NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2016-2017

EE6101 – DIGITAL COMMUNICATION SYSTEMS

November/ December 2016 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 6 questions and comprises 8 pages.
2. Answer any 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. A table of Fourier transform properties is provided in Appendix 1 (Page 7).
6. A Fourier transform table is provided in Appendix 2 (Page 8).

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1. A random signal $X(t)$ can be expressed as

$$X(t) = \begin{cases} A\left|\sin 2\pi f_0 t\right| & -T_0/2 \le t \le T_0/2 \\ 0 & \text{otherwise,} \end{cases}$$

where A is a positive-valued random variable and $T_0 = 1/f_0$.

(a) Plot the waveform $X(t)$ for a given value of A. Classify the signal as energy signal or power signal. Find the normalized energy or power of the signal in terms of A.

(4 Marks)

Note: Question No. 1 continues on page 2

(b) The random variable A has a probability density function (PDF)

$$p_A(a) = k\,e^{-2a}u(a),$$

where $u(a)$ is the unit step function and k is a constant. Determine the value of k.

(4 Marks)

(c) Suppose the signal $X(t)$ is sampled by an ideal sampling function

$$T_\delta(t) = \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{nT_0}{4}\right)$$

followed by an integrator. Compute the expected value of the integrator output, $\mathrm{E}\left[\int_{-\infty}^{\infty} X(t) \times T_\delta(t)\,dt\right]$, where the PDF has been defined in part (b).

(4 Marks)

(d) At the time instant $t = T_0/4$, $Z = X(T_0/4)$ is applied to an electronic device characterized by

$$Y = \begin{cases} Z^2 & Z \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Plot the graph of Y versus Z. Calculate the PDF of the output Y of the electronic device.

(8 Marks)

2. Figure 1 on page 3 shows the diagram of a synchronous receiver. The input $r(t) = s(t) + n(t)$ consists of two independent components, where the signal component

$$s(t) = \pm\sqrt{\frac{2E_b}{T}}\cos 2\pi f_c t, \quad 0 \leq t \leq T,$$

may be positive or negative with signal energy E_b during the bit duration T and the additive white Gaussian noise (AWGN) component $n(t)$ has zero mean and two-sided power spectral density (PSD) $N_0/2$. It is assumed that the positive and negative signal components are equally likely. The demodulating signal is $v(t) = 2\cos 2\pi f_c t$.

Note: Question No. 2 continues on page 3

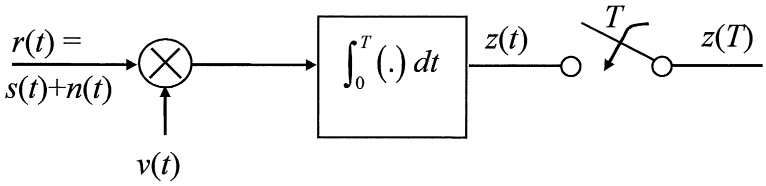

Figure 1

(a) Find the signal components in the output $z(T)$ due to positive and negative signal components, respectively. Determine the optimum threshold γ_0 for this detector.

(5 Marks)

(b) Determine the mean and variance of the output noise when AWGN is applied to the input of the receiver.

(5 Marks)

(c) Derive the error probability of the receiver in terms of Q-function.

(6 Marks)

(d) If the demodulating function is changed to $v(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$, discuss its impact on the optimum threshold and the error probability of the receiver.

(4 Marks)

3. An information source generates a quadrature-amplitude modulation (QAM) signal once every T_s seconds. Each QAM signal is selected from an alphabet of 6 symbols with equal probability. The signal structure of the symbols is shown in Figure 2 on page 4, where $\varphi_x(t)$ and $\varphi_y(t)$ are the two orthonormal bases for the signal space. The signal is transmitted over an additive white Gaussian noise (AWGN) channel with two-sided power spectral density $N_0/2$.

Note: Question No. 3 continues on page 4

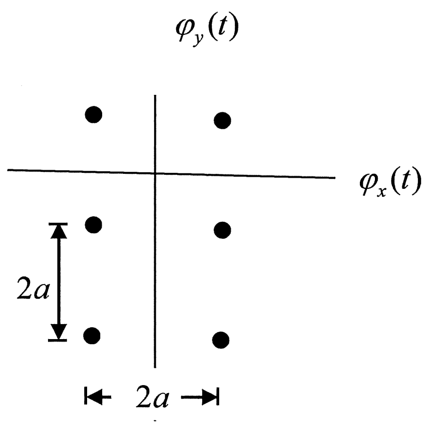

Figure 2

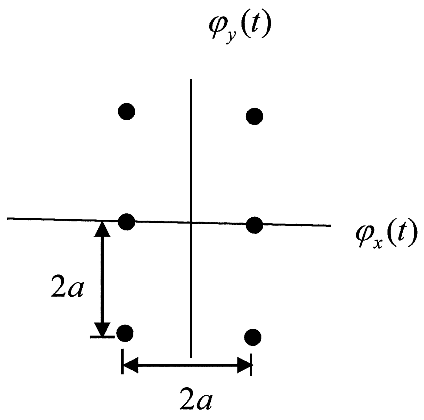

Figure 3

(a) Determine the optimum receiver and decision regions for the AWGN channel. Describe the detection steps of the optimum receiver.

(6 Marks)

(b) Derive the average error probability of the optimum receiver in terms of $\varepsilon = Q\left(a/\sqrt{N_0/2}\right)$.

(10 Marks)

(c) Suppose the signal structure is modified to that shown in Figure 3. *Discuss* its impact on the transmitted energy per message and *comment* on the average error probability of the QAM transmission.

(4 Marks)

4. (a) Let $g(X) = 1 + X^2 + X^4 + X^6 + X^8$ denote a polynomial over the binary field.

(i) Find the lowest-rate cyclic code whose generator polynomial is $g(X)$. What is the rate of this code?

(ii) Determine the generator matrix **G** from $g(X)$ in systematic form.

(iii) Determine the parity check matrix **H**, and hence or otherwise obtain the possible minimum distance of the code.

(16 Marks)

(b) A (63, 36) Bose-Chadhuri-Hocquenghem (BCH) code can correct five errors. A (7, 4) code can correct a single error, and hence nine of these (7, 4) codes arranged in parallel can correct nine errors in a block of 63 bits. Both the (63, 36) BCH code and the nine (7, 4) codes have the same code rate. Since the nine (7, 4) codes can correct more errors, is it more powerful than the BCH code? Explain.

(4 Marks)

5. A convolutional code is described by

$$g_1 = [\,0\ 0\ 1\,]$$
$$g_2 = [\,1\ 1\ 0\,]$$

(a) Draw the encoder diagram corresponding to this code.

(3 Marks)

(b) Draw the state diagram for this code.

(3 Marks)

(c) Find the transfer function, $T(D)$, and the free distance of the code.

(6 Marks)

(d) Suppose a sequence (starting from left to right) 010100110110 is received at the Viterbi decoder of the code through a binary symmetric channel. Trace the decisions (all the possible survivors' paths) on the trellis diagram and label the survivors' Hamming distance metric at each node level. If a tie occurs in the metrics required for a decision, always choose the upper path. If the decoded sequence is chosen from the path with smallest path metric, determine the decoded data bits.

(8 Marks)

6. The cellular network shown in Figure 4 has 10 omni-directional cells and a total bandwidth of 1.68 MHz. All cells are designed to support the same number of voice users with data rate of 20 kbps per user.

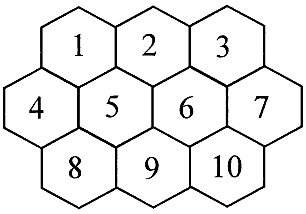

Figure 4

(a) If the above cellular network implements a frequency division multiple access (FDMA) system with a per-channel bandwidth of 20 kHz and a 3-cell re-use pattern in the uplink, compute the total uplink capacity of the FDMA system. You may assume that all re-use cells are sufficiently separated and hence do not interfere with each other.

(5 Marks)

Note: Question No. 6 continues on page 6

(b) If the above cellular network implements a code division multiple access (CDMA) system with:

- voice-activated discontinuous transmission with voice activity factor of 3/8,
- perfect uplink power control,
- 20% cell loading between neighbouring cells (e.g., Cell 5 and Cell 6),
- zero interference between non-neighbouring cells (e.g., Cell 2 and Cell 4),
- minimum receiver threshold $E_b/I_0 = 7$ dB for the uplink.

Compute the total uplink capacity of this CDMA system.

(7 Marks)

(c) Will increasing the number of cells in Figure 4 on page 5 give the CDMA system greater uplink capacity than the FDMA system? Explain your answer *quantitatively*.

(4 Marks)

(d) Will increasing the cell sizes in Figure 4 give the CDMA system greater uplink capacity than the FDMA system? Explain your answer *qualitatively*.

(4 Marks)

Appendix 1

Summary of Properties of the Fourier Transform

Item	Property	Mathematical Description
1.	Linearity	$ag_1(t)+bg_2(t)\longleftrightarrow aG_1(f)+bG_2(f)$ where a and b are constants
2.	Time scaling	$g(at)\longleftrightarrow\frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$ where a is a constant
3.	Duality	If $g(t)\longleftrightarrow G(f)$, then $G(t)\longleftrightarrow g(-f)$
4.	Time shifting	$g(t-t_0)\longleftrightarrow G(f)\exp(-j2\pi ft_0)$
5.	Frequency shifting	$\exp(j2\pi f_c t)g(t)\longleftrightarrow G(f-f_c)$
6.	Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)dt=G(0)$
7.	Area under $G(f)$	$g(0)=\int_{-\infty}^{\infty} G(f)\,df$
8.	Differentiation in the time domain	$\frac{d}{dt}g(t)\longleftrightarrow j2\pi fG(f)$
9.	Integration in the time domain	$\int_{-\infty}^{t} g(\tau)d\tau\longleftrightarrow\frac{1}{j2\pi f}G(f)+\frac{G(0)}{2}\delta(f)$
10.	Conjugate functions	If $g(t)\longleftrightarrow G(f)$, then $g^*(t)\longleftrightarrow G^*(-f)$
11.	Multiplication in the time domain	$g_1(t)g_2(t)\longleftrightarrow\int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\,d\lambda$
12.	Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\,d\tau\longleftrightarrow G_1(f)G_2(f)$

Appendix 2

Fourier Transform Pairs

Time Function	Fourier Transform
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\ \mathrm{sinc}(fT)$
$\mathrm{sinc}(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t),\quad a>0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\|t\|),\quad a>0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\Delta\left(\frac{t}{T}\right)=\begin{cases}1-\frac{\|t\|}{T}, & \|t\|<T\\ 0, & \|t\|\geq T\end{cases}$	$T\ \mathrm{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t-t_0)$	$\exp(-j2\pi ft_0)$
$\exp(j2\pi f_c t)$	$\delta(f-f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty}\delta(t-iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_0}\right)$

END OF PAPER

EE6101 DIGITAL COMMUNICATION SYSTEMS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.