577- Time Series

Final Project - Version 2

Evaluating Car Data

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Load the data and look at it

In [23]:
```
source('https://nmimoto.github.io/R/TS-00.txt')
```

In [24]:
```
D <- read.csv("https://nmimoto.github.io/datasets/car.csv", header=T)
D
```

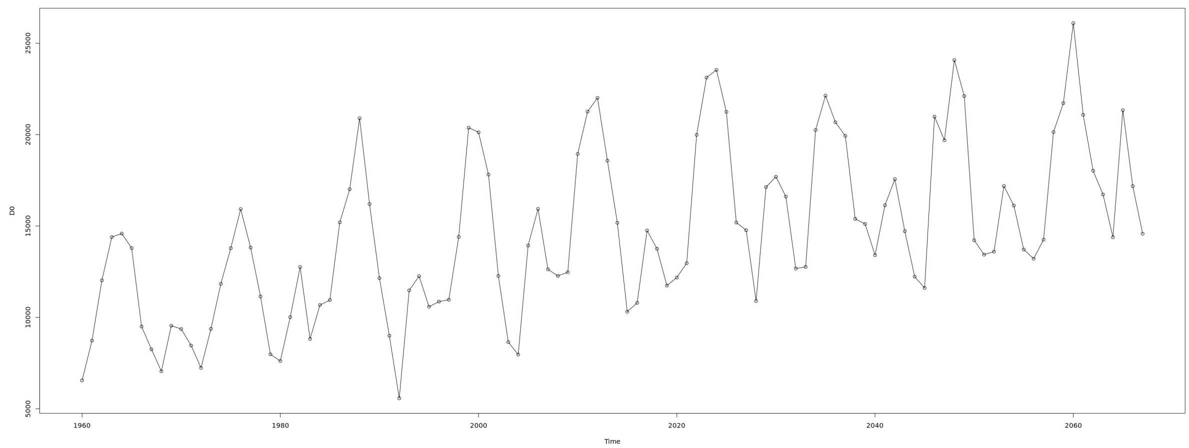
A data.frame: 108 × 2

Month	Monthly.car.sales.in.Quebec.1960.1968
<fct>	<int>
1960-01	6550
1960-02	8728
1960-03	12026
1960-04	14395
1960-05	14587
1960-06	13791
1960-07	9498
1960-08	8251
1960-09	7049
1960-10	9545
1960-11	9364
1960-12	8456
1961-01	7237
1961-02	9374
1961-03	11837
1961-04	13784
1961-05	15926
1961-06	13821
1961-07	11143
1961-08	7975
1961-09	7610
1961-10	10015
1961-11	12759
1961-12	8816
1962-01	10677
1962-02	10947
1962-03	15200
1962-04	17010
1962-05	20900
1962-06	16205
⋮	⋮
1966-07	15388
1966-08	15113
1966-09	13401
1966-10	16135
1966-11	17562
1966-12	14720
1967-01	12225

First try it without using seasonal

In [4]:
```
D0 <- ts(D[,2], start=c(1960,1), freq=1)
```

In [5]:
```
options(repr.plot.width=30, repr.plot.height=12)
plot(D0, type='o')
```



Looking at the graph, it appears that it is not stationary, but instead is gradually increasing. It also appears that it may have a seasonal component.

In [6]:
```
Fit0 <- auto.arima(D0, stepwise = FALSE, approximation = FALSE)
Fit0
```

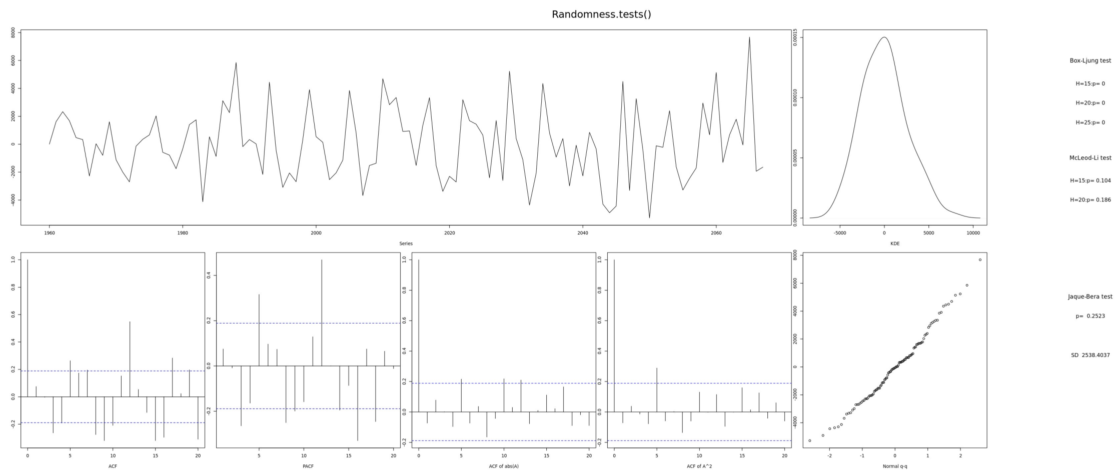
```
Series: D0
ARIMA(2,1,2) with drift

Coefficients:
         ar1      ar2      ma1     ma2    drift
      1.4951  -0.7798  -1.8793  0.9149  88.4266
s.e.  0.0882   0.0660   0.1211  0.1354  34.3670

sigma^2 estimated as 6759362:  log likelihood=-992.94
AIC=1997.88   AICc=1998.72   BIC=2013.92
```

In [7]:
```
Randomness.tests(Fit0$residuals)
```

```
  B-L test H0: the series is uncorrelated
  M-L test H0: the square of the series is uncorrelated
  J-B test H0: the series came from Normal distribution
  SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
0	0	0	0.104	0.186	0.252	2538.404



ACF and PACF graphs are showing a lot of correlation at higher lags, especially lag 12, indicating the data is seasonal. The Box-Ljung numbers are all 0 confirming that there is still a lot of correlation. Jaque-Bera number is high, indicating the errors follow a normal distribution.

Try with lambda = 0

In [8]:
```
Fit0 <- auto.arima(D0, lambda = 0, stepwise = FALSE, approximation = FALSE)
Fit0
```

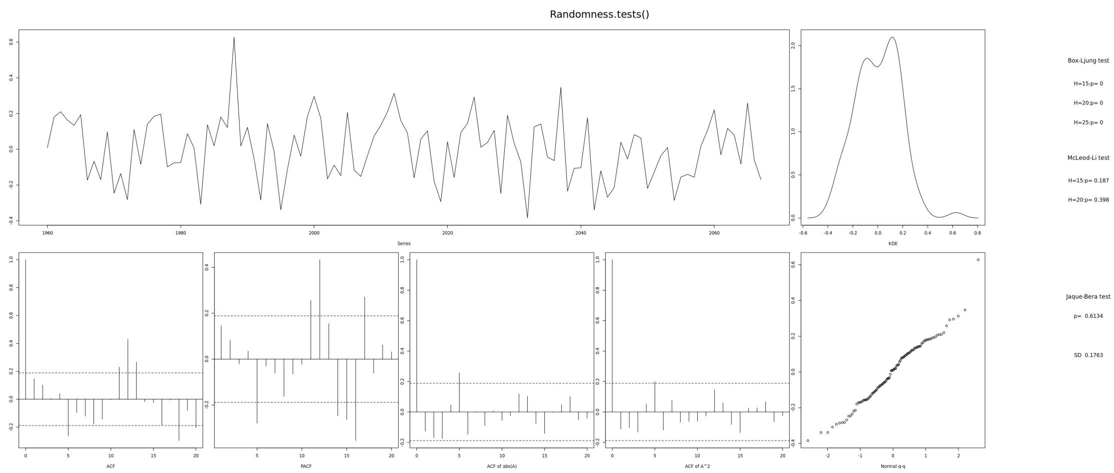
```
Series: D0
ARIMA(0,1,5) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ma1      ma2      ma3      ma4     ma5   drift
      -0.3587  -0.4776  -0.4709  -0.4136  0.8225  0.0059
s.e.   0.0715   0.0757   0.0745   0.0791  0.0926  0.0018

sigma^2 estimated as 0.03294:  log likelihood=29.89
AIC=-45.77   AICc=-44.64   BIC=-27.06
```

In [9]:
```
Randomness.tests(Fit0$residuals)
```

```
B-L test H0: the series is uncorrelated
M-L test H0: the square of the series is uncorrelated
J-B test H0: the series came from Normal distribution
SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
0	0	0	0.187	0.398	0.613	0.176



AICc is alot better. Still have issues with residuals

Try forcing auto.arima() to look at higher p and q values

In [10]:
```
Fit00 <- auto.arima(D0, stepwise = FALSE, lambda=0, approximation = FALSE, max.p =
15, max.q = 15)
Fit00
```

```
Series: D0
ARIMA(0,1,5) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ma1      ma2      ma3      ma4     ma5   drift
      -0.3587  -0.4776  -0.4709  -0.4136  0.8225  0.0059
s.e.   0.0715   0.0757   0.0745   0.0791  0.0926  0.0018

sigma^2 estimated as 0.03294:  log likelihood=29.89
AIC=-45.77   AICc=-44.64   BIC=-27.06
```

It returned the same model

Try manually forcing higher p and q values

In [11]:
```
Fit01 <- Arima(D0, lambda=0, order=c(15,1,15), include.drift = TRUE)
Fit01
```

```
Warning message in sqrt(diag(x$var.coef)):
“NaNs produced”

Series: D0
ARIMA(15,1,15) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4     ar5      ar6      ar7      ar8
       -1.127  -0.1508  -0.1006  -0.2970  0.0219  -0.1305  -0.2531  -0.0498
s.e.    0.391   0.1075   0.3149   0.2128  0.2737   0.2433   0.2112   0.2619
           ar9     ar10     ar11    ar12    ar13    ar14    ar15     ma1      ma2
       -0.0926  -0.3050  0.0106  0.8427  0.8700  0.1074  0.0086  0.6369  -0.6977
s.e.    0.2299   0.2115  0.2691  0.2383  0.3574     NaN  0.1358  0.3914      NaN
           ma3  ma4      ma5     ma6     ma7      ma8     ma9    ma10     ma11
       -0.3320    0  -0.3181  0.1836  0.0311  -0.5491  0.1702  0.4052  -0.1131
s.e.    0.4992  NaN   0.2658  0.1691  0.2095   0.1996  0.3231  0.2045   0.3032
          ma12     ma13    ma14    ma15   drift
       -0.4818  -0.3539  0.2098  0.2135  0.0062
s.e.    0.2981   0.3603  0.2615  0.2687  0.0008

sigma^2 estimated as 0.01033:  log likelihood=95.3
AIC=-126.59   AICc=-98.05   BIC=-41.06
```

ar15, ar14 are not significant, remove them.

In [12]:
```
Fit01 <- Arima(D0, lambda=0, order=c(13,1,15), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(13,1,15) with drift
Box Cox transformation: lambda= 0

Coefficients:
           ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
       -1.2240  -0.3239  -0.2372  -0.5087  -0.2864  -0.2839  -0.4346  -0.3613
s.e.    0.2637   0.4983   0.5226   0.5353   0.5464   0.5151   0.5100   0.5304
           ar9     ar10     ar11    ar12    ar13     ma1      ma2      ma3
       -0.2575  -0.4923  -0.2821  0.6637  0.7352  0.7044  -0.6039  -0.2365
s.e.    0.5531   0.5020   0.5111  0.5612  0.3127  0.3091   0.4169   0.3003
          ma4      ma5      ma6      ma7      ma8      ma9    ma10    ma11
       0.1856  -0.1224  -0.0747  -0.0323  -0.2463  -0.0771  0.2757  0.0541
s.e.   0.2544   0.2289   0.2299   0.1837   0.2648   0.2002  0.3476  0.2331
          ma12     ma13     ma14     ma15   drift
       -0.3877  -0.2719  -0.0098  -0.1293  0.0062
s.e.    0.4248   0.1883   0.5550   0.2650  0.0007

sigma^2 estimated as 0.01088:  log likelihood=94.55
AIC=-129.1   AICc=-104.62   BIC=-48.91
```

ma15, ma14 are not significant, remove them.

In [13]:
```
Fit01 <- Arima(D0, lambda=0, order=c(13,1,13), include.drift = TRUE)
Fit01
```

```
Warning message in sqrt(diag(x$var.coef)):
“NaNs produced”

Series: D0
ARIMA(13,1,13) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1     ar2      ar3      ar4     ar5      ar6      ar7     ar8
      -0.9127  0.0142  -0.0189  -0.2999  0.0917  -0.0738  -0.2161  0.0038
s.e.      NaN     NaN      NaN      NaN     NaN      NaN      NaN     NaN
          ar9     ar10     ar11     ar12    ar13     ma1      ma2      ma3
      -0.0109  -0.2883  0.0786  0.8772  0.6485  0.3479  -0.7434  -0.1853
s.e.      NaN      NaN     NaN     NaN     NaN     NaN   0.1781      NaN
         ma4      ma5     ma6     ma7      ma8     ma9    ma10     ma11     ma12
      0.1324  -0.3867  0.0572  0.0248  -0.4403  0.0965  0.3631  -0.054  -0.3088
s.e.  0.1246      NaN     NaN  0.1230      NaN     NaN     NaN     NaN      NaN
        ma13   drift
      0.1048  0.0062
s.e.     NaN  0.0008

sigma^2 estimated as 0.01153:  log likelihood=91.03
AIC=-126.06   AICc=-105.24   BIC=-51.22
```

Remove ar13 and ma13

In [14]:
```
Fit01 <- Arima(D0, lambda=0, order=c(12,1,12), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(12,1,12) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4     ar5      ar6      ar7      ar8
      -0.1082  -0.4564  -0.0889  -0.6123  0.0514  -0.5872  -0.1176  -0.4617
s.e.   0.4756   0.1774   0.4175   0.1630  0.4711   0.1642   0.4605   0.1766
          ar9     ar10    ar11    ar12      ma1     ma2      ma3     ma4
      -0.0692  -0.6436  0.0704  0.3780  -0.5221  0.3522  -0.4283  0.5150
s.e.   0.4176   0.1585  0.4828  0.1642   0.4758  0.3169   0.3777  0.3895
          ma5     ma6      ma7     ma8      ma9    ma10     ma11     ma12
      -0.4940  0.6006  -0.5540  0.3149  -0.3604  0.7258  -0.5763  -0.0470
s.e.   0.4486  0.3288   0.3484  0.3155   0.3142  0.3734   0.4723   0.3371
       drift
      0.0062
s.e.  0.0014

sigma^2 estimated as 0.0113:  log likelihood=89.25
AIC=-126.5   AICc=-108.95   BIC=-57.01
```

Remove ma12

In [15]:
```
Fit01 <- Arima(D0, lambda=0, order=c(12,1,11), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(12,1,11) with drift
Box Cox transformation: lambda= 0

Coefficients:
         ar1      ar2      ar3      ar4      ar5      ar6      ar7     ar8
      0.0225  -0.2200  -0.2527  -0.2424  -0.0735  -0.2600  -0.3166  0.0060
s.e.  0.3659   0.2316   0.2555   0.2001   0.2265   0.1829   0.2173  0.2301
          ar9     ar10    ar11    ar12      ma1     ma2      ma3      ma4
      -0.2928  -0.3876  0.2069  0.3602  -0.6953  0.2317  -0.0565  -0.0205
s.e.   0.1736   0.2259  0.2469  0.1829   0.3708  0.3438   0.2676   0.1606
         ma5      ma6     ma7      ma8     ma9     ma10     ma11   drift
      0.0076  -0.0314  0.2380  -0.7293  0.7762  -0.1416  -0.2437  0.0061
s.e.  0.1314   0.0999  0.1039   0.1405  0.2824   0.3487   0.2142  0.0014

sigma^2 estimated as 0.01193:  log likelihood=86.27
AIC=-122.55   AICc=-106.5   BIC=-55.73
```

Remove ma11, ma10

In [16]:
```
Fit01 <- Arima(D0, lambda=0, order=c(12,1,9), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(12,1,9) with drift
Box Cox transformation: lambda= 0

Coefficients:
         ar1      ar2      ar3      ar4     ar5      ar6     ar7      ar8
      0.1350  -0.1312  -0.1587  -0.1942  0.2136  -0.4542  0.1309  -0.1233
s.e.  0.4042   0.3180   0.3421   0.3103  0.3602   0.2848  0.3854   0.2926
          ar9     ar10    ar11    ar12      ma1     ma2      ma3     ma4
      -0.1494  -0.1761  0.1774  0.4844  -0.7414  0.1235  -0.0127  0.0543
s.e.   0.3311   0.2964  0.3350  0.2551   0.4105  0.2762   0.1924  0.2406
          ma5     ma6      ma7     ma8     ma9   drift
      -0.5087  0.6158  -0.6797  0.2000  0.1532  0.0062
s.e.   0.1918  0.1998   0.2838  0.2899  0.2371  0.0016

sigma^2 estimated as 0.01235:  log likelihood=85.54
AIC=-125.07   AICc=-111.77   BIC=-63.6
```

Remove ma9, ma8

In [17]:
```
Fit01 <- Arima(D0, lambda=0, order=c(12,1,7), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(12,1,7) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.6167  -0.6701  -0.6877  -0.8160  -0.4735  -0.8433  -0.5812  -0.6614
s.e.   0.3893   0.3111   0.2795   0.3231   0.3799   0.2437   0.3650   0.3188
          ar9     ar10     ar11    ar12     ma1     ma2     ma3     ma4
      -0.6663  -0.7492  -0.4571  0.0970  0.0225  0.1821  0.0987  0.2516
s.e.   0.2987   0.3094   0.3445  0.2346  0.3862  0.1534  0.1009  0.1563
          ma5     ma6      ma7   drift
      -0.3311  0.3397  -0.3854  0.0062
s.e.   0.1618  0.1882   0.2165  0.0015

sigma^2 estimated as 0.0121:  log likelihood=85.52
AIC=-129.04   AICc=-118.17   BIC=-72.91
```

Remove ar12

In [18]:
```
Fit01 <- Arima(D0, lambda=0, order=c(11,1,7), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(11,1,7) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.8946  -0.7827  -0.8886  -0.8913  -0.8580  -0.7797  -0.9058  -0.8708
s.e.   0.1255   0.0801   0.0911   0.0999   0.0847   0.0979   0.0877   0.0746
          ar9     ar10     ar11     ma1     ma2     ma3     ma4     ma5
      -0.7864  -0.9633  -0.6512  0.2691  0.0800  0.1643  0.0318  0.1909
s.e.   0.0817   0.0734   0.0978  0.1596  0.1289  0.1384  0.1671  0.1134
          ma6     ma7   drift
      -0.1344  0.3080  0.0063
s.e.   0.1497  0.1509  0.0020

sigma^2 estimated as 0.01398:  log likelihood=79.9
AIC=-119.8   AICc=-110.03   BIC=-66.34
```

Remove ma7

In [19]:
```
Fit01 <- Arima(D0, lambda=0, order=c(11,1,6), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(11,1,6) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.6743  -0.6720  -0.8975  -0.6551  -0.7723  -0.7439  -0.7130  -0.7431
s.e.   0.2513   0.1038   0.1133   0.1992   0.1259   0.1322   0.1503   0.1401
          ar9     ar10     ar11      ma1     ma2     ma3      ma4     ma5
      -0.7172  -0.8332  -0.4951  -0.0254  0.1350  0.3407  -0.2606  0.3262
s.e.   0.1080   0.1343   0.1701   0.2765  0.1602  0.1261   0.1982  0.1519
          ma6   drift
      -0.1652  0.0062
s.e.   0.1903  0.0017

sigma^2 estimated as 0.01394:  log likelihood=79.52
AIC=-121.04   AICc=-112.31   BIC=-70.26
```

Remove ma6

In [20]:
```
Fit01 <- Arima(D0, lambda=0, order=c(11,1,5), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(11,1,5) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.8431  -0.8332  -0.7934  -0.9784  -0.7470  -0.8421  -0.8306  -0.8488
s.e.   0.1342   0.0754   0.1042   0.0793   0.1083   0.0778   0.1009   0.0638
          ar9     ar10     ar11    ma1     ma2      ma3     ma4      ma5
      -0.7912  -0.9264  -0.6448  0.166  0.1998  -0.0050  0.2661  -0.1011
s.e.   0.0933   0.0686   0.1004  0.165  0.1383   0.1308  0.1409   0.1097
        drift
       0.0062
s.e.   0.0016

sigma^2 estimated as 0.01343:  log likelihood=81.1
AIC=-126.2   AICc=-118.43   BIC=-78.09
```

Remove ma5

In [21]:
```
Fit01 <- Arima(D0, lambda=0, order=c(11,1,4), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(11,1,4) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.8609  -0.8421  -0.8121  -0.9721  -0.7848  -0.8501  -0.8408  -0.8608
s.e.   0.1277   0.0717   0.0898   0.0883   0.0947   0.0759   0.0946   0.0635
          ar9     ar10     ar11     ma1     ma2     ma3     ma4   drift
      -0.8069  -0.9327  -0.6588  0.1908  0.2158  0.0298  0.2741  0.0062
s.e.   0.0854   0.0683   0.0910  0.1646  0.1293  0.1230  0.1484  0.0018

sigma^2 estimated as 0.0134:  log likelihood=80.68
AIC=-127.35   AICc=-120.48   BIC=-81.92
```

Remove ma4

In [22]:
```
Fit01 <- Arima(D0, lambda=0, order=c(11,1,3), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(11,1,3) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.7683  -0.8092  -0.8281  -0.8576  -0.7425  -0.8308  -0.7851  -0.8114
s.e.   0.1481   0.0756   0.0950   0.0903   0.1016   0.0762   0.1028   0.0789
          ar9     ar10     ar11     ma1     ma2     ma3   drift
      -0.7708  -0.8904  -0.6130  0.0569  0.2521  0.0350  0.0062
s.e.   0.0877   0.0753   0.1075  0.1757  0.1293  0.1164  0.0015

sigma^2 estimated as 0.01363:  log likelihood=79.38
AIC=-126.76   AICc=-120.72   BIC=-84
```

Remove ma3

In [23]:
```
Fit01 <- Arima(D0, lambda=0, order=c(11,1,2), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(11,1,2) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.7551  -0.8155  -0.8133  -0.8532  -0.7375  -0.8267  -0.7764  -0.8091
s.e.   0.1557   0.0711   0.0880   0.0933   0.1049   0.0767   0.1046   0.0810
          ar9     ar10     ar11     ma1     ma2   drift
      -0.7639  -0.8873  -0.6061  0.0430  0.2625  0.0062
s.e.   0.0912   0.0769   0.1127  0.1846  0.1328  0.0015

sigma^2 estimated as 0.0135:  log likelihood=79.34
AIC=-128.67   AICc=-123.4   BIC=-88.58
```

Remove ma2

```
In [24]: Fit01 <- Arima(D0, lambda=0, order=c(11,1,1), include.drift = TRUE)
         Fit01
```

```
Series: D0
ARIMA(11,1,1) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.8702  -0.7645  -0.8497  -0.8914  -0.7636  -0.8302  -0.8202  -0.8245
s.e.   0.1043   0.0718   0.0697   0.0763   0.0875   0.0721   0.0844   0.0697
          ar9     ar10     ar11     ma1    drift
      -0.8058  -0.9111  -0.6383  0.1433  0.0062
s.e.   0.0711   0.0735   0.0839  0.1209  0.0013

sigma^2 estimated as 0.01395:  log likelihood=77.22
AIC=-126.44   AICc=-121.87   BIC=-89.02
```

Remove ma1

```
In [25]: Fit01 <- Arima(D0, lambda=0, order=c(11,1,0), include.drift = TRUE)
         Fit01
```

```
Series: D0
ARIMA(11,1,0) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.7753  -0.7258  -0.8075  -0.8413  -0.7061  -0.7946  -0.7647  -0.7844
s.e.   0.0788   0.0668   0.0645   0.0674   0.0774   0.0663   0.0753   0.0653
          ar9     ar10     ar11   drift
      -0.7597  -0.8638  -0.5765  0.0062
s.e.   0.0646   0.0653   0.0779  0.0012

sigma^2 estimated as 0.01399:  log likelihood=76.56
AIC=-127.13   AICc=-123.21   BIC=-92.38
```

Try adding back in ar12

In [26]:
```
Fit01 <- Arima(D0, lambda=0, order=c(12,1,0), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(12,1,0) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.6843  -0.5903  -0.6915  -0.7190  -0.5876  -0.6722  -0.6547  -0.6540
s.e.   0.0973   0.1095   0.0985   0.1034   0.1081   0.1023   0.1028   0.1061
          ar9     ar10     ar11    ar12   drift
      -0.6358  -0.7491  -0.4575  0.1523  0.0062
s.e.   0.1024   0.0984   0.1089  0.0980  0.0014

sigma^2 estimated as 0.0138:  log likelihood=77.75
AIC=-127.5   AICc=-122.94   BIC=-90.08
```

Try adding back in ar13

In [27]:
```
Fit01 <- Arima(D0, lambda=0, order=c(13,1,0), include.drift = TRUE)
Fit01
```

```
Series: D0
ARIMA(13,1,0) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.7182  -0.4915  -0.5280  -0.5864  -0.4466  -0.5323  -0.5120  -0.5276
s.e.   0.0958   0.1156   0.1216   0.1178   0.1234   0.1183   0.1194   0.1183
          ar9     ar10     ar11    ar12    ar13   drift
      -0.4820  -0.6043  -0.3279  0.2981  0.2117  0.0062
s.e.   0.1222   0.1170   0.1216  0.1161  0.0963  0.0017

sigma^2 estimated as 0.01328:  log likelihood=80.09
AIC=-130.18   AICc=-124.91   BIC=-90.09
```

Remove ar13

In [28]:
```
Fit01 <- Arima(D0, lambda=0, order=c(13,1,0), include.drift = TRUE)
Fit01
```

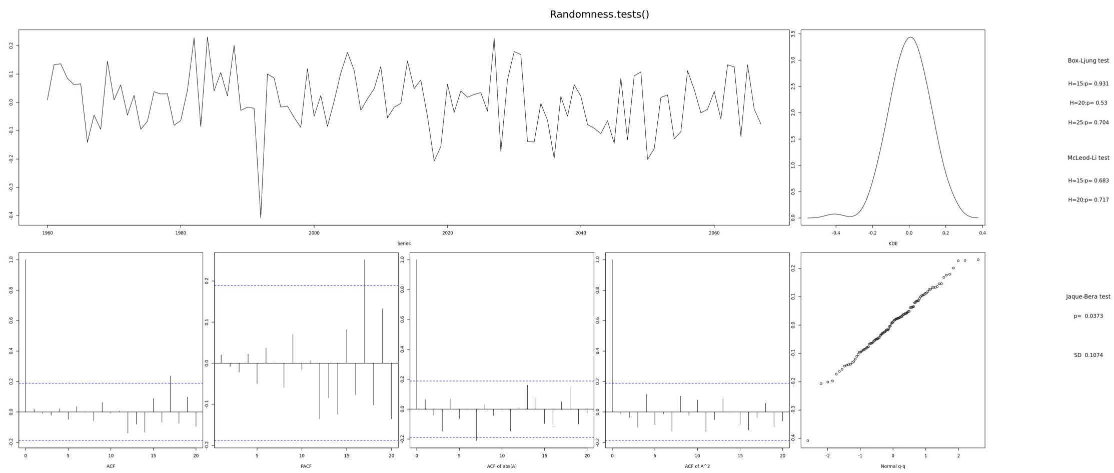
```
Series: D0
ARIMA(13,1,0) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.7182  -0.4915  -0.5280  -0.5864  -0.4466  -0.5323  -0.5120  -0.5276
s.e.   0.0958   0.1156   0.1216   0.1178   0.1234   0.1183   0.1194   0.1183
          ar9     ar10     ar11    ar12    ar13   drift
      -0.4820  -0.6043  -0.3279  0.2981  0.2117  0.0062
s.e.   0.1222   0.1170   0.1216  0.1161  0.0963  0.0017

sigma^2 estimated as 0.01328:  log likelihood=80.09
AIC=-130.18   AICc=-124.91   BIC=-90.09
```

In [29]:
```
Randomness.tests(Fit01$residuals)
```

```
B-L test H0: the series is uncorrelated
M-L test H0: the square of the series is uncorrelated
J-B test H0: the series came from Normal distribution
SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
0.931	0.53	0.704	0.683	0.717	0.037	0.107



The one Box-Ljung number is low indicating remaining correlation. ACF and PACF plots still indicate correlation at lag 14 and lag 17. The Jaque-Bera p value is low, indicating the residuals are not normal.

Try pushing p value up higher.

In [30]:
```
Fit01 <- Arima(D0, lambda=0, order=c(17,1,0), include.drift = TRUE)
Fit01
```

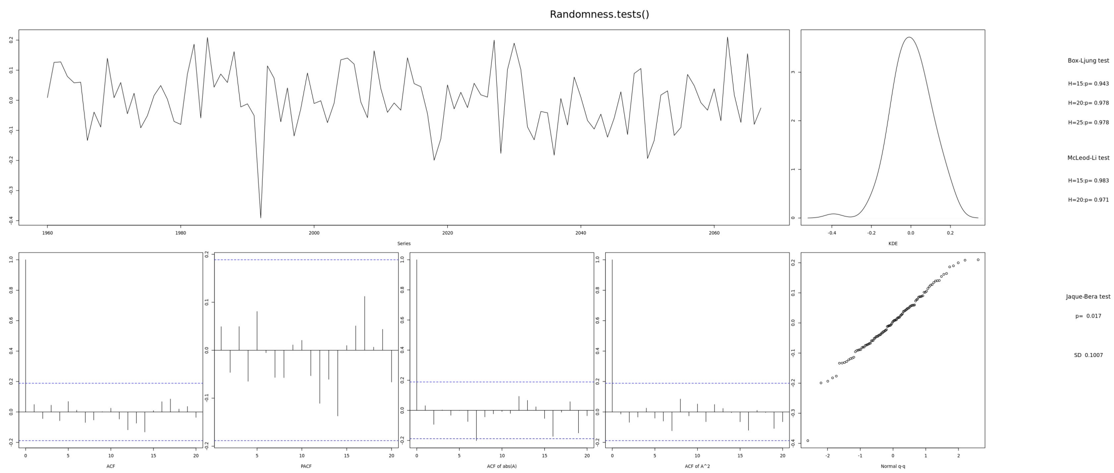
```
Series: D0
ARIMA(17,1,0) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ar1      ar2      ar3      ar4      ar5      ar6      ar7      ar8
      -0.6054  -0.4911  -0.5291  -0.5161  -0.4809  -0.5428  -0.4013  -0.4801
s.e.   0.0967   0.1136   0.1233   0.1343   0.1415   0.1442   0.1528   0.1475
          ar9     ar10     ar11    ar12    ar13    ar14    ar15     ar16
      -0.3973  -0.5611  -0.2182  0.3147  0.2118  0.0392  0.1156  -0.0783
s.e.   0.1506   0.1478   0.1534  0.1456  0.1421  0.1347  0.1245   0.1144
        ar17   drift
      0.2005  0.0063
s.e.  0.0967  0.0019

sigma^2 estimated as 0.01219:  log likelihood=86.12
AIC=-134.25   AICc=-125.51   BIC=-83.46
```

In [31]:
```
Randomness.tests(Fit01$residuals)
```

```
  B-L test H0: the series is uncorrelated
  M-L test H0: the square of the series is uncorrelated
  J-B test H0: the series came from Normal distribution
  SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
0.943	0.978	0.978	0.983	0.971	0.017	0.101



The residuals look good, however ar11, ar13, ar14, ar15, and ar16 are not significant. The Jaque-Bera p-value is low indicating the residual are not normal.

Try with linear trend

In [11]:
```
Fit02 <- auto.arima(D0, d=0, D=0, lambda=0, xreg=time(D0), stepwise=FALSE, approxim
ation=FALSE, max.p = 17, max.q = 17)
Fit02
```

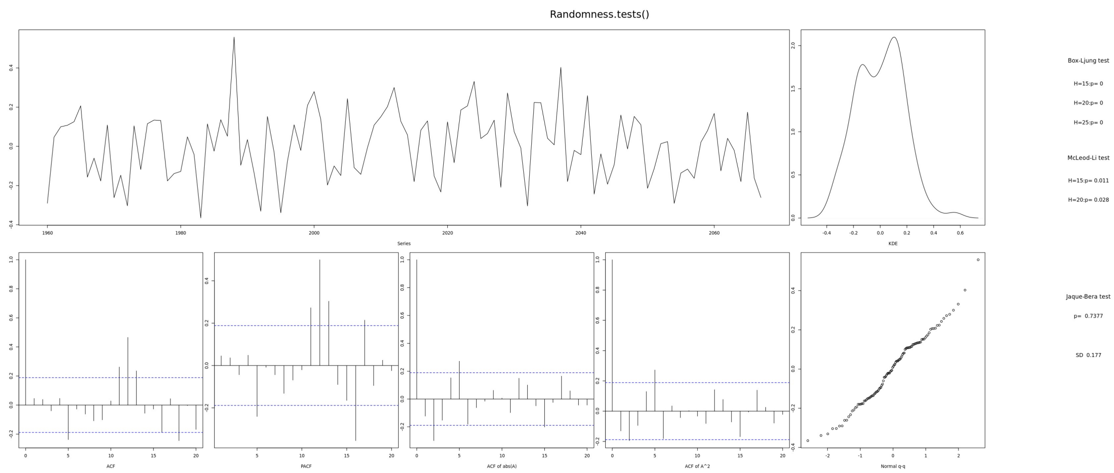
```
Series: D0
Regression with ARIMA(0,0,4) errors
Box Cox transformation: lambda= 0

Coefficients:
         ma1     ma2      ma3      ma4  intercept    xreg
      0.7280  0.2743  -0.2116  -0.6900    -2.7933  0.0061
s.e.  0.0863  0.1324   0.1238   0.1113     1.2748  0.0006

sigma^2 estimated as 0.03288:  log likelihood=31.97
AIC=-49.94   AICc=-48.82   BIC=-31.16
```

In [12]:
```
Randomness.tests(Fit02$residuals)
```

```
  B-L test H0: the series is uncorrelated
  M-L test H0: the square of the series is uncorrelated
  J-B test H0: the series came from Normal distribution
  SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
0	0	0	0.011	0.028	0.738	0.177



Try linear trend and manually forcing higher p and q values

In [13]:
```
Fit02 <- Arima(D0, lambda=0, order=c(17,0,0), include.drift=FALSE,  xreg=time(D0))
Fit02
```

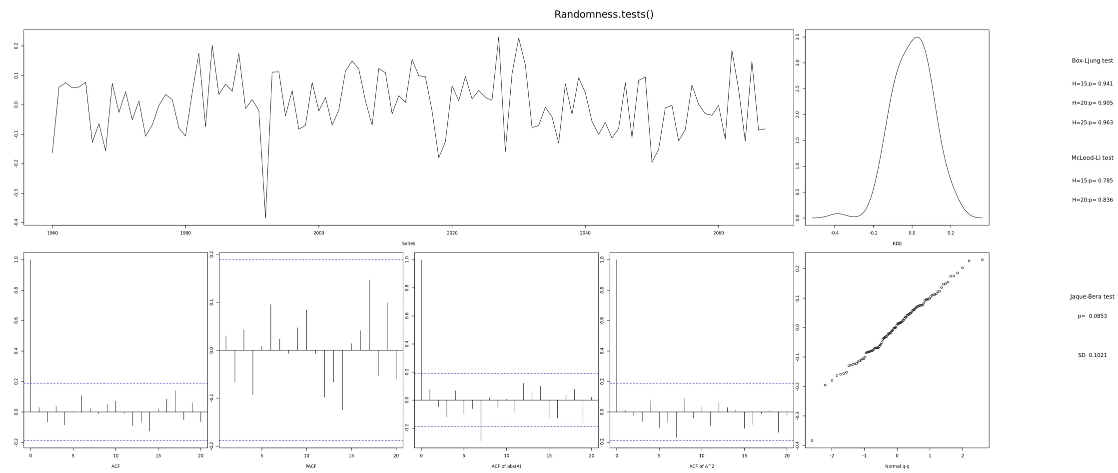
```
Series: D0
Regression with ARIMA(17,0,0) errors
Box Cox transformation: lambda= 0

Coefficients:
         ar1     ar2      ar3     ar4     ar5      ar6     ar7      ar8     ar9
      0.3137  0.1473  -0.0681  0.0410  0.0528  -0.1984  0.0416  -0.0651  0.0346
s.e.  0.0934  0.0965   0.0992  0.0965  0.0948   0.0752  0.0662   0.0724  0.0655
         ar10    ar11    ar12     ar13     ar14    ar15     ar16    ar17
      -0.1779  0.2993  0.5534  -0.1185  -0.1885  0.0683  -0.2383  0.1819
s.e.   0.0689  0.0683  0.0753   0.0943   0.0939  0.0953   0.0898  0.0694
      intercept    xreg
        -2.9609  0.0062
s.e.     1.4564  0.0007

sigma^2 estimated as 0.01254:  log likelihood=86.82
AIC=-133.63   AICc=-123.98   BIC=-79.99
```

In [14]:
```
Randomness.tests(Fit02$residuals)
```

```
  B-L test H0: the series is uncorrelated
  M-L test H0: the square of the series is uncorrelated
  J-B test H0: the series came from Normal distribution
  SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
0.941	0.905	0.963	0.785	0.836	0.085	0.102



The residuals look good, the AICc is much better.

Try as seasonal time series

In [25]:
```
D1 <- ts(D[,2], start=c(1960,1), freq=12)
```

See what auto.arima() gives

In [35]:
```
Fit2 <- auto.arima(D1, lambda=0, stepwise = FALSE, approximation = FALSE)
Fit2
```

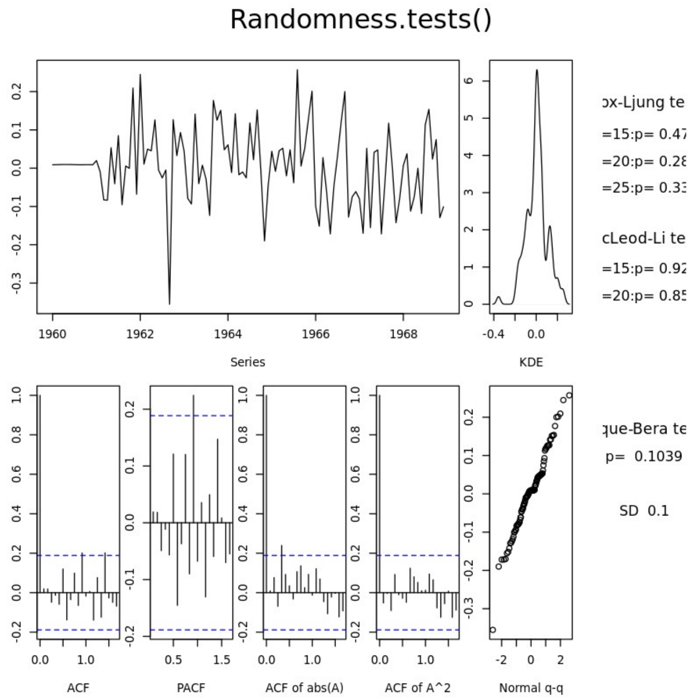
```
Series: D1
ARIMA(2,0,0)(0,1,2)[12] with drift
Box Cox transformation: lambda= 0

Coefficients:
         ar1     ar2      sma1     sma2   drift
      0.2283  0.2408   -0.5055  -0.2896  0.0062
s.e.  0.1014  0.1024    0.1724   0.1587  0.0008

sigma^2 estimated as 0.01179:  log likelihood=74.43
AIC=-136.85   AICc=-135.91   BIC=-121.46
```

In [36]:
```
Randomness.tests(Fit2$residuals)
```

```
B-L test H0: the series is uncorrelated
M-L test H0: the square of the series is uncorrelated
J-B test H0: the series came from Normal distribution
SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
0.473	0.286	0.332	0.926	0.856	0.104	0.1



Auto.arima() gives a model of ARIMA(2,0,0)(0,1,2)[12] with drift.

The residuals look good, however the ACF and PACF graphs are showing some correlation at lag 11 and 17

The sma2 term included in the model does not seems to be significant. Will try removing it, but first make sure taking a seasonal difference if appropriate

Check if seasonal difference is appropriate

In [31]:
```
Fit21 <- Arima(D1, lambda = 0, order=c(0,0,0), seasonal=c(0,1,1))
Fit21
```

```
Series: D1
ARIMA(0,0,0)(0,1,1)[12]
Box Cox transformation: lambda= 0

Coefficients:
         sma1
      -0.0087
s.e.   0.0887

sigma^2 estimated as 0.02237:  log likelihood=46.7
AIC=-89.4   AICc=-89.27   BIC=-84.27
```

sma1 is not 1, but try it with drift

In [32]:
```
Fit21 <- Arima(D1, lambda = 0, order=c(0,0,0), seasonal=c(0,1,1), include.drift = TRUE)
Fit21
```

```
Series: D1
ARIMA(0,0,0)(0,1,1)[12] with drift
Box Cox transformation: lambda= 0

Coefficients:
         sma1   drift
      -0.3932  0.0062
s.e.   0.1281  0.0007

sigma^2 estimated as 0.01485:  log likelihood=65.9
AIC=-125.81   AICc=-125.54   BIC=-118.11
```

sam1 is still not close to 1, so seasonal differencing is appropriate

In [33]:
```
Arima(D1, lambda=0, order=c(0,0,12), seasonal=c(0,1,0))
```

```
Series: D1
ARIMA(0,0,12)(0,1,0)[12]
Box Cox transformation: lambda= 0

Coefficients:
         ma1     ma2     ma3     ma4     ma5     ma6     ma7      ma8     ma9
      0.4183  0.3819  0.2899  0.1989  0.2537  0.3472  0.0785  -0.0279  0.2581
s.e.  0.1071  0.1337  0.1206  0.1227  0.1170  0.1420  0.1217   0.1275  0.1271
        ma10    ma11     ma12
      0.1514  0.4090  -0.3441
s.e.  0.1133  0.1149   0.1142

sigma^2 estimated as 0.01194:  log likelihood=76.27
AIC=-126.53   AICc=-122.1   BIC=-93.2
```

In [34]:
```
Arima(D1, lambda=0, order=c(0,0,12), seasonal=c(0,1,0), include.drift=TRUE)
```

```
Series: D1
ARIMA(0,0,12)(0,1,0)[12] with drift
Box Cox transformation: lambda= 0

Coefficients:
         ma1     ma2     ma3     ma4     ma5     ma6      ma7      ma8     ma9
      0.3276  0.2862  0.1627  0.0648  0.1057  0.1762  -0.0852  -0.1656  0.1241
s.e.  0.1082  0.1223  0.1161  0.1176  0.1133  0.1318   0.1186   0.1279  0.1242
        ma10    ma11     ma12   drift
      0.0301  0.3033  -0.4534  0.0062
s.e.  0.1066  0.1075   0.1213  0.0016

sigma^2 estimated as 0.01126:  log likelihood=79.94
AIC=-131.88   AICc=-126.69   BIC=-95.98
```

The ma's are not close to 1, so this looks good.

Try model that auto.arima() gave with sma2 term removed

In [19]:
```
Fit22 <- Arima(D1, lambda=0, order=c(2,0,0), seasonal=c(0,1,1), include.drift = TRU
E)
Fit22
```

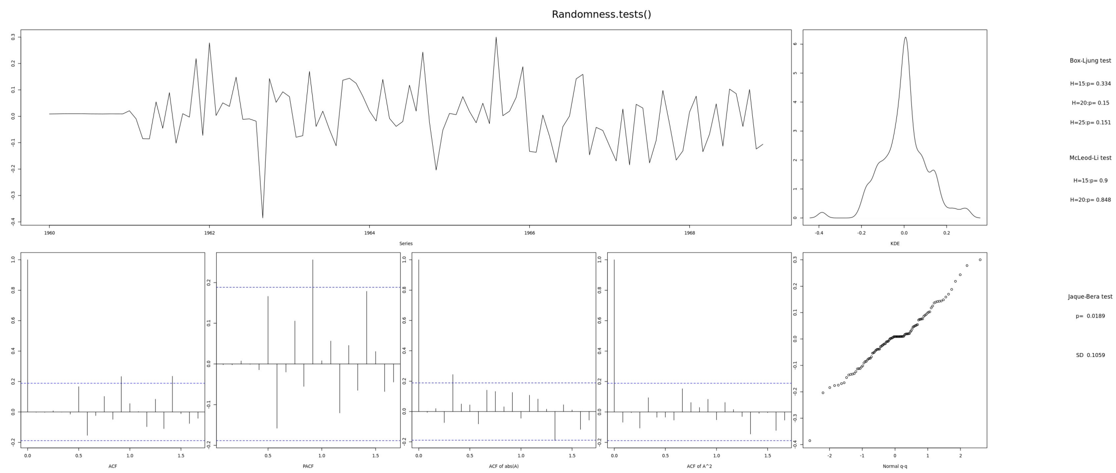
```
Series: D1
ARIMA(2,0,0)(0,1,1)[12] with drift
Box Cox transformation: lambda= 0

Coefficients:
         ar1     ar2     sma1   drift
      0.2420  0.2104  -0.4977  0.0062
s.e.  0.1074  0.1020   0.1447  0.0010

sigma^2 estimated as 0.01304:  log likelihood=72.36
AIC=-134.71   AICc=-134.05   BIC=-121.89
```

In [10]:
```
Randomness.tests(Fit22$residuals)
```

```
B-L test H0: the series is uncorrelated
M-L test H0: the square of the series is uncorrelated
J-B test H0: the series came from Normal distribution
SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
0.334	0.15	0.151	0.9	0.848	0.019	0.106



All the terms in the model are now significant.

However two of the Box-Ljung tests are showing correlation now, and the ACF and PACF plots have gotten worse, confirming this.

The Jaque-Bera p value is low indicating the residuals are not normal

There seem to be some indicators that there may be correlation at lag 11. Try forcing bigging p and q values.

Try manually forcing p and q to be higher

In [11]:
```
Fit23 <- Arima(D1, lambda=0, order=c(11,0,11), seasonal=c(0,1,1), include.drift = T
RUE)
Fit23
```

```
Warning message in sqrt(diag(x$var.coef)):
“NaNs produced”

Series: D1
ARIMA(11,0,11)(0,1,1)[12] with drift
Box Cox transformation: lambda= 0

Coefficients:
         ar1      ar2      ar3     ar4      ar5     ar6      ar7     ar8
      0.6138  -0.2008  -0.3283  0.4131  -0.3835  0.3725  -0.4509  0.3036
s.e.  0.0834   0.0523   0.0511  0.0692   0.0528  0.0687   0.0417  0.0570
         ar9     ar10     ar11      ma1     ma2     ma3      ma4     ma5
      0.2145  -0.7116   0.8512  -0.0887  0.1462  0.4282  -0.1977  0.4032
s.e.  0.0438   0.0539   0.0891   0.1987     NaN  0.2112   0.2777     NaN
          ma6     ma7      ma8      ma9    ma10     ma11     sma1   drift
      -0.2531  0.3279  -0.2437  -0.2091  0.6769  -0.3850  -0.9987  0.0062
s.e.   0.2150  0.3668   0.1770      NaN  0.2132   0.0873   0.1721  0.0009

sigma^2 estimated as 0.00842:  log likelihood=89.88
AIC=-129.76   AICc=-111.19   BIC=-65.65
```

None of the ma terms are significant except ma10. Try removing ma.

In [12]:
```
Fit24 <- Arima(D1, lambda=0, order=c(11,0,0), seasonal=c(0,1,1), include.drift = TR
UE)
Fit24
```

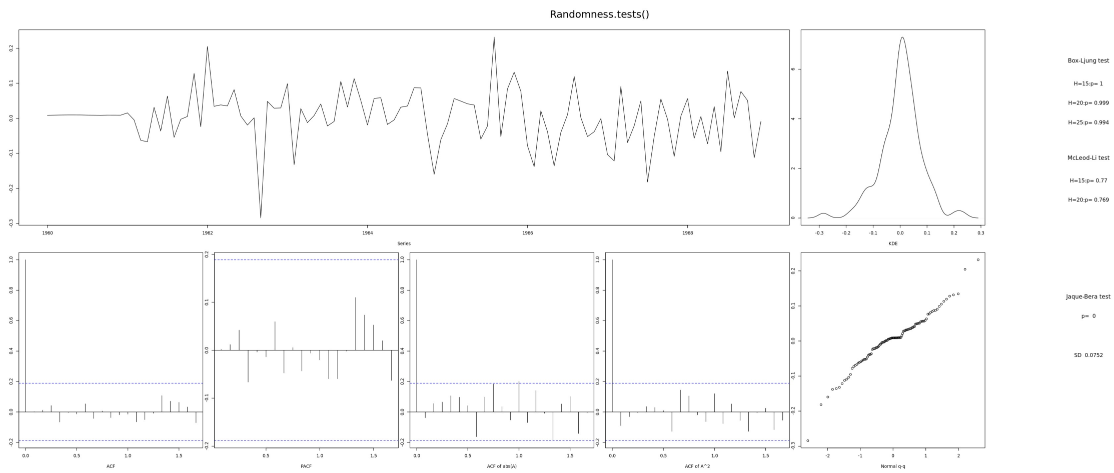
```
Series: D1
ARIMA(11,0,0)(0,1,1)[12] with drift
Box Cox transformation: lambda= 0

Coefficients:
         ar1     ar2      ar3     ar4      ar5     ar6      ar7      ar8
      0.3427  0.1225  -0.0185  0.0554  -0.0806  0.1814  -0.2220  -0.0005
s.e.  0.1036  0.1065   0.1031  0.1037   0.1025  0.1032   0.0987   0.1030
         ar9     ar10    ar11     sma1   drift
      0.1653  -0.0997  0.3101  -0.6995  0.0062
s.e.  0.1052   0.1047  0.1054   0.1387  0.0012

sigma^2 estimated as 0.01162:  log likelihood=80.67
AIC=-133.34   AICc=-128.15   BIC=-97.44
```

In [13]:
```
Randomness.tests(Fit22$residuals)
```

```
  B-L test H0: the series is uncorrelated
  M-L test H0: the square of the series is uncorrelated
  J-B test H0: the series came from Normal distribution
  SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
1	0.999	0.994	0.77	0.769	0	0.075



The AR11 term is significant. And the residuals look good, the ACF and PACF graphs now do not have any issues.

The Jaque-Bera p value is 0, indicating the residuals are not normal.

However the terms ar2, ar3, ar4, ar5, ar6, ar8, ar9, and ar10 are not significant. Because of all of these extra terms not contributing to the model, the AICc score is higher.

Try changing the seasonal length to 11

Since we do not have a sar term, and there seems to be correlation at lag 11, try using a seasonal length of 11 instead of 12

In [19]:
```
D11 <- ts(D[,2], start=c(1960,1), freq=11)
```

In [20]:
```
Fit25 <- auto.arima(D11, lambda=0, stepwise = FALSE, approximation = FALSE)
Fit25
```

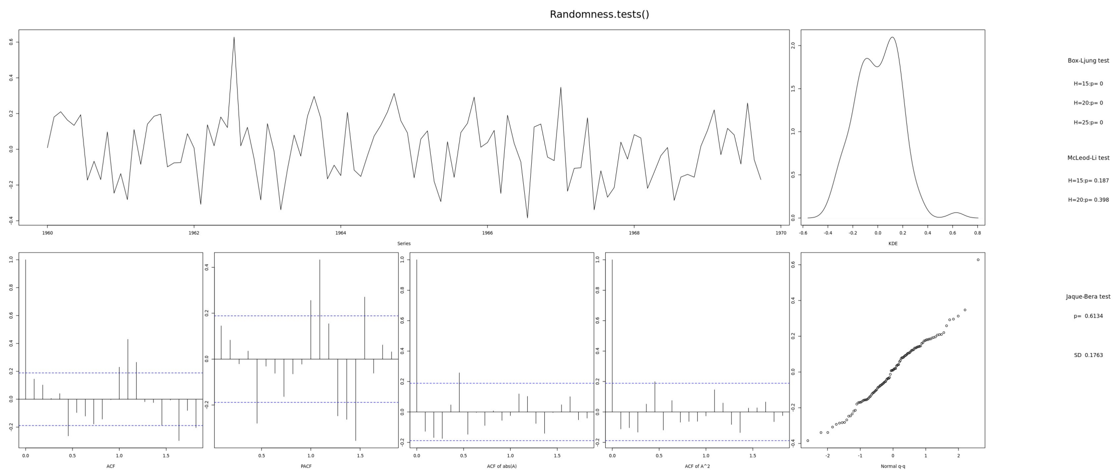
```
Series: D11
ARIMA(0,1,5) with drift
Box Cox transformation: lambda= 0

Coefficients:
          ma1      ma2      ma3      ma4     ma5   drift
      -0.3587  -0.4776  -0.4709  -0.4136  0.8225  0.0059
s.e.   0.0715   0.0757   0.0745   0.0791  0.0926  0.0018

sigma^2 estimated as 0.03294:  log likelihood=29.89
AIC=-45.77   AICc=-44.64   BIC=-27.06
```

In [22]:
```
Randomness.tests(Fit25$residuals)
```

```
B-L test H0: the series is uncorrelated
M-L test H0: the square of the series is uncorrelated
J-B test H0: the series came from Normal distribution
SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

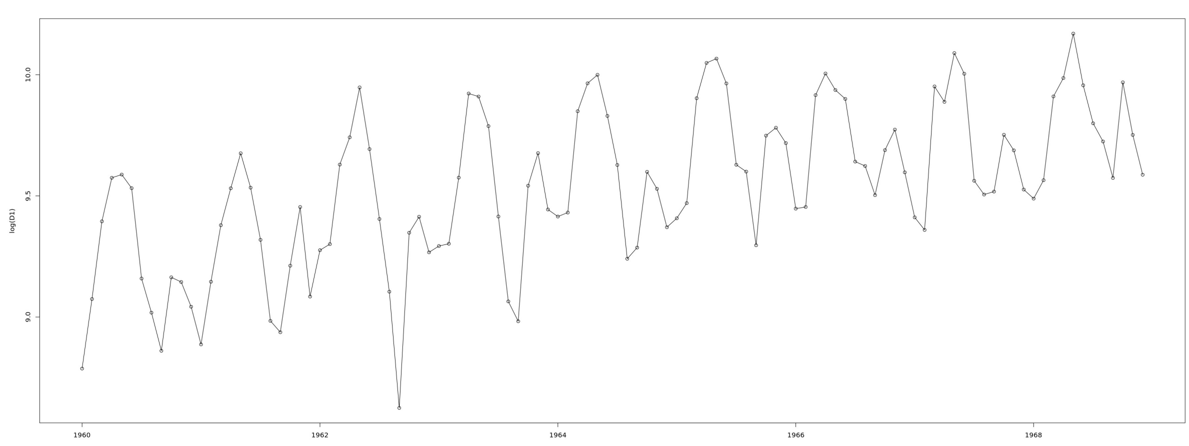
BL15	BL20	BL25	ML15	ML20	JB	SD
0	0	0	0.187	0.398	0.613	0.176



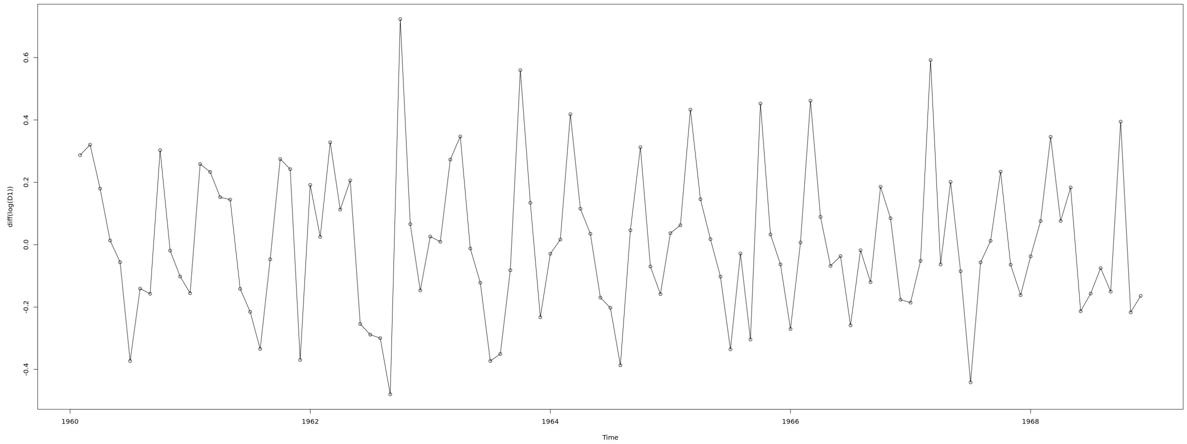
There is a lot of correlation at lag 12, more correlation than we where seeing at lag 11 when we had the seasonal length set to 12. The Box-Ljung numbers are all 0, confirming that there is a lot of correlation. The AICc number is higher. Everything indicates that this is a worse model.

Investigate d and D

In [9]:
```
plot(log(D1), type='o')
```



In [10]:
```
plot(diff(log(D1)), type='o')
```



In [11]:
```
Stationarity.tests(diff(log(D1)))
```

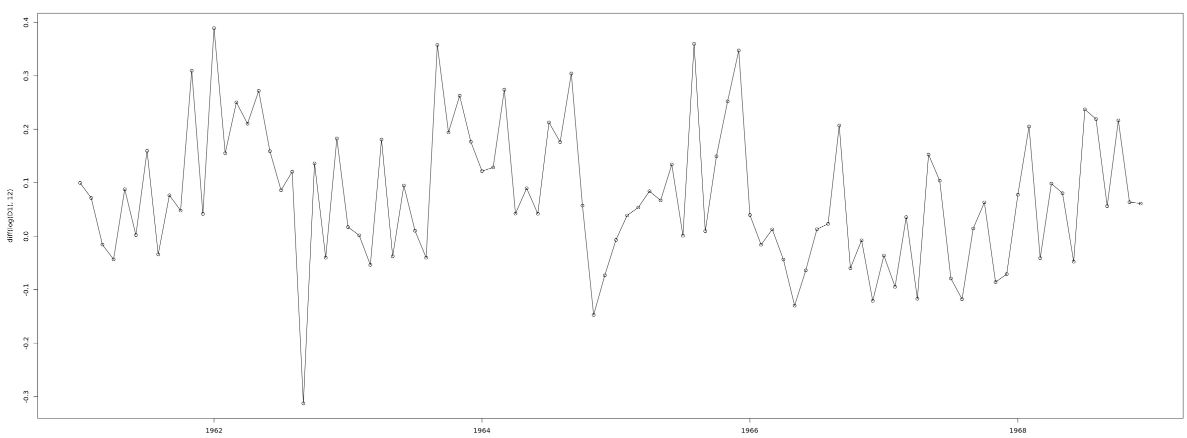
```
Warning message in adf.test(A):
“p-value smaller than printed p-value”
Warning message in pp.test(A):
“p-value smaller than printed p-value”
Warning message in kpss.test(A):
“p-value greater than printed p-value”
```

A matrix: 1 × 3 of type dbl

	KPSS	ADF	PP
p-val:	0.1	0.01	0.01

It is stationary with d=1

In [12]:
```
plot(diff(log(D1), 12), type='o')
```



In [13]:
```
Stationarity.tests(diff(log(D1), 12))
```

```
Warning message in pp.test(A):
“p-value smaller than printed p-value”
Warning message in kpss.test(A):
“p-value greater than printed p-value”
```

A matrix: 1 × 3 of type dbl

	KPSS	ADF	PP
p-val:	0.1	0.024	0.01

It is stationary with D=1.

Try it with just d=1 (D=0)

In [14]:
```
Fit3 <- auto.arima(D1, d=1, D=0, lambda=0, stepwise = FALSE, approximation = FALSE)
Fit3
```

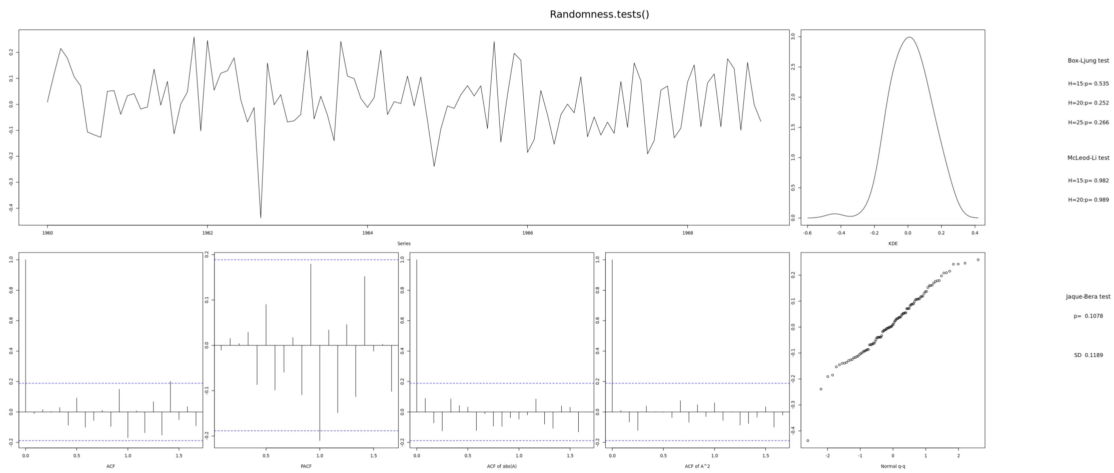
```
Series: D1
ARIMA(0,1,3)(1,0,0)[12]
Box Cox transformation: lambda= 0

Coefficients:
          ma1     ma2      ma3    sar1
      -0.7108  0.0702  -0.2772  0.8592
s.e.   0.1014  0.1206   0.0905  0.0439

sigma^2 estimated as 0.01491:  log likelihood=66.93
AIC=-123.85   AICc=-123.26   BIC=-110.49
```

In [15]:
```
Randomness.tests(Fit3$residuals)
```

```
B-L test H0: the series is uncorrelated
M-L test H0: the square of the series is uncorrelated
J-B test H0: the series came from Normal distribution
SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
0.535	0.252	0.266	0.982	0.989	0.108	0.119



The suggested model is ARIMA(0,1,3)(1,0,0)[12]

The residuals still look good, though the PACF graph is still slightly concerning.

The AICc value is higher.

Try with a linear trend

In [15]:
```
Fit4 <- auto.arima(D1, d=0, D=0, lambda=0, xreg=time(D1), stepwise=FALSE, approxima
tion=FALSE)
Fit4
```

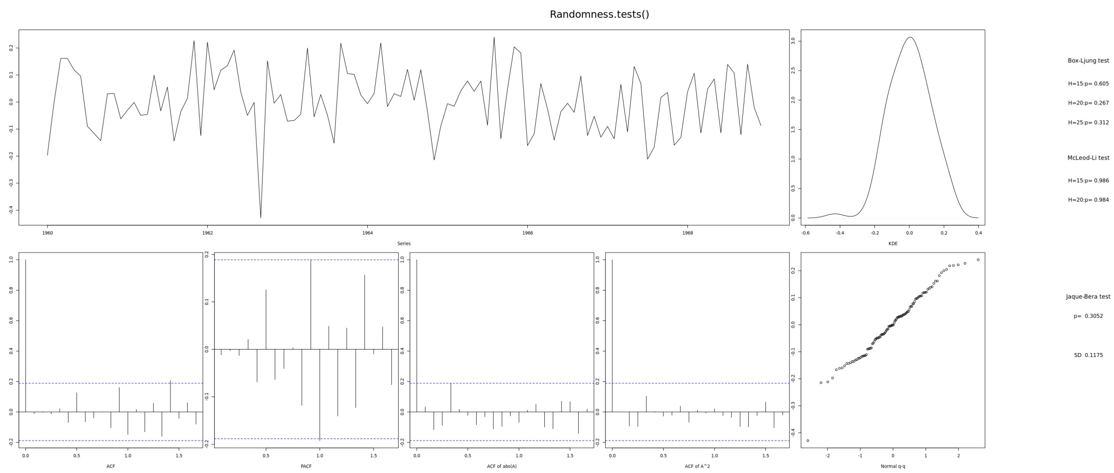
```
Series: D1
Regression with ARIMA(0,0,2)(1,0,0)[12] errors
Box Cox transformation: lambda= 0

Coefficients:
         ma1     ma2    sar1  intercept    xreg
      0.2462  0.2935  0.8437  -135.9359  0.0740
s.e.  0.0993  0.0924  0.0457    28.6874  0.0146

sigma^2 estimated as 0.01436:  log likelihood=70.87
AIC=-129.73   AICc=-128.9   BIC=-113.64
```

```
In [16]: Randomness.tests(Fit4$resid)
```

```
B-L test H0: the series is uncorrelated
M-L test H0: the square of the series is uncorrelated
J-B test H0: the series came from Normal distribution
SD         : Standard Deviation of the series
```

A matrix: 1 × 7 of type dbl

BL15	BL20	BL25	ML15	ML20	JB	SD
0.605	0.267	0.312	0.986	0.984	0.305	0.118



Evaluate Possible models

Summary of possible models

	Model	Sigma^2	AICc	BL	ML	JB	ACF/PACF
1	ARIMA(0,1,5) with drift	0.03294	-44.64	Bad	Good	Good	Correlation
2	ARIMA(13,1,0) with drift	0.01328	-124.91	Good	Good	Bad	Correlation
3	ARIMA(17,1,0) with drift	0.01219	-125.51	Good	Good	Bad	No Correlation
4	Regression with ARIMA(0,0,4)	0.03288	-48.82	Bad	Bad	Good	Correlation
5	Regression with ARIMA(17,0,0)	0.01254	-123.98	Good	Good	Marginal	No Correlation
6	ARIMA(2,0,0)(0,1,2)[12] with drift	0.01179	-135.91	Good	Good	Good	Correlation
7	ARIMA(11,0,0)(0,1,1)[12] with drift	0.01162	-128.15	Good	Good	Bad	No Correlation
8	Regression with ARIMA(0,0,2)(1,0,0)[12]	0.01436	-128.9	Good	Good	Good	Correlation

The two models we will choose to evaluate are ARIMA(2,0,0)(0,1,2)[12] with drift, and Regression with ARIMA(0,0,2)(1,0,0)[12].

Model ARIMA(2,0,0)(0,1,2)[12] with drift

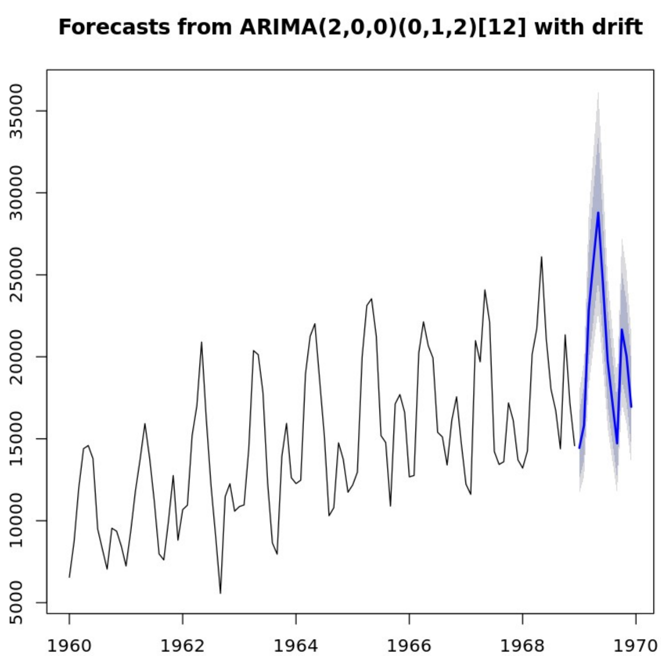
In [5]:
```
FitE1 <- Arima(D1, lambda=0, order=c(2,0,0), seasonal=c(0,1,2), include.drift = TRU
E)
FitE1
```

```
Series: D1
ARIMA(2,0,0)(0,1,2)[12] with drift
Box Cox transformation: lambda= 0

Coefficients:
         ar1     ar2     sma1     sma2   drift
      0.2283  0.2408  -0.5055  -0.2896  0.0062
s.e.  0.1014  0.1024   0.1724   0.1587  0.0008

sigma^2 estimated as 0.01179:  log likelihood=74.43
AIC=-136.85   AICc=-135.91   BIC=-121.46
```

In [6]:
```
forecast1 <- forecast(FitE1, 12)
forecast1
```

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Jan 1969	14440.60	12553.40	16611.51	11656.37	17889.88
Feb 1969	15827.88	13710.13	18272.74	12706.29	19716.34
Mar 1969	22839.59	19670.94	26518.66	18175.60	28700.41
Apr 1969	25879.64	22267.68	30077.48	20564.41	32568.68
May 1969	28794.70	24760.37	33486.35	22858.85	36271.93
Jun 1969	24477.69	21044.56	28470.89	19426.62	30842.09
Jul 1969	19734.09	16964.90	22955.30	15659.93	24868.20
Aug 1969	17225.26	14807.73	20037.49	13668.50	21707.55
Sep 1969	14719.60	12653.61	17122.90	11680.06	18550.13
Oct 1969	21665.08	18624.25	25202.40	17191.31	27303.08
Nov 1969	20029.83	17218.84	23299.73	15894.19	25241.57
Dec 1969	16949.63	14571.08	19716.45	13450.21	21359.52

In [7]:
```
plot(forecast1)
```



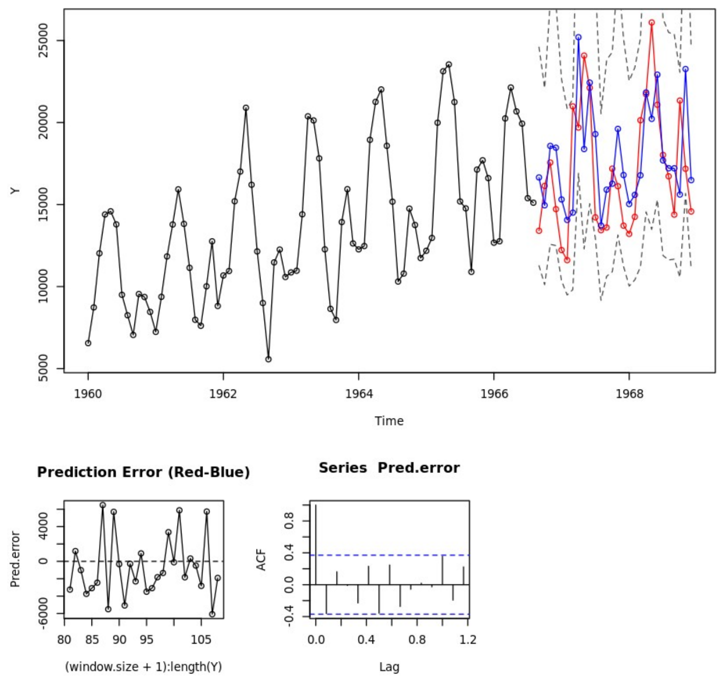
The 95% CI for the next observation is 11,656.37 to 17,889.88

In [9]:
```
Y <- D1
window.size <- 80
Arima.order <- c(2,0,0)
pred.plot <- TRUE
include.mean = TRUE
include.drift = TRUE
lambda = 0
xreg = FALSE
seasonal = c(0, 1, 2)

Rolling1step.forecast(Y, window.size, Arima.order, pred.plot, include.mean, includ
e.drift, lambda, xreg, seasonal)
```

```
Last  28  obs fit retrospectively
    with Rolling 1-step prediction
  Average prediction error:  -730.574
  root Mean Squared Error:   3470.825
```

A matrix: 1 × 2 of type dbl

mean pred error	rMSE
-730.574	3470.825



Model Regression with ARIMA(0,0,2)(1,0,0)[12]

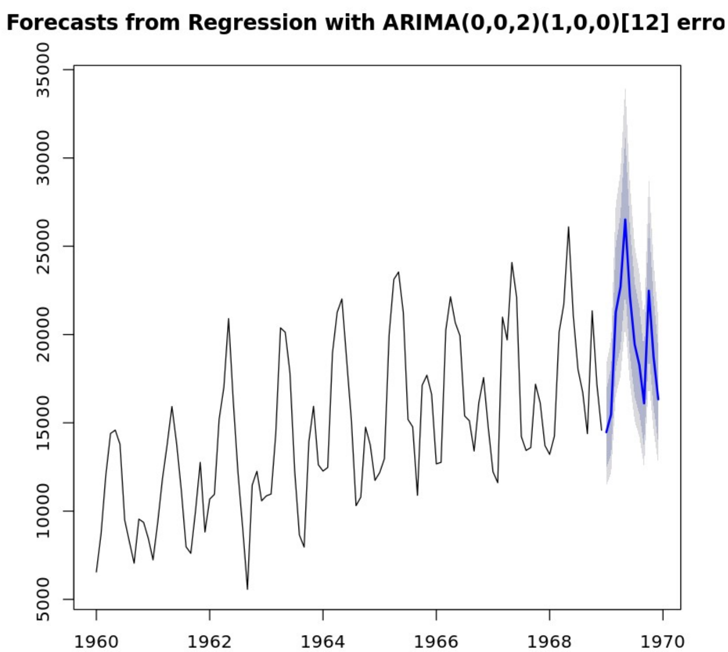
In [14]:
```
FitE2 <- Arima(D1, lambda=0, order=c(0,0,2), seasonal=c(1,0,0), xreg=time(D1))
FitE2
```

```
Series: D1
Regression with ARIMA(0,0,2)(1,0,0)[12] errors
Box Cox transformation: lambda= 0

Coefficients:
         ma1     ma2    sar1  intercept    xreg
      0.2462  0.2935  0.8437  -135.9359  0.0740
s.e.  0.0993  0.0924  0.0457    28.6874  0.0146

sigma^2 estimated as 0.01436:  log likelihood=70.87
AIC=-129.73   AICc=-128.9   BIC=-113.64
```

In [20]:
```
h = 12
forecast2 <- forecast(FitE2, xreg=last(time(D1))+(1:h)/frequency(D1))
forecast2
```

```
         Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
Jan 1969       14468.14 12408.24 16870.01 11439.31 18298.92
Feb 1969       15469.75 13206.54 18120.80 12145.76 19703.43
Mar 1969       21264.82 18039.83 25066.34 16535.61 27346.59
Apr 1969       22691.21 19249.90 26747.73 17644.78 29180.94
May 1969       26515.00 22493.78 31255.10 20618.17 34098.34
Jun 1969       22167.71 18805.80 26130.64 17237.70 28507.72
Jul 1969       19439.26 16491.14 22914.43 15116.05 24998.93
Aug 1969       18265.19 15495.12 21530.46 14203.08 23489.06
Sep 1969       16102.04 13660.03 18980.61 12521.01 20707.25
Oct 1969       22482.91 19073.19 26502.18 17482.80 28913.06
Nov 1969       18740.51 15898.36 22090.76 14572.70 24100.33
Dec 1969       16330.35 13853.72 19249.73 12698.55 21000.86
```

In [21]:
```
plot(forecast2)
```



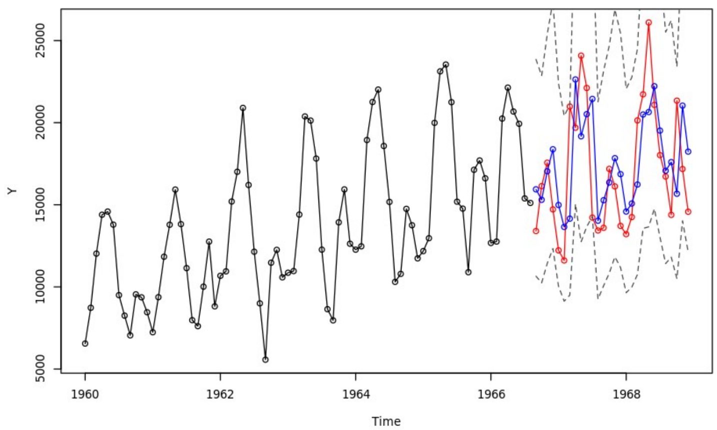
The 95% CI for the next observation is 11,439.31 to 18298.92

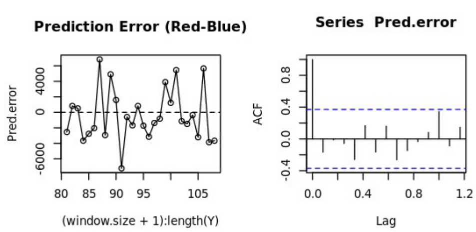
In [22]:
```
Y <- D1
window.size <- 80
Arima.order <- c(0,0,2)
pred.plot <- TRUE
include.mean = TRUE
include.drift = FALSE
lambda = 0
xreg = TRUE
seasonal = c(1, 0, 0)

Rolling1step.forecast(Y, window.size, Arima.order, pred.plot, include.mean, includ
e.drift, lambda, xreg, seasonal)
```

```
Last  28  obs fit retrospectively
    with Rolling 1-step prediction
  Average prediction error:  -445.2373
  root Mean Squared Error:   3311.3023
```

A matrix: 1 × 2 of type dbl

mean pred error	rMSE
-445.2373	3311.302





Conclusion

The ARIMA(11,0,0)(0,1,1)[12] with drift has the smallest 95% CI for the next prediction, however it has a larger rMSSE. These two models are close, since they are close I would choose the Regression with ARIMA(0,0,2)(1,0,0)[12] model since it has less terms (simpler model).