Basic image processing

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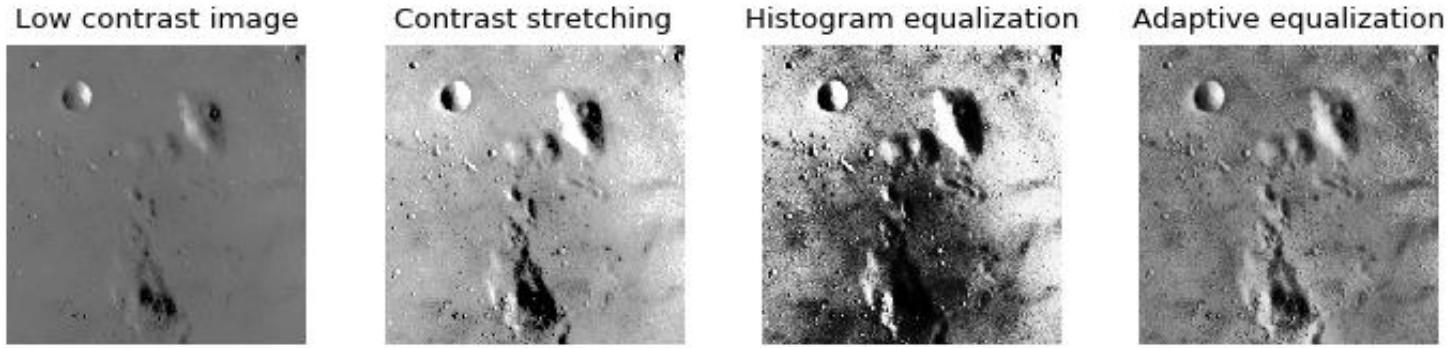
References

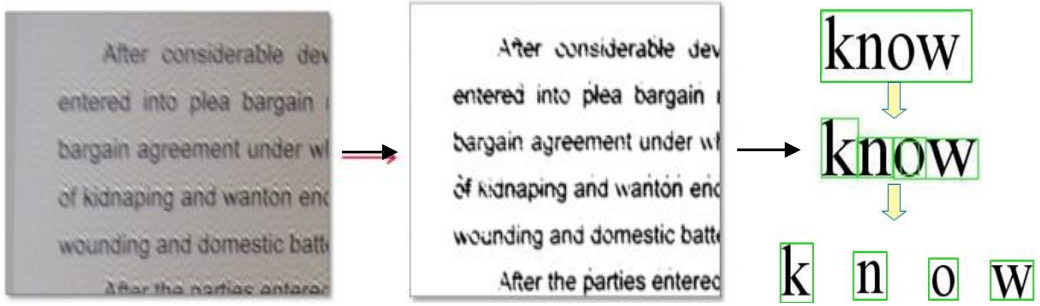
- http://szeliski.org/Book/
- http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html
- http://www.cs.cmu.edu/~16385/

Some motivation

Art
(Photoshop color grading)



Science and space
(image enhancement)



Robotics
(OCR – optical character recognition)

Agriculture
(color ripeness detection)

contents

- **Image representation**
- Pixel-wise operations
- Histogram equalization
- Template matching
- Morphology operators
- Connected components
- Color space

Image representation

- We can think of an image as a 3d matrix of discrete RGB values.
- The values mark the intensity of each color channel and are usually of type uint8 = $\{0, \dots, 255\}$.

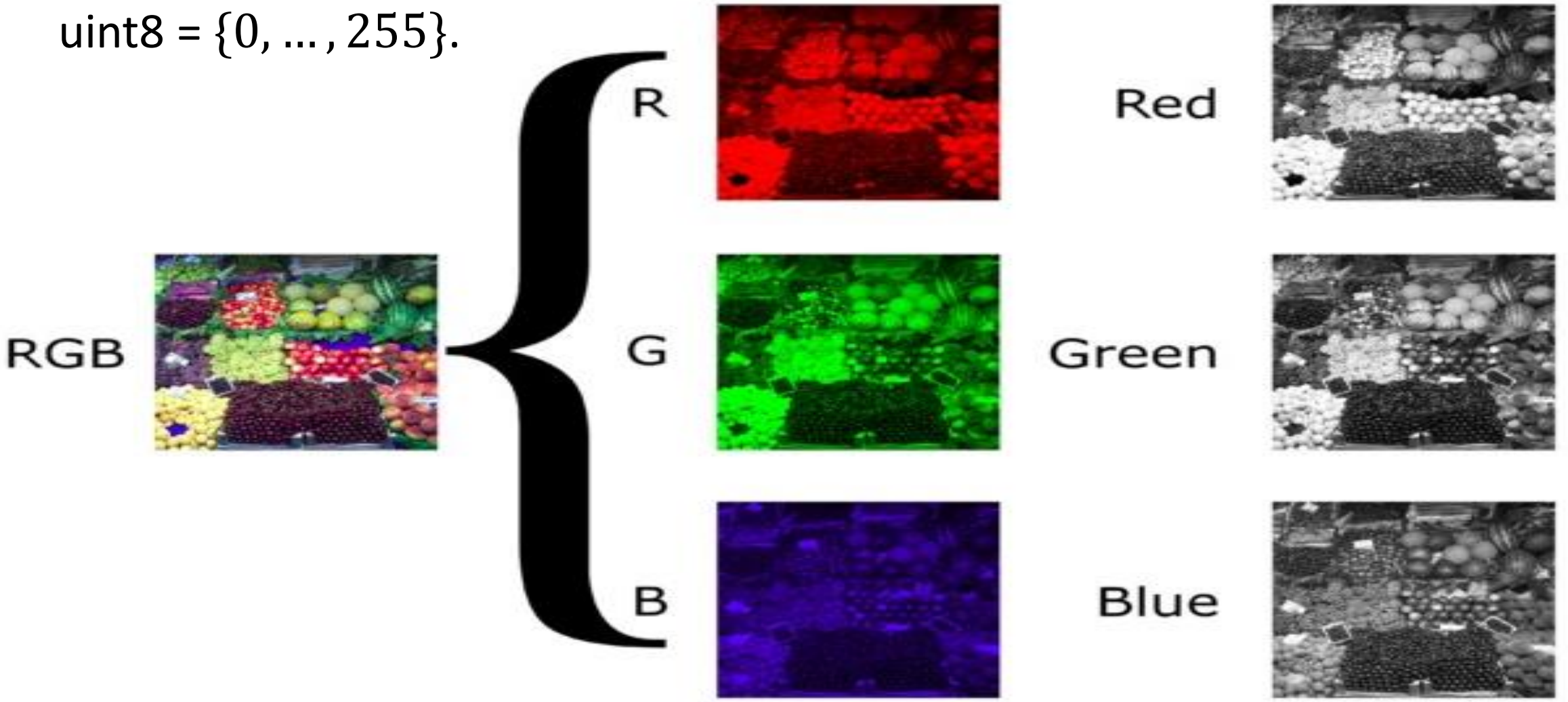
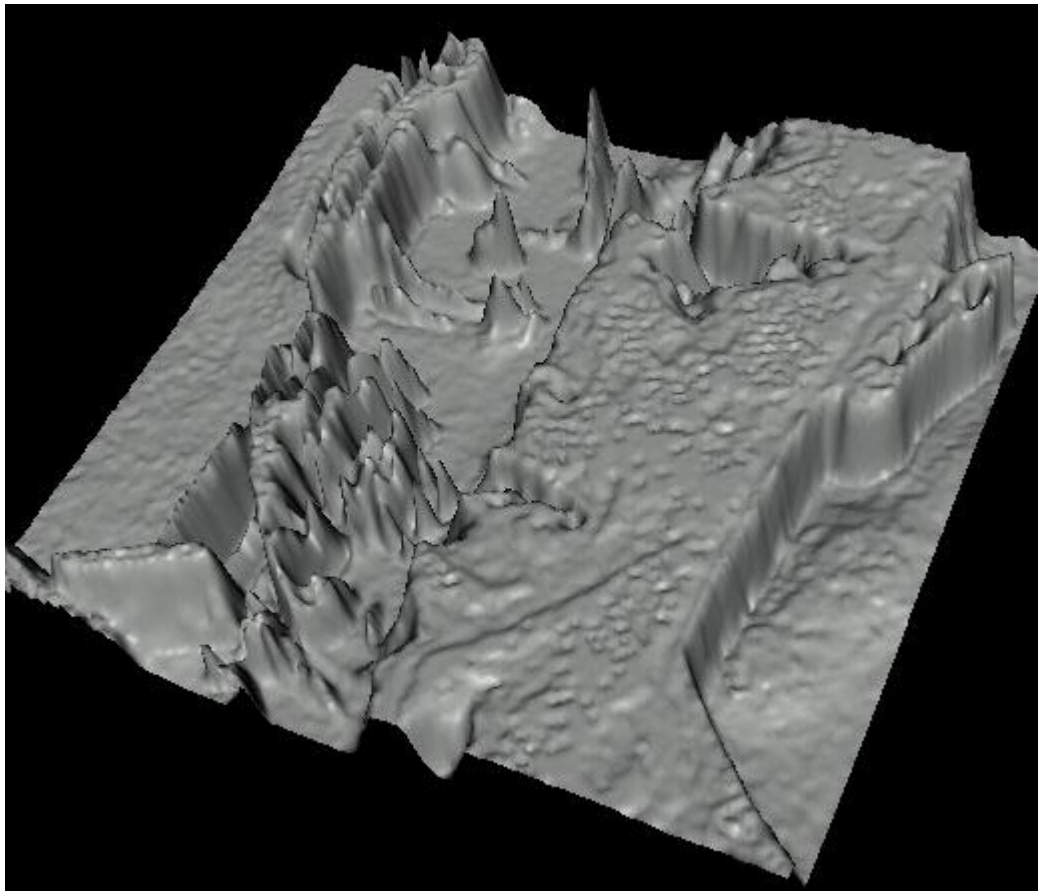


Image representation

- We can also think of an image as a function $f(x, y)$.



contents

- Image representation
- **Pixel-wise operations**
- Histogram equalization
- Template matching
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Pixel-wise operators

- Pixel-wise operators, or point operators, are defined as such that each output pixel’s value depends on only the corresponding input pixel value.

Pixel-wise operators

original

x

darken

lower contrast

Gamma compression

invert

lighten

raise contrast

Gamma expansion

Pixel-wise operators

original

x

darken

lower contrast

Gamma compression

invert

$255 - x$

lighten

raise contrast

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

Gamma compression

invert

$255 - x$

lighten

raise contrast

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast

$x \times 2$

Gamma expansion

Contrast

- **Contrast** in visual perception is the difference in appearance of two or more parts of a seen field.
- The human visual system is more sensitive to contrast than absolute luminance;
- **Contrast ratio**, or **dynamic range**, is the ratio between the largest and smallest values of the image or:

$$CR = \frac{V_{max}}{V_{min}}$$

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

invert

$255 - x$

lighten

$x + 128$

raise contrast

$x \times 2$

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

$\left(\frac{x}{255}\right)^{1/3} \times 255$

invert

$255 - x$

lighten

$x + 128$

raise contrast

$x \times 2$

Gamma expansion

Pixel-wise operators

original

x

darken

$x - 128$

lower contrast

$\frac{x}{2}$

Gamma compression

$\left(\frac{x}{255}\right)^{1/3} \times 255$

invert

$255 - x$

lighten

$x + 128$

raise contrast

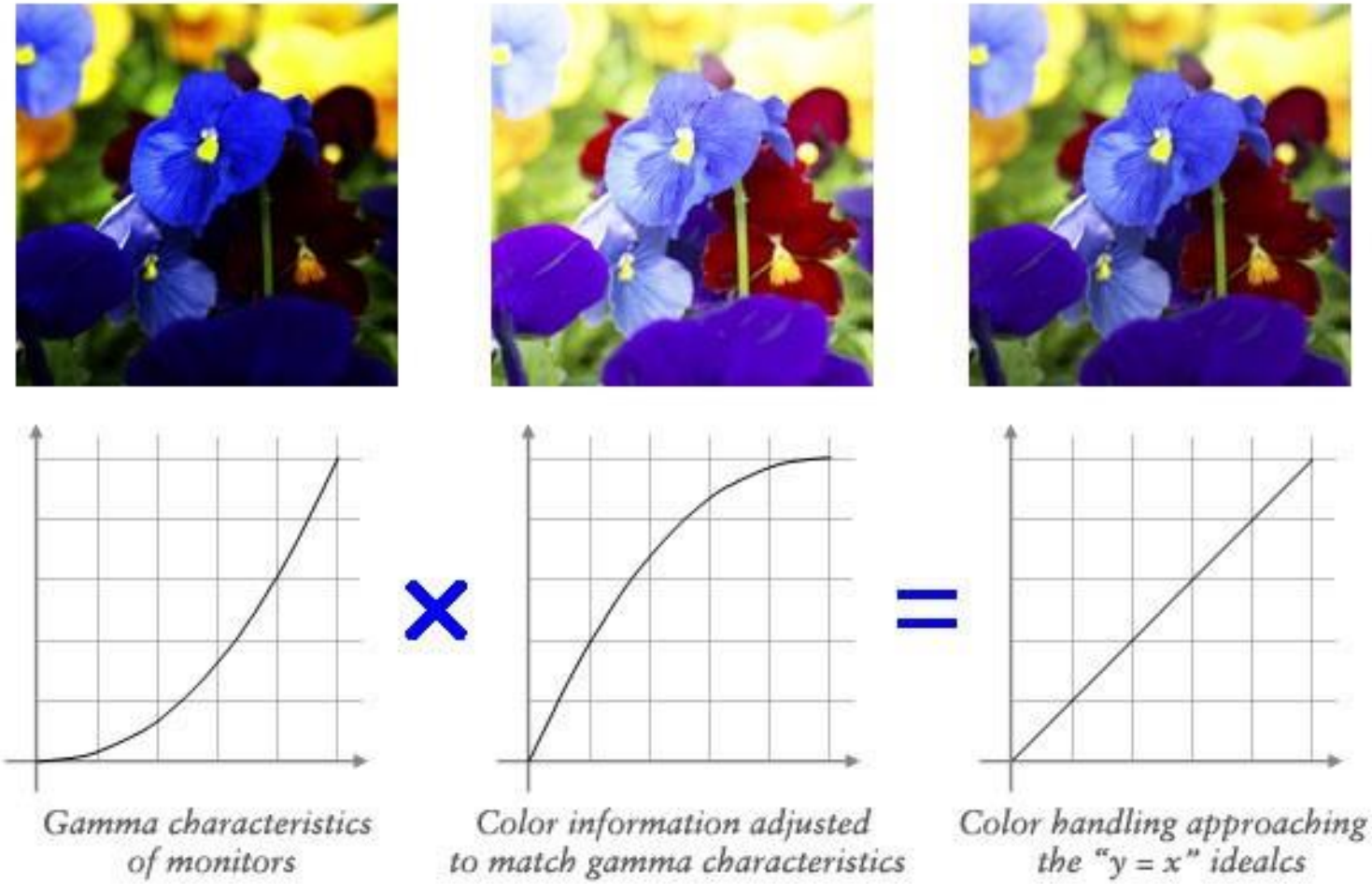
$x \times 2$

Gamma expansion

$\left(\frac{x}{255}\right)^{2} \times 255$

Gamma correction

- Originally, Due to non-linearities in the old CRT televisions, intensities was seen different then they are.

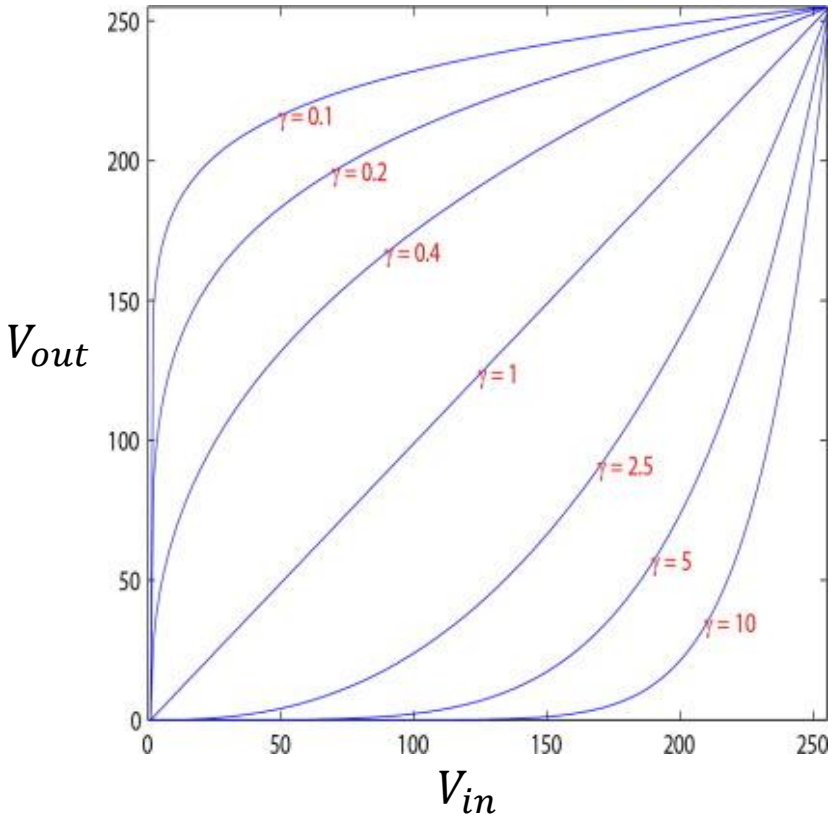


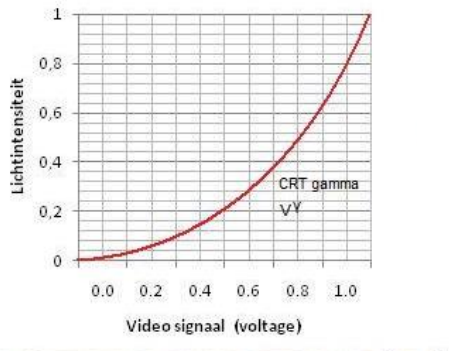
Gamma correction

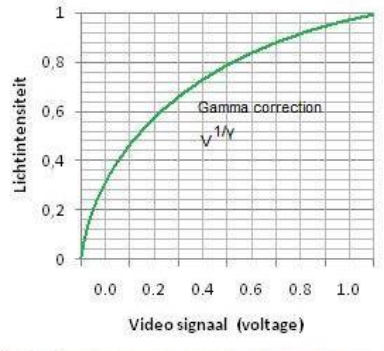
- To correct this non-linear transformation, gamma correction was done:

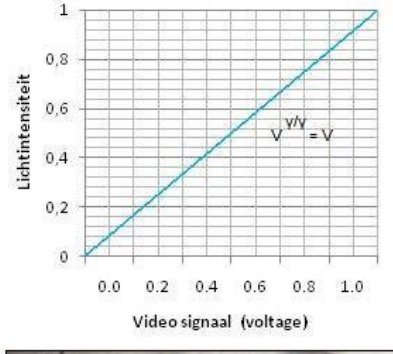
$$V_{out} = \left(\frac{V_{in}}{255}\right)^{\gamma} \cdot 255 \qquad (V_{in}, V_{out} \in \{0,1,\dots,255\})$$

- This is, of course, also applicable for image enhancements.









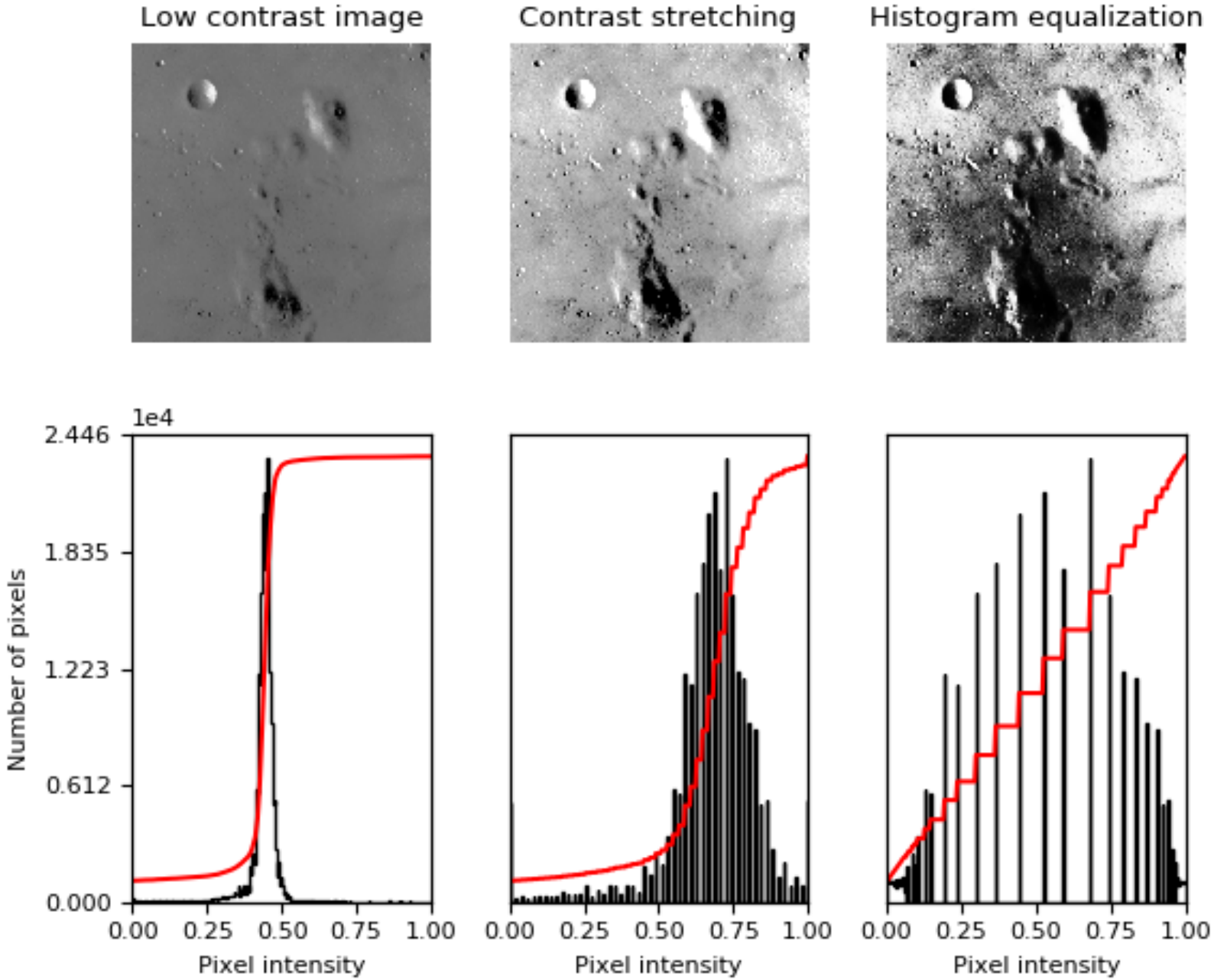
Some more point- wise operators

contents

- Image representation
- Pixel-wise operations
- **Histogram equalization**
- Template matching
- Morphology operators
- Connected components
- Color space

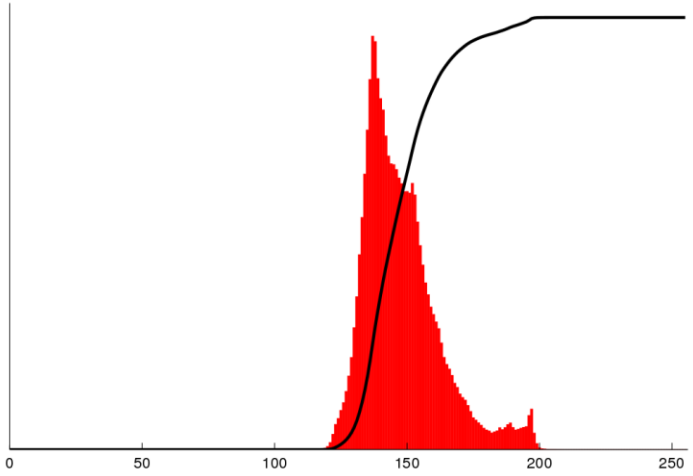
Histogram equalization

- **Histogram equalization** is a method in image processing of contrast adjustment using the image's histogram.
- This method is used to increase the global contrast of an image and is useful in images with backgrounds and foregrounds that are both bright or both dark.
- **Histogram equalization accomplishes this by effectively spreading out the most frequent intensity values.**


Low contrast image
Contrast stretching
Histogram equalization
1e4
2.446
1.835
1.223
0.612
0.000
Number of pixels
0.00
0.25
0.50
0.75
1.00
Pixel intensity

Histogram equalization

- A histogram is a discrete form representation of the distribution of numerical data.
- We will assume at first that our image is continues in the range [0,255] for better understanding.
- Instead of a histogram we will talk about the **probability density function** (**PDF**) $f_X(x)$ of the data.



Reminder: PDF and CDF

- **cumulative distribution function (CDF)** of a real-valued random variable X is the probability that X will take a value less than or equal to x:

$$F_X(x) = P(X \leq x)$$

- Properties of CDF:
  - $\lim_{x \to -\infty} F_X(x) = 0, \quad \lim_{x \to +\infty} F_X(x) = 1$
  - Monotonically non decreasing.
- The **probability density function (PDF)** of a continuous random variable can be determined from the cumulative distribution function by differentiating.

$$f_X(x) = \frac{dF_X(x)}{dx} \quad \text{OR} \quad F_X(x) = \int_{-\infty}^{x} f_X(t)\, dt$$

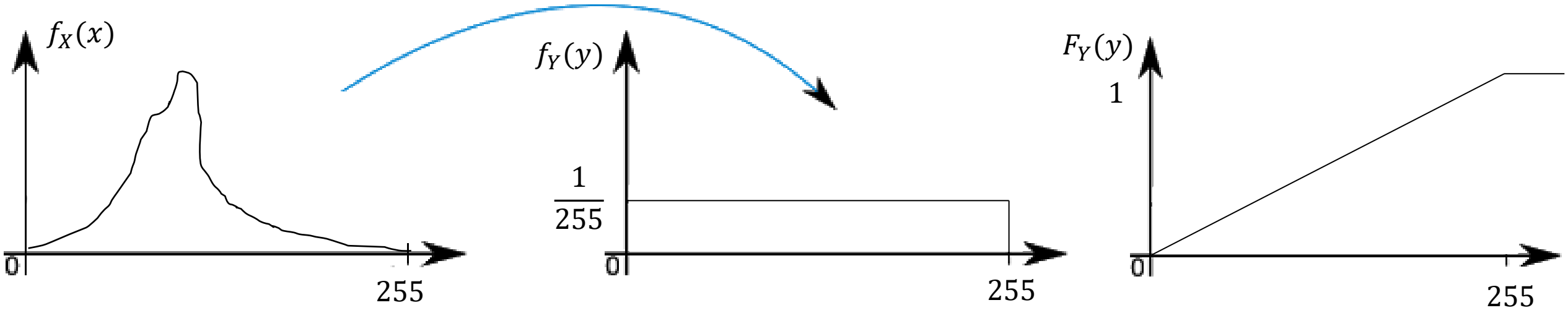
PDF definition- Wikipedia

- **Probability density function (PDF)** can be interpreted as providing a ”relative likelihood” that the value of the random variable would equal that sample.
  - The *absolute likelihood* for a continuous random variable to take on any particular value is 0 (since there are an infinite set of possible values to begin with).
- In a more precise sense, the PDF is used to specify the probability of the random variable falling within a particular range of values. This probability is given by the integral of this variable's PDF over that range and is actually the **CDF.**

PDF equalization

- We want that our resulting PDF $[f_Y(y)]$ will be constant for any value in the range [0,255].
- If the PDF is constant, that means that the CDF is linear in [0,255] (integration of constant is a linear function), and so we get the final CDF as:

$$F_Y(y) = P(Y \leq y) = \begin{cases} 0 & : \ y < 0 \\ \frac{y}{255} & : \ 0 \leq y \leq 255 \\ 1 & : \ y > 255 \end{cases}$$



PDF equalization

- So we are looking for a transformation function of the random variable X to Y such that:

$$Y = T(X)$$

- In the interesting area [0,255]:

$$P(Y \leq y) = y$$
$$P(T(X) \leq y) = y$$
$$assuming\ T\ is\ invertible\colon\quad P\left(X \leq T^{-1}(y)\right) = y$$
$$change\ of\ variables\ z = T^{-1}(y)\colon\quad P(X \leq z) = \mathrm{T}(\mathrm{z})$$

- The answer is $F_X(x) = T(x)$
  - and in fact T is invertible since F_X is Monotonically non decreasing

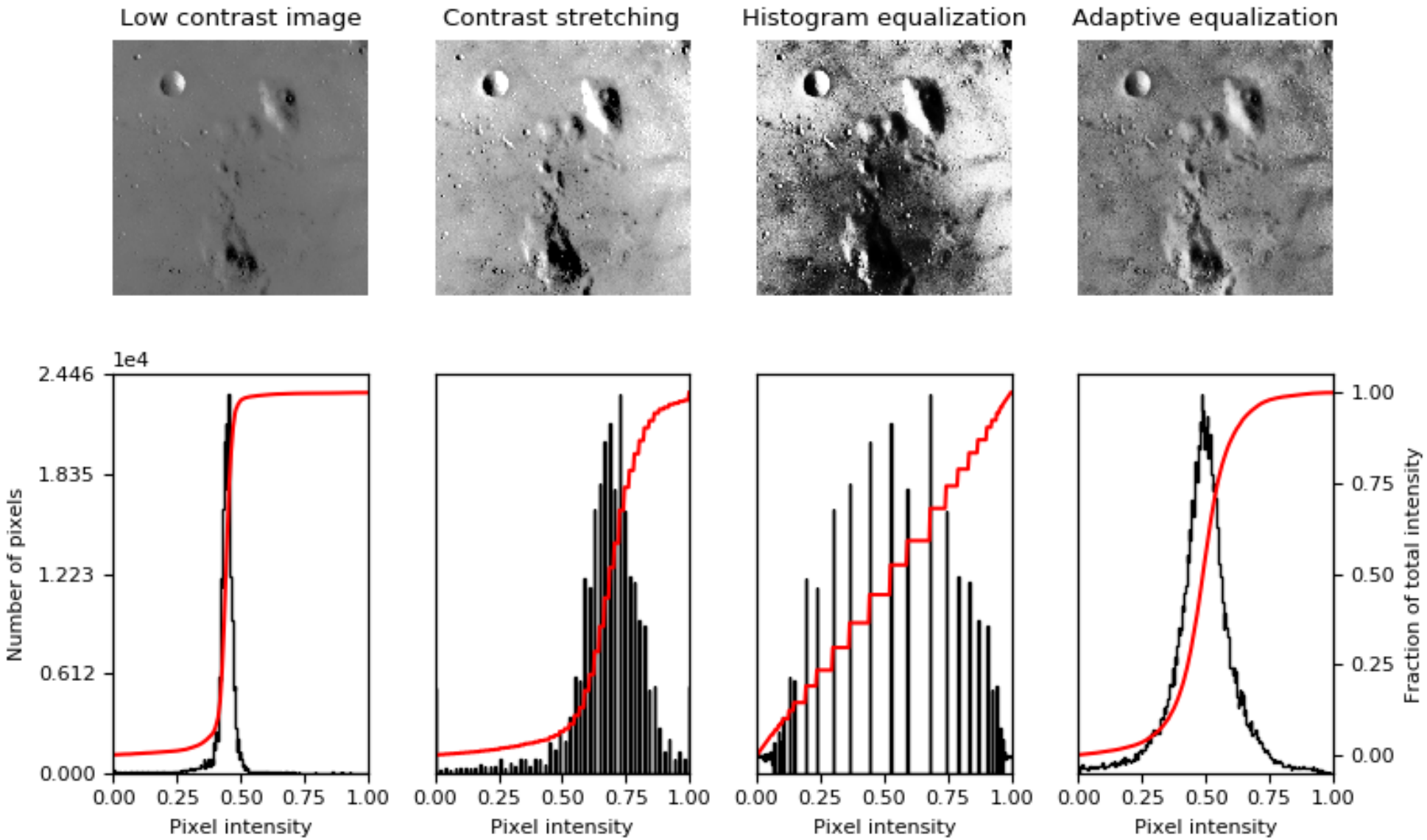
Back to histogram equalization

- The same result is also applicable for discrete space like actual images and their histograms.

- Build histogram of a given image.
- To make the histogram act like a discrete pdf, we will normalize the PDF: divide each bin by the sum of all bins.
- Build the discrete CDF.
- Un-normalize the CDF and round the results back to uint8:

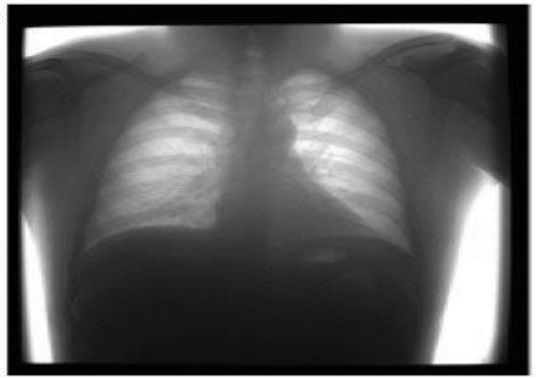
$$f_{eq}(x) = round(CDF(x) * 255)$$

Other variants of histogram equalization

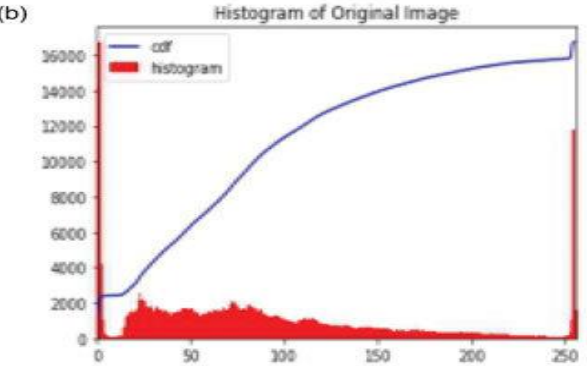


Other variants of histogram equalization

(a)

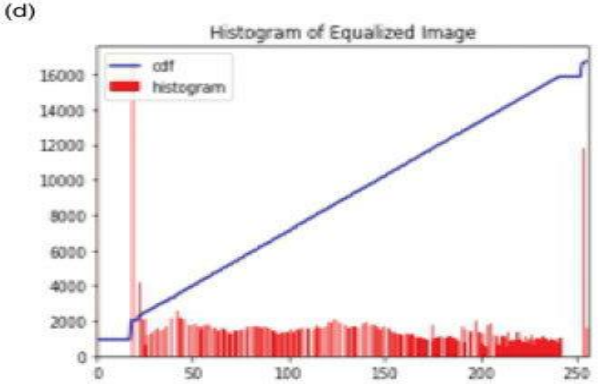


(b)

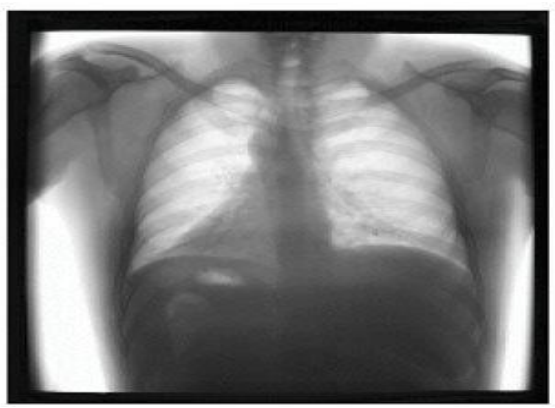


(c)

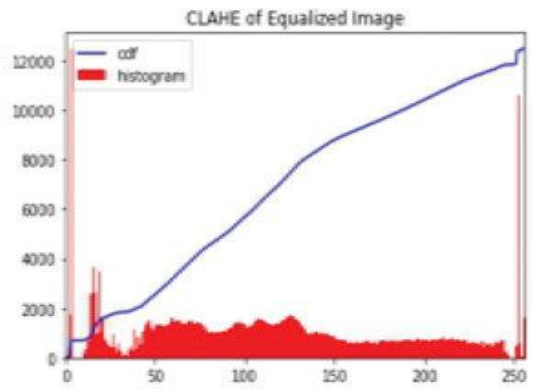
(d)



(e)



(f)



contents

- Image representation
- Pixel-wise operations
- Histogram equalization
- **Template matching**
- Morphology operators
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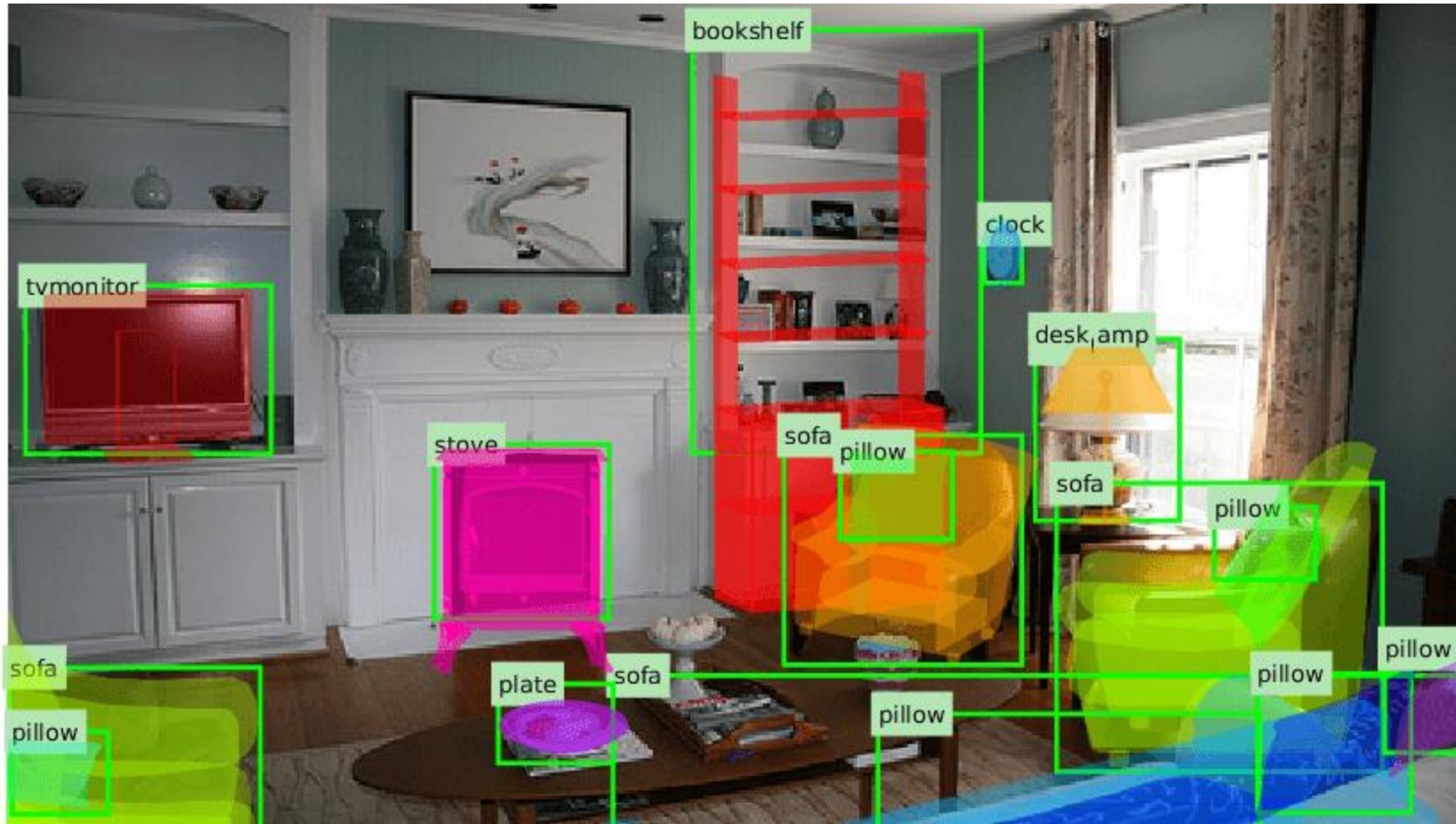
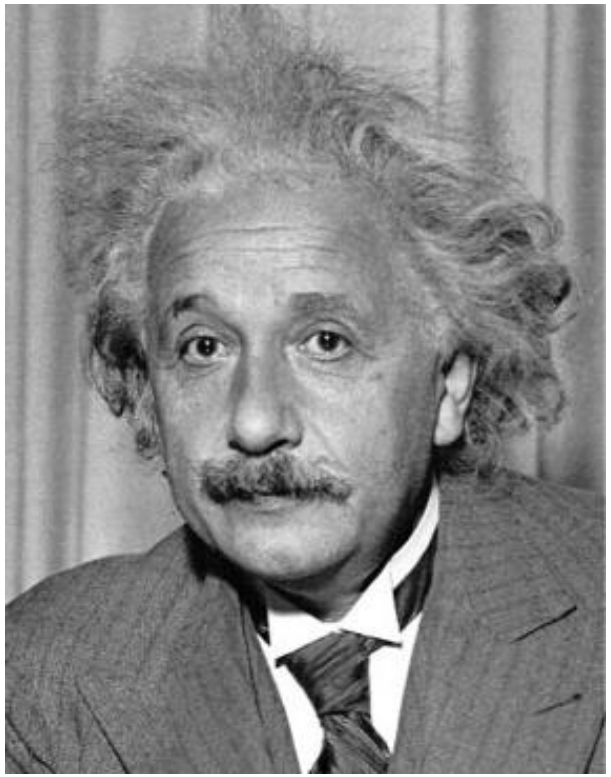
Template matching

- Given an image template- find it in another image.
- Template matching is a sub-field in **object recognition**.
  - We will see it **a lot** of this topic in this course:
    - Cross correlation
    - Feature based – SIFT
    - Neural networks



CC – cross correlation

How do we detect the template 👁 in he following image?



filter 👁

output

$$h[m,n] = \sum_{k,j} g[k,l] f[m+k, n+l]$$

image

What will the output look like?

Run the filter

kernel

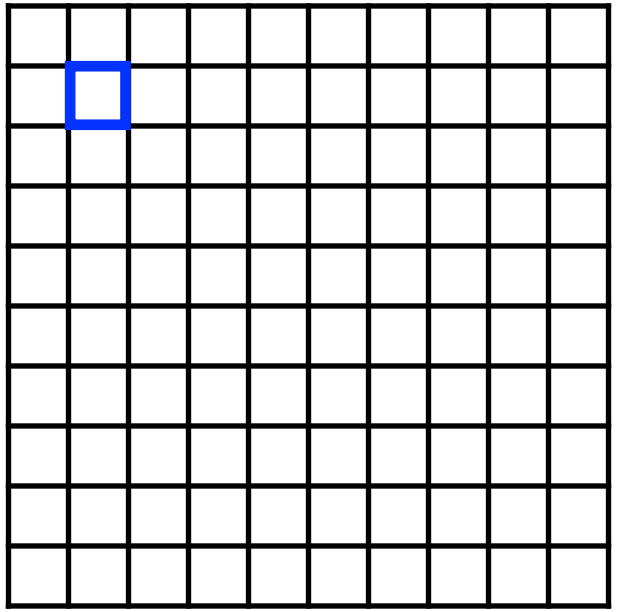
$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output



Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

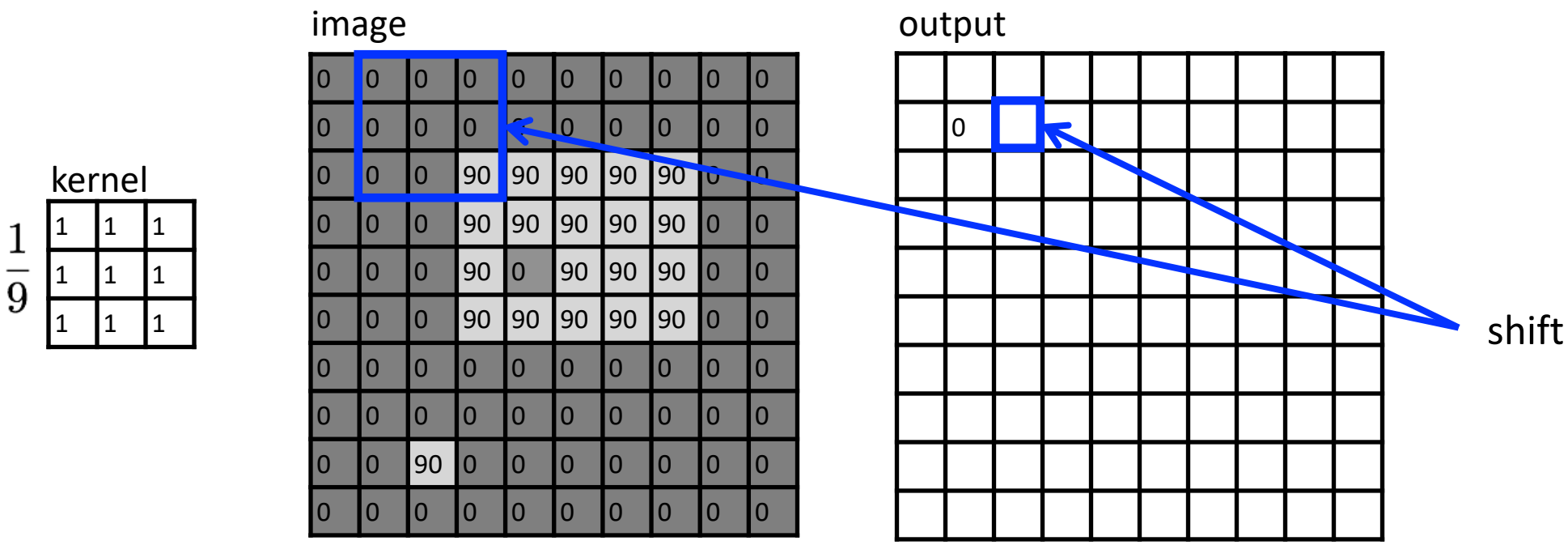
image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0								

Run the filter



Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10							

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20						

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30					

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30				

Run the filter

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30	30			

… and the result is

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

image

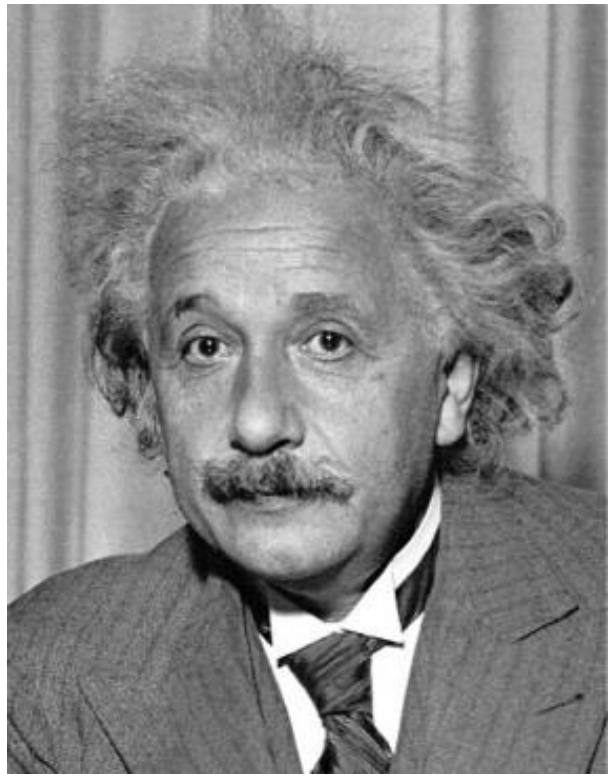
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	0	10	20	30	30	30	20	10	
	10	10	10	10	0	0	0	0	
	10	10	10	10	0	0	0	0	

CC – cross correlation

How do we detect the template 👁 in he following image?



filter 👁

output

$$h[m,n] = \sum_{k,j} g[k,l] f[m+k, n+l]$$

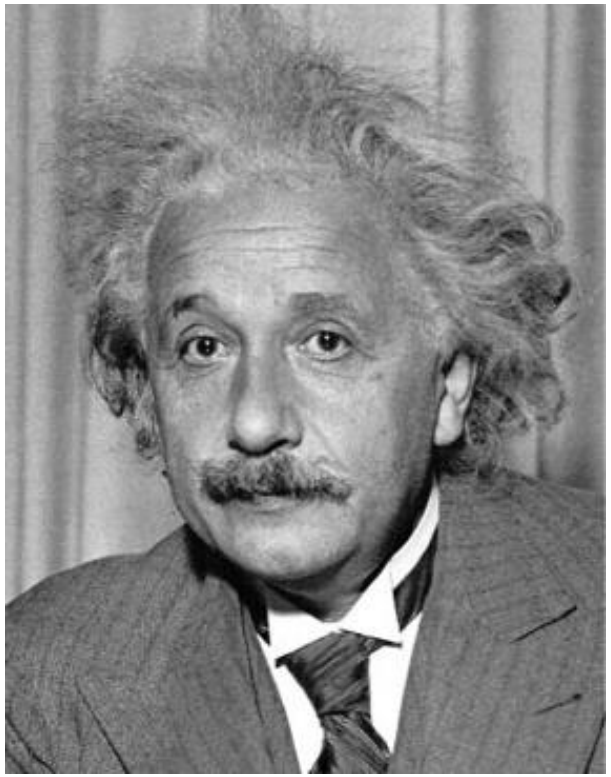
image

What will the output look like?

Cross correlation can also be more simply denoted as $h = g \star f$

CC – cross correlation

How do we detect the template  in he following image?

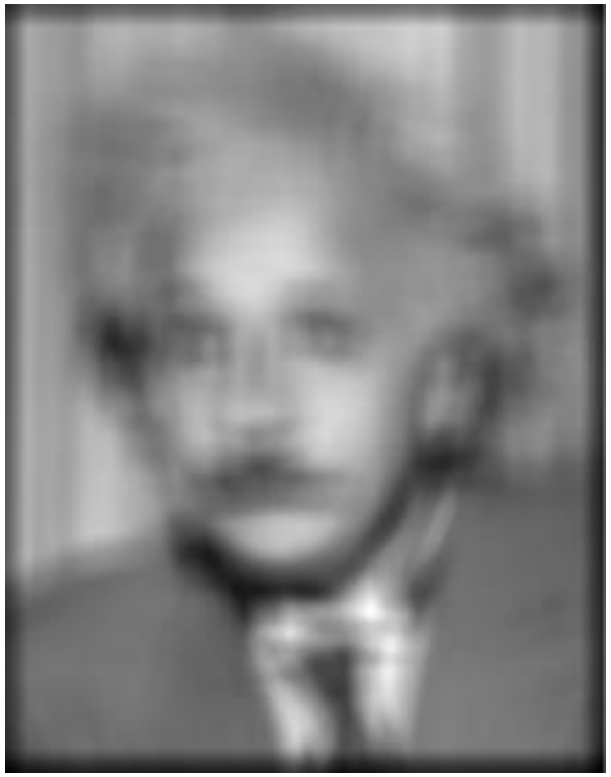


filter 

output

$$h[m,n] = \sum_{k,j} g[k,l] f[m+k, n+l]$$

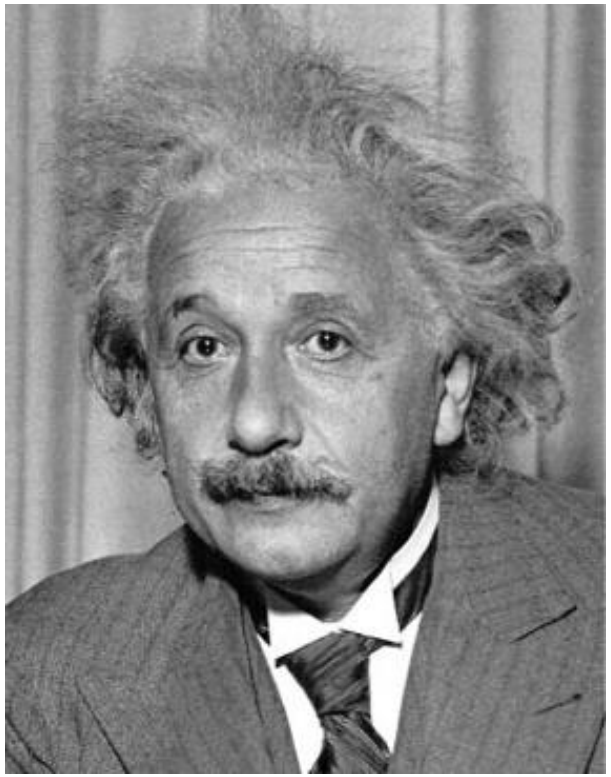
image



Is this good for template matching?

CC – cross correlation

How do we detect the template  in he following image?

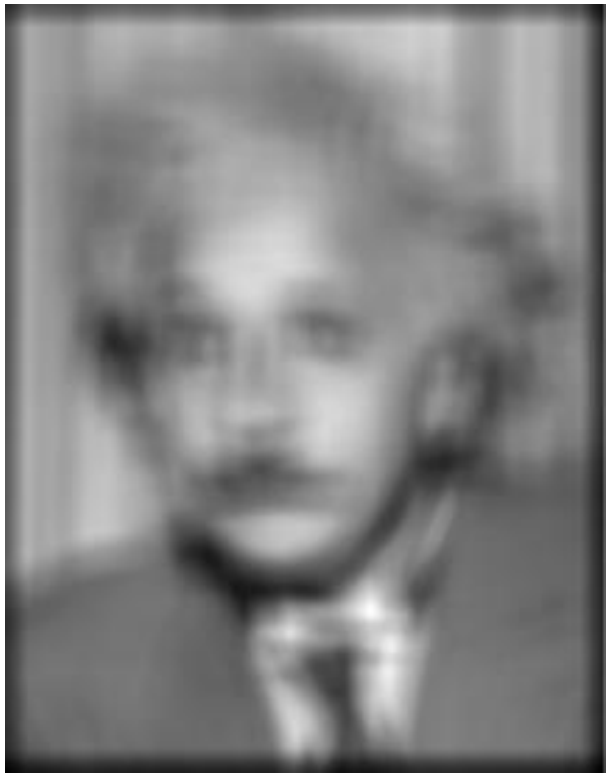


filter 

output

$$h[m,n] = \sum_{k,j} g[k,l] f[m+k, n+l]$$

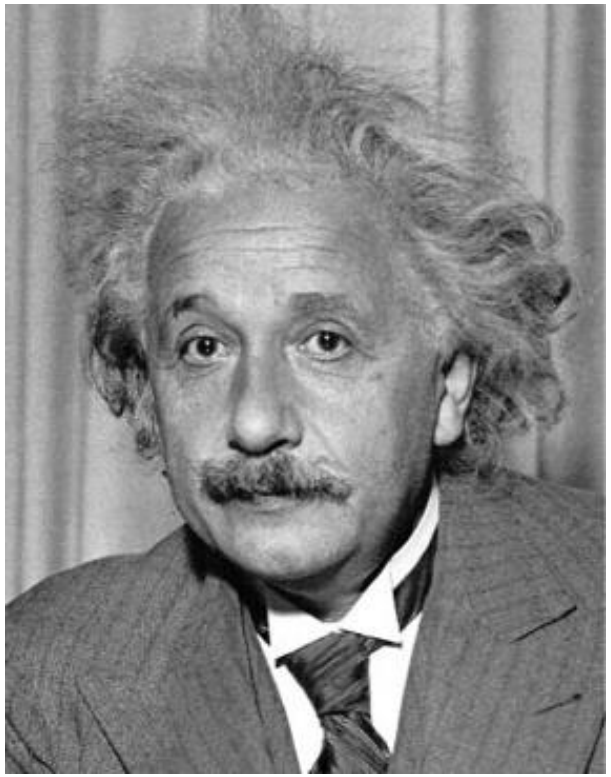
image



Increases for higher local intensities.

Zero mean cross correlation

How do we detect the template  in he following image?



filter 

template mean

output

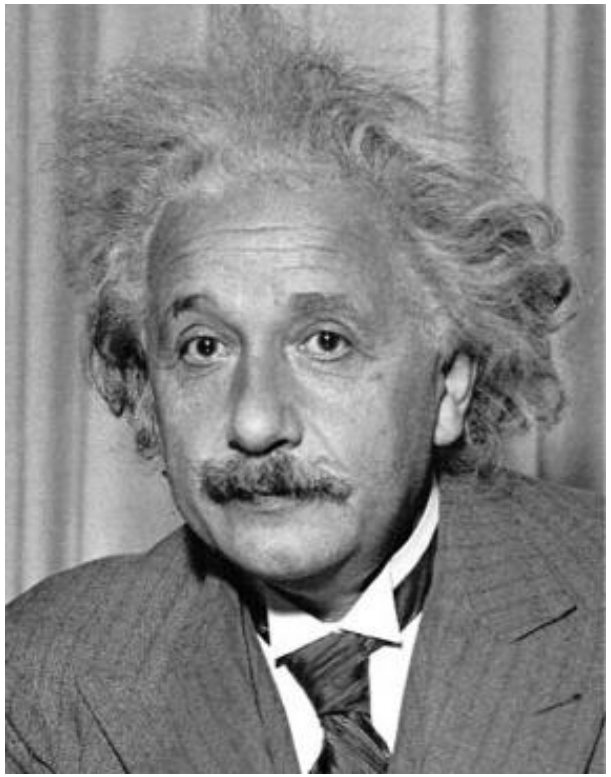
$$h[m,n] = \sum_{k,l} (g[k,l] - \bar{g}) f[m+k, n+l]$$

image

What will the output look like?

Zero mean cross correlation

How do we detect the template  in he following image?



output

filter 

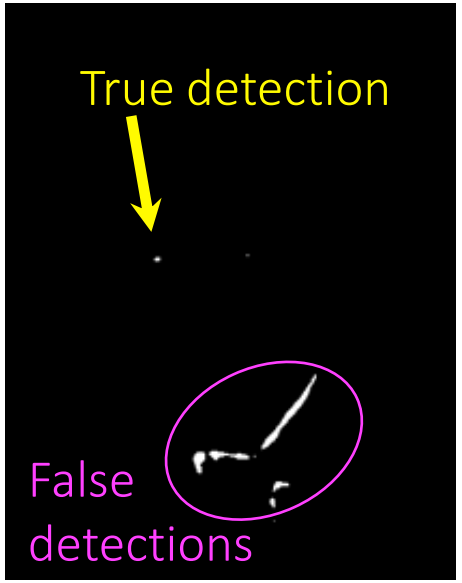
template mean

output

$$h[m,n] = \sum_{k,l}(g[k,l] - \bar{g})f[m+k, n+l]$$

thresholding

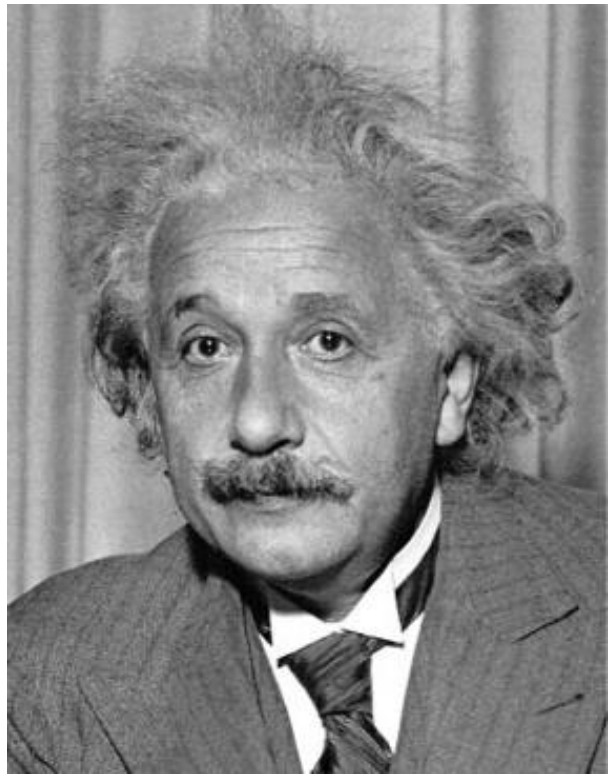
image



What went wrong?

Zero mean cross correlation

How do we detect the template in he following image?



output

filter

template mean

output

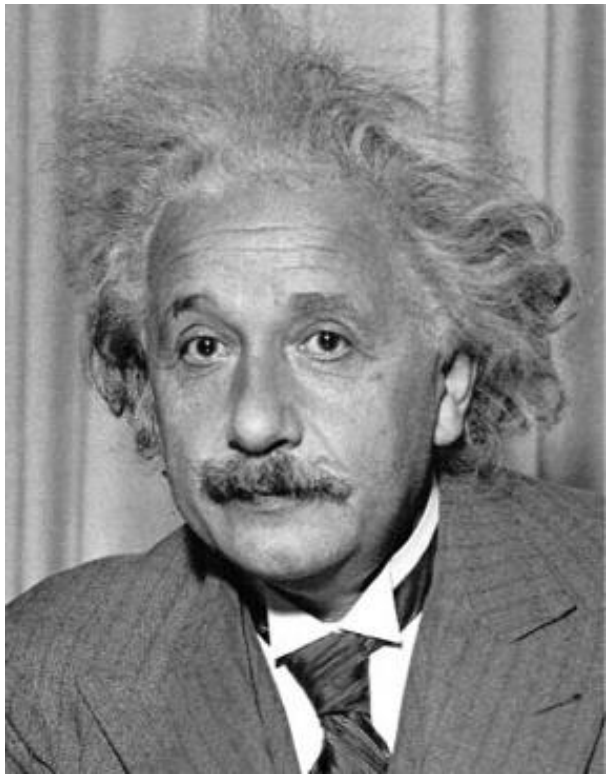
$$h[m,n] = \sum_{k,l} (g[k,l] - \bar{g}) f[m+k, n+l]$$

image

Not robust to high-contrast areas

ZNCC – zero mean normalized cross correlation

How do we detect the template  in he following image?



What will the output look like?

filter 

template mean

$$\text{output } h[m,n] = \frac{\sum_{k,l}(g[k,l]-\bar{g})(f[m+k,n+l]-\bar{f}_{m,n})}{\sqrt{(\sum_{k,l}(g[k,l]-\bar{g})^2 \sum_{k,l}(f[m+k,n+l]-\bar{f}_{m,n})^2)}}$$

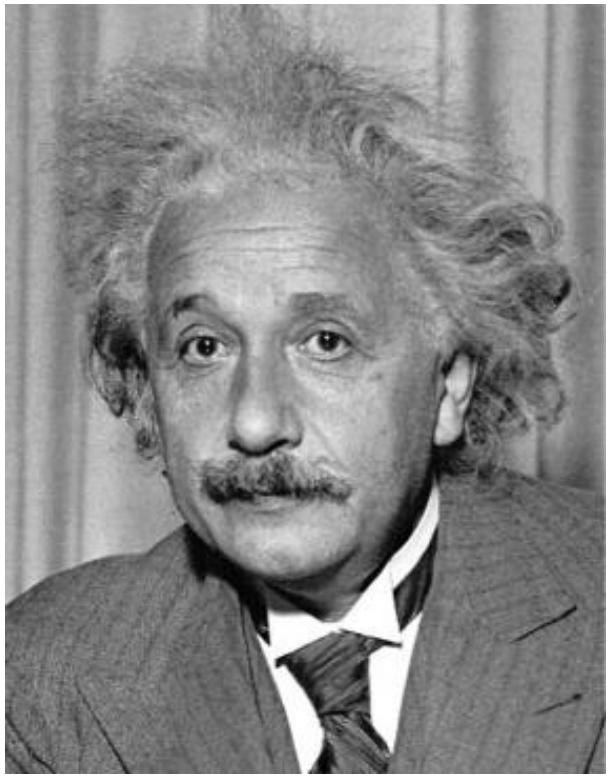
image

local patch mean

The below square root is actually the product of both template and patch STD.

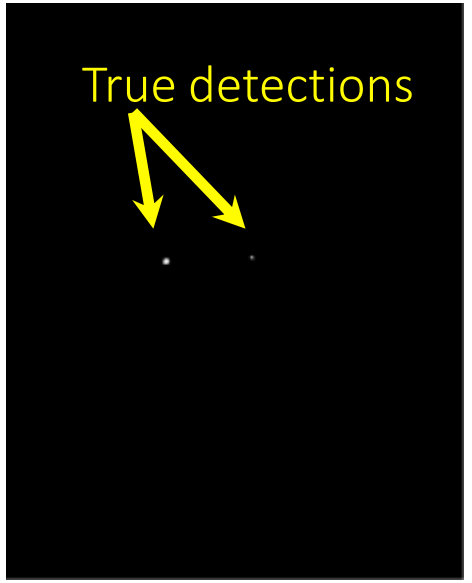
ZNCC – zero mean normalized cross correlation

How do we detect the template  in he following image?



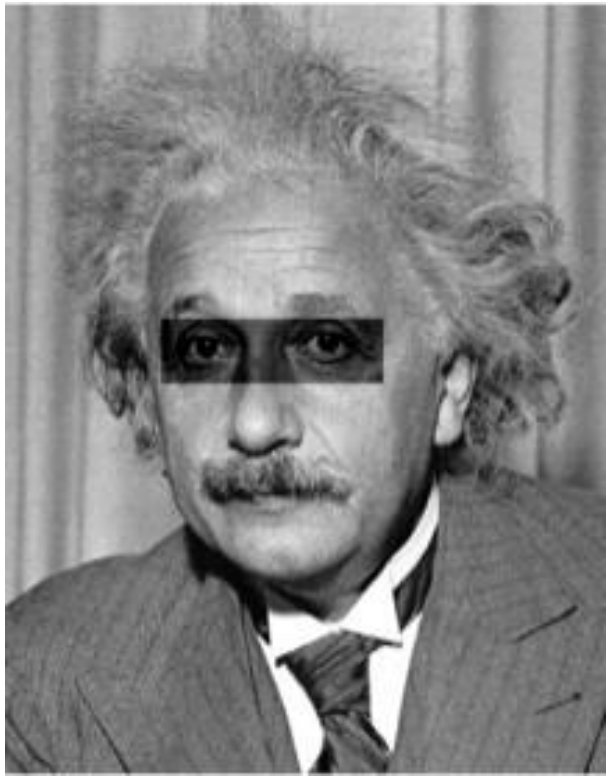
output

thresholding

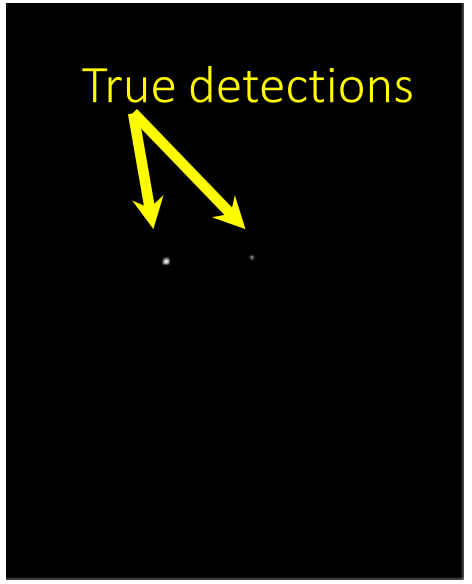


ZNCC – zero mean normalized cross correlation

How do we detect the template  in he following image?



output

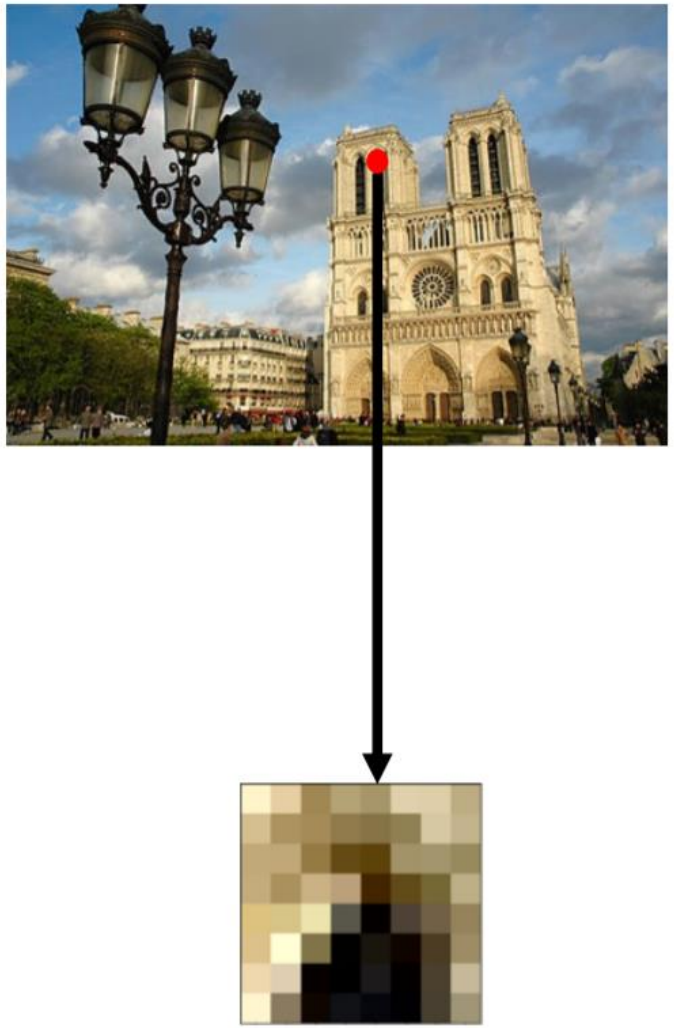


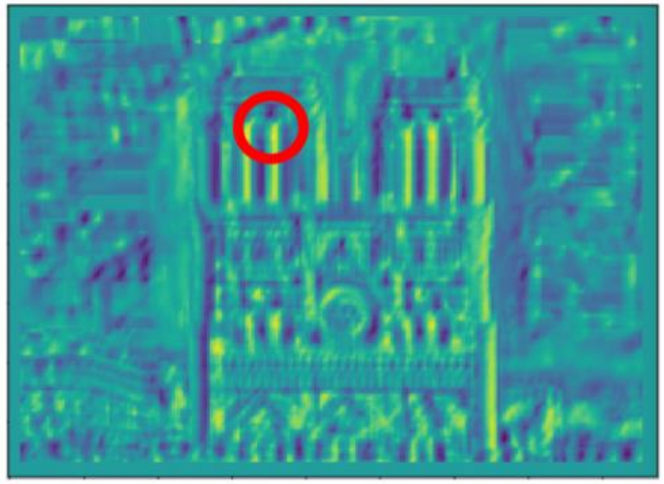
thresholding

robust to change in intensities

Template matching- ZNCC

- Good for very carefully constructed scenarios.
- Can’t handle change in rotation and scale.





contents

- Image representation
- Pixel-wise operations
- Histogram equalization
- Template matching
- **Morphology operators**
- Connected components
- Color space

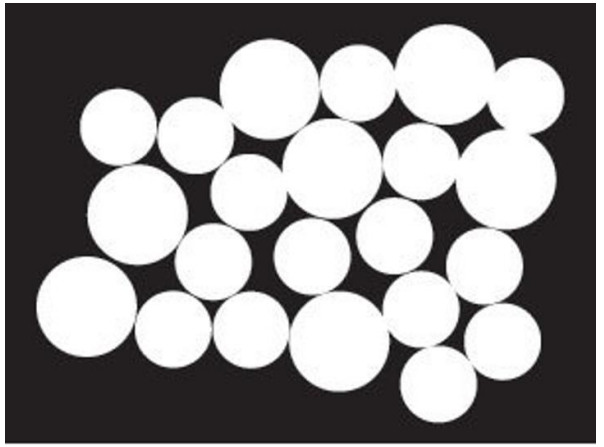
Morphology

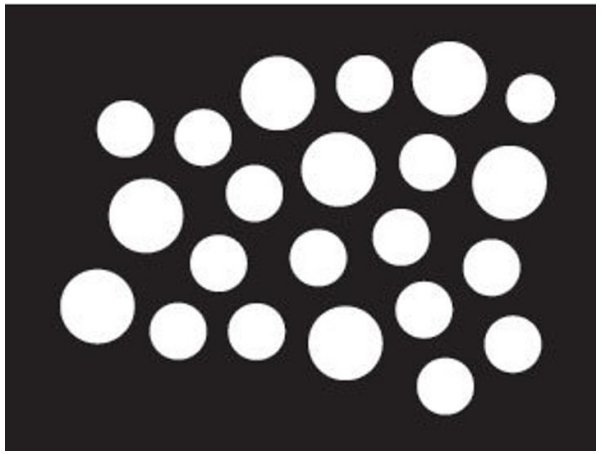
Examples:

Image cleaning

Style

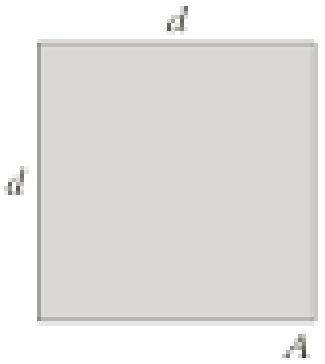
Coin counting
(using connected components)



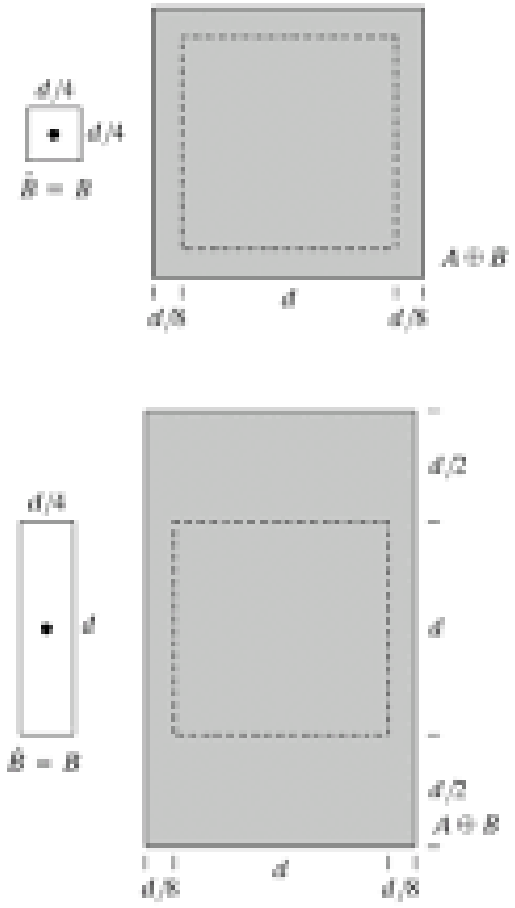


Morphology

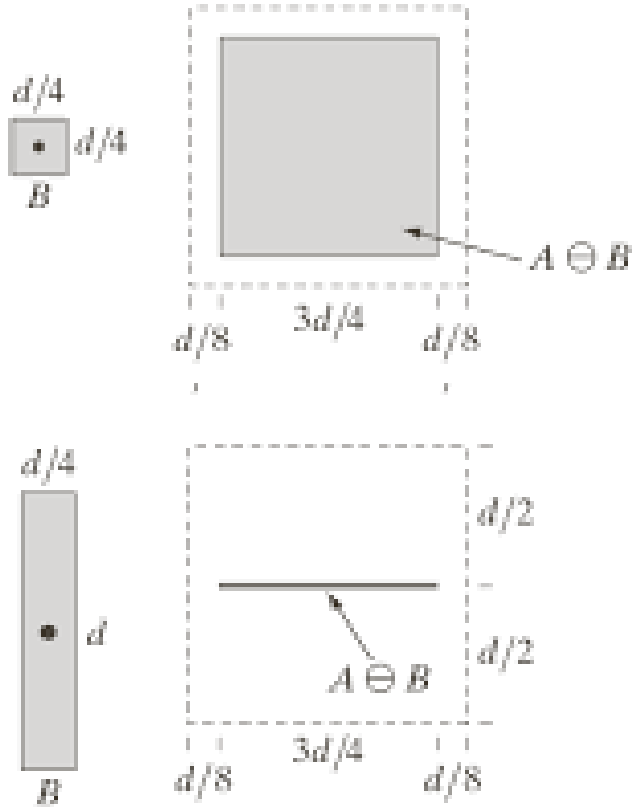
- Two basic morphology operators



Dilation: “inflating” a shape with a given kernel



Erosion: “reducing” a shape with a given kernel



Morphology: geometric interpretation

- Each kernel has an anchor point (usually in the kernel center).
- Dilation: the final shape is all points where the anchor point can be placed in which **the kernel touches a part of the original shape**.
- Erosion: the final shape is all points where the anchor point can be placed in which **all kernel points touch the original shape**.

Morphology: algorithm

- Each morphology operator is constructed as such:
  1. Select a structure element (binary kernel)

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Morphology: algorithm

- Each morphology operator is constructed as such:
  1. Select a structure element (binary kernel)
  2. Cross-correlate with input binary image $g = f \star s$

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Morphology: algorithm

- Each morphology operator is constructed as such:
  1. Select a structure element (binary kernel)
  2. Cross-correlate with input binary image $g = f \star s$
  3. Threshold the output

$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

Morphology: algorithm

- Each morphology operator is constructed as such:
  1. Select a structure element (binary kernel)
  2. Cross-correlate with input binary image $g = f \star s$
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$$s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\theta_{TH}(x,t) = \begin{cases} 1 & if \quad x \geq t, \\ 0 & else \end{cases}$$

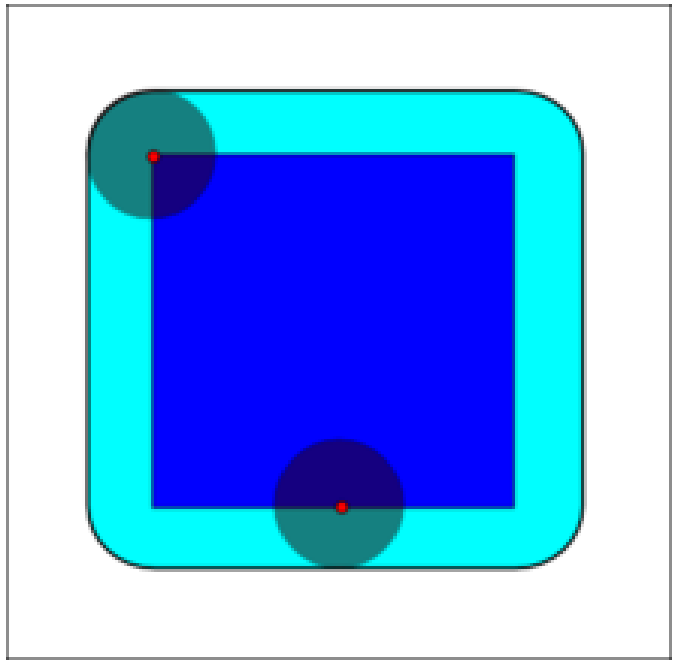
- Overall morphologic operation should look like so:

$$k = \theta_{TH}(f \star s, t)$$

Dilation

- $k = \theta_{TH}(f \star s, t)$

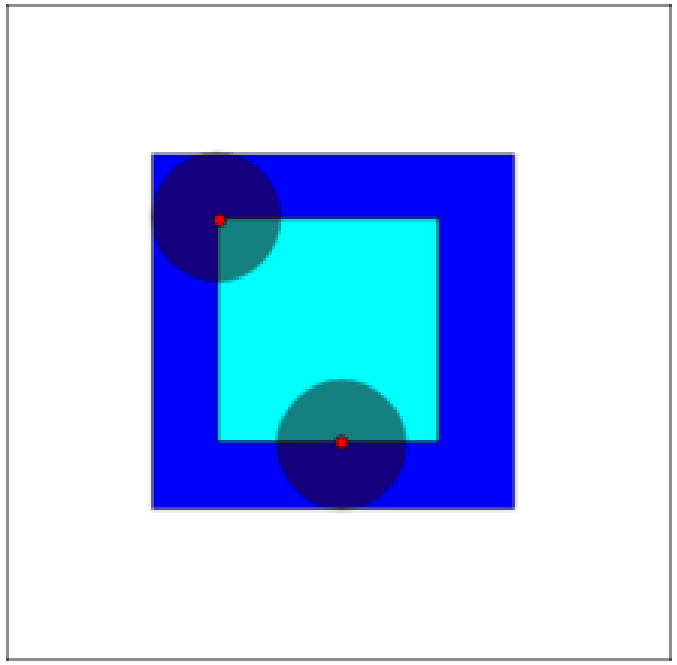
- Dilation: $t = 1$



Erosion

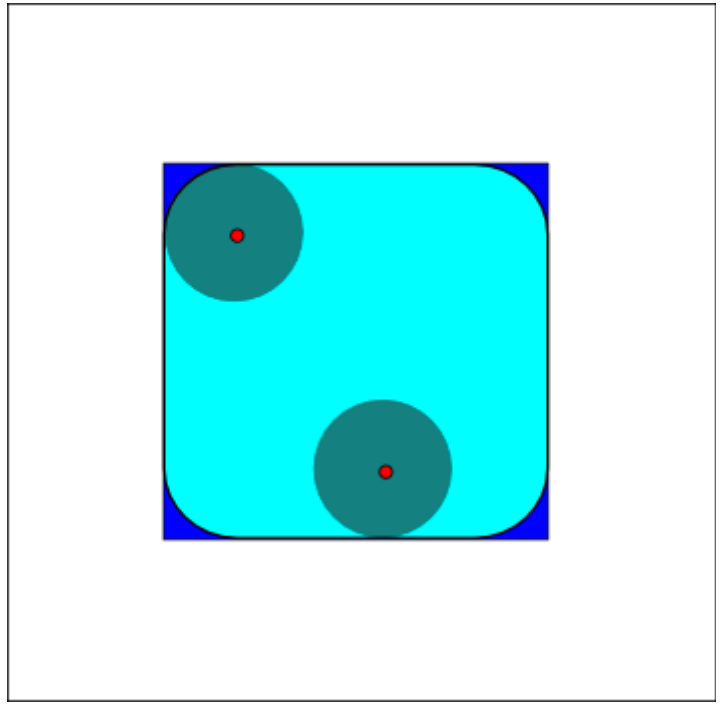
- $k = \theta_{TH}(f \star s, t)$

- Erosion: $t = sum(s)$



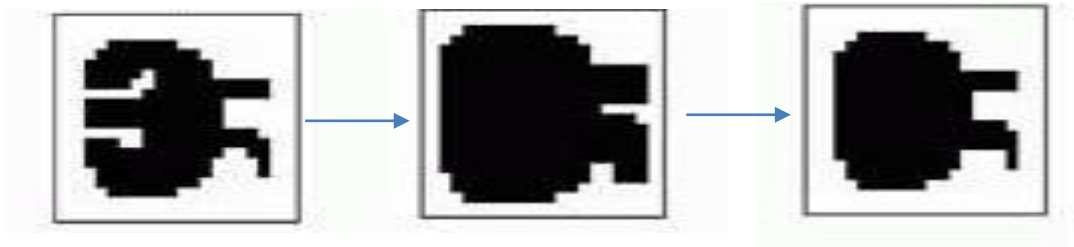
Opening

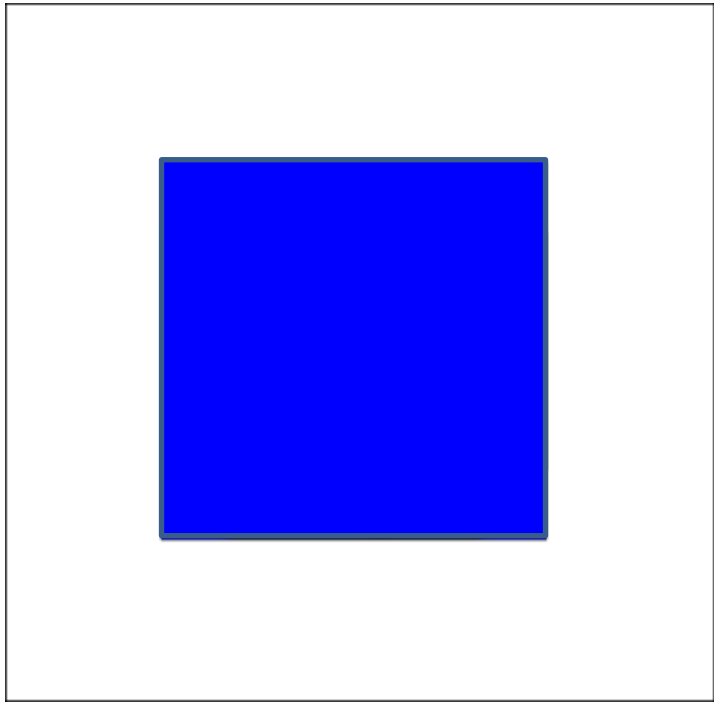
- Erosion followed by dilation.
  - The effect is of rounding off sharp edges.



Closing

- Dilation followed by erosion.
  - The effect is of closing of narrow gaps and holes.



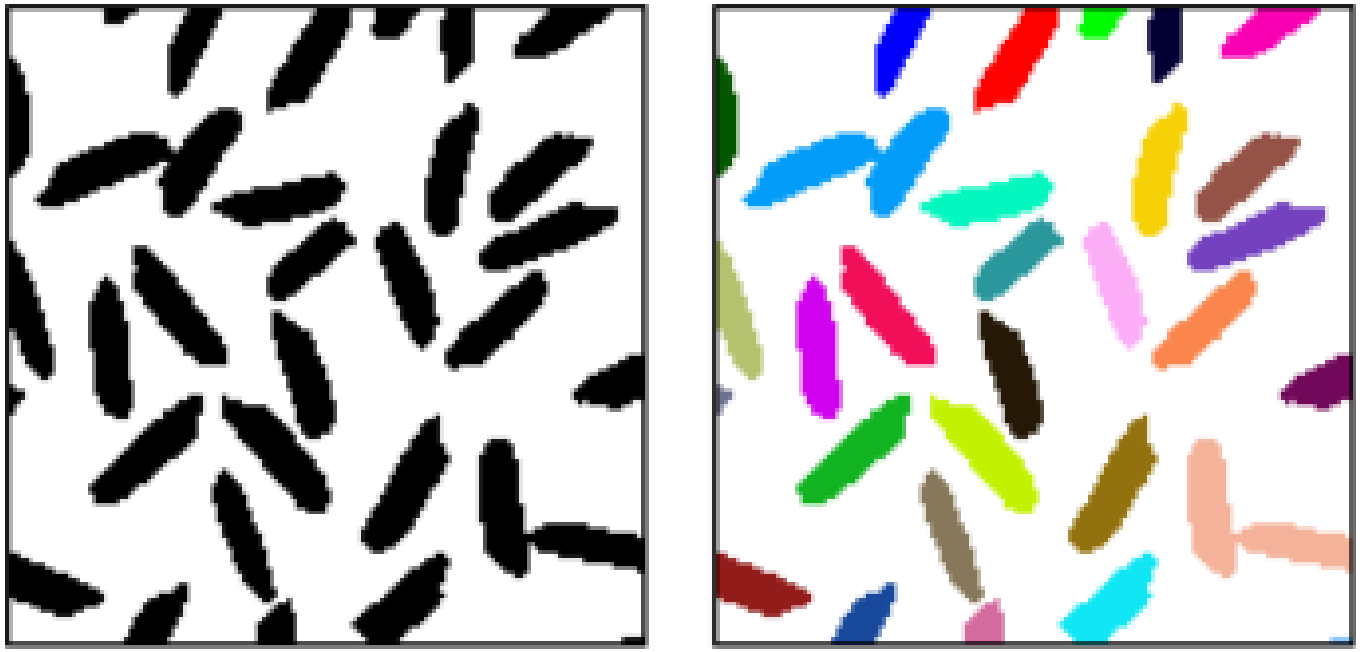


contents

- Image representation
- Pixel-wise operations
- Histogram equalization
- Template matching
- Morphology operators
- **Connected components**
- Color space

Connected components

- Defined as regions of adjacent pixels that have the same value.
- Commonly used with binary images to find stand alone objects.
  - e.g.: letters in a document.



contents

- Image representation
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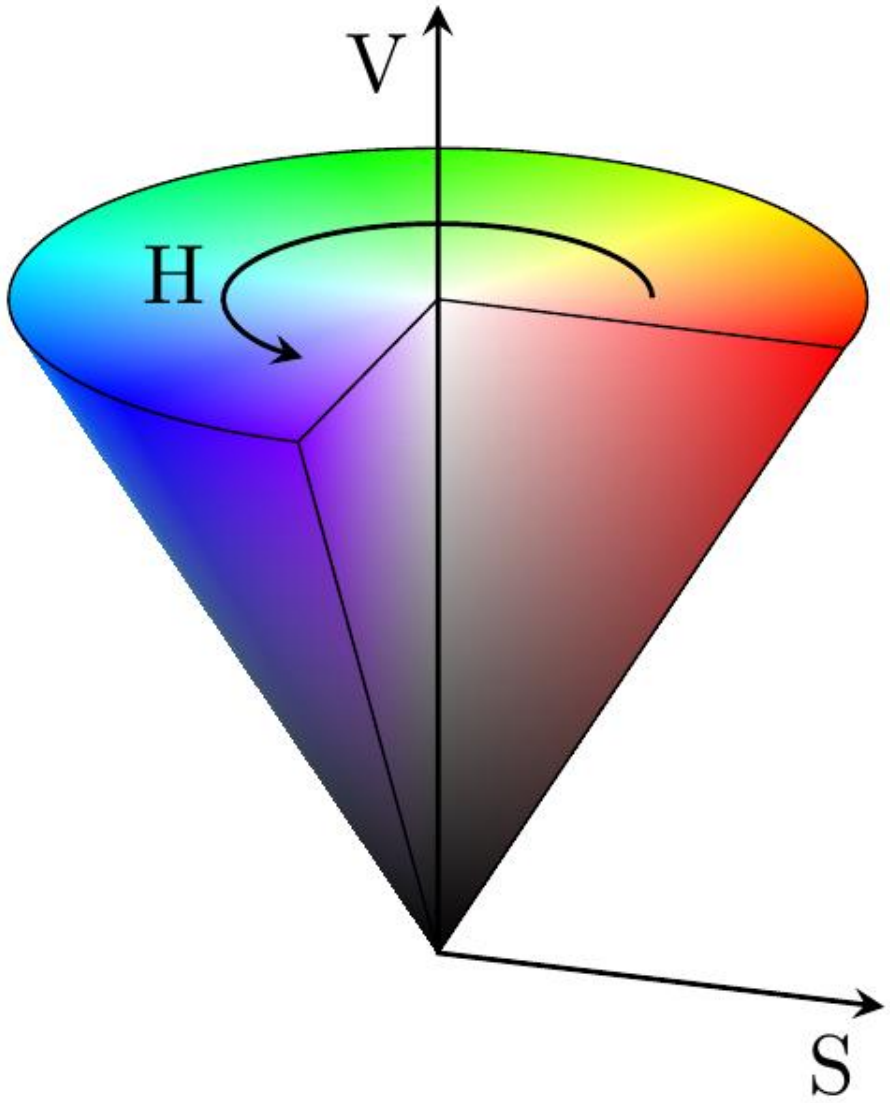
- Connected components
- **Color space**

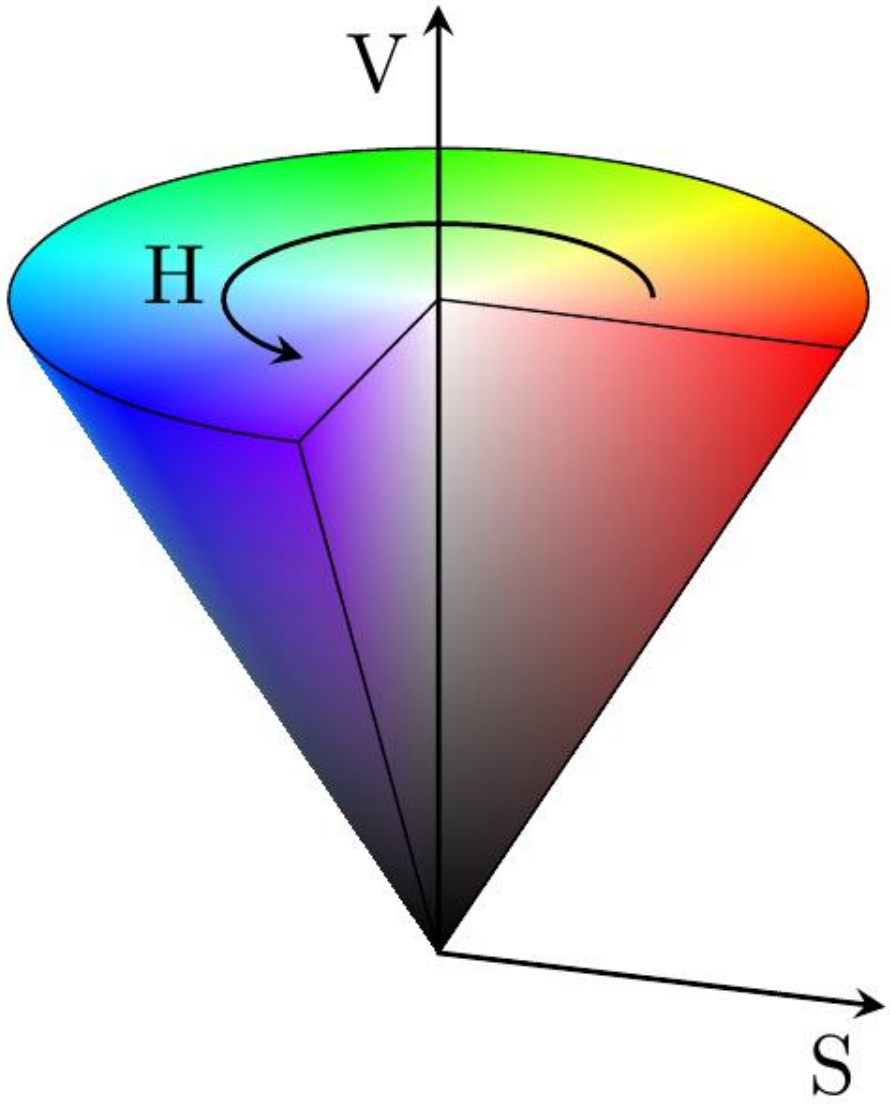
HSV

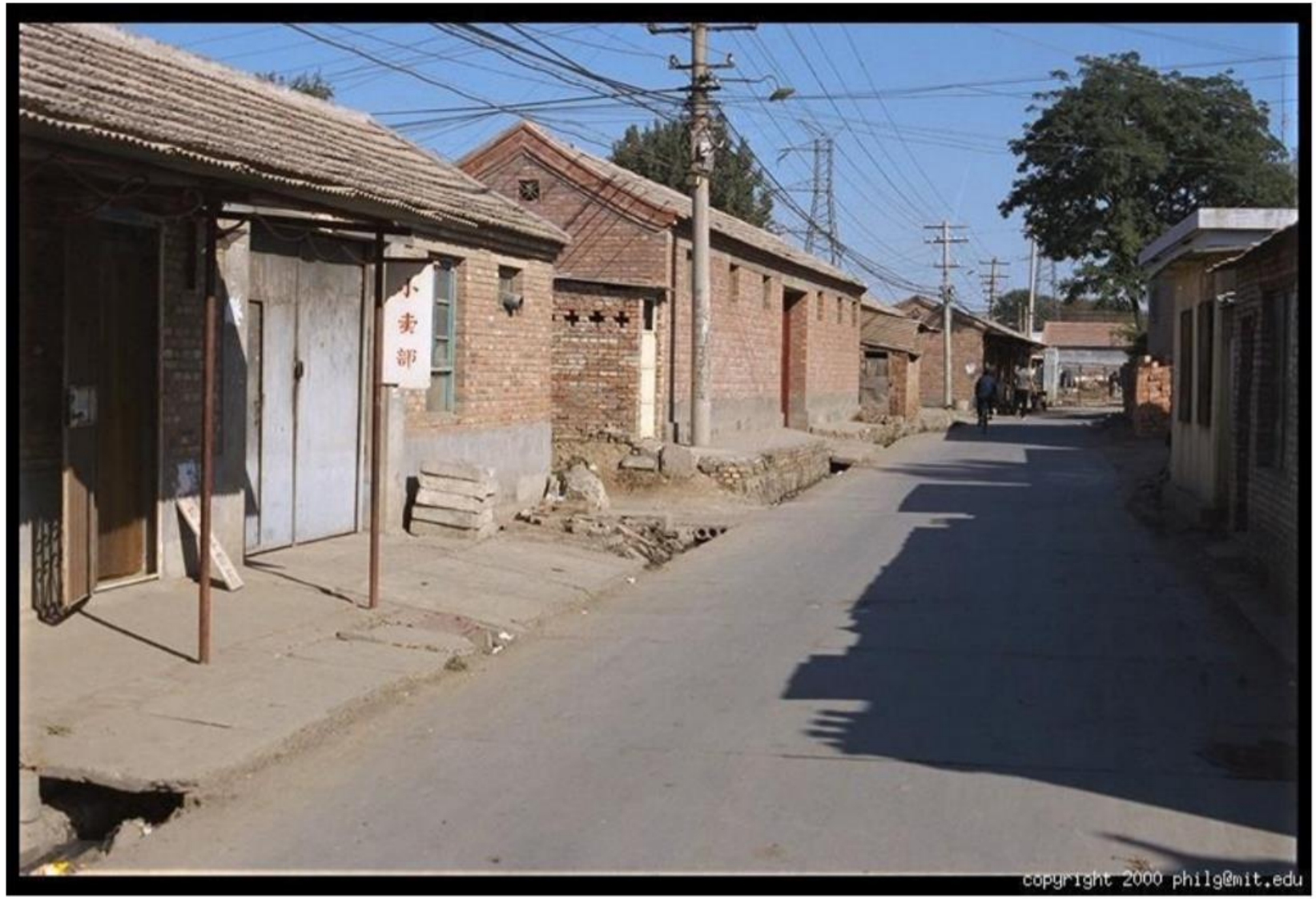


https://cs.brown.edu/courses/csci1430/lectures/2020Spring_08_CamerasLightAndColor.pdf

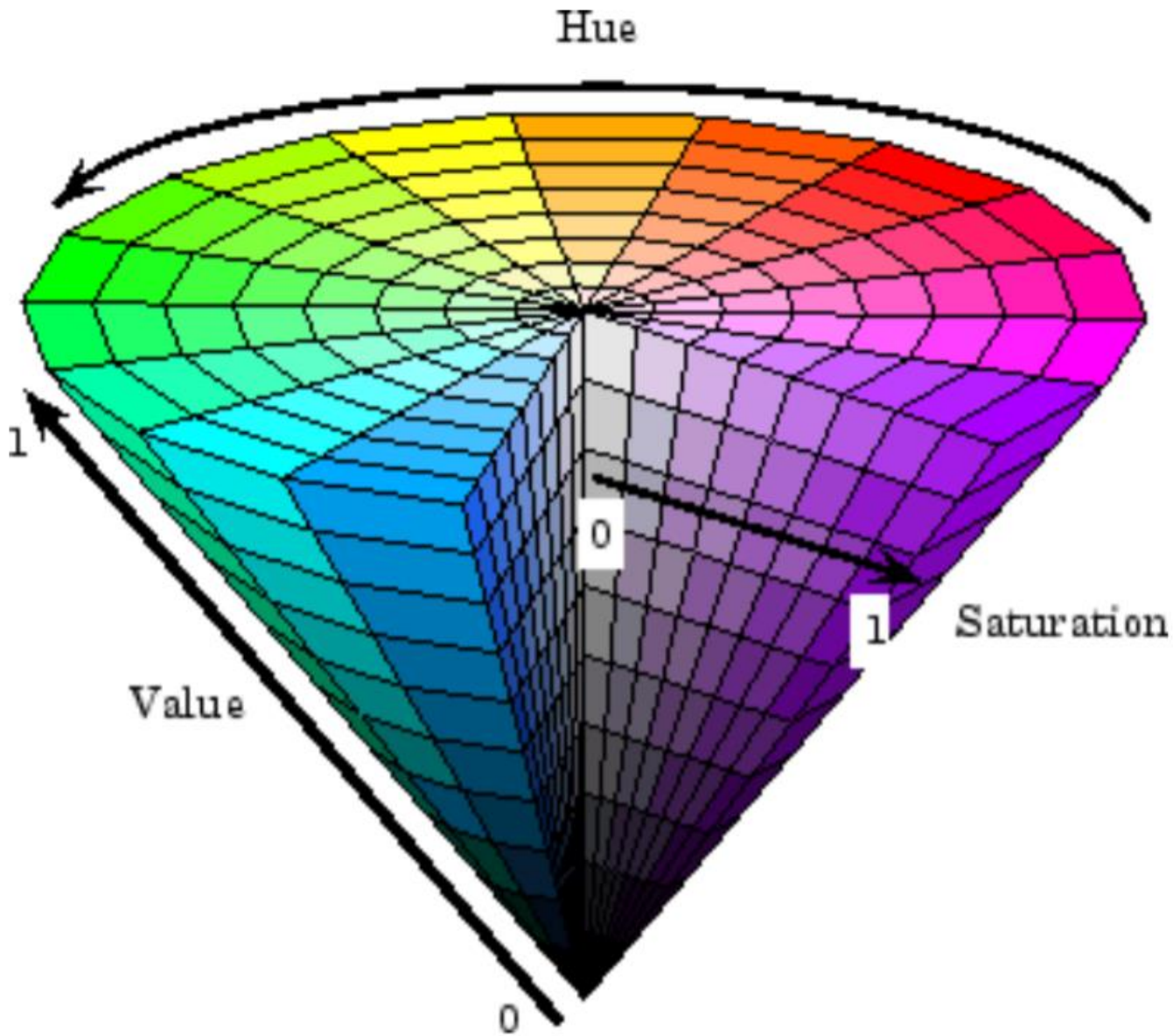
HSV

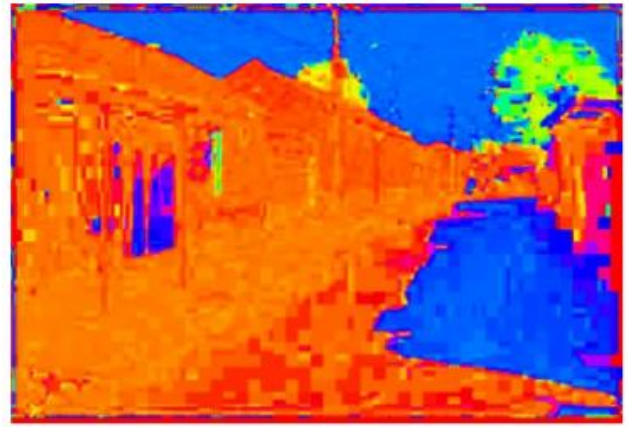
- **Hue**: The "attribute of a visual sensation according to which an area appears to be similar to one of the perceived colors: red, yellow, green, and blue, or to a combination of two of them"
- **Saturation**: The "colorfulness of a stimulus relative to its own brightness"
- **Value**: The "brightness relative to the brightness of a similarly illuminated white“. Can also be called **brightness or intensity**.
  - [Wikipedia]





Original image





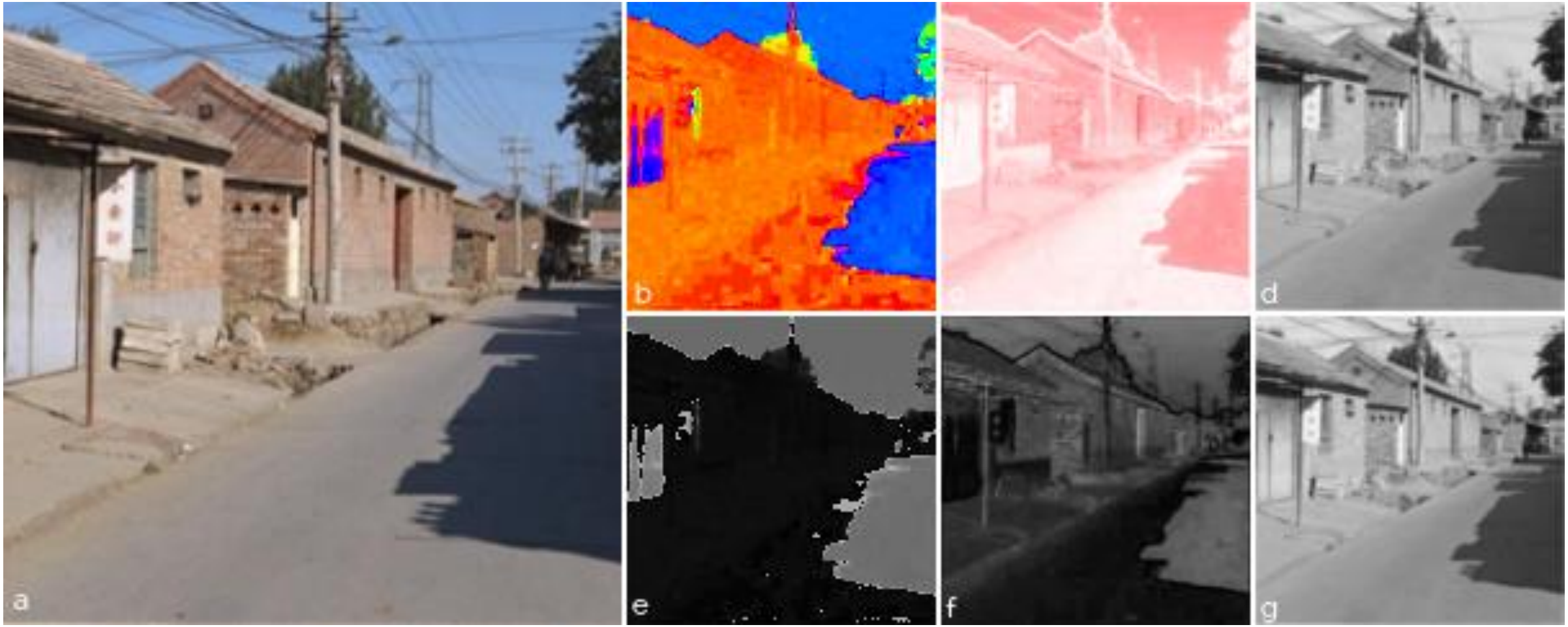
H
(S=1,V=1)

S
(H=1,V=1)

V
(H=1,S=0)

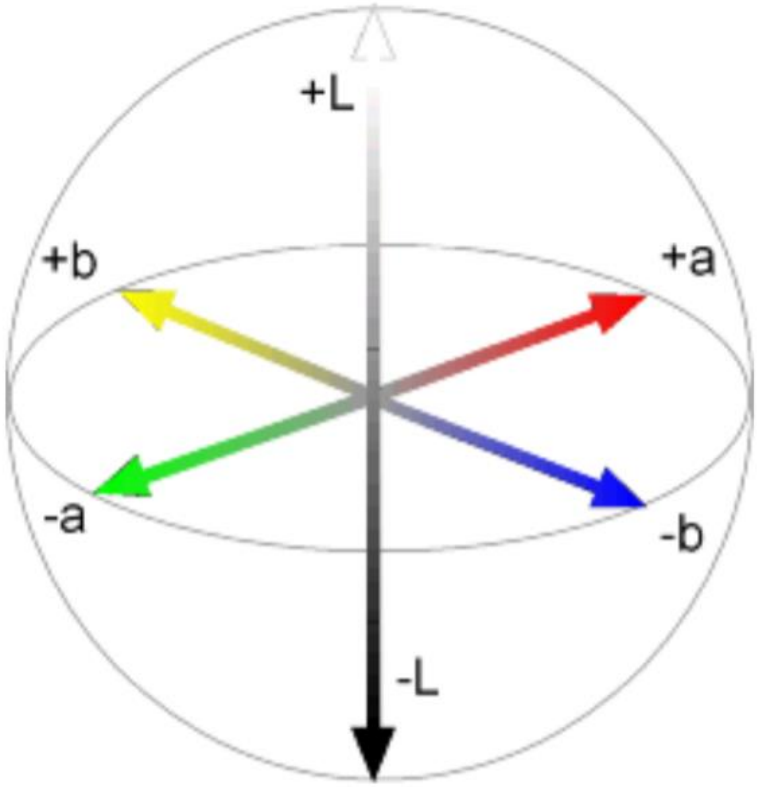
HSV

- In e, f, g: single channel image representation.
- Conclusion: people are much more responsive to intensity then chroma.



More color spaces: LAB

- L: lightness from black (0) to white (100).
- A: from green (−) to red (+).
- B: from blue (−) to yellow (+).



L
(a=0,b=0)

a
(L=65,b=0)

b
(L=65,a=0)

More color spaces: YUV

- Y: brightness/ intensity.
- U: blue projection.
- V: red projection.

- [Similar to YCbCr]