NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

EE7403 – IMAGE ANALYSIS & PATTERN RECOGNITION

April/May 2018 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 3 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. An output image $g(x,y)$ is produced by processing the digital image $f(x,y)$ as given by:

$$g(x,y) = f(x-1,y-1) + 2f(x-1,y) + f(x-1,y+1) \\ -f(x+1,y-1) - 2f(x+1,y) - f(x+1,y+1).$$

(a) Derive the impulse response $h(x,y)$ of the processing system and show the filter mask indicating the x- and y-axes besides the mask.

(10 Marks)

(b) Compute the Fourier transform of the impulse response of the filter $H(u,v)$ given by:

$$H(u,v) = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} h(x,y) \exp[-j2\pi(ux+vy)]$$

and determine the properties of the filter.

(10 Marks)

2. An edge detection algorithm produces a binary image

$$f(x,y)=\delta(x,y-1)+\delta(x-1,y-2)+\delta(x+1,y-2)+\delta(x-2,y-3)+\delta(x+2,y-3).$$

The possible pixels (x, y) on the edge are indicated by $f(x,y)>0$. A Hough transform $g(a,b)$ of model $y=ax+b$ is applied to detect the lines in $f(x,y)$.

(a) Based on the Hough transform $g(a,b)$, determine the number of lines that have more than one pixels in image and determine the equations of lines $y=ax+b$ that have the most number of pixels.

(10 Marks)

(b) Let, $G(a,b)=\begin{cases} g(a,b), & \text{if } g(a,b)>1 \\ 0, & \text{otherwise} \end{cases}$

Determine $G(a,b)$.

(10 Marks)

3. Suppose that the class prior probability and the class-conditional probability density function (PDF) are given as $p(\omega_i)$ and $p(\mathbf{x}\mid\omega_i)$ for $\omega_i, i=1,2,\ldots,c$.

(a) Express the probability of the error $p(e_i\mid\mathbf{x})$ of classifying an input sample $\mathbf{x}$ to class ω_i, and hence figure out the decision rule that minimizes $p(e_i\mid\mathbf{x})$.

(10 Marks)

(b) Show how to use training samples to estimate $p(\omega_i)$ and $p(\mathbf{x}\mid\omega_i)$ and $p(\mathbf{x})$ so that the decision rule that minimizes the probability of the wrong decision $p(e_i\mid\mathbf{x})$ leads to the simple *k*-nearest neighbor classifier.

(10 Marks)

4. A dataset contains 5 two-dimensional samples: $\mathbf{x}_1 = [5 \quad 1]^T$, $\mathbf{x}_2 = [4 \quad 2]^T$, $\mathbf{x}_3 = [3 \quad 3]^T$, $\mathbf{x}_4 = [2 \quad 4]^T$ and $\mathbf{x}_5 = [1 \quad 5]^T$.

(a) Compute the sample covariance matrix of the dataset.

(5 Marks)

(b) Plot the five samples as 5 points in the data space $\mathbf{x}$.

(5 Marks)

(c) Obtain the eigenvalues and the unit-length eigenvectors of the covariance matrix.

(5 Marks)

(d) Interpret the meanings of the eigenvalue and the eigenvector of a covariance matrix that support the results in (a), (b) and (c).

(5 Marks)

5. (a) A RBF neural network of two hidden neurons is applied to a classification problem where two samples $X_1 = [1 \ 1]^T$, $X_2 = [2 \ 2]^T$ of class A and two samples $X_3 = [1 \ 2]^T$, $X_4 = [2 \ 1]^T$ of class B are given to train the network. Assume the centers of two hidden neurons are set to: $C_1 = [1 \ 1]^T$, $C_2 = [2 \ 2]^T$, and the width of the Gaussian basis function is set to $2\sigma^2 = 1$. Determine the network parameters trained by the 4 samples and check if the trained network classify the 4 training samples correctly.

(12 Marks)

(b) The relation of the output y_j and the input x_i of a layer of neural network has a general form as:

$$y_j = \sum_{i=1}^{n} w_{ij} x_i$$

We want to design a convolutional network so that the output y_j, $1 \le j \le n-2$, is the convolution of input by a filter mask $h^1 = [a \quad b \quad a]$, and y_j, $n-1 \le j \le 2n-4$, is the convolution of input by another filter mask $h^2 = [c \quad d \quad c]$. Specify the parameters w_{ij} of such a network in terms of filter coefficients. How many percent of the parameters are definitely zero for $n = 100$?

(8 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.