NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

EE6402 – REAL-TIME DSP DESIGN AND APPLICATIONS

November/December 2017 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. (a) The word-length of an infinite precision signal $x(n)$ is reduced to 10 bits by rounding $x(n)$ to the nearest discrete level. The weight of the 10^{th} bit is 2^{-10}. The rounding process produces a rounding error $e(n)$ at time n. Draw a diagram showing the probability density function for $e(n)$.

(3 Marks)

(b) Let $x_8(n)$ and $x_{10}(n)$ be 8-bit and 10-bit signals, respectively, at time n. The weights of the least significant bits of $x_8(n)$ and $x_{10}(n)$ are 2^{-8} and 2^{-10}, respectively. The value of $x_8(n)$ is obtained from that of $x_{10}(n)$ by rounding $x_{10}(n)$ to the nearest 8-bit value whenever possible; if the value of $x_{10}(n)$ is at the midpoint of two consecutive 8-bit values, $x_{10}(n)$ is rounded up, i.e. rounded to the larger 8-bit value. Let $v(n) = x_8(n) - x_{10}(n)$.

(i) Draw a diagram showing the probability density function for $v(n)$.

(ii) Determine the d.c. value of $v(n)$.

(iii) Determine the a.c. noise power of $v(n)$.

(10 Marks)

Note: Question No. 1 continues on page 2

(c) A 10-bit Analog-to-Digital Converter (ADC) samples at 4 GS/s (Giga samples per second). Discuss how the quantization noise of the ADC can be made equivalent to that of a 12-bit ADC by reducing the signal band-width.

(7 Marks)

2. Let $H(e^{j\omega})$ denotes the frequency response of a symmetrical impulse response FIR filter with length N odd, i.e. $h(n) = h(N-1-n)$ where $h(n)$ is its n^{th} coefficient value. Suppose that $h(n)$ is rounded to $[h(n)]$ with quantization step size Q. The rounding process introduces an error of $e(n)$ for each coefficient. After rounding $h(n)$ to $[h(n)]$, its frequency response becomes $[H(e^{j\omega})]$. Let the difference between $H(e^{j\omega})$ and $[H(e^{j\omega})]$ be $\xi(e^{j\omega})$.

(a) Derive an expression for $\xi(e^{j\omega})$ as a function of $e(n)$.

(4 Marks)

(b) What is the absolute maximum bound for $|\xi(e^{j\omega})|$.

(4 Marks)

(c) What is the absolute maximum bound for $|\xi(e^{j\omega})|$ at $\omega = \frac{\pi}{2}$, i.e. at quarter sampling frequency?

(4 Marks)

(d) Find an expression for the expected value of $|\xi(e^{j\omega})|^2$ assuming that $e(n)$ and $e(m)$ are uncorrelated for $n \neq m$.

(8 Marks)

3. (a) Answer parts (a)(i) through (a)(iv) assuming that

$X = [x_7, x_6, \ldots, x_2, x_1, x_0, x_{-1}, x_{-2}, \ldots]$ is rounded to an integer $K = [k_7, k_6, \ldots, k_2, k_1, k_0]$ using the following non-exact rounding technique. X and K are given by

$$X = x_7 \times 2^7 + \ \ldots \ + x_1 \times 2^1 + x_0 \times 2^0 + x_{-1} \times 2^{-1} + x_{-2} \times 2^{-2} + \ldots$$

$$K = k_7 \times 2^7 + \ \ldots \ + k_1 \times 2^1 + k_0 \times 2^0$$

where $x_n = 0$ or 1, for $n = 7, 6, \ldots, 1, 0, -1, -2, \ldots$ and $k_n = 0$ or 1, for $n = 7, 6, \ldots, 1, 0$.

Note: Question No. 3 continues on page 3

Let $M = x_7 \times 2^7 + x_6 \times 2^6 \ldots x_2 \times 2^2 + x_1 \times 2^1 + x_0 \times 2^0 + x_{-1} \times 2^0$. This is "0"

For the non-exact rounding scheme, $[k_7, k_6, k_5, k_4, k_3] = [x_7, x_6, x_5, x_4, x_3]$.
If $x_2 = x_1 = x_0 = x_{-1} = 1$, then $k_2 = k_1 = k_0 = 1$; otherwise,
$k_2 \times 2^2 + k_1 \times 2 + k_0 = x_2 \times 2^2 + x_1 \times 2 + x_0 + x_{-1}$.

Let the difference between K and M be e, i.e. $e = K - M$.
If the condition $x_2 = x_1 = x_0 = x_{-1} = 1$ is not satisfied, then $K = M$.
If the condition $x_2 = x_1 = x_0 = x_{-1} = 1$ is satisfied, K will be different from M.

(i) Determine e if the condition $x_2 = x_1 = x_0 = x_{-1} = 1$ is satisfied.

(ii) What is the probability that the condition $x_2 = x_1 = x_0 = x_{-1} = 1$ is satisfied?

(iii) What is the expected value of e?

(iv) What is the noise power of e?

(10 Mark)

(b) A baseband signal is carrier modulated at frequency f_c and sampled at frequency f_s such that $f_s \geq 2B$. The signal is bandlimited to $\pm B/2$.

(i) Let $f_s = (2f_c - B)/m$, where m is a positive integer. Show that the baseband signal can be recovered without aliasing. You should elaborate your answer using the sampled signal spectrum.

(ii) Show what happens to the sampled signal spectrum when f_s is decreased and determine conditions for no spectral aliasing.

(iii) If $f_c = 16\,MHz$ and $B = 3\,MHz$, find the lowest value for the sampling frequency which avoids spectral aliasing.

(10 Marks)

4. (a) What are the difficulties associated with designing Digital-to-Analog (DAC) converters with high accuracy? How would an over-sampled sigma-delta ($\Sigma - \Delta$) modulator architecture helps in overcoming some of these difficulties.

(6 Marks)

(b) Primary means for Digital Signal Processors (DSPs) to communicate with input-output devices are using interrupts. Figure 1 shows a possible interrupt scheme. Explain the operation of this scheme and discuss how it could be improved for using with DSPs.

(8 Marks)

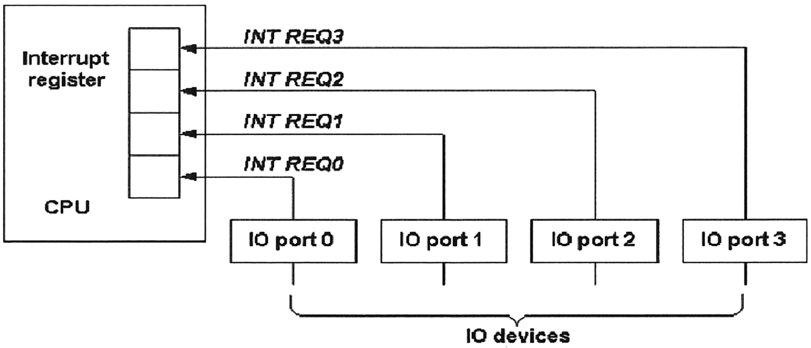

Figure 1

(c) Overflow is a potential problem in fixed-point DSPs. Explain the methods that are commonly used in DSPs to protect against overflow.

(6 Marks)

5. (a) Draw a diagram to show the operation of a symmetric even length finite impulse response (FIR) filter. Explain how the C54x digital signal processor (DSP) architecture can be used in efficient implementation of the symmetric FIR filter.

(5 Marks)

(b) A symmetric even length FIR filter was implemented on a C54x DSP. A fragment of the program which uses the FIRS instruction is shown below. Explain the operation of the program fragment when used with the above filter.

```
ADD     *AR3+, *AR2-, A
RPTZ    B, #(L/2-1)
FIRS    *AR3+, *AR2-, #coeff
```

(5 Marks)

(c) A symmetric FIR filter was implemented on a C54x DSP in block processing mode with a block size of 64. The DSP executes an instruction in 10 ns. The symmetric FIR filter routine requires $43 + B \times (15 + L)$ clocks where L and B are the filter length and the block size, respectively. What would be the largest value of L if the A/D converter sampling frequency is 480 kHz?

(5 Marks)

(d) A DSP system designed for a real time filter application is shown in Figure 2. The DSP can read from and write to the buffers at a much higher rate than the A/D and the D/A converter sampling rates. Emphasizing on the methods used for fast real time application, explain the operation of the DSP system.

(5 Marks)

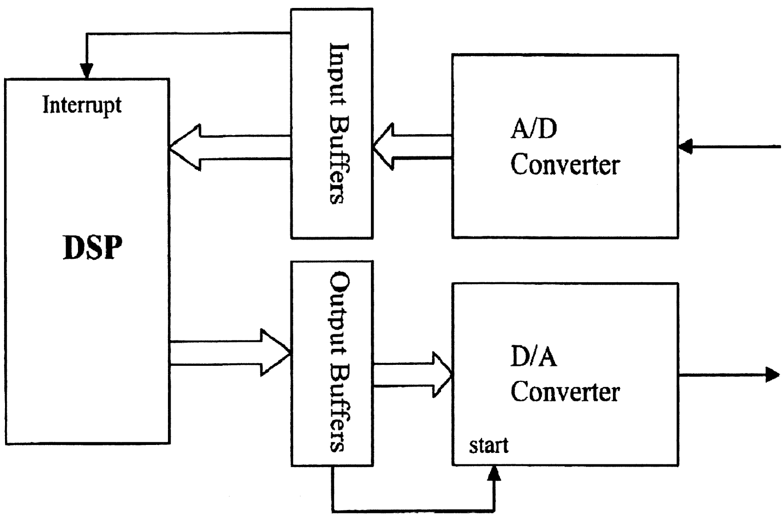

Figure 2

END OF PAPER

EE6402 REAL-TIME DSP DESIGN & APPLICATIONS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.