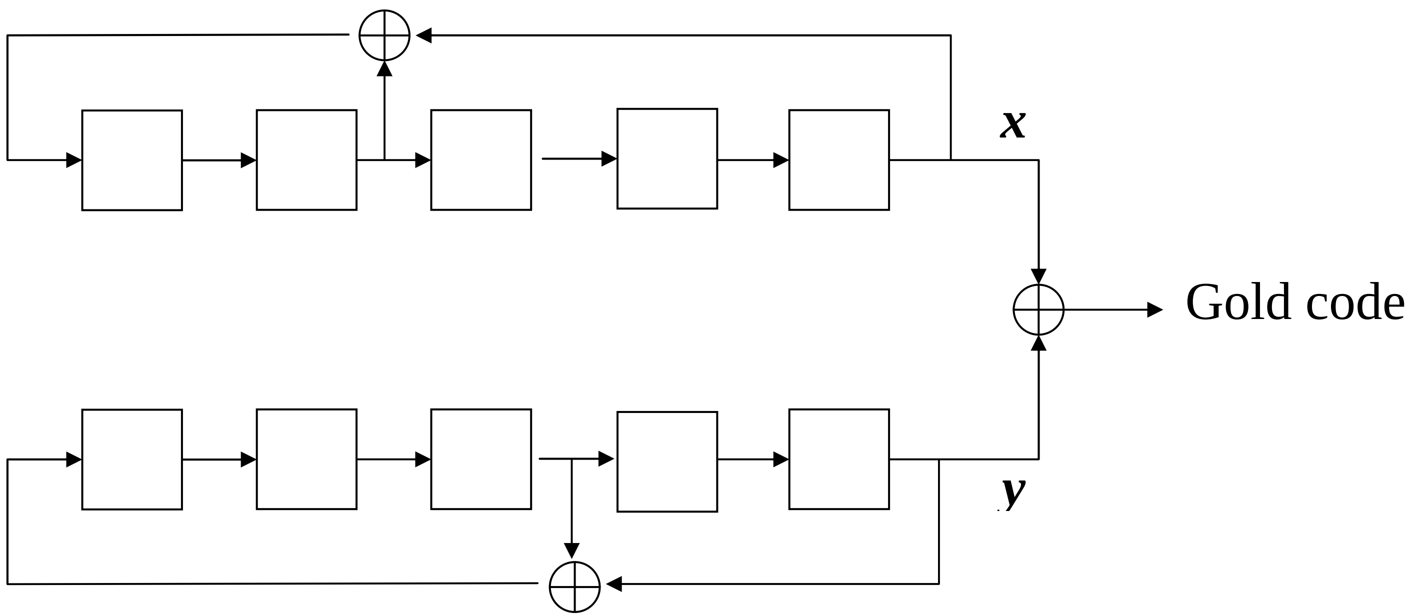
Gold Code/Sequence

- One important class of binary PN sequence which can provide large set of sequences with good, bounded cross-correlation values. Invented in 1967 by R.Gold.

- Constructed by modulo-2 addition of 2 selected m-sequences called the "preferred pair".



Construction of Gold Sequence

1. Identify a **preferred pair** of m-sequences from a set of m-sequences with the same length but constructed using different LFSR generators. A preferred pair of m-sequence has **only 3 crosscorrelation values** of:

   $\phi_{xy}(\mathrm{k}) = -1, -t(m)$ or $t(m)-2$ for all values of k,

   where

   $t(m) = 1 + 2^{\lfloor (m+2/)2 \rfloor}$ with $\lfloor x \rfloor$ denotes rounding x down to the next smaller integer

2. Denoting the preferred pair as $\boldsymbol{x}$ and $\boldsymbol{y}$, the set of Gold sequences will be:

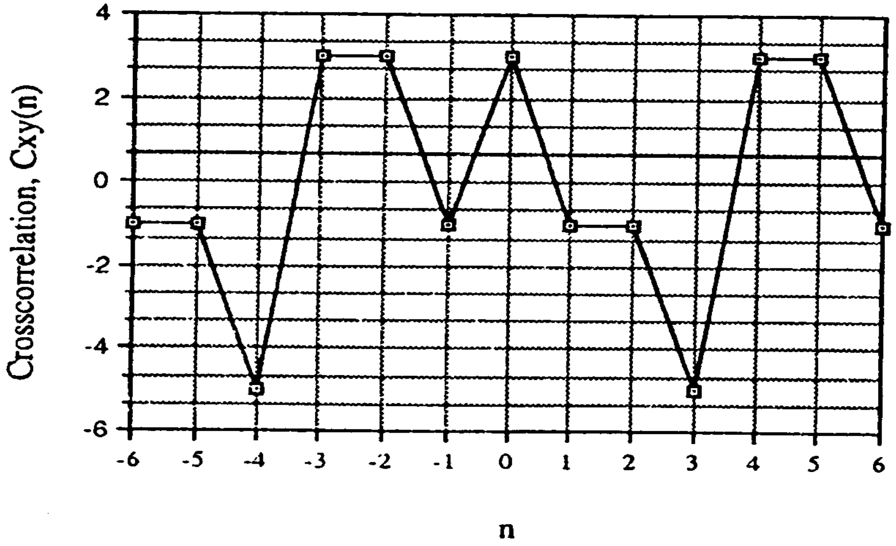
   $\{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{x} \oplus \boldsymbol{y}, \boldsymbol{x} \oplus$ all shifted versions of $\boldsymbol{y}\}$

   Since $\boldsymbol{x}$ and $\boldsymbol{y}$ are each m-sequence with period $N=2^m-1$, there are $N+2=2^m+1$ Gold sequences in the same set.

3. It is known that **no** preferred pairs exist for m-sequence with m = 4, 8, 12, 16.

Correlation Properties of Gold Sequence

4. Cross-correlation of Gold sequences in the same set is the same as that of the preferred pair used to generate them, ie. there are only 3 crosscorrelation values of $\phi_{xy}(\mathrm{k}) = -1, -t(m)$ or $t(m)-2$ for all values of k



5. The max. cross-correlation = $t(m)/N \cong 2^{-m/2}$ approaches zero as m increases → Good

6. Except for the preferred pair ***x*** and ***y***, the off-peak autocorrelation of Gold sequences also has the same 3 values as the X-correlation ← worse than auto-correlation of m-sequence of the same length ← classical trade-off between auto- and X-correlation of any PN sequence

Eg. Length-7 Gold codes

m-seq$_1$ = [–1 +1 +1 -1 +1 –1 –1] $\rightarrow$ auto-corr $\phi_{11}(1) = \phi_{11}(2) = ... = \phi_{11}(6) = -1$

m-seq$_2$ = [+1 –1 –1 –1 +1 –1 +1] $\rightarrow$ auto-corr $\phi_{22}(1) = \phi_{22}(2) = ... = \phi_{22}(6) = -1$

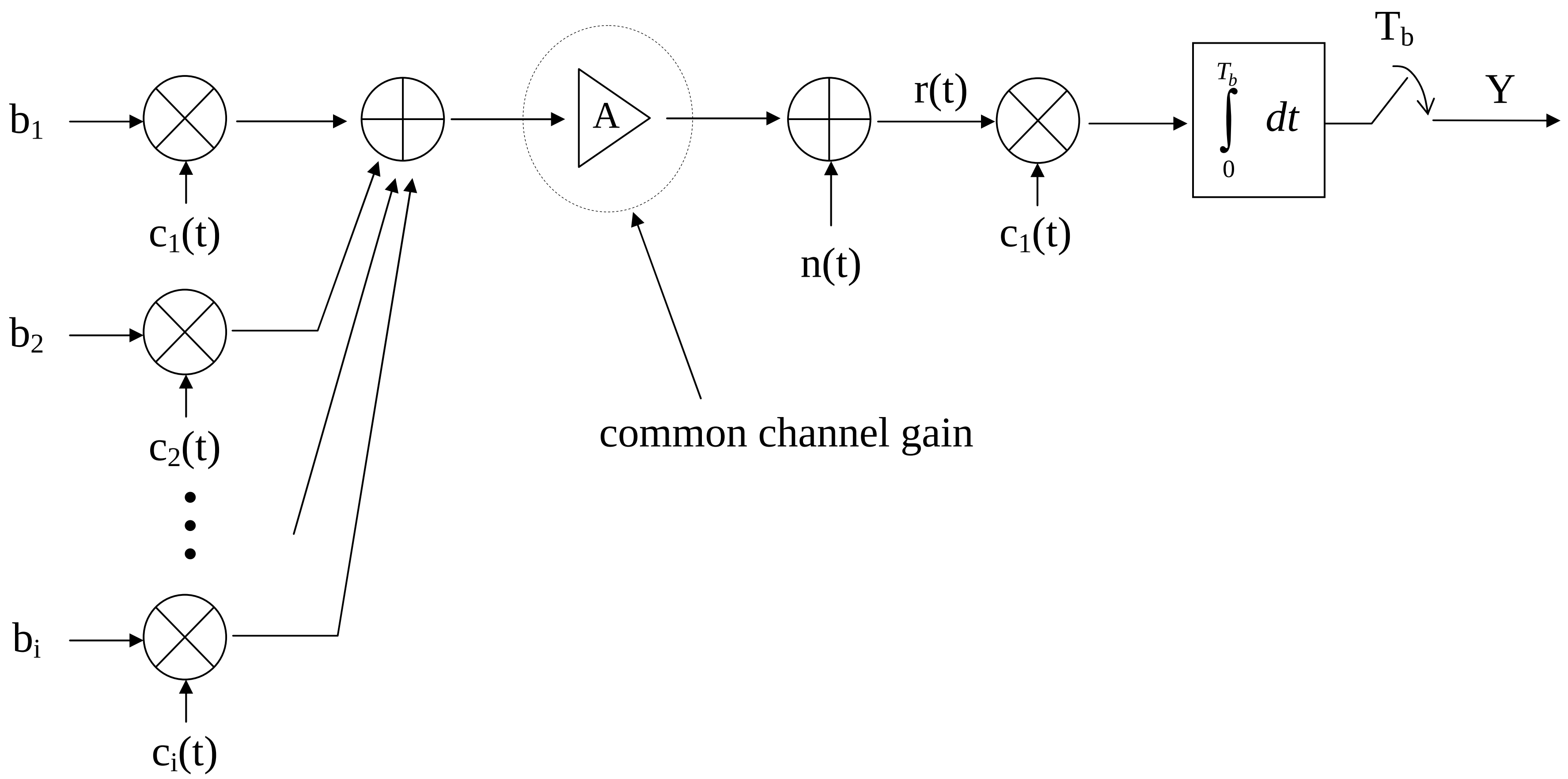
Gold seq = [–1 –1 –1 +1 +1 +1 –1] $\rightarrow$ auto-corr $\phi_{33}(1) = 3$, $\phi_{33}(2) = -1$, $\phi_{33}(3) = -5$

$\rightarrow$ cross-correlation $\phi_{13}(0) = -1$, $\phi_{23}(5) = -5$, $\phi_{12}(3) = -1$, $\phi_{23}(-3) = 3$

CDMA Wireless Systems

Synchronous CDMA Channel: Downlink

Since the downlink is a hub-to-user communication channel, all user signals can be properly aligned/synchronized and combined before being transmitted. If there exists a dominant signal propagation path from the transmitter to every user without major reflections, then all users receive a synchronous CDMA signal and every user's communication link can be modelled as:



Signal Correlation vs. Sequence Correlation

- Cross-correlation of **2** time **functions/signals** $\phi_{ij}(\tau) = \int_0^{T_b} c_i(t)c_j(t-\tau)dt$
- Cross-correlation of 2 **sequences** $\phi_{ij}(k) = \sum_{n=1}^{N} c_i(n)c_j(n-k)$
- Auto-correlation of **a** time function/signal $\phi_{ii}(\tau) = \int_0^{T_b} c_i(t)c_i(t-\tau)dt$
- Auto-correlation of a sequence $\phi_{ii}(k) = \sum_{n=1}^{N} c_i(n)c_i(n-k)$

What are their ideal values?

Signal Correlation

$$= \int_0^{T_b} c_i(t)c_j(t-\tau)dt = T_c \times \sum_{n=1}^{N} c_i(n)c_j(n-k)$$

$$= T_c \times \text{Sequence Correlation}$$

Effect of MAI on Synchronous CDMA Performance

$$r(t) = Ab_1 c_1(t) + A\sum_{i=2}^{M}\left[b_i c_i(t)\right] + n(t)$$

$$Y = \int_0^{T_b} r(t) c_1(t)\, dt$$

$$= Ab_1 \int_0^{T_b} c_1^{\,2}(t)\, dt + A\sum_{i=2}^{M}\left[b_i \int_0^{T_b} c_i(t) c_1(t) dt \right] + \int_0^{T_b} n(t) c_1(t)\, dt$$

$$= (\pm) AT_b \qquad\qquad + AT_c \sum_{i=2}^{M}\left[b_i\, \phi_{i1}(k=0)\right] \; + \int_0^{T_b} n(t) c_1(t)\, dt$$

$$= \text{useful signal} \qquad + \text{MAI} \qquad\qquad + \;\text{noise}$$

<u>Case 1: Orthogonal Spreading</u>

If full-period WH spreading is used, $\phi_{i1}(0) = 0$ → MAI = 0 irregardless of the sign of $b_i(t)$, so

$$\text{BER} = Q\left(\sqrt{\frac{A^2 T_b^{\,2}}{N_0 T_b / 2}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$ ← as if there are no other users in the system!

Case 2: Non-Orthogonal Spreading

If full-period Gold-code spreading is used,

→ $\phi_{i1}(0) = -1$ or $-t(m)$ or $t(m)-2$

→ MAI is random because b_i can be +1 or -1

→ If the number of CDMA users (M) is large, Central Limit Theorem suggests that MAI is approximately Gaussian.

→ This MAI then combine with the thermal noise to result in a net AWGN interference component with larger power, ie. Y = useful signal + AWGN (MAI + thermal noise)

Since MAI is assumed to be Gaussian, we only need to know its mean $E[\bullet]$ and variance $VAR[\bullet]$

→ $E[MAI] = 0$ since $E[b_i] = 0$

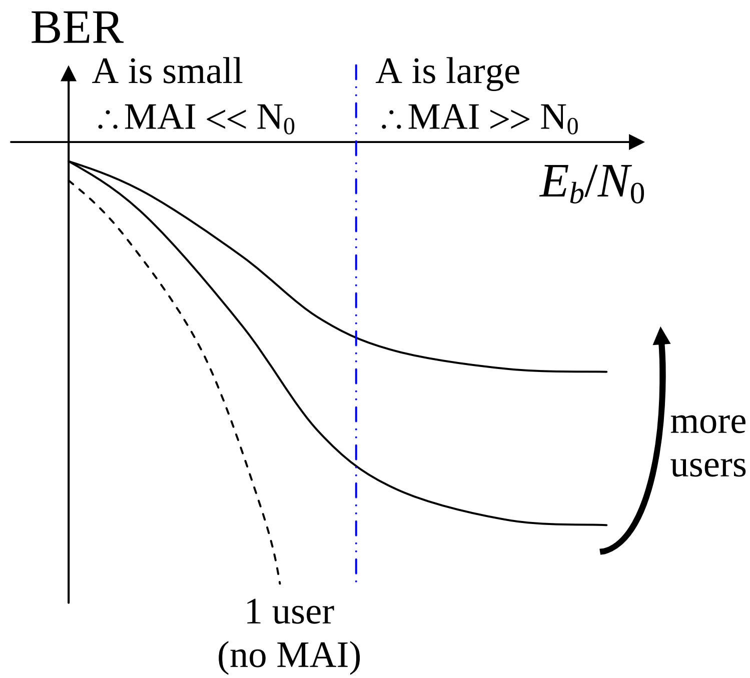
→ $Var[MAI] = E[MAI^2] - \cancelto{0}{E^2[MAI]}$

$$= A^2\, T_c^2\, E\left[\sum_{i=2}^{M} b_i^2\, \phi_{il}^2\big(k=0\big)\right] = A^2\, T_c^2 \sum_{i=2}^{M} E\left[\phi_{il}^2\big(k=0\big)\right]$$

As a result of MAI with mean = 0 but variance ≠ 0,

$$\text{BER} = Q\left(\sqrt{\frac{A^2 T_b^2}{A^2 T_c^2 \sum_{i=2}^{M} E\left[\phi_{il}^2(k=0)\right] + N_0 T_b / 2}}\right) = Q\left(\sqrt{\frac{2E_b}{MAI_0 + N_0}}\right)$$

where $MAI_0 = \frac{2}{T_b} \times A^2 T_c^2 \sum_{i=2}^{M} E\left[\phi_{il}^2(k=0)\right]$ = MAI energy per bit



Obviously, the presence of **MAI degrades the BER** of the system. Worse, it introduces **irreducible BER floor at high E_b/N_0 values** when $MAI_0 >> N_0$, which can only be reduced by increasing the processing gain of the system.

$$\text{BER} = Q\left(\sqrt{\frac{A^2 T_b^2}{A^2 T_c^2 \sum_{i=2}^{M} E\left[\phi_{il}^2(k=0)\right]}}\right) = Q\left(\frac{\text{PG}}{\sqrt{\sum_{i=2}^{M} E\left[\phi_{il}^2(k=0)\right]}}\right)$$

where PG (processing gain) = T_b / T_c

Scrambled Spreading Codes

1. To scramble = to randomize

2. A CDMA spreading code (eg. WH code) can be scrambled by mod-2 addition with a PN code (eg. a long m-sequence) of the same rate.

   Eg. length-8 WH: 1 0 1 0 1 0 1 0 *1 0 1 0 1 0 1 0 … (repeated)*
   length-15 m-seq: 1 1 1 1 0 1 0 1 1 0 0 1 0 0 0 *1 1 1 1 0 1 0 …*
   ➔ scrambled WH: 0 1 0 1 1 1 1 1 0 0 1 1 1 0 1 1 …

   <u>Observations</u>:
   - The scrambled code appear random and has longer repetition period
   - Chip rate (hence processing gain) does not change after scrambling

3. The scrambled code can then be used for spreading the user data. At the receiver, use the same scrambled code to de-spread.

4. Different base stations may use different scrambling codes (long m-seq) to scramble a WH code set (eg. WH-64) to generate different spreading code sets for different cells
   → code re-use

5. Scrambled WH codes within a synchronous CDMA cell are still orthogonal because they are scrambled by the same spreading code.

Eg. scrambled WH from last page: $\mathbf{c_1} = 0\ \ 1\ \ 0\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1$

[0 0 0 0 1 1 1 1] scrambled by the same m-seq: $\mathbf{c_2} = 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0\ \ 1\ \ 0$

➔ cross-correlation of $\mathbf{c_1}$ and $\mathbf{c_2}$ = –1 +1 –1 +1 +1 –1 +1 –1 = 0

<u>Proof</u>

($\mathbf{w}_i$: WH code, $\mathbf{m}_i$: long m-sequence, $\mathbf{w}_i\mathbf{m}_i$: WH code scrambled by the long m-seq)

$\mathbf{c_1} = \mathbf{w_1 m_1}$, $\mathbf{c_2} = \mathbf{w_2 m_1}$

Cross correlation of $\mathbf{c_1}$ and $\mathbf{c_2}$

$= \sum(\mathbf{c_1} \cdot \mathbf{c_2})$

$= \sum(\mathbf{w_1 m_1} \cdot \mathbf{w_2 m_1})$

$= \sum(\mathbf{w_1} \cdot \mathbf{w_2} \cdot \mathbf{m_1}^2)$

$= \sum(\mathbf{w_1} \cdot \mathbf{w_2})$ because $\mathbf{m_1}^2 = [1\ 1\ \ldots\ 1]$, i.e. all ones

$= 0$ because the sequences $\mathbf{w_1}$ and $\mathbf{w_2}$ are orthogonal

6. Scrambled WH codes from different CDMA cells are scrambled by different scrambling codes, this generally reduces their MAI.

Eg. Unscrambled length-8 WH from Cell A: 1 0 1 0 1 0 1 0
Same unscrambled length-8 WH from Cell B: 1 0 1 0 1 0 1 0
➔ cross-correlation = +1 +1 +1 +1 +1 +1 +1 +1 = +8 (large MAI)

Cell A's WH code scrambled by [1 1 1 1 0 1 0 1]: 0 1 0 1 1 1 1 1
Same Cell B's WH code scrambled by [1 1 0 1 0 1 1 0]: 0 1 1 1 1 1 0 0
➔ cross-correlation = +1 +1 -1 +1 +1 +1 –1 –1 = +2 (smaller MAI)

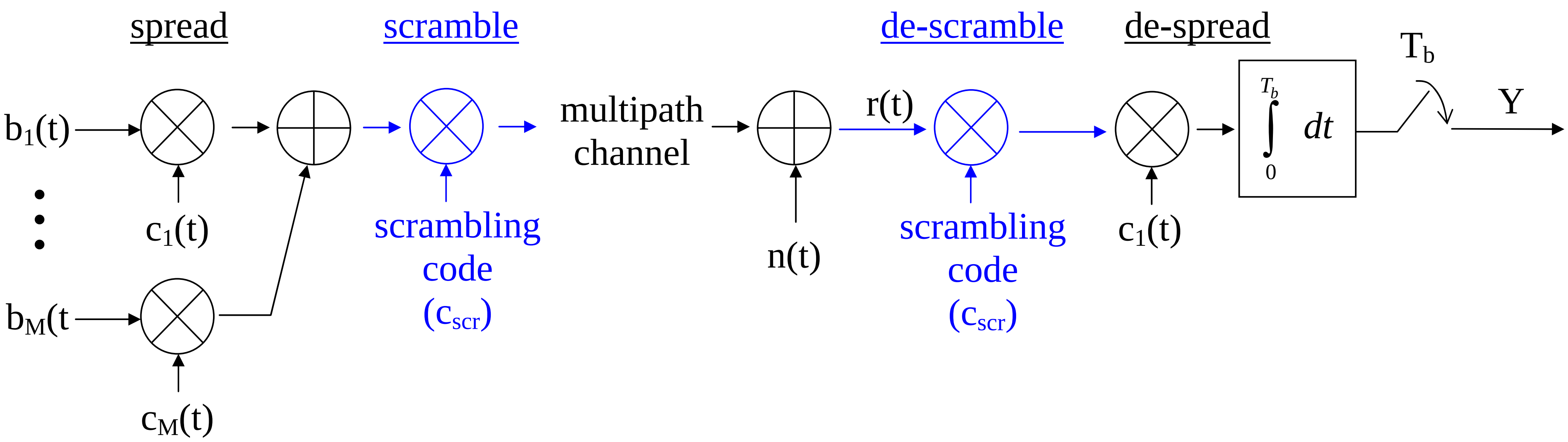
7. Scrambling also reduces the multipath ISI of some CDMA codes which have high autocorrelation values (eg. WH codes)

Eg. Unscrambled length-8 WH from Cell A: 1 0 1 0 1 0 1 0
Its multipath signal delayed by 1 chip time: 0*(or 1)* 1 0 1 0 1 0 1
➔ cross-correlation = –1*(or+1)* –1 –1 –1 –1 –1 –1 –1 = –8 or –6 (large ISI)

Cell A's WH code scrambled by [1 1 1 1 0 1 0 1]: 0 1 0 1 1 1 1 1
Its multipath signal delayed by 1 chip time: 0*(or 1)* 0 1 0 1 1 1 1
➔ cross-correlation = +1*(or –1)* -1 -1 -1 +1 +1 +1 +1 = +2 or 0 (smaller ISI)

Effect of Multipath ISI

To reduce MAI and multipath ISI, the downlink signal can be scrambled before transmission:



First, assume that the channel has 1 synchronized direct path + 1 delayed path with phase θ_i and delay equal to integer number of T_c

$$r(t) = \underbrace{A\, b_1\, c_1(t) c_{scr}(t) + A \sum_{i=2}^{M} b_i\, c_i(t)\, c_{scr}(t)}_{\text{Direct path}} + \underbrace{A' \sum_{i=1}^{M} b'_i\, c'_i(t)\, c'_{scr}(t) \cos\theta_i}_{\text{Delayed path}} + n(t)$$

$$Y = \int_0^{T_b} r(t) c_1(t) c_{scr}(t)\, dt$$

$$= Ab_1 \int_0^{T_b} c_1^2 c_{scr}^2 \, dt + A \sum_{i=2}^{M} \left[b_i \int_0^{T_b} c_i(t) c_1(t) c_{scr}^2 \, dt \right] + A' \sum_{i=1}^{M} \left[b'_i \cos\theta_i \int_0^{T_b} c'_i(t) c'_{scr}(t) \cdot c_1(t) c_{scr}(t) dt \right] + \text{noise}$$

$$= (\pm) AT_b \quad + \quad AT_c \sum_{i=2}^{M} b_i\, \phi_{i1}(k=0) \quad + \quad A' \sum_{i=1}^{M} \left[b'_i \cos\theta_i \int_0^{T_b} (\text{random} \pm 1 \text{ pulses}) dt \right] \; + \; \text{noise}$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \; \underline{A'T_c \sum_{i=1}^{M} \left[b'_i \cos\theta_i \sum_{n=1}^{PG} (\text{random} \pm 1) \right]} \; \ldots\ldots\ldots\ldots$$

Mean = 0, Variance = $A'^2\, T_c^2 \sum_{i=1}^{M} \left[E\left[b'^2_i\right] E\left[\cos^2\theta_i\right] \sum_{n=1}^{PG} E\left[(\pm 1)^2\right] \right] = \dfrac{A'^2\, T_c^2\, M \cdot \text{PG}}{2}$

In general, if the channel has L paths, Variance = $\sum_{l=2}^{L} A_l^2 \cdot \dfrac{T_c^2\, M \cdot \text{PG}}{2}$

At high E_b/N_0 values (i.e., ignoring channel noise),

$$\mathrm{BER} = Q\left(\sqrt{\frac{A^2 T_b^2}{\underbrace{A^2 T_c^2 \sum_{i=2}^{M} E\left[\phi_{i1}^2(k=0)\right]}_{\text{MAI}} + \underbrace{\sum_{l=2}^{L} A_l^2 \cdot \frac{T_c^2\, M \cdot \mathrm{PG}}{2}}_{\text{ISI}}}}\right) = Q\left(\frac{\mathrm{PG}}{\sqrt{\sum_{i=2}^{M} E\left[\phi_{i1}^2(k=0)\right] + \sum_{l=2}^{L}\left(\frac{A_l}{A}\right)^2 \frac{M \cdot \mathrm{PG}}{2}}}\right)$$

<u>Conclusion</u>:

Multipaths give rise to additional interference which degrades the BER and capacity of CDMA downlink.

Asynchronous CDMA Channel: Uplink

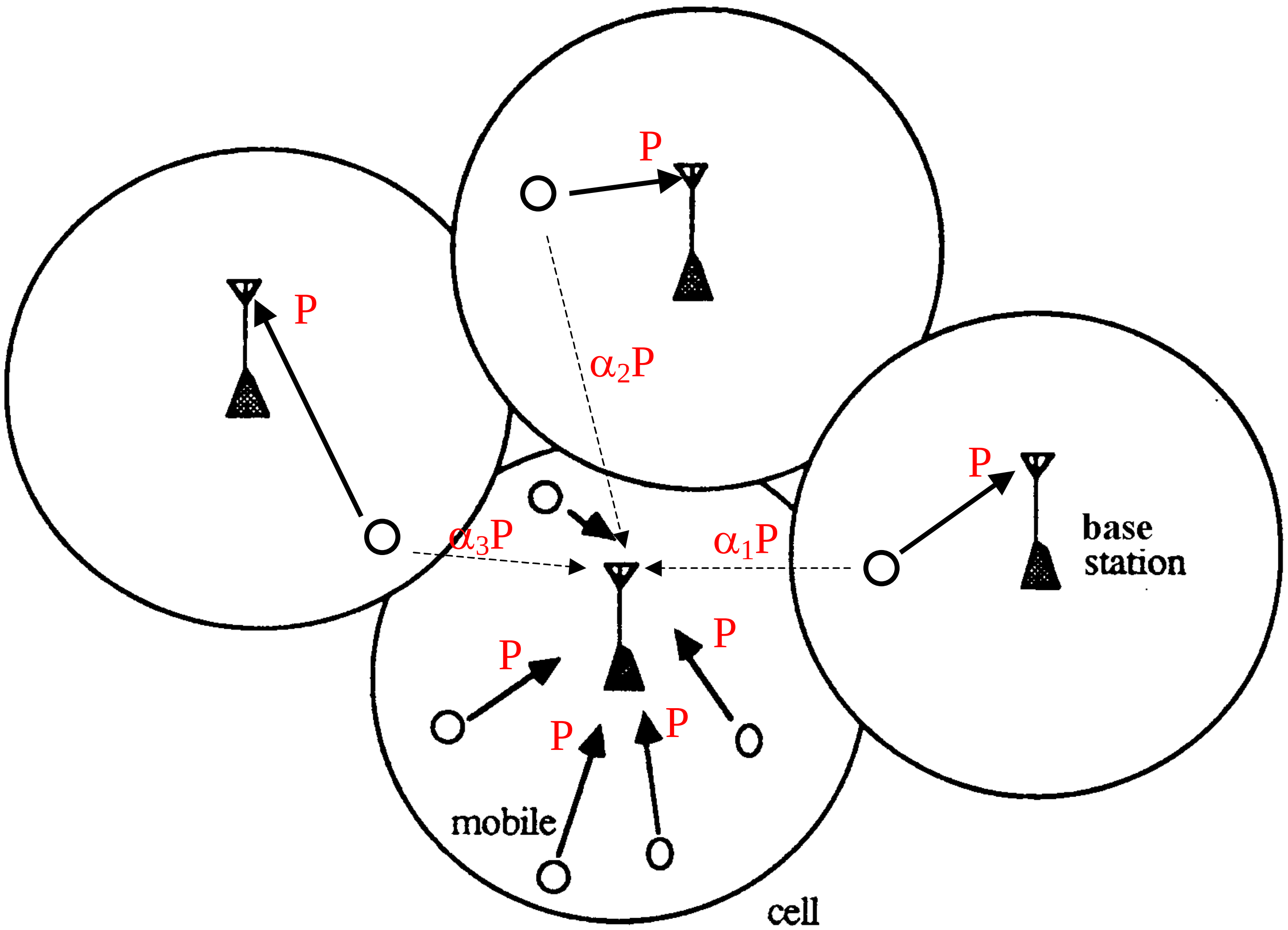
- In contrast to the downlink, the uplink is a user-to-hub communication channel.

- All users transmit asynchronously and their signals arrive at the base station not only with different time delays, but also different carrier phase.

- Signals from the same cell can be **power-controlled** by their base station so as to be received with the same amplitude, but signals from other cells generally arrive with different amplitudes, which are subject to free-space path loss as well as shadowing fluctuation.

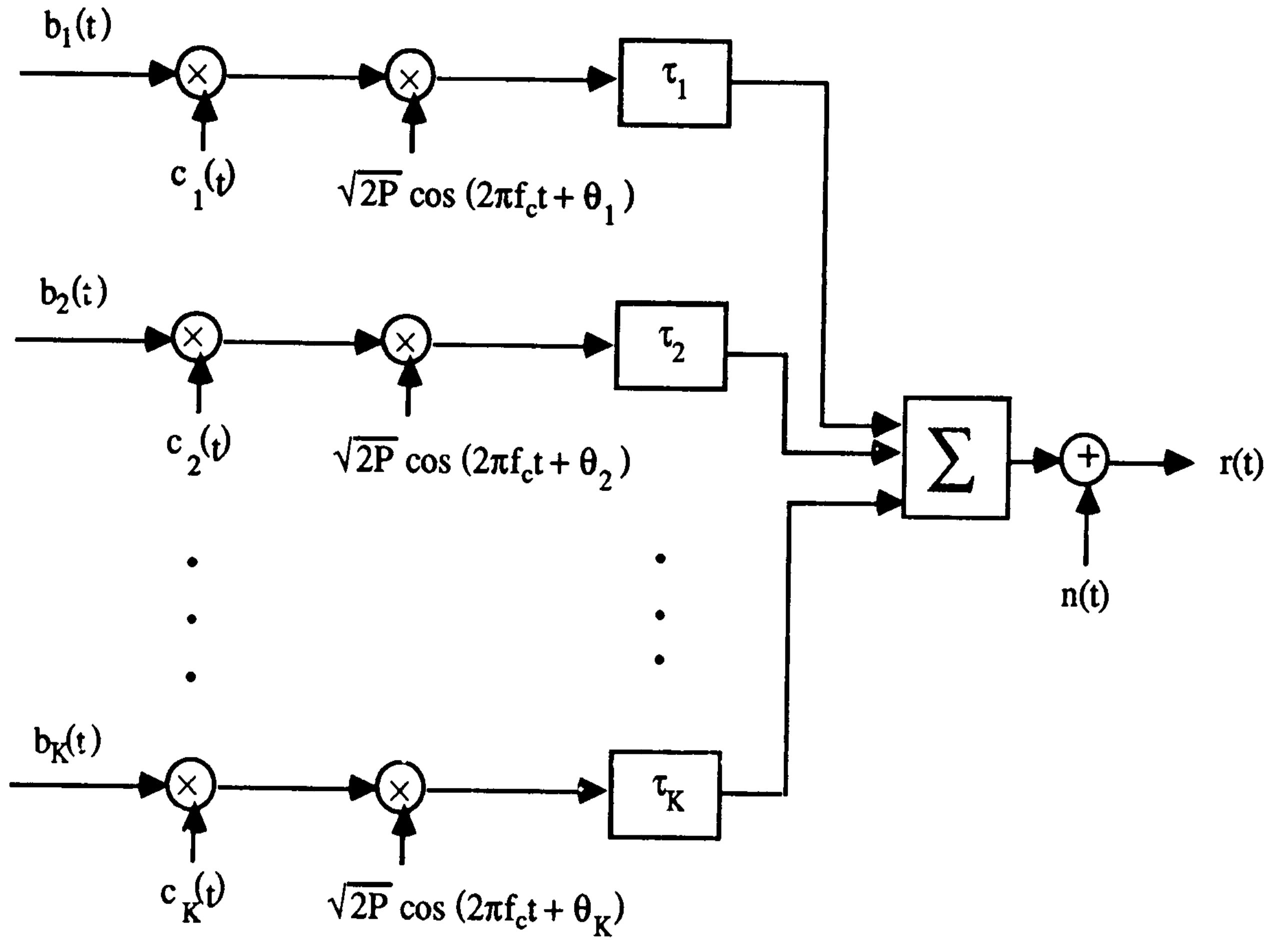
- In the **downlink**, MAI **on a user** comes **from neighbouring base stations**. In the **uplink**, MAI **on a base station** comes **from ALL USERS** in the systems, as shown overleaf.

Uplink Interference in a Cellular DS-CDMA System

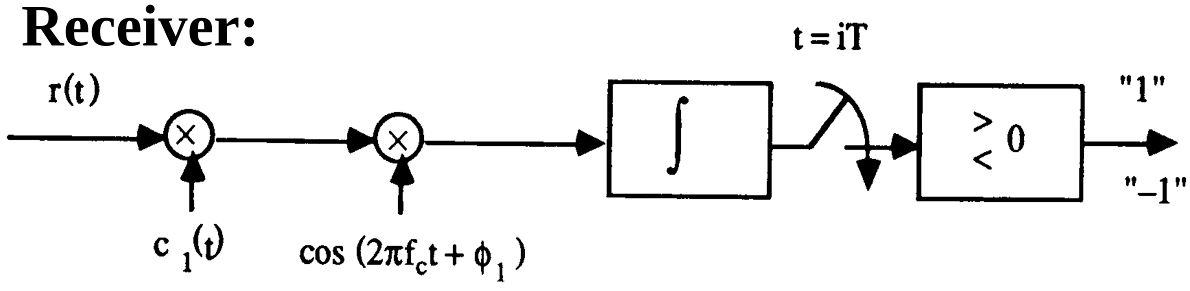


DS-CDMA Uplink Channel Model

Transmitter:



Receiver:



CDMA System Capacity

- System capacity = user capacity = **total number of users supported** by a cellular system while maintaining some specified performance criterion, eg. BER $\leq 10^{-3}$ for voice communication, BER $\leq 10^{-6}$ for data communication.

- For cellular CDMA systems with mainly **symmetrical voice traffic** over both the uplink and downlink, the downlink tends to have a larger capacity because it has better BER due to its predominantly synchronous nature. The uplink is highly asynchronous, suffers a lot of MAI and ISI and hence can only support lower capacity.

- As a result, cellular CDMA system capacity is typically limited by the uplink capacity.

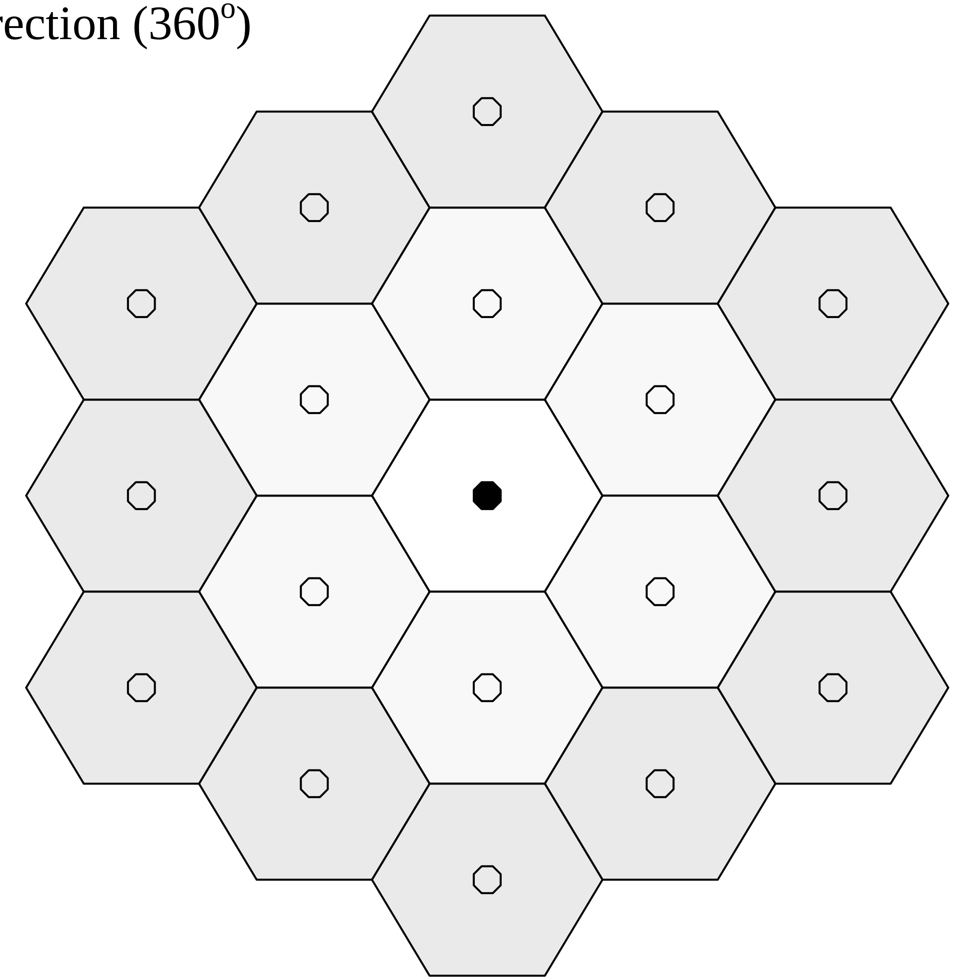
CDMA Uplink Capacity

- The most accurate way to evaluate system capacity is to develop detailed BER expression in terms of number of users in the system and setting BER $\leq 10^{-3}$. This approach is however rather complex for the CDMA uplink as it requires the knowledge of the aperiodic auto- and cross-correlation characteristics of the spreading codes used, among other things.

- A simplified approach widely used by industry to give a good estimate to the CDMA uplink capacity advantage over FDMA or TDMA systems will be adopted in this course. The most important assumptions of this approach is that **random spreading codes** are used (this can be approached in practice by adopting a partial-period spreading per data bit using a very long *m*-sequence, eg. with period = $2^{23} - 1$). With random spreading, all MAI contributions are random with mean = 0 and variance $\propto \dfrac{1}{PG}$.

MAI Reduction Techniques for CDMA Capacity Enhancement

Consider a cellular DS-CDMA system with regular, identical cell shapes and *M* users uniformly distributed within every cell. The desired user is located in the central cell. All base station antennas are omni-direction (360^{o})



Let the signal power of the desired user received by the central base station be P. Assuming perfect power control for simplicity sake, there are M – 1 interferers each with power P from within the same cell, so the signal to noise power ratio (SNR) at the central base station is:

$$\mathrm{SNR} = \frac{P}{(M-1)P + N_0 W}$$

own-cell MAI

There are 6 neighbouring cells in the first cell-ring. Assuming each interferer from these cells contributes an MAI power of ηP to the central base station, where η = average loading factor of a neighbouring cell, then its SNR becomes:

other-cell MAI

$$\mathrm{SNR} = \frac{P}{(M-1)P + 6\eta M P + N_0 W}$$

Here we assume for simplicity sake that the outer cell-ring contributes negligible MAI to the central base station due to their large geographical separation. In other words, the value of η for users in the outer cell-ring is close to 0.

Voice-Activated Discontinuous Transmission

Obviously, mobile users do not talk all the time (they need to listen and breathe!). From statistical studies, most of them talk only 3/8 of the time. This implies that at any one time, only about 3/8 of the users in the system are talking. Since CDMA systems are interference-limited, it is clearly beneficial to suppress mobile transmit power when the user is not talking, so that effective MAI can be reduced to 3/8 of the original. This facility, called **voice-activated discontinuous transmission**, can be implemented in the mobile phone equipped with voice-activity detection such that it transmits power only when the user is speaking. Denoting the voice activity factor (or voice duty cycle) by α, the SNR improves to:

$$\text{SNR} = \frac{P}{[(1+6\eta)M-1]\alpha P + N_0 W}$$

Sector/Directional Antenna

Another effective technique to reduce MAI is to use **sector (or directional) antenna** with much narrower beamwidth than the omni-directional antenna. With this, every cell becomes a sector (or sectored cell) which receives only a fraction of the total MAI as before. Since most practical sector antennas have sidelobes or backlobes which may not be regularly shaped, a convenient way to characterize them is to assume that they have an effective antenna gain of 1 for λ^{o} (which may somewhat overlap with the neighbouring sectors) and 0 otherwise. Therefore, MAI is reduced to $\beta = (\lambda^{o}/360^{o})$ of its original value and the SNR is further improved:

$$\text{SNR} = \frac{P}{\left[(1+6\eta)M-1\right]\alpha\,\beta\,P + N_0 W} \quad \text{where } \beta = \text{antenna factor}$$

$$\boxed{\text{SNR} = \frac{\text{signal power}}{\text{total (noise + interference) power}} = \frac{E_b\, r_b}{(N_0 + I_0)W} = \frac{E_b}{(N_0 + I_0)PG}}$$

where

r_b = user data rate (typically in Kbps)

W = system BW = signal BW after spreading

PG = processing gain = $\dfrac{T_b}{T_C} = \dfrac{W}{r_b}$

Usually $I_o >> N_o$ (since there are many users in the system), so

$$\frac{E_b}{(N_0 + I_0)PG} \cong \frac{E_b}{(I_0)PG} = \frac{P}{[(1+6\eta)M - 1]\alpha\,\beta\,P}$$

$$\rightarrow \quad \frac{E_b}{I_0} = \text{signal to interference energy ratio} \cong \frac{PG}{[(1+6\eta)M - 1]\alpha\,\beta}$$

For the IS-95 CDMA standard, a minimum receiver $\frac{E_b}{I_0}$ threshold of 7dB is required for satisfactory functioning of the base station receiver, so

$$\frac{PG}{[(1+6\eta)M-1]\alpha\beta} > 7\text{dB} = 10^{7/10} = 5.01$$

$\rightarrow$ user capacity per cell $M < \left(\frac{PG}{5.01\alpha\beta}+1\right)\times\frac{1}{(1+6\eta)}$

Alternative Analysis

Assuming that BER < 10^{-3} is required, and BER = $Q\left(\sqrt{2E_b/I_0}\right)$.

From the Q-function table, $Q(3.1) \approx 10^{-3}$.

So, $\sqrt{2E_b/I_0} = 3.1 \rightarrow E_b/I_0 = 4.805 \rightarrow$ user capacity per cell $M < \left(\frac{PG}{4.805\alpha\beta}+1\right)\times\frac{1}{(1+6\eta)}$

TUTORIAL

Please do Tutorial 3 before we discuss it next week.