The Distribution of Wealth in a Life-Cycle Model with Durables

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1 Introduction

The importance of distributional questions
Such models provide the basis for the analysis of redistributional policies. They shed light on the underlying mechanisms leading to unequal distributions of wealth and consumption. In order to improve policy making it is crucial in understanding these mechanisms.

The main goal of this paper is to contribute to the net-worth literature by investigating the empirical prediction of the net-worth cross-section in the US. Doing so, using detailed micro-data for income as well as the net-worth distributions for three different age groups thus allows me to accurately illustrate, what part of the distribution can be explained by the model and where the model presents shortcomings. Moreover, I look at two important determinants, the income risk and the LTV-ratio by simulating unexpected changes to illustrate how these affect the distribution across different age groups and the consumption-savings decision across the life-cycle. Finally, I show that the initial portfolio allocation is particularly important to predict the the net-worth distribution of the youngest age group when calibrating the model.

I use detailed consumer data from Hintermaier and Koeniger (2011) to evaluate the out of sample prediction of the evolution of the net-worth distribution in the US in a model abstracting from durables. Modeling durables is particularly interesting, since they exhibit a dual-role. On the one hand consumers derive consumption services by holding durables, one the other hand, they may use these as collateral to borrow and thus transfer wealth across time. FernÃ¡ndez-Villaverde and Krueger (2011) show that this dual role is important to reproduce the non-durable consumption life-cycle profile. Díaz and Luengo-Prado (2010) investigates the role of housing in the wealth-distribution in the US using an infinite-horizon model. I show that my model is able to accurately reproduce life-cycle profiles of the portfolio composition as well as non-druable consumption over the life-cycle observed in the data. In order to do so, I compare the simulated life-cycle profiles to the empirical data presented in other literature.

2 Modeling the wealth distribution

2.1 Net-Worth

The modeling of distributions requires some form of heterogeneity. The standard model used to achieve heterogeneity and thus differences among agents is the standard incomplete markets model (SIM) (Heathcote, Storesletten, and Violante, 2009). In this model, ex-ante identical consumers are subject to uninsurable idiosyncratic income risks. In order to insure themselves against these income shocks agents accumulate precautionary savings. Since income histories vary according to the agent, these accumulate different amounts of savings resulting in an endogenous wealth distribution. This model is mainly based on the works of Bewley, Aiyagari, Hugget and Imorphoglu and thus sometimes refered to as Bewely-Aiyagari-Hugget-Imorghoroglu model.

An important application of these models is the evaluation of the empirical prediction of the wealth-distribution.[1] Investigate empirical predictions of the life-cycle incomplete markets model. Gourinchas and Parker (2002); Castaneda, Diaz-Gimenez, and Rios-Rull (2003) for predictions of the cross-section or Hintermaier and Koeniger (2011) for the study of distributional dynamics. More recent literature has primarily focused on modeling the the right tail of the wealth distribution, which is particularly challenging, as the canonical SIM fails to produce the extreme density of wealth found among the richest in empirical data (Fella and De Nardi, 2017). A commonly used approach entails modeling extreme income shocks in order to provoke precautionary savings to match the vast accumulation of assets by the very top of the wealth distributionand is based Castaneda et al. (2003) ((Díaz and Luengo-Prado, 2010) or (Kaymak and Poschke, 2016) for a more recent implementations). Krusell and Smith (1998) were the first to point out that a stochastic discount factor across dynasties can account for the variance of the cross-sectional distribution of wealth and does increase the wealth concentration among the richest. Other approaches are: A heterogeneous interest rates (Benhabib, Bisin, and Luo, 2015), a richer earnings process (De Nardi, Fella, and Pardo, 2016) or entrepreneurship (Cagetti and De Nardi, 2009). De Nardi and Yang (2014) show that adding voluntary bequests and intergenerational transmission of earnings drastically improves the model's prediction for the empirical cross-sectional differences in wealth at retirement as well as their correlation with lifetime incomes. Although these additional modeling features are able to better approximate the right tail of the wealth distribution, they still fail to produce a satisfying representation. [2]
These studies, for the exception of.........., use life-cycle models.

2.2 Modeling Durables

Including durables into the canonical SIM-Model with one asset is interesting for several reasons. Durables exhibit a dual role. Firstly, they are a consumption good and,

[1]The application of such models ranges from ... to... CITE.

[2]For a more indebted review and the shortcomings of the different modeling approaches of this literature, see (Fella and De Nardi, 2017).

therefore consumers draw utility services from that good. Secondly, durables can be used as collateral and thus facilitate borrowing as the individual durable stock rises. FernÃ¡ndez-Villaverde and Krueger (2011) show that due to the dual role durables are important to replicate average non-durable consumption over the life-cycle observed in the data. An abstraction from durables may therefore bias life-cycle profiles of non-durable consumption and thus the accumulation of net-worth in a model with one asset. Moreover, they point out that durables help to explain why households with higher life-cycle income save proportionally more than poor households. Yang (2009) shows that borrowing constraints are important in explaining the accumulation of housing assets early in life. Gruber and Martin (2003) show that durables are important for the accumulation of precautionary savings and Díaz and Luengo-Prado (2010), show that an infinite-horizon model with durables can replicate portfolio changes with the level of wealth. They further investigate distributional consequences of changes in financial liberalization, simulating changes in the down payment. Cho (2012) show that changes in the LTV ratio can account for 40% of the difference in homeownership between Korea and the US.

3 Research Strategy

As discussed above, the modeling of the top of the wealth distribution is still unsatisfactory, unless one models extreme income shocks. The resulting income process is, however, far from realistic. Conclusions, drawn from such a model may therefore be biased.

I will thus chose the former strategy. Since calibrating the income process separately from the wealth distribution and thus reproducing a more realistic modeling of income risk does not allow for a representation of the right tail of the wealth distribution I focus on the distribution up to the 9th percentile. As Hintermaier and Koeniger (2011) argue this strategy is robust if general equilibrium feedback effects on the consumers excluded do not adversely affect the result.

; Yang (2009); Kaplan and Violante (2010);), thus contribute to the literature that (note copy paste.... demonstrate that a plausibly parameterized version of their models can quantitatively explain empirical findings as arising from rational choices of consumers facing an increasing wage profile and income uncertainty.)

The first quantitative was based on the following the work of Kydland and Prescott (1982) was based on the representative agent paradigm. One of the main reasons of abstracting from heterogeneity and incomplete markets was the lack of dynamic tools to solve such models. Heathcote et al. (2009) Over the past twenty years computational power has increased substantially and numerous dynamic methods have been

developed, thus allowing to solve more complex models. This development is crucial for the modeling of distributions.
Modeling heterogeneity is important for several reasons: precautionary savings as a consequence of idiosyncratic uncertainty increases aggregate savings and, thus, decreases the interest rate **?**
The choice of an exogenously determined imperfect market structure allows for a straight forward answer. It comes down to the economic question one poses. The aim here is to demonstrate that such a model can quantitatively explain the empirical net-wealth distribution in the US. The role the imperfect market structure, entering as a limited choice of assets to fully insure against idiosyncratic wage shocks, poses, is thus to produce an endogenous wealth distribution. The savings are fist and foremost driven by a precautionary motive to insure for future wage shocks. Knowing that the income may drop in the future consumers save today to achieve a more stable consumption across their live. In a complete market framework consumers could perfectly diversify away idiosyncratic risks (Tideman and Weber, 2010) (and thus would not build a buffer stock of savings for future shocks.) (???????? is this exact?????) The hypothesis of the complete market framework has been rejected by the vast majority of empirical research (Tideman and Weber, 2010). Deaton and Paxson (1994) show that the life cycle profile of consumption inequality is increasing with age, a fact that would be inconsistent with complete markets. Moreover, the assuming away markets, where individuals can trade contingent claims to insure themselfs against the idiosyncratic wage shocks is justified by moral hazard problems (FernÃ¡ndez-Villaverde and Krueger, 2011). While in some rare cases - such as when only studying aggregates, it may still be sufficient to consider a complete market framework, when considering wealth-distributive questions, the imperfect market assumption is both necessary from a theoretical perspective and well-founded from an empirical perspective. It remains to discuss the whether it makes sens to treat distributive issues from a life-cycle point of view.

4 Literature Overview

The model is based on the classic income-fluctuation problem in which the consumer faces a stochastic income process and decides in every period how much to save and how much to consume. Important contributions to the literature are Deaton (1991), Carroll (1992,1997), Gourinchas and Parker (2002). (note copy paste... Following Bewley (1986), this problem has been embedded by Huggett (1993) and Aiyagari (1994) into a general equilibrium framework, giving rise to the endogenous determination of the interest rate as well as a nontrivial income, wealth, and consumption distribution

at equilibrium.) (Note copy paste... Furthermore, this paper relates to other literature such as: Endogenous borrowing constraint: The specification of the borrowing constraint in FernÃ¡ndez-Villaverde and Krueger (2011) is adapted from the recent endogenous incomplete-markets literature: Kehoe and Levine (1993), Kocherlakota (1996), Krueger and Perri (1999, 2006), and Alvarez and Jermann (2000). Lustig (2004) also has a model with durable assets and an endogenous borrowing constraint to explain the equity premium puzzle, however agents have full access to Arrow securities and are infinitely lived. Literature on optimal portfolio choice in the presence of consumer durables: Grossman and Laroque (1990), Eberly (1994), Chah et al. (1995), and Flavin and Yamashita (2002).)
Note: missing citations, not all of the above are cited properly!!!!!
Literature on wealth distribution!!!!!

5 Facts about the wealth distribution

Changes across age groups Net worth Portfolio composition Financial liberalization

6 Research strategy

In the following I explain the strategy I use to shed light on the effect of different parametrisations on the wealth distribution as well as the life-cycle behavior of consumers.
As I will further discuss below, one crucial driver of heterogeneity, which manifests itself through different portfolio holdings in these models, is the income process. Permanent uninsurable income shocks lead to different choices in the accumulation of assets and eventually lead to a wealth distribution. Hence, the modeling of the income process is crucial.
Two general approaches can be distinguished. For one, there is the approach to closely match the individual income process and hence calibrating the income process independantly of wealth distributional apsects. Hintermaier and Koeniger (2011) or Hintermaier and Koeniger (2016) estimate age effects from the data and then take the variance of the residuals the retrieve the income shocks. The big plus of such an approach is to closely model the income process on a micro level. Changes in determinants of this process can thus be directly incorporated in the analysis. The downside is, that such models are not able to match the top part of the wealth distribution Fella and De Nardi (2017). Another approach commonly used in the literature is thus to calibrate the income process to match features of the income- as well as the wealth distribution at the same time. This approach is base on a paper from Castaneda et al. (2003). (OR

ADDING PARETO SHAPED TAIL PIKETTY AND SAEZ 2003)

6.1 Prime Age Sample

Moreover, I will focus on consumers aged between 26 and 55, where income risk plays an important role for the accumulation of savings.

7 Generating a wealth distribution

In the following section I discuss the economic model considered and outline the most important considerations made within the literature. Firstly, I discuss the life-cycle modeling and the literature, which applies life-cycle and to be more exact imperfect markets models in the context of wealth distributions. Secondly, I present my modeling choices and solution methods applied for this particular problem.

7.1 Modeling Literature

7.2 The Model

Before going into a more detailed description of the model at hand it is important to understand the modelling choice. Why a life-cycle model with an imperfect market structure? Why include durables as an additional asset choice to the more standard approach, which does summarize all assets in a one period bond, such as for example Hintermaier and Koeniger (2011)?
To answer the first question I will mainly refer to the the literature. The second will be discussed in a more detailed manner in the following section, which treads the life-cycle profiles and especially the importance of modeling durables.

7.2.1 Imperfect market structure

7.2.2 Life-Cycle

As Deaton and Paxson (1994) show, consumption inequality seems to vary with age. Further empirical research does support these findings, indicating that some of the differences in earnings, income, and wealth across households can be attributed to differences in the household's age Rios-Rull, Kuhn, et al. (2016) (FIND BETTER REFERENCES?). Hintermaier and Koeniger (2011) Show that the average net-wealth increases with wealth and that the dispersion of wealth falls with age. Moreover, there is a vast theoretical literature using a different versions of the life-cycle model to discuss distributive questions. Gourinchas and Parker (2002); Cagetti (2003); Castaneda et al. (2003); Yang (2009); Kaplan and Violante (2010);Hintermaier and Koeniger (2011)

(CITE MORE RECENT LITERATURE!!!) while there are also some that use infinite horizon models....
Finally, the choice to study the empirical validity of a life-cycle model, which produces a net-wealth distribution arising from rational choices of consumers subject to income uncertainty, moreover allows for a detailed analysis of consumption and savings behavior over a person's live-cycle. A coherent framework incorporating the behavior of household's over their lifespan is paramount for the study of re-distributive policies. (Cite....) These factors thus support the choice of a life-cycle model.

7.2.3 Partial Equilibrium

Last but not least, partial equilibrium as modeling choice has to be discussed briefly. (????????????????)
(copy paste from hintermaier 2011 Note that we assume that changes in the domestic supply of assets do not affect do not affect the interest rate. As in a small-open economy, this price is determined exogenously (on the world markets.) [3])

7.2.4 The Income Process

Kaplan and Violante (2010) discuss various features of the income process in more detail.

7.3 The determinants of savings

There are different factors contributing to savings. Understanding these factors and their importance for the accumulation of wealth is crucial in understanding distributional issues and for the provision of policy advice, where such issues are the subject of discussion, both normative and positive.
In the model presented in this work a number of determinants are considered, all of which precedent work has shown to be of importance in the context of the accumulation of wealth. The present paragraph thus gives an overview of these determinants, to whom I will refer to when discussing simulation results.

Consumption Smoothing Early life-cycle models predict consumption (SLOWLY DECREASING???) to be flat over the life-cycle. This is the case, when perfect consumption smoothing can be achieved. With an income profile observed above, agents would thus accumulate debt, when expected future earnings are high to substitute

[3] This assumption is not restrictive for our purposes since we observe the price in our ex post analysis for the period 1983-2007. Hence, the supply of assets and the observed price suffice to determine the equilibrium asset quantities. This allows us to be agnostic about the price elasticity of asset demand.

this higher income for lower income today. And high current income is converted into savings when future higher earnings are low. (CITE A MODEL OF THIS TYPE)

Precautionary Savings Motive Idiosyncratic income risk as for example in (Aiyagari, 1994) is an additional savings motive. Savings are used as a buffer for future income shocks. If, uncertainty is high, agents save more and thus cut down on consumption to do so in order to save for rainy days.

Borrowing Constraints A third determinant are the borrowing constraints. These prevent agents from borrowing and thus do not allow for perfect consumption smoothing across time or income states.

Durables (FernÃ¡ndez-Villaverde and Krueger, 2011) show that including durables into a life-cycle model is an important determinant of the consumption behavior along an agents life cycle. The relative price of durables does affect savings-consumption (IS IT LIKE THAT???) decisions.

7.4 Why model durables? How do durables affect savings?

1. FernÃ¡ndez-Villaverde and Krueger (2011)
   (a) empirical observation that durables constitute a large fraction of households' asset holdings - hh hold 35% of their total assets in durables and only 28% in equity (flow of funds account 2nd quarter)
   (b) the life-cycle pattern of nondurable consumption may be closely related to the life-cycle pattern of the accumulation of durables and thus any abstraction to net wealth may severely bias the study of asset accumulation (and life-cycle profile of nondurable consumption

7.4.1 Consumer's Problem

As established above, the baseline model in question is a life-cycle model with an imperfect market structure. I closely follow Hintermaier and Koeniger (2010).[4] While there algorithm allows for both a live-cycle and infinite-horizon models, the example model presented in their paper uses the latter. For reasons discussed above, I will in this case opt for a live-cycle formulation. Finally, the baseline model does not include adjustment costs. These are considered in a later section of the model.
There is a continuum of risk averse consumers with a finite time horizon. At age 90 consumers die with certainty, younger consumers between the age of 26 and 90 may die

[4] In order to facilitate comparability, I will closely follow the notation used in their paper.

with probability $\pi_j < 1$ at age j. Until retirement the labor income of each individual i y_{ij} is stochastic. After the age 65 is completed, they retire with certainty and receive individual-specific retirement benefits b_i.[5] Consumers derive utility from a durable good d and a non-durable good c. The utility function $U(c,d)$ is strictly concave, increasing and non-separable in c and d (REALLY??? When not??? all three true???). In each period consumption c_j, the durable good d_{j+1} and the financial risk-free asset a_{j+1} are chosen after receiving the income. Agents then derive utility from consumption and durable services, before the assets pay returns. While the liquid assets return an interest rate r durables depreciate at rate *delta*. As previously discussed, such a market structure implies that markets are incomplete and agents will accumulate a buffer stock in order to save for rainy days. The ex-ante identical consumers thus will thus, depending on their income trajectory, end up with different portfolios of the endogenous state variables. The model is therefore suited to match empirical facts about net worth positions since it does produce an endogenous distribution of portfolios, which differs in age and income. Moreover, (THE ADVANTAGE OF DURABLES) besides providing services and thus allowing consumers to derive utility from durables, their durability implies, that they may be used as collateral. It is assumed that all credit needs to be collateralized.[6]
The collateral constraint takes the form:

$$\underbrace{\mu(1-\delta)d_{j+1} + \gamma \underline{y}}_{collateral} \geq -(1+r)a_{j+1} \tag{1}$$

where $\underline{y}$ is the minimum labor income realization [7] and $\mu \in [0,1) and \gamma \in [0,1)$ are the respective fractions of the durable stock and of minimum labor income, which can be collateralized. [8]
It is useful to rewrite the constraint in terms of total wealth and future durable holdings. Total wealth available to a consumer in each period is

$$x_j \equiv (1+r)a_j + (1-\delta)d_j, \tag{2}$$

rewritten the constraint then looks like this:

[5] The construction of these benefits is discussed in the Appendix.

[6] About 85% of household debt in the Survey of Consumer Finances 2004 is secured by collateral.(Hintermaier and Koeniger, 2010)

[7] CHECK THIS WITH CODE!!!

[8] It is important to note that this constraint guarantees full repayment by consumers and thus acts as non-bankruptcy constraint. This is assured by the timing assumptions made above –The lender does know the financial portfolio choice (a_{j+1}, d_{j+1}) and the minimum of the support of the income distribution $\underline{y}$, however future individual income draws y_{j+1} do remain obscure to him. Finally, note that only choices in j determine, whether the constraint is binding or not.

$$x_{j+1} \geq -\gamma \underline{y} + (1-\mu)(1-\delta)d_{j+1} \tag{3}$$

One now sees that total wealth needs to be larger than the negative value of the minimum fraction of income that can be collateralized and the fraction of wealth which cannot be used as collateral. Furthermore, under the assumption that $\mu < 1$ d_{j+1} is determined for a given x_{j+1}, when the constraint binds.
Although in my baseline model they are set to 0, I also consider the case where durables can only be adjusted with some costs:

$$\Psi(d_{t+1}, d_t) = \frac{\alpha}{2}\left(\frac{d_{t+1} - (1-\delta)d_t}{d_t}\right)^2 d_t. \tag{4}$$

9

Finally, the specification of the budget constraint, follows from the above:

$$a_{j+1} + d_{j+1} + c_j + \Psi(d_{t+1}, d_t) = x_j + y_j \tag{5}$$

The recursive formulation of the household problem I will proceed by depicting the case prior to retirement. (SPECIFY RETIREMENT CASE!!!) We let T^r denote the first period of retirement. The Bellman equation prior to retirement, if $j < T^r$thus is:

$$v_j(x_j, d_j, y_j) = \max_{a_{j+1}, d_{j+1}} \left[U(\underbrace{x_j + y_j - a_{j+1} - d_{j+1} - \Psi(d_{t+1}, d_t)}_{c_j}, d_j) + \hat{v}_j(x_{j+1}, d_{j+1}, y_j) \right] \tag{6}$$

where the expected next period value function is discouted by the product of the probability of survival $(1-\pi_j)$ and the discount factor β

$$\hat{v}_j(x_{j+1}, d_{j+1}, y_j) \equiv \beta(1-\pi_j)E_j v_{j+1}(x_{j+1}, d_{j+1}, y_{j+1}),^{10} \tag{7}$$

with the constraints:

$$a_{j+1} + d_{j+1} + c_j + \Psi(d_{t+1}, d_t) = x_j + y_j, \tag{8}$$

[9](Hintermaier and Koeniger, 2010) use the same specification, which is based on the investment literature (COPY PASTE(see, for example, **?**,p.192)). Costs are differntiable in d_{t+1} and d_t. Moreover, by letting the durable stock depreciate, consumers can avoid these costs. In this particular specification, adjustment costs do not affect the collateral constraint, as it is assumed that these costs do not have an impact on the sales of collateral seized by lenders. NOTE (Hintermaier and Koeniger, 2010) EXPLAIN IN APPENDIX VERSION WHERE THIS IS THE CASE.

$$x_{j+1} = (1+r)a_{j+1} + (1-\delta)d_{j+1}, \tag{9}$$

$$x_{j+1} \geq -\gamma \underline{y} + (1-\mu)(1-\delta)d_{j+1}, \tag{10}$$

(WHAT HAPPENS WITH MIN Y IN RETIREMENT????)

$$d_{j+1} \geq d_{min}, \tag{11}$$

FROM RED PAPER HELP TO SPECIFY CASE WITH RETIREMENT!!! We specify our model in discrete time. Rearranging the budget constraint, consumption of individual i with age j is

$$c_{ij} = \begin{cases} (1+r)a_{i,j-1} - a_{ij} + y_{ij} & \text{if } j < T^r, \\ (1+r)a_{i,j-1} - a_{ij} + b(z_{i,T^r-1}) & \text{if } j \geq T^r, \end{cases} \tag{12}$$

where $b(z_{i,T^r-1})$ are the retirement benefits. These benefits depend on the last realization of labor income before retirement z_{i,T^r-1} as we explain further in the next section. Defining cash-on-hand as

$$x_{ij} = \begin{cases} (1+r)a_{i,j-1} + y_{ij} & \text{if } j < T^r, \\ (1+r)a_{i,j-1} + b(z_{i,T^r-1}) & \text{if } j \geq T^r, \end{cases} \tag{13}$$

we can write the Bellman equation as

$$V_j(x_{ij}, y_{ij}) = \max_{a_{ij} \geq 0} [u(\underbrace{x_{ij} - a_{ij}}_{c_{ij}}) + \beta(1-\delta_j)E_j V_{j+1}(x_{i,j+1}, y_{i,j+1})], \tag{14}$$

where we restrict the choice of assets to $a_{ij} \geq 0$, as motivated in the previous section.)

8 Calibration

The calibration strategy demands to be in line with the aim of the paper - discussing the importance of modeling durables in context of the distribution of net worth. It is thus crucial to satisfy a high degree of comparability with other work dealing with net-worth distributions. Moreover, the calibration ought to replicate some features of the data, without containing to much information on the features the model is asked to show. (CITE SOME BOOK/ PAPER). In order to give insight on the underlying parameters of the cross-section of inequality I will thus proceed in the following way. As discussed above, I base the parametrisation of model on Hintermaier and Koeniger (2011) and apply the algorithm introduced by Hintermaier and Koeniger (2010) to solve the model with durables.

8.1 The income process

As discussed above the income process is an important driver of the accumulation of savings in the form of precautionary savings. Its calibration does thus decisively influence the outcome. Hintermaier and Koeniger (2011) use the SCF cross sections to construct a measure for labor earnings risk before retirement purging labor earnings from age effects for consumers between age 26 and age 65. The authors assume that log of earnings y_{ij} of individual i at age j before retirement is additively separable in a deterministic age polynomial ϕ_j and an idiosyncratic income shock z_{ij}:[11]

$$y_{ij} = \phi_j + z_{ij}$$

,

where the shock z_{ij} follows an AR(1) process

$$z_{ij} = \rho z_{i,j-1} + \epsilon_{ij}$$

After retirement the income process is deterministic. Thus, each individual receives retirement benefits from social security, the level of which is determined by the last period's income resulting in a replacement ratio of benefits over gross income of 52% for the median income in the last period before retirement. The approximation takes into account the US social security legislation(http://www.ssa.gov). [12] The age polynomial ϕ_j is obtained by regression the log of earnings on a quartic age polynomial in each survey year of the SCF data between ages 26 and 65.[13] The then obtained standard deviation of the residuals are used to calibrate the distribution of earnings shocks z_{ij}, with the assumption that the shocks are drawn from a normal distribution. This means that in the calibration to the SCF 1983, $z_{1983} \sim \mathcal{N}(0, 0.498)$, and in the calibration to the SCF 2004, $z_{2004} \sim \mathcal{N}(0, 0.607)$. (POSSIBLY COMPARE THIS RISK TO OTHER STUDIES ALSO SEE KRUEGER AND PERRI 2006) The calculations for the experience premium displays a concave relationship between the base age and the rising age it is compared to. Moreover, the experience premium has risen since the 1980s. The autocorrelation of the log-earnings shocks are calibrates as $\rho = 0.95$, which implies a variance for the innovations of $\epsilon_{ij} = 0.048$. (HERE ALSO COMPARE TO OTHER, MORE RECENT LITERATURE) The AR(1) process for z_{ij} is then approximated by a Markov chain with 21 income states via the Rouwenhorst method.

[11]This approach is quite standard in the literature (see others! NEWERS THAN YANG AND KAPLAN VIOLANTE)

[12]The reader may consult Hintermaier and Koeniger (2011) for a more detailed discription of the calibration of the retirement benefits.

[13]The table in appendix A displays the experience obtained by Hintermaier and Koeniger (2011).

Finally, the authors adjust for growth in life-cycle income to convert the cross-sectional age-earnings pattern into life-cycle profiles considering a growth factor of $1.015^{age-baseage)}$, where the base age is a reference age to make income units comparable across cohorts of different years.

The choice for the comparison of the SCF 2004 with the SCF 1983 is motivated by the fact that both surveys have been undertaken at similar points in the business cycle.

Controlling for the age distribution is another important determinant for the dynamics!

It is worth noting that the smallest income is larger than 0!!!!

Income Process LOOK AT GOURINCHASandPARKER(2002) AND AT THE ADDAandCOOPER BOOK!

1. (FernÃ¡ndez-Villaverde and Krueger, 2011) stochastic labor income process up to age 65, where they impose mandatory retirement. Differs from other specifications such as Abowd and Card (1989), Carroll (1992), or Gourinchas and Parker (2002) in two aspects:

   (a) do not allow labor income to go to zero- very low probability Carroll (1992) estimates it at 0.003 for a year. However, its effects on the intertemporal allocations are quite big, since hh will not borrow any positive amount since they might face a lifelong sequence of zero labor income and might be unable to consume a positive amount and repay their debt in some period. (Moreover, argument of public income-support programs in the US -> labor in the model is after-tax, after-government transfer labor income.

   (b) unit root in AR process is not imposed (is something that is debated -> they show that their results are not very sensitive toward this issue -> increase ρ toward unity (LOOK AT STORESLETTEN ET AL: (2001) -> do not reject unit root.

2. Check Hintermaier and Koeniger (2010) and Hintermaier and Koeniger (2011)

8.2 Utility parameters

I consider a utility function that is non-separable in durable- and non-durable consumption, strictly increasing in both consumption types, strictly concave and obeying the INADA-Conditions with respect to nondurable consumption. It is the same class of preferences as Hintermaier and Koeniger (2010), which is commonly used in the literature considering durable and non-durable consumption (CITE MORE). [14]

$$U(c,d) = \frac{\psi(c,d)^{1-\sigma} - 1}{1-\sigma} \quad \text{where} \quad \psi(c,d) = c^{\theta}(d+\epsilon_d)^{1-\theta}, \tag{15}$$

where I assume $\epsilon_d > 0$, which thus indicates a number small enough to be irrelevant for the quantitative exercise at hand but nonetheless larger than zero. The CRRA utility function with the Cobb-Douglas specification of the consumption index, thus allows for zero consumption of durables, while the INADA-Condition ensures that people always consume some non-durables, i.e. intuitively this means consumers cannot survive without food, but are allowed to survive without houses and cars.[15] (WHAT IS THE CONSEQUENCE OF OMITTING THE EPSILON????
(FernÃ¡ndez-Villaverde and Krueger, 2011) utility form death is normalized to 0 ?????????
The risk-aversion σ typically takes values from 1-5 in the literature (Yang, 2009). Following the aforementioned author I use $\sigma = 1.5$ as estimated by Attanasio, Banks, Meghir, and Weber (1999) and Gourinchas and Parker (2002) from consumption data. The discount factor β and the weight on non-durable consumption θ are calibrated to match the average net-worth holding and average durable holding of the prime age population up to the 90th percentile, respectively. The data moments are 2.95 for the durable holdings and 2.39 for the net-worth in terms of median labor-income. TO MATCH HOW EXACT: SHOW WITH SMALL TABLE (BE CAREFUL TO BE SPECIFIC ENOUGH!) To calibrate the two parameters I solved the for a grid of (??????) for θ and a grid of (???????) for β. The choice to match the 90th percentile and the population from 26 to 55 years old is motivated by the fact that calibrating the model for the whole data-set would lead to overestimating the net-worth holdings of the prime sample –as is both the population above 55 years of age and above the 90th percentile hold more wealth on average than in the prime age sample. The estimation procedure is explained in more detail in appendix B.

Table 1: This table depicts the parameter estimates matching the average durable holdings and average total worth of the prime age population in the data.

Parameters	Baseline	Alternative
β	0.991	??
θ	0.761	???

[14]Note that this choice is in line with a more generic formulation of the utility function (FernÃ¡ndez-Villaverde and Krueger, 2011).

[15]For the recursive problem it means that when allowing the potentially optimal choice $d' = d_{min}$, the constraint for minimal durable holdings needs to be accounted for explicitly, as the INADA-Conditions do not implicitly exclude zero durable consumption.

(BE MINDFUL OF THE FACT THAT HINTERMAIER 2016 HAS 0.76 FOR HOUSING - SAME SAMPLE!!!!!!) The estimation results are somewhat different from what other authors with similar models have obtained. (NOTE DIFFERENCE INF HORIZON AND LIFE CYCLE) FernÃ¡ndez-Villaverde and Krueger (2011) estimate a weight on non-durable consumption of 0.81, Yang (2009) 0.8615 and Hintermaier and Koeniger (2010) 0.8092. (CITE NEWER STUDIES) Whereas studies solely considering housing, obtain estimates more closely to mine (CITE THESE STUDIES). This should, however, not be surprising, as I used data moments which correspond to the means of the above defined prime-age sample, contrary to these other studies, who calibrated their to the whole sample. Díaz and Luengo-Prado (2010) among others, show that the empirical portfolio composition changes over life-time, as will be discussed more in detail below. Young households take on credit to invest in durables and older households accumulate assets to finance retirement. The fraction of durables of total wealth, as the two data moments show 2.95/2.39 is larger than one for the prime-age sample, since the average liquid assets are negative. Where as for the whole sample the fraction of average durable holdings in terms of average total wealth 4.45/6.33 is smaller than one (Hintermaier and Koeniger, 2010). The estimated discount factorβ is quite similar to the one inHintermaier and Koeniger (2011).

8.3 Further inputs

As is common in the literature dealing with the period of the great moderation and supported by empirical evidence, the interest rate r is set to 4%. (CITING LITERATURE). As discussed above, this partial equilibrium approach entails the assumption of a small open economy. Furthermore, I set the loan-to-value ratio $\mu = 0.8$. Although commonly used in the literature, this is a rather conservative assumption for the loan-to-value ratio: CAPLIN ET AL (1997) claim that it is almost impossible for a hh to obtain a house without paying at least 10%, moreover (OTHER AUTHOR IN HINTERMAIER2010 max LTV). However, this will allow me to investigate the effects of recent years, which lead to a relaxation of the LTV (IacovielloandPavan 2013) or (THAT OTHER PAPER ABOUT THE DENSITIES OF LTV). Finally, as in Hintermaier and Koeniger (2010) $\gamma = 0.97$ is chosen to be smaller than 1 in order to assure positive consumption at the smallest gridpoint of next periods net-worth x'. The probabilities of death as in Hintermaier and Koeniger (2011) are taken from Table 1 of the decennial life tables 1979-1982 and 1999-2001 for the calibration of 1983 and 2004, respectively.[16] (COMMENT ON WHY THESE ARE IMPORTANT!)

[16]They are published by the National Center for Health Statistics at $http : //www.cdc.gov/nchs/products/life_tables.htm$.

9 Numerical algorithm

The problem above does not provide a closed-form solution (CITE!!!!) and thus the solution has to be approximated numerically. Hintermaier and Koeniger (2010) offer a solution for problems of this type. They expand the endogenous gridpoints method (EGM) proposed by Carroll (2006) for a problem with two states and thus providing a very efficient algorithm to solve models with durables and collateralized debt. As their algorithm is formulated recursively, it is well suited to solve life-cycle models. [17]
I iterate over the policy function starting in period $j = T$ where the consumer sells all liquid assets and durables (TIME ASSUMPTION OF DURABLES IS IMPORTANT HERE - agents retrieve utility from the durable stock before it is sold, and can consume afterwards????) to consume them before death. It thus holds that $x' = d' = a' = 0$ and thus the initial consumption policy is $c(x, d_j, y_{kj}) = x + y_{j,k}$ (NOTE CASE WITHOUT ADC). Each iteration n on the policy function then gives the the solution for the period $T - n$, where in my case $n = 65$ which corresponds to ages 90 to 26.
The exogenous grids for d and x are chosen as follows:
(NOTE THAT ALGORITHM MUST BE CHANGED FOR CASE WITH ADJUSTMENT COSTS)
For each period t < T-1, the algorithm proceeds as follows. In a first step it maps x' into d' and in a second step it solves for the current choice c and state x from optimal future combinations (x',d'). Moreover, the values for the exogenous grid are chosen such that the results are not affected. (BE MORE PRECISE INDICATE DIFFERENCES BETWEEN POST/PRE RETIREMENT (expectation)!

9.0.1 Comparing the simulation output with SCF data

Data (Initializing DATA + probability of death (PLOT!)

10 Life-Cycle Profiles

As discussed above, the choice of a life-cycle model is mainly motivated by the fact, that age plays an important role in questions related to wealth distribution and in particular to the net-wealth distribution. This section will give insight into the accumulation of wealth by agents over their lifetime. As we will see, this also affects consumption. FernÃ¡ndez-Villaverde and Krueger (2011) have pointed out that it is paramount to

[17] Solving such models has been considered particularly expensive and challenging, as durables enter as additional state (if non-separability utility from durables and non-durable or adjustemnt costs are allowed for) and durables increase the dimensionality of the portfolio choice.(Hintermaier and Koeniger, 2010)

consider durables in order to properly model the life-cycle profile of consumption, which is hump shaped. The authors underline the importance of modeling durables in order to accurately reproduce the the shape of consumption over the life cycle observed in the data. He shows that including durables in a non-separable way in the utility function into the canonical incomplete markets model with one asset may achieve a better representation of the hump shape. Namely, durables account for 50% of the hump size, while the other half is attributed to changes in the household size, a fact pointed out by Attanasio et al. (1999). The first section discusses empirical results as well as life-cycle profiles produced by models from theoretical research. In the second section I will produce and comment on the life-cycle profile of my baseline model and then go over to a more detailed investigation of certain factors in the third section. LOOK AT GOURINCHASandPARKER(2002) for more insight

10.1 Life-Cycle profiles in the data and theoretical literature

This section discusses a number of important issues discussed by the theoretical literature related to empirical patterns. I thereby focus on issues discussed in relation to durables.

10.1.1 Consumption smoothing

The standrad life-cycle model with complete financial markets predicts, that individuals smooth consumption over their lifetimes and across states of the world.[18] This is true if the one period utility function and the time discount factor are constant. As households will then choose consumption plans in order to equate marginal utility across time and states of the world. Potentially they might do this with some growth rate, depending on the relative size of the real interst rate and the discount factor. (COPY PASTE (FernÃ¡ndez-Villaverde and Krueger, 2011): "Under CRRA (Constant Relative Risk Aversion), period utility consumption growth itself should be constant across time. With complete markets, consumption smoothing can be achieved through the transfer of contingent claims across periods and states. (this statement relies on the further assumption that leisure and consumption are separable in the period utility function.") However, the data shows that consumption of both, non-durables and durables, is hump shaped over the life-cycle.[19] (amongst others CITE MORE). As these authors

[18] NEED TO FIND GOOD REFERENCES!!! SEE also: Further empirical literature about the life-cycle profile of consumption observed in cross-sectional micro data: Blundell et al. (1994), Attanasio and Browning (1995), Attanasio and Weber (1995), Gourinchas and Parker (2002).

[19] In the Consumer Expenditure Survey (CEX), when the head of household is 50, the average household spends 63% more when he or she is 25 and 70% more when he or she is 65 (FernÃ¡ndez-Villaverde and Krueger, 2011).

point out it is important, especially for policy considerations, such as social security reforms, public provision of saving incentives, or welfare consequences of progressive taxation, as these will vary by age group. FernÃ¡ndez-Villaverde and Krueger (2011) show that 50% of the consumption hump can be accounted for by adjustment through family size. However, the other 50% can only be explained by including durables. Namely, if the period utility function is separable in nondurable consumption and durable consumption, the real interst rate will be equal to the time discount factor and remain constant over time. It thus follows that optimizing households will buy the desired stock of durables early in life and then only replace depreciation from there on. However, as the data shows, durable consumption is hump shaped, which is consistent with work, which has documented liquidity constraints in the purchases of consumption durables [20] or argued that is vital to consider nonseparabilities in the utility function [21].

In this first section, FernÃ¡ndez-Villaverde and Krueger (2011) thus contradict Blundell et al. (1994), Attanasio and Browning (1995) and Attanasio et al. (1999), who argue that if one adjusted the consumption data for changes in household size (which is also hump-shaped over the life-cycle) the hump in life cycle consumption disappears.

Income tracking consumption (COPY PASTE :Deaton (1992) provides an overview for the literature answering this question. The main stylized fact emerging in this literature was that consumption seems to track income over the life cycle, changing only when income changes and not when it becomes known that income will change, as economic theory predicts. was interpreted as indication for liquidity constraints or other financial market imperfections. The empirical observation that consumption initially underresponds to income shocks and then adjusts with a delay. Flavin (1985), Campbell and Deaton (1989), Jappelli and Pistaferri (2010))

10.1.2 Portfolio Composition

As FernÃ¡ndez-Villaverde and Krueger (2011) point out households' portfolio composition do vary over the lifetime. Young households only little liquid assets and hold most of their wealth in consumer durables, whereas later in life, they accumulate big amounts of financial assets to save up for retirement. Durables make up 35%of the aggregate composition of wealth, while equity only accounts for 28% (FernÃ¡ndez-Villaverde and Krueger, 2011).The same authors thus show that these facts can better be accounted for, when one considers liquid assets and durables separately, instead of combining

[20] NOTE COPY PASTE FROM FV& K2011 Eberly (1994); Alessie et al. 1997; Barrow and McGranahan (2000); Attanasio et al. (2005)

[21] NOTE COPY PASTE FROM FV& K2011 Attanasio and Weber (1995)

them in one asset such as net worth. The interaction of the two assets thus is able to reproduce the abserved pattern in the data. Where the consumer durables provide both consumption and collateral services as well as an endogenous borrowing constraint.

10.2 Profiles predicted by the model

NOTE THE NUMBERS AND FIGURES NEED TO BE ADJUSTED SINCE THERE WERE SOME OBSERVATIONS DROPPED TO CALCULATE THESE PROFILES!!!!!
In the following section I will discuss the baseline life-cycle profiles produced by the model more in detail and compare it to other work and empirical results presented therein. I will mainly focus on two aspects discussed in the literature. The consumption and income profiles and comparing the asset profiles, which entails a discussion about portfolio allocations over the life-cycle.
The life-cycle presented in figure 1 and figure 2 show averages of the variables of interest over the simulated population of 100000 consumers aged between 26 and 90. With the exception of net-labor earnings all variables are in units of average net-labor earnings per adult-equivalent. (BE MORE PRECISE HERE)

10.3 Income and Consumption

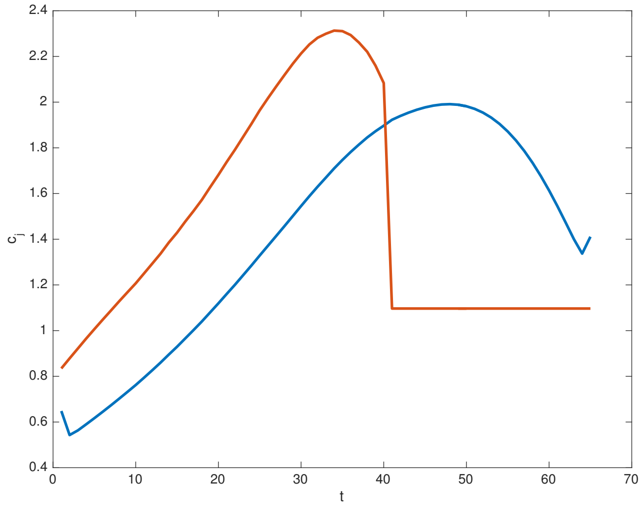
Income Figure 1 shows the hump of the income profile arising by construction as the process is determined exogenously. Note that this hump shaped profile arises from the experience premium captured by the estimated age polynomial as well as economic growth, up to age 65. Consumers earnings rise until the age of 49, when they reach the maximum log net-income equivalent of 2.3131. Afterwards the experience premium decreases, as the experience factor is dominated by a decrease in productivity to the age of 65, where consumers earn 2.0834, thus around 10% less than when they are at their top. At this point the average income decreases sharply, as every consumer retires after the age of 65 and receives deterministic retirement benefits from then on, with an average of 1.0966. [22]

Consumption As income consumption is hump shaped and seems to follow the behavior of income with a slight time lag and its decrease is smooth. Non-durable consumption peaks on average at the age of 74. At that point agents consume 1.9911 in terms of the average net labor-earnings equivalent, which is almost four times as much as at the age of 27, when their non-durable consumption is lowest. [23]

[22] Following Hintermaier and Koeniger (2011) the growth adjustment is neglected for the retirement period, therefore the curve does not increase for ages 66 to 90. (CHECK IF WOULD HAVE AN EFFECT! OR IS THERE A REASON FOR THIS???)

[23] Note this first fall of consumption is due to adaptation from initiating durables and net worth.

Figure 1: Average Income and Consumption over the Life-Cycle



Consumption is in terms of average net labor-earning equivalents and and the labor earnings are as calibrated, the log of net labor earnings-equivalents?????? JUST VERY STANDARD! NEEDS TO BE UPDATED

Clearly, agents do not achieve perfect consumption smoothing over their life-cycle. Instead, their consumption depends on other factors. The first observation, consumption traking income, is due to borrowing constraints. In principle, agent's expected future income rises with certainty, due to the higher experience premium. Thus, they would like to accumulate debt early in live to substitute future higher income for todays lower income. In the beginning of their live, as their durable stock is low, and thus the borrowing constraint is tight. Consumption does therefore quite closely track the deterministic, averaged part of the income early in live. [24]As they grow older accumulating a stock of durables, the constraint loosens and agents may borrow more, which facilitates consumption smoothing thus permitting consumption to behave contrarily to income.

As discussed above, (FernÃ¡ndez-Villaverde and Krueger, 2011) show, dourables contribute to a large extend to this hump shape of consumption. The double role of durables, which provides consumption services and may be utilized to substitute wealth across periods, as well as the non-separability of durable and non-durable consumption are responsible for this behavior. In the early part of life consumers prefer to stack durables over consuming more non-durables as marginal utilities are similar, however durables may be used as collateral and itself provides the benefits of investment. The non-separability is responsible for a rise in non-durable consumption as the early in-

[24]this model does display excess sensitivity of consumption to income (income tracking consumption to closely SEE LITERATURE!!!!), which contradicts the predictions of the basic life-cycle model.See DEATON (1992) for a review

crease in durables decreases the latters marginal utility and thus the relative utility benefits of consumption rises as agents grow older.
Later in live, when death becomes more likely and utility is discounted to a larger extend consumption decreases again. [25]

Comparing to results reported in the literature FernÃ¡ndez-Villaverde and Krueger (2011) and Yang (2009) have looked at life-cycle profiles using very similar models. They both find similar shapes. However, differing in the size and period of the peak.
Using data from the Consumer expenditure survey (CEX) and correcting it for the family size, FernÃ¡ndez-Villaverde and Krueger (2011) show that the average household spends 65% more than when the head of the household is 50 compared to when he or she is 25 and around 70% more, when he or she is 65.[26] Their model predicts a peak around the age of 45, with a size that is about 40% bigger than at the age of 20. Yang (2009) reports similar results: non-housing consumption at age 50, where it peaks, is 80% more than that of age 20, after the peak, the decrease is steady and at age 75, people consume 25% less on average.

Notably, the increase of consumption is much higher than reported by other, similar models and the data. Moreover, consumption peeks much later in life. One important difference between the two papers discussed above and my own analysis is the model calibration involving the estimation of preference parameters. As explained above, the estimation of the the discount factor (SURE ABOUT THIS?????) and the weight on non-durable consumption are calibrated to match the prime-age sample, which leads to a lower theta and a higher beta. (SHOW THIS!!!!!!!!!!) (MAYBE ANALYSIS WITH POLICY FUNCTIONS!!!) Trading off durable and non-durable consumption has a lot to do with the discount factor!!!!

10.4 Portfolio Composition over the Life-Cycle

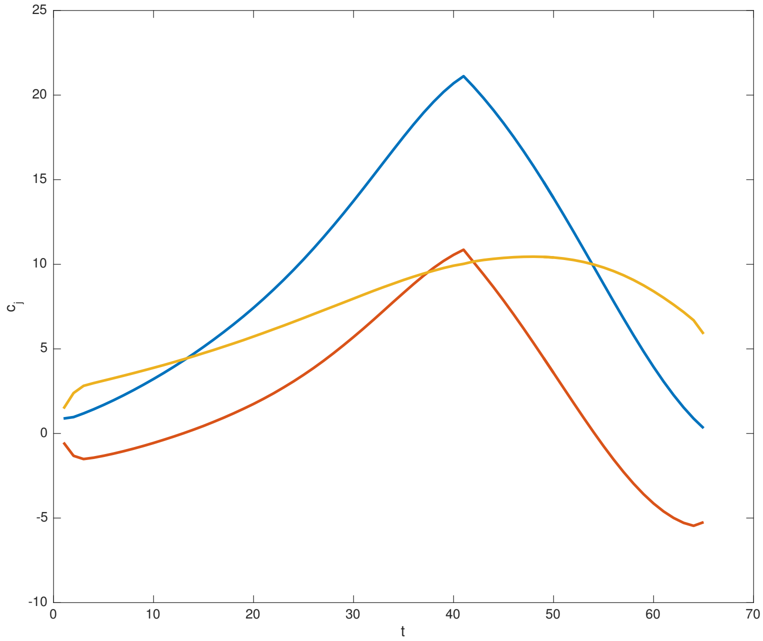
As discussed in the previous section, average non-durable consumption over the life-cycle is influenced by the capacity to accumulate assets as well as the choice of asset

[25]The late rise in consumption is due to life-time uncertainty. As consumers are never entirely sure at what point they will die, they hold a small buffer even when retired and subject to deterministic income. In the last period, they die with certainty and thus sell all of their assets increasing consumption.

[26]FernÃ¡ndez-Villaverde and Krueger (2011) use data from 1998. Yang (2009) reports similar findings for estimating consumption profiles from CEX 2001 data. Adjusting for household size, he finds an peak at age 55, with an increase of roughly 60% compared to age 20.

accumulation. The present section discusses the average portfolio composition over the life-cycle.

Figure 2: Average Asset Holdings over the Life-Cycle



NEED TO INDICATE THE MEASURES. JUST VERY STANDARD! NEEDS TO BE UPDATED

Wealth Portfolio All three curves display humps. Liquid assets and net-worth peak at age 66, the period when consumers retire and durables peak at 73, exhibiting a later and somewhat slower decline.

Early on in live households borrow as much as possible to accumulate durables, which leads to a sharp rise in the average durable stock early on in live. Savings in liquid assets thus start out to be negative and then rise as consumers start to save up for retirement. As durables can also be used to insure against idiosyncratic shocks, assets are primarily used for life-cycle purposes. Moreover, assets only become relatively more interesting as the average stock of durables rises and its marginal utility, the return on durables, shrinks, while the interest rate is fixed.

What does the literature say? Yang (2009) documents life-cycles for net worth, housing stock and non-housing assets controlling for cohort and time effects by constructing synthetic sohorts by using six waves of the SCF (1983-1998). [27]. He shows that housing value increases until age sixty-five and then flattens out until the end of

[27] Details of the estimation are available in **?**

the life cycle. [28] Moreover, he shows that young agents tend to hold little wealth, borrowing early in life to buy houses and once they have accumulated stocks of houses start to save in financial assets. Financial assets surpass housing assets in the early 40ies. Moreover, net-worth and liquid assets peak at the age of 70 and then stay constant until the end of life.

While the model captures the between durables and financial assets quite well and predicts the timing of the peaks in a reasonable manner, two factors are at odds. Durables, seem to be favored to an extend not found in the data (LOOK FOR A BETTER WAY TO DESCRIBE THIS!) and in the model, all assets decline again. While, the decline in durables may be explained by the data to some extent and only of small magnitude (FernÃ¡ndez-Villaverde and Krueger, 2011), liquid assets and net worth decline by a very big margin. Liquid assets become negative again at the age of 80, and net-worth reaches zero at 90, when all agents die with certainty. For the very obvious reason, that agent do not derive utility from leaving any assets behind.
(FernÃ¡ndez-Villaverde and Krueger, 2011) look at data and show that durables rise quite early until around the age of 30 and then peak in the mid-40ies, while declining afterwards.
The model reproduces a few important factors of the life-cycle composition of wealth:

1. (COPY PASTE (FernÃ¡ndez-Villaverde and Krueger, 2011): Young households save and thus net worth becomes positive at the age of 35 (MINE IS NEVER NEGATIVE!!!) but they do not save in financial assets, but rather accumulate consumer durables.

2. As the households get older, they do start accumulating more assets, which are important in retirement. They do hold a lot of net worth until high ages in order to insure against living too long. Elderly households seem to overacumulate assets. (Seems not to be the case in my model!!!)

Figure **??** shows how net net worth evolves over the life-cycle.

Durables Note that the durables are rather different from (FernÃ¡ndez-Villaverde and Krueger, 2011) (COPY PASTE:Consumer durables: the stock follows a hump shape, differing from a complete-markets model where the desired stock is built up in the first period and only an amount equalto depreciation is spent each period thereafter. Comparing durable consumption (model) and durable expenditure (the data):

[28] He shows that in order to achieve this one must model adjustment costs. Otherwise, the key prediction of the standard life-cycle model is that ratio housing stock to non-housing consumption stays constant over time, does arise, which is inaccurate.

the model generates a pattern of consumer durables that somehow diverges from the observed pattern: there is a big peak in the first years and then it falls, even though it is possible to see something of a hump after the first spike. One possible explanation is that in the data young families obtain bequests, which in large part come as consumer durables. A second possible explanation is the endogenous formation of households in the data. In our model, all households enter their active economic life at age 20, a time period where they want to build up the desired stock of capital. In the data, however, economically active (in the sense of our model) households are created endogenously due to differences in marriage timing and education. This endogeneity smooths out the first big spike of durable expenditures in the data and leads to the pattern of life-cycle durables expenditure reported in section 2. (this divergence between data and model may also indicate that our borrowing constraint is specified too loosely, allowing households to invest in consumer durables at too rapid a pace when young.)
Yang (2009) shows that modelling adjustment costs is important to account for the slow downsizing of durables (housing in his case) as agents get older.Since the high transaction costs for trading houses prevents the households from decreasing their stock quickly later in life. He also shows that when agents are young they build a housing stock and compromise on non-housing consumption. Moreover, as they age they start to decrease non-housing consumption because time the time preference is higher than the interest rate and mortality rates are increasing along the life cycle.

10.5 What drives the life-cycle profiles within the model?

Yang (2009) investigates the quantitative relevance of transaction costs, borrowing constraints and bequest motives. He shows (COPY PASTE: that borrowing constraints are essential in explaining the accumulation of housing assets early in life, the existence of transaction costs is crucial in explaining the slwo downsizing of housing profile later in life.) Moreover, he shows that the bequest motives play a role in determining total lifetime wealth, but not the housing profile!!!! Certain features Yang (2009) has abstracted from and his justification:

1. no inter-vivos transfers (parents lend children money if they need some), data from Health and Retirement Study suggest that these transfers are fairly small, this would lead to higher durable and non-durable consumption

2. yang uses exogenous borrowing constraints and not endogenous ones. FernÃ¡ndez-Villaverde and Krueger (2011) compare the two and show that life-cycle profiles are identical. only exception, with endogenous borrowing constraints net worth of average young agents is slightly negative

3. no housing rental market

4. no health shocks for elderly, which might be important for accumulation of wealth stock

What I want to look at:

1. Check if income uncertainty zero
2. Check what happens when death proba is always zero
3. Use Ad-Hoc constraint
4. With adjustment costs!

11 What features of the Wealth Distribution are explained by the model?

In the previous section we have seen what durables may contribute. Since they do have an impact on life-cycle consumption it seems likely that they will affect the wealth distribution created by the model in one way or the other. In the first part of this chapter I will discuss certain features of the wealth distribution that the baseline model is able to capture. In a second part I will conduct experiments on some parameter changes and analyze how this affects the wealth distribution. Namely, I will drop the adjustment costs, then simulate a model where durables do not provide any utility to provide an insight for the importance of durables. Moreover, I will try different versions of the borrowing constraint, remove the probability of death and finally look at some features of the income process.

11.1 Creating a cross-section from simulated life-cycle profiles

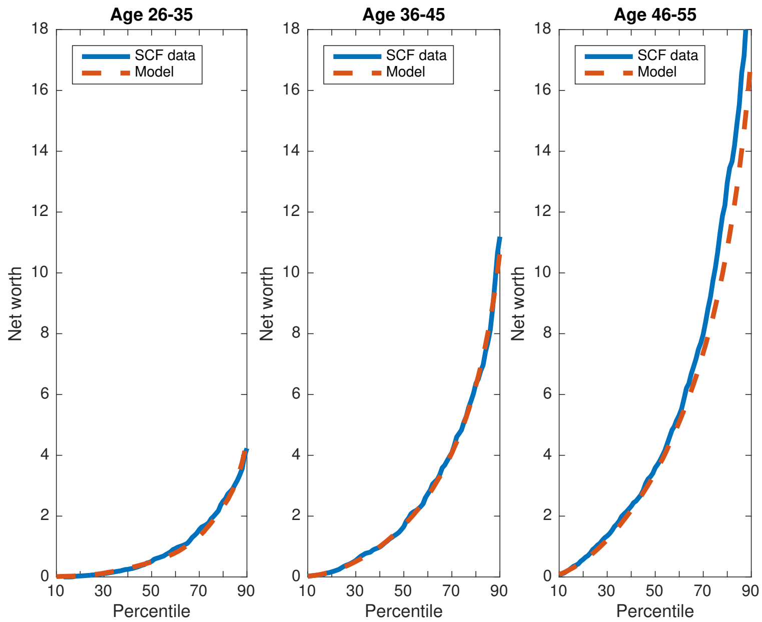
In order to proceed with the distributional analysis and to compare the model distribution to the empirical distribution, the above life-cycle profiles have to be converted into a cross-section. In order to do so I follow Hintermaier and Koeniger (2011), who reproduce the age-composition of the relevant data sample and then account for cohort effects resulting from income growth.[29]

11.2 Model Predictions

Figure 3 shows the net-wealth distribution for three different age groups from the 10th to the 90th percentile. The blue represents the SCF-Data from the year 2004

[29] I hereby use the matlab function, compose_survey.m, provided by Hintermaier and Koeniger (2016).

Figure 3: Average Durable Holdings over the Life-Cycle



CHANGE THE BLOODY COLOURS!!!! IT IS THE SAME AS IN HINTERMAIERS! NEED TO INDICATE THE MEASURES. JUST VERY STANDARD! NEEDS TO BE UPDATED

and the dotted orange line shows the distribution reproduced by the model. Table 2 shows the corresponding gini-indices. The ones for the SCF-Data are directly taken from Hintermaier and Koeniger (2011) whereas the model indices and averages are calculated from the net-wealth distribution resulting from the model simulation.[30] The evidence suggests that the model does manage to reproduce a decent representation of the wealth distribution. For the youngest age group it does uderpredict inequality, while for the other two the model does quite accurately reproduce the ginis. While one may argue about the precision it is evident, that it reproduces a known feature (AS DISCUSSED IN THE FACTS PART) of the empirical distribution, namely that while average wealth holdings increase with age, the concentration of wealth holdings decreases. The means are such that the model predicts average net-worth of the younger two age groups quite accurately, however, underpredicts average worth for the oldest age group.

COMPARE RESULTS TO OTHER STUDIES!

[30] As I allow for negative net-worth holdings in the model, the gini-indices have to be normalized, as otherwise indices of a magnitude larger than 1 were possible. I hereby follow Chen, Tsaur, and Rhai (1982). Note that this may slightly bias the gini-indices compared to the ones calculated without normalization. BE MORE PRECISE!!!

Table 2: As Table 6 in Hintermaier and Koeniger (2011) this Table shows the Gini and Means for the distribution of different age groups up to and with the 90th percentile of the net-worth distribution.

	Data	Model
Age 26-35		
Mean	0.80	0.8137
Gini coefficient	0.713	0.6273
Age 36-45		
Mean	2.36	2.3609
Gini coefficient	0.596	0.5850
Age 46-55		
Mean	4.81	4.2821
Gini coefficient	0.564	0.5464

12 The decomposition of determinants

In this section I perform a series of experiments to simulate, how one-time unexpected changes in the determinants introduced above affect both the wealth distributions of the age groups, as well as the life-cycle profiles.

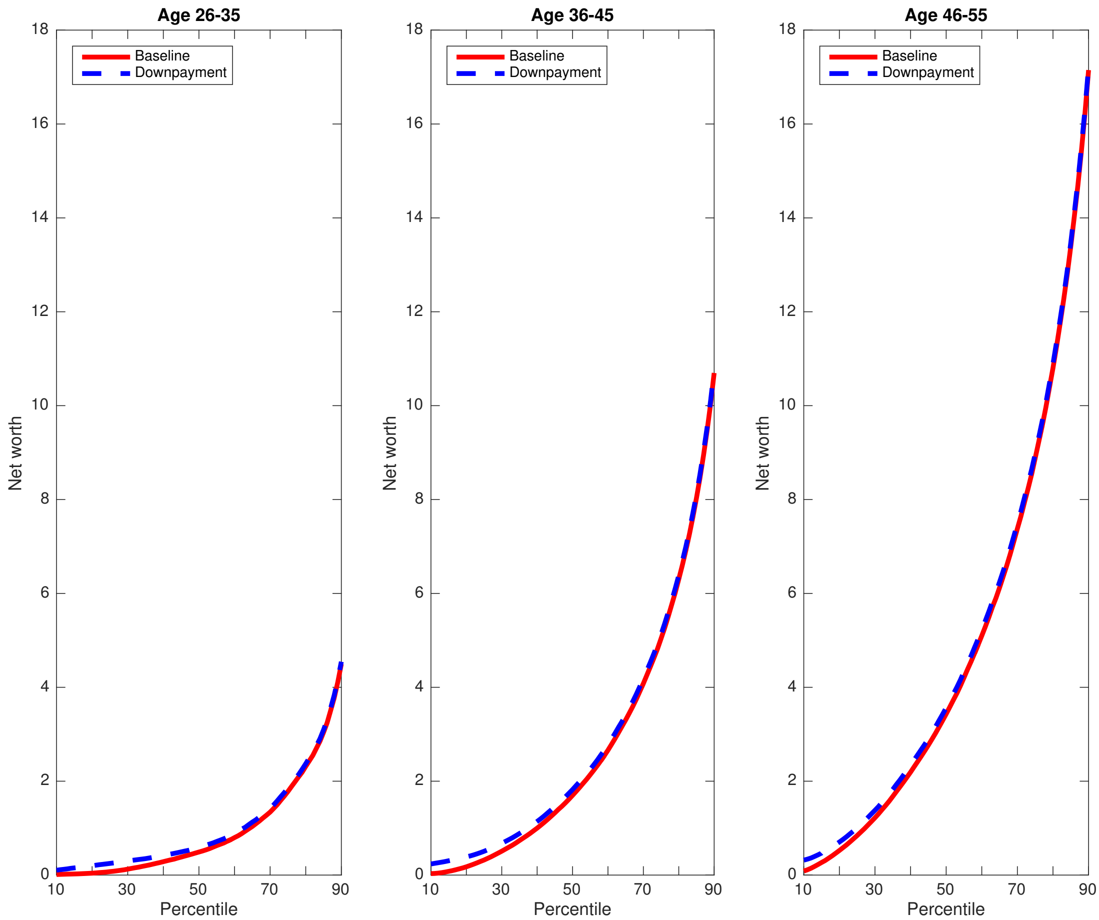
12.1 Loan-to-value ratio

An important determinant, when considering durables is the loan-to value ratio (LTV), which is equal to the part of a durable that is collaterizable. A lower loan to value ratio, would mean that when purchasing a house the amount of credit needed by a consumer would be higher. As is evident, the loan to value ratio has increased by quite a margin during financial libaration. I here simulate the effect of a one time change and unexpected change of the loan to value ratio. A decrease in the LTV to 0.8 (Show increase instead and cite papers. Possible to go over 1?) .

12.1.1 The distribution

Figure 4 shows the impact of such a change on the distributions. The red line represents the results obtained from the baseline calibration and the dotted blue line the steady state distribution for the prime age sample after a change of the LTV ratio. All three age groups seem to be affected. However, only the poorer consumers are affected. With a tighter borrowing constraint, poor agents cannot borrow as much and thus keep more net-assets. This tightening does not affect the rich, who have already accumulated enough assets to borrow at their desired rate.

Figure 4: Average Durable Holdings over the Life-Cycle



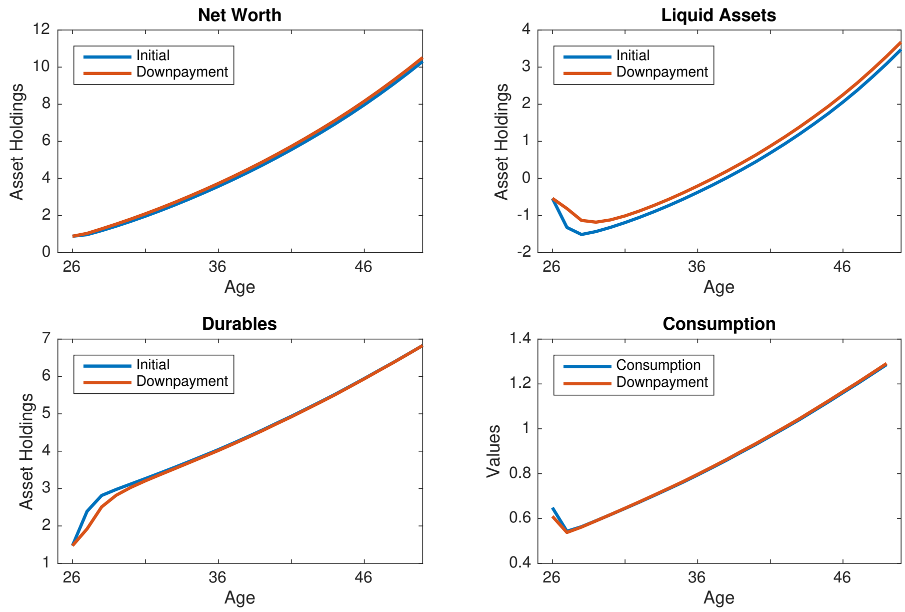
CHANGE THE BLOODY COLOURS!!!! IT IS THE SAME AS IN HINTERMAIERS! NEED TO INDICATE THE MEASURES. JUST VERY STANDARD! NEEDS TO BE UPDATED

Look at different ginis: change of gini of net-worth, gini of durables and gini of liquid assets

12.1.2 Over the life-cycle

Figure 5 shows how the change affects consumers over the life-cycle. A lower loan to value ratio forces agents to pros-pone the accumulation of durables for a few years. As they now can finance a smaller part with debt, they cannot immediately increase the durable stock but need to accumulate it step by step to be able to increase borrowing. Note that only in the very first period, consumption is affected. It does therefore not affect consumption smoothing??? This trade-off is mainly one, between durables and liquid assets. At around age 30 the durable level is the same as in the baseline case, however, the liquid assets stock stays higher for quite some time and thus net worth is affected over this period.

Figure 5: Average Durable Holdings over the Life-Cycle



CHANGE THE BLOODY COLOURS!!!! IT IS THE SAME AS IN HINTERMAIERS! NEED TO INDICATE THE MEASURES. JUST VERY STANDARD! NEEDS TO BE UPDATED

look at policies!
NOTE SIMULATE OVERBORROWING i.e. a negative downpayment

12.2 Income risk

12.3 The experience premium

12.3.1 Adjustment Costs

Yang (2009) shows and explains the impact of the adjustment costs on the life-cycle profiles:

1. Transaction Costs and Borrowing constraints
   (a) Helps to slow down decrease of non-housing consumption later in life
   (b) Comparing ratio of housing to non-housing consumption
      i. Without transaction costs, ratio becomes flatter over the life cycle
      ii. ratio is increasing early in life since borrowing constraints bind more likely than later in live

iii. borrowing constraints are essential in explaining accumulation of housing assets early in life

(c) average life cycle profiles of financial assets and total net worth

i. the evolution of wealth portfolio over the life cycle is similar

ii. shift from financial assets to housing assets when transaction costs are reduced

(d) small effect on net worth profile!!!

2. explains that it is not that important in explaining the net worth distribution!

Díaz and Luengo-Prado (2010) Interaction Between Adjustment Costs and the Collateral Constraint Effect of zero adjustment costs: Home ownership rate rises to 89%, the Gini indices of ouses and financial assets increase significantly for the group of homeowners Gini index of houses for total pop falls from 0.58 to 0.52 in the benchmark economy Since houses are liquid, fraction of wealth held increases as houses increases for all wealth quintiles (although relatively more for the poor) thus effect on overall wealth distribution is minimal With lower adjustment costs, houses are liquid and there is less need to accumulate financial assets for precautionary reasons Zero down payment it follows that households accumulate less financial assets because they are not required to keep a down payment

12.3.2 Durables

12.3.3 Collateral Constraint

12.3.4 Income Process

In connection with the relative degrees of inequality:
Moreover, Hintermaier and Koeniger (2010) and Díaz and Luengo-Prado (2010) both show that a modeling of persistent earnings is necessary to reproduce higher inequality in the durables and thus to more closely match the data.

12.3.5 Determinants

What I can look at

1. Diaz and Luengo-Prado(2010)

(a) the role of superstars and how it affects other results

(b) interaction between adjustment costs and the collateral constraint

(c) the role of persistent earnings

What are the savings motives? Hence why do households accumulate wealth?

1. Kaymak and Poschke (2015): the relative strength of each motive depends on a household's productivity

    (a) precautionary savings motive to insure against life-cycle income risk

    (b) consumption smoothing motive to save for retirement

    (c) bequest motive to endow estates for their offsprings

12.3.6 The literature

12.3.7 Implementing Castañeda

13 Conclusion

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A Experience Premium

Table 3: The annualized experience premium of labor income at ages 36, 46 and 56 relative to age 26.
Source: Corresponds to Table 4 in Hintermaier and Koeniger (2011) and displays the results of the authors' calculation based on SCF Data.

Annualized experience premium		1983	2004
Age –26:	10 years	1.81%	2.60%
	20 years	1.33%	2.18%
	30 years	0.99%	1.83%

B Estimation

Due to my estimation strategy, to estimate the model for the prime age sample and thus excluding consumers, which are situated in the top 10% of the wealth distribution, the means estimation does not suffer from the bias Cagetti (2003) puts forward.

ADDA and COOPER: Model is just identified. Choice of weighting matrix imp when model is overidentified.
The moment condition holds for both moments:

$$E(d_i - D(\beta_0, \theta_0)) = 0$$
$$E(d_i - X(\beta_0, \theta_0)) = 0$$

where D and X are the average of durable stock and net worth holdings of the prime age population respectively. As a consequence, the simulated method of moments can be applied (Duffie and Singleton, 1993).

Therefore I look for

$$\widehat{\Theta} = \arg\min[(m(\Theta) - \nu)' I (m(\Theta) - \nu)]$$

where Θ is a vector containing β and θ, m contains the simulated moments for a specific Θ and $\nu = [2.95, 2.39]$ the empirical counterpart.
$\hat{\beta}$ and $\hat{\theta}$ that
Heathcote et al. (2010): (COPY PASTE: As Christiano and Eichenbaum argue, the exact identification strategy allows for a clear separation between what the model is

restricted to match and what it is designed to explain. The exactly identified strategy amounts to a weighting matrix that sets positive and equal weight only on certain moments, based on the investigator's prior about the (first-order) dimensions of the data that the model should fit.)

C Data

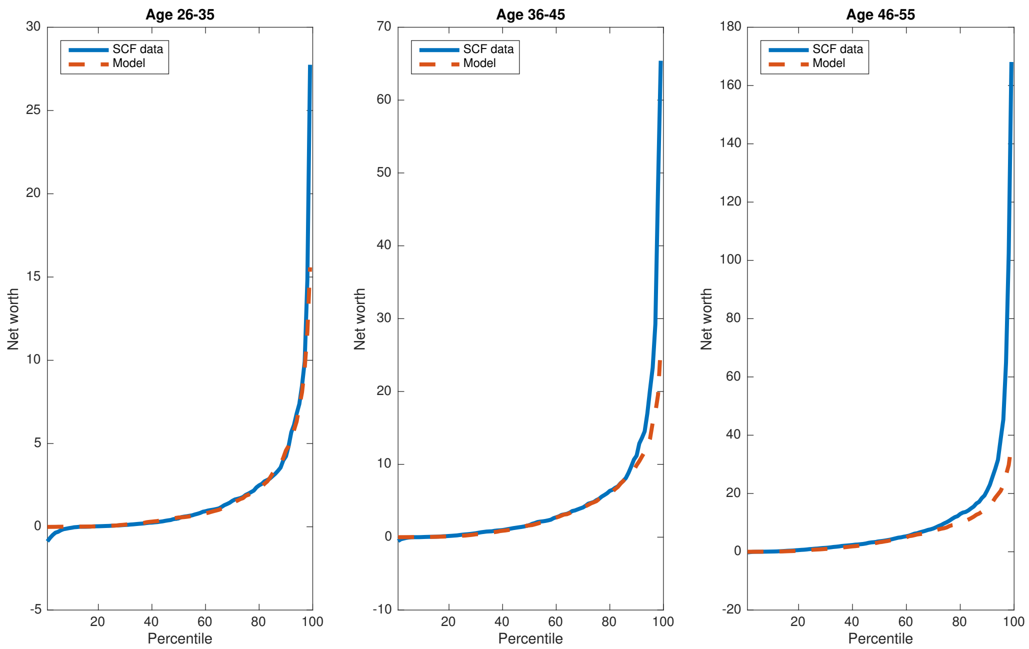
D The full distribution and the issue with the tails of the distribution

Figure 6 shows the full distribution of the three age groups. Note that the scale of the y-axis varies between the age groups and compared to 3 in order to illustrate the whole outline of the empirical distribution. In comparison to the previous graph, two differences immediately meet the eye. For one, the model is not able to correctly predict the lowest part of the distribution. While in the model the poorest agents' net-worth holdings are around 0 for all ages, the SCF-Data displays negative values, which are more pronounced for the two younger generations. The second notable difference is the top of the distribution. Clearly, the model is unable to generate high enough savings for the richest 10%. This is a known problem in the literature.[31]

The richest One of the reasons is that people in the later stages of their lifes do not have enough incentives to accumulate assets. The main driving forces during their life-cycle is the accumulation of assets for retirement as well as precautionary savings, i.e. to be able to deal with wage shocks. However, as the agent approaches death, the incentive to substitute assets for consumption becomes stronger and stronger. As seen in the life-cycle profiles REFERENCE GRAPH, the point where agents start decumulating liquid assets and durables with 65, when entering the retirement period. There are different ways in dealing with this issue. One approach is to model a bequest motive. Agents will thus draw utility from the capital stock remaining at the time of death and therefore tend to accumulate more and longer over the life-cycle (CITE DE NARDI!). Another way is to model extreme income risk. (Castaneda et al., 2003) calibrates the income process to match the ginis of both the income as well as the net-worth distribution. The resulting income states contain an extreme income state of a magnitude about 100 times larger than the second state and 1000 times larger

[31]See Fella and De Nardi (2017) for a review and modeling choices that are able to deal with this issue.

Figure 6: Average Durable Holdings over the Life-Cycle



CHANGE THE BLOODY COLOURS!!!! IT IS THE SAME AS IN HINTERMAIERS! NEED TO INDICATE THE MEASURES. JUST VERY STANDARD! NEEDS TO BE UPDATED. NOTE THAT THE LOWEST PART OF THE DISTRIBUTION IS NOT VISIBLE!

than the worst state. The issue with this approach is that the income process then is not founded on micro-data of the income, which may be counterproductive for policy recommendation.

MAYBE INCLUDE GRAPH WITH SAVING RATE FROM DE NARDI

The poorest The main reason for the poor reproduction of the lowest 10% is the calibration. When Hintermaier and Koeniger (2011) estimated their preference parameters they did not consider all observation with negative net-worth as their estimation approach did not permit for negative values.

E Relative degrees of inequality

So far I have only discussed net-worth in the distribution section. However, the model does also reproduce a distribution of consumption as well as the two assets, which make up net-worth, independently. Moreover, there is the distribution of earnings, which is endogenously determined. It is well known that heterogeneous agent models of this type are able to reproduce the ordinal ranking of inequalities for consumption,

Table 4: Gini indices for the different distributions

	Consumption	Income	Durables	Liquid Assets	Net Worth
Model	0.3250	0.4062	0.3678	0.9523	0.6674
Data	????	0.427	0.67	0.97	0.81

income and net-worth in line with empirical evidence.Aiyagari (1994) shows that the model reproduces the (COPY PASTE: empirical plausible relative degrees of inequality), where consumption exhibits the least inequality, followed by income and capital is most unequal. (NOTE AIYAGARI IS A INFINITE HORIZON MODEL)
Table 4 displays the gini indices produced by the model and the one found in the data.[32] The model manages to match the ordering for the different inequalities only considering the types of the assets as well as net-worth. Moreover, it also shows that the consumption inequality is lowest. However, it does not manage to show that durables are more concentrated than income. Díaz and Luengo-Prado (2010) show, in a model with housing, that this may be overcome by explicitly modeling a rental market. Housing makes up the biggest part of durables (CITE AND SHOW NUMBERS). As a consequence of rental markets, wealth poor households will decide to rent instead of owning, thus satisfying their durable consumption needs without being able to benefit from the collateral value of the durable object. As the rented object does not account for the housing part of the durable holdings, wealth poor will hold less durables and thus the gini index of durables increases, while, as Díaz and Luengo-Prado (2010) show, the other indices change only slightly are only marginally affected. As I will show below. The concentration on durables largely depends on the persistence of the earning process. A change in the income process thus helps to improve on the prediction of the gini index of durables by the model.

[32]The empirical indices for durables, liquid assets and net worth are taken from Hintermaier and Koeniger (2010) and the index for income from Hintermaier and Koeniger (2011)