1.15 If the main wing and horizontal stabilizer both have zero mounting angles, compute the angle of attack and elevator deflection required to trim the aircraft in a steady climb at an altitude of 5,000 ft and a climb angle of 20 deg at a speed of 200 mph.

$\gamma = 20^\circ \quad V_\infty = 293.33$ ft/s

$\rho = 0.002048$

$\alpha_{oh} = 0 \quad \alpha_{ow} = 0$

$$\begin{bmatrix} C_{L,\alpha} & C_{L,\delta_e} \\ C_{m,\alpha} & C_{m,\delta_e} \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{L0} \\ -C_{m0} \end{bmatrix}$$

$$C_L = \frac{\cos(20)\,5375}{\frac{1}{2}(0.002048)(293.33^2)\,244}$$

$$\underline{C_L = 0.366}$$

$$C_{L0} = C_{Lw,\alpha}(-\alpha_{Low}) = 4.62\left(-2.2 \cdot \frac{\pi}{180}\right) = \underline{-0.1774}$$

$$C_L - C_{L0} = \underline{0.5434}$$

$$C_{L,\alpha} = \underline{5.13}$$

$$C_{L,\delta_e} = \frac{S_h}{S_w}(1)\,C_{Lh,\alpha}\,\varepsilon_e = \frac{31}{244}(4.06(0.6)) = \underline{0.30949}$$

$$C_{m,\alpha} = \frac{S_h l_h}{S_w c_w} = \underline{1.4139}$$

$$C_{m,\delta_e} = \frac{S_h \bar{c}_h}{S_w \bar{c}_w}(1)C_{mh,\delta_e} - \frac{S_h l_h}{S_w c_w}(1)\,C_{Lh,\alpha}\,\varepsilon_e = \frac{31(2.913)}{244(6.625)}(-0.55) - \frac{31(18.16)}{244(6.625)}4.06(0.6)$$

$$C_{m,\delta_e} = \underline{-0.879}$$

$$C_{m0} = \cancel{C_{mw}} - \frac{l_w}{c_w}C_{Lw,\alpha}\cancel{(\alpha_{ow} - \alpha_{Low})} - \cancel{\frac{S_h l}{S_w}(\alpha_{oh} - \varepsilon_0)} \qquad C_{m0} = 0$$

$$\begin{bmatrix} 5.13 & 0.3095 \\ 1.4139 & -0.879 \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} 0.5434 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} \alpha = 0.096 \text{ rad} \\ \delta_e = 0.1553 \end{array}$$

$S_w = \text{Area} = \bar{c}_w \cdot L_w$

$R_{Aw} = \frac{L_w}{c_w}$

$$\underline{\alpha = 5.500^\circ}$$

$$\underline{\delta_e = 8.898^\circ}$$

$\eta_h =$ ______

1.10 Combine the solutions from problems 1.6, 1.7, 1.8, and 1.9 to develop equations for the lift coefficient and pitching-moment of the aircraft as a function of wing and horizontal stabilizer geometric and aerodynamic properties, as well as the elevator deflection.

$$C_L = \underbrace{\left[C_{Lw,\alpha} + \eta_h \frac{S_h}{S_w} C_{Lh,\alpha}(1-\varepsilon_{d,\alpha})\right]}_{C_{L,\alpha}}\alpha + \underbrace{\left[\frac{S_h}{S_w}\eta_h C_{Lh,\alpha}\varepsilon_e\right]}_{C_{L,\delta_e}}\delta_e =$$

$$C_W \cos\gamma - \underbrace{C_{Lw,\alpha}(\alpha_{0w}-\alpha_{L0w}) - \frac{S_h}{S_w}\eta_h C_{Lh,\alpha}(\alpha_{0h}-\alpha_{L0h}-\varepsilon_{d0})}_{C_{L0}\ \text{"geometric."}}$$

$$C_{m_w} + \frac{S_h \bar{c}_h}{S_w \bar{c}_w}\eta_h C_{mh,\delta_e}\delta_e - \frac{l_w}{\bar{c}_w}C_{Lw,\alpha}(\alpha + \alpha_{0w} - \alpha_{L0w}) - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{Lh,\alpha}\left[(1-\varepsilon_{d,\alpha})\alpha + \alpha_{0h} - \varepsilon_{d0} + \varepsilon_e \delta_e\right] = 0$$

1.11 Starting from the pitching-moment equation developed in problem 1.10, develop an expression for the pitch stability criteria as a function of the wing and horizontal stabilizer geometric and aerodynamic properties.

$$\left[-\frac{l_w}{\bar{c}_w}C_{Lw,\alpha} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{Lh,\alpha}(1-\varepsilon_{d,\alpha})\right]\alpha + \left[\frac{S_h \bar{c}_h}{S_w \bar{c}_w}\eta_h C_{mh,\delta_e} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{Lh,\alpha}\varepsilon_e\right]\delta_e =$$

$$-\left[C_{m_w} - \frac{l_w}{\bar{c}_w}C_{L,\alpha}(\alpha_{0w}-\alpha_{L0w}) - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{Lh,\alpha}(\alpha_{0h}-\varepsilon_{d0})\right] \quad C_{m,\alpha}$$

$$C_{m,\alpha} = [\quad\quad] < 0$$

1.12 For an aircraft to be trim, both equations in problem 1.10 must be satisfied. This provides a system of two equations that can be expressed in terms of two unknown operating paramters, α and δ_e as

$$\begin{bmatrix} C_{L,\alpha} & C_{L,\delta_e} \\ C_{m,\alpha} & C_{m,\delta_e} \end{bmatrix} \begin{Bmatrix} \alpha \\ \delta_e \end{Bmatrix} = \begin{bmatrix} C_L - C_{L0} \\ -C_{m0} \end{bmatrix}$$

where the known geometric and aerodynamic information of the aircraft is contained in the variables

$$C_{L,\alpha} = C_{Lw,\alpha} + \frac{S_h}{S_w}\eta_h C_{Lh,\alpha}(1-\varepsilon_{d,\alpha})$$

$$C_{L,\delta_e} = \frac{S_h}{S_w}\eta_h C_{Lh,\alpha}\varepsilon_e$$

$$C_L = \frac{W}{\frac{1}{2}\rho V_\infty^2 S_w}$$

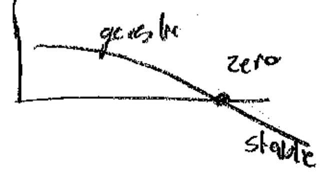
$$C_{L0} = C_{Lw,\alpha}(\alpha_{0w}-\alpha_{L0w}) + \frac{S_h}{S_w}\eta_h C_{Lh,\alpha}(\alpha_{0h}-\varepsilon_{d0})$$

$$C_{m,\alpha} = -\frac{l_w}{\bar{c}_w}C_{Lw,\alpha} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{Lh,\alpha}(1-\varepsilon_{d,\alpha})$$

$$C_{m,\delta_e} = \frac{S_h \bar{c}_h}{S_w \bar{c}_w}\eta_h C_{mh,\delta_e} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{Lh,\alpha}\varepsilon_e$$

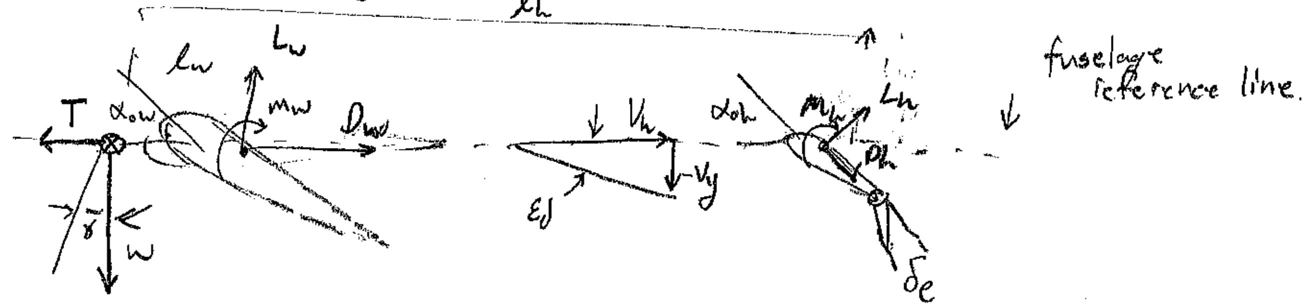
$$C_{m0} = C_{m_w} - \frac{l_w}{\bar{c}_w}C_{Lw,\alpha}(\alpha_{0w}-\alpha_{L0w}) - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{Lh,\alpha}(\alpha_{0h}-\varepsilon_{d0})$$

C_{m_α}



C_{m_α}

1.3 Consider a conventional aircraft with a main wing and horizontal tail. Assume the main wing, horizontal stabilizer, and center of gravity all lie along the fuselage reference line, and that the thrust and fuselage reference line are aligned with the direction of flight. Draw a side view of the aircraft with the longitudinal forces and moments labeled including the forces and moments on each lifting surface.



[w h]

$$\boxed{\alpha_{o_{all}} = \alpha_{oh} + \alpha_{ow}}$$

$\frac{1}{2}\rho V^2 \bar{c}_w S_w$

α_{oh}

$\sin(\theta) = \theta$

4.3)

1.4 Using the aircraft given in problem 1.3 write an equation for the force balance in the direction of lift if the aircraft is trimmed with a climb angle of γ. Apply the small-angle approximation for the downwash angle and drop very small terms.

Eq 4.3.2

ε_d = downwash

Small angle consideration

$$L = L_w + L_h \cos \varepsilon_d - D_h \sin \varepsilon_d = W \cos \gamma$$

$$\boxed{L = L_w + L_h - D_h \varepsilon_d = W \cos \gamma} \quad 4.3.2$$

1.5 Using the aircraft given in problem 1.3, write an equation for the pitching-moment if the aircraft is trimmed with a climb angle of γ. Apply the small-angle approximation for the downwash angle and drop very small terms.

$$m = m_w + m_h - l_w L_w - l_h L_h \cos \varepsilon_d + l_h D_h \sin \varepsilon_d = 0$$

$$m = m_w + m_L - l_w L_w - l_h L_h + \cancel{l_h D_h \varepsilon_d} = 0$$