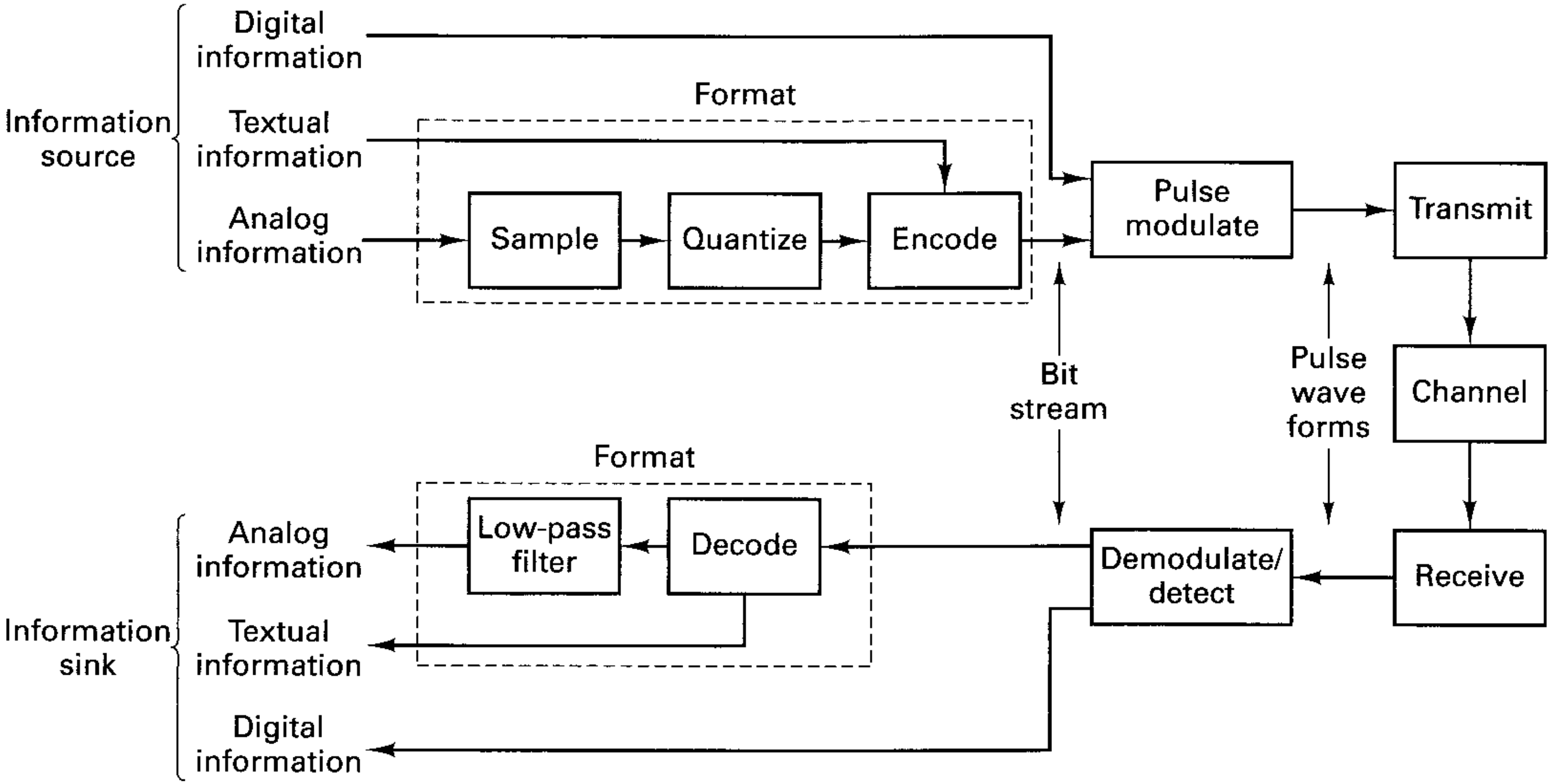
Signals

A **baseband signal** has a spectral magnitude that is nonzero for frequencies in the vicinity of the origin (i.e., $f = 0$), usually less than a few megahertz, and negligible elsewhere.

A **bandpass signal** has a spectral magnitude that is nonzero for frequencies in some band concentrated about $f = \pm f_c$, where f_c is the carrier frequency. The spectral magnitude is negligible elsewhere.

Baseband System



Information Source

- **Data** already in a digital format would bypass the formatting function.
- **Textual information** is transformed into binary digits by use of a coder.
- **Analog information** is formatted using three separate processes: **sampling**, **quantization**, and **coding**.

In all cases, the formatting step results in a sequence of binary digits.

Waveform Encoder

The resulting binary digits are transformed by the **waveform encoder** or **baseband modulator** to digital waveforms that are compatible with the baseband channel. The output of the waveform encoder is typically a sequence of pulses with characteristics that correspond to the binary digits being sent.

Baseband Channel

Pulses are transmitted through the baseband channel, which is normally a pair of wires or a coaxial cable.

After transmission through the channel, the received waveforms are detected by the **waveform detector** to produce estimates of the transmitted binary digits, which are then converted to the desired format required by the **information sink**.

Example

Message (text): "THINK"

Character coding (6-bit ASCII):	T		H		I		N		K	
	0 0 1	0 1 0	0 0 0	1 0 0	1 0 0	1 0 0	0 1 1	1 0 0	1 1 0	1 0 0
8-ary digits (symbols):	1	2	0	4	4	4	3	4	6	4
8-ary waveforms:	$s_1(t)$	$s_2(t)$	$s_0(t)$	$s_4(t)$	$s_4(t)$	$s_4(t)$	$s_3(t)$	$s_4(t)$	$s_6(t)$	$s_4(t)$

(a)

Character coding (6-bit ASCII): T H I N K

0 0 1 0 1 0 0 0 0 1 0 0 1 0 0 1 0 0 0 1 1 1 0 0 1 1 0 1 0 0

	0 0 1 0 1	0 0 0 0 1	0 0 1 0 0	1 0 0 0 1	1 1 0 0 1	1 0 1 0 0
32-ary digits (symbols):	5	1	4	17	25	20
32-ary waveforms:	$s_5(t)$	$s_1(t)$	$s_4(t)$	$s_{17}(t)$	$s_{25}(t)$	$s_{20}(t)$

(b)

The Sampling Theorem

If a bandlimited signal contains no frequency components above f_m Hz, then the signal is completely described by instantaneous sample values uniformly spaced in time with sampling period

$$T_s \leq \frac{1}{2 f_m}$$

The **Nyquist criterion** states that the signal can be exactly reconstructed from its samples by passing them through an ideal low-pass filter.

The sampling rate

$$f_s = \frac{1}{T_s} \geq 2 f_m$$

The sampling rate $f_s = 2 f_m$ is called the **Nyquist rate**.

Note that the Nyquist criterion is a theoretically sufficient condition to allow an analog signal to be reconstructed completely from its discrete-time samples.

Example

We would like to determine the Fourier transform of

$$x_\delta(t) \equiv \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

where T_s is the sampling period. Since the signal is a periodic function, it can be expressed in the form of Fourier series as

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_s t}$$

where $f_s = 1/T_s$ and

$$C_n = \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} x_\delta(t)\, e^{-j2\pi n f_s t}\; dt$$

$$= \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} \delta(t)\, e^{-j2\pi n f_s t}\; dt = \frac{1}{T_s}$$

The Fourier series expression is

$$x_\delta(t) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} e^{j2\pi n f_s t}$$

Taking Fourier transform on both sides, we have

$$X_\delta(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta(f - nf_s)$$

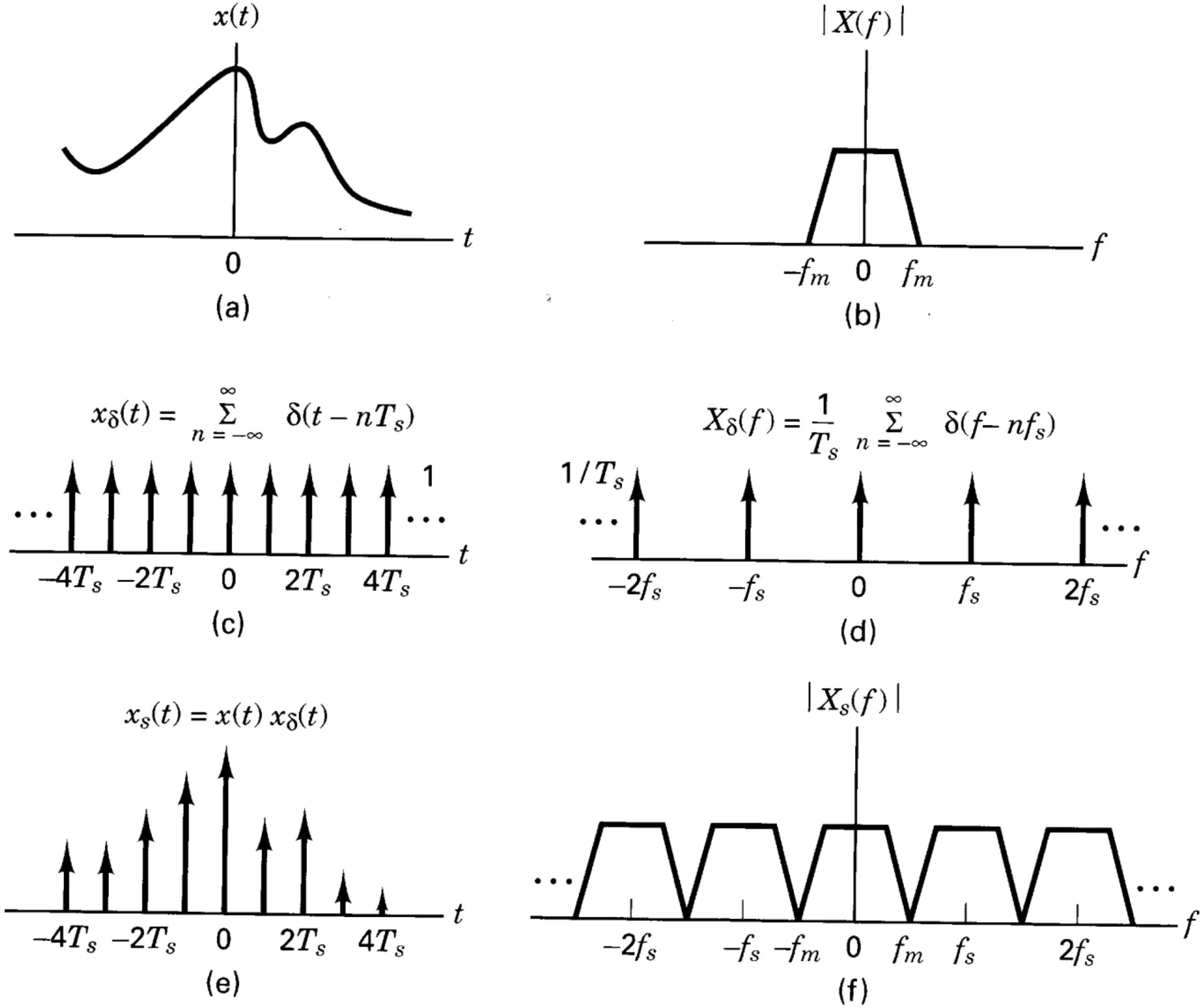
☑

Impulse Sampling

Here we consider the representation of an analog signal, $x(t)$, by an ideal sampling function $x_\delta(t)$. The signal $x(t)$ is assumed to be bandlimited to f_m. The sampling of $x(t)$ can be viewed as

$$x_s(t) = x(t) \times x_\delta(t)$$

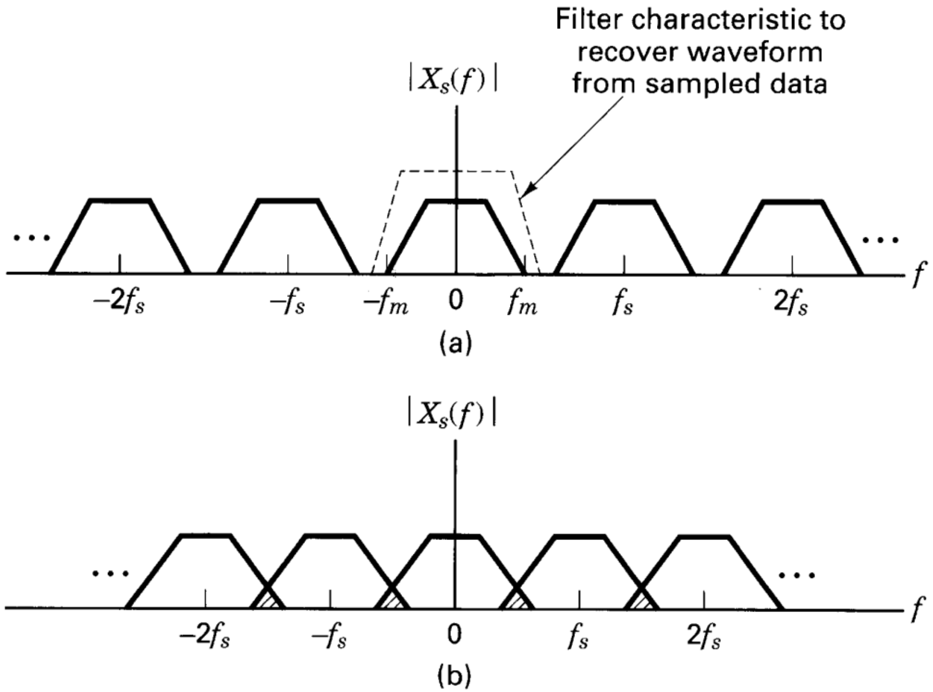
where $x_\delta(t)$ is the impulse train defined in the previous example. In the frequency domain, we have to perform convolution as follows.



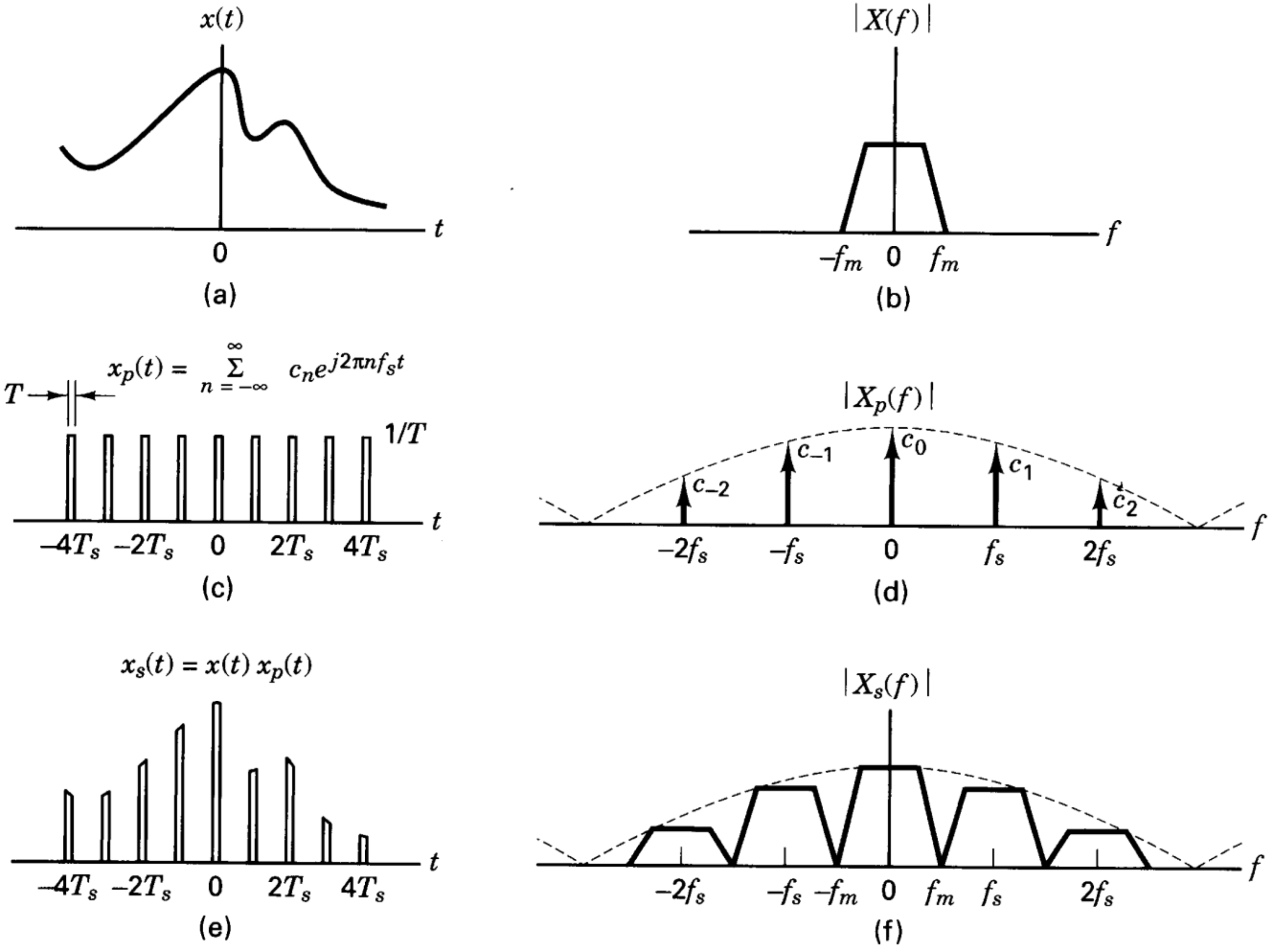
Sampling theorem using the frequency convolution property of the Fourier transform.

$$
\begin{aligned}
X_s(f) &= X(f) \otimes X_\delta(f) \\
&= X(f) \otimes \left[\frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \right] \\
&= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \left[X(f) \otimes \delta(f - nf_s) \right] \\
&= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - nf_s)
\end{aligned}
$$

The sampling process creates a periodic repetition of $X(f)$ in the frequency domain with a frequency spacing f_s. If $f_s \geq 2f_m$, then the analog waveform $x(t)$ can be recovered perfectly from the samples, by using low-pass filtering. If $f_s < 2f_m$, the terms will overlap in frequency, and there is no apparent way to recover $x(t)$ without distortion. This phenomenon is called **aliasing**. To avoid aliasing, the Nyquist criterion $f_s \geq 2f_m$ must be satisfied.



Spectra for various sampling rates. (a) Sampled spectrum ($f_s > 2f_m$). (b) Sampled spectrum ($f_s < 2f_m$).



Sampling theorem using the frequency shifting property of the Fourier transform.

Example

We would like to determine the Fourier transform of

$$x_p(t) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \mathrm{rect}\left(\frac{t - nT_s}{T} \right)$$

where T_s is the sampling period and

$$\mathrm{rect}\left(\frac{t}{T} \right) \equiv \begin{cases} 1 & |t| \le T/2 \\ 0 & \text{otherwise} \end{cases}$$

Since the signal is a periodic function, it can be expressed as a Fourier series in the form

$$x_p(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_s t}$$

where $f_s = 1/T_s$ and

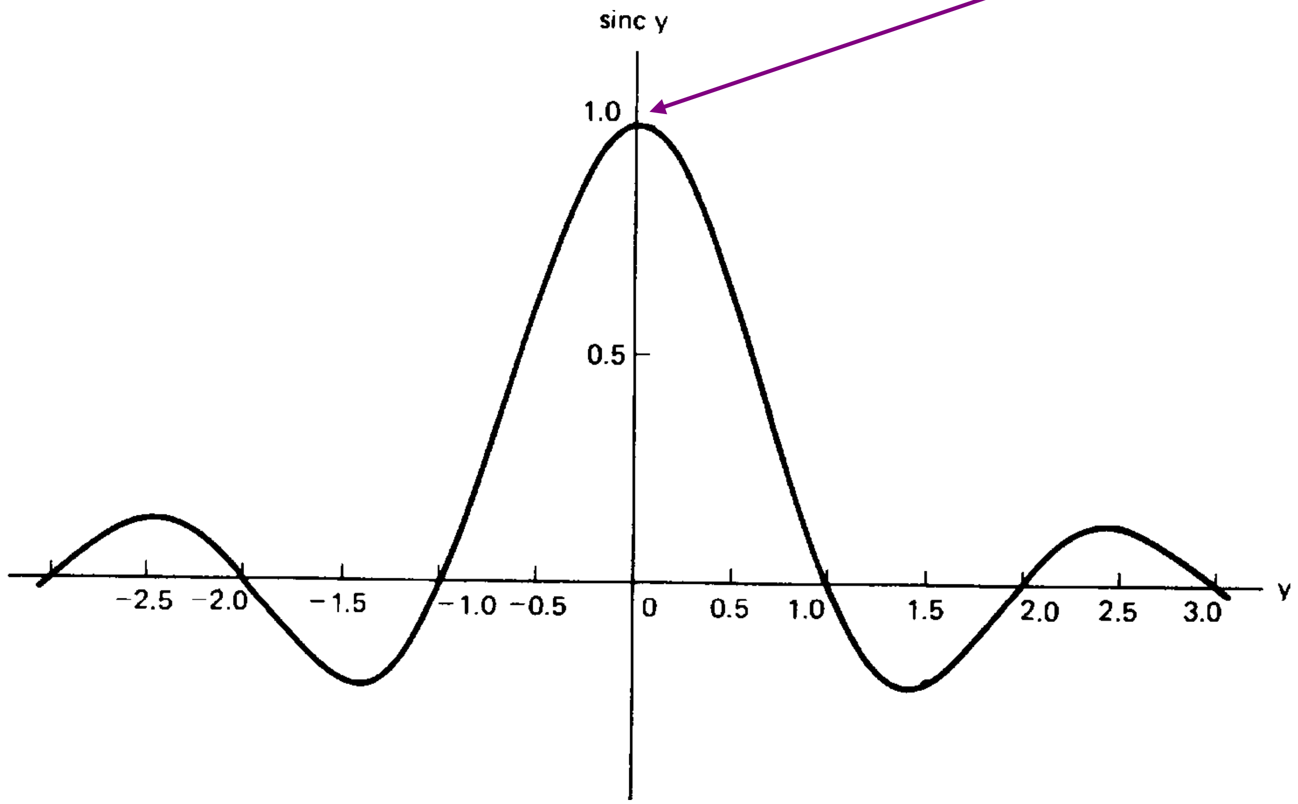
$$\begin{aligned} C_n &= \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} x_p(t)\, e^{-j2\pi n f_s t} \quad dt \\ &= \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} \frac{1}{T}\operatorname{rect}\left(\frac{t}{T}\right) e^{-j2\pi n f_s t} \quad dt \\ &= \frac{1}{T_s}\operatorname{sinc}\left(\frac{nT}{T_s}\right) \end{aligned}$$

The sinc function is defined as

$$\mathrm{sinc}(y) \equiv \frac{\sin \pi y}{\pi y}$$

$$\mathrm{sinc}(0) = \left.\frac{\pi \cos \pi y}{\pi}\right|_{y=0}$$

$$= \cos 0 = 1$$



The Fourier series expression is

$$x_p(t) = \sum_{n=-\infty}^{\infty} C_n \, e^{j2\pi n f_s t} = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{nT}{T_s}\right) e^{j2\pi n f_s t}$$

Taking Fourier transform on both sides, we have

$$X_p(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{nT}{T_s}\right) \delta(f - nf_s)$$

☑

Natural Sampling

The resulting sampled-data sequence, $x_s(t)$, can be expressed as

$$x_s(t) = x(t) \times x_p(t)$$

where $x_p(t)$ is the periodic pulse train defined in the previous example. This sample scheme is called **natural sampling** because the top of each pulse in the $x_s(t)$ sequence retains the shape of its corresponding analog segment during the pulse duration. The sampling rate f_s is chosen to be $2f_m$, so that the Nyquist criterion is just satisfied. In the frequency domain,

$$
\begin{aligned}
X_s(f) &= X(f) \otimes X_p(f) \\
&= X(f) \otimes \left[\sum_{n=-\infty}^{\infty} C_n \delta(f - nf_s) \right] \\
&= \sum_{n=-\infty}^{\infty} C_n \left[X(f) \otimes \delta(f - nf_s) \right] = \sum_{n=-\infty}^{\infty} C_n \, X(f - nf_s) \\
&= \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{nT}{T_s} \right) X(f - nf_s)
\end{aligned}
$$

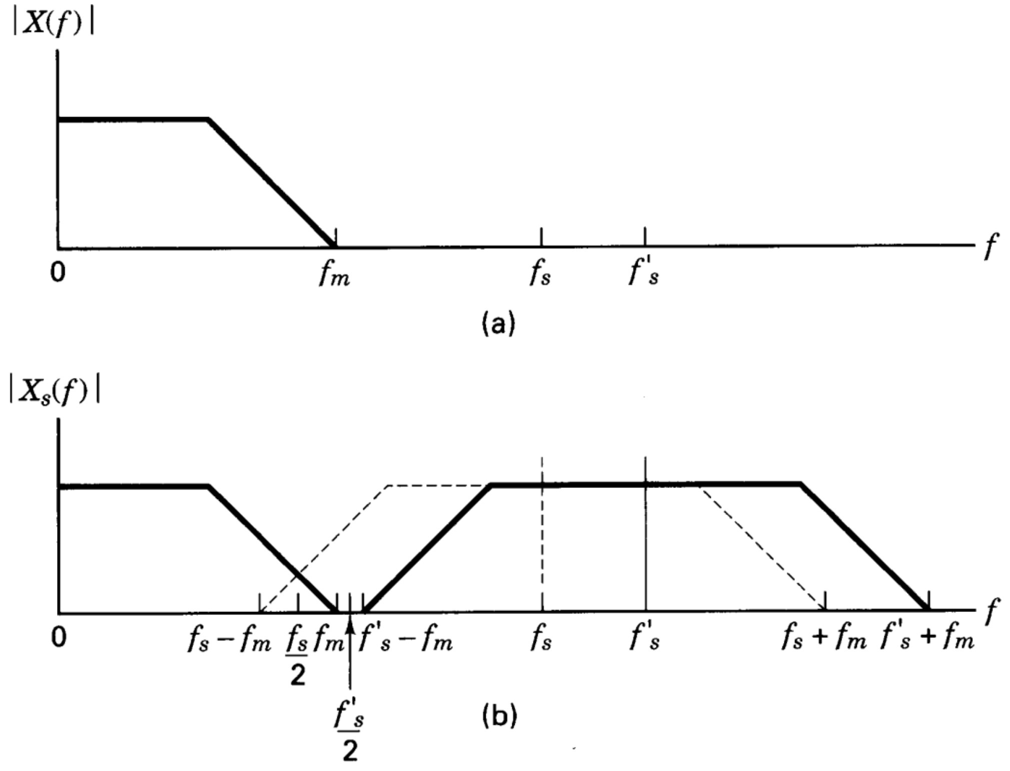
As the pulse width T approaches zero, the sinc function in the above expression is equal to 1 and $X_s(f)$ converges to that of impulse sampling.

Aliasing

Aliasing in the frequency domain is due to undersampling. To avoid aliasing, we may

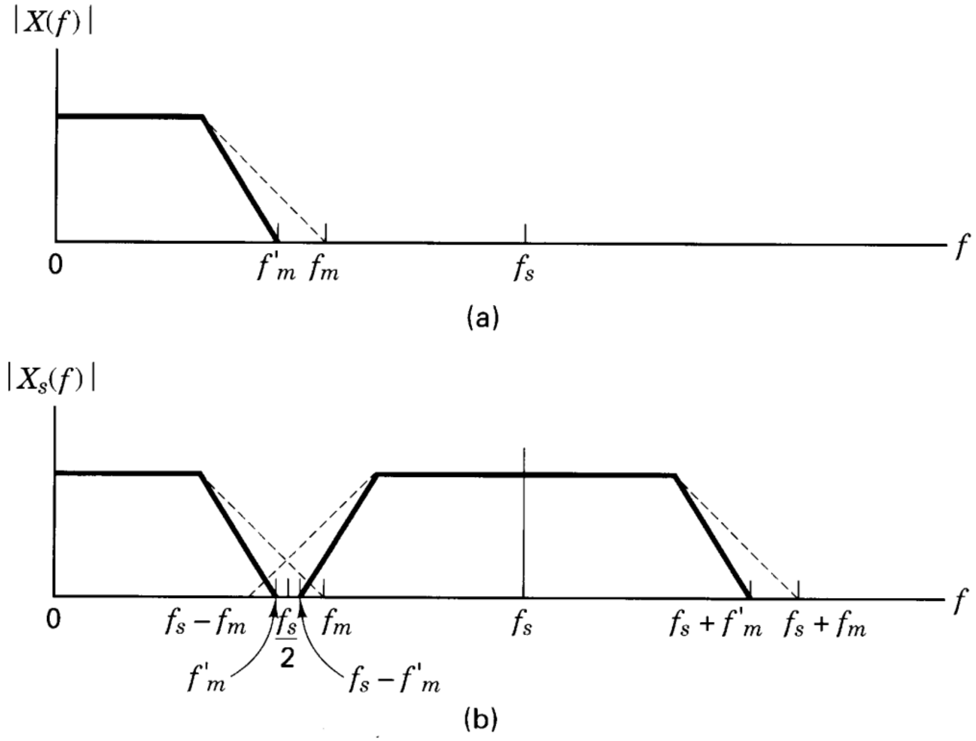
- increase the sampling rate f_s to satisfy the Nyquist criterion.
- prefilter the signal so that the new maximum frequency of the signal is less than or equal to $f_s/2$. This is normally good engineering practice.
- postfilter the sampled data when the signal structure is well known.

Increase Sampling Rate



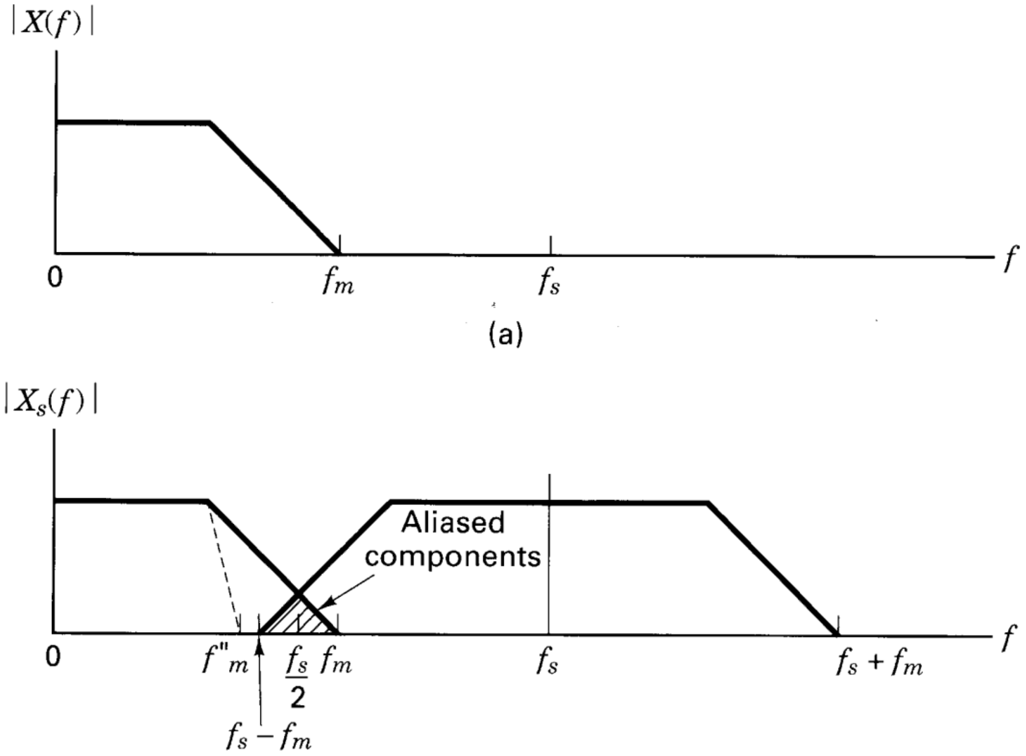
Higher sampling rate eliminates aliasing. (a) Continuous signal spectrum. (b) Sampled signal spectrum.

Prefiltering



Sharper-cutoff filters eliminate aliasing. (a) Continuous signal spectrum. (b) Sampled signal spectrum.

Postfiltering



Postfilter eliminates aliased portion of spectrum. (a) Continuous signal spectrum. (b) Sampled signal spectrum.

Observations

- Note that the last two methods will result in a loss of some of the signal information.
- Realizable filters require a nonzero bandwidth for the transition between the passband and the stopband. In many systems we need to make the transition bandwidth between 10% and 20% of the signal bandwidth. If we account for the 20% transition bandwidth of the filter, we have an engineer's version of the Nyquist sampling rate: $f_s \geq 2.2 f_m$.

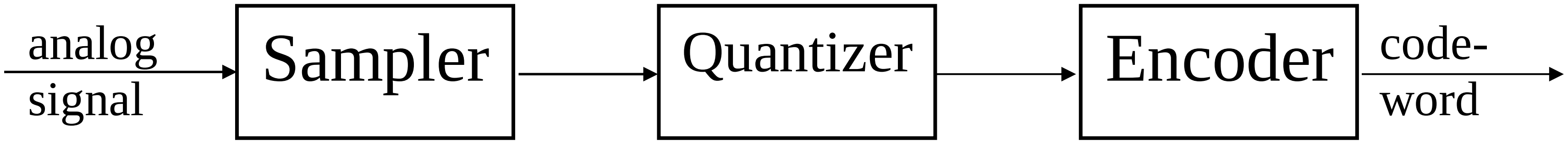
Example

For all practical purpose, a signal can be considered to be essentially bandlimited at some value f_m, the choice of which depends on the accuracy desired. A practical example of this is speech signal. Theoretically, a speech signal, being a finite time signal, has an infinite bandwidth. But frequency components beyond 3 kHz contribute a negligible fraction of the total energy. When speech signals are sampled and transmitted, they are first passed through a low-pass filter of bandwidth 3500 Hz, and the resulting signal is sampled at a rate of 8 kilosamples/s.

For a high-quality music source, the bandwidth is normally set at 20 kHz. By the engineer's version of Nyquist rate, the sampling rate should be greater than 44.0 kilosamples/s. As a matter of comparison, the standard sampling rate for the compact disc digital audio player is 44.1 kilosamples/s, and the standard sampling rate for studio-quality audio is 48.0 kilosamples/s.

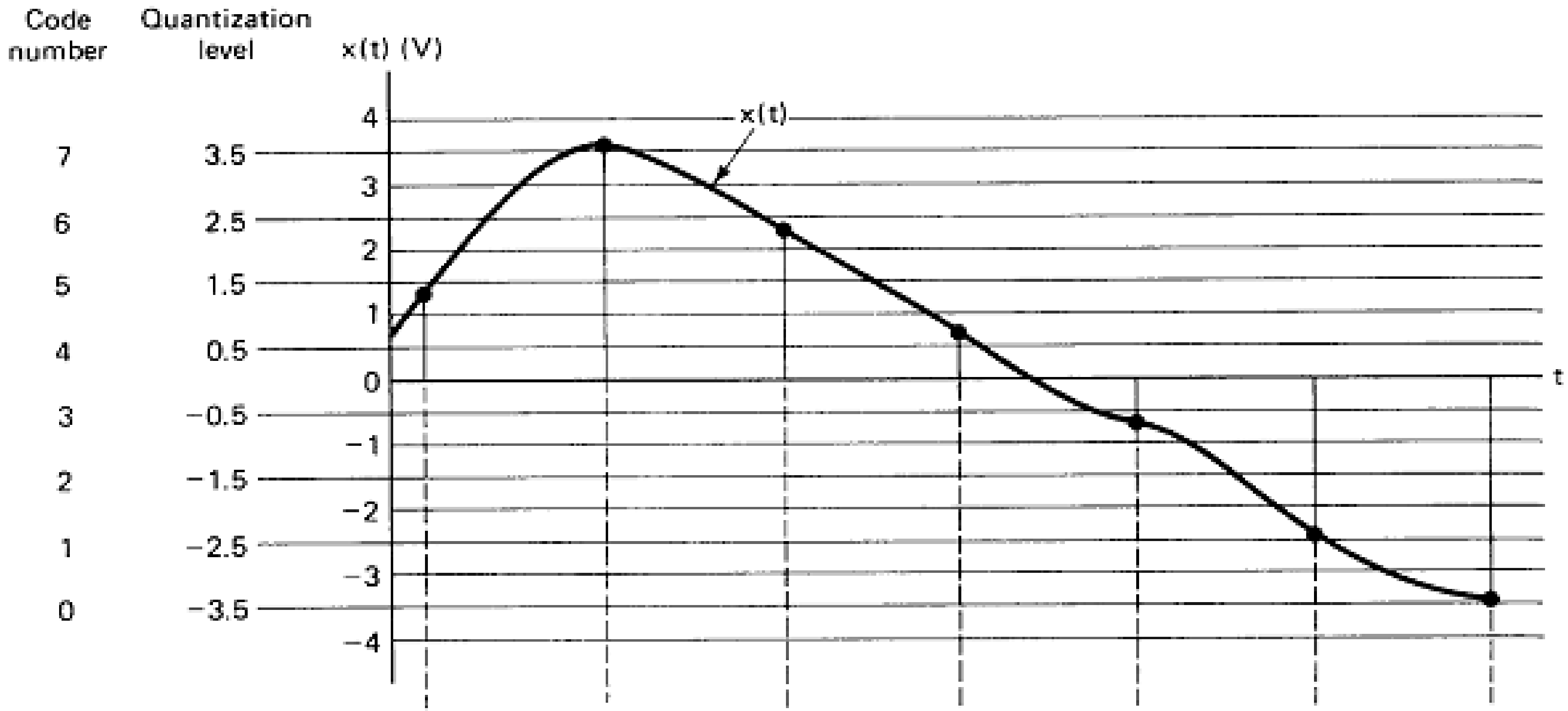
Pulse Code Modulation

Pulse code modulation (PCM) consists of three parts as follows.



The analog signal is first sampled. The sampled values are then quantized to one of L levels. Each of these quantized samples is digitally encoded into an l-bit codeword, where $l = \log_2 L$. For baseband transmission, the codeword bits will then be mapped to pulses for transmission.

Example

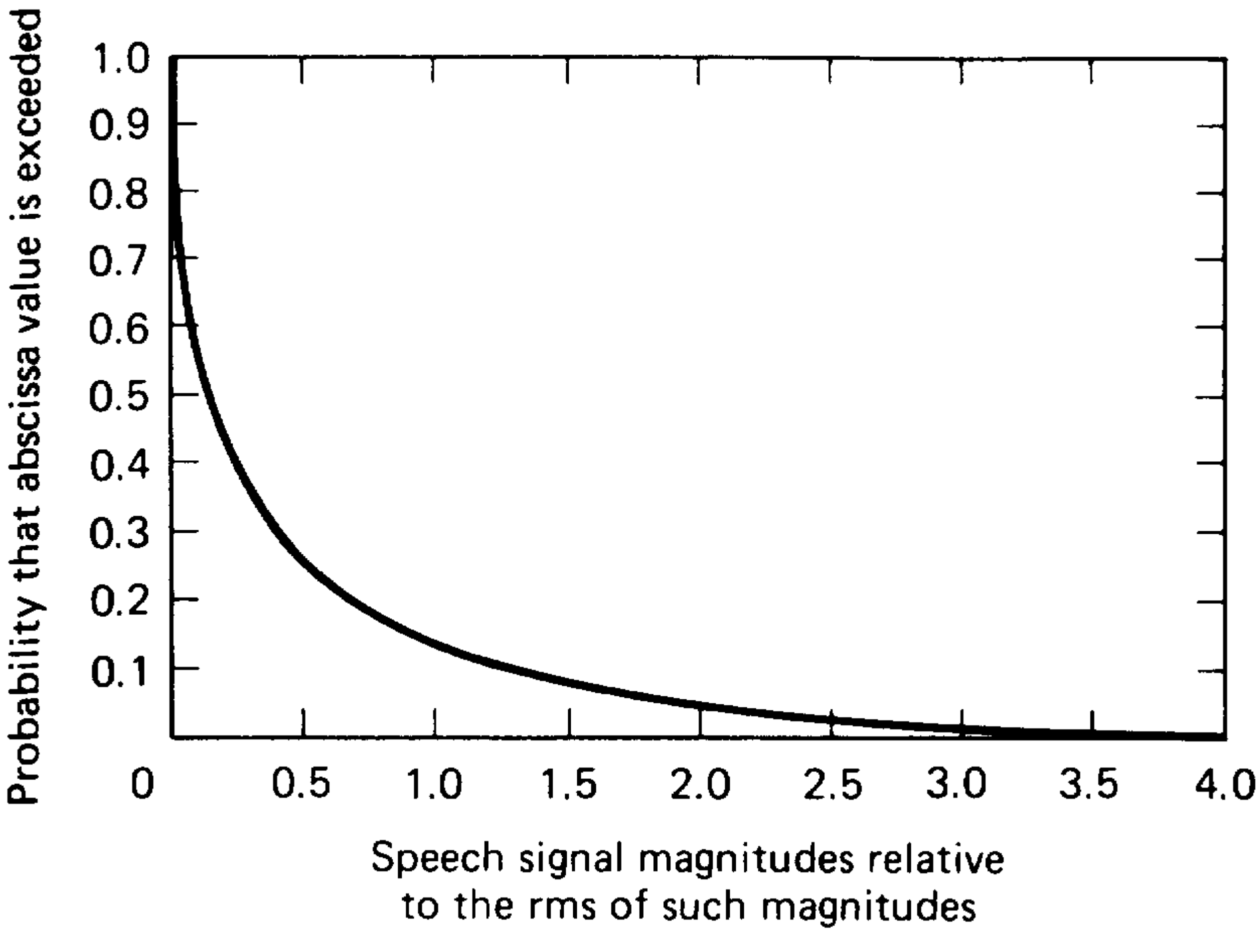


Natural sample value	1.3	3.6	2.3	0.7	−0.7	−2.4	−3.4
Quantized sample value	1.5	3.5	2.5	0.5	−0.5	−2.5	−3.5
Code number	5	7	6	4	3	1	0
PCM sequence	101	111	110	100	011	001	000

We assume that the analog signal $x(t)$ is limited in its excursions to the range from –4 to +4 volts. We have set the quantile interval between quantization levels at 1 volt. Eight quantization levels are employed, and they are located at –3.5, -2.5, …, +3.5 volts. We assign the code number 0 to the level at –3.5 volts, the code number 1 to the level at –2.5 volts, etc., until the level at +3.5 volts, which is assigned the code number 7. Each code number has its binary representation from 000 for code number 0 to 111 for code number 7.

This is an example of uniform quantization.

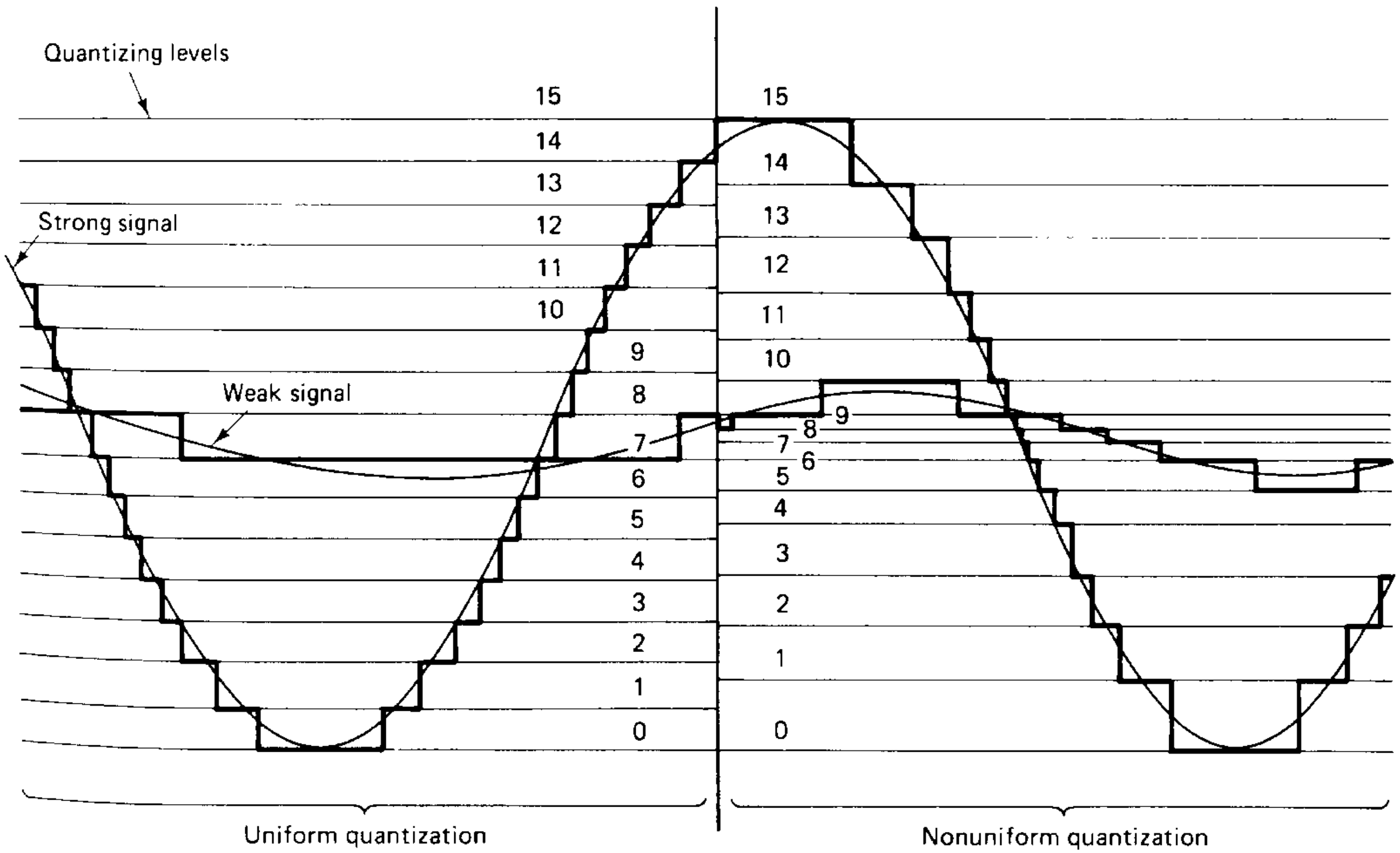
Statistics of Speech Signals



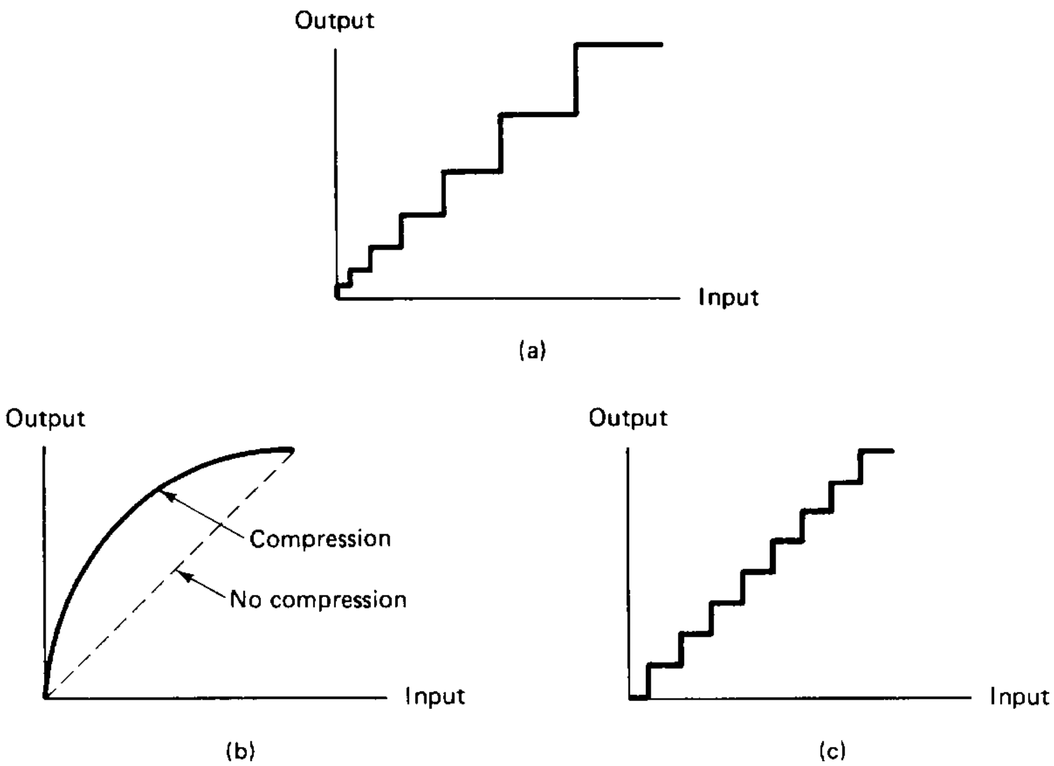
For a speech waveform, there exist a high probability for smaller amplitudes and a low probability for larger amplitudes.

If we use a uniform quantizer for speech signals, many of the quantization levels would rarely be used. Moreover, the signal-to-noise ratio (SNR) for a weak signal may not be large enough. Hence, it makes sense to design a quantizer with more quantization levels at low amplitudes and less quantization levels at large amplitudes. The resulting quantizer will be a **nonuniform quantizer**.

Example (Uniform and Nonuniform Quantization of Signals)

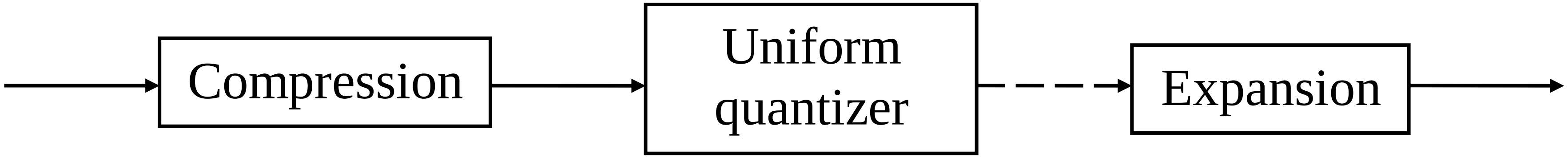


Nonuniform PCM



(a) Nonuniform quantizer characteristic.
(b) Compression characteristic.
(c) Uniform quantizer characteristic.

The usual method for performing nonuniform quantization is to pass the samples through a nonlinear element that compress the large amplitudes and then perform a uniform quantization on the output. At the receiving end, the inverse of this nonlinear operation, called **expansion**, is applied so that the overall transmission is not distorted. This process is usually referred to as **companding** (compression and expansion).



Companding Characteristics

There are two types of compander that are widely used for speech coding. The μ-law compander used in North America employs the logarithmic function at the transmitting side with

$$y = y_{\max} \frac{\ln\left[1 + \mu(|x| / x_{\max})\right]}{\ln\left(1 + \mu\right)} \operatorname{sgn}(x)$$

where the signum (or sign) function is defined as

$$\mathrm{sgn}(x) = \begin{cases} +1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

The parameter μ controls the amount of compression and expansion. The standard PCM system in North America uses a compressor with μ = 255, followed by a uniform quantizer with 128 levels (7 bits/sample). Use of compander in this system improves the performance of the system by about 24 dB.

The second widely used logarithmic compressor is the *A*-law compander. The characteristic of this compander is given by

$$y' = \begin{cases} \dfrac{1+\ln A|x'|}{1+\ln A}\,\mathrm{sgn}(x') & \dfrac{1}{A} \le |x'| \le 1 \\ \dfrac{A|x'|}{1+\ln A}\,\mathrm{sgn}(x') & 0 \le |x'| \le \dfrac{1}{A} \end{cases}$$

where $x' = x/x_{\mathrm{max}}$, $y' = y/y_{\mathrm{max}}$ and A is chosen to be 87.56.