Basic image processing



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www.AlisMath.com

References

- http://szeliski.org/Book/
- http://www.cs.cornell.edu/courses/cs5670/2019sp/lectures/lectures.html
- http://www.cs.cmu.edu/~16385/

Some motivation



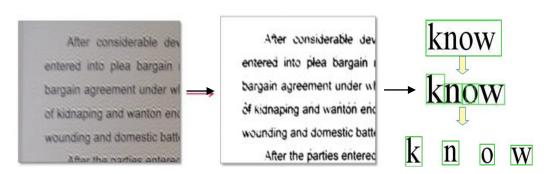
Low contrast image

Contrast stretching

Histogram equalization



Art (Photoshop color grading)



Robotics (OCR – optical character recognition)

Science and space (image enhancement)



Agriculture (color ripeness detection)

contents

- Image representation
- Pixel-wise operations
- Histogram equalization
- Template matching
- Morphology operators
- Connected components
- Color space

Image representation

• We can think of an image as a 3d matrix of discrete RGB values.

The values mark the intensity of each color channel and are usually of type

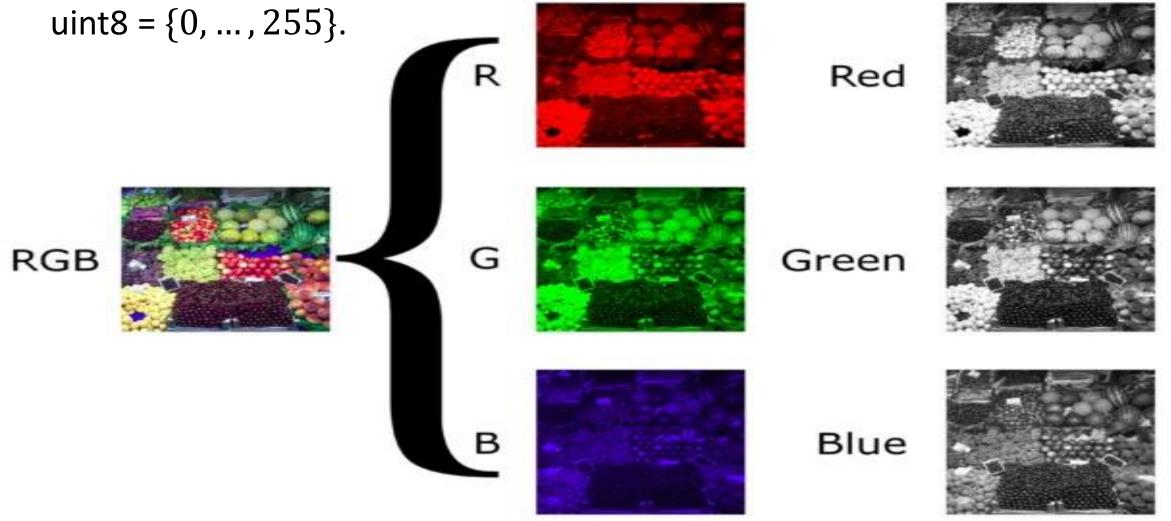
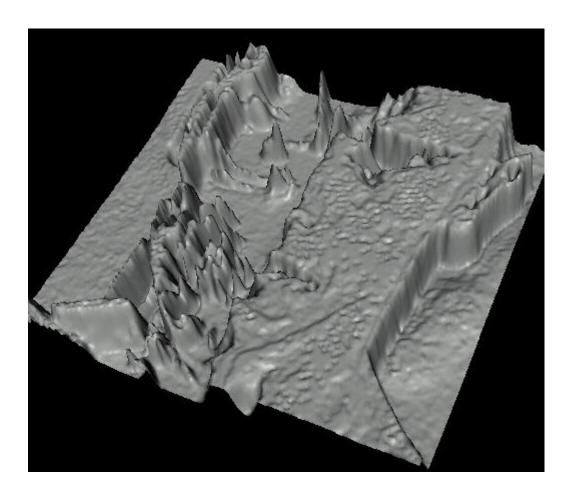


Image representation

• We can also think of an image as a function f(x, y).





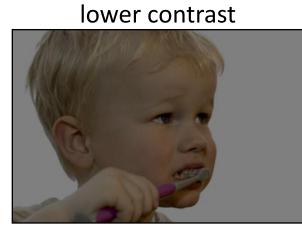
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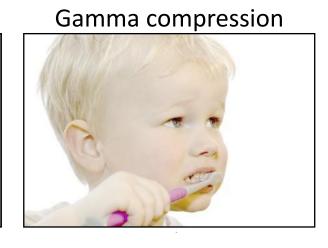
- Image representation
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• Pixel-wise operators, or point operators, are defined as such that each output pixel's value depends on only the corresponding input pixel value.

original

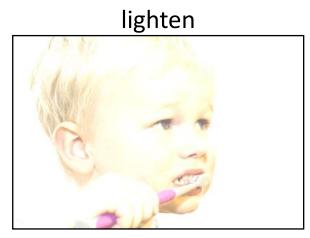


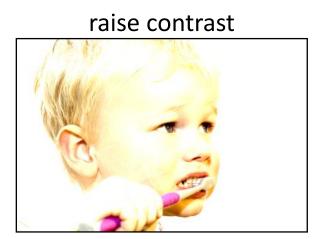




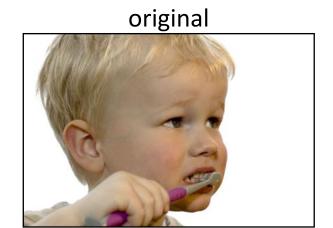
 \boldsymbol{x}

invert

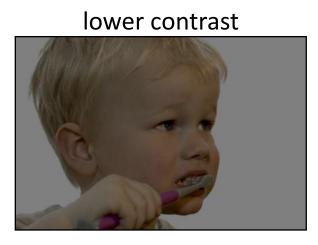






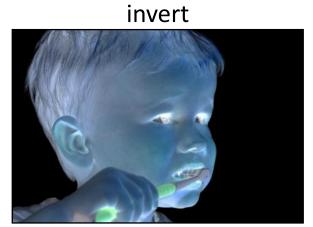


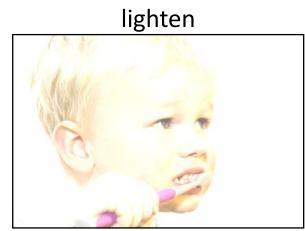


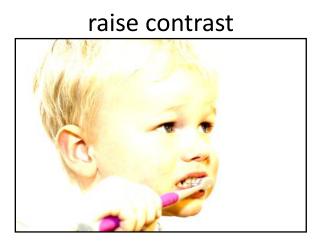




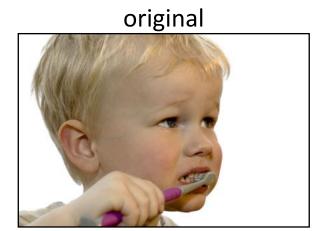
 \boldsymbol{x}



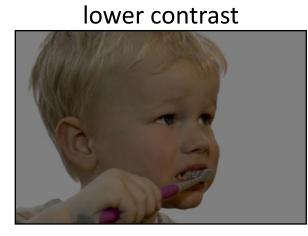


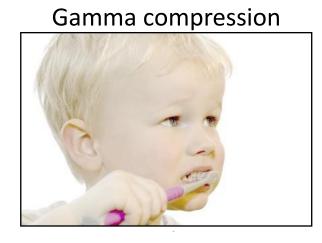






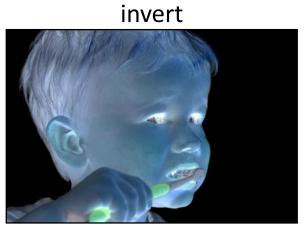


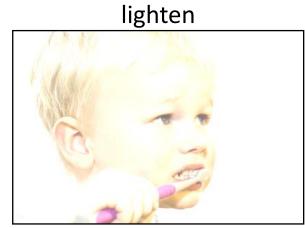


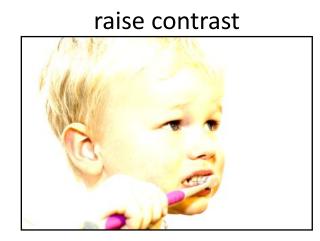


 \boldsymbol{x}

x - 128

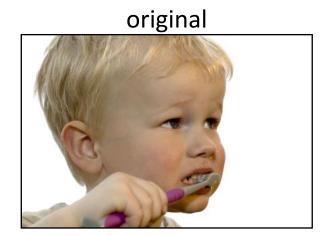




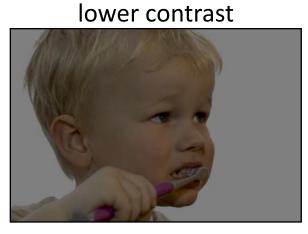




255 - x



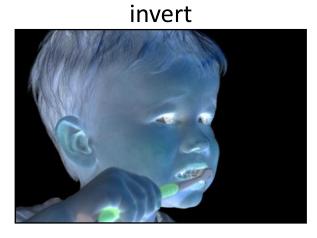


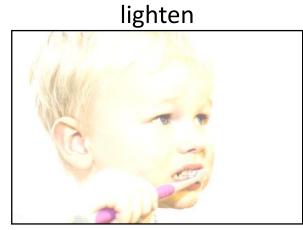


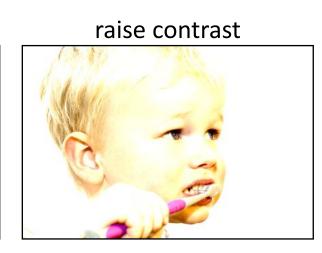


 \boldsymbol{x}

x - 128



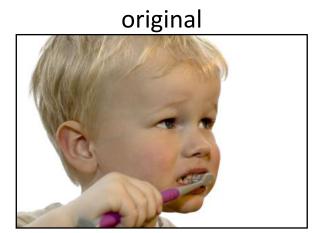




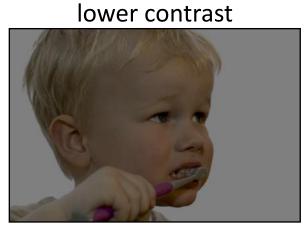


255 - x

x + 128









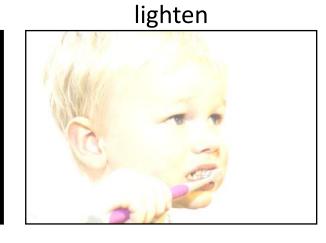
x

x - 128

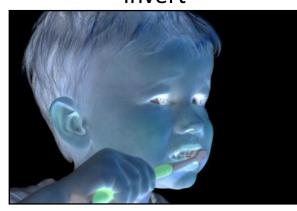
 $rac{x}{2}$



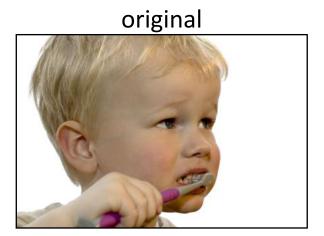
invert



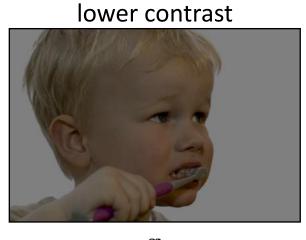


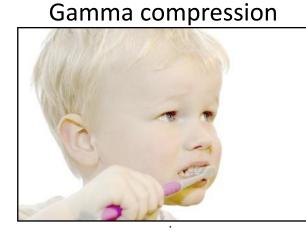


255 - x x + 128









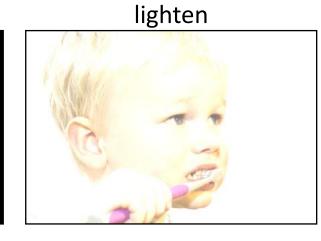
x

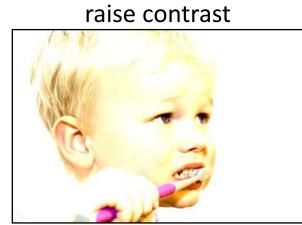
x - 128

 $rac{x}{2}$



invert





 $255 - x x + 128 x \times 2$

Contrast

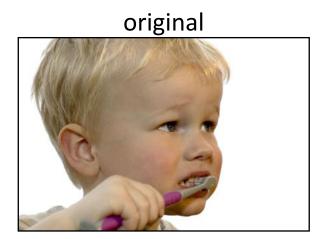
- Contrast in visual perception is the difference in appearance of two or more parts of a seen field.
- The human visual system is more sensitive to contrast than absolute luminance;
- Contrast ratio, or dynamic range, is the ratio between the largest and smallest values of the image or:

$$CR = \frac{V_{max}}{V_{min} + \epsilon}$$

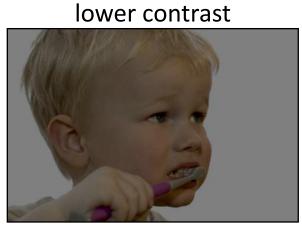


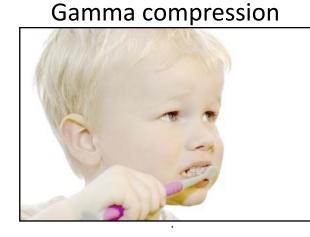
Contrast

- Example of calculating contrast ratio in determining website accessibility:
 - https://contrast-ratio.com/#%23000000-on-white
 - https://www.accessibility-developer-guide.com/knowledge/colours-andcontrast/how-to-calculate/









x

x - 128

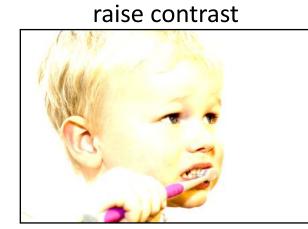
 $\frac{x}{2}$

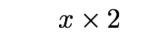


invert



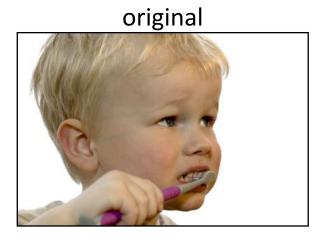




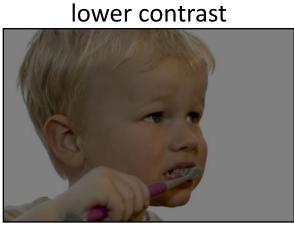


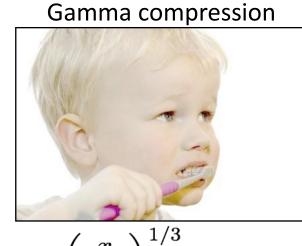
255 - x

x + 128









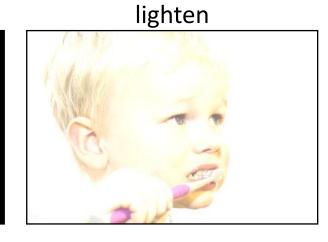
x

x-128

 $\frac{x}{2}$

 $\left(\frac{x}{255}\right)^{1/3} \times 255$





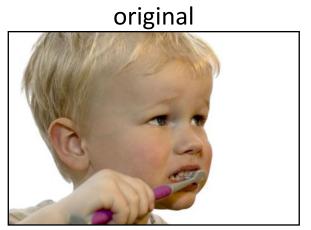




255 - x

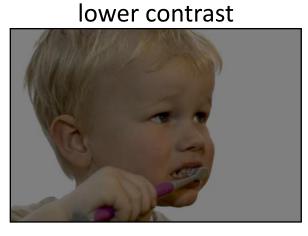
x + 128

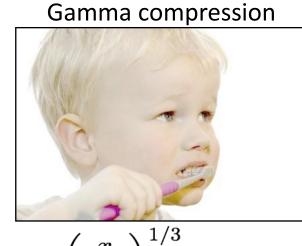
 $x \times 2$





x - 128

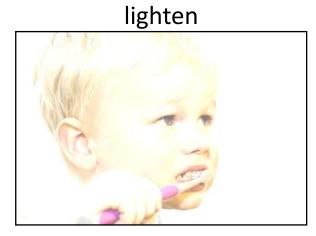


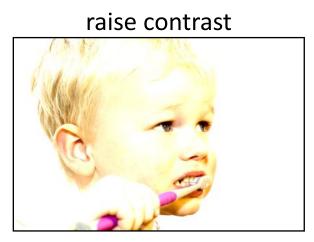


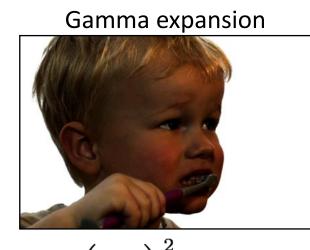
 $\times 255$

 $\times 255$









255 - x

x + 128

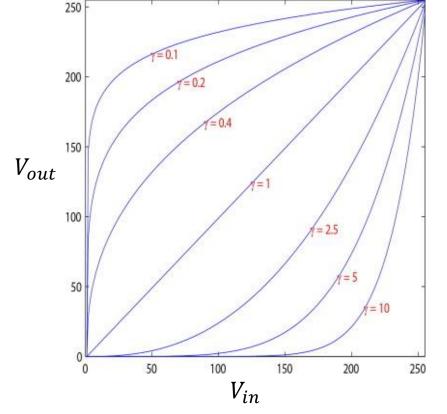
 $x \times 2$

Gamma correction

• To correct this non-linear transformation, gamma correction was done:

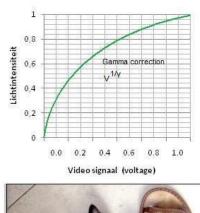
$$V_{out} = \left(\frac{V_{in}}{255}\right)^{\gamma} \cdot 255$$
 $(V_{in}, V_{out} \in \{0, 1, ..., 255\})$

• This is, of course, also applicable for image enhancements.

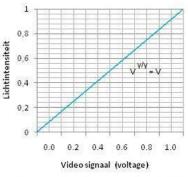








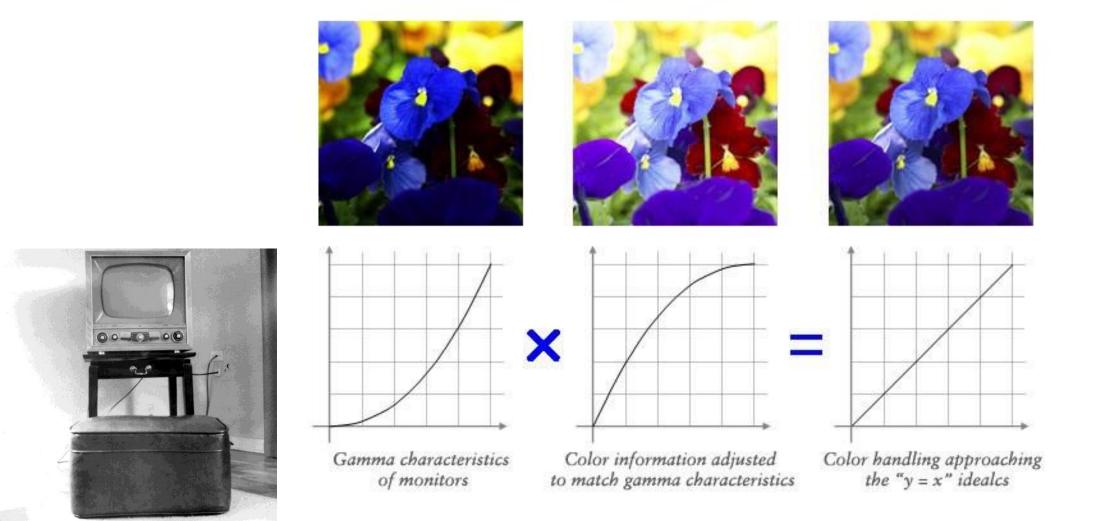






Gamma correction

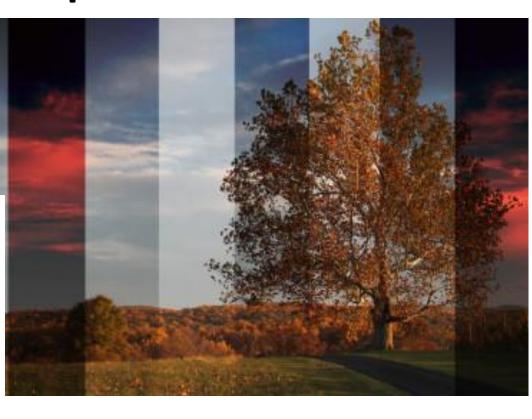
• Originally, Due to non-linearities in the old CRT televisions, intensities was seen different then they are.





Some more point- wise operators





contents

- Image representation
- Pixel-wise operations
- Histogram equalization
- Template matching
- Morphology operators
- Connected components
- Color space

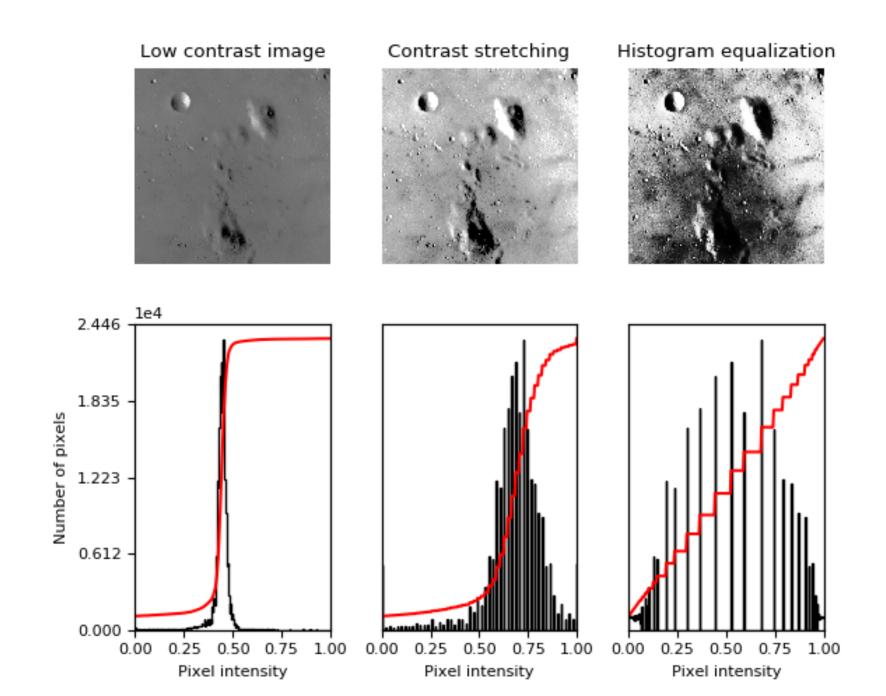
Histogram equalization

- **Histogram equalization** is a method in image processing of contrast adjustment using the image's histogram.
- This method is used to increase the global contrast of an image and is useful in images with backgrounds and foregrounds that are both bright or both dark.

Histogram equalization accomplishes this by effectively spreading out the

most frequent intensity values.

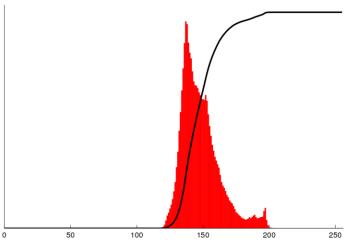




Histogram equalization

- A histogram is a discrete form representation of the distribution of numerical data.
- We will assume at first that our image is continues in the range [0,255] for better understanding.
- Instead of a histogram we will talk about the **probability density function (PDF)** $f_X(x)$ of the data.





Reminder: PDF and CDF

• cumulative distribution function (CDF) of a real-valued random variable X is the probability that X will take a value less than or equal to x:

$$F_X(x) = P(X \le x)$$

Properties of CDF:

$$-\lim_{x \to -\infty} F_X(x) = 0, \quad \lim_{x \to +\infty} F_X(x) = 1$$

- Monotonically non decreasing.
- The **probability density function (PDF)** of a continuous random variable can be determined from the cumulative distribution function by differentiating.

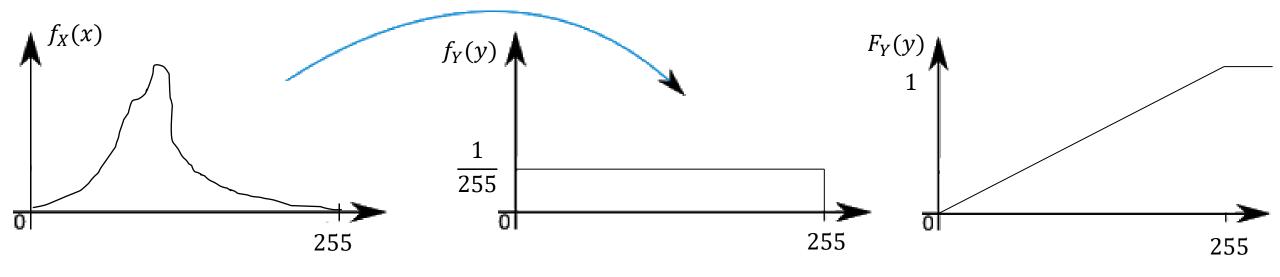
$$f_X(x) = \frac{dF_X(x)}{dx}$$
 OR $F_X(x) = \int_{-\infty}^x f_X(t) dt$

PDF definition- Wikipedia

- **Probability density function (PDF)** can be interpreted as providing a "relative likelihood" that the value of the random variable would equal that sample.
 - The absolute likelihood for a continuous random variable to take on any particular value is 0 (since there are an infinite set of possible values to begin with).
- In a more precise sense, the PDF is used to specify the probability of the random variable falling within a particular range of values. This probability is given by the integral of this variable's PDF over that range and is actually the **CDF**.

- We want that our resulting PDF $[f_Y(y)]$ will be constant for any value in the range [0,255].
- If the PDF is constant, that means that the CDF is linear in [0,255] (integration of constant is a linear function), and so we get the final CDF as:

$$F_Y(y) = P(Y \le y) = \begin{cases} 0 & : \ y < 0 \\ \frac{y}{255} & : \ 0 \le y \le 255 \\ 1 & : \ y > 255 \end{cases}$$



 So we are looking for a transformation function of the random variable X to Y such that:

$$Y = T(X)$$

$$P(Y \le y) = \frac{y}{255}$$

• So we are looking for a transformation function of the random variable X to Y such that:

$$Y = T(X)$$

$$P(Y \le y) = \frac{y}{255}$$
$$255 \cdot P(T(X) \le y) = y$$

 So we are looking for a transformation function of the random variable X to Y such that:

$$Y = T(X)$$

$$P(Y \le y) = \frac{y}{255}$$

$$255 \cdot P(T(X) \le y) = y$$

$$255 \cdot P(X \le T^{-1}(y)) = y \quad (assuming T is invertible)$$

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$$255 \cdot P(X \le T^{-1}(y)) = y \quad (assuming T is invertible)$$

$$255 \cdot P(X \le z) = T(z) \quad (change of variables z = T^{-1}(y))$$

 So we are looking for a transformation function of the random variable X to Y such that:

$$Y = T(X)$$

• In the interesting area [0,255]:

$$P(Y \le y) = \frac{y}{255}$$

$$255 \cdot P(T(X) \le y) = y$$

$$255 \cdot P(X \le T^{-1}(y)) = y \quad (assuming T is invertible)$$

$$255 \cdot P(X \le z) = T(z) \quad (change of variables z = T^{-1}(y))$$

$$T(x) = F_X(x) \cdot 255$$

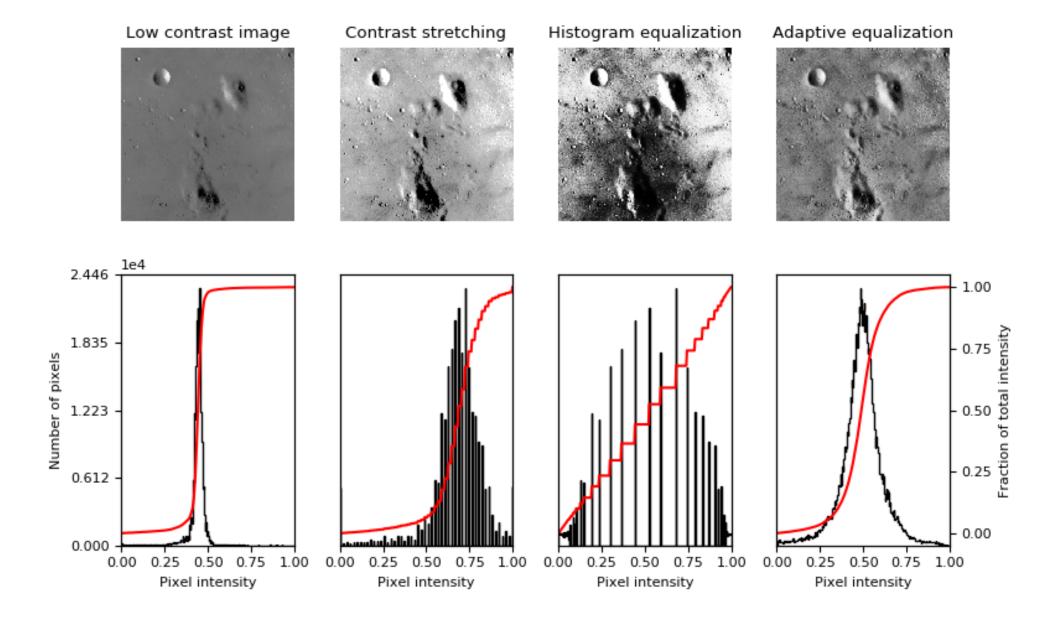
• In fact T is invertible since F_X is Monotonically non decreasing.

Back to histogram equalization

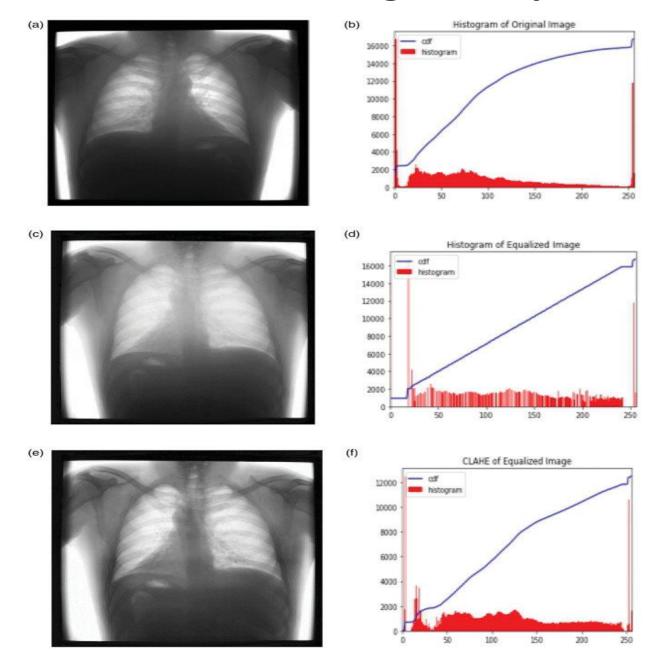
- The same result is also applicable for discrete space like actual images and their histograms.
 - Build a histogram of a given image.
 - To make the histogram act like a discrete PDF- divide each bin by the sum of all bins.
 - Cumulative sum the PDF to get the discrete CDF.
 - Un-normalize the CDF and round the results back to uint8:

$$f_{eq}(x) = round(CDF(x) \cdot 255)$$

Other variants of histogram equalization



Other variants of histogram equalization

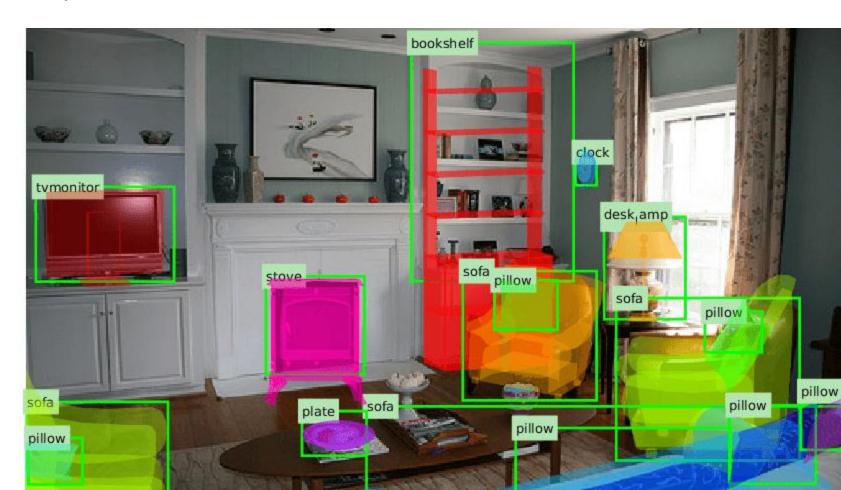


contents

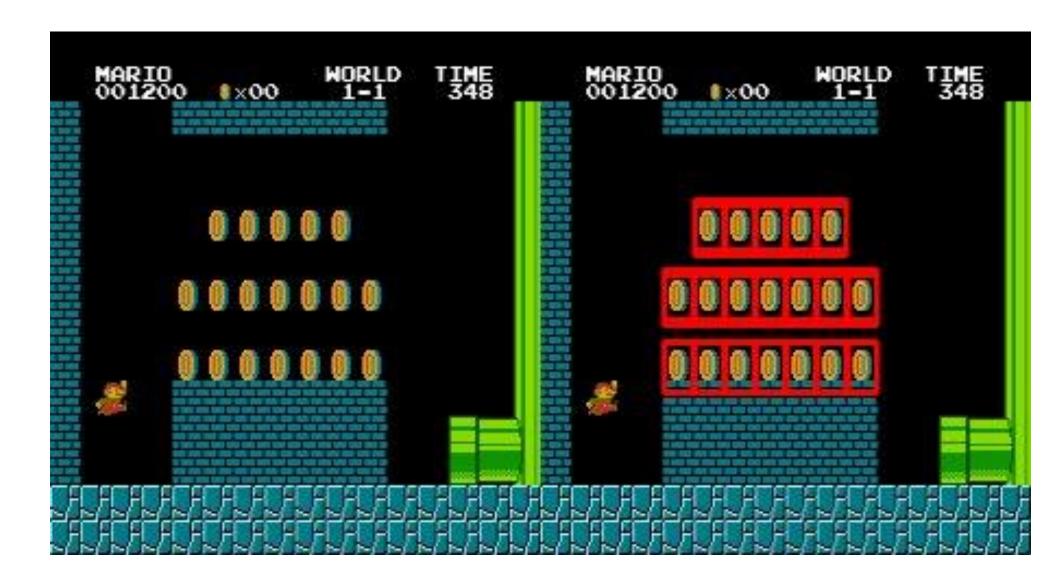
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Template matching

- Given an image template- find it in another image.
- Template matching is a sub-field in object recognition.
 - We will see it a lot of this topic in this course:
 - Cross correlation
 - Feature based SIFT
 - Neural networks



Example output





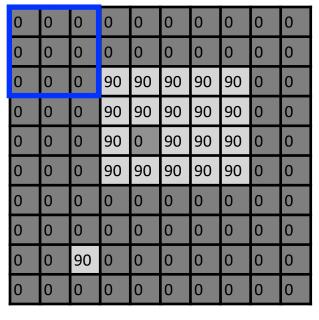
First-let's understand what is cross-correlation



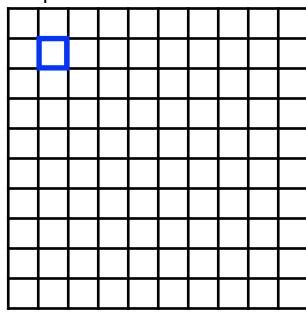
KCITICI							
1 9	$\frac{1}{9}$	1 9					
1	1	1					
9	9	9					
1	1	1					
9	9	9					

a

image



output



ţ



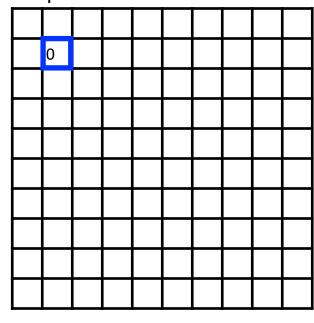
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$						
9	9	9						
1	1	1						
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$						
1	1	1						
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$						

a

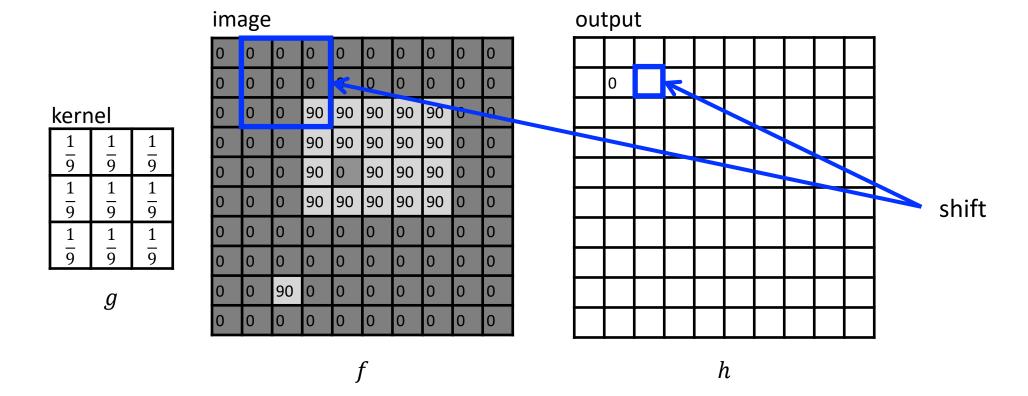
image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output



f



kernel

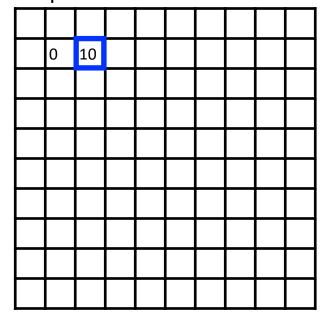
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
1 9	1 9	$\frac{1}{9}$
1 9	1 9	$\frac{1}{9}$

a

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output



f



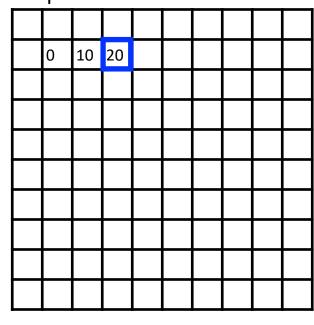
kernel								
$\frac{1}{9}$	1 9	$\frac{1}{9}$						
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$						
$\frac{1}{9}$	1 9	$\frac{1}{9}$						

g

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output



ţ



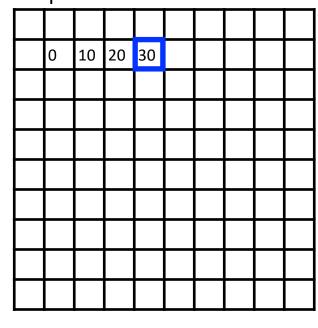
kernel								
1	1	1						
- 9	- 9	9						
1	1	1						
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$						
1	1	1						
- 9	- 9	9						

g

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output



f



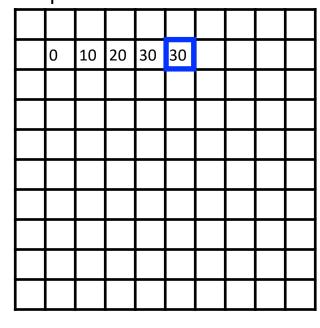
kernel								
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$						
1	1	1						
9	9	9						
$\frac{1}{9}$	1	1						
9	9	9						

g

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

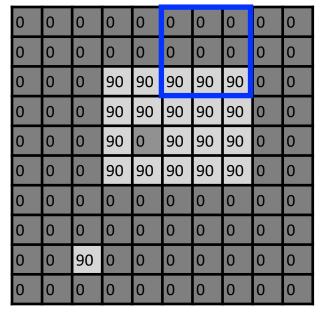


f

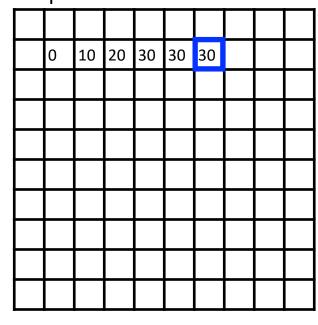
kernel								
1	1	1						
9	9	9						
1	1	1						
9	9	9						
1	1	1						
9	9	9						

g

image



output



f

... and the result is

kerr	kernel							
1 9	1 9	$\frac{1}{9}$						
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$						
1	1	1						
9	9	9						

a

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

output

0	10	20	30	30	30	20	10	
0	20	40	60	60	60	40	20	
0	30	50	80	80	90	60	30	
0	30	50	80	80	90	60	30	
0	20	30	50	50	60	40	20	
0	10	20	30	30	30	20	10	
10	10	10	10	0	0	0	0	
10	10	10	10	0	0	0	0	

h

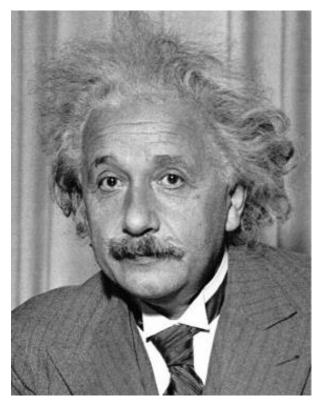
f

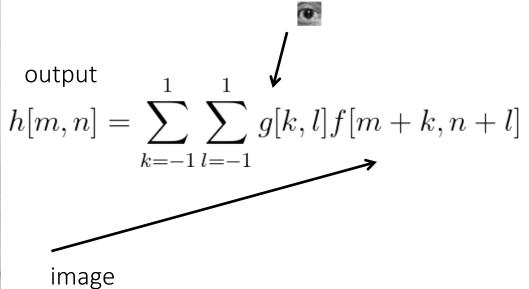
Cross correlation can also be more simply denoted as $h=g\star f$

The full mathematical notation is this:

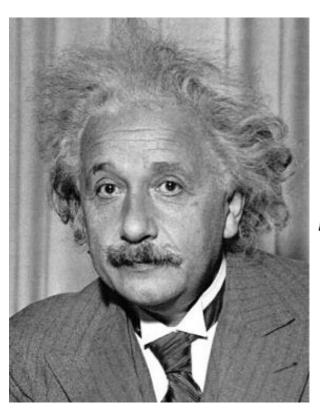
$$h[m,n] = \sum_{k=-1}^{1} \sum_{l=-1}^{1} g[k,l] f[m+k,n+l]$$

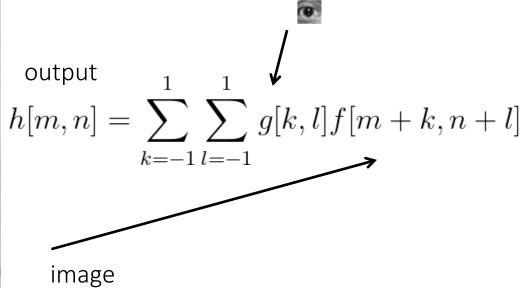
Can cross-correlation be good for template matching? Let's take our template that we want to find as our kernel



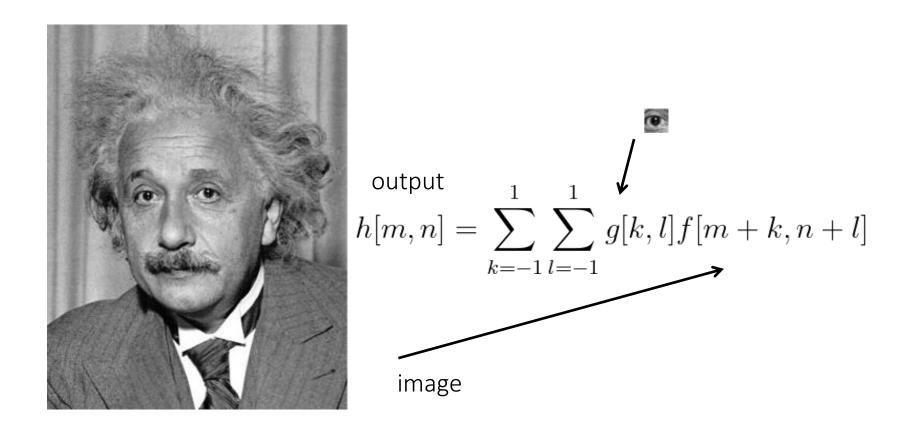


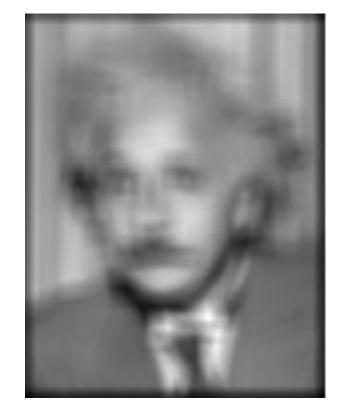
What will the output look like?



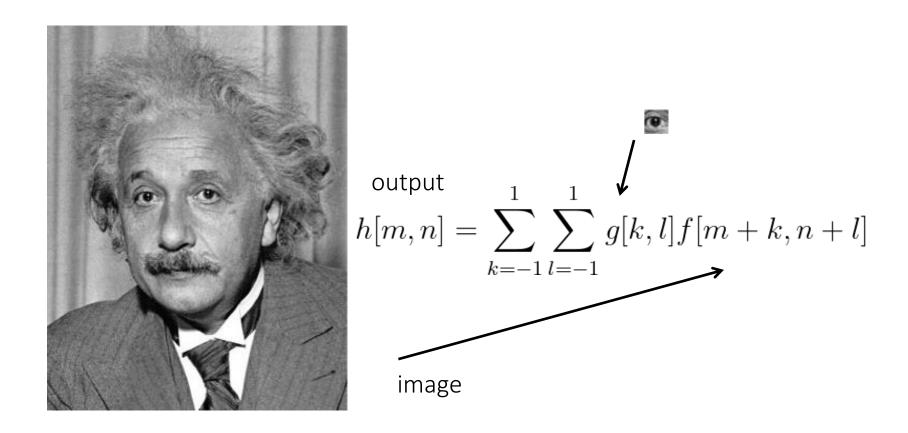


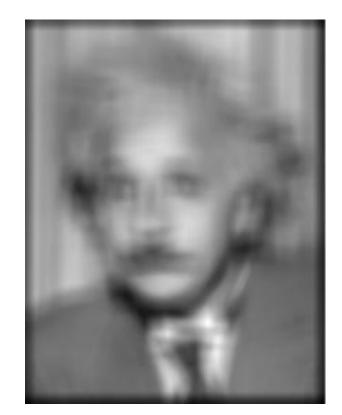
What will the output look like?





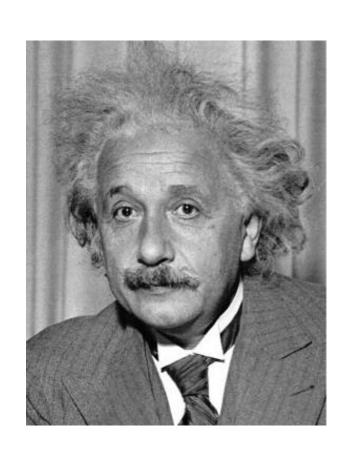
Is this good for template matching?





Increases for higher local intensities.

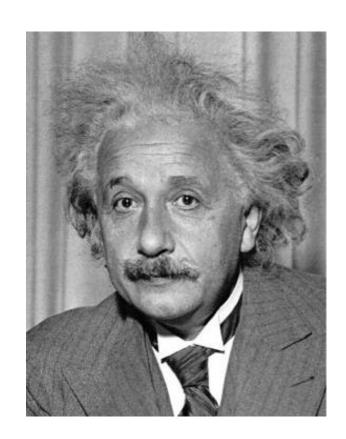
Zero mean cross correlation



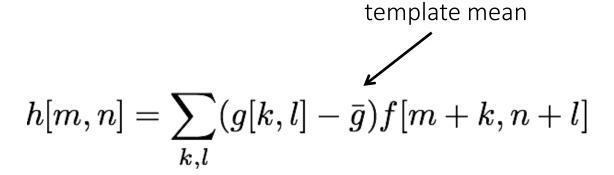
template mean
$$h[m,n] = \sum_{k,l} (g[k,l] - \bar{g}) f[m+k,n+l]$$

What will the output look like?

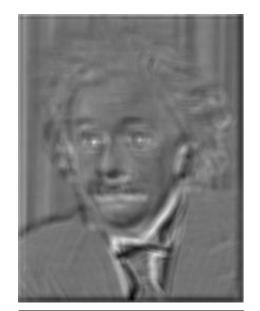
Zero mean cross correlation

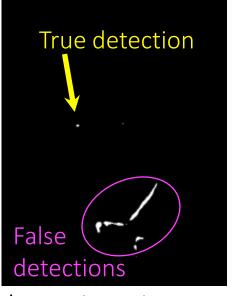


output



thresholding





Zero mean CC is good enough for most problems but can also cause false detections in high contrast areas.

CC

	255	255	255		255	255	255	
	255	0	255	*	255	255	255	→
	255	255	255		255	255	255	
•		g		'		f		•

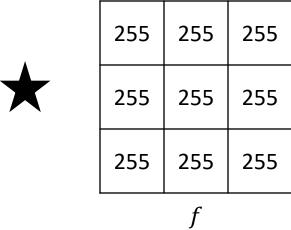
255 ²	255 ²	255 ²		
255 ²	0	255 ²	 8 * 255 ²	
255 ²	255 ²	255 ²	h	

1. Scalar multiplication

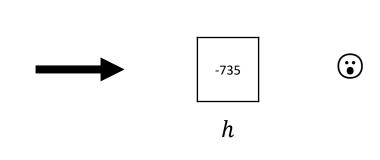
2. Summation

Zero mean CC

28	28	28				
28	-227	28				
28	28	28				
$g-ar{g}$						



7140	7140	7140
7140	-57855	7140
7140	7140	7140

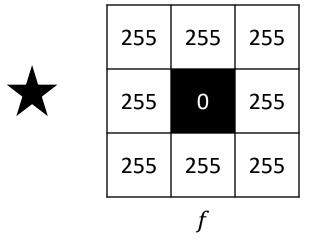


1. Scalar multiplication

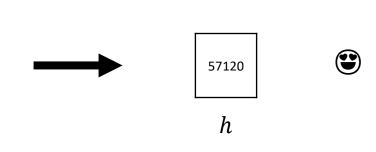
2. Summation

Zero mean CC

28	28	28				
28	-227	28				
28	28	28				
$g - \bar{g}$						



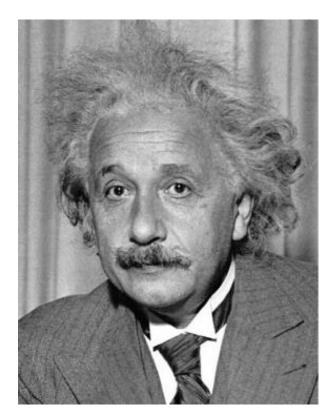
7140	7140	7140
7140	0	7140
7140	7140	7140



1. Scalar multiplication

2. Summation

ZNCC - zero mean normalized cross correlation

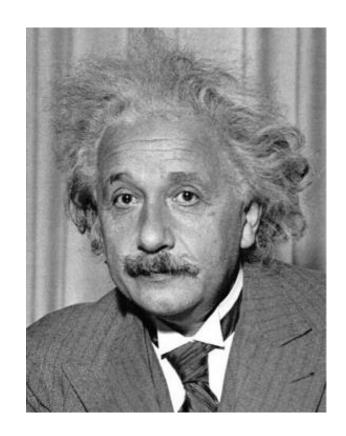


What will the output look like?

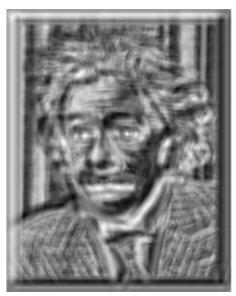
$$h[m,n] = \frac{\sum_{k,l} (g[k,l] - \bar{g})(f[m+k,n+l] - \bar{f}_{m,n})}{\sqrt{(\sum_{k,l} (g[k,l] - \bar{g})^2 \sum_{k,l} (f[m+k,n+l] - \bar{f}_{m,n})^2)}}$$

The below square root is the product of both template and patch STD.

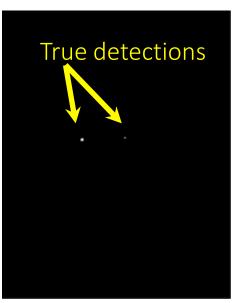
ZNCC – zero mean normalized cross correlation



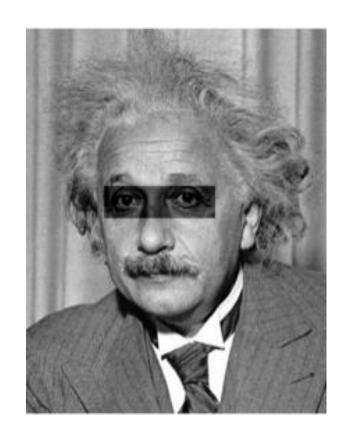
output



thresholding



ZNCC – zero mean normalized cross correlation

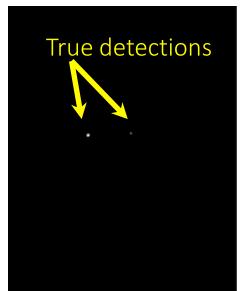


output



thresholding





contents

- Image representation
- Pixel-wise operations
- Histogram equalization
- Template matching
- Morphology operators
- Connected components
- Color space

Morphology

Examples:

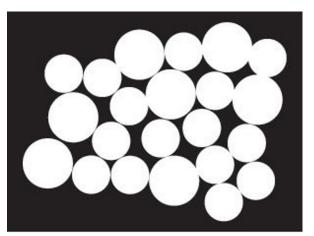
Image cleaning

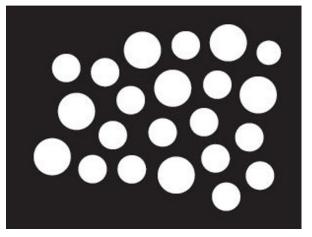
Style

Coin counting (using connected components)









The 4 basic operators

Dilate Erode

Open (Erode ⇒ Dilate)

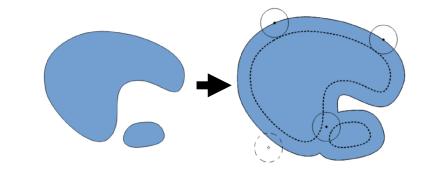


Close (Dilate ⇒ Erode)

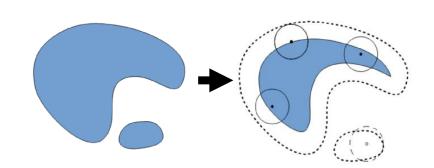


Morphology: geometric interpretation

- Each kernel (g) has an anchor point (usually in the kernel center).
- <u>Dilation</u>: the final shape is all points where the anchor point can be placed in which **the kernel touches a part of the original shape**.

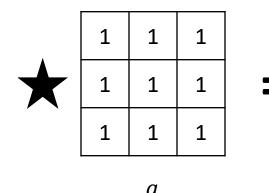


 <u>Erosion</u>: the final shape is all points where the anchor point can be placed in which all kernel points touch the original shape.



1. Cross-correlation with the kernel

0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0
0	1	1	1	1	0	0	0
0	1	1	1	1	1	1	0
0	1	1	1	1	0	0	0
0	1	1	1	1	0	0	0
0	1	1	0	0	0	0	0
0	1	1	0	0	0	0	0



2	თ	თ	2	2	1	
4	6	6	5	5	3	
6	9	9	7	5	2	
6	9	9	7	5	2	
6	8	7	4	2	0	
6	7	5	2	1	0	

f

2. Threshold the result

For dilation- threshold with 1

2	3	3	2	2	1	
4	6	6	5	5	3	
6	9	9	7	5	2	
6	9	9	7	5	2	
6	8	7	4	2	0	
6	7	5	2	1	0	

$$\geq 1 =$$

1	1	1	1	1	1	
1	1	1	1	1	1	
1	1	1	1	1	1	
1	1	1	1	1	1	
1	1	1	1	1	0	
1	1	1	1	1	0	

2. Threshold the result

For **erosion-** threshold with the sum of the kernel

2	3	3	2	2	1	
4	6	6	5	5	3	
6	9	9	7	5	2	
6	9	9	7	5	2	
6	8	7	4	2	0	
6	7	5	2	1	0	

$$\geq sum(g) =$$

0	0	0	0	0	0	
0	0	0	0	0	0	
0	1	1	0	0	0	
0	1	1	0	0	0	
0	0	0	0	0	0	
0	0	0	0	0	0	

Morphology: algorithm

- Each morphology operator is constructed as such:
 - 1. Select a structure element (binary kernel)
 - 2. Cross-correlate with input binary image $h = f \star g$
 - 3. Threshold the output

$$\theta_{TH}(x,t) = \begin{cases} 1 & if & x \ge t, \\ 0 & else \end{cases}$$

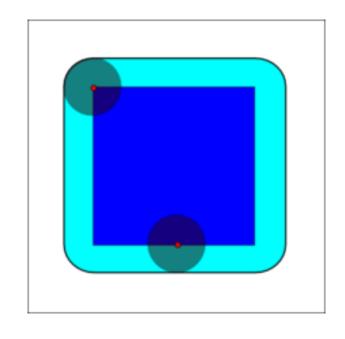
Overall morphologic operation should look like so:

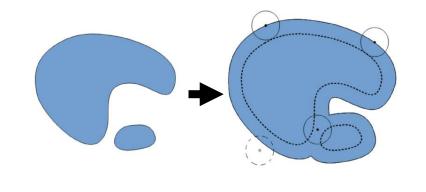
$$k = \theta_{TH}(f \star g, t)$$

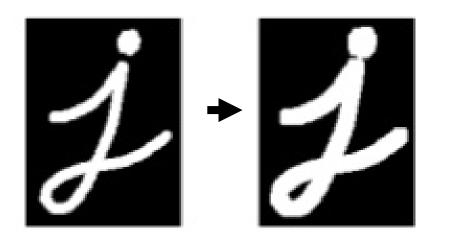
$$g = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Dilation- examples

•
$$k = \theta_{TH}(f \star g, t = 1)$$

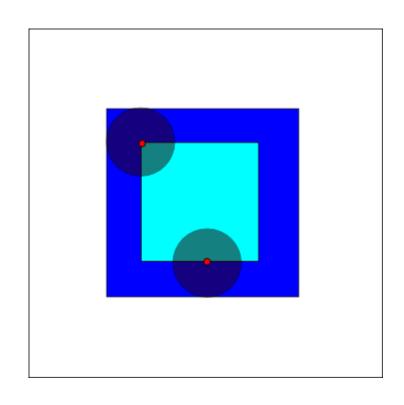


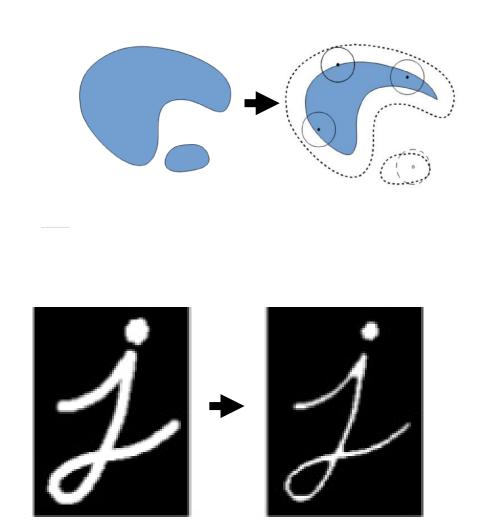




Erosion- examples

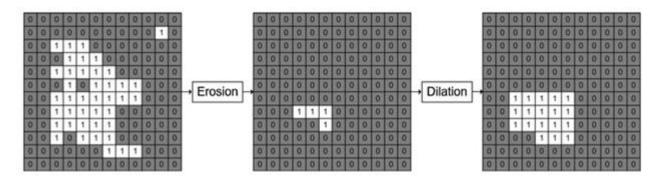
• $k = \theta_{TH}(f \star g, t = sum(g))$





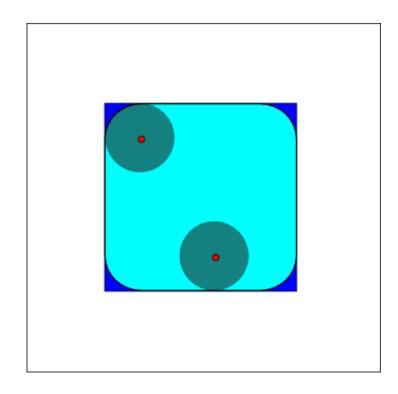
Opening

- Erosion followed by dilation.
 - The effect is of removing noise or sharp edges.



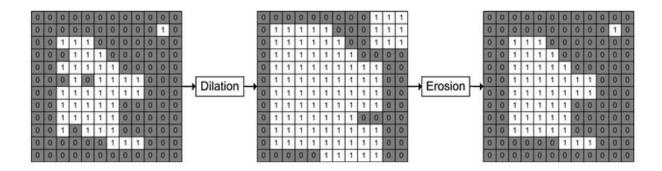
Open (Erode ⇒ Dilate)





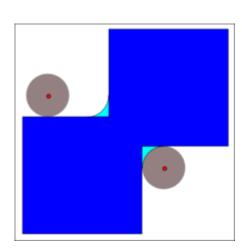
Closing

- Dilation followed by erosion.
 - The effect is of closing of narrow gaps and holes.

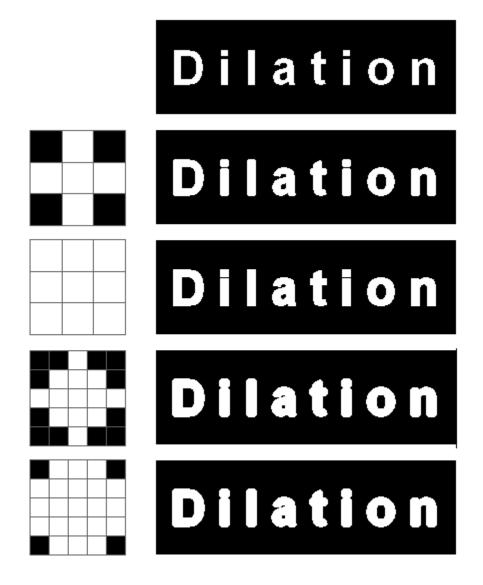


Close (Dilate ⇒ Erode)





Affect of different kernels



We will see non-symmetrical kernels in the HW

contents

- Image representation
- Pixel-wise operations
- Histogram equalization
- Template matching
- Morphology operators
- Connected components
- Color space

Connected components

- Defined as regions of adjacent pixels that have the same value.
- Commonly used with binary images to find stand alone objects.
 - e.g.: letters in a document.

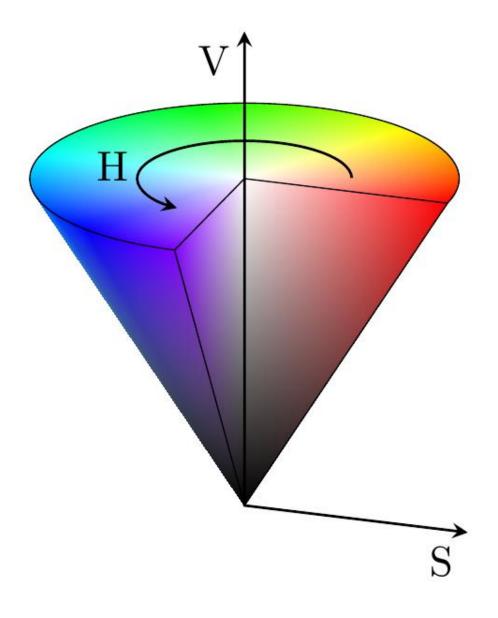




contents

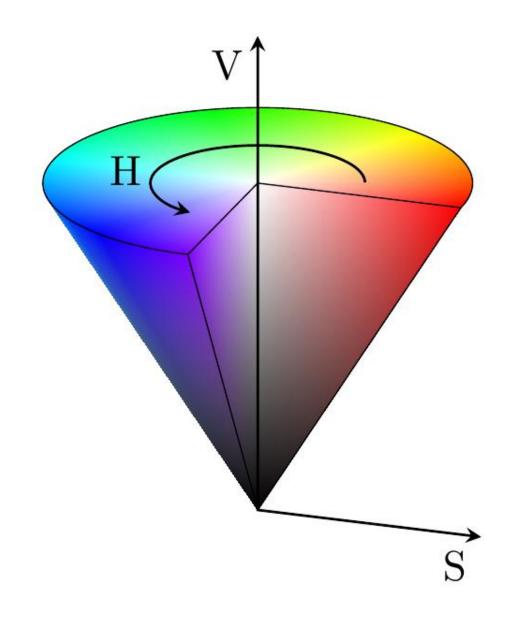
- Image representation
- Pixel-wise operations
- Histogram equalization
- Template matching
- Morphology operators
- Connected components
- Color space





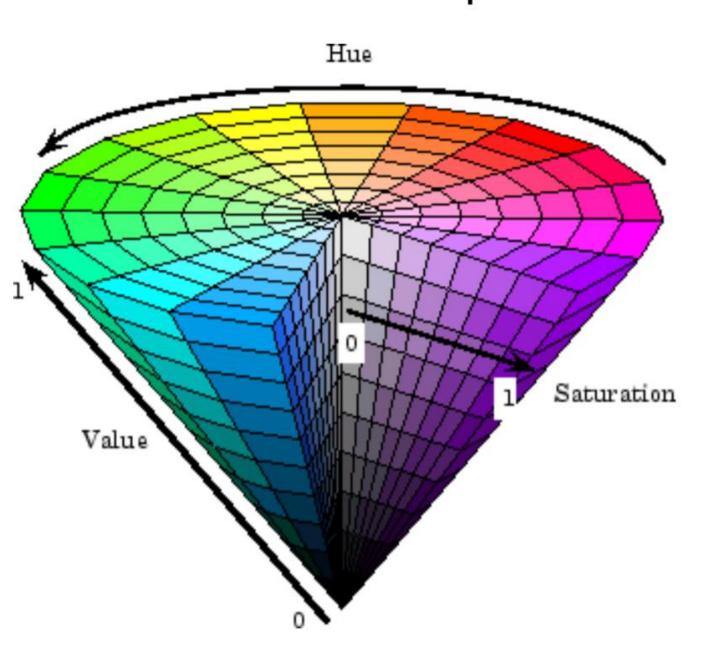
HSV

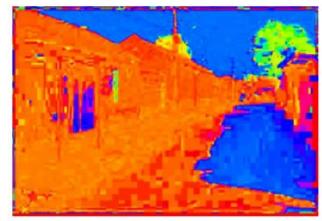
- Hue: The "attribute of a visual sensation according to which an area appears to be similar to one of the perceived colors: red, yellow, green, and blue, or to a combination of two of them"
- **Saturation**: The "colorfulness of a stimulus relative to its own brightness"
- Value: The "brightness relative to the brightness of a similarly illuminated white". Can also be called brightness or intensity.
 - [Wikipedia]





Original image





H (S=1,V=1)



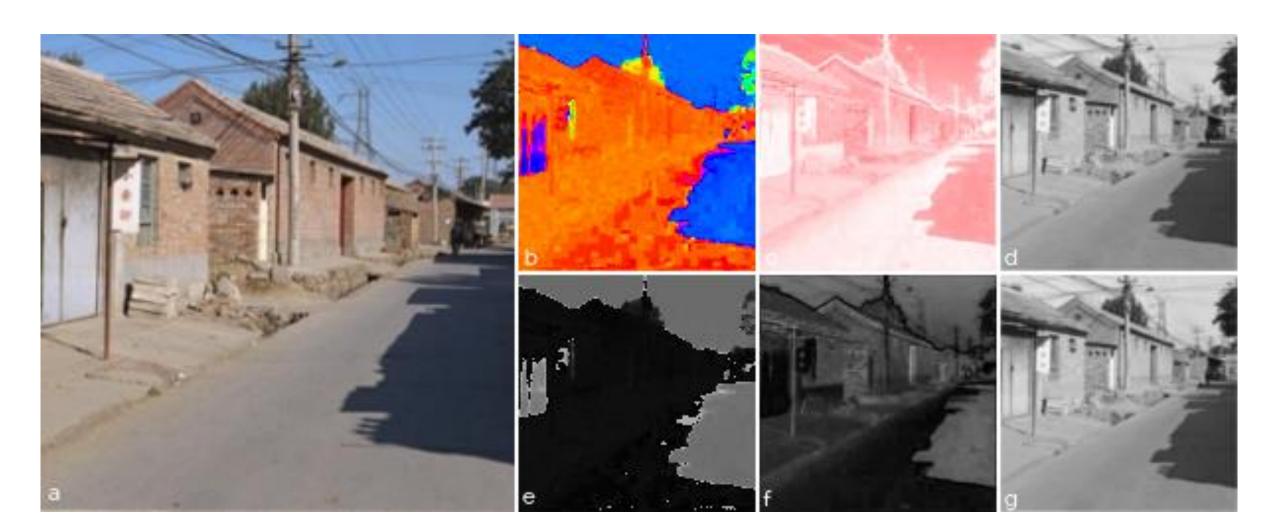
S (H=1,V=1)



V (H=1,S=0)

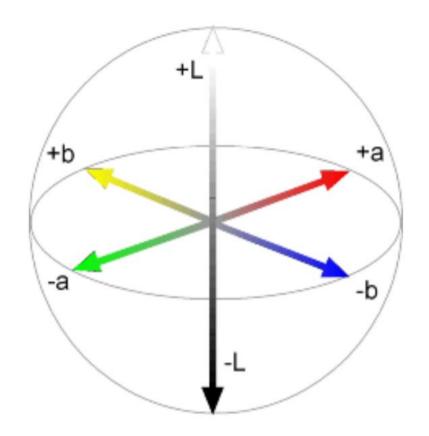
HSV

- In e, f, g: single channel image representation.
- Conclusion: people are much more responsive to intensity then chroma.



More color spaces: LAB

- L: lightness from black (0) to white (100).
- A: from green (-) to red (+).
- B: from blue (-) to yellow (+).





(a=0,b=0)



a (L=65,b=0)



b (L=65,a=0)

More color spaces: YUV

- Y: brightness/ intensity.
- U: blue projection.
- V: red projection.
- [Similar to YCbCr]

