MAE 5510 : Exercise Set 1 Solutions

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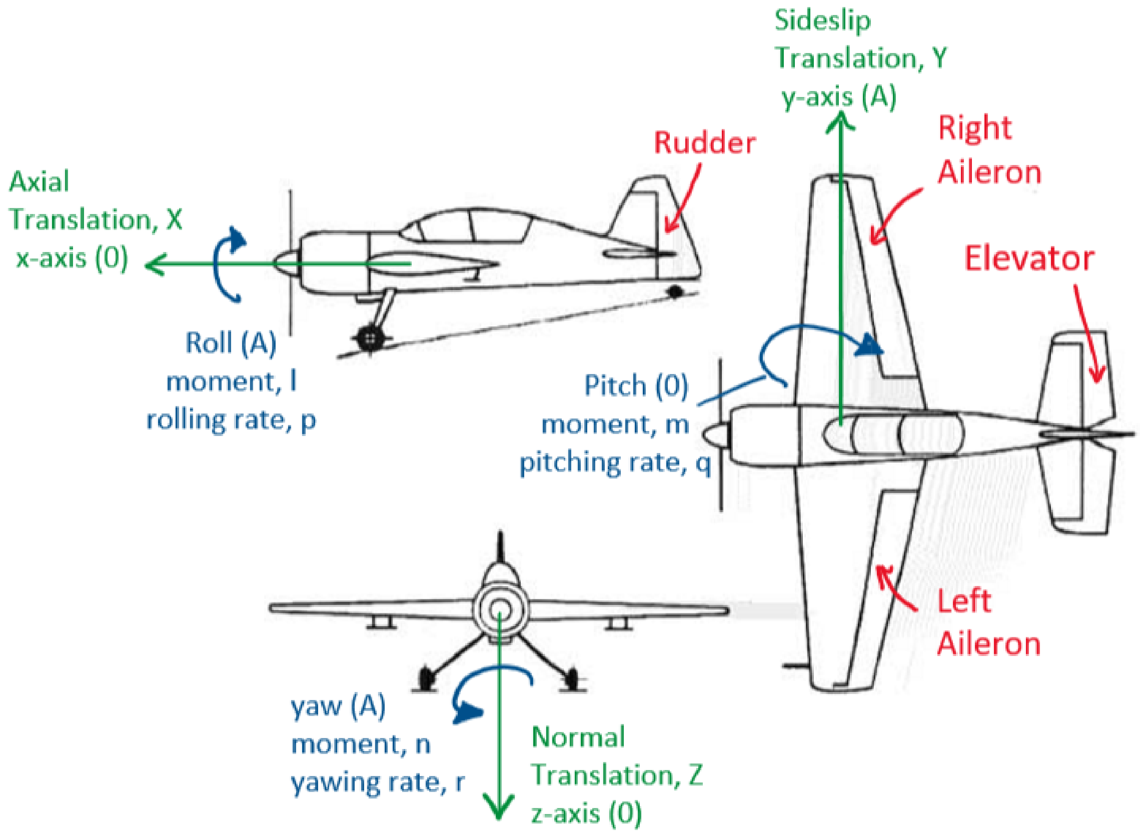
1.1 Draw a 3-view of an aircraft and label the control surfaces, translational axes, and moment about each axis. Next to each of the axes, include the letter symbol used to denote the force along that axis, the moment about that axis, and the rotation rate about the axis. Label each component as longitudinal (O) or lateral (A).



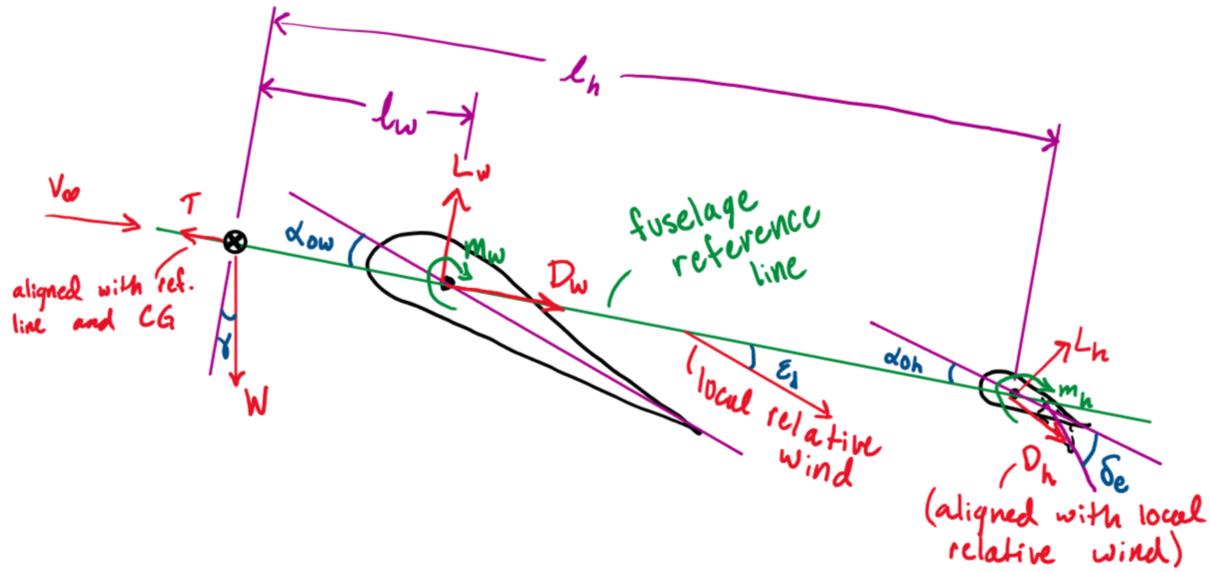
Reference in Book: Figure 4.1.1, Figure 4.1.3, Box on page 729

1.2 Write the equation that expresses the requirement for an aircraft to be stable in pitch.

$$\frac{\partial C_m}{\partial \alpha} \equiv C_{m,\alpha} < 0$$

Reference in Book: Equation 4.2.8

1.3 Consider a conventional aircraft with a main wing and horizontal tail. Assume the main wing, horizontal stabilizer, and center of gravity all lie along the fuselage reference line, and that the thrust and fuselage reference line are aligned with the direction of flight. Draw a side view of the aircraft with the longitudinal forces and moments labeled including the forces and moments on each lifting surface.



Reference in Book: Figure 4.3.3

1.4 Using the aircraft given in problem 1.3 write an equation for the force balance in the direction of lift if the aircraft is trimmed with a climb angle of γ. Apply the small-angle approximation for the downwash angle and drop very small terms.

Force Balance Equation:

$$L_w + L_h \cos \varepsilon_d - D_h \sin \varepsilon_d = W \cos \gamma$$

Small Angle Approximation: The downwash angle is typically small so $D_h \sin \varepsilon_d << L_h$ and $\cos \varepsilon_d \approx 1$.

$$L_w + L_h = W \cos \gamma$$

Reference in Book: Equation 4.3.2 and paragraph 2 on page 387

1.5 Using the aircraft given in problem 1.3, write an equation for the pitching-moment if the aircraft is trimmed with a climb angle of γ. Apply the small-angle approximation for the downwash angle and drop very small terms.

Moment Balance Equation (no net moment):

$$m = m_w + m_h - L_w l_w - L_h \cos \varepsilon_d l_h + D_h \sin \varepsilon_d l_h = 0$$

Small Angle Approximation: The downwash angle is typically small so $D_h \sin \varepsilon_d << L_h$ and $\cos \varepsilon_d \approx 1$.

$$m = m_w + m_h - L_w l_w - L_h l_h = 0$$

Reference in Book: Equation 4.3.3 and paragraph 2 on page 387

1.6 Write the solutions to problems 1.4 and 1.5 in nondimensional form.

$$\eta_h \equiv \frac{0.5\rho V_h^2}{0.5\rho V_\infty^2}$$

is the tail efficiency, which is a ratio of the dynamic pressures on the tail and main wing. [See top of page 388]
Nondimensionalize 1.4:

$$L = L_w + L_h = W\cos\gamma$$
$$\frac{L}{0.5\rho V_\infty^2 S_w} = \frac{L_w}{0.5\rho V_\infty^2 S_w} + \frac{0.5\rho V_h^2 S_h}{0.5\rho V_\infty^2 S_w}\frac{L_h}{0.5\rho V_h^2 S_h} = \frac{W\cos\gamma}{0.5\rho V_\infty^2 S_w}$$
$$C_L = C_{L_w} + \frac{S_h}{S_w}\eta_h C_{L_h} = \frac{W\cos\gamma}{0.5\rho V_\infty^2 S_w}$$

Reference in Book: Equation 4.3.4- 4.3.7
Nondimensionalize 1.5:

$$m = m_w + m_h - L_w l_w - L_h l_h = 0$$
$$\frac{m}{0.5\rho V_\infty^2 S_w \bar{c}_w} = \frac{m_w}{0.5\rho V_\infty^2 S_w \bar{c}_w} + \frac{0.5\rho V_h^2 S_h \bar{c}_h}{0.5\rho V_\infty^2 S_w \bar{c}_w}\frac{m_h}{0.5\rho V_h^2 S_h \bar{c}_h} - \frac{L_w l_w}{0.5\rho V_\infty^2 S_w \bar{c}_w} - \frac{0.5\rho V_h^2 S_h l_h}{0.5\rho V_\infty^2 S_w \bar{c}_w}\frac{L_h}{0.5\rho V_h^2 S_h} = 0$$
$$C_m = C_{m_w} + \frac{S_h \bar{c}_h}{S_w \bar{c}_w}\eta_h C_{m_h} - \frac{l_w}{\bar{c}_w}C_{L_w} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h} = 0$$

Reference in Book: Equation 4.3.8- 4.3.9

1.7 Applying the small-angle approximation, write the expression for the lift coefficient of a main wing as a function of lift slope, mounting angle, and zero-lift angle of attack.

$$C_{L_w} = C_{L_w,\alpha}(\alpha + \alpha_{0w} - \alpha_{L0w})$$

Reference in Book: Equation 4.3.16

1.8 Applying the small-angle approximation, write the expression for the lift coefficient of a horizontal stabilizer as a function of lift slope, mounting angle, zero-lift angle of attack, downwash, elevator effectiveness, and elevator deflection.

$$C_{L_h} = C_{L_h,\alpha}(\alpha + \alpha_{0h} - \alpha_{L0h} - \varepsilon_d + \varepsilon_e\delta_e)$$

The downwash angle can be rewritten as a linear function of alpha

$$\varepsilon_d = \varepsilon_{d0} + \varepsilon_{d,\alpha}\alpha$$

Substituting this into the equation above yields

$$C_{L_h} = C_{L_h,\alpha}((1 - \varepsilon_{d,\alpha})\alpha + \alpha_{0h} - \alpha_{L0h} - \varepsilon_{d0} + \varepsilon_e\delta_e)$$

Reference in Book: Equation 4.3.20 (note that the book makes the assumption of a symmetric airfoil on the horizontal stabilizer so the term $\alpha_{L0h} = 0$.

1.9 Assuming a linear relationship between control-surface deflection and pitching moment, write the expression for the pitching-moment coefficient on the horizontal stabilizer as a function of elevator deflection.

$$C_{m_h} = C_{m_h 0} + C_{m_h,\delta_e}\delta_e$$

Reference in Book: Equation 4.3.21

1.10 Combine the solutions from problems 1.6, 1.7, 1.8, and 1.9 to develop equations for the lift coefficient and pitching-moment of the aircraft as a function of wing and horizontal stabilizer geometric and aerodynamic properties, as well as the elevator deflection.

Lift:

$$C_L = C_{L_w,\alpha}(\alpha + \alpha_{0w} - \alpha_{L0w}) + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}((1-\varepsilon_{d,\alpha})\alpha + \alpha_{0h} - \alpha_{L0h} - \varepsilon_{d0} + \varepsilon_e\delta_e) = \frac{W\cos\gamma}{0.5\rho V_\infty^2 S_w}$$

Reference in Book: Equation 4.3.22 (some modifications)
Pitching Moment:

$$\begin{aligned} C_m =& C_{m_w} + \frac{S_h\bar{c}_h}{S_w\bar{c}_w}\eta_h(C_{m_h 0} + C_{m_h,\delta_e}\delta_e) - \frac{l_w}{\bar{c}_w}C_{L_w,\alpha}(\alpha + \alpha_{0w} - \alpha_{L0w}) \\ &- \frac{S_h l_h}{S_w\bar{c}_w}\eta_h C_{L_h,\alpha}((1-\varepsilon_{d,\alpha})\alpha + \alpha_{0h} - \alpha_{L0h} - \varepsilon_{d0} + \varepsilon_e\delta_e) = 0 \end{aligned}$$

Reference in Book: Equation 4.3.23 (some modifications)

1.11 Starting from the pitching-moment equation developed in problem 1.10, develop an expression for the pitch stability criteria as a function of the wing and horizontal stabilizer geometric and aerodynamic properties.

Take the derivative of the pitching moment equation with respect to alpha to find the pitching moment derivative.

$$C_{m,\alpha} = -\frac{l_w}{\bar{c}_w}C_{L_w,\alpha} - \frac{S_h l_h}{S_w\bar{c}_w}\eta_h C_{L_h,\alpha}(1-\varepsilon_{d,\alpha}) < 0$$

Reference in Book: Equation 4.3.30

1.12 For an aircraft to be trim, both equations in problem 1.10 must be satisfied. This provides a system of two equations that can be expressed in terms of two unknown operating paramters, α and δ_e as

$$\begin{bmatrix} C_{L,\alpha} & C_{L,\delta_e} \\ C_{m,\alpha} & C_{m,\delta_e} \end{bmatrix} \begin{Bmatrix} \alpha \\ \delta_e \end{Bmatrix} = \begin{bmatrix} C_L - C_{L0} \\ -C_{m0} \end{bmatrix}$$

where the known geometric and aerodynamic information of the aircraft is contained in the variables

$$
\begin{aligned}
C_{L,\alpha} &= C_{L_w,\alpha} + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(1-\varepsilon_{d,\alpha}) \\
C_{L,\delta_e} &= \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}\varepsilon_e \\
C_L &= \frac{W\cos\gamma}{0.5\rho V_\infty^2 S_w} \\
C_{L0} &= C_{L_w,\alpha}(\alpha_{0w}-\alpha_{L0w}) + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(\alpha_{0h}-\alpha_{L0h}-\varepsilon_{d0}) \\
C_{m,\alpha} &= -\frac{l_w}{\bar{c}_w}C_{L_w,\alpha} - \frac{S_h l_h}{S_w\bar{c}_w}\eta_h C_{L_h,\alpha}(1-\varepsilon_{d,\alpha}) \\
C_{m,\delta_e} &= \frac{S_h\bar{c}_h}{S_w\bar{c}_w}\eta_h C_{m_h,\delta_e} - \frac{S_h l_h}{S_w\bar{c}_w}\eta_h C_{L_h,\alpha}\varepsilon_e \\
C_{m0} &= C_{m_w} + \frac{S_h\bar{c}_h}{S_w\bar{c}_w}\eta_h C_{m_h0} - \frac{l_w}{\bar{c}_w}C_{L_w,\alpha}(\alpha_{0w}-\alpha_{L0w}) - \frac{S_h l_h}{S_w\bar{c}_w}\eta_h C_{L_h,\alpha}(\alpha_{0h}-\alpha_{L0h}-\varepsilon_{d0})
\end{aligned}
$$

Reference in Book: Equation 4.3.22- 4.3.23

Consider a version of the British Spitfire with the following geometric and aerodynamic characteristics:

$$S_w = 244 \text{ ft}^2, \quad b_w = 36.83 \text{ ft}, \quad C_{L_w,\alpha} = 4.62, \quad \alpha_{L0_w} = -2.2°, \quad C_{m_w} = -0.053,$$
$$S_h = 31 \text{ ft}^2, \quad b_h = 10.64 \text{ ft}, \quad C_{L_h,\alpha} = 4.06, \quad \varepsilon_e = 0.60, \quad C_{m_h,\delta_e} = -0.55,$$
$$W = 8,375 \text{ lbf}, \quad l_h - l_w = 18.16 \text{ ft}$$

For the following problems, assume that the center of gravity lies at the quarter-chord of the main wing, the horizontal stabilizer has a symmetric airfoil, and neglect any effects from downwash.

1.13 Find the mounting angle of the main wing and horizontal stabilizer required for the aircraft to be trim in steady-level flight at sea level at a velocity of 200 mph with zero elevator deflection and zero angle of attack.

Things to note [See information given in problem 4.3]:
- The Spitfire has a symmetric airfoil for the horizontal tail so $\alpha_{L0h} = 0$
- Standard air density at sea level is $\rho = 2.3769 * 10^{-3}\text{slugs/ft}^3$ [See back of book]
- Steady level flight indicates that $\gamma = 0$.
- CG is at quarter chord of main wing therefore $l_w = 0$.
- Because we are neglecting downwash $\eta_h = 1$.
- 200 mph = 293.333 fps

Using the equations we developed above, plug in the information we have given.

Lift equation:

$$[C_{L_w,\alpha} + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(1-\varepsilon_{d,\alpha})]\alpha + [\frac{S_h}{S_w}\eta_h C_{L_h,\alpha}\varepsilon_e]\delta_e$$
$$= \frac{W\cos\gamma}{0.5\rho V_\infty^2 S_w} - [C_{L_w,\alpha}(\alpha_{0w} - \alpha_{L0w}) + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(\alpha_{0h} - \alpha_{L0h} - \varepsilon_{d0})]$$
$$[4.62 + \frac{31}{244}4.06](0) + [\frac{31}{244}4.06 * 0.60](0)$$
$$= \frac{8375}{0.5(2.3769*10^{-3})293.333^2 * 244} - [4.62(\alpha_{0w} - (-2.2\frac{\pi}{180})) + \frac{31}{244}4.06(\alpha_{0h})]$$
$$0 = 0.335654308 - [4.62\alpha_{0w} + 0.177395 + 0.51582\alpha_{0h}]$$
$$0.158259 = 4.62\alpha_{0w} + 0.51582\alpha_{0h}$$

Moment Equation:

$$[-\frac{l_w}{\bar{c}_w}C_{L_w,\alpha} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}(1-\varepsilon_{d,\alpha})]\alpha + [\frac{S_h\bar{c}_h}{S_w\bar{c}_w}\eta_h C_{m_h,\delta_e} - \frac{S_h l_h}{S_w\bar{c}_w}\eta_h C_{L_h,\alpha}\varepsilon_e]\delta_e$$
$$= -[C_{m_w} + \frac{S_h\bar{c}_h}{S_w\bar{c}_w}\eta_h C_{m_h 0} - \frac{l_w}{\bar{c}_w}C_{L_w,\alpha}(\alpha_{0w} - \alpha_{L0w}) - \frac{S_h l_h}{S_w\bar{c}_w}\eta_h C_{L_h,\alpha}(\alpha_{0h} - \alpha_{L0h} - \varepsilon_{d0})]$$
$$[-\frac{0}{\frac{244}{36.83}}4.62 - \frac{31*18.16}{244*\frac{244}{36.83}}4.06](0) + [\frac{31*\frac{31}{10.64}}{244\frac{244}{36.83}} * -0.55 - \frac{31*18.16}{244\frac{244}{36.83}}4.06*0.60](0)$$
$$= -[-0.053 + \frac{31*31/10.64}{244*244/36.83}(0) - \frac{0}{244/36.83}4.62(\alpha_{0w} - (-2.2\frac{\pi}{180})) - \frac{31*18.16}{244*244/36.83}4.06(\alpha_{0h})]$$
$$0 = -[-0.053 - \frac{31*18.16}{244*244/36.83}4.06(\alpha_{0h})]$$
$$0 = 0.053 + 1.413922604\alpha_{0h}$$
$$\alpha_{0h} = -0.03748 \text{ rad} = -2.1477 \text{ deg}$$

Plugging the solution for α_{0h} into the lift equation yields:

$$
\begin{aligned}
0.158259 &= 4.62\alpha_{0w} + 0.51582(-0.03748) \\
0.177592 &= 4.62\alpha_{0w} \\
\alpha_{0w} &= 0.03844 \text{ rad} = 2.20244 \text{ deg}
\end{aligned}
$$

1.14 Compute the aircraft static margin.

The static margin can be computed using equation 4.4.8 in the book.

$$\frac{l_{np}}{\bar{c}_w} = -\frac{C_{m,\alpha}}{C_{L,\alpha}}$$

Use the equation for the pitch stability derivative developed above

$$
\begin{aligned}
C_{m,\alpha} &= -\frac{l_w}{\bar{c}_w} C_{L_w,\alpha} - \frac{S_h l_h}{S_w \bar{c}_w} \eta_h C_{L_h,\alpha}(1 - \varepsilon_{d,\alpha}) \\
C_{m,\alpha} &= -\frac{0}{4}.62 - \frac{31 * 18.16}{244 * 244/36.83} 4.06 \\
C_{m,\alpha} &= -1.41392
\end{aligned}
$$

Take the derivative of the lift equation with respect to α and compute.

$$
\begin{aligned}
C_{L,\alpha} &= C_{L_w,\alpha} + \frac{S_h}{S_w} \eta_h C_{L_h,\alpha}(1 - \varepsilon_{d,\alpha}) \\
C_{L,\alpha} &= 4.62 + \frac{31}{244} 4.06 \\
C_{L,\alpha} &= 5.1358
\end{aligned}
$$

Now compute the static margin for the aircraft.

$$-\frac{C_{m,\alpha}}{C_{L,\alpha}} = -\frac{-1.41392}{5.1358} = 0.275306 = 27.53 \text{ percent}$$

1.15 If the main wing and horizontal stabilizer both have zero mounting angles, compute the angle of attack and elevator deflection required to trim the aircraft in a steady climb at an altitude of 5,000 ft and a climb angle of 20 deg at a speed of 200 mph.

Things to note [See information given in problem 4.3]:

- The Spitfire has a symmetric airfoil for the horizontal tail so $\alpha_{L0h} = 0$
- Standard air density at 5000 ft is $\rho = 2.048 * 10^{-3}$slugs/ft^3 [See back of book]
- CG is at quarter chord of main wing therefore $l_w = 0$.
- Because we are neglecting downwash $\eta_h = 1$.
- The weight of the aircraft is 8,375 lbf.
- 200 mph = 293.333 fps

Using the equations we developed above, plug in the information we have given.

$$\begin{bmatrix} C_{L,\alpha} & C_{L,\delta_e} \\ C_{m,\alpha} & C_{m,\delta_e} \end{bmatrix} \begin{Bmatrix} \alpha \\ \delta_e \end{Bmatrix} = \begin{bmatrix} C_L - C_{L0} \\ -C_{m0} \end{bmatrix}$$

$$\begin{bmatrix} C_{L_w,\alpha} + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(1-\varepsilon_{d,\alpha}) & \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}\varepsilon_e \\ -\frac{l_w}{\bar{c}_w}C_{L_w,\alpha} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}(1-\varepsilon_{d,\alpha}) & \frac{S_h \bar{c}_h}{S_w \bar{c}_w}\eta_h C_{m_h,\delta_e} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}\varepsilon_e \end{bmatrix} \begin{Bmatrix} \alpha \\ \delta_e \end{Bmatrix}$$
$$= \begin{bmatrix} \frac{W\cos\gamma}{0.5\rho V_\infty^2 S_w} - C_{L_w,\alpha}(\alpha_{0w} - \alpha_{L0w}) + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(\alpha_{0h} - \alpha_{L0h} - \varepsilon_{d0}) \\ -[C_{m_w} + \frac{S_h \bar{c}_h}{S_w \bar{c}_w}\eta_h C_{m_h 0} - \frac{l_w}{\bar{c}_w}C_{L_w,\alpha}(\alpha_{0w} - \alpha_{L0w}) - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}(\alpha_{0h} - \alpha_{L0h} - \varepsilon_{d0})] \end{bmatrix}$$

$$\begin{bmatrix} 4.62 + \frac{31}{244}4.06 & \frac{31}{244}4.06 * 0.60 \\ -\frac{0}{244/36.83}4.62 - \frac{31*18.16}{244*244/36.83}4.06 & \frac{31*31/10.64}{244*244/36.83}(-0.55) - \frac{31*18.16}{244*244/36.83}4.06*0.60 \end{bmatrix} \begin{Bmatrix} \alpha \\ \delta_e \end{Bmatrix}$$
$$= \begin{bmatrix} \frac{8375\cos 20}{0.5(2.048*10^{-3})293.333^2 244} - 4.62(2.2\frac{\pi}{180}) + \frac{31}{244}4.06(0) \\ -[-0.053 + \frac{31*31/10.64}{244*244/36.83}(0) - \frac{0}{244/36.83}4.62(2.2\frac{\pi}{180}) - \frac{31*18.16}{244*244/36.83}4.06(0)] \end{bmatrix}$$

$$\begin{bmatrix} 5.13582 & 0.309492 \\ -1.41392 & -0.879084 \end{bmatrix} \begin{Bmatrix} \alpha \\ \delta_e \end{Bmatrix} = \begin{bmatrix} 0.366066 - 0.177395 \\ 0.053 \end{bmatrix}$$

$$\begin{Bmatrix} \alpha \\ \delta_e \end{Bmatrix} = \begin{bmatrix} 0.044702 \\ -0.132189 \end{bmatrix}$$

Thus,

$$\alpha = 0.044702 \text{ rad} = 2.56124 \text{ deg}$$
$$\delta_e = -0.132189 \text{ rad} = -7.57388 \text{ deg (up)}$$