NANYANG TECHNOLOGICAL UNIVERSITY
School of Electrical & Electronic Engineering

E6101 Digital Communication Systems

Continuous Assessment (Year 2021/2022)

The 4 questions below have been selected for the CONTINUOUS ASSESSEMENT. Each question carries equal mark. You are required to solve the problems yourself. Note that plagiarism will result in serious consequences. Write down your solutions neatly on A4-size paper sheets with dark ball pens. Please scan the document and submit your solutions to NTULearn under Assignment folder before Thursday, 7 October 2021 at Week 8. Late penalty will be imposed, and no marks will be given for late submission of more than 1 week.

1. (a) The random variable Z is a decision variable for the binary detection in a receiver, and is given by the following uniform probability density function (pdf) $f_Z(z)$ as shown below.

$$f_Z(z) = \begin{cases} k & a_1 \le z \le a_2 \\ 0 & otherwise, \end{cases}$$

where k, a_1 and a_2 are fixed parameters.

(i) Determine the value of k for this pdf.

(ii) Let the value of $a_1 = -1$ and $a_2 = 2$, then calculate the value of P($|Z| \le ½$).

(iii) Find the mean value and variance of Z.

(8 Marks)

(b) A Wide Sense Stationary (WSS) random process **X(t)** has a Power Spectral Density (PSD) **S$_X$(f)**. It is applied to a differentiator to produce the output derivative random process **Y(t)** as shown in Figure 1.

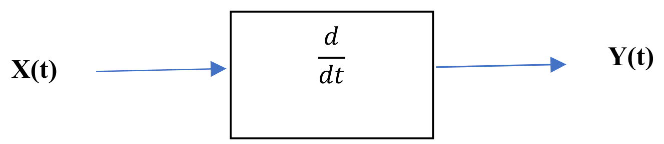

Figure 1

(i) The autocorrelation function $R_X(\tau) = E[X(t)X(t+\tau)]$. Show the autocorrelation function $R_Y(\tau)$ of the output **Y(t)** is related as shown below. Determine the expression for the output power spectral density **$S_Y(f)$** of **Y(t)**.

$$R_Y(\tau) = -\frac{d^2 R_X(\tau)}{d\tau^2}.$$

(ii) If the autocorrelation function of **X(t)** is given by $R_X(\tau) = 16\ \text{sinc}(4\tau)$, determine the PSD of **Y(t)** and calculate its average power. Explain the significance of your results for higher frequency component of the output **Y(t).**

(12 Marks)

2. (a) If the Fourier transform of an energy signal $x(t)$ is denoted by $X(f)$, then

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df .$$

State the physical meaning of the above relation. Provide the details of the derivation.

(8 Marks)

(b) Let

$$x(t) = \left[\frac{2}{T_b}\text{sinc}\left(\frac{2t}{T_b}\right)\right] \otimes \left[\frac{1}{T_b}\text{sinc}\left(\frac{t-T_b}{T_b}\right)\right],$$

where T_b is a constant, $\otimes$ is the convolution operator and

$$\text{sinc}(t) \equiv \frac{\sin(\pi t)}{\pi t}.$$

Find the *spectrum* $X(f)$ of $x(t)$ and determine its energy E_x.

(6 Marks)

(c) The signal $x(t)$ is sampled by an ideal sampling function

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s),$$

where T_s is the sampling period and $\delta(t)$ is the unit impulse function. Design a low-pass filter (LPF) to recover the desired signal $x(t)$ without distortion. What are the bandwidth and the gain of the LPF? Draw the transfer function $H(f)$ of the LPF for illustration,

(6 Marks)

3. (a) The random variable X is normally distributed with probability density function (PDF)

$$p_X(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(x-1)^2}{2}\right).$$

Obtain the mean and variance of the random variable X by observation. Suppose X is applied to an electronic limiter with output Y characterized by

$$Y = \begin{cases} 3x/2, & x > -2, \\ -3, & x \le -2. \end{cases}$$

Plot the graph of Y versus X. Determine the PDF of the output Y of the limiter.

(10 Marks)

(b) The random variable V has a cumulative distribution function (CDF)

$$F_V(v) = \Pr\left(V \le v\right) = (1 - e^{-2v})u(v),$$

where $u(v)$ is the unit step function. Plot the curve of $F_V(v)$ versus v and find the PDF of V.

Define a new random variable W in terms of V by

$$W = F_V(V) = (1 - e^{-2V})u(V).$$

Determine the CDF $F_W(w)$ of W for $w < 0$, $0 \le w \le 1$ and $w > 1$. Finally, plot the CDF $F_W(w)$ and obtain the PDF of W.

(10 Marks)

4. Consider a binary signal detector with the input

$$r = \pm a + n,$$

where the signal component may be $+a$ or $-a$ with equal probability. The noise component n is described by the probability density function (PDF)

$$p_n(n) = \frac{1}{\sqrt{2}} \exp\left(-\sqrt{2}\,|\,n\,|\right).$$

(a) Plot the conditional PDFs $p_r(r\,|+a)$ and $p_r(r\,|-a)$ together. Determine the optimum threshold γ_0 of the binary signal detector and label the position of γ_0 clearly on the same graph.

(5 Marks)

(b) Given that the signal component $-a$ is received at the detector, derive the probability of bit error, i.e.,

$$\Pr\left(\text{error}\,|-a\right) = \Pr\left(r > \gamma_0\,|-a\right).$$

(6 Marks)

(c) Compute the overall probability of bit error of the binary signal detector.

(6 Marks)

(d) Suppose that the noise PDF is replaced by a Gaussian PDF with the same mean and variance. Without going through the detailed calculation, will the overall probability of bit error of the binary detector be bigger or smaller? Justify your answer.

(3 Marks)

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