E6101 - Part III: Spread-Spectrum Communications and CDMA

Assoc Prof Erry Gunawan, 67905392, egunawan@ntu.edu.sg

School of Electrical and Electronic Engineering, Nanyang Technological University.

Text

- Alex W Lam & S Tantaratana, *Theory & Applications of Spread Spectrum Systems*, IEEE Inc, 1994. (Chapters 3, 7 & 10)
- Valery P Ipatov, *Spread Spectrum and CDMA*, Wiley, 2005.

CONTENT

Why Code Division Multiple Access (CDMA)?

Multiple Access (MA)

- Definition: Dynamic sharing of a common pool of communication channels by a large number of users in a pre-defined, orderly manner.

- Examples of MA channels:
  - submarine cables for international calls
  - optical fibre backbone for internet traffic (SingaporeOne)
  - marine/mobile satellite communication systems
  - **mobile radio communication systems ← focus of this course**

- Goals of MA systems:
  - more simultaneous users
  - higher data rates

    more capacity
  - little/no interference (cross-talk) between users, ie. good channel separation at receiver.

Mobile Radio Communication Systems

- Eg. paging, cordless phone, walkie talkie, cellular phone, wireless LAN, mobile satellite etc.
- Trends in mobile communication applications: voice → data (SMS, fax) → multimedia (MMS, video)

Mobile Phone Systems in Singapore

- AMPS (Advanced Mobile Phone System, US origin)
- ETACS (Extended Total Access Communication System, UK)
- GSM 900/1800 (Global System for Mobile Communication, Europe)
- IS-95 or Qualcomm CDMA (EIA/TIA Interim Standard 95, US) ← M1 Chat
- GPRS, EDGE – “2.5G” systems
- 3G cellular mobile radio system: WCDMA (Europe/Japan), cdma2000 (US, Korea) etc.

The MA schemes used in different mobile radio systems are shown on the next pages.

Major Mobile Radio Standards in North America

Standard	Type	Year of Introduction	Multiple Access	Frequency Band	Modulation	Channel Bandwidth
AMPS	Cellular	1983	FDMA	824-894 MHz	FM	30 kHz
NAMPS	Cellular	1992	FDMA	824-894 MHz	FM	10 kHz
USDC	Cellular	1991	TDMA	824-894 MHz	$\pi/4$-DQPSK	30 kHz
CDPD	Cellular	1993	FH/ Packet	824-894 MHz	GMSK	30 kHz
IS-95	Cellular/ PCS	1993	CDMA	824-894 MHz 1.8-2.0 GHz	QPSK/ BPSK	1.25 MHz
GSC	Paging	1970's	Simplex	Several	FSK	12.5 kHz
POCSAG	Paging	1970's	Simplex	Several	FSK	12.5 kHz
FLEX	Paging	1993	Simplex	Several	4-FSK	15 kHz
DCS-1900 (GSM)	PCS	1994	TDMA	1.85-1.99 GHz	GMSK	200 kHz
PACS	Cordless/ PCS	1994	TDMA/ FDMA	1.85-1.99 GHz	$\pi/4$-DQPSK	300 kHz
MIRS	SMR/PCS	1994	TDMA	Several	16-QAM	25 kHz

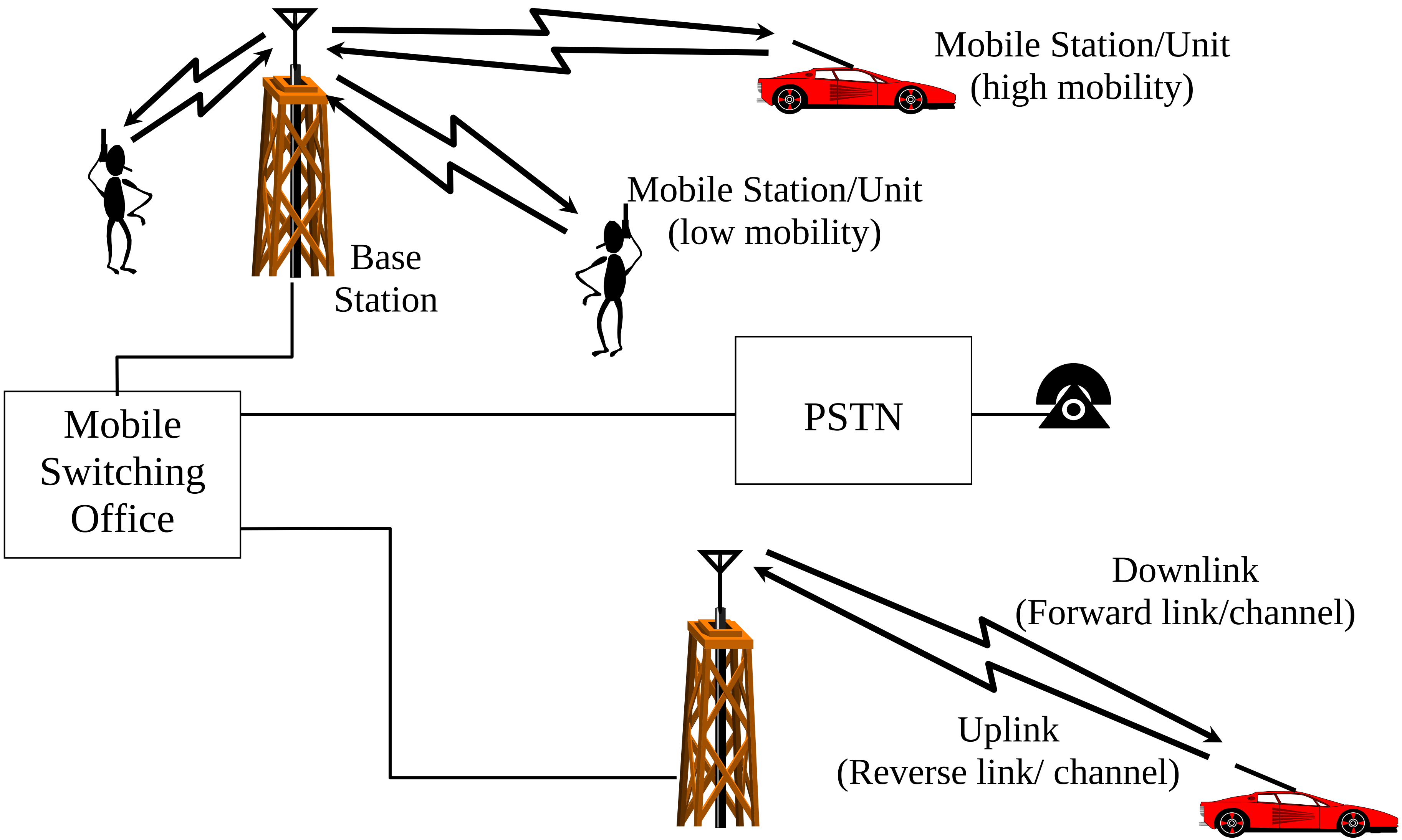
Major Mobile Radio Standards in Europe

Standard	Type	Year of Introduction	Multiple Access	Frequency Band	Modulation	Channel Bandwidth
E-TACS	Cellular	1985	FDMA	900 MHz	FM	25 kHz
NMT-450	Cellular	1981	FDMA	450-470 MHz	FM	25 kHz
NMT-900	Cellular	1986	FDMA	890-960 MHz	FM	12.5 kHz
GSM	Cellular /PCS	1990	TDMA	890-960 MHz	GMSK	200 kHz
C-450	Cellular	1985	FDMA	450-465 MHz	FM	20 kHz/ 10 kHz
ERMES	Paging	1993	FDMA	Several	4-FSK	25 kHz
CT2	Cordless	1989	FDMA	864-868 MHz	GFSK	100 kHz
DECT	Cordless	1993	TDMA	1880-1900 MHz	GFSK	1.728 MHz
DCS-1800	Cordless /PCS	1993	TDMA	1710-1880 MHz	GMSK	200 kHz

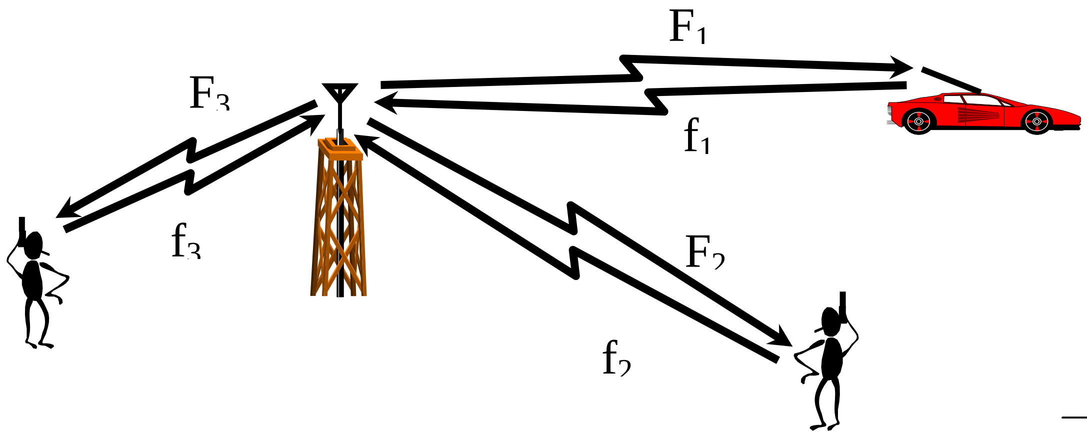
Major Mobile Radio Standards in Japan

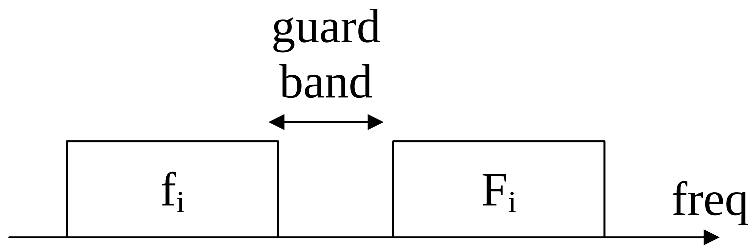
Standard	Type	Year of Introduction	Multiple Access	Frequency Band	Modulation	Channel Bandwidth
JTACS	Cellular	1988	FDMA	860-925 MHz	FM	25 kHz
PDC	Cellular	1993	TDMA	810-1501 MHz	$\pi/4$-DQPSK	25 kHz
NTT	Cellular	1979	FDMA	400/800 MHz	FM	25 kHz
NTACS	Cellular	1993	FDMA	843-925 MHz	FM	12.5 kHz
NTT	Paging	1979	FDMA	280 MHz	FSK	12.5 kHz
NEC	Paging	1979	FDMA	Several	FSK	10 kHz
PHS	Cordless	1993	TDMA	1895-1907 MHz	$\pi/4$-DQPSK	300 kHz

Mobile Communication Terminology



Qn: Why no interference between uplinks & downlinks?





- They use different freq bands
  - FDD or Freq Division Duplex (duplex ≡ simultaneous 2-way communication)
  - Eg. GSM, 890MHz < uplink < 915MHz, 935MHz < downlink < 960MHz.
- Time Division Duplex is also possible (mostly in indoor cordless phone systems, also in 3G).
- Guard bands are usually used to prevent interference (eg. 20MHz in GSM)
- No interference between uplinks and downlinks, hence they can be separately analyzed.

Qn: In just the uplink (or downlink), how to separate different user signals?

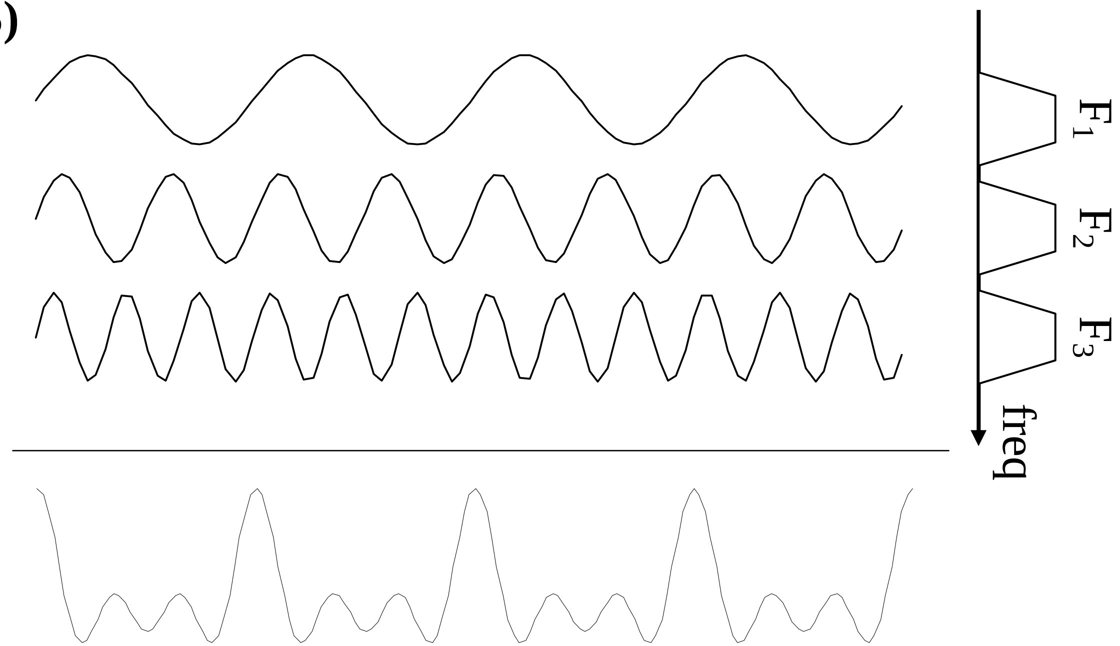
- Use MA schemes such as FDMA, TDMA, CDMA, SDMA or their hybrids
  (F-frequency, T-time, C-code, S-space, DMA-division multiple access)

FDMA (eg. AMPS, TACS)

user 1's signal

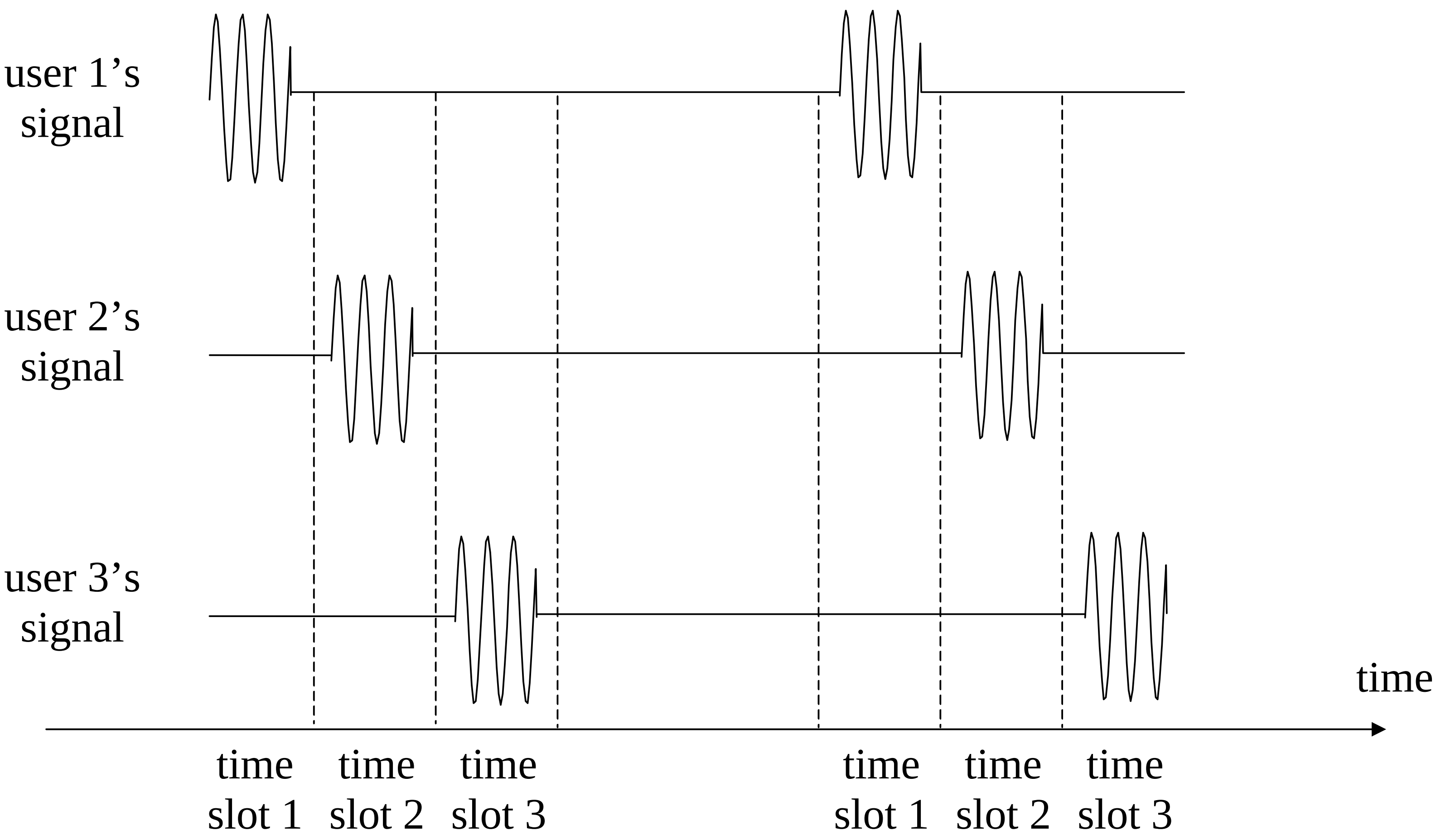
user 2's signal

user 3's signal



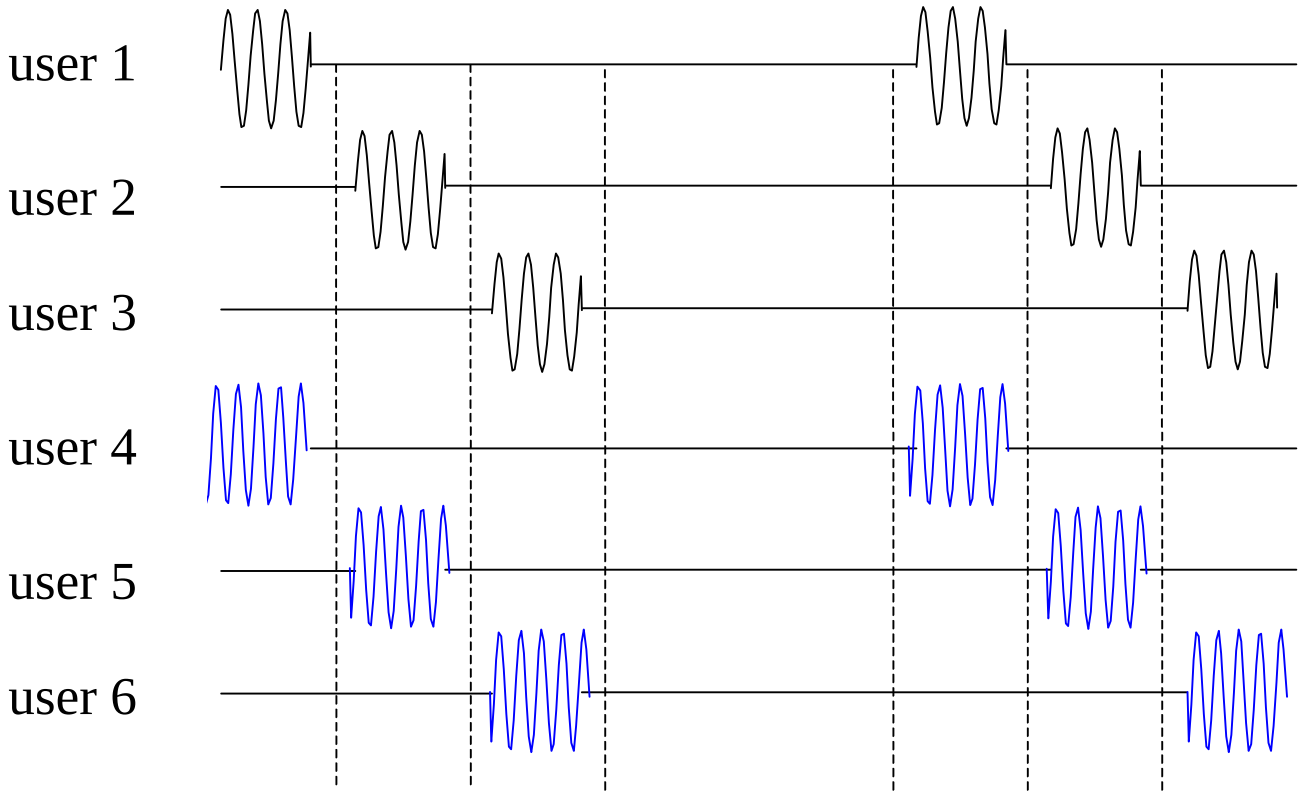
- Users transmit signal **continuously** in **different freq bands (on different carrier freqs)**
- At the receiver, use different bandpass filters to separate the superimposed signals.

TDMA



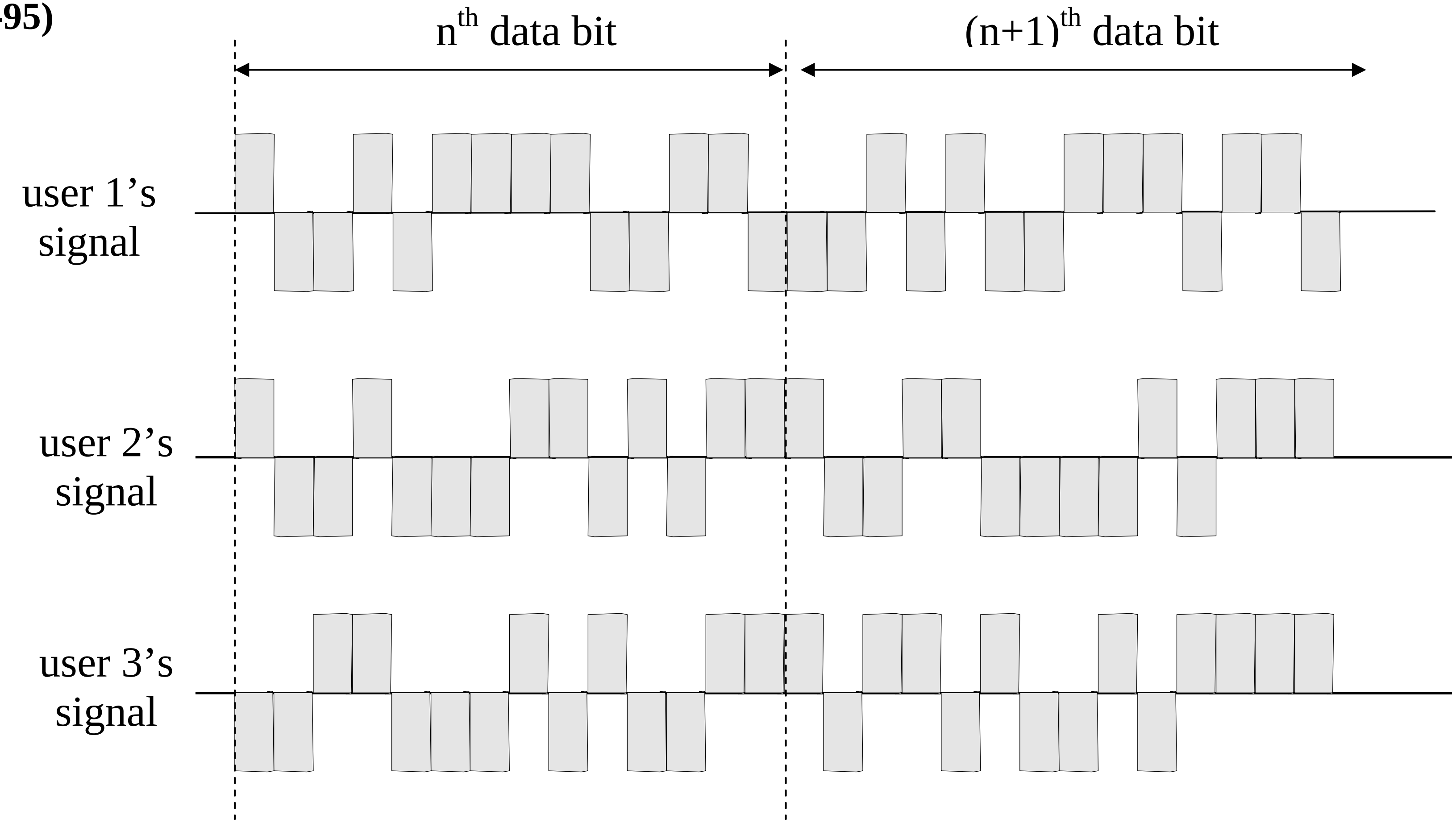
- Users transmit on the same carrier frequency, but in different assigned **time slots (periodic)**
- Synchronize receiver at correct time slots

Hybrid FDMA/TDMA (eg. GSM900, DCS1800, DECT)



- Sub-division of every FDMA channels into multiple time slots for different users
- At receiver, use correct **combination** of filter band & time slot to separate signals.

CDMA (eg. IS-95)



- Different users transmit data using different time/frequency patterns
- Use appropriate correlators/matched filters (pattern identifiers) to separate signals

Qn:

If there are many people talking in the same room, how may they communicate effectively?

(use MA techniques just mentioned)

Qn:

Imagine you are to design a FDMA cellular system for Singapore, eg. using AMPS which allocates 30KHz bandwidth to every user, how much total BW is needed for 1000 users? Repeat your design using hybrid FDMA/TDMA system, such as GSM with 8 time slots on every 200KHz band.

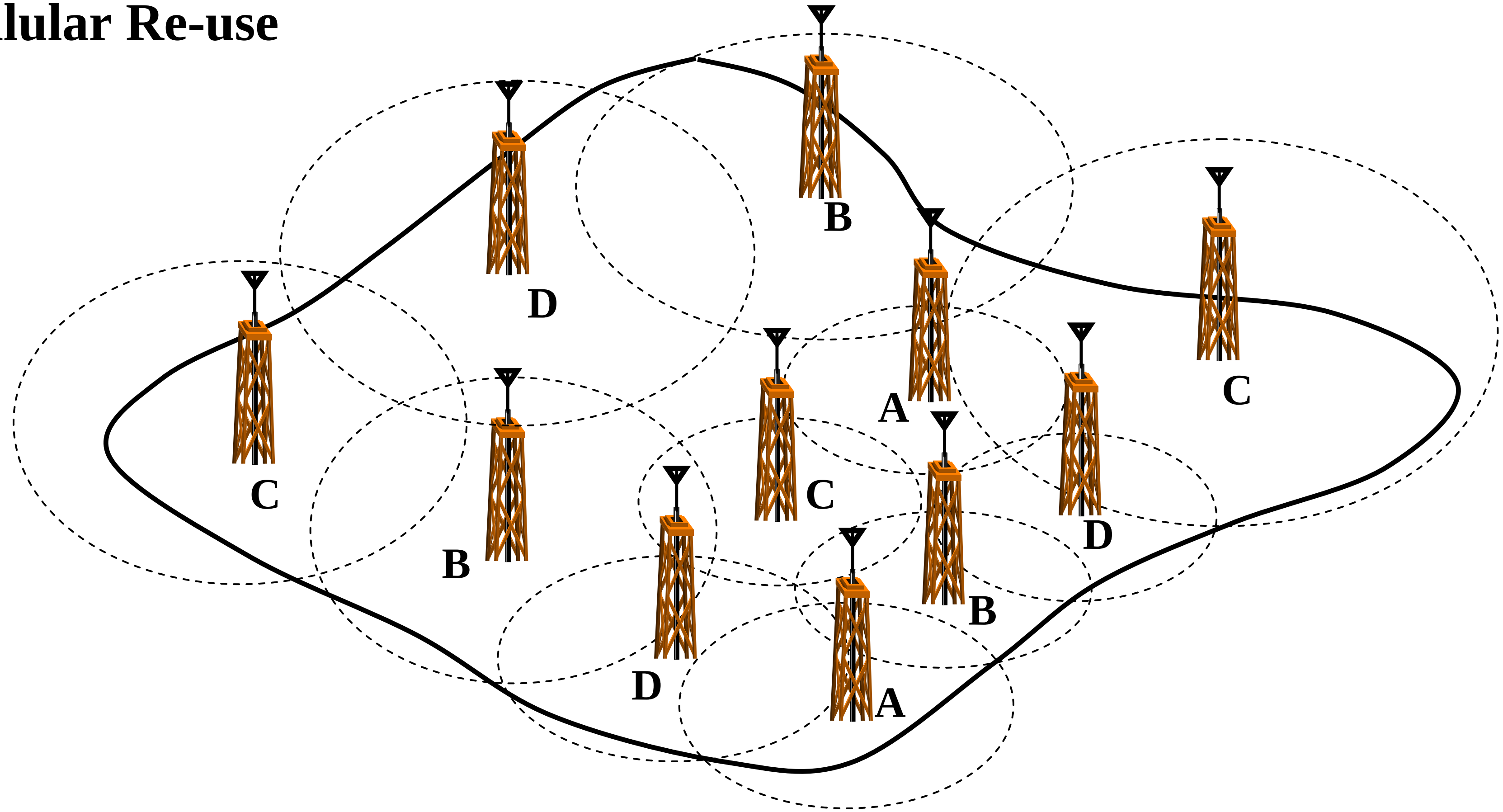
- AMPS: 30KHz × 1000 = 30MHz in total

  (Note: total BW in AMPS = 25MHz)
- GSM: 200KHz ÷ 8 × 1000 = 25MHz in total

  (Note: total BW in GSM = 25MHz)

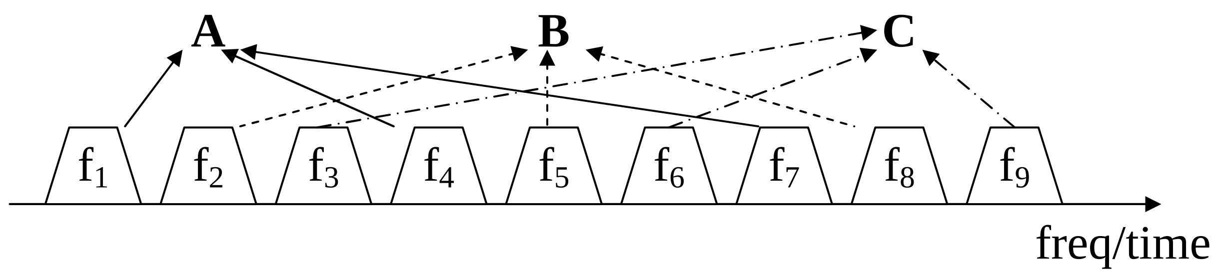
Qn: How do SingTel, M1 and StarHub support millions of users in Spore using GSM?

Cellular Re-use



cell coverage area

A, B, C ... freq/time re-use group

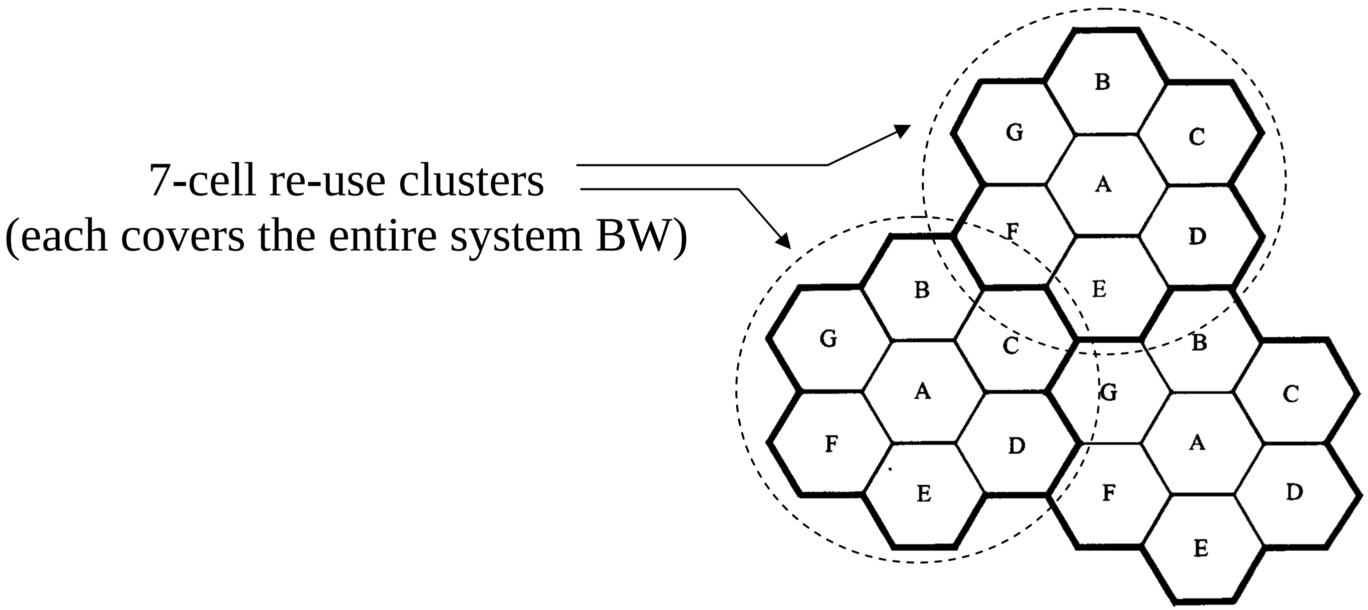


Cellular Re-use in FDMA/TDMA Systems

- Without cellular re-use, mobile telephony would not be what it is today.
- Cellular re-use offers high capacity (more user channels) without requiring excessive BW
- Before cellular era, mobile comm systems rely on a few high-power base station transmitters (ie. large cells)
- Cellular systems use many low-power base station transmitters, each providing radio coverage to a small service area (ie. small cells)
- Total system BW is divided into a few groups of frequency channels
- Every cell is given a group of channels for use by its users
- Neighbouring cells are given different groups of channels to minimize interference
- But cells which are far apart can be assigned the same frequency group, since natural signal attenuation (propagation loss, path loss) helps reduce interference.
- Since frequencies are re-used, total BW requirement is reduced.
- Obviously, the re-use concept **can be used on time slots** too.

Cellular Re-use Patterns

- Assuming classical hexagonal cell shapes over a 2-D service area and omni-directional (360^o) BS antennas, possible re-use patterns include 12-cell, 7-cell, 4-cell and 3-cell re-use patterns.
- Typically, neighbouring cells should not use the same freq group



- Cellular re-use brings capacity advantage, but also technological challenges such as handover, frequency planning, radio coverage engineering and interference management.

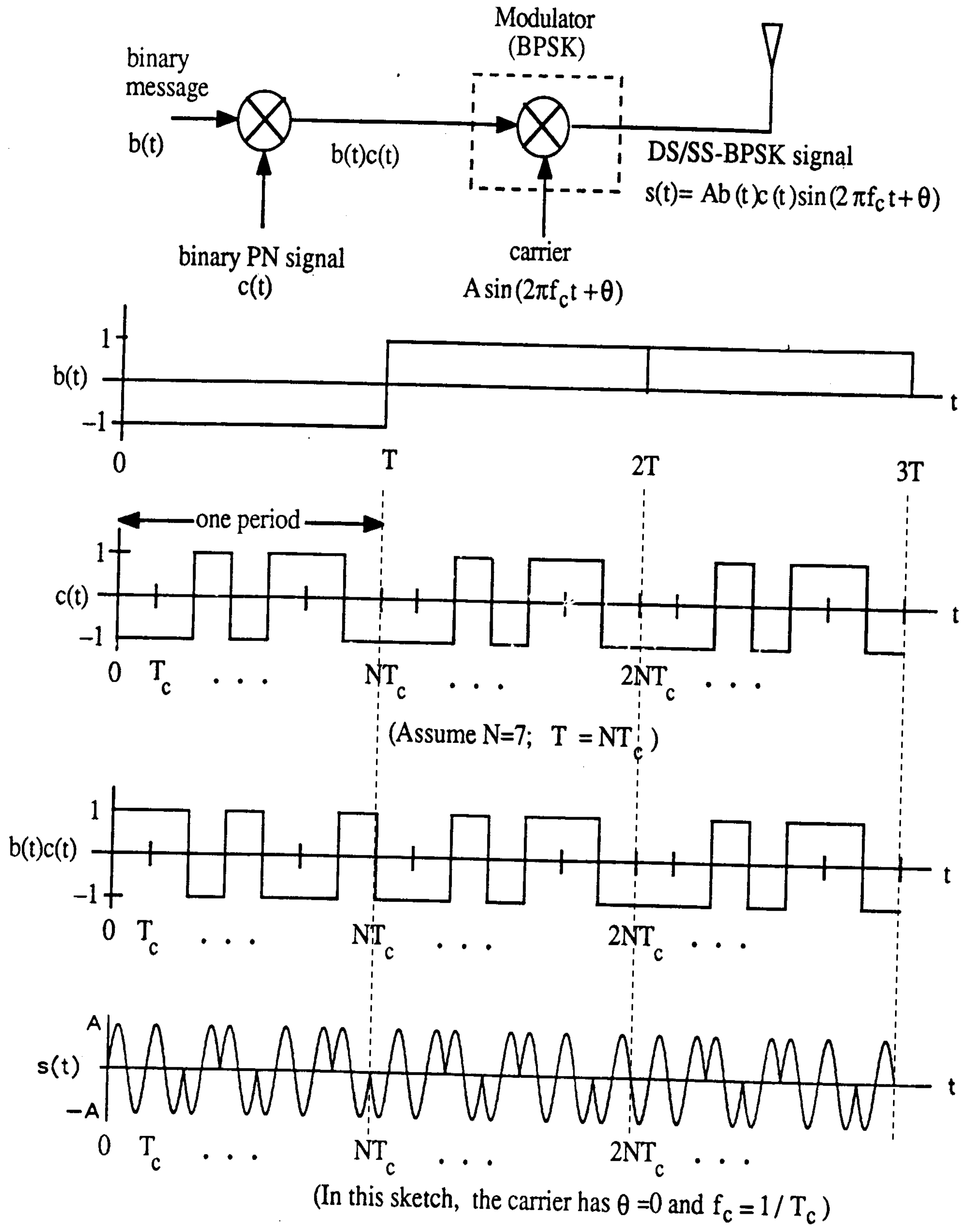
Code Division Multiple Access (CDMA) - Introduction

- In CDMA systems, every user in every cell uses the same carrier freq and BW
  - **Universal frequency re-use**
  - Will they not interfere with each other? *Yes!*
  - How can they communicate? *Using **orthogonal spreading codes/sequences***

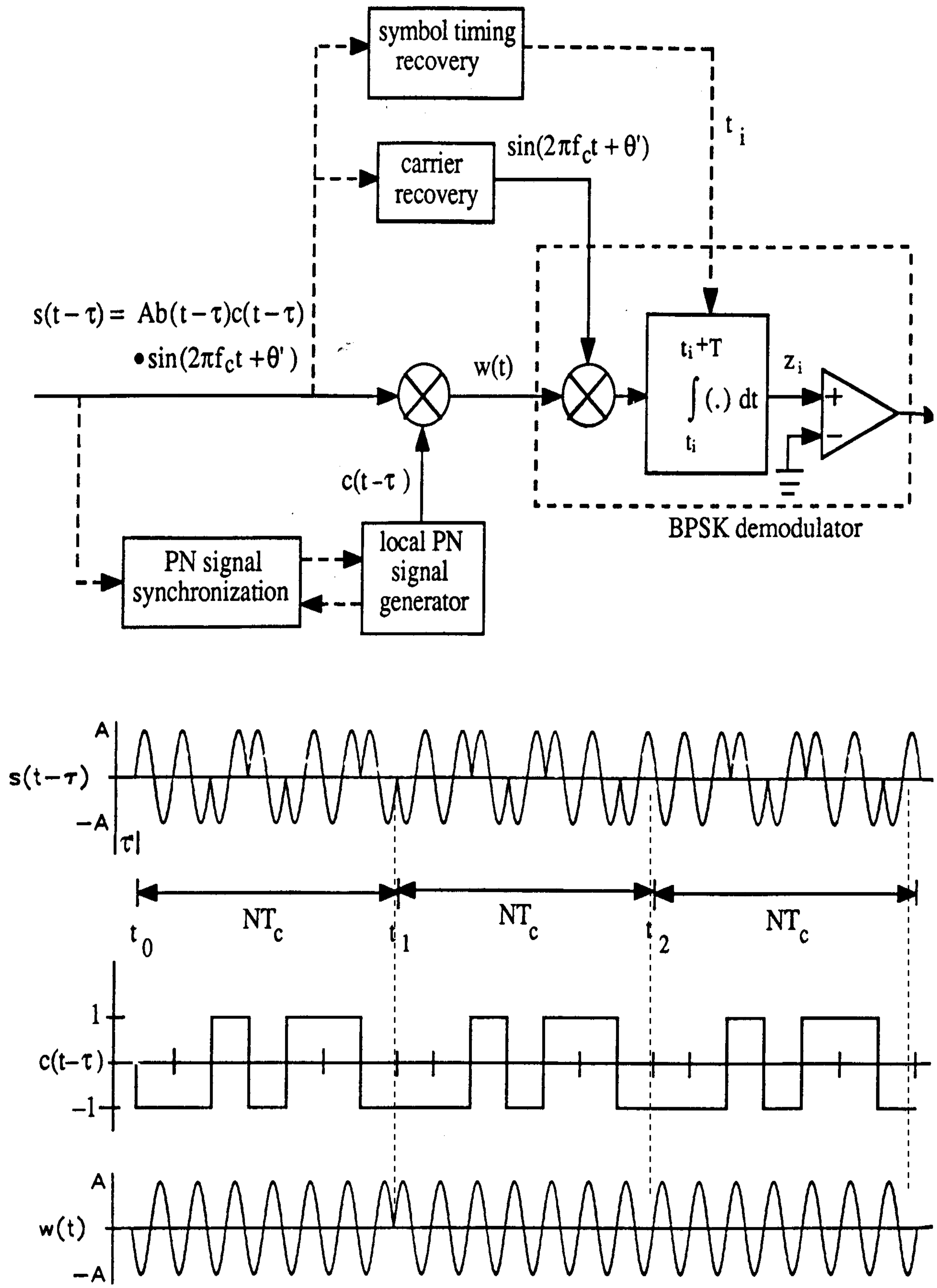
CDMA – HOW?

- In CDMA systems, the message signal is directly multiplied by a high-rate signal called the **spreading code/signal**. This operation is called **direct-sequence (DS) spreading,** which is a form of spread-spectrum modulation. The system is therefore also called DS-CDMA system.
- Different users are assigned different spreading codes: code division. The codes are orthogonal (eg. Walsh Hadamard sequence) or near-orthogonal (eg. Gold sequence).
- All user signals use the same carrier frequency (universal frequency re-use) and may be transmitted at the same time. This results in inter-user interference.
- The receiver extracts the desired message signal from the received signal (which contains thermal noise, inter-user interference and other channel impairments) by **correlating** the received signal with an exact **synchronised copy** of spreading code used by the transmitter. This operation is called **despreading**.
- With proper selection of spreading codes, the despreading operation can remove or suppress the otherwise very large inter-user interference and other channel impairments.

Fundamentals of Direct Sequence Spread-Spectrum Communications



Block diagram of DS/SS-BPSK transmitter.



Block diagram of a DS/SS-BPSK receiver.

Example

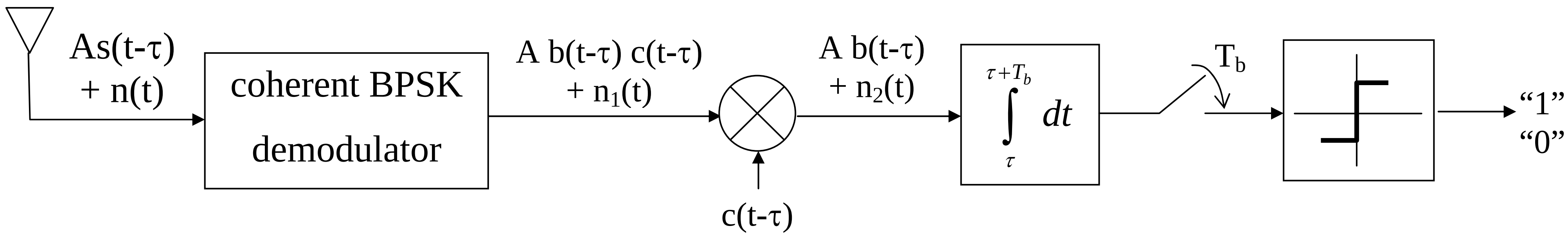
User 1:	Transmit bit 0	:	$+1$
	Walsh Hadamard code $\mathbf{w}_1 = [0\ 0\ 1\ 1]$	:	$+1\quad +1\quad -1\quad -1$
	Transmitted signal, $\mathbf{s}_1$	:	$+1\quad +1\quad -1\quad -1$
User 2:	Transmit bit 1	:	-1
	Walsh Hadamard code $\mathbf{w}_2 = [0\ 1\ 1\ 0]$	:	$+1\quad -1\quad -1\quad +1$
	Transmitted signal, $\mathbf{s}_2$	:	$-1\quad +1\quad +1\quad -1$

Supposed $\mathbf{s}_1$ and $\mathbf{s}_2$ are transmitted synchronously. At the wireless channel, both are added, $\mathbf{r} = \mathbf{s}_1 + \mathbf{s}_2 = [\ 0 \quad +2 \quad 0 \quad -2\ ]$

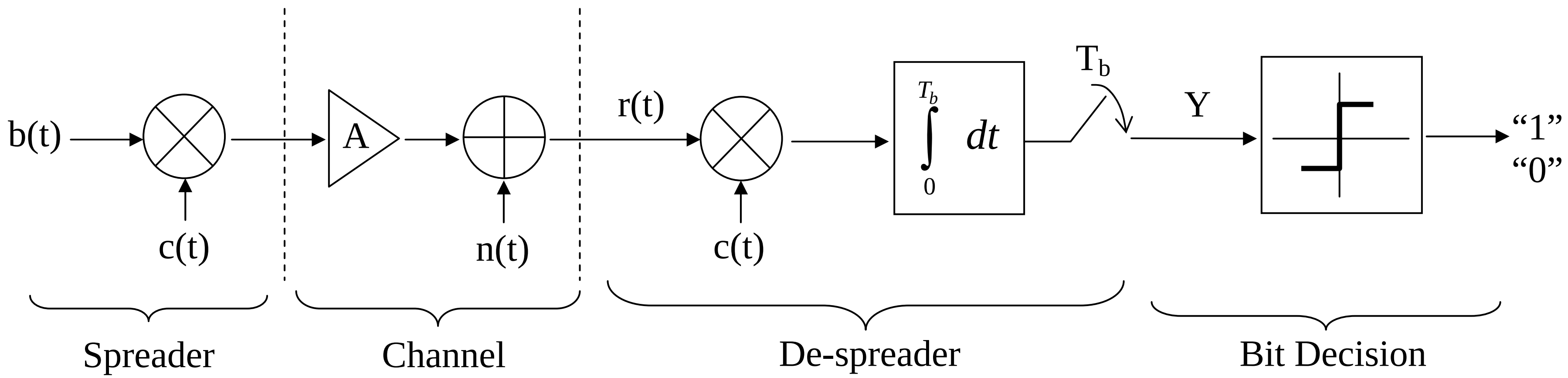
A user who would like to detect data transmitted by User 1:
$r \times (\mathbf{w}_1)^T = [\ 0 \quad +2 \quad 0 \quad +2]$ Integrator output $= 0 + 2 + 0 + 2 = +4 \rightarrow$ bit 0.

A user who would like to detect data transmitted by User 2:
$r \times (\mathbf{w}_2)^T = [\ 0 \quad -2 \quad 0 \quad -2]$ Integrator output $= 0 - 2 + 0 - 2 = -4 \rightarrow$ bit 1.

Performance of Coherent DS-BPSK System in Noisy Channels



The $\sin(2\pi f_c t)$ frequency carrier can be assumed completely removed by the downconverter. Hence the entire coherent DS-BPSK system –transmitter (spreader) + channel + receiver (de-spreader + bit decision) – can be modelled by the following block diagram:

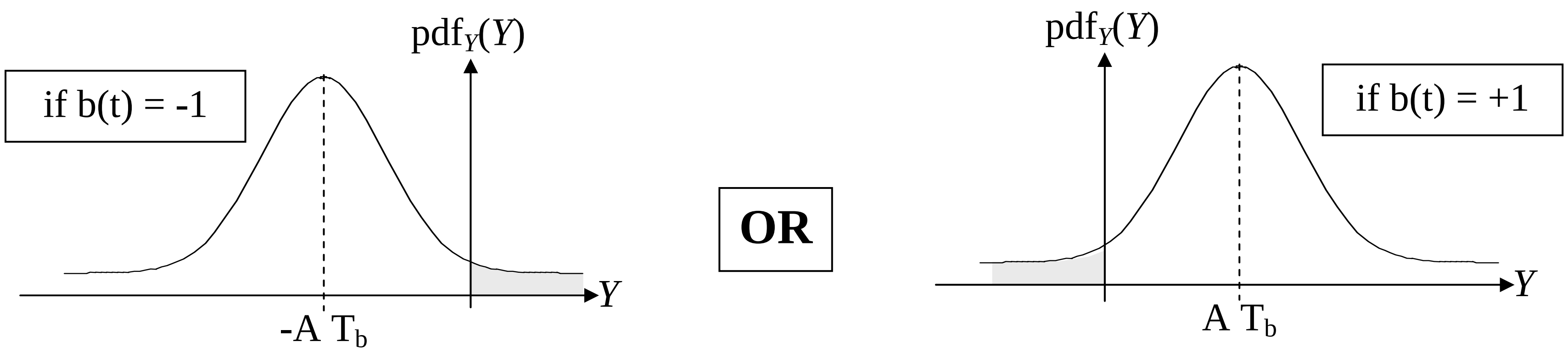


BER of Coherent DS-BPSK System (Single-User)

In digital communication system, an important figure of merit is the bit error rate (BER) or bit error probability (BEP). If n(t) is Additive White Gaussian Noise (AWGN) with zero mean and 2-sided power spectral density (PSD) of $N_o/2$,

$$
\begin{aligned}
r(t) &= Ab(t)c(t) + n(t) \\
Y &= \int_0^{T_b} r(t)c(t)\, dt \\
&= \int_0^{T_b} Ab(t)c^2(t)\, dt + \int_0^{T_b} n(t)c(t)\, dt \\
&= (\pm)AT_b \qquad\qquad\quad + \text{noise term}
\end{aligned}
$$

Since $n(t)$ is Gaussian distributed with zero mean, Y is a random variable with a probability density function (pdf) of



That is, Y is Gaussian distributed with mean $E(Y)$ and variance $Var(Y)$, where

$$E[Y] = E\left[(\pm)AT_b + \int_0^{T_b} n(t)c(t)\,dt\right] = (\pm)AT_b + \int_0^{T_b} E[n(t)]c(t)\,dt = (\pm)AT_b \quad \text{since } E[n(t)] = 0$$

$$\begin{aligned}
Var[Y] &= Var[(\pm)AT_b\ ] + Var\left[\int_0^{T_b} n(t)c(t)\,dt\right] \\
&= \quad 0 \quad + E\left[\left(\int_0^{T_b} n(t)c(t)\,dt\right)^2\right] \quad - E^2\left[\int_0^{T_b} n(t)c(t)\,dt\right] \\
&= \quad E\left[\int_0^{T_b} n(t)c(t)\,dt \times \int_0^{T_b} n(u)c(u)\,du\right] \quad - \quad 0 \\
&= E\left[\int_0^{T_b}\int_0^{T_b} n(t)c(t) \times n(u)c(u)\,dt\,du\right] \\
&= \int_0^{T_b}\int_0^{T_b} E[n(t)n(u)] \times c(t)c(u)\,dt\,du \\
&= \int_0^{T_b}\int_0^{T_b} \frac{N_0}{2}\delta[t-u] \times c(t)c(u)\,dt\,du \\
&= \frac{N_0}{2}\int_0^{T_b} c(t)c(t)\,dt \\
&= \frac{N_0 T_b}{2}
\end{aligned}$$

If $b(t) = -1$ was sent, the BER is

$$P_{eb}\big| -1 = \text{shaded area in plot of pdf}_{\mathrm{y}}(Y)$$

$$= Q\left(\frac{E[Y]}{std[Y]}\right) = Q\left(\frac{AT_b}{\sqrt{N_0 T_b / 2}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

where E_b = bit energy = $A^2\, T_b$

and $$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt$$

Since it is equally likely to have $b(t) = -1$ and $b(t) = +1$, the average BER is:

$$P_e = \frac{1}{2}\left(P_{eb}\big| -1\right) + \frac{1}{2}\left(P_{eb}\big| +1\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \text{since } \left(P_{eb}\big| -1\right) = \left(P_{eb}\big| +1\right)$$

<u>Note</u>

1. Processing Gain/Spreading Ratio/Spreading Factor of the DS-BPSK system

   = number of spreading chips per input data bit

   = T_b/T_c i.e. bit duration over chip duration

   = r_c/r_b i.e. baseband chip rate over baseband bit rate

2. In a multi-user CDMA system, every user is assigned a unique spreading code $c_i(t)$.

3. In general, 2 types of spreading can be used:
   - Full-period spreading: PG = length of spreading
   - Partial-period spreading: PG < length of spreading

Q-Function Q(x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5.000E-01	4.960E-01	4.920E-01	4.880E-01	4.840E-01	4.801E-01	4.761E-01	4.721E-01	4.681E-01	4.641E-01
0.1	4.602E-01	4.562E-01	4.522E-01	4.483E-01	4.443E-01	4.404E-01	4.364E-01	4.325E-01	4.286E-01	4.247E-01
0.2	4.207E-01	4.168E-01	4.129E-01	4.090E-01	4.052E-01	4.013E-01	3.974E-01	3.936E-01	3.897E-01	3.859E-01
0.3	3.821E-01	3.783E-01	3.745E-01	3.707E-01	3.669E-01	3.632E-01	3.594E-01	3.557E-01	3.520E-01	3.483E-01
0.4	3.446E-01	3.409E-01	3.372E-01	3.336E-01	3.300E-01	3.264E-01	3.228E-01	3.192E-01	3.156E-01	3.121E-01
0.5	3.085E-01	3.050E-01	3.015E-01	2.981E-01	2.946E-01	2.912E-01	2.877E-01	2.843E-01	2.810E-01	2.776E-01
0.6	2.743E-01	2.709E-01	2.676E-01	2.643E-01	2.611E-01	2.578E-01	2.546E-01	2.514E-01	2.483E-01	2.451E-01
0.7	2.420E-01	2.389E-01	2.358E-01	2.327E-01	2.296E-01	2.266E-01	2.236E-01	2.206E-01	2.177E-01	2.148E-01
0.8	2.119E-01	2.090E-01	2.061E-01	2.033E-01	2.005E-01	1.977E-01	1.949E-01	1.922E-01	1.894E-01	1.867E-01
0.9	1.841E-01	1.814E-01	1.788E-01	1.762E-01	1.736E-01	1.711E-01	1.685E-01	1.660E-01	1.635E-01	1.611E-01
1.0	1.587E-01	1.562E-01	1.539E-01	1.515E-01	1.492E-01	1.469E-01	1.446E-01	1.423E-01	1.401E-01	1.379E-01
1.1	1.357E-01	1.335E-01	1.314E-01	1.292E-01	1.271E-01	1.251E-01	1.230E-01	1.210E-01	1.190E-01	1.170E-01
1.2	1.151E-01	1.131E-01	1.112E-01	1.093E-01	1.075E-01	1.056E-01	1.038E-01	1.020E-01	1.003E-01	9.853E-02
1.3	9.680E-02	9.510E-02	9.342E-02	9.176E-02	9.012E-02	8.851E-02	8.692E-02	8.534E-02	8.379E-02	8.226E-02
1.4	8.076E-02	7.927E-02	7.780E-02	7.636E-02	7.493E-02	7.353E-02	7.215E-02	7.078E-02	6.944E-02	6.811E-02
1.5	6.681E-02	6.552E-02	6.426E-02	6.301E-02	6.178E-02	6.057E-02	5.938E-02	5.821E-02	5.705E-02	5.592E-02
1.6	5.480E-02	5.370E-02	5.262E-02	5.155E-02	5.050E-02	4.947E-02	4.846E-02	4.746E-02	4.648E-02	4.551E-02
1.7	4.457E-02	4.363E-02	4.272E-02	4.182E-02	4.093E-02	4.006E-02	3.920E-02	3.836E-02	3.754E-02	3.673E-02
1.8	3.593E-02	3.515E-02	3.438E-02	3.362E-02	3.288E-02	3.216E-02	3.144E-02	3.074E-02	3.005E-02	2.938E-02
1.9	2.872E-02	2.807E-02	2.743E-02	2.680E-02	2.619E-02	2.559E-02	2.500E-02	2.442E-02	2.385E-02	2.330E-02
2.0	2.275E-02	2.222E-02	2.169E-02	2.118E-02	2.068E-02	2.018E-02	1.970E-02	1.923E-02	1.876E-02	1.831E-02
2.1	1.786E-02	1.743E-02	1.700E-02	1.659E-02	1.618E-02	1.578E-02	1.539E-02	1.500E-02	1.463E-02	1.426E-02
2.2	1.390E-02	1.355E-02	1.321E-02	1.287E-02	1.255E-02	1.222E-02	1.191E-02	1.160E-02	1.130E-02	1.101E-02
2.3	1.072E-02	1.044E-02	1.017E-02	9.903E-03	9.642E-03	9.387E-03	9.137E-03	8.894E-03	8.656E-03	8.424E-03
2.4	8.198E-03	7.976E-03	7.760E-03	7.549E-03	7.344E-03	7.143E-03	6.947E-03	6.756E-03	6.569E-03	6.387E-03
2.5	6.210E-03	6.037E-03	5.868E-03	5.703E-03	5.543E-03	5.386E-03	5.234E-03	5.085E-03	4.940E-03	4.799E-03
2.6	4.661E-03	4.527E-03	4.397E-03	4.269E-03	4.145E-03	4.025E-03	3.907E-03	3.793E-03	3.681E-03	3.573E-03
2.7	3.467E-03	3.364E-03	3.264E-03	3.167E-03	3.072E-03	2.980E-03	2.890E-03	2.803E-03	2.718E-03	2.635E-03
2.8	2.555E-03	2.477E-03	2.401E-03	2.327E-03	2.256E-03	2.186E-03	2.118E-03	2.052E-03	1.988E-03	1.926E-03
2.9	1.866E-03	1.807E-03	1.750E-03	1.695E-03	1.641E-03	1.589E-03	1.538E-03	1.489E-03	1.441E-03	1.395E-03
3.0	1.350E-03	1.306E-03	1.264E-03	1.223E-03	1.183E-03	1.144E-03	1.107E-03	1.070E-03	1.035E-03	1.001E-03
3.1	9.677E-04	9.355E-04	9.043E-04	8.741E-04	8.448E-04	8.164E-04	7.889E-04	7.623E-04	7.364E-04	7.114E-04
3.2	6.872E-04	6.637E-04	6.410E-04	6.190E-04	5.977E-04	5.771E-04	5.571E-04	5.378E-04	5.191E-04	5.010E-04
3.3	4.835E-04	4.665E-04	4.501E-04	4.343E-04	4.189E-04	4.041E-04	3.898E-04	3.759E-04	3.625E-04	3.495E-04
3.4	3.370E-04	3.249E-04	3.132E-04	3.018E-04	2.909E-04	2.803E-04	2.701E-04	2.603E-04	2.508E-04	2.416E-04
3.5	2.327E-04	2.241E-04	2.158E-04	2.078E-04	2.001E-04	1.927E-04	1.855E-04	1.785E-04	1.718E-04	1.654E-04
3.6	1.591E-04	1.531E-04	1.473E-04	1.417E-04	1.364E-04	1.312E-04	1.261E-04	1.213E-04	1.166E-04	1.122E-04
3.7	1.078E-04	1.037E-04	9.964E-05	9.577E-05	9.204E-05	8.844E-05	8.498E-05	8.165E-05	7.844E-05	7.535E-05
3.8	7.237E-05	6.951E-05	6.675E-05	6.409E-05	6.154E-05	5.908E-05	5.671E-05	5.444E-05	5.225E-05	5.014E-05
3.9	4.812E-05	4.617E-05	4.429E-05	4.249E-05	4.076E-05	3.909E-05	3.749E-05	3.595E-05	3.447E-05	3.305E-05
4.0	3.169E-05	3.037E-05	2.911E-05	2.790E-05	2.674E-05	2.562E-05	2.455E-05	2.352E-05	2.253E-05	2.158E-05

Walsh Hadamard Codes - Orthogonal Spreading Codes

- If cross-correlation = 0 for all pairs of $c_i(t)$ and $c_j(t)$ ($i \neq j$), then the set of spreading codes are **orthogonal**.

- By assigning users different orthogonal codes, their signals can be recovered without MAI at all. So orthogonal codes can provide **perfect channel separation** in CDMA systems, much in the same way as filtering or time gating in FDMA and TDMA systems respectively.

- A well-known class of binary orthogonal code is the Walsh Hadamard (WH) Code. WH spreading code and its modified versions are used in the downlink of IS95 and next-generation WCDMA systems to separate user signals from within the same cell.

WH Codes – Generation using Hadamard Matrix

$$\mathbf{H}_0 = [0] \qquad \mathbf{H}_n = \begin{bmatrix} \mathbf{H}_{n-1} & \mathbf{H}_{n-1} \\ \mathbf{H}_{n-1} & \overline{\mathbf{H}}_{n-1} \end{bmatrix}$$

H_2

H_1

$$\mathbf{H}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

map

0 → +1

1 → -1

→ WH Codes /Functions

❖**Qn: Any more direct method?**

WH Codes – Properties

- Length of code is always 2^k with $k \geq 1$ and k = integer
- There are 2^k WH codes of length 2^k (since WH matrix is always square)
- 1st WH row/code always has a constant DC value (all zeros code)
- Cross-correlation = 0 so long as code pairs are synchronous.
- What is the cross-correlation of WH codes which are not synchronous? Try out yourself ...
- How about its autocorrelation? Try out yourself ...
- Cross-correlation of CDMA codes determines their MAI. Higher cross-correlation, higher MAI (see page 28).
- Auto-correlation of a CDMA code determines its multipath ISI rejection and synchronisation capabilities.

Near-Orthogonal Code/Sequence: Pseudo-Noise (PN) Sequence

- Although WH codes are perfectly orthogonal, they are quite easy to predict, hence not secure. Furthermore, they are also difficult to synchronize and track due to bad auto-correlation characteristics (with large peaks at non-zero time-shift).

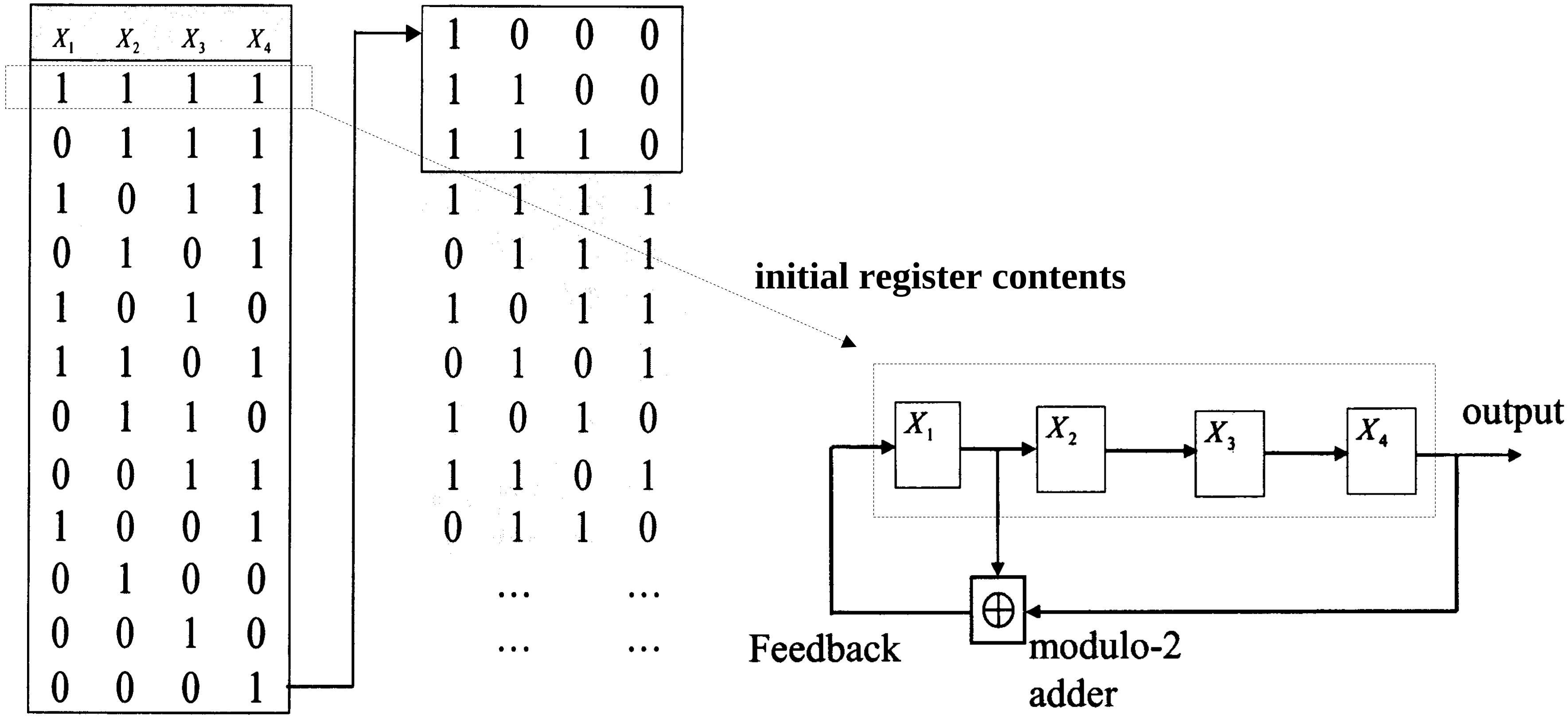
- PN sequences have better security features and auto-correlation characteristics. They are not completely orthogonal, but they can be designed to be **near-orthogonal** (i.e. their cross correlations are non-zero, but small).

- **Pseudo** noise means **fake, man-made** noise, ie. it appears like random noise to other users but can be reproduced exactly by the intended user. True noise cannot be reproduced. For obvious reasons, pseudo-noise sequences are also called **pseudo-random** sequences.

- The most important class of PN sequence is the **binary** maximal-length sequence**,** or **m-seq**.

Maximal-Length Sequence (m-sequence)

The binary m-sequences (with values 0 and 1) can be generated using linear feedback shift registers (LFSR) and boolean Exclusive-OR gates (XOR) or modulo-2 adders.



m-sequence generated: [1 1 1 1 0 1 0 1 1 0 0 1 0 0 0]

Binary Primitive Polynomial Coefficients for degrees $3 \le m \le 10$

$m = 3$

1011	1101

$m = 4$

10011	11001

$m = 5$

100101	101001
110111	111011
101111	111101

$m = 6$

1000011	1100111
1011011	1101101
1100001	1110011

$m = 7$

10000011	10001001	10001111
10010001	10011101	10100111
10101011	10111001	10111111
11000001	11001011	11010011
11010101	11100101	11101111
11110001	11110111	11111101

$m = 8$

100011101	100101011	100101101	101001101
101011111	101100011	101100101	101101001
101110001	110000111	110001101	110101001
111000011	111001111	111100111	111110101

$m = 9$

1000010001	1000011011	1000100001	1000101101
1000110011	1001011001	1001011111	1001101001
1001101111	1001110111	1001111101	1010000111
1010010101	1010100011	1010100101	1010101111
1010110111	1010111101	1011001111	1011010001
1011011011	1011110101	1011111001	1100010011
1100010101	1100011111	1100100011	1100110001
1100111011	1101001111	1101011011	1101100001
1101101011	1101101101	1101110011	1101111111
1110000101	1110001111	1110110101	1110111001
1111000111	1111001011	1111001101	1111010101
1111011001	1111100011	1111101001	1111111011

$m = 10$

10000001001	10000011011	10000100111	10000101101	10001100101
10001101111	10010000001	10010001011	10011000101	10011010111
10011100111	10011110011	10011111111	10100001101	10100011001
10100100011	10100110001	10100111101	10101000011	10101010111
10101101011	10110000101	10110001111	10110010111	10110100001
10111000111	10111100101	10111110111	10111111011	11000010011
11000010101	11000100101	11000110111	11001000011	11001001111
11001011011	11001111001	11001111111	11010001001	11010110101
11011000001	11011010011	11011011111	11011111101	11100010111
11100011101	11100100001	11100111001	11101000111	11101001101
11101010101	11101011001	11101100011	11101111101	11110001101

Some Important Properties of M-Sequence

1. **Period** = $2^m - 1$ where m = number of stages of shift registers

2. **(Near) Balanced:** There are exactly 2^{m-1} number of "1"s and $2^{m-1}-1$ number of "0"s in a period (ie. there is one more "1" than "0").

3. **Runs (consecutive "0"s and "1"s):** In each period, ½ of the runs are of length one, ¼ are of length two etc., finally with 1 run of length m.

4. **Shift:** A cyclic shift of a m-sequence is the same m-sequence generated using a different set of initial register contents.

5. **Shift & Add:** The modulo-2 sum of two shifted versions of the same m-sequence is the same m-sequence with yet another shift.

Randomness Properties

(1). **Balance property** — In each period of the sequence, the number of binary ones differs from the number of binary zeros by at most one digit.

e.g., a 15 chip PN sequence
`111100010011010`
number of 1's = 8
number of 0's = 7

(2). **Run property**

> A **run** is defined as a sequence of single type of binary digit(s).
> e.g., `...01110001110011...`
> run of 3   run of 2

In each period, half of the runs of consecutive zeros or ones are of length one, one-fourth are of length 2, one-eighth are of length 3, and so on.

e.g., for the 15-chip PN sequence defined above.

runs of 4	1
runs of 3	1
runs of 2	2
runs of 1	4
Total	8

3. **Shift-and-add property**—The modulo-2 sum of an *m*-sequence and any shifted version of the same sequence is another shifted version of the same *m*-sequence

Example

shift *three* chips and add

```
...111101011001000...
...000111101011001...
---------------------
...111010110010001...
```

4. If a window of width m is slid along the sequence for $2^m - 1$ shifts, each m-tuple except the all-zero m-tuple [Why?] will appear **exactly once**. For example,

Example

...$\boxed{1111}$010$\boxed{1100}$1000...

1st window — 8th window

5. Periodic autocorrelation function

Define

$$\phi(k) = \sum_{j=1}^{N} c_j \, c_{j+k}; \quad 0 \le k \le N-1$$

and

$$R(k) = \frac{1}{N}\sum_{j=1}^{N} c_j \, c_{j+k}; \quad 0 \le k \le N-1$$

where $\phi(k)$ is the autocorrelation function, $R(k)$ is the normalized autocorrelation function and $\boxed{c_k \in \{\pm 1\}}$.

For m-sequences,

$$\phi(k) = \begin{cases} N & k = 0 \\ -1 & 1 \le k \le N-1 \end{cases}$$

and

$$R(k) = \begin{cases} 1 & k = 0 \\ -1/N & 1 \le k \le N-1 \end{cases}$$

For large values of N or long sequences, the off-peak values of $\phi(k)$ relative to the peak value

$$\phi(k)/\phi(0) = -1/N$$

Autocorrelation function—The **normalized** autocorrelation function $R_c(\tau)$ of an m-sequence **waveform** $c(t)$ with period T_o is given by

$$R_C(\tau) = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} c(t)\,c(t+\tau)\,dt$$

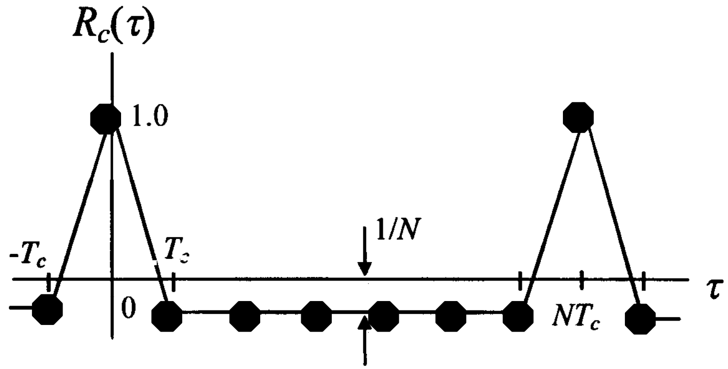
$$= \begin{cases} 1 - \dfrac{\tau}{T_c}(1 + 1/N) & |\tau| \le T_c \\ -1/N & T_c < \tau < (N-1)T_c \end{cases}$$

Note that $T_o = NT_c$, where T_c is the chip duration. It is apparent that the autocorrelation function is periodic with triangular pulses of width $2T_c$ repeated every NT_c seconds and that a correlation level of $-1/N$ occurs between these triangular pulses.

Note:

● = $R(k)$

= autocorrelation of the *sequence* instead of the waveform



Correlation Properties of M-Sequence

- The **impulse-like periodic autocorrelation** characteristic of m-sequences approaches that of a purely random white noise sequence. It can be shown to be the best periodic autocorrelation achievable with any man-made binary sequence. In fact, it is the **most important distinguishing feature** of a m-sequence.

- This near-ideal autocorrelation property makes the m-sequence ideal for **synchronisation** purpose.

- The cross-correlation values of m-sequences are however much worse: there are many high–valued peaks instead of the ideal zero value for all shifts. Different m-sequences (generated by different LFSR structures) are therefore hardly used directly as CDMA spreading codes.