MAE 5510 Flight Dynamics: Exam 1 - Longitudinal Trim and Stability **Name** Eric Larsen

A02176917

Open book. Open notes. Closed neighbor, videos, internet, AI, or any other resource. All work done neatly and logically. Calculators, Excel, or personal code allowed.

I did not communicate about this exam to anyone who had already started ~~the~~ exam, and I will not talk to anyone about this exam until after it is due. Sign here if this is true: ____

1. Short Answer (50 Points)

1.1 For an aircraft to be trim, the forces about the center of gravity must sum to be (greater than | less than | equal to) zero.

trim $\Sigma F_x = \Sigma F_z = \Sigma M_y = 0$

1.2 For an aircraft to be trim, the moments about the center of gravity must sum to be (greater than | less than | equal to) zero.

1.3 For an aircraft to be longitudinally stable, the change in pitching moment with respect to angle of attack must be (greater than | less than | equal to) zero. $C_{m,\alpha} < 0$

1.4 Another name for the aerodynamic center of an aircraft is the Neutral Point.

1.5 Rank the following in order of magnitude for a typical aircraft (1 = largest, 2 = next largest, 3 = smallest)

1 The lift slope of the airfoil section used on the main wing, $\tilde{C}_{L,\alpha}$

2 The lift slope of the main wing, $C_{L_w,\alpha}$

3 The lift slope of the entire aircraft, $C_{L,\alpha}$

$\sigma = \frac{l_{np}}{\bar{c}_w}$

1.6 The static margin can be computed by subtracting the location of the Neutral point from the location of the Center of gravity and dividing by the Main wing average chord ($\bar{c}_w$)

1.7 Alternately, the static margin can be found by the negative of the ratio of the pitch stability derivative to the lift slope. $\sigma \approx -\frac{C_{m,\alpha}}{C_{L,\alpha}}$

1.8 A common rule of thumb is that the static margin should be near $\sigma \approx$ 0.05 or 5%.

1.9 (True | False) The simplified linear longitudinal analysis, in which all components lie along the fuselage reference line, predicts that the pitch stability derivative is independent of angle of attack.

1.10 (True | False) The generalized longitudinal analysis, in which nonlinearities and vertical offsets are included, predicts that the pitch stability derivative is independent of angle of attack.

shows up in sin and cos throughout.

2.3 Estimate the static margin of this aircraft.

$$\sigma \approx \frac{-C_{m,\alpha}}{C_{L,\alpha}}$$

$\sigma =$

$$C_{m,\alpha} = -\frac{l_w}{\bar{c}_w} C_{Lw,\alpha} - \frac{S_h l_h}{S_w \bar{c}_w} \eta_h C_{Lh,\alpha}(1-\varepsilon_{d,\alpha}) = \underbrace{\frac{0.52}{10} 3.73}_{0.19396} - \underbrace{\frac{63.7 \cdot 16.61}{300 \cdot 10} (1)(3.20)(1-0.8008)}_{0.224816}$$

$$= -0.030856$$

$$C_{L,\alpha} = C_{Lw,\alpha} + \frac{S_h}{S_w} \eta_h C_{Lh,\alpha}(1-\varepsilon_{d,\alpha}) = 3.73 + \underbrace{\frac{63.7}{300}(1) 3.20 (1-0.8008)}_{0.1353}$$

$C_{Lw,\alpha} = 3.73$

$$= 3.865$$

$C_{Lh,\alpha} = 3.20$

$$\boxed{\sigma = \frac{-(-0.030856)}{3.865} = 0.00798 \quad \text{or} \quad 0.798\,\%}$$

For the following two problems, assume the aircraft is operating near take-off conditions at a velocity of 225 ft/s, a weight of 20,500 lbf, in steady-level flight at sea level. lift must equal the weight of aircraft.

2.4 Compute the lift coefficient for the complete aircraft at this operating condition.

$\gamma = 0$

$C_L =$

$W = 20{,}500$ lbf

$$\frac{W \cos\gamma}{\frac{1}{2} \rho V_\infty^2 S_w} = \frac{20{,}500(\cos(0))}{\frac{1}{2}(0.0023769 \cdot 225^2 \cdot 300)}$$

$V_\infty = 225$ ft/s

$\rho_{air} = 0.0023769$ slug/ft³

$$\boxed{C_L = 1.13576}$$

Aero Exam 1

Problem 2.2 Solve for $\varepsilon_d = \varepsilon_{d_0} + \varepsilon_{d,\alpha}\alpha$

Need k_v, k_p, k_s, k_b

$$\varepsilon_d = \frac{k_v k_p k_s}{k_b}\frac{C_{L,w}}{R_{Aw}}$$

$$C_{L_w} = C_{L_{w,\alpha}}(\alpha + \alpha_{ow} - \alpha_{Low} - \cancel{\varepsilon_u})$$

$R_{Tw} = 0.2275$

$R_{Aw} = b_w/\bar{c}_w = 30/10 = 3$

$\bar{C}_w = S_w/b_w = 300/30 = 10\ ft$

$k_v \approx 1.04$

$k_b \approx 0.75$

values not on chart for R_{Aw} so approximated w/ best Guess.

$$k_p = \frac{2k_b^2}{\pi^2(\bar{y}^2 + k_b^2)}\left[1 + \frac{\bar{x}(\bar{x}^2 + \bar{y}^2 + k_b^2)}{(\bar{x}^2 + \bar{y}^2)\sqrt{\bar{x}^2 + \bar{y}^2 + k_b^2}}\right] = \frac{2\cdot \cancel{0.75}^2}{\pi^2(0.75^2)}\left[1 + \frac{1.142(1.142^2 + 0.75^2)}{(1.142^2)\sqrt{(1.142^2 + 0.75^2)}}\right]$$

$$\bar{y} = \frac{y}{b_w/2} = \frac{0}{15} = 0$$

$$\bar{x} = \frac{x}{b_w/2} = \frac{17.13}{15} = 1.142$$

$k_p \approx 0.445$

$$k_s = 1 + \frac{\frac{\bar{x} - s}{t} + \bar{x}(\bar{r}t\ldots\ldots)}{\ldots}$$

"used personal code"

$k_s = 1.04397 \approx 1.044$

check w/ chart ✓✓

$k_v = 1.04$

$k_p = 0.445$

$k_b = 0.75$

$k_s = 1.044$

$$\varepsilon_d = \frac{1.04 \cdot 0.445 \cdot 1.044}{0.75}\cdot\frac{C_{Lw}}{3}$$

$$\underline{\varepsilon_d = 0.2147\ C_{Lw}}$$

$$\varepsilon_d = 0.2147(C_{Lw_0} + C_{Lw,\alpha}\alpha)$$

$$C_{Lw_0} = C_{Lw,\alpha}(\alpha_{ow} - \alpha_{Low}) = 3.73(1.9 + 1.62^\circ)$$

$$3.73\left(3.52\cdot\frac{\pi}{180}\right) = C_{Lw_0}$$

$$C_{Lw_0} = 0.22915$$

Problem 2.2 Continued

$$C_{Lw0} = 0.22915 = 3.73\left(3.52 \cdot \frac{\pi}{180}\right) = c_{Lw,\alpha}(\alpha_{0w} - \alpha_{L0w})$$

$$\varepsilon_d = \varepsilon_{d_0} + \varepsilon_{d,\alpha}\,\alpha \qquad \varepsilon_{d_\alpha} = C_{Lw,\alpha} \cdot 0.2147 = 3.73 \cdot 0.2147$$

$$\varepsilon_{d_\alpha} = 0.8008$$

$$\varepsilon_{d_0} = 0.2147 \cdot C_{Lw0} = 0.2147 \cdot 0.22915 = 0.0492$$

$$\varepsilon_d = \underbrace{0.0492}_{\varepsilon_{d_0}} + \underbrace{0.8008}_{\varepsilon_{d_\alpha}}\,\alpha$$