Appendix 1

Summary of Properties of the Fourier Transform

Item	Property	Mathematical Description
1.	Linearity	$ag_1(t)+bg_2(t)\longleftrightarrow aG_1(f)+bG_2(f)$ where a and b are constants
2.	Time scaling	$g(at)\longleftrightarrow\frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$ where a is a constant
3.	Duality	If $g(t)\longleftrightarrow G(f)$, then $G(t)\longleftrightarrow g(-f)$
4.	Time shifting	$g(t-t_0)\longleftrightarrow G(f)\exp(-j2\pi ft_0)$
5.	Frequency shifting	$\exp(j2\pi f_c t)g(t)\longleftrightarrow G(f-f_c)$
6.	Area under $g(t)$	$\int_{-\infty}^{\infty}g(t)dt=G(0)$
7.	Area under $G(f)$	$g(0)=\int_{-\infty}^{\infty}G(f)\,df$
8.	Differentiation in the time domain	$\frac{d}{dt}g(t)\longleftrightarrow j2\pi fG(f)$
9.	Integration in the time domain	$\int_{-\infty}^{t}g(\tau)d\tau\longleftrightarrow\frac{1}{j2\pi f}G(f)+\frac{G(0)}{2}\delta(f)$
10.	Conjugate functions	If $g(t)\longleftrightarrow G(f)$, then $g^*(t)\longleftrightarrow G^*(-f)$
11.	Multiplication in the time domain	$g_1(t)g_2(t)\longleftrightarrow\int_{-\infty}^{\infty}G_1(\lambda)G_2(f-\lambda)\,d\lambda$
12.	Convolution in the time domain	$\int_{-\infty}^{\infty}g_1(\tau)g_2(t-\tau)\,d\tau\longleftrightarrow G_1(f)G_2(f)$

Appendix 2

Fourier Transform Pairs

Time Function	**Fourier Transform**
$\text{rect}\left(\frac{t}{T}\right)$	$T\ \text{sinc}(fT)$
$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t),\ \ a>0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\lvert t\rvert),\ \ a>0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\Delta\left(\frac{t}{T}\right)=\begin{cases}1-\frac{\lvert t\rvert}{T}, & \lvert t\rvert<T\\ 0, & \lvert t\rvert\geq T\end{cases}$	$T\ \text{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t-t_0)$	$\exp(-j2\pi ft_0)$
$\exp(j2\pi f_c t)$	$\delta(f-f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)]$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\text{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty}\delta(t-iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_0}\right)$

END OF PAPER