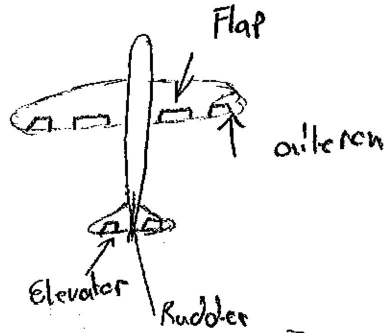
Eric Larsen
A02176917

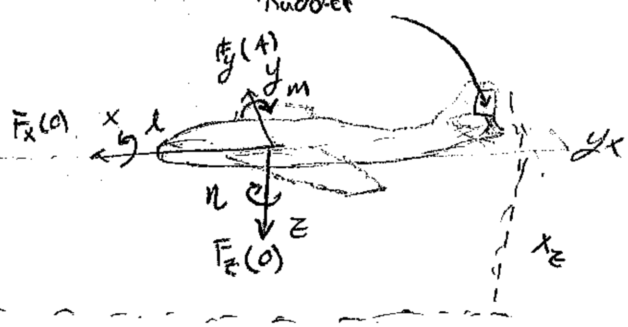
MAE 5510 : Exercise Set 1

Group	8				
Date	1/10/2024	1/12/2024			
Leader	Ben	Payton			
Member	Payton	Eric C			
Member	Eric	Ben			
Member	~	~			

1.1 Draw a 3-view of an aircraft and label the control surfaces, translational axes, and moment about each axis. Next to each of the axes, include the letter symbol used to denote the force along that axis, the moment about that axis, and the rotation rate about the axis. Label each component as longitudinal (O) or lateral (A).



axis	moment	rate	velocity	Force
x	ℓ	p	u	axial
y	M	q	v	sideslip
z	n	r	w	Normal



Longitudinal: F_x, F_z, M_y

Lateral: F_y, M_x, M_z

C_m = Pitching moment

c

$$\frac{M}{\frac{1}{2}\rho V^2 \bar{c} S_w}$$

1.2 Write the equation that expresses the requirement for an aircraft to be stable in pitch.

$$\frac{dM}{d\alpha} < 0 \quad \Rightarrow \quad C_{m,\alpha} < 0$$

$$\frac{\partial C_m}{\partial \alpha} = \frac{\partial C_{m_{ac}}}{\partial \alpha} - \frac{l_w}{c}\frac{\partial C_L}{\partial \alpha} < 0$$

$$\boxed{-\frac{l_w}{\bar{c}} C_{L,\alpha} < 0} \qquad l_w > 0$$

1.6 Write the solutions to problems 1.4 and 1.5 in nondimensional form.

$$C_L = C_{L_w} + \frac{S_h}{S_w} \eta_h C_{L_h} = C_w \cos\gamma$$

$$C_m = C_{m_w} + \frac{S_h}{S_w} \frac{\bar{c}_h}{\bar{c}_w} \eta_h C_{m_h} - \frac{l_w}{\bar{c}_w} C_{L_w} - \frac{l_h}{\bar{c}_w} \frac{S_h}{S_w} \eta_h C_{L_h} = 0$$

1.7 Applying the small-angle approximation, write the expression for the lift coefficient of a main wing as a function of lift slope, mounting angle, and zero-lift angle of attack.

$$C_{L_w} = C_{L_w,\alpha} (\alpha + \alpha_{0w} - \alpha_{L0w})$$

1.8 Applying the small-angle approximation, write the expression for the lift coefficient of a horizontal stabilizer as a function of lift slope, mounting angle, zero-lift angle of attack, downwash, elevator effectiveness, and elevator deflection.

$$C_{L_h} = C_{L_{h,\alpha}} (\alpha + \alpha_{0h} - \alpha_{L0h} - \varepsilon_{d0} - \varepsilon_{d,\alpha}\alpha + \varepsilon_e \delta_e)$$

1.9 Assuming a linear relationship between control-surface deflection and pitching moment, write the expression for the pitching-moment coefficient on the horizontal stabilizer as a function of elevator deflection.

$$C_{m_h} = C_{m_{h0}} + C_{m_h,\delta_e} \delta_e$$

$\eta_h = \frac{\frac{1}{2}\rho V_h^2}{\frac{1}{2}\rho V_\infty^2} \approx 1.0$ $\quad l_w \approx 0$ $\quad$ ~~8375~~ $\frac{W \cos\gamma}{\frac{1}{2}\rho_a V_\infty^2 S_w} = C_W$

Consider a version of the British Spitfire with the following geometric and aerodynamic characteristics:

$\bar{c}_w = 6.625$ ft
$\bar{c}_h = 2.9135$ ft

$S_w = 244\ \text{ft}^2$, $\quad b_w = 36.83$ ft, $\quad C_{L_w,\alpha} = 4.62$, $\quad \alpha_{L0_w} = -2.2°$, $\quad C_{m_w} = -0.053$,
$S_h = 31\ \text{ft}^2$, $\quad b_h = 10.64$ ft, $\quad C_{L_h,\alpha} = 4.06$, $\quad \varepsilon_e = 0.60$, $\quad C_{m_h,\delta_e} = -0.55$,
$W = 8,375$ lbf, $\quad l_h - l_w = 18.16$ ft $\quad l_h = 18.16$

For the following problems, assume that the center of gravity lies at the quarter-chord of the main wing, the horizontal stabilizer has a symmetric airfoil, and neglect any effects from downwash.

C_{m} $\quad \varepsilon_d = 0$

$\alpha_{oh} \quad \alpha_{ow}$

1.13 Find the mounting angle of the main wing and horizontal stabilizer required for the aircraft to be trim in steady-level flight at sea level at a velocity of 200 mph with zero elevator deflection and zero angle of attack.

$L = h \neq 0$

$\gamma = 0 \quad \alpha = 0 \quad \rho = 0.0023769 \quad \delta_e = 0$

$200 \frac{mi}{hr} \cdot \frac{5280\ ft}{1\ mile} \cdot \frac{hr}{3600\ s} = 293.33\ ft/s$

$C_L = \frac{8375}{\frac{1}{2}\rho V_\infty^2 S_w}$

when degrees and when radians.

$C_W = \frac{8375}{\frac{1}{2}\ 0.0023769 \cdot 293.33\ ft/s^2 \cdot 244\ ft^2} = 0.335$

zero lift angle

$C_W = C_W \cos\gamma - C_{L_w,\alpha}(\alpha_{ow} - \alpha_{L0w}) - \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(\alpha_{oh} - \alpha_{L0h} - \cancel{\varepsilon_{d0}})$

2 eqs 2 unknowns.

$0 = -\left[C_{m_w} - \frac{l_w}{c_w} C_{L,\alpha}(\alpha_{ow} - \alpha_{L0w}) - \frac{S_w l_h}{S_w c_w}\eta_h C_{L_h,\alpha}(\alpha_{oh} - \cancel{\varepsilon_{d0}})\right]$

Eq. rearranged.

$0 = -4.62(\alpha_{ow} + 2.2°) - \frac{31}{244}(1)\ 4.06(\alpha_{oh})$

$\boxed{\alpha_{oh} = -3.715 \text{ degrees}}$

$\boxed{10.164° = -4.62\,\alpha_{ow} - 0.5158\,\alpha_{oh}}$ eq. 1

Eq 2 $\quad 0 = -0.335 + \cancel{C_{L,\alpha}(\alpha_{Lw})} + \underbrace{\frac{31}{244}\frac{18.16}{6.625}}_{0.3482}(1)\ 4.06(\alpha_{oh})$

$0.335 = 1.4139\ \alpha_{oh}$

$\boxed{\alpha_{oh} = 0.2369 \text{ rad.}}$ $= 13.57$ degrees.

1.14 Compute the aircraft static margin.

$\frac{l_{np}}{\bar{c}_w} \cong \frac{-C_{m,\alpha}}{C_{L,\alpha}}$

$$\frac{-\frac{l_w}{c_w}C_{L_w,\alpha} + \frac{S_h l_h}{S_w c_w}\eta_h C_{L_h,\alpha}(1 - \cancel{\varepsilon_{d,\alpha}})}{C_{L_w,\alpha} + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(1 - \cancel{\varepsilon_{d,\alpha}})}$$

$$\frac{-\cancel{\frac{18.16}{6.625}4.62} + \frac{31}{244}\frac{18.16}{6.625}(1)\ 4.06\ (1)}{4.62 + \frac{31}{244}(1)\ 4.06(1)} = \frac{1.4139}{5.13}$$

$= 0.2756 \quad \underline{27\%}$

1.15 If the main wing and horizontal stabilizer both have zero mounting angles, compute the angle of attack and elevator deflection required to trim the aircraft in a steady climb at an altitude of 5,000 ft and a climb angle of 20 deg at a speed of 200 mph.

$\gamma = 20^\circ \quad V_\infty = 293.33 \text{ ft/s}$

$\rho = 0.002048$ $\qquad \alpha_{oh} = 0 \quad \alpha_{ow} = 0$

$$\begin{bmatrix} C_{L,\alpha} & C_{L,\delta_e} \\ C_{m,\alpha} & C_{m,\delta_e} \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_L - C_{L0} \\ -C_{m0} \end{bmatrix} \qquad C_L = \frac{\cos(20)\,6375}{\frac{1}{2}(0.002048)(293.33^2)\,244}$$

$$C_L = \underline{0.366}$$

$$C_{L0} = C_{L_w,\alpha}(-\alpha_{L0w}) = 4.62\left(-2.2 \cdot \frac{\pi}{180}\right) = \underline{-0.1774}$$

$$C_L - C_{L0} = \underline{0.5434}$$

$$C_{L,\alpha} = \underline{5.13}$$

$$C_{L,\delta_e} = \frac{S_h}{S_w}(1)\,C_{Lh,\alpha}\,\varepsilon_e = \frac{31}{244}(4.06(0.6)) = \underline{0.30949}$$

$$C_{m,\alpha} = \frac{S_h l_h}{S_w c_w} = \underline{1.4139}$$

$$C_{m,\delta_e} = \frac{S_h \bar{c}_h}{S_w \bar{c}_w}(1)C_{m_h,\delta_e} - \frac{S_h l_h}{S_w c_w}(1)\,C_{Lh,\alpha}\,\varepsilon_e = \frac{31(2.913)}{244(6.625)}(-0.55) - \frac{31(18.16)}{244(6.625)}\,4.06\,(0.6)$$

$$C_{m,\delta_e} = \underline{-0.879}$$

$$C_{m0} = \cancel{C_{m_w}} - \frac{l_w}{c_w}\,\cancel{C_{Lw,\alpha}(\alpha_{0w} - \alpha_{L0w})} - \cancel{\frac{S_h l}{S_w}}(\alpha_{0h} - \varepsilon_0) \qquad C_{m0} = 0$$

$$\begin{bmatrix} 5.13 & 0.3095 \\ 1.4139 & -0.879 \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} 0.5434 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} \alpha = 0.096 \text{ rad} \\ \delta_e = 0.1553 \end{matrix}$$

$S_w = \text{Area} = \bar{c}_w \cdot l_w \qquad \underline{\alpha = 5.500^\circ}$

$R_{Aw} = \frac{l_w}{c_w} \qquad \underline{\delta_e = 8.898^\circ}$

$\eta_h =$

1.10 Combine the solutions from problems 1.6, 1.7, 1.8, and 1.9 to develop equations for the lift coefficient and pitching-moment of the aircraft as a function of wing and horizontal stabilizer geometric and aerodynamic properties, as well as the elevator deflection.

$$C_L = \underbrace{\left[C_{L_w,\alpha} + \eta_h \frac{S_h}{S_w} C_{L_h,\alpha}(1-\varepsilon_{d,\alpha})\right]}_{C_{L,\alpha}}\alpha + \underbrace{\left[\frac{S_h}{S_w}\eta_h C_{L_h,\alpha}\varepsilon_e\right]}_{C_{L,\delta_e}}\delta_e =$$

$$C_W \cos\gamma - \underbrace{C_{L_w,\alpha}(\alpha_{0w}-\alpha_{L0w}) - \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(\alpha_{0h}-\alpha_{L0h}-\varepsilon_{d0})}_{C_{L0} \text{ "geometric."}}$$

$$C_{m_w} + \frac{S_h \bar{c}_h}{S_w \bar{c}_w}\eta_h C_{m_h,\delta_e}\delta_e - \frac{l_w}{\bar{c}_w} C_{L_w,\alpha}(\alpha + \alpha_{0w} - \alpha_{L0w}) - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}\left[(1-\varepsilon_{d,\alpha})\alpha + \alpha_{0h} - \varepsilon_{d0} + \varepsilon_e \delta_e\right] = 0$$

1.11 Starting from the pitching-moment equation developed in problem 1.10, develop an expression for the pitch stability criteria as a function of the wing and horizontal stabilizer geometric and aerodynamic properties.

$$\underbrace{\left[-\frac{l_w}{\bar{c}_w}C_{L_w,\alpha} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}(1-\varepsilon_{d,\alpha})\right]}_{C_{m,\alpha}}\alpha + \left[\frac{S_h \bar{c}_h}{S_w \bar{c}_w}\eta_h C_{m_h,\delta_e} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}\varepsilon_e\right]\delta_e =$$

$$-\left[C_{m_w} - \frac{l_w}{\bar{c}_w}C_{L,\alpha}(\alpha_{0w}-\alpha_{L0w}) - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}(\alpha_{0h}-\varepsilon_{d0})\right]$$

$$C_{m,\alpha} = [\quad] < 0$$

1.12 For an aircraft to be trim, both equations in problem 1.10 must be satisfied. This provides a system of two equations that can be expressed in terms of two unknown operating paramters, α and δ_e as

$$\begin{bmatrix} C_{L,\alpha} & C_{L,\delta_e} \\ C_{m,\alpha} & C_{m,\delta_e} \end{bmatrix} \begin{Bmatrix} \alpha \\ \delta_e \end{Bmatrix} = \begin{bmatrix} C_L - C_{L0} \\ -C_{m0} \end{bmatrix}$$

where the known geometric and aerodynamic information of the aircraft is contained in the variables

$$C_{L,\alpha} = C_{L_w,\alpha} + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(1-\varepsilon_{d,\alpha})$$

$$C_{L,\delta_e} = \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}\varepsilon_e$$

$$C_L = \frac{W}{\frac{1}{2}\rho V_\infty^2 S_w}$$

$$C_{L0} = C_{L_w,\alpha}(\alpha_{0w}-\alpha_{L0w}) + \frac{S_h}{S_w}\eta_h C_{L_h,\alpha}(\alpha_{0h}-\varepsilon_{d0})$$

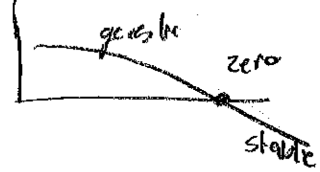
$$C_{m,\alpha} = -\frac{l_w}{\bar{c}_w}C_{L_w,\alpha} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}(1-\varepsilon_{d,\alpha})$$

$$C_{m,\delta_e} = \frac{S_h \bar{c}_h}{S_w \bar{c}_w}\eta_h C_{m_h,\delta_e} - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}\varepsilon_e$$

$$C_{m0} = C_{m_w} - \frac{l_w}{\bar{c}_w}C_{L_w,\alpha}(\alpha_{0w}-\alpha_{L0w}) - \frac{S_h l_h}{S_w \bar{c}_w}\eta_h C_{L_h,\alpha}(\alpha_{0h}-\varepsilon_{d0})$$

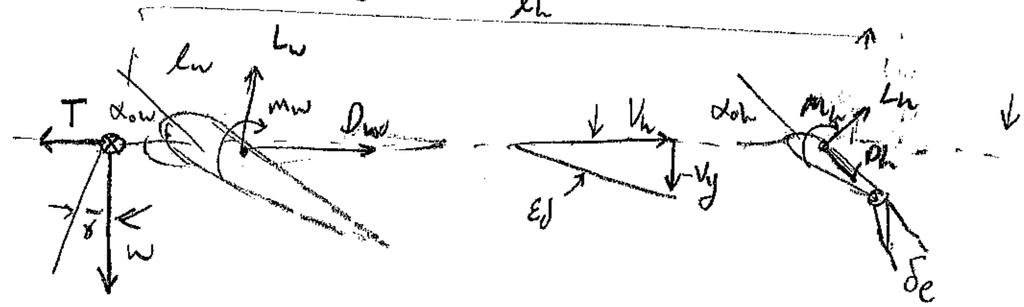
$\frac{W}{\frac{1}{2}\rho V_\infty^2 S_w} \quad \frac{W}{\frac{1}{2}\rho V^2 S_w} \quad C_W = \frac{W}{\frac{1}{2}\rho V^2 S_w}$

$C_{m,\alpha}$



$C_{m,\alpha}$

1.3 Consider a conventional aircraft with a main wing and horizontal tail. Assume the main wing, horizontal stabilizer, and center of gravity all lie along the fuselage reference line, and that the thrust and fuselage reference line are aligned with the direction of flight. Draw a side view of the aircraft with the longitudinal forces and moments labeled including the forces and moments on each lifting surface.



[w h]

$$\boxed{\alpha_{0_{all}} = \alpha_{0h} + \alpha_{0w}}$$

$\frac{1}{2}\rho V^2 \bar{c}_w S_w$

α_{0h}

$\sin(\theta) = \theta$

4.3)

1.4 Using the aircraft given in problem 1.3 write an equation for the force balance in the direction of lift if the aircraft is trimmed with a climb angle of γ. Apply the small-angle approximation for the downwash angle and drop very small terms.

Eq 4.3.2

ε_d = downwash

small angle consideration

$$L = L_w + L_h \cos \varepsilon_d - D_h \sin \varepsilon_d = W \cos \gamma$$

$$\boxed{L = L_w + L_h - D_h \varepsilon_d = W \cos \gamma} \quad 4.3.2$$

1.5 Using the aircraft given in problem 1.3, write an equation for the pitching-moment if the aircraft is trimmed with a climb angle of γ. Apply the small-angle approximation for the downwash angle and drop very small terms.

$$m = m_w + m_h - l_w L_w - l_h L_h \cos \varepsilon_d + l_h D_h \sin \varepsilon_d = 0$$

$$m = m_w + m_h - l_w L_w - l_h L_h + \cancel{l_h D_h \varepsilon_d} = 0$$