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MAE 5510 : Exercise Set 1

Group	8				
Date	1/10/2024	1/12/2024			
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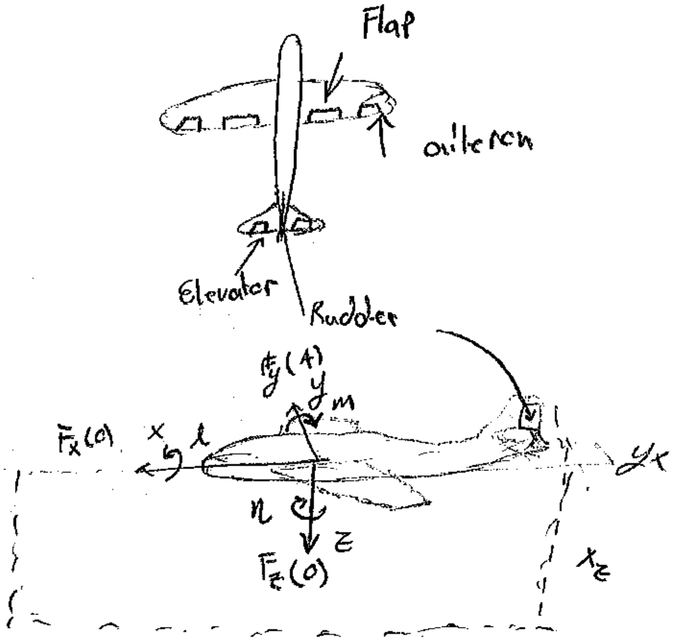
1.1 Draw a 3-view of an aircraft and label the control surfaces, translational axes, and moment about each axis. Next to each of the axes, include the letter symbol used to denote the force along that axis, the moment about that axis, and the rotation rate about the axis. Label each component as longitudinal (O) or lateral (A).



axis	moment	rate	velocity	force
x	ℓ	p	u	axial
y	m	q	v	sideslip
z	n	r	w	Normal

Longitudinal

F_x, F_z, M_y

Lateral

F_y, M_x, M_z

C_m = Pitching moment

c

$$\frac{M}{\frac{1}{2}\rho V^2 \bar{c} S_w}$$

1.2 Write the equation that expresses the requirement for an aircraft to be stable in pitch.

$$\frac{dM}{d\alpha} < 0 \quad \Rightarrow \quad C_{m,\alpha} < 0$$

$$\frac{d C_m}{d\alpha} = \cancel{\frac{\partial C_{m_{ac}}}{\partial \alpha}} - \frac{l_w}{c}\frac{\partial C_L}{\partial \alpha} < 0$$

$$\boxed{-\frac{l_w}{\bar{c}} C_{L,\alpha} < 0} \qquad l_w > 0$$

1.6 Write the solutions to problems 1.4 and 1.5 in nondimensional form.

$$C_L = C_{L_w} + \frac{S_h}{S_w} \eta_h C_{L_h} = C_W \cos\gamma$$

$$C_m = C_{m_w} + \frac{S_h}{S_w} \frac{\bar{c}_h}{\bar{c}_w} \eta_h C_{m_h} - \frac{l_w}{\bar{c}_w} C_{L_w} - \frac{l_h}{\bar{c}_w} \frac{S_h}{S_w} \eta_h C_{L_h} = 0$$

1.7 Applying the small-angle approximation, write the expression for the lift coefficient of a main wing as a function of lift slope, mounting angle, and zero-lift angle of attack.

$$C_{L_w} = C_{L_{w,\alpha}} (\alpha + \alpha_{0w} - \alpha_{L0w})$$

1.8 Applying the small-angle approximation, write the expression for the lift coefficient of a horizontal stabilizer as a function of lift slope, mounting angle, zero-lift angle of attack, downwash, elevator effectiveness, and elevator deflection.

$$C_{L_h} = C_{L_{h,\alpha}} (\alpha + \alpha_{0h} - \alpha_{L0h} - \varepsilon_{d0} - \varepsilon_{d,\alpha}\alpha + \varepsilon_e \delta_e)$$

1.9 Assuming a linear relationship between control-surface deflection and pitching moment, write the expression for the pitching-moment coefficient on the horizontal stabilizer as a function of elevator deflection.

$$C_{m_h} = C_{m_{h0}} + C_{m_{h,\delta_e}} \delta_e$$

$$\eta_h = \frac{\frac{1}{2}\rho V_h^2}{\frac{1}{2}\rho V_\infty^2} \approx 1.0 \qquad l_w \approx 0 \qquad \frac{W\cos\gamma}{\frac{1}{2}\rho_\alpha V_\infty^2 S_w} = C_w$$

Consider a version of the British Spitfire with the following geometric and aerodynamic characteristics:

$\bar{c}_w = 6.625$ ft, $\bar{c}_h = 2.9135$ ft

$S_w = 244\ \text{ft}^2$, $b_w = 36.83$ ft, $C_{L_w,\alpha} = 4.62$, $\alpha_{L0_w} = -2.2°$, $C_{m_w} = -0.053$,
$S_h = 31\ \text{ft}^2$, $b_h = 10.64$ ft, $C_{L_h,\alpha} = 4.06$, $\varepsilon_e = 0.60$, $C_{m_h,\delta_e} = -0.55$,
$W = 8,375$ lbf, $l_h - l_w = 18.16$ ft $l_h = 18.16$

For the following problems, assume that the center of gravity lies at the quarter-chord of the main wing, the horizontal stabilizer has a symmetric airfoil, and neglect any effects from downwash.

$\varepsilon_d = 0$ C_m

$\alpha_{0h} \quad \alpha_{0w}$

1.13 Find the mounting angle of the main wing and horizontal stabilizer required for the aircraft to be trim in steady-level flight at sea level at a velocity of 200 mph with zero elevator deflection and zero angle of attack.

$L = h \neq 0$

$\gamma = 0 \quad \alpha = 0 \quad \rho = 0.0023769 \qquad \delta_e = 0$

$$200\ \frac{\text{mi}}{\text{h}} \cdot \frac{5280\ \text{ft}}{1\ \text{mile}} \cdot \frac{1\text{hr}}{3600\text{s}} = 293.33\ \text{ft/s} \qquad C_L = \frac{8375}{\frac{1}{2}\rho V_\infty^2 S_w}$$

when degrees and when radians.

$$C_w = \frac{8375}{\frac{1}{2}\ 0.0023769 \cdot 293.33\ \text{ft/s}^2 \cdot 244\ \text{ft}^2} = 0.335$$

zero lift angle

$$C_{L_{eq}} = C_w \cancel{\cos\gamma} - C_{L_w,\alpha}(\alpha_{0w} - \alpha_{L0w}) - \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(\alpha_{0h} - \alpha_{L0h} - \cancel{\varepsilon_{d0}})$$

2 eqs 2 unknowns.

$$0 = -\left[C_{m_w} - \frac{l_w}{c_w}C_{L,\alpha}(\alpha_{0w} - \alpha_{L0w}) - \frac{S_w l_h}{S_w c_w}\eta_h C_{L_h,\alpha}(\alpha_{0h} - \cancel{\varepsilon_{d0}})\right]$$

Eq rearranged.

$$0 = -4.62(\alpha_w + 2.2°) - \frac{31}{244}(1)\ 4.06(\alpha_{0h})$$

$$\boxed{\alpha_{0h} = -3.715 \text{ degrees}}$$

$$\boxed{\frac{10.164°}{180} = -4.62\,\alpha_{0w} - 0.5158\,\alpha_{0h}} \quad \text{eq 1}$$

Eq 2

$$0 = -0.335 + \cancel{C_{L,\alpha}(\alpha_{Lw})} + \underbrace{\frac{31}{244}\frac{18.16}{6.625}}_{0.3482}(1)\ 4.06(\alpha_{0h})$$

$$\boxed{\alpha_{0h} = 0.2369 \text{ rad.}} \qquad 0.335 = 1.4139\,\alpha_{0h}$$

$= 13.57$ degrees.

1.14 Compute the aircraft static margin.

$$\frac{l_{np}}{c_w} \cong \frac{-C_{m,\alpha}}{C_{L,\alpha}} = \frac{-\frac{l_w}{c_w}C_{L_w,\alpha} + \frac{S_w l_h}{S_w c_w}\eta_h C_{L_h,\alpha}(1 - \cancel{\varepsilon_{d,\alpha}})}{C_{L_w,\alpha} + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(1 - \cancel{\varepsilon_{d,\alpha}})}$$

$$\frac{-\cancel{\frac{18.16}{6.625}4.62} + \frac{31}{244}\frac{18.16}{6.625}(1)\ 4.06\ (1)}{4.62 + \frac{31}{244}(1)\ 4.06\ (1)} = \frac{1.4139}{5.13}$$

$$= 0.2756 \qquad \underline{27\%}$$