NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2020-2021

EE6101 – DIGITAL COMMUNICATION SYSTEMS

November / December 2020 Time Allowed: 2 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 7 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.
6. A table of Fourier transform properties is provided in Appendix 1 (Page 6).
7. A table of Fourier transform pairs is provided in Appendix 2 (Page 7).

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1. (a) The random variable Z is a decision variable for the binary detection in a receiver, and is given by the following uniform probability density function (pdf) $f_Z(z)$ as shown below.

$$f_Z(z) = \begin{cases} k & if\ a_1 \leq z \leq a_2, \\ 0 & otherwise, \end{cases}$$

where k, a_1 and a_2 are fixed parameters.

(i) Determine the value of k for this pdf.
(ii) Let the value of $a_1 = -1$ and $a_2 = 2$. Calculate the value of P($|Z| \leq ½$).
(iii) Find the mean and variance of Z.

(10 Marks)

Note: Question No. 1 continues on page 2.

(b) A Wide Sense Stationary (WSS) random process **X(t)** has a Power Spectral Density (PSD) **S_X(f)**. It is applied to a differentiator to produce the output derivative random process **Y(t)** as shown in Figure 1.

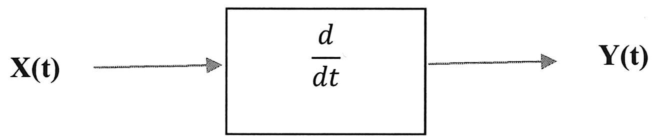

Figure 1

(i) The autocorrelation function $R_X(\tau)$ = E[X(t)X(t+τ)]. Show that the autocorrelation function $R_Y(\tau)$ of the output **Y(t)** is related as shown below. Determine the expression for the output power spectral density **S_Y(f)** of **Y(t)**.

$$R_Y(\tau) = -\frac{d^2 R_x(\tau)}{d\tau^2}.$$

(ii) If the autocorrelation function of **X(t)** is given by $R_X(\tau)$ = 16 sinc(4τ), determine the PSD of **Y(t)** and calculate its average power. Explain the significance of your results for higher frequency components of the output **Y(t)**.

(15 Marks)

2. Consider a baseband bipolar binary communication system. The received signal $r(t) = s_i(t) + n(t), i = 1, 2$, where the transmitted signals,

$$s_1(t) = \quad A \qquad 0 \le t \le T_b$$
$$s_2(t) = -A \qquad 0 \le t \le T_b,$$

are equally likely, A is the amplitude in volts, and T_b is the bit duration. The noise component $n(t)$ is the additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$. The linear filter (known *as integrate-and-dump detector*) for the receiver is shown in Figure 2 on page 3.

(a) Compute E_b the energy per bit and energy difference $E_d = \int_0^{T_b} [s_1\,(t) - s_2\,(t)]^2\, dt$.

(5 Marks)

(b) Determine the signal components a_i for $i = 1, 2$, contained in the output $r(T_b)$ in terms of A and T_b . Determine the mean and variance of the output noise when $n(t)$ is applied at the input of this receiver.

(8 Marks)

Note: Question No. 2 continues on page 3.

(c) Suppose the threshold is $\gamma_0 = (a_1 + a_2)/2$. The decision is $s_1(t)$ if $r(T_b) > \gamma_0$. Otherwise, the decision is $s_2(t)$. Derive the error probability of the receiver in terms of the Q-function, N_0 and E_b. Comment on the result obtained.

(7 Marks)

(d) Suppose the amplitude $A = 2$ Volts for the input signal is $s_i(t)$ for $i = 1, 2$, and the added noise AWGN with power spectral density $N_0/2 = 10^{-5}$ W/Hz. Determine the maximum bit rate that can be sent with a bit error probability $P_e \leq 10^{-4}$. Note that $Q(3.72) = 10^{-4}$.

(5 Marks)

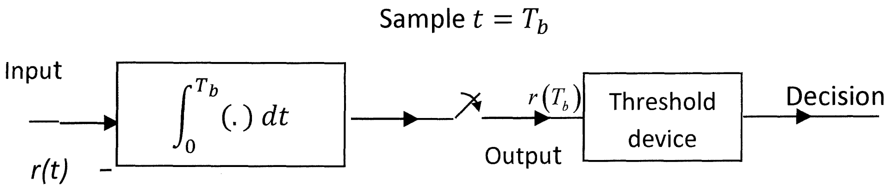

Figure 2

3. (a) Consider the (7,4) Hamming code defined by the generator polynomial:

$$g(X) = 1 + X + X^3.$$

Find the generator matrix **G** for this code in systematic form and hence, determine the parity check matrix **H**.

(9 Marks)

(b) Determine the syndrome for the received codeword 0101001 for the Hamming code defined in part (a).

(4 Marks)

(c) The state diagram of a convolutional code is given in Figure 3 on page 4.

Note: Question No. 3 continues on page 4.

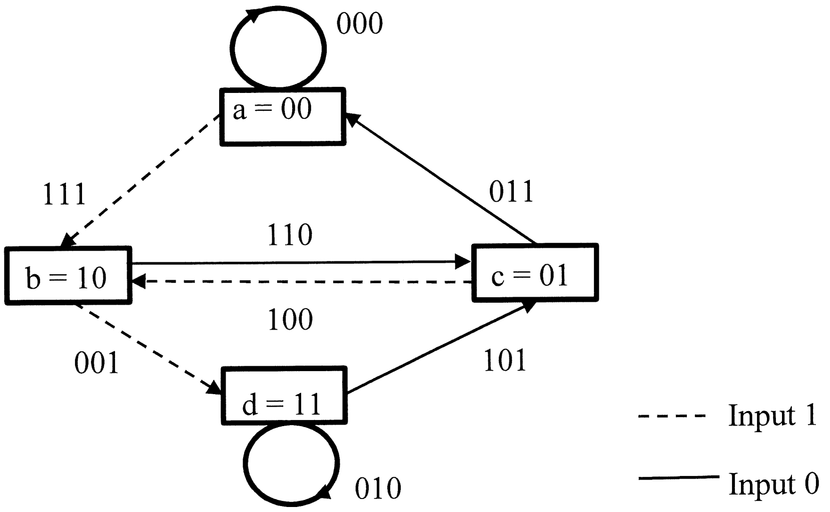

Figure 3

Decode the received sequence (starting from left to right), **r** = [111 000 001 101 110 011] using the stack algorithm. Illustrate, using the tree diagram and accumulated path metrics stack table, the steps involved. You may assume that the metric for the jth branch of the rth path is

$$u_j^{(r)} = 1 - 2d$$

where d is the Hamming distance between the received bits and the branch bits.

(12 Marks)

4. The block diagram of a downlink CDMA system operating at baseband in AWGN channel with 2 paths (one direct and one delayed of one T_c paths) of equal amplitude of 1 is shown in Figure 4 on page 5, where b_i is the binary antipodal data of i^{th} user with values $\{\pm 1\}$, $c_i(t)$ is the spreading code of i^{th} user, M is the system capacity, T_b is the data bit duration of 10^{-3} seconds, and T_c is the spreading chip duration. The AWGN power spectral density of $n(t)$, $N_0 = 10^{-4}$ W/Hz . The received signal $r(t) = b_1 c_1(t) + \sum_{i=2}^{M} b_i c_i(t) + \sum_{i=1}^{M} b_i c_i(t - T_c) + n(t)$.

Note: Question No. 4 continues on page 5.

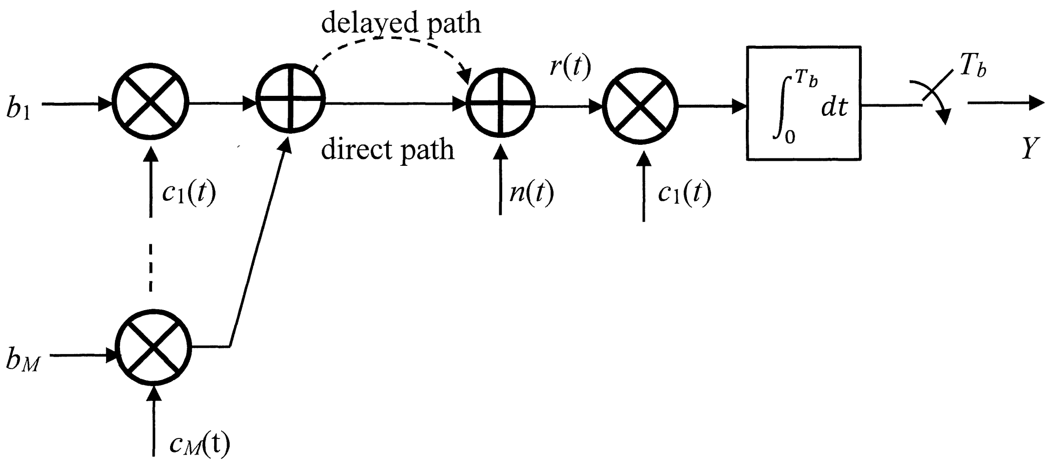

Figure 4

(a) Show that the expression of the output Y can be reduced to:

$$Y = b_1 T_b + T_c \sum_{i=2}^{M} b_i \phi_{i1}(0) + T_c \sum_{i=1}^{M} b_i \phi_{i1}(1) + N$$

and specify the meanings of the terms $\phi_{i1}(0)$, $\phi_{i1}(1)$ and N in the expression.

(11 Marks)

(b) If the spreading codes are obtained from the full period of Walsh Hadamard codes of length 32 and assuming that the code cross-correlation has an average variance of 25, determine the system capacity to maintain the bit-error-rate (BER) per user of at least 10^{-3}. You may assume that the inter-symbol interference (ISI) can be approximated with Gaussian distribution with zero mean. Note that $Q(3.09) \leq 10^{-3}$.

(14 Marks)

Appendix 1

Summary of Properties of the Fourier Transform

Item	Property	Mathematical Description
1.	Linearity	$ag_1(t)+bg_2(t) \longleftrightarrow aG_1(f)+bG_2(f)$ where a and b are constants
2.	Time scaling	$g(at) \longleftrightarrow \frac{1}{\lvert a \rvert} G\left(\frac{f}{a}\right)$ where a is a constant
3.	Duality	If $g(t) \longleftrightarrow G(f)$, then $G(t) \longleftrightarrow g(-f)$
4.	Time shifting	$g(t-t_0) \longleftrightarrow G(f)\exp(-j2\pi f t_0)$
5.	Frequency shifting	$\exp(j2\pi f_c t)g(t) \longleftrightarrow G(f-f_c)$
6.	Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)dt = G(0)$
7.	Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\, df$
8.	Differentiation in the time domain	$\frac{d}{dt} g(t) \longleftrightarrow j2\pi f G(f)$
9.	Integration in the time domain	$\int_{-\infty}^{t} g(\tau)d\tau \longleftrightarrow \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2}\delta(f)$
10.	Conjugate functions	If $g(t) \longleftrightarrow G(f)$, then $g^*(t) \longleftrightarrow G^*(-f)$
11.	Multiplication in the time domain	$g_1(t)g_2(t) \longleftrightarrow \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\, d\lambda$
12.	Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\, d\tau \longleftrightarrow G_1(f)G_2(f)$

Fourier Transform Pairs

Time Function	**Fourier Transform**
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\ \mathrm{sinc}(fT)$
$\mathrm{sinc}(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t),\quad a>0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\lvert t\rvert),\quad a>0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\Delta\left(\frac{t}{T}\right)=\begin{cases}1-\frac{\lvert t\rvert}{T}, & \lvert t\rvert<T\\ 0, & \lvert t\rvert\geq T\end{cases}$	$T\ \mathrm{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t-t_0)$	$\exp(-j2\pi ft_0)$
$\exp(j2\pi f_c t)$	$\delta(f-f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty}\delta(t-iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_0}\right)$

END OF PAPER

EE6101 DIGITAL COMMUNICATION SYSTEMS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.