**NANYANG TECHNOLOGICAL UNIVERSITY**

**SEMESTER 1 EXAMINATION 2018-2019**

**EE6101 – DIGITAL COMMUNICATION SYSTEMS**

November/ December 2018 Time Allowed: 3 hours

**INSTRUCTIONS**

1. This paper contains 6 questions and comprises 8 pages.
2. Answer any 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. A table of Fourier transform properties is provided in Appendix 1 (Page 7).
6. A Fourier transform table is provided in Appendix 2 (Page 8).

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1. (a) If the Fourier transform of an energy signal $x(t)$ is denoted by $X(f)$, then

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(f)|^2 \, df .$$

State the physical meaning and prove the above relationship.

(8 Marks)

Note: Question No. 1 continues on page 2

(b) Let

$$x(t) = \left[\frac{2}{T_b}\text{sinc}\left(\frac{2t}{T_b}\right)\right] \otimes \left[\frac{1}{T_b}\text{sinc}\left(\frac{t-T_b}{T_b}\right)\right],$$

where $T_b$ is a constant, $\otimes$ is the convolution operator and

$$\text{sinc}(t) \equiv \frac{\sin(\pi t)}{\pi t}.$$

Find the *spectrum* $X(f)$ of $x(t)$ and determine its energy $E_x$.

(6 Marks)

(c) The signal $x(t)$ in part (b) is sampled by an ideal sampling function

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s),$$

where $T_s$ is the sampling period and $\delta(t)$ is the unit impulse function. Design a low-pass filter (LPF) to recover the desired signal $x(t)$ without distortion. What are the bandwidth and the gain of the LPF? Draw the transfer function $H(f)$ of the LPF for illustration.

(6 Marks)

2. (a) The random variable $X$ is normally distributed with probability density function (PDF)

$$p_X(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(x-1)^2}{2}\right).$$

Obtain the mean and variance of the random variable $X$ by observation. Suppose $X$ is applied to an electronic limiter with output $Y$ characterized by

$$Y = \begin{cases} 3X/2, & X > -2, \\ -3, & X \le -2. \end{cases}$$

Plot the graph of $Y$ versus $X$. Determine the PDF of the output $Y$ of the limiter.

(10 Marks)

Note: Question No. 2 continues on page 3

(b) The random variable $V$ has a cumulative distribution function (CDF)

$$F_V(v) = \Pr\left(V \le v\right) = (1 - e^{-2v})u(v),$$

where $u(v)$ is the unit step function. Plot the curve of $F_V(v)$ versus $v$ and find the PDF of $V$.

Define a new random variable $W$ in terms of $V$ by

$$W = F_V(V) = (1 - e^{-2V})u(V).$$

Determine the CDF $F_W(w)$ of $W$ for $w < 0$, $0 \le w \le 1$ and $w > 1$. Finally, plot the CDF $F_W(w)$ and obtain the PDF of $W$.

(10 Marks)

3. Consider a binary signal detector with the input

$$r = \pm a + n,$$

where the signal component may be $+a$ or $-a$ with equal probability. The noise component $n$ is described by the probability density function (PDF)

$$p_n(n) = \frac{1}{\sqrt{2}} \exp\left(-\sqrt{2}\,|\,n\,|\right).$$

(a) Plot the conditional PDFs $p_r(r\,|+a)$ and $p_r(r\,|-a)$ together. Determine the optimum threshold $\gamma_0$ of the binary signal detector and label the position of $\gamma_0$ clearly on the same graph.

(5 Marks)

(b) Given that the signal component $-a$ is received at the detector, derive the probability of bit error, i.e.

$$\Pr\left(\text{error}\,|-a\right) = \Pr\left(r > \gamma_0\,|-a\right).$$

(6 Marks)

(c) Compute the overall probability of bit error of the binary signal detector.

(6 Marks)

Note: Question No. 3 continues on page 4

(d) Suppose that the noise PDF is replaced by a Gaussian PDF with the same mean and variance. Without going through the detailed calculation, will the overall probability of bit error of the binary detector be bigger or smaller? Justify your answer.

(3 Marks)

4. A block code encoder appends a single even parity bit $\rho_0$ to each block of 3 message bits $(m_0, m_1, m_2)$.

(a) Determine all the possible codewords of this code.

(4 Marks)

(b) Determine the generator matrix, **G**, and the parity-check matrix, **H**, of this code in systematic form.

(6 Marks)

(c) Show that $\mathbf{c}\mathbf{H}^T = \mathbf{0}$, where **c** is any valid codeword. Determine the syndrome for any single error received vector **r**.

(7 Marks)

(d) What is the probability of message error when using the code if the channel bit error probability is $10^{-3}$? Consider only the first term of the equation.

(3 Marks)

5. The convolutional encoder diagram is shown in Figure 1.

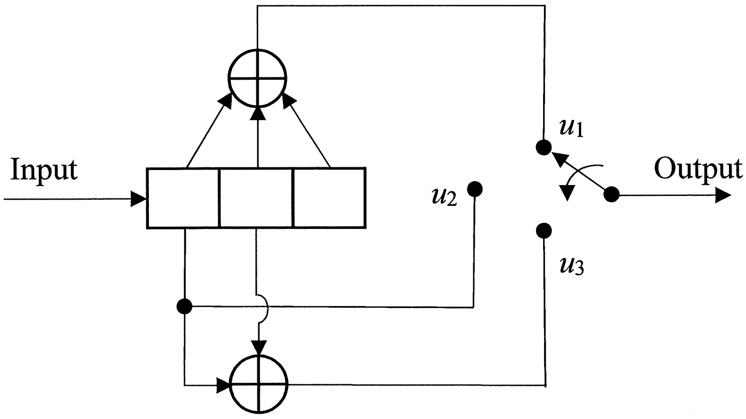


**Figure 1**

(a) Draw the state diagram.

(4 Marks)

(b) Determine the transfer function, $T(D)$, of the encoder and, hence, the minimum free distance of the code.

(8 Marks)

(c) The received sequence (starting from left to right) corresponding to an output sequence of the encoder is 110 101 011 111. Using the Viterbi algorithm, find the encoded sequence (from the path with smallest path metric). If a tie occurs, take the upper branch.

(8 Marks)

6. Consider a direct-sequence single-cell CDMA system.

(a) For downlink, it is common that the user data is first spread by a Walsh-Hadamard (WH) code and then scrambled by a Gold sequence.

(i) Given that the Hadamard, $\mathbf{H}_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, construct the matrix $\mathbf{H}_3$ to obtain 8 WH codes of length 8. Explain why WH codes are not suitable for uplink and multipath channel.

(ii) A preferred pair of *m*-sequences $\mathbf{m}_1$ can be generated using primitive polynomial 101111 and initial shift-register content 10001, and $\mathbf{m}_2$ can be generated using primitive polynomial 111101 and initial shift-register content 00110. Determine and write down the first 8 chips of the Gold code generated from $\mathbf{m}_1$ and $\mathbf{m}_2$. (Note: No need to compute the entire code.) What is the purpose of scrambling the WH using a Gold sequence?

(10 Marks)

(b) The users in the cell of the CDMA system are distributed uniformly within it. The base-station (BS) is equipped with a 4-sector antenna with effective beamwidth of 100°. The voice data rate per user is 10 kbps and the system bandwidth is 2 MHz. Assume voice-activated discontinuous transmission is activated with half of the users in the system speaking $\frac{3}{8}$ of the time and the rest speaking only 20% of the time. Assume also that perfect uplink power-control is implemented. The radio transmission channel is an additive white Gaussian noise (AWGN) channel. If only one terminal transmits the signal, the received energy per bit to noise spectral density ratio, $\frac{E_b}{N_0}$, at the BS is 25 dB. If the required $\frac{E_b}{N_0+I_0}$, where $I_0$ is the total interference power spectral density, is 7 dB to operate satisfactorily, how many equal-power users in the cell can be supported?

(10 Marks)

**Appendix 1**

## Summary of Properties of the Fourier Transform

| Item | Property | Mathematical Description |
|---|---|---|
| 1. | Linearity | $ag_1(t)+bg_2(t)\longleftrightarrow aG_1(f)+bG_2(f)$<br>where $a$ and $b$ are constants |
| 2. | Time scaling | $g(at)\longleftrightarrow \frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$<br>where $a$ is a constant |
| 3. | Duality | If $g(t)\longleftrightarrow G(f)$,<br>then $G(t)\longleftrightarrow g(-f)$ |
| 4. | Time shifting | $g(t-t_0)\longleftrightarrow G(f)\exp(-j2\pi f t_0)$ |
| 5. | Frequency shifting | $\exp(j2\pi f_c t)g(t)\longleftrightarrow G(f-f_c)$ |
| 6. | Area under $g(t)$ | $\int_{-\infty}^{\infty} g(t)dt = G(0)$ |
| 7. | Area under $G(f)$ | $g(0)=\int_{-\infty}^{\infty} G(f)\, df$ |
| 8. | Differentiation in the time domain | $\frac{d}{dt}g(t)\longleftrightarrow j2\pi f G(f)$ |
| 9. | Integration in the time domain | $\int_{-\infty}^{t} g(\tau)d\tau \longleftrightarrow \frac{1}{j2\pi f}G(f)+\frac{G(0)}{2}\delta(f)$ |
| 10. | Conjugate functions | If $g(t)\longleftrightarrow G(f)$,<br>then $g^*(t)\longleftrightarrow G^*(-f)$ |
| 11. | Multiplication in the time domain | $g_1(t)g_2(t)\longleftrightarrow \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\, d\lambda$ |
| 12. | Convolution in the time domain | $\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\, d\tau \longleftrightarrow G_1(f)G_2(f)$ |

**Appendix 2**

**Fourier Transform Pairs**

| **Time Function** | **Fourier Transform** |
|---|---|
| $\mathrm{rect}\left(\frac{t}{T}\right)$ | $T\ \mathrm{sinc}(fT)$ |
| $\mathrm{sinc}(2Wt)$ | $\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$ |
| $\exp(-at)u(t),\quad a>0$ | $\frac{1}{a+j2\pi f}$ |
| $\exp(-a\lvert t\rvert),\quad a>0$ | $\frac{2a}{a^2+(2\pi f)^2}$ |
| $\exp(-\pi t^2)$ | $\exp(-\pi f^2)$ |
| $\Delta\left(\frac{t}{T}\right)=\begin{cases}1-\frac{\lvert t\rvert}{T}, & \lvert t\rvert<T\\ 0, & \lvert t\rvert\geq T\end{cases}$ | $T\ \mathrm{sinc}^2(fT)$ |
| $\delta(t)$ | $1$ |
| $1$ | $\delta(f)$ |
| $\delta(t-t_0)$ | $\exp(-j2\pi f t_0)$ |
| $\exp(j2\pi f_c t)$ | $\delta(f-f_c)$ |
| $\cos(2\pi f_c t)$ | $\frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$ |
| $\sin(2\pi f_c t)$ | $\frac{1}{2j}[\delta(f-f_c)-\delta(f+f_c)]$ |
| $\mathrm{sgn}(t)$ | $\frac{1}{j\pi f}$ |
| $\frac{1}{\pi t}$ | $-j\,\mathrm{sgn}(f)$ |
| $u(t)$ | $\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$ |
| $\sum_{i=-\infty}^{\infty}\delta(t-iT_0)$ | $\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T_0}\right)$ |

**END OF PAPER**

# EE6101 DIGITAL COMMUNICATION SYSTEMS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.