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NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

EE6101 – DIGITAL COMMUNICATION SYSTEMS

November/December 2015 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 6 questions and comprises 8 pages.
2. Answer any 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. A table of Fourier transform properties is provided in Appendix 1 (Page 7).
6. A Fourier transform table is provided in Appendix 2 (Page 8).

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1. Consider the RC filter shown in Figure 1.

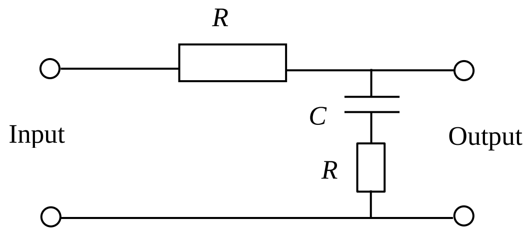

Figure 1

(a) Determine the transfer function $H(f)$ of the filter and its impulse response $h(t)$. Find the frequency f_3 of the filter at which $|H(f_3)|^2 = 0.5$.

(6 Marks)

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Note: Question No. 1 continues on page 2

(b) Suppose a rectangular pulse

$$x(t) = \begin{cases} 2 & |t| \le T \\ 0 & \text{otherwise} \end{cases}$$

is applied at the input of the filter. Find its output $y(t)$.

(10 Marks)

(c) From the viewpoint of filter bandwidth and input spectrum, comment on the shape of the waveform $y(t)$ as compared to the input $x(t)$ for

(i) $2\pi RC \ll T$ and (ii) $2\pi RC \gg T$.

(4 Marks)

2. An analog signal $x(t)$ with bandwidth B Hz is sampled using a rectangular pulse train

$$x_p(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t - nT_s}{T_0}\right)$$

where T_s is the sampling period and T_0 is the width of each rectangular pulse. It is assumed that $T_0 \ll T_s$. The rectangular function is defined as

$$\text{rect}\left(\frac{t}{T_0}\right) = \begin{cases} 1 & |t| \le T_0/2 \\ 0 & \text{otherwise} \end{cases}$$

The sampling frequency f_s is at the Nyquist rate, i.e., $f_s = 1/T_s = 2B$. The sampled-data signal $x_s(t) = x(t)\,x_p(t)$ is used to recover the desired signal $x(t)$.

(a) Find the Fourier series expansion of $x_p(t)$ in terms of sinc function $\text{sinc}(y) = \dfrac{\sin \pi y}{\pi y}$. Then obtain the Fourier transform of $x_p(t)$ in terms of sinc function and delta function.

(5 Marks)

(b) Determine the spectrum of the sampled-data signal $x_s(t)$.

(8 Marks)

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Note: Question No. 2 continues on page 3

(c) Design a low-pass filter (LPF) to perfectly recover the desired signal $x(t)$. Draw the transfer function of the LPF and label the bandwidth and filter gain properly.

(4 Marks)

(d) Suppose the periodic rectangular pulse train is replaced with a periodic triangular pulse train. Is it possible to recover the desired signal as illustrated in part (c)? Justify your answer.

(3 Marks)

3. Consider a binary phase-shift keying (BPSK) receiver shown in Figure 2. The received incoming waveform is $r(t) = s_i(t) + n(t)$, where

$$s_i(t) = (-1)^{i-1}\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t \qquad i = 1,2 \quad 0 \le t \le T_b$$

Note that E_b is the bit energy and T_b is the bit duration. The noise component $n(t)$ is assumed to be additive white Gaussian noise (AWGN) with mean zero and two-sided power spectral density $N_0/2$. The demodulating function is given by $v(t) = 2\cos\left(2\pi f_c t + \theta\right)$, where θ is treated as a constant.

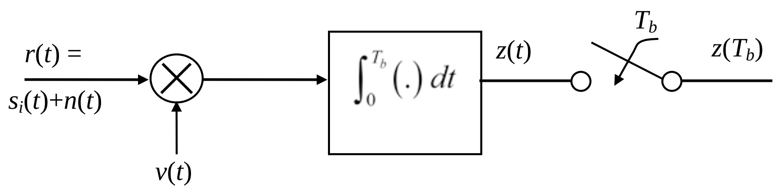

Figure 2

(a) Compute the energy levels of the two possible received signals, $s_1(t)$ and $s_2(t)$.

(3 Marks)

(b) Determine the optimum threshold using the maximum-likelihood (ML) criterion.

(5 Marks)

(c) Find the mean and variance of the noise component contained in $z(T_b)$.

(4 Marks)

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Note: Question No. 3 continues on page 4

(d) Compute the bit-error rate (BER) of the BPSK system in terms of Q-function, E_b, N_0 and θ.

(5 Marks)

(e) If the phase angle θ is uniformly distributed over $[0, 2\pi)$, derive the *average BER expression* of the BPSK system. You do not need to simplify the expression.

(3 Marks)

4. To reverse engineer a systematic cyclic encoder device, a message vector **m** = [10101] is input to the device. The output from the device is found to be:

$$\mathbf{U} = [01010101]$$

(a) Determine the generator polynomial $g(X)$ of the encoder.

(10 Marks)

(b) Determine the generator matrix **G** from $g(X)$ in systematic form.

(6 Marks)

(c) What is the possible minimum distance of the code?

(4 Marks)

5. A convolutional code is described by

$$\mathbf{g}_1 = [110]$$

$$\mathbf{g}_2 = [011]$$

$$\mathbf{g}_3 = [111]$$

(a) Draw the encoder corresponding to this code.

(3 Marks)

(b) Draw the state diagram for this code.

(3 Marks)

(c) Find the transfer function, $T(D)$, and the free distance of this code.

(6 Marks)

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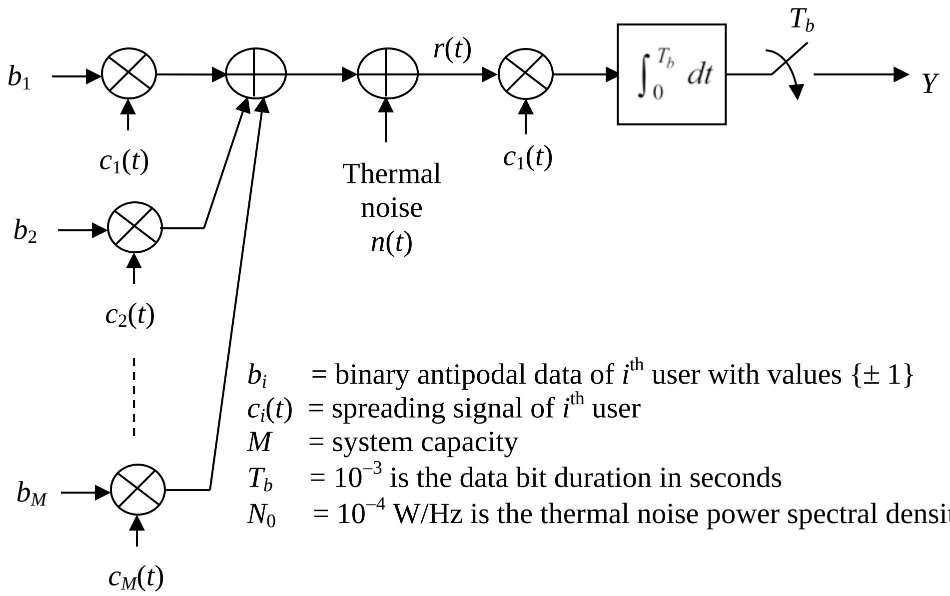
Note: Question No. 5 continues on page 5

(d) Suppose a sequence (starting from left to right) 111 111 111 111 111 111 is received at the Viterbi decoder of the code through a binary symmetric channel. Trace the decisions (all the possible survivors' paths) on the trellis diagram and label the survivors' Hamming distance metric at each node level. If a tie occurs in the metrics required for a decision, always choose the lower path.

(8 Marks)

6. The block diagram of a synchronous CDMA system operating at baseband in AWGN channel is shown in Figure 3, where $r(t) = \sum_{i=1}^{M} b_i c_i(t) + n(t)$ is the received signal and $Y = \int_0^{T_b} r(t) c_1(t)\, dt$ is the receiver output for the user 1.



b_i = binary antipodal data of i^{th} user with values $\{\pm 1\}$
$c_i(t)$ = spreading signal of i^{th} user
M = system capacity
T_b = 10^{-3} is the data bit duration in seconds
N_0 = 10^{-4} W/Hz is the thermal noise power spectral density

Figure 3

(a) Show that the expression of Y can be reduced to:

$$Y = b_1 T_b + T_c \sum_{i=2}^{M} b_i\, \phi_{i1}(k=0) + N$$

and specify the meanings of the terms T_c, ϕ_{i1} and N in the expression.

(5 Marks)

Note: Question No. 6 continues on page 6

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(b) If the spreading signal $c_i(t)$ is obtained from a full period of Walsh Hadamard code with length equals 32, what is the system capacity M if bit-error rate (BER) of at least 10^{-3} must be obtained for every user? You may approximate the Q-function as follows:

$$Q(x) \approx \frac{1}{\sqrt{2\pi x^2}} \exp\left(-\frac{x^2}{2}\right)$$

(7 Marks)

(c) The system next operates under a fixed 2-path channel with equal path amplitudes of 1, equal path phase, and differential path delay of 1 T_c. Assuming that Walsh Hadamard code correlation has an average variance of 25, what will be the new system capacity that can maintain the same BER per user of at least 10^{-3} ? You may assume that the inter-symbol interference (ISI) can be approximated with Gaussian distribution with zero mean. Note that $Q(3.09) \leq 10^{-3}$.

(8 Marks)

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Appendix 1

Summary of Properties of the Fourier Transform

Property	Mathematical Description
1. Linearity	$ag_1(t)+bg_2(t) \rightleftharpoons aG_1(f)+bG_2(f)$ where a and b are constants
2. Time scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t-t_0) \rightleftharpoons G(f)\exp(-j2\pi ft_0)$
5. Frequency shifting	$\exp(j2\pi f_c t)g(t) \rightleftharpoons G(f-f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\,df$
8. Differentiation in the time domain	$\frac{d}{dt}g(t) \rightleftharpoons j2\pi fG(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f)+\frac{G(0)}{2}\delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\,d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\,d\tau \rightleftharpoons G_1(f)G_2(f)$

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Appendix 2

Fourier Transform Pairs

Time Function	Fourier Transform
$\text{rect}\left(\frac{t}{T}\right)$	$T\ \text{sinc}(fT)$
$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t),\quad a>0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\lvert t\rvert),\quad a>0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\Delta\left(\frac{t}{T}\right)=\begin{cases}1-\frac{\lvert t\rvert}{T}, & \lvert t\rvert<T\\ 0, & \lvert t\rvert\geq T\end{cases}$	$T\ \text{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t-t_0)$	$\exp(-j2\pi ft_0)$
$\exp(j2\pi f_c t)$	$\delta(f-f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)]$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\text{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty}\delta(t-iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_0}\right)$

END OF PAPER