0.01, learning rate: 0.5
Episode 33500 finished after 1264 time steps, cumulated reward: 27.0, exploring rate: 0.01, learning rate: 0.5
Episode 34000 finished after 1545 time steps, cumulated reward: 35.0, exploring rate: 0.01, learning rate: 0.5
Episode 34500 finished after 2168 time steps, cumulated reward: 51.0, exploring rate: 0.01, learning rate: 0.5
Episode 35000 finished after 324 time steps, cumulated reward: 2.0, exploring rate: 0.01, learning rate: 0.5
len frames: 325
Moviepy - Building video __temp__.mp4.
Moviepy - Writing video __temp__.mp4
```

```
Moviepy - Done !
Moviepy - video ready __temp__.mp4
```



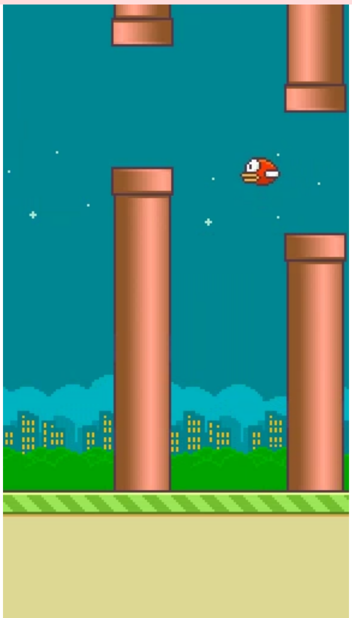
```
Episode 35500 finished after 73 time steps, cumulated reward: -4.0, exploring rate: 0.01, learning rate: 0.5
Episode 36000 finished after 1680 time steps, cumulated reward: 38.0, exploring rate: 0.01, learning rate: 0.5
Episode 36500 finished after 3314 time steps, cumulated reward: 82.0, exploring rate: 0.01, learning rate: 0.5
Episode 37000 finished after 751 time steps, cumulated reward: 14.0, exploring rate: 0.01, learning rate: 0.5
Episode 37500 finished after 329 time steps, cumulated reward: 3.0, exploring rate: 0.01, learning rate: 0.5
Episode 38000 finished after 324 time steps, cumulated reward: 2.0, exploring rate: 0.01, learning rate: 0.5
Episode 38500 finished after 627 time steps, cumulated reward: 10.0, exploring rate: 0.01, learning rate: 0.5
Episode 39000 finished after 586 time steps, cumulated reward: 9.0, exploring rate: 0.01, learning rate: 0.5
Episode 39500 finished after 6589 time steps, cumulated reward: 169.0, exploring rate: 0.01, learning rate: 0.5
```

In [10]:

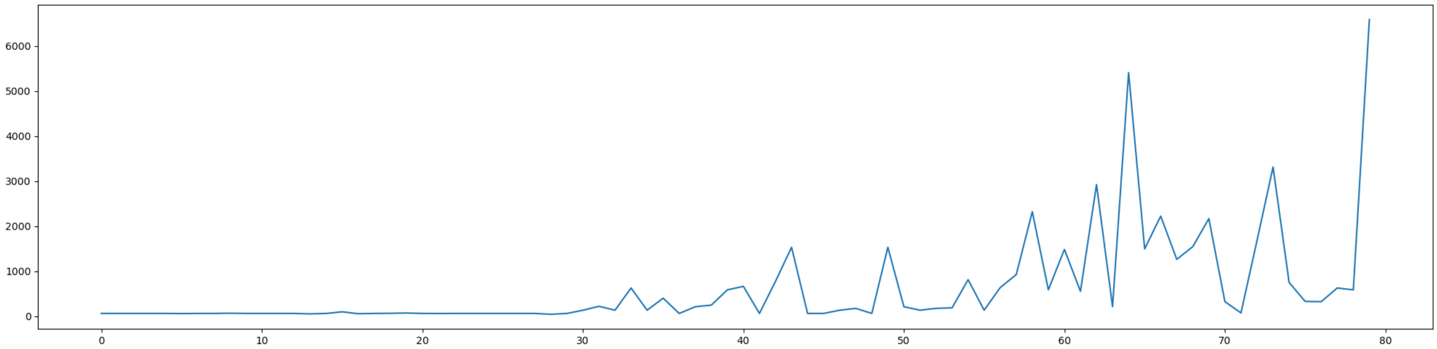
```python
def demo():
    # Reset the environment
    env.reset_game()

    # record frame
    frames = [env.getScreenRGB()]

    # shutdown exploration to see performance of greedy action
    agent.shutdown_explore()

    # the initial state
    state = game.getGameState()

    while not env.game_over():
        # select an action
        action = agent.select_action(state)

        # execute the action and get reward
        reward = env.act(env.getActionSet()[action])

        frames.append(env.getScreenRGB())
```

```
        # observe the result
        state_prime = game.getGameState()  # get next state

        # Setting up for the next iteration
        state = state_prime

    clip = make_anim(frames, fps=60, true_image=True).rotate(-90)
    display(clip.ipython_display(fps=60, autoplay=1, loop=1))


demo()
```

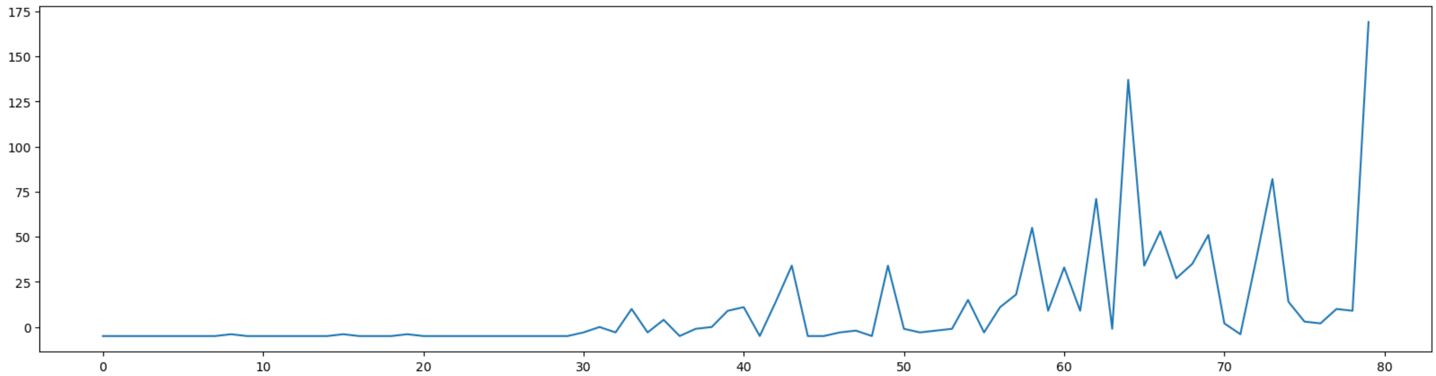
```
Moviepy - Building video __temp__.mp4.
Moviepy - Writing video __temp__.mp4

Moviepy - Done !
Moviepy - video ready __temp__.mp4
```



In [11]:
```
# plot life time against training episodes
fig, ax1 = plt.subplots(figsize=(20, 5))
plt.plot(range(len(lifetime_per_epoch)), lifetime_per_epoch)
fig.tight_layout()
plt.show()
```



In [12]:
```
# plot reward against training episodes
fig, ax1 = plt.subplots(figsize=(20, 5))
plt.plot(range(len(reward_per_epoch)), reward_per_epoch)
plt.show()
```



Reference Reading:

Toturials:

- An example of value iteration
- An example of Q-learning(Flappy Bird)
- on-policy vs off-policy

- Cliff Walking(Q-learning vs SARSA)

Book：

- Reinforcement Learning: An Introduction

Assignment

What you should do:

- Change the update rule from Q-learning to SARSA(**with the same episodes**).
- Give a brief report to discuss the result(compare Q-learning with SARSA based on the game result).

Requirements:

- Write a brief report in the notebook
- Upload both ipynb and html to google drive
    - Lab14_{student_id}.ipynb
    - Lab14_{student_id}.mp4
- Share your drive's link via eeclass
    - Please make sure that TA can access your google drive!!!
- Deadline: 2022-12-15(Thur) 23:59.

In [ ]: