NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

EE6101 – DIGITAL COMMUNICATION SYSTEMS

November / December 2019 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 6 questions and comprises 8 pages.
2. Answer any 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. A table of Fourier transform properties is provided in Appendix 1 (Page 7).
6. A table of Fourier transform pairs is provided in Appendix 2 (Page 8).

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1. Consider a high-pass RC filter as shown in Figure 1.

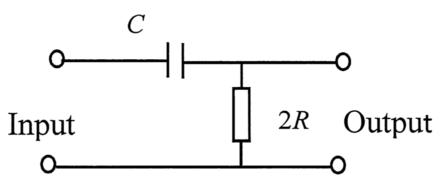

Figure 1

(a) Determine the transfer function of the filter, $H(f)$, and its impulse response, $h(t)$. Find the frequency at which half of the input power can pass through the filter.

(7 Marks)

Note: Question No. 1 continues on page 2.

(b) Suppose a rectangular waveform

$$x(t) = \begin{cases} 1 & 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$$

is applied at the input of the high-pass filter in part (a). Find the output waveform $y(t)$ of the filter.

(8 Marks)

(c) From the viewpoint of filter bandwidth, comment on the output energy of $y(t)$ for (i) $4\pi RC \gg T$ and (ii) $4\pi RC \ll T$.

(5 Marks)

2. Pulse code modulation (PCM) is the class of baseband signals obtained from the quantized analogue signals by encoding each quantized sample into a digital codeword. The source information is first sampled and quantized to one of the L levels. Then each quantized sample is digitally encoded into an l-bit PCM codeword, where $l = \log_2 L$. For baseband transmission, the codeword bits will then be transformed to pulse waveforms. Consider a PCM system with $L = 8$ levels. The essential parameters of the PCM system are included in Table 1 on page 3.

(a) Assume that the analogue signal $x(t)$ is limited in its excursions to the range -4 V and 4 V. Suppose 10 sample values are collected: -3.4, -2.9, -2.4, -0.7, 0.2, 0.7, 2.3, 3.6, 1.3 and 0.0. Create a table to list sample values, quantization levels, code numbers and PCM codewords.

(10 Marks)

(b) When the input signal is converted into one of the quantization levels, the random error Y is assumed to be uniformly distributed over the interval. Compute the mean value $E\{Y\}$ and the quantization noise power $E\{Y^2\}$. If the 3-bit PCM codeword is increased to 4 bits, calculate the 4-bit quantization noise power.

(10 Marks)

Note: Question No. 2 continues on page 3.

Table 1

Input signal (in V)	Quantization level (in V)	Code number	PCM codeword
$[-4.0, -3.0)$	-3.5	0	000
$[-3.0, -2.0)$	-2.5	1	001
$[-2.0, -1.0)$	-1.5	2	010
$[-1.0, 0.0)$	-0.5	3	011
$[0.0, 1.0)$	0.5	4	100
$[1.0, 2.0)$	1.5	5	101
$[2.0, 3.0)$	2.5	6	110
$[3.0, 4.0)$	3.5	7	111

3. Consider a binary communication system. The received signal $(t) = s_i(t) + n(t)$, $i = 1, 2$, where the transmitted signals

$$s_1(t) = A\cos(2\pi f_c t) \qquad 0 \le t \le T_b$$
$$s_2(t) = -2s_1(t) \qquad 0 \le t \le T_b$$

are equal likely, A is the amplitude, T_b is the bit duration and f_c is carrier frequency. The component $n(t)$ is the additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$. The matched-filter receiver is shown in Figure 2 on page 4.

(a) Compute the energy $E_1 = \int_0^{T_b} s_1^2(t)dt$ of the signal $s_1(t)$.

(4 Marks)

(b) If the input signal is $s_i(t)$ for $i = 1, 2$, find the signal components a_i contained in the output $r(T_b)$ in terms of E_1.

(6 Marks)

Note: Question No. 3 continues on page 4.

(c) Determine the mean and variance of the output noise, in terms of E_1, when $n(t)$ is applied at the input of the receiver.

(4 Marks)

(d) Suppose the threshold is $\gamma_0 = (a_1 + a_2)/2$. The decision is $s_1(t)$ if $r(T_b) > \gamma_0$. Otherwise, the decision is $s_2(t)$. Derive the error probability of the receiver in terms of Q-function, N_0 and E_1. Comment on the result obtained. Can we say that the error probability is better than that of BPSK?

(6 Marks)

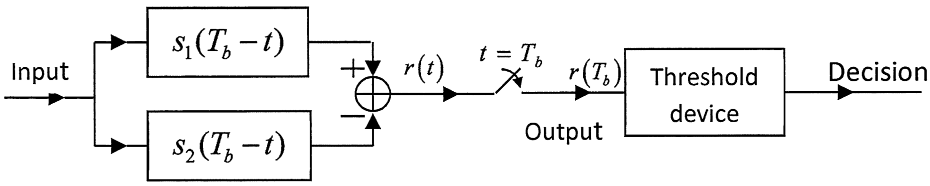

Figure 2

4. We would like to design an $(n, k) = (5, 2)$ linear block code.

(a) List all the possible codewords and choose from the list the valid codewords in systematic form with the goal of maximising the minimum distance. What is the minimum distance of the code?

(10 Marks)

(b) Determine the generator matrix **G** in systematic form from the valid codewords chosen in part (a).

(3 Marks)

(c) Determine the parity-check matrix **H**.

(2 Marks)

(d) What is the error-correcting capability of the code? Make a syndrome table for the correctable error patterns.

(5 Marks)

5. The state diagram of a binary convolutional encoder is given in Figure 3.

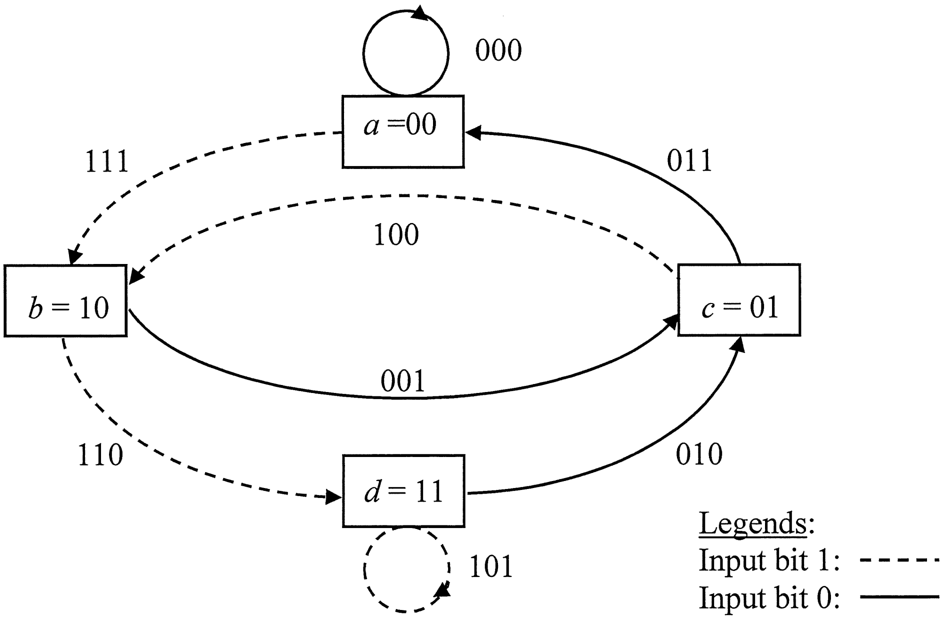

Figure 3

(a) Sketch the encoder diagram.

(4 Marks)

(b) Determine the transfer function of the encoder, $T(D)$, and hence, the minimum free distance of the code.

(7 Marks)

(c) A sequence (starting from left to right), $\mathbf{Z}$ = (111101010000110 and the rest all "0") is received at the Viterbi decoder of the code through a binary symmetric channel. Find the maximum likelihood path through the trellis diagram by labelling the survivors' Hamming distance metric at each node level. If a tie occurs between any two merged paths, choose the upper branch entering the particular node. Determine the first 5 decoded information bits.

(9 Marks)

6. (a) A preferred pair of m-sequences $\mathbf{m}_1$ can be generated using primitive polynomial 101111 and initial shift-register content 10001, and $\mathbf{m}_2$ can be generated using primitive polynomial 111101 and initial shift-register content 00110. Determine and write down the first 8 chips of the Gold code generated from $\mathbf{m}_1$ and $\mathbf{m}_2$. (Note: No need to compute the entire code.)

(7 Marks)

(b) Consider a direct-sequence single-cell CDMA system. For the downlink it is common that the user data is spread by a scrambled Walsh Hadamard code. The scrambling code is a Gold code generated in part (a). There are 2 users in the cell. The cell base-station (BS) wants to transmit bit 1 to user A and assigns a Walsh Hadamard code [0110 0110]. At the same time, the BS wants to transmit bit 0 to user B and assigns another Walsh Hadamard code [1111 0000]. Determine the CDMA signal vector to be transmitted by the BS. Express your answer using the signal mapping $0 \rightarrow +1$ and $1 \rightarrow -1$.

(5 Marks)

(c) Now consider the CDMA uplink system. The bit-error rate (BER) performance of the RAKE receiver with maximal-ratio combining (MRC) under the frequency-selective fading channel with L equal-power slow Rayleigh paths is approximately given by:

$$\text{BER} \approx \binom{2L-1}{L} \prod_{i=1}^{L} \left[\left(\frac{4E_b}{N_0 + I_0} \right)_i \right]^{-1}$$

where E_b, N_0 and I_0 are the average energy per bit, channel noise power spectral density and average interference power spectral density, respectively. With only one user in the cell transmitting signal, the received $\frac{E_b}{N_0}$ at the cell BS receiver is 15 dB. The interference due to the other users in the cell, I_0, is 20% of E_b. Calculate the BER of the BS receiver if the BS uses:

i) A conventional receiver without RAKE combining.

ii) An MRC-RAKE combiner with 3 RAKE fingers.

(8 Marks)

Appendix 1

Summary of Properties of the Fourier Transform

Item	Property	Mathematical Description
1.	Linearity	$ag_1(t)+bg_2(t)\longleftrightarrow aG_1(f)+bG_2(f)$ where a and b are constants
2.	Time scaling	$g(at)\longleftrightarrow \frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$ where a is a constant
3.	Duality	If $g(t)\longleftrightarrow G(f)$, then $G(t)\longleftrightarrow g(-f)$
4.	Time shifting	$g(t-t_0)\longleftrightarrow G(f)\exp(-j2\pi ft_0)$
5.	Frequency shifting	$\exp(j2\pi f_c t)g(t)\longleftrightarrow G(f-f_c)$
6.	Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)dt = G(0)$
7.	Area under $G(f)$	$g(0)=\int_{-\infty}^{\infty} G(f)\, df$
8.	Differentiation in the time domain	$\frac{d}{dt}g(t)\longleftrightarrow j2\pi fG(f)$
9.	Integration in the time domain	$\int_{-\infty}^{t} g(\tau)d\tau\longleftrightarrow \frac{1}{j2\pi f}G(f)+\frac{G(0)}{2}\delta(f)$
10.	Conjugate functions	If $g(t)\longleftrightarrow G(f)$, then $g^*(t)\longleftrightarrow G^*(-f)$
11.	Multiplication in the time domain	$g_1(t)g_2(t)\longleftrightarrow \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\, d\lambda$
12.	Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\, d\tau \longleftrightarrow G_1(f)G_2(f)$

Appendix 2

Fourier Transform Pairs

Time Function	**Fourier Transform**
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\ \mathrm{sinc}(fT)$
$\mathrm{sinc}(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t),\quad a>0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\lvert t\rvert),\quad a>0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\Delta\left(\frac{t}{T}\right)=\begin{cases}1-\frac{\lvert t\rvert}{T}, & \lvert t\rvert<T\\ 0, & \lvert t\rvert\geq T\end{cases}$	$T\ \mathrm{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t-t_0)$	$\exp(-j2\pi ft_0)$
$\exp(j2\pi f_c t)$	$\delta(f-f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)]$
$\mathrm{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty}\delta(t-iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_0}\right)$

END OF PAPER

EE6101 DIGITAL COMMUNICATION SYSTEMS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.