The Distribution of Wealth in a Life-Cycle Model with Durables

Master Thesis

in Economics

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1 Ïntroduction

1.1 UnterÜberschrift

UnterUnterÜberschrift

Hier steht mal ein Text. Eine Möglichkeit des Zitierens ist, direkt im Text die Quelle anzugeben (see Name, 2006, pp.225-369). Andererseits schreiben Mustermann and Musterfrau (2006), dass man auch so zitieren kann.
In der Matheumgebung kann der oben (im Latex-Quellcode) genannte Shortcut verwendet werden, um aus einem normalen β ein fettes $\boldsymbol{\beta}$ zu machen. Wichtige Gleichungen, die nochmal verwendet werden, sollten nummeriert werden, z.B.

$$b = (x'x)x'y \; . \tag{1}$$

Nebensächlicheres, auf das man sich nicht mehr bezieht, bleibt unnummeriert, also

$$a = 1 \; .$$

Nun kann man direkt auf die erste Gleichung als Gleichung (1) verweisen mittels des zugewiesenen labels. In gleicher Weise kann man auf die Graphik 1 bzw. Graphik 2 verweisen. Die Tilde zwischen „Graphik“ und „`\ref{fig:andereGraphik}`“ verhindert, dass bei Zeilenumbrüchen die Zahl als erstes alleine in die neue Zeile rutscht. Ganz analog für die Tabelle 1.

2 Benötigte Programme

unter Windows:

- Miktex (`http://miktex.org/`)
- ein Editor, je nach Geschmack z.B. WinEdt (`http://www.winedt.com/`; kostenpflichtige Studentenversion) oder einen der vielen anderen verfügbaren, z.B. TeXnicCenter (`www.texniccenter.org/`)
- ghostview und ghostscript (`http://pages.cs.wisc.edu/~ghost/`

unter Linux:

- Latex ist in den meisten Verteilungen enthalten, z.B. tetex in Suse (ggf. über yast nachinstallieren)
- als Editor empfiehlt sich z.B. Kile

für die Literatur:
z.B. JabRef (`http://jabref.sourceforge.net/`)

3 Präsentationen

Beispiele für Präsentationen mit dem Beamer-Style:
`http://www.informatik.uni-freiburg.de/~frank/latex-kurs/latex-kurs-3/Latex-Kurs-3.html`

4 Literature Overview

Investigate empirical predictions of the life-cycle incomplete markets model (Gourinchas and Parker (2002); Cagetti (2003); Castaneda, Diaz-Gimenez, and Rios-Rull (2003); Yang (2009); Kaplan and Violante (2010); Hintermaier and Koeniger (2011)), thus contribute to the literature that (note copy paste.... demonstrate that a plausibly parameterized version of their models can quantitatively explain empirical findings as arising from rational choices of consumers facing an increasing wage profile and income uncertainty.) The model is based on the classic income-fluctuation problem in which the consumer faces a stochastic income process and decides in every period how much to save and how much to consume. Important contributions to the literature are Deaton (1991), Carroll (1992,1997), Gourinchas and Parker (2002). (note copy paste... Following Bewley (1986), this problem has been embedded by Huggett (1993) and Aiyagari (1994) into a general equilibrium framework, giving rise to the endogenous determination of the interest rate as well as a nontrivial income, wealth, and consumption distribution at equilibrium.) (Note copy paste... Furthermore, this paper relates to other literature such as: Endogenous borrowing constraint: The specification of the borrowing constraint in FernÃ¡ndez-Villaverde and Krueger (2011) is adapted from the recent endogenous incomplete-markets literature: Kehoe and Levine (1993), Kocherlakota (1996), Krueger and Perri (1999, 2006), and Alvarez and Jermann (2000). Lustig (2004) also has a model with durable assets and an endogenous borrowing constraint to explain the equity premium puzzle, however agents have full access to Arrow securities and are infinitely lived. Literature on optimal portfolio choice in the presence of consumer durables: Grossman and Laroque (1990), Eberly (1994), Chah et al. (1995), and Flavin and Yamashita (2002).)
Note: missing citations, not all of the above are cited properly!!!!!
Literature on wealth distribution!!!!!

5 The Life-Cycle Model

In the following section I discuss the economic model considered and outline the most important considerations made within the literature. Firstly, I discuss the life-cycle modeling and the literature, which applies life-cycle and to be more exact imperfect markets models in the context of wealth distributions. Secondly, I present my modeling choices and solution methods applied for this particular problem.

5.1 Modeling Literature

5.2 The Model

Before going into a more detailed description of the model at hand it is important to understand the modelling choice. Why a life-cycle model with an imperfect market structure? Why include durables as an additional asset choice to the more standard approach, which does summarize all assets in a one period bond, such as for example Hintermaier and Koeniger (2011)?
To answer the first question I will mainly refer to the the literature. The second will be discussed in a more detailed manner in the following section, which treads the life-cycle profiles and especially the importance of modeling durables.

5.2.1 Imperfect market structure

The choice of an exogenously determined imperfect market structure allows for a straight forward answer. It comes down to the economic question one poses. The aim here is to demonstrate that such a model can quantitatively explain the empirical net-wealth distribution in the US. The role the imperfect market structure, entering as a limited choice of assets to fully insure against idiosyncratic wage shocks, poses, is thus to produce an endogenous wealth distribution. The savings are fist and foremost driven by a precautionary motive to insure for future wage shocks. Knowing that the income may drop in the future consumers save today to achieve a more stable consumption across their live. In a complete market framework consumers could perfectly diversify away idiosyncratic risks (Tideman and Weber, 2010) (and thus would not build a buffer stock of savings for future shocks.) (???????? is this exact?????) The hypothesis of the complete market framework has been rejected by the vast majority of empirical research (Tideman and Weber, 2010). Deaton and Paxson (1994) show that the life cycle profile of consumption inequality is increasing with age, a fact that would be inconsistent with complete markets. While in some rare cases - such as when only studying aggregates, it may still be sufficient to consider a complete market framework, when considering wealth-distributive questions, the imperfect market assumption is both necessary from a theoretical perspective and well-founded from an empirical perspective. It remains to discuss the whether it makes sens to treat distributive issues from a life-cycle point of view.

5.2.2 Life-Cycle

As Deaton and Paxson (1994) show, consumption inequality seems to vary with age. Further empirical research does support these findings, indicating that some of the differences in earnings, income, and wealth across households can be attributed to differences in the household's age Rios-Rull, Kuhn, et al. (2016) (FIND BETTER REFERENCES?). Hintermaier and Koeniger (2011) Show that the average net-wealth increases with wealth and that the dispersion of wealth falls with age. Moreover, there is a vast theoretical literature using a different versions of the life-cycle model to discuss distributive questions. Gourinchas and Parker (2002); Cagetti (2003); Castaneda et al. (2003); Yang (2009); Kaplan and Violante (2010);Hintermaier and Koeniger (2011) (CITE MORE RECENT LITERATURE!!!) while there are also some that use infinite horizon models....

Finally, the choice to study the empirical validity of a life-cycle model, which produces a net-wealth distribution arising from rational choices of consumers subject to income uncertainty, moreover allows for a detailed analysis of consumption and savings behavior over a person's live-cycle. A coherent framework incorporating the behavior of household's over their lifespan is paramount for the study of re-distributive policies. (Cite....) These factors thus support the choice of a life-cycle model.

5.2.3 Partial Equilibrium

Last but not least, partial equilibrium as modeling choice has to be discussed briefly. (????????????????)

(copy paste from hintermaier 2011 Note that we assume that changes in the domestic supply of assets do not affect do not affect the interest rate. As in a small-open economy, this price is determined exogenously (on the world markets.) [1])

5.2.4 Consumer's Problem

As established above, the baseline model in question is a life-cycle model with an imperfect market structure. I closely follow Hintermaier and Koeniger (2010).[2] While there algorithm allows for both a live-cycle and infinite-horizon models, the example model presented in their paper uses the latter. For reasons discussed above, I will in this case opt for a live-cycle formulation. Finally, the baseline model does not include adjustment costs. These are considered in a later section of the model.

[1] This assumption is not restrictive for our purposes since we observe the price in our ex post analysis for the period 1983-2007. Hence, the supply of assets and the observed price suffice to determine the equilibrium asset quantities. This allows us to be agnostic about the price elasticity of asset demand.

[2] In order to facilitate comparability, I will closely follow the notation used in their paper.

There is a continuum of risk averse consumers with a finite time horizon. At age 90 consumers die with certainty, younger consumers between the age of 26 and 90 may die with probability $\pi_j < 1$ at age j. Until retirement the labor income of each individual i y_{ij} is stochastic. After the age 65 is completed, they retire with certainty and receive individual-specific retirement benefits b_i.[3] Consumers derive utility from a durable good d and a non-durable good c. The utility function $U(c, d)$ is strictly concave, increasing and non-separable in c and d (REALLY??? When not??? all three true???). In each period consumption c_j, the durable good d_{j+1} and the financial risk-free asset a_{j+1} are chosen after receiving the income. Agents then derive utility from consumption and durable services, before the assets pay returns. While the liquid assets return an interest rate r durables depreciate at rate *delta*. As previously discussed, such a market structure implies that markets are incomplete and agents will accumulate a buffer stock in order to save for rainy days. The ex-ante identical consumers thus will thus, depending on their income trajectory, end up with different portfolios of the endogenous state variables. The model is therefore suited to match empirical facts about net worth positions since it does produce an endogenous distribution of portfolios, which differs in age and income. Moreover, (THE ADVANTAGE OF DURABLES) besides providing services and thus allowing consumers to derive utility from durables, their durability implies, that they may be used as collateral. It is assumed that all credit needs to be collateralized.[4]
The collateral constraint takes the form:

$$\underbrace{\mu(1-\delta)d_{j+1} + \gamma \underline{y}}_{collateral} \geq -(1+r)a_{j+1} \tag{2}$$

where $\underline{y}$ is the minimum labor income realization [5] and $\mu \in [0, 1) and \gamma \in [0, 1)$ are the respective fractions of the durable stock and of minimum labor income, which can be collateralized. [6]
It is useful to rewrite the constraint in terms of total wealth and future durable holdings. Total wealth available to a consumer in each period is

$$x_j \equiv (1+r)a_j + (1-\delta)d_j, \tag{3}$$

[3]The construction of these benefits is discussed in the Appendix.

[4]About 85% of household debt in the Survey of Consumer Finances 2004 is secured by collateral.(Hintermaier and Koeniger, 2010)

[5]CHECK THIS WITH CODE!!!

[6]It is important to note that this constraint guarantees full repayment by consumers and thus acts as non-bankruptcy constraint. This is assured by the timing assumptions made above –The lender does know the financial portfolio choice (a_{j+1}, d_{j+1}) and the minimum of the support of the income distribution $\underline{y}$, however future individual income draws y_{j+1} do remain obscure to him. Finally, note that only choices in j determine, whether the constraint is binding or not.

rewritten the constraint then looks like this:

$$x_{j+1} \geq -\gamma \underline{y} + (1-\mu)(1-\delta)d_{j+1} \tag{4}$$

One now sees that total wealth needs to be larger than the negative value of the minimum fraction of income that can be collateralized and the fraction of wealth which cannot be used as collateral. Furthermore, under the assumption that $\mu < 1$ d_{j+1} is determined for a given x_{j+1}, when the constraint binds.
Finally, the specification of the budget constraint, follows from the above:

$$a_{j+1} + d_{j+1} + c_j = x_j + y_j \tag{5}$$

The recursive formulation of the household problem I will proceed by depicting the case prior to retirement. (SPECIFY RETIREMENT CASE!!!) We let T^r denote the first period of retirement. The Bellman equation prior to retirement, if $j < T^r$thus is:

$$v_j(x_j, d_j, y_j) = \max_{a_{j+1}, d_{j+1}} \left[U(\underbrace{x_j + y_j - a_{j+1} - d_{j+1}}_{c_j}, d_j) + \hat{v}_j(x_{j+1}, d_{j+1}, y_j) \right] \tag{6}$$

where the expected next period value function is discouted by the product of the probability of survival $(1-\pi_j)$ and the discount factor β

$$\hat{v}_j(x_{j+1}, d_{j+1}, y_j) \equiv \beta(1-\pi_j)E_j v_{j+1}(x_{j+1}, d_{j+1}, y_{j+1}),^{7} \tag{7}$$

with the constraints:

$$a_{j+1} + d_{j+1} + c_j = x_j + y_j, \tag{8}$$

$$x_{j+1} = (1+r)a_{j+1} + (1-\delta)d_{j+1}, \tag{9}$$

$$x_{j+1} \geq -\gamma \underline{y} + (1-\mu)(1-\delta)d_{j+1}, \tag{10}$$

(WHAT HAPPENS WITH MIN Y IN RETIREMENT????)

$$d_{j+1} \geq d_{min}, \tag{11}$$

FROM RED PAPER HELP TO SPECIFY CASE WITH RETIREMENT!!! We specify our model in discrete time. Rearranging the budget constraint, consumption of individual i with age j is

$$c_{ij} = \begin{cases} (1+r)a_{i,j-1} - a_{ij} + y_{ij} & \text{if } j < T^r, \\ (1+r)a_{i,j-1} - a_{ij} + b(z_{i,T^r-1}) & \text{if } j \geq T^r, \end{cases} \tag{12}$$

where $b(z_{i,T^r-1})$ are the retirement benefits. These benefits depend on the last realization of labor income before retirement z_{i,T^r-1} as we explain further in the next section. Defining cash-on-hand as

$$x_{ij} = \begin{cases} (1+r)a_{i,j-1} + y_{ij} & \text{if } j < T^r, \\ (1+r)a_{i,j-1} + b(z_{i,T^r-1}) & \text{if } j \geq T^r, \end{cases} \tag{13}$$

we can write the Bellman equation as

$$V_j(x_{ij}, y_{ij}) = \max_{a_{ij} \geq 0} [u(\underbrace{x_{ij} - a_{ij}}_{c_{ij}}) + \beta(1-\delta_j) E_j V_{j+1}(x_{i,j+1}, y_{i,j+1})], \tag{14}$$

where we restrict the choice of assets to $a_{ij} \geq 0$, as motivated in the previous section.)

5.2.5 Numerical algorithm

The problem above does not provide a closed-form solution and thus the solution has to be approximated numerically. As already mentioned above, the model is based on Hintermaier and Koeniger (2010) and I can therefore directly apply their algorithm. They expand the endogenous gridpoints method (EGM) proposed by Carroll (2006) for a problem with two states and thus providing a very efficient algorithm to solve models with durables and collateralized debt. [8]
(NOTE THAT ALGORITHM MUST BE CHANGED FOR CASE WITH ADJUSTMENT COSTS)
Given the life-time sequences for income (as described in the income process section) y_{kj} and survival probabilities (CHECK ON HOW TO BEST WRITE DOWN!) $(1-\pi_j)$, $\forall j = 1, ..., T$ (AND ALL MARKOV STATES k) the algorithm starts with an exogenous grid for the future state x', $G_{x'} \equiv x'_1, x'_2, ..., x'_I$, the current state d, $G_d \equiv d_1, d_2, ..., d_J$. The algorithm starts iterates recursively over the policy function starting with the last period $j = T$ in which the consumer sells all durables and liquid assets to consume before death. It thus holds that $x' = d' = a' = 0$ and thus the initial consumption policy is $c(x, d_j, y_{kj}) = x + y_{j,k}$.For each period t < T-1, the algorithm proceeds as follows. In a first step it maps x' into d' and in a second step it solves for the current choice c and state x from optimal future combinations (x',d'). Moreover, the values for the

[8]Solving such models has been considered particluarly expensive and challenging, as durables enter as additional state (if non-separability utility from durables and non-durable or adjustemnt costs are allowed for) and durables increase the dimensionality of the portfolio choice.(Hintermaier and Koeniger, 2010)

exogenous grid are chosen such that the results are not affected. (BE MORE PRECISE INDICATE DIFFERENCES BETWEEN POST/PRE RETIREMENT (expectation)! NOT NECESSARY FROM HERE ON. HOWEVER MAYBE BE MORE PRECISE ABOVE!

For each period t < T-1, the algorithm proceeds as follows:

Step 1: Mapping x' into d' Using the optimal relationship,

$$K_{ijk}\left((1+r)\left(1+\frac{\partial\Psi(d'_{ijk},d_j)}{\partial d'}\right)-\mu(1-\delta)\right)-\eta_{ijk}=-\left(\frac{\partial\Psi(d'_{ijk},d_j)}{\partial d'}(1+r)+r+\delta\right)\frac{\partial\hat{v}(x'_i,d'_{ijk},y_k)}{\partial x'}+\frac{\partial\hat{v}(x'_i,d'_{ijk},y_k)}{\partial d'}, \quad (15)$$

in the algorithm determines the values of, d'_{ijk} for a given y_k and d_j, which correspond to each of the exogenous gridpoints x'_i.[9]

There are three possible cases for each y_k and d_j, to compute the optimal d'_{ijk}.[10]

Case 1 –Neither the collateral nor the constraint on durables are binding Hence, $K_{ijk} = \eta_{ijk} = 0$ and thus the there exists a d'_{ijk} for which the right hand side of 15 is 0.

Case 2 –the right hand side of 15 is positive jklh

Case 3 –the right hand side of 15 is negative lkjg

Step:2 Obtaining current choice c and state x from optimal future combinations (x',d')

[9]The partial derivatives are obtained by exploiting the envelope conditions. When considering adjustment costs one therefore has to solve for $\partial\Psi(d'',d')/\partial d'$. In the first iteration this is done by making use of terminal condition $d''=0$. For every iteration that follows d'' is then replaced by the previously computed optimal policy.(COPY PASTE: Note that with adjustment costs, full decumulation is feasible for all x,d and y if $\mu < 1-(1-\delta)\alpha/2$, which is satisfied for the chosen parameters below!!!!!!!!!!!

[10]Note that the optimal solution necessarily lies in the interval imposed by the collateral constraint and the lower bound on d', $[d_{min},\bar{d}_i]$, where $\bar{d}_i \equiv \frac{x'_i+\gamma y}{(1-\mu)(1-\delta}$.

5.2.6 Comparing the simulation output with SCF data

5.2.7 Calibration

6 Life-Cycle Profiles

7 The Wealth Distribution

8 Conclusion

References

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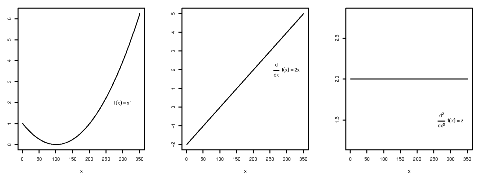
Figure 1: titel der Graphik



die Graphik sollte beschrieben werden, sodass man ohne den Text vorne zu lesen weiß, worum es geht: panel 1 zeigt die Funktion, panel 2 die erste Ableitung und Panel 3 die zweite Ableitung

Figure 2: titel der Graphik

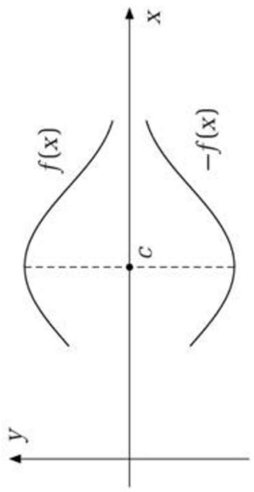


Table 1: Der Title der Tabelle

Eine	kleine	Tabelle
Text links	mittig	oder rechts
	unterstrichen	
über zwei Spalten		dritte Spalte