MAE 5510 Flight Dynamics: Exam 1 - Longitudinal Trim and Stability **Name** Eric Jarsen

A02176917

Open book. Open notes. Closed neighbor, videos, internet, AI, or any other resource. All work done neatly and logically. Calculators, Excel, or personal code allowed.

I did not communicate about this exam to anyone who had already started ~~the~~ exam, and I will not talk to anyone about this exam until after it is due. Sign here if this is true: ____________

1. Short Answer (50 Points)

1.1 For an aircraft to be trim, the forces about the center of gravity must sum to be (greater than | less than | equal to) zero.

trim $\Sigma F_x = \Sigma F_z = \Sigma M_y = 0$

1.2 For an aircraft to be trim, the moments about the center of gravity must sum to be (greater than | less than | equal to) zero.

1.3 For an aircraft to be longitudinally stable, the change in pitching moment with respect to angle of attack must be (greater than | less than | equal to) zero. $C_{m,\alpha} < 0$

1.4 Another name for the aerodynamic center of an aircraft is the Neutral Point.

1.5 Rank the following in order of magnitude for a typical aircraft (1 = largest, 2 = next largest, 3 = smallest)

1 The lift slope of the airfoil section used on the main wing, $\tilde{C}_{L,\alpha}$

2 The lift slope of the main wing, $C_{L_w,\alpha}$

3 The lift slope of the entire aircraft, $C_{L,\alpha}$

$\sigma = \frac{l_{np}}{\bar{c}_w}$

1.6 The static margin can be computed by subtracting the location of the Neutral point from the location of the Center of gravity and dividing by the Main wing average chord ($\bar{c}_w$)

1.7 Alternately, the static margin can be found by the negative of the ratio of the pitch stability derivative to the lift slope. $\sigma \approx -\frac{C_{m,\alpha}}{C_{L,\alpha}}$

1.8 A common rule of thumb is that the static margin should be near $\sigma \approx$ 0.05 or 5%.

1.9 (True | False) The simplified linear longitudinal analysis, in which all components lie along the fuselage reference line, predicts that the pitch stability derivative is independent of angle of attack.

1.10 (True | False) The generalized longitudinal analysis, in which nonlinearities and vertical offsets are included, predicts that the pitch stability derivative is independent of angle of attack.

shows up in sin and cos throughout.

2. Work-Out Problems (50 Points)

One version of the Lockheed Martin F-16 Fighting Falcon has the following geometric and aerodynamic properties:

$\bar{c}_w = 10$ ft
$\bar{c}_h = 5.4913$

$$S_w = 300\text{ ft}^2,\quad b_w = 30.0\text{ ft},\quad R_{T_w} = 0.2275,\quad \Lambda_{c/4_w} = 32\text{ deg},$$
$$S_h = 63.7\text{ ft}^2,\quad b_h = 11.6\text{ ft},\quad R_{T_h} = 0.39,\quad \Lambda_{c/4_h} = 32\text{ deg},$$
$$C_{L_w,\alpha} = 3.73,\quad \alpha_{0_w} = 1.9°,\quad \alpha_{L0_w} = -1.62°,\quad C_{m_w} = -0.041,$$
$$C_{L_h,\alpha} = 3.20,\quad \alpha_{0_h} = 0.0°,\quad \alpha_{L0_h} = 0.0°,\quad \varepsilon_e = 1.0,\quad C_{m_h,\delta_e} = 0.0$$

The longitudinal components all lie directly on the fuselage reference line, and their locations are known relative to the nose of the aircraft as follows:

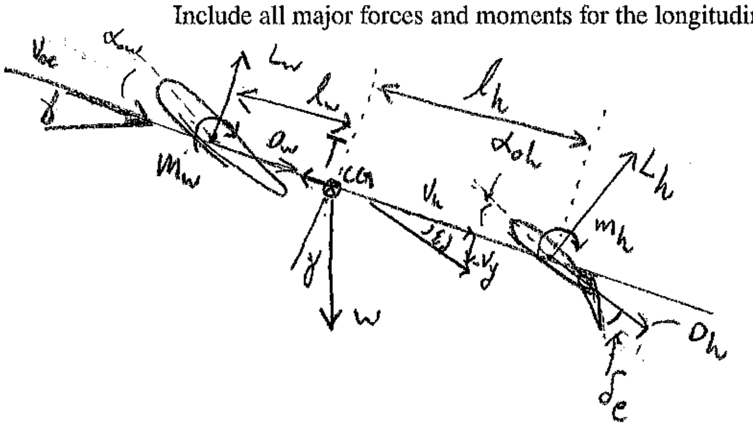
Component	Distance Aft of Nose [ft]
CG	26.94
Main Wing Aerodynamic Center	26.42
Horizontal Tail Aerodynamic Center	43.55

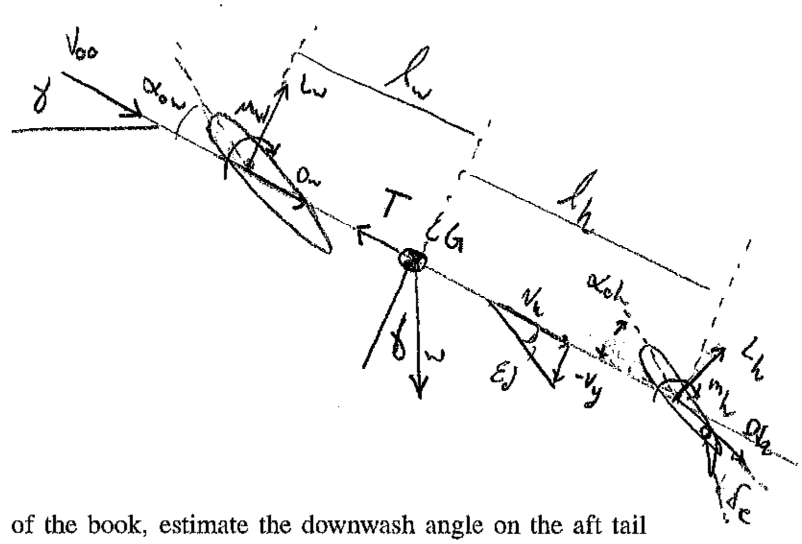
$l_w = 26.42 - 26.94 = -0.52'$
$l_h = 43.55 - 26.94 = 16.61'$
$l_{hw} = l_h - l_w = 17.13'$

Notice that this aircraft has an all-flying tail, i.e. $\varepsilon_e = 1.0$. For the following analysis, neglect the influence of the fuselage and engine, assume the tail efficiency factor, η_h, is unity, and neglect the upwash on the main wing due to the horizontal tail.

$\eta_h = 1$ $\qquad$ ε_u = Neglected

2.1 Draw a free-body diagram of this aircraft using the simplified model discussed in Section 4.3 of the book. Include all major forces and moments for the longitudinal components.





2.2 Using the simplified analysis given in Section 4.5 of the book, estimate the downwash angle on the aft tail as a function of angle of attack, $\varepsilon_d = \varepsilon_{d0} + \varepsilon_{d,\alpha}\alpha$. Compute values for

$$\varepsilon_d = \frac{k_v k_p k_s}{K_b} \cdot \frac{C_{Lw}}{R_{Aw}}$$

$$\varepsilon_{d0} = \frac{k_v k_p k_s}{K_b R_{Aw}} \cdot C_{Lw_0} = 0.2147(0.22915) = 0.0492$$

$k_p =$

$$\varepsilon_{d,\alpha} = \frac{k_v k_p k_s}{k_b R_{Aw}} C_{Lw,\alpha} = 0.8008$$

$$\boxed{\varepsilon_d = 0.0492 + 0.8008\,\alpha}$$

$\varepsilon_{d,0}$ $\qquad$ $\varepsilon_{d,\alpha}$ 2

2.3 Estimate the static margin of this aircraft.

$$\sigma \approx \frac{-C_{m,\alpha}}{C_{L,\alpha}}$$

$\sigma =$

$$C_{m,\alpha} = -\frac{l_w}{\bar{c}_w} C_{Lw,\alpha} - \frac{S_h l_h}{S_w \bar{c}_w} \eta_h C_{Lh,\alpha}(1-\varepsilon_{d,\alpha}) = \underbrace{\frac{0.52}{10} 3.73}_{0.19396} - \underbrace{\frac{63.7 \cdot 16.61}{300 \cdot 10} (1)(3.20)(1-0.8008)}_{0.224816}$$

$$= -0.030856$$

$$C_{L,\alpha} = C_{Lw,\alpha} + \frac{S_h}{S_w} \eta_h C_{Lh,\alpha}(1-\varepsilon_{d,\alpha}) = 3.73 + \underbrace{\frac{63.7}{300}(1) 3.20 (1-0.8008)}_{0.1353}$$

$C_{Lw,\alpha} = 3.73$

$$= 3.865$$

$C_{Lh,\alpha} = 3.20$

$$\boxed{\sigma = \frac{-(-0.030856)}{3.865} = 0.00798 \quad \text{or} \quad 0.798\,\%}$$

For the following two problems, assume the aircraft is operating near take-off conditions at a velocity of 225 ft/s, a weight of 20,500 lbf, in steady-level flight at sea level. lift must equal the weight of aircraft.

2.4 Compute the lift coefficient for the complete aircraft at this operating condition.

$\gamma = 0$

$C_L =$

$W = 20{,}500$ lbf

$$\frac{W \cos\gamma}{\frac{1}{2} \rho V_\infty^2 S_w} = \frac{20{,}500 (\cos(0))}{\frac{1}{2}(0.0023769 \cdot 225^2 \cdot 300)}$$

$V_\infty = 225$ ft/s

$\rho_{air} = 0.0023769$ slug/ft³

$$\boxed{C_L = 1.13576}$$

2.5 Compute the angle of attack and elevator deflection **in degrees** required to trim the aircraft at this operating condition.

$$\begin{bmatrix} C_{L,\alpha} & C_{L,\delta_e} \\ C_{m,\alpha} & C_{m,\delta_e} \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{L_0} \\ -C_{m_0} \end{bmatrix}$$

$$\boxed{\begin{aligned} \alpha &= 13.766^\circ \\ \delta_e &= 0.965^\circ \end{aligned}}$$

$C_{L,\alpha} = 3.865$ [Prob 2.3]

$C_{m,\alpha} = -0.030856$ [Prob 2.3]

$C_L = 1.13576$ [Prob 2.4]

Need C_{L,δ_e}, C_{m,δ_e}, C_{L_0}, C_{m_0}

$$C_{L,\delta_e} = \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}\varepsilon_e = \frac{63.7}{300}(1)(3.2)(1) = \underline{0.67946}$$

$$C_{m,\delta_e} = \frac{S_h \bar{c}_h}{S_w \bar{c}_w}\eta_h C_{m_h,\delta_e} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}\varepsilon_e = \frac{63.7 \cdot 5.4913}{300 \cdot 10}(1)(0.0) - \frac{63.7}{300}\frac{16.61}{10}(1)(3.20)(1) = \underline{-1.1286}$$

$$C_{L_0} = C_{L_w,\alpha}(\alpha_{0w} - \alpha_{L0w}) + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(\alpha_{0h} - \varepsilon_0) =$$

$$\underbrace{3.73\left(3.52 \cdot \frac{\pi}{180}\right)}_{0.229155} + \underbrace{\frac{63.7}{300}(1)(3.2)(0 - 0.0492)}_{-0.03342} = \underline{0.1957}$$

$$C_{m_0} = C_{m_w} - \underbrace{\frac{l_w}{\bar{c}_w} C_{L_w,\alpha}(\alpha_{0w} - \alpha_{L0w})} - \underbrace{\frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}(\alpha_{0h} - \varepsilon_0)}$$

$$-0.041 - (-0.011916) - (-0.0555) = \underline{0.02644}$$

$$\begin{bmatrix} 3.865 & 0.67946 \\ -0.030856 & -1.1286 \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} 0.94006 \\ -0.02644 \end{bmatrix}$$ "Calc RREF"

$$\boxed{\begin{aligned} \alpha &= 0.24026 \text{ Rad} = 13.766 \text{ degrees} \\ \delta_e &= 0.0168 \text{ Rad} = 0.965 \text{ degrees} \end{aligned}}$$

Aero Exam 1

Problem 2.2 Solve for $\varepsilon_d = \varepsilon_{d_0} + \varepsilon_{d,\alpha}\alpha$

Need k_v, k_p, k_s, k_b

$$\varepsilon_d = \frac{k_v k_p k_s}{k_b}\frac{C_{L,w}}{R_{Aw}}$$

$$C_{L_w} = C_{L_{w,\alpha}}(\alpha + \alpha_{ow} - \alpha_{Low} - \cancel{\varepsilon_u})$$

$R_{Tw} = 0.2275$

$R_{Aw} = b_w/\bar{c}_w = 30/10 = 3$

$\bar{C}_w = S_w/b_w = 300/30 = 10$ ft

$k_v \approx 1.04$

$k_b \approx 0.75$

values not on chart for R_{Aw} so approximated w/ best Guess.

$$k_p = \frac{2k_b^2}{\pi^2(\bar{y}^2+k_b^2)}\left[1 + \frac{\bar{x}(\bar{x}^2+\bar{y}^2+k_b^2)}{(\bar{x}^2+\bar{y}^2)\sqrt{\bar{x}^2+\bar{y}^2+k_b^2}}\right] = \frac{2\cdot 0.75^2}{\pi^2(0.75^2)}\left[1+\frac{1.142(1.142^2+0.75^2)}{(1.142^2)\sqrt{(1.142^2+0.75^2)}}\right]$$

$$\bar{y} = \frac{y}{b_w/2} = \frac{0}{15} = 0$$

$$\bar{x} = \frac{x}{b_w/2} = \frac{17.13}{15} = 1.142$$

$k_p \approx 0.445$

$$k_s = 1 + \frac{\frac{\bar{x}-s}{t} + \bar{x}(\bar{r}+\ldots)}{\ldots}$$

"used personal code"

$k_s = 1.04397 \approx 1.044$

check w/ chart ✓✓

$k_v = 1.04$

$k_p = 0.445$

$k_b = 0.75$

$k_s = 1.044$

$$\varepsilon_d = \frac{1.04 \cdot 0.445 \cdot 1.044}{0.75}\cdot\frac{C_{Lw}}{3}$$

$$\underline{\varepsilon_d = 0.2147\, C_{Lw}}$$

$$\varepsilon_d = 0.2147(C_{Lw_0} + C_{Lw,\alpha}\alpha)$$

$$C_{Lw_0} = C_{Lw,\alpha}(\alpha_{ow} - \alpha_{Low}) = 3.73(1.9 + 1.62°)$$

$$3.73\left(3.52\cdot\frac{\pi}{180}\right) = C_{Lw_0}$$

$$C_{Lw_0} = 0.22915$$

Problem 2.2 continued

$$C_{Lw0} = 0.22915 = 3.73\left(3.52 \cdot \frac{\pi}{180}\right) = C_{Lw,\alpha}(\alpha_{0w} - \alpha_{L0w})$$

$$\varepsilon_d = \varepsilon_{d_0} + \varepsilon_{d,\alpha}\,\alpha \qquad \varepsilon_{d_\alpha} = C_{Lw,\alpha} \cdot 0.2147 = 3.73 \cdot 0.2147$$

$$\varepsilon_{d_\alpha} = 0.8008$$

$$\varepsilon_{d_0} = 0.2147 \cdot C_{Lw0} = 0.2147 \cdot 0.22915 = 0.0492$$

$$\underline{\varepsilon_d = \overset{\varepsilon_{d_0}}{0.0492} + \overset{\varepsilon_{d_\alpha}}{0.8008}\,\alpha}$$