NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

EE7403 – IMAGE ANALYSIS & PATTERN RECOGNITION

April/May 2017 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 3 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. Consider an image $g(x,y)$ that is an image $f(x,y)$ degraded by $h(x,y)$ and $\eta(x,y)$ in the spatial domain as given by:

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

(a) Express the degraded image in the frequency domain, $G(u,v)$.

(5 Marks)

(b) The degradation function in the spatial domain is given by

$$h(x,y) = \begin{cases} 1, & -5 \le x \le 5, -8 \le y \le 8 \\ 0, & \text{otherwise} \end{cases}$$

Design a generalized inverse filter $R(u,v)$ in the frequency domain for image restoration from the degraded image $g(x,y)$ so that $|R(u,v)| < 5$.

(8 Marks)

(c) Supposing that the power spectra of the image $f(x,y)$ and noise $\eta(x,y)$ are respectively given by $S_f(u,v) = \dfrac{1}{2u^2 + 3v^2}$ and $S_\eta(u,v) = 0.5$, design the Wiener filter for the image restoration.

(7 Marks)

2. A binary image $f(x,y)$ is given by $f(0,0)=1, f(1,3)=1, f(3,1)=1, f(2,2)=1$ and $f(x,y)=0$ for all other pixels. The result of the Hough transform of $f(x,y)$, where $y=ax+b$ is applied, is stored in the image $g(a,b)$ of size 20×20. Suppose that there is no discretizing error in the Hough transform $g(a,b)$.

   (a) What is the sum of all values of $g(a,b)$ over all *a*- and *b*-values?

   (5 Marks)

   (b) How many points in $g(a,b)$ have the value 2?

   (5 Marks)

   (c) Compute the histogram $p_g(g)$ of the image $g(a,b)$.

   (5 Marks)

   (d) If $g_m(a_m,b_m)=\max\{g(a,b)\}$, what are the values of g_m, a_m, b_m?

   (5 Marks)

3. The linear classifier is the most frequently applied classifier in the practice, which can be formulated by the discriminant function of class ω_i as

$$g_i(\mathbf{x})=\mathbf{w}_i^T\mathbf{x}+b_i$$

where $\mathbf{x}=[x_1 \quad x_2 \quad \cdots \quad x_n]^T$ is the feature vector to be classified. Another simple classifier is minimum weighted distance classifier, which can be expressed by the discriminant function of class ω_i as

$$g_i(\mathbf{x})=-\sum_{j=1}^{n}(x_j-\mu_{ij})^2/\sigma_{ij}^2+c_i,$$

where μ_{ij} and σ_{ij}^2 are the class-conditional mean and variance of x_j for class ω_i.

   (a) Derive the conditions and weighting vector $\mathbf{w}_i$ with which the linear classifier is the optimal classifier.

   (10 Marks)

   (b) Derive the conditions under which the minimum weighted distance classifier is the optimal classifier.

   (6 Marks)

   (c) Compare the complexity of the above two classifiers in the design/training phase and in the online classification phase.

   (4 Marks)

Hint:
An optimal classifier minimizes the probability of the misclassification. This is achieved by choosing the class that has the maximum probability after the observation of $\mathbf{x}$.

4. (a) A matrix $\mathbf{X}$ has m rows and n columns. Determine the size of the matrix $\mathbf{C}$, $\mathbf{C} = \mathbf{X}\mathbf{X}^T$ and show whether $\mathbf{C}$ is symmetric or not.

(4 Marks)

(b) Prove that eigenvectors ϕ_k corresponding to distinct eigenvalues λ_k of a symmetric matrix $\mathbf{\Sigma}$ are orthogonal. (Hint: an eigenvalue λ_k and its corresponding eigenvector ϕ_k of a matrix $\mathbf{\Sigma}$ satisfy $\mathbf{\Sigma}\phi_k = \lambda_k \phi_k$.)

(7 Marks)

(c) The Mahalanobis distance between a sample vector X and its class mean vector M of size $n \times 1$ is given by

$$d(X, M) = (X - M)^T \mathbf{\Sigma}^{-1} (X - M)$$

To facilitate the analysis of its characteristics, we need to covert the distance into scalar form:

$$d(X, M) = (X - M)^T \mathbf{\Sigma}^{-1} (X - M) = \sum_{k=1}^{n} w_k (y_k - \mu_k)^2$$

Derive w_k and y_k and μ_k from $\mathbf{\Sigma}$, X and M.

(9 Marks)

5. (a) A two-layer feed-forward neural network contains two inputs, two hidden neurons and one output neuron. Draw the diagram of this network and show all network weights in the diagram.

(6 Marks)

(b) The above network has nonlinear activation function of $f(s) = \dfrac{1}{1 + \exp(-s)}$ for the hidden neurons and linear activation function of $g(s) = s$ for the output neuron. Express the output y of the network as a function of the input vector $[x_1\ x_2]$.

(6 Marks)

(c) In a step-by-step manner, derive a learning rule that adjusts the weights of the output neuron to make the squared error between the network output y and the desired output t smaller.

(8 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.