NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2015-2016

EE7403 – IMAGE ANALYSIS & PATTERN RECOGNITION

April/May 2016 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 4 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. A noisy digital image $g(x,y)$ is given by:

$$g(x,y) = f(x,y) + n(x,y),$$

$$\text{where } f(x,y) = \begin{cases} 9, & \text{for } x > 8 \\ 0, & \text{for } x \le 8 \end{cases}$$

$$\text{and } n(x,y) = 9\delta(x-4,y) - 9\delta(x-15).$$

(a) A mean filter of size 3 x 3 is applied to the image $g(x,y)$. Find the output image $g_a(x,y)$.

(8 Marks)

(b) Express the impulse response of the mean filter of size 3 x 3.

(5 Marks)

(c) A median filter of size 3 x 3 is applied to the image $g(x,y)$. Find the output image $g_m(x,y)$.

(7 Marks)

2. (a) Express the following 6 operations of binary morphological image processing.

(i) $A - B = ?$

(ii) $(B)_z = ?$

(iii) $A \ominus B = ?$

(iv) $A \oplus B = ?$

(v) $A \circ B = ?$

(vi) $A \bullet B = ?$

(8 Marks)

(b) A set A contains the shaded squares in Figure 2.1. Let $C = A - (A \ominus B)$, where B contains the shaded squares in Figure 2.2.

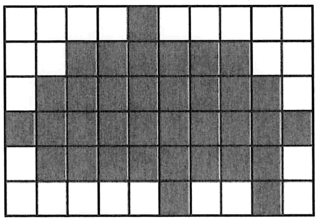

Figure 2.1

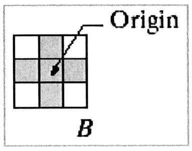

Figure 2.2

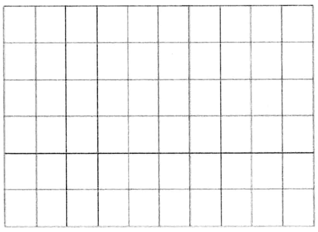

Figure 2.3

(i) Copy Figure 2.3 to the answer sheet and shade the elements of set C.

(6 Marks)

(ii) Suppose B is changed to a 3 x 3 mask as shown in Figure 2.4. Copy Figure 2.3 to the answer sheet and shade the elements of set C.

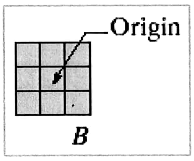

Figure 2.4

(6 Marks)

3. An image is modeled locally by $f(x,y) = 5 + \sin(0.1\sqrt{3}\pi x - 0.1\pi y)$, where x and y are continuous variable.

   (a) What is the orientation ϕ, along which $f(x,y)$ has the least amount of change?

   (5 Marks)

   (b) Compute the phase of image gradient $\varphi(x,y)$.

   (5 Marks)

   (c) Let $F(m,n)$ be the discrete image obtained by sampling $f(x,y)$ with a sampling interval 2. A gradient operator $[-1 \quad 0 \quad 1]$ and $[-1 \quad 0 \quad 1]^T$ is applied to the discrete image $F(m,n)$ to estimate the gradient of the image. Compute the phase of image gradient $\alpha(m,n)$ and the estimation error if we use it to estimate the orientation of the image.

   (10 Marks)

4. (a) For a two-class classification problem, express the decision rule in terms of the posterior probability that minimizes the probability of misclassification. Also, express the decision rule in terms of class-conditional probability density.

   (5 Marks)

   (b) Suppose the class-conditional probability densities are multi-dimensional Gaussian functions. Simplify the decision rule with detailed steps. Is the classifier linear or nonlinear?

   (5 Marks)

   (c) Suppose the two classes have the same covariance matrix. Further simplify the decision rule. Is the classifier linear or nonlinear? Justify your answer.

   (5 Marks)

   (d) In what condition, the classifier can be simplified to a minimum (Euclidian) distance classifier? From which of the above questions ((a), (b), (c) or (d)), the classifier can be called the minimum Mahalanobis distance classifier? Is the minimum Mahalanobis distance classifier linear or nonlinear?

   (5 Marks)

5. (a) A data set consists of L samples. Each sample is represented by an n-dimensional column vector $\mathbf{x}_i, 1 \le i \le L$. Express the data covariance matrix $\mathbf{\Sigma}$ in terms of $\mathbf{x}_i$.

(3 Marks)

(b) Suppose that the sample mean of the data set is zero and the data set is represented by the data matrix $\mathbf{X} = [\mathbf{x}_1 \quad \mathbf{x}_2 \quad \ldots \quad \mathbf{x}_L]$. Express the data covariance matrix $\mathbf{\Sigma}$ in terms of the matrix $\mathbf{X}$.

(3 Marks)

(c) An eigenvalue λ_k and its corresponding eigenvector ϕ_k of the data covariance matrix $\mathbf{\Sigma}$ are defined by $\mathbf{\Sigma}\phi_k = \lambda_k \phi_k$. Prove that the eigenvalue λ_k is the variance of the data $\mathbf{X}$ projected to the eigenvector ϕ_k.

(6 Marks)

(d) Suppose we have n distinct eigenvalues λ_k of the data covariance matrix $\mathbf{\Sigma}_x$ and their corresponding eigenvectors ϕ_k, forming a matrix, $\mathbf{\Phi} = [\phi_1 \quad \phi_2 \quad \ldots \quad \phi_n]$. All eigenvectors have unit length, $\| \phi_k \|_2 = \phi_k^T \phi_k = 1$. Let the kth component of the column vector $\mathbf{y}_i$ be the sample $\mathbf{x}_i$ projected to the eigenvector ϕ_k. Express the covariance matrix $\mathbf{\Sigma}_y$ of the projected data $\mathbf{y}_i, 1 \le i \le L$ and simplify it to the simplest form.

(8 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompained by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.