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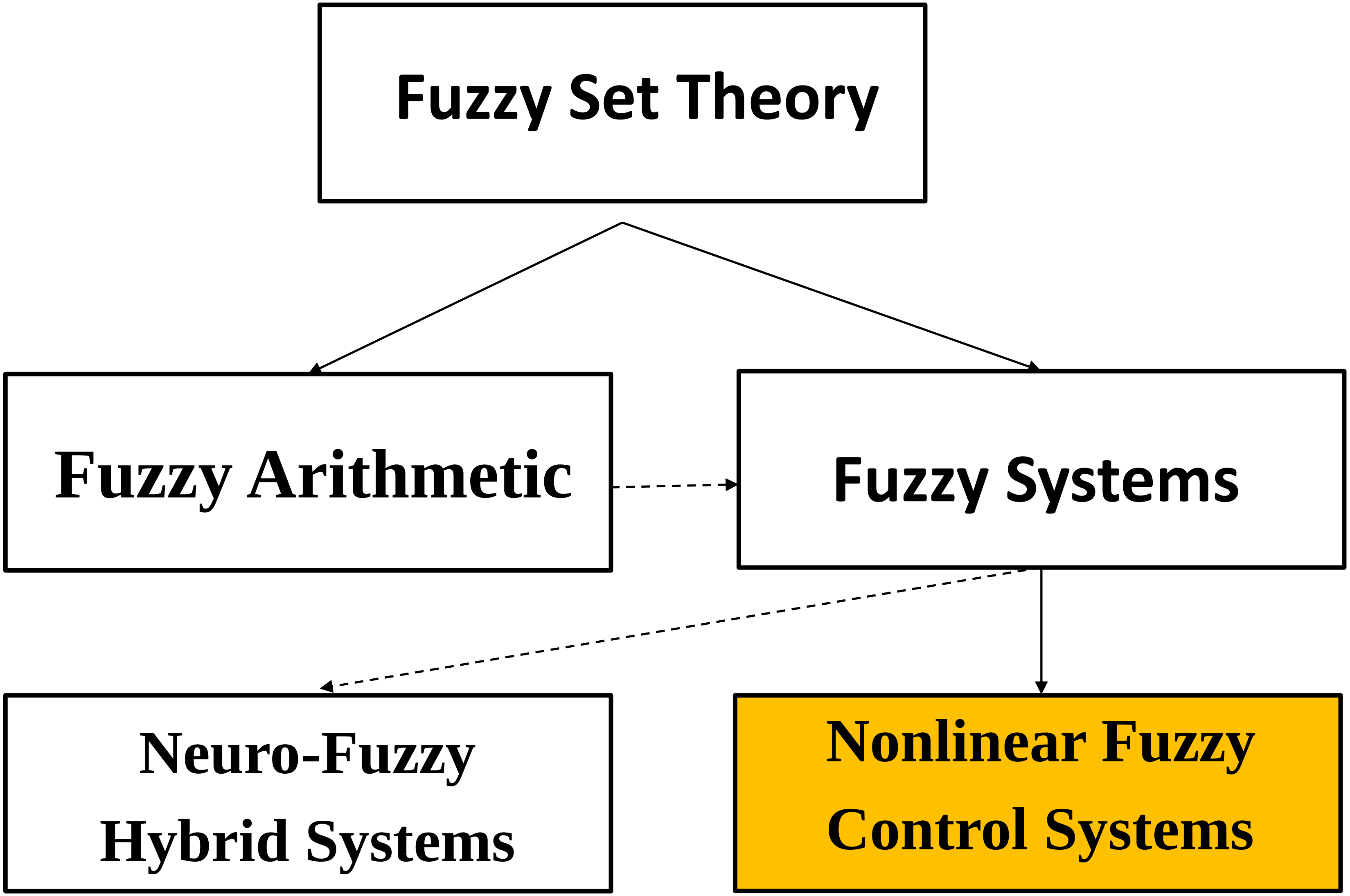
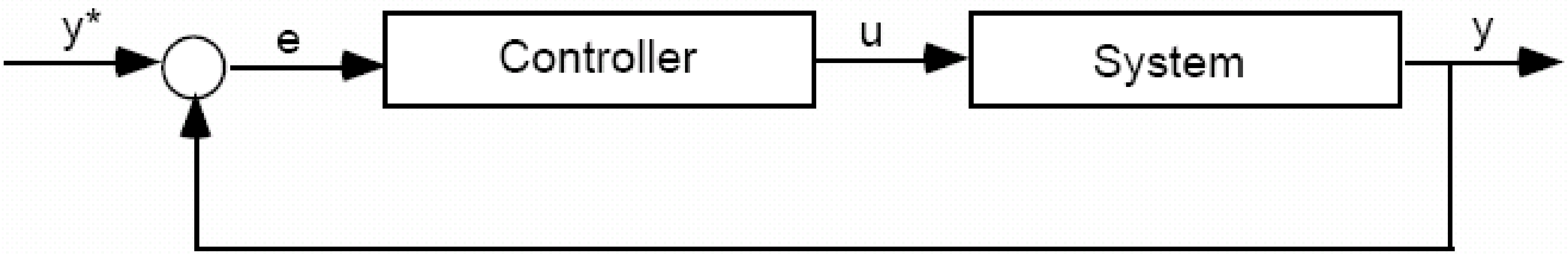
4. Fuzzy Control Systems


Fuzzy Set Theory
Fuzzy Arithmetic
Fuzzy Systems
Neuro-Fuzzy Hybrid Systems
Nonlinear Fuzzy Control Systems

4.1 Motivation for Fuzzy Control



A basic feedback control system..

The output of the controller (which is the input of the system) is the control action *u*.

The purpose of the feedback controller is to guarantee a desired response of the output *y*.

Conventional control:

Use process model to design controllers based on specifications of the desired closed-loop.

This approach may fall short if the model of the process is difficult to obtain, (partly) unknown, or highly nonlinear.

Some simple control actions cannot be modeled mathematically, e.g.

Driving a car or grasping a fragile object continues to be a challenge for robotics, while these tasks are easily performed by human beings

In fact, many processes controlled by human operators in industry cannot be automated using conventional control techniques, since the performance of these controllers is often inferior to that of the operators. There are two main reasons:

•Linear controllers, which are commonly used in conventional control, are not appropriate for nonlinear plants.

•Humans aggregate various kinds of information and combine control strategies, that cannot be integrated into a single analytic control law

The underlying principle of *knowledge-based (expert) control* is to capture and implement experience and knowledge available from experts (e.g., process operators).

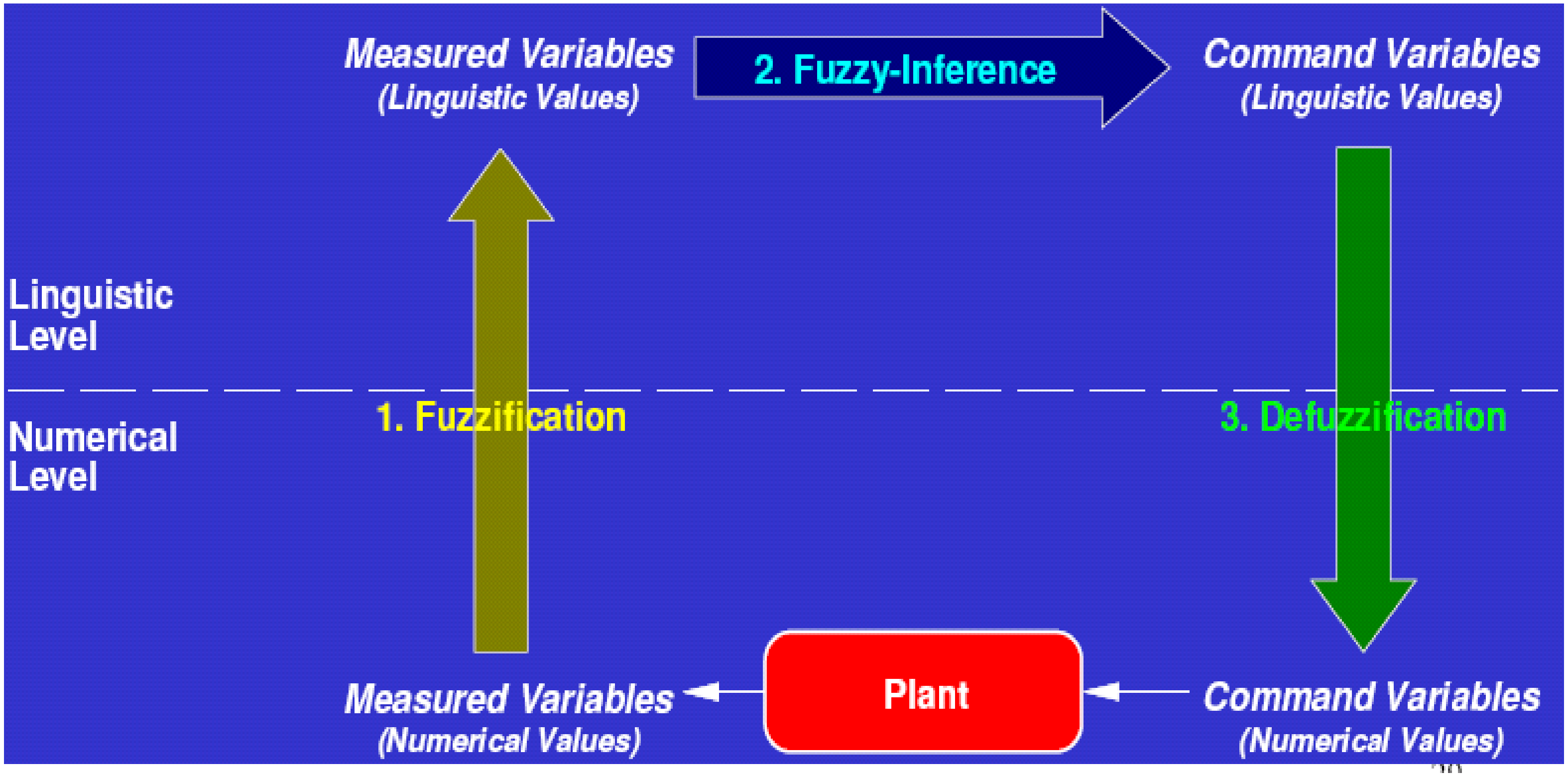
A specific type of knowledge-based control is the fuzzy rule-based control.

Main motivation of fuzzy control:

The linguistic nature of fuzzy control makes it possible to express process knowledge concerning how the process should be controlled or how the process behaves.

Definition (Fuzzy Controller):
A fuzzy controller is a controller that contains a (nonlinear) mapping that has been defined by using fuzzy if-then rules.

Fuzzy controllers are the most important applications of fuzzy theory.



4.2 Fuzzy Control: Background

- Controller designed by using If-Then rules instead of mathematical formulas (knowledge-based control, 1970)

  Early motivation: mimic experienced operators

- Supervisory control (1980)
- Controllers designed on the basis of a fuzzy model (model-based fuzzy control) (1990)
- Emphasis on stability analysis, robustness (2000)

Basic Fuzzy Control Schemes

_ Direct (low-level, Mamdani) fuzzy control

_ Fuzzy supervisory (high-level, Takagi-Sugeno) control

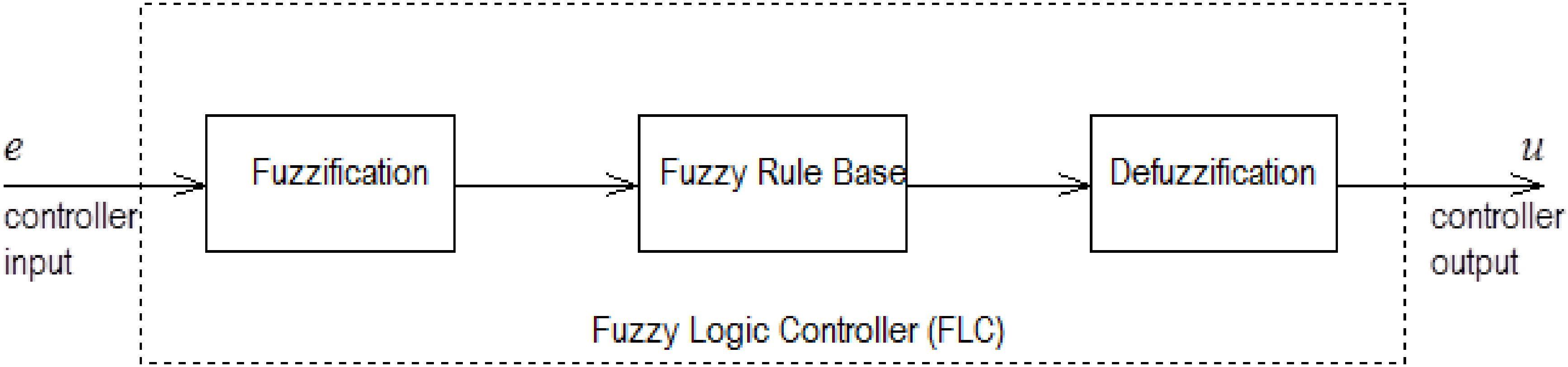
_ Fuzzy model-based control

Direct Fuzzy Control

It is quite close in nature to manual control.

Expert knowledge is used instead of differential equations to describe a system.

A direct fuzzy logic controller usually consists of 3 parts.



(i) Fuzzification module:

It transforms the physical values of the process signal into a normalized **fuzzy set** consisting of

- an interval (for the range of the input values); and
- an associate membership function (describing the degrees of the confidence of the input belonging to this range).

(ii) **Inference engine**: A typical design is a set of IF-THEN rules:

R^1: **IF** input e_1 is E_{11} **AND** ... **AND** input e_n is E_{1n}
THEN output u_1 is U_1
⋮
R^m: **IF** input e_1 is E_{m1} **AND** ... **AND** input e_n is E_{mn}
THEN output u_m is U_m

This is the key part which is decision-making unit and gives the dynamic behavior of the fuzzy controller.

(iii) **Defuzzification** module: It transforms the fuzzy signals back to crisp values

This is a three-step design routine:

Fuzzification – Rule Base – Defuzzification

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Major Advantages of Fuzzy Control:

- No need for mathematical model
- Less sensitive to system fluctuations
- Based on intuitions and judgments.
- Design objectives difficult to express mathematically can be incorporated in a fuzzy controller by linguistic rules.
- Relatively simple, fast and adaptable
- Implementation is simple and straight forward.

Merits of Conventional Control

- Engineers are comfortable with the classical control designs.
- Well-established technologies
- Verifiable overall system stability
- System’s reliability can be evaluated.

4.3 Design Aspects of Direct Mamdani Fuzzy Control

Fuzzy control system design is based on empirical methods, basically a methodical approach to trial-and-error. The general process is as follows:

- 1) Determine Inputs and Outputs

The inputs of the controller are usually the tracking error and its derivative or integral.

It can also be the plant output(s), measured or reconstructed states, measured disturbances or other external variables.

It is, however, important to realize that with an increasing number of inputs, the complexity of the fuzzy controller (i.e., the number of linguistic terms and the total number of rules) increases dramatically.

•2) Define Membership Functions

The linguistic terms and their membership functions are a part of the inference engine.

- *First, the designer must decide, how many linguistic terms per input variable will be used.*

The linguistic terms have usually some meaning, i.e. they express magnitudes of some physical variables, such as *Small*, *Medium*, *Large*, etc.

For interval domains symmetrical around zero, the magnitude is combined with the sign, e.g. *Positive small* or *Negative medium*.

The number of rules needed for defining a complete rule base increases exponentially with the number of linguistic terms per input variable. In order to keep the rule base maintainable, the number of terms per variable should be low.

On the other hand, with few terms, the flexibility in the rule base is restricted with respect to the achievable nonlinearity in the control mapping.

The number of terms should be carefully chosen, considering different settings for different variables according to their expected influence on the control strategy. A good choice may be to start with a few terms (e.g. 2 or 3 for the inputs and 5 for the outputs) and increase these numbers when needed.

•*Secondly, the designer must choose membership functions.*

The membership functions may be a part of the expert’s knowledge, e.g., the expert knows approximately what a “High temperature” means in a particular application.

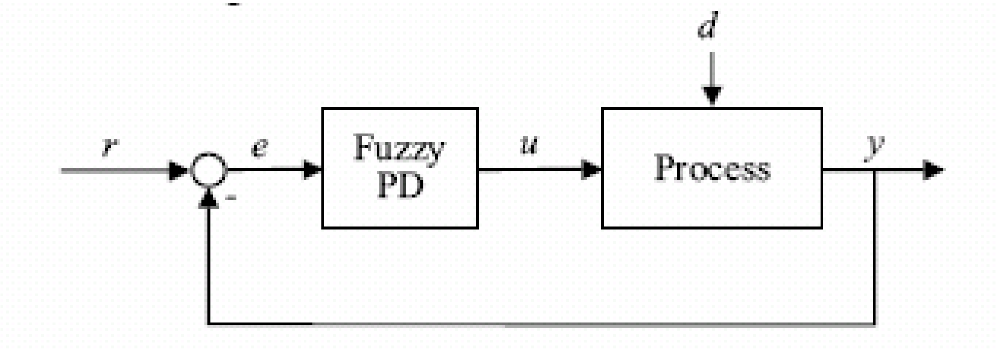
If such knowledge is not available, membership functions of the same shape, uniformly distributed over the domain can be used as an initial setting and can be tuned later.

For computational reasons, triangular and trapezoidal membership functions are usually preferred to bell-shaped Gaussian functions

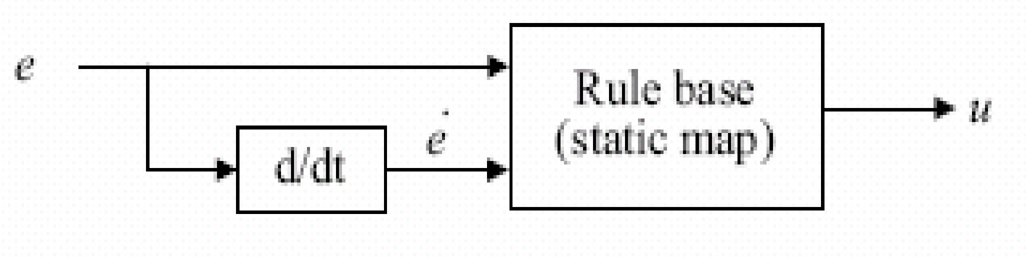
- 3) Design the Rule Base
  (sometimes summarized in a Tabular Form).
- 4) Determine the defuzzification method.
- 5) Run through test suite to validate system, adjust details as required.

***Example*:**

A control scheme with a fuzzy PD (proportional-derivative) controller including the process is shown in the following block diagram:



The internal structure of the fuzzy controller is:



The rule base has

two inputs – the error e, and the error change (derivative) $\dot{e}$

one output – the control action u.

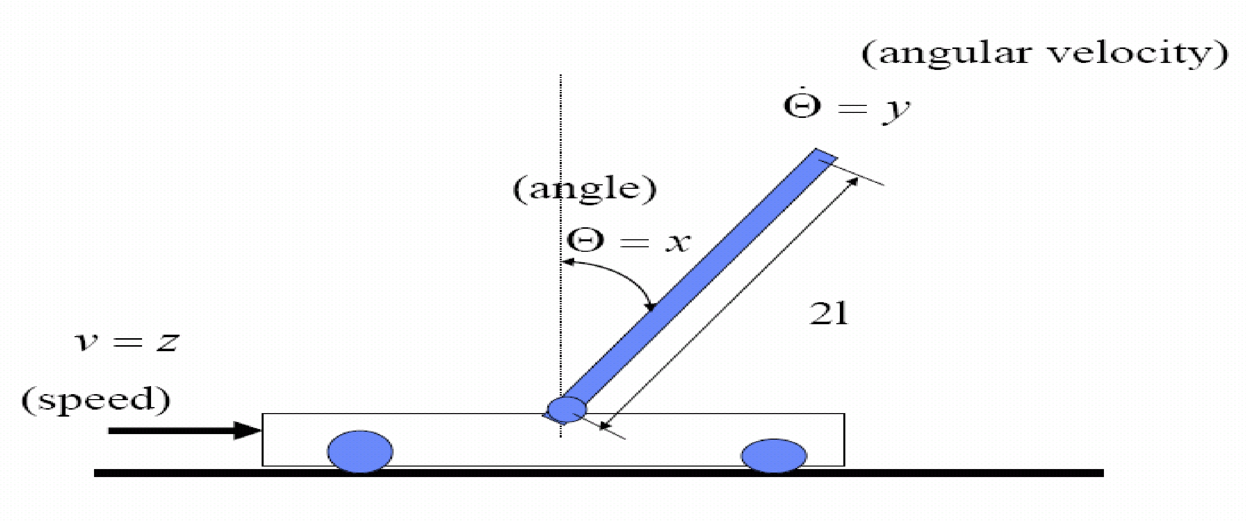
One possible rule base in the following Tabular Form is:

e \ $\dot{e}$	NB	NS	ZE	PS	PB
NB	NB	NB	NS	NS	ZE
NS	NB	NS	NS	ZE	PS
ZE	NS	NS	ZE	PS	PS
PS	NS	ZE	PS	PS	PB
PB	ZE	PS	PS	PB	PB

Five linguistic terms are used for each variable,
(NB – *Negative big*, NS – *Negative small*, ZE – *Zero*,
PS – *Positive small* and PB – *Positive big*).
Each entry of the table defines one rule, e.g.

R_{23}: **If** e is NS and $\dot{e}$ is ZE **then** u is NS.

An Design Example: Inverted Pendulum

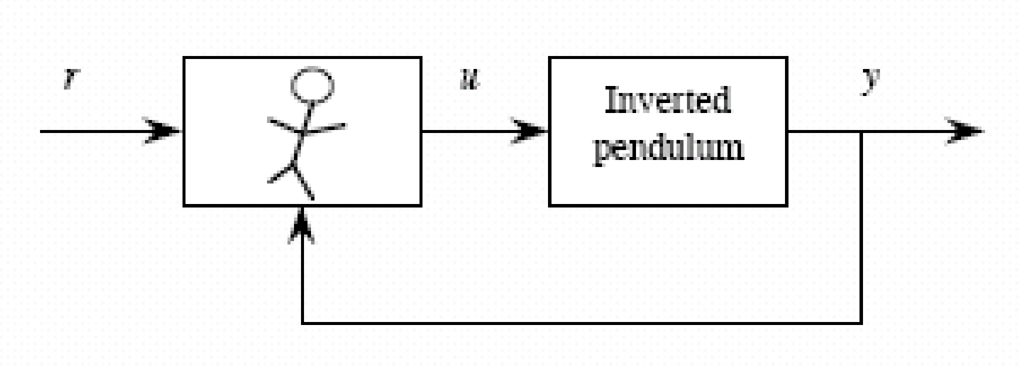


Problem Formulation:
Balance a pole on a mobile platform that can move in only two directions, to the left or to the right.

Assumption:
In the beginning, the pole is in a *nearly upright* position so that an angle greater than, say, 45 degrees in any direction, can never occur.

Choosing Outputs and Inputs

Consider a human-in-the-loop whose responsibility is to control the pendulum, as shown in the following figure:



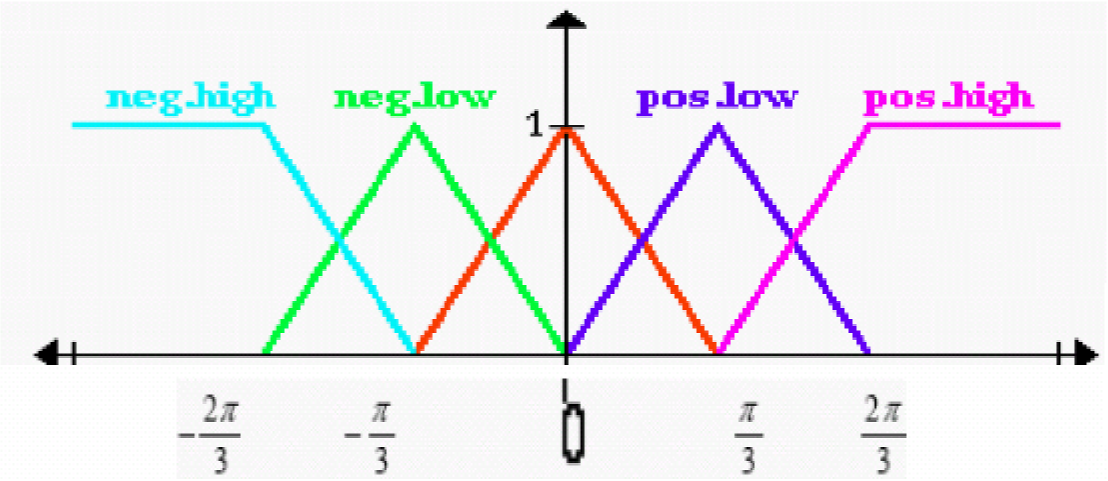
Suppose a human expert (this could be you!) is successful at this task. A fuzzy controller is to be designed to automate how the expert would control the system.

First, the expert tells the designers of the fuzzy controller what information she or he will use as inputs to the decision-making process

Inputs:

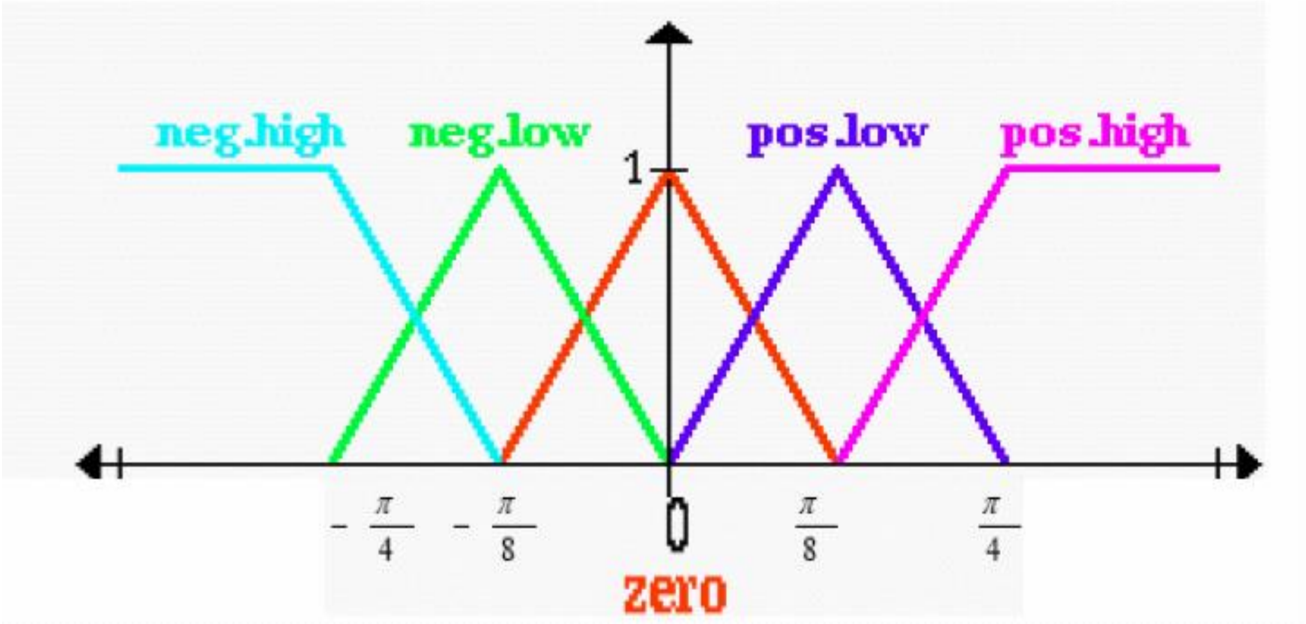
The expert says that two inputs will be used as inputs to the fuzzy decision making process, viz, ***the angle*** between the pendulum and the platform normal, and ***the rate of change of this angle*** or ***angular velocity***.

Membership Functions for the **Angle** between the Platform Normal and Pendulum



where “neg.high”, “neg.low”, “zero”, “pos.low” and “pos.high” are fuzzy sets giving linguistic descriptions “negative high in size” etc.

Membership Functions for **Angular Velocity of Pendulum**

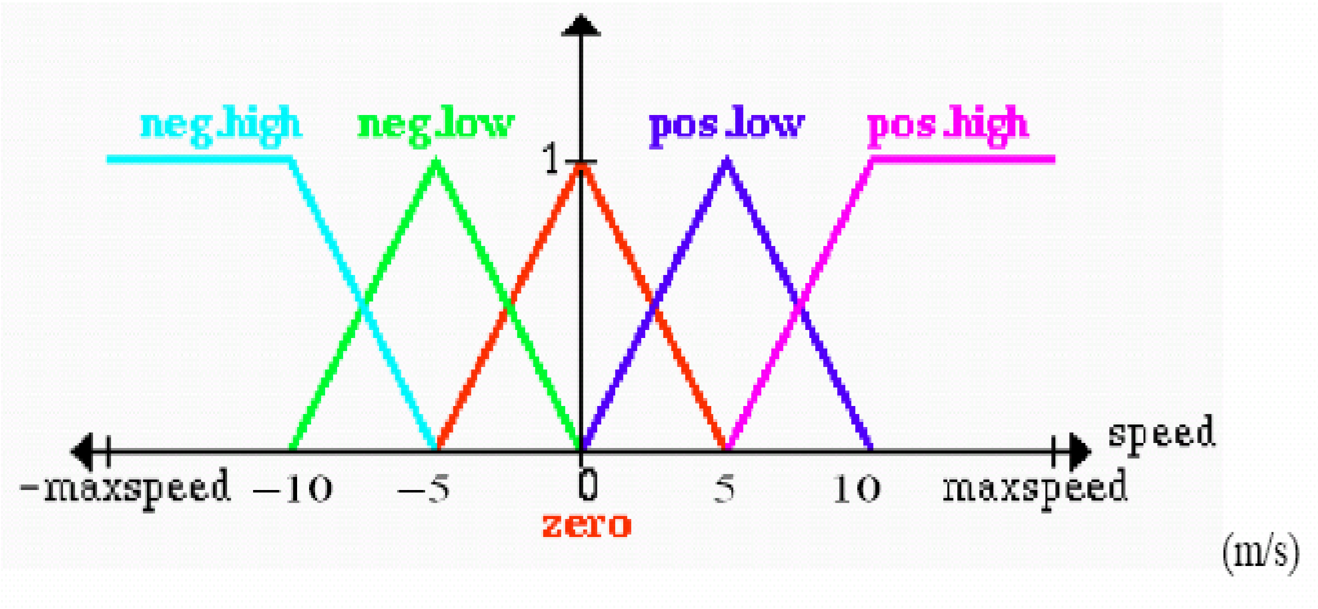


Output(s):

The expert says that controlled variable is ***the speed of the cart***

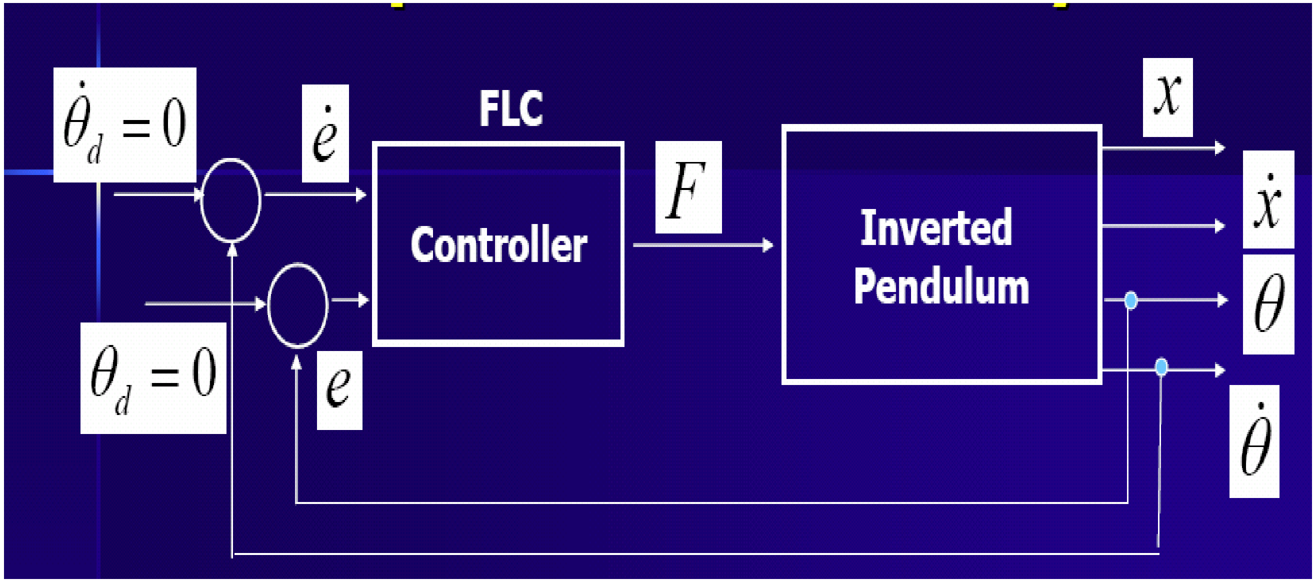
First of all, we have to define (subjectively) what *high* speed, *low* speed etc. of the platform is; this is done by specifying the membership functions for the fuzzy sets.

Membership Functions for the Output Speed of Platform



Once the fuzzy controller inputs and outputs are chosen, you must determine what the reference inputs are.

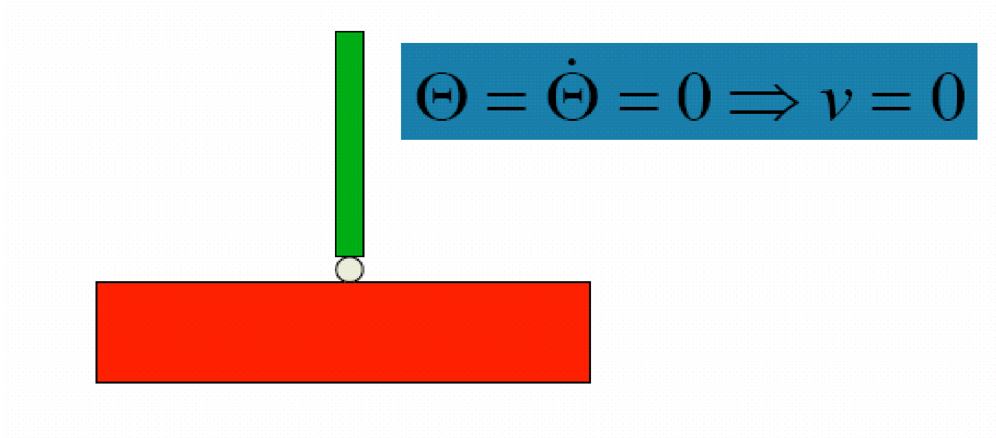
For the inverted pendulum, the reference inputs $\theta_d = 0$ and $\dot{\theta}_d = 0$. The overall fuzzy control system for the inverted pendulum is shown in the following block diagram:



Setting up Rules

Consider for example that the pole is in the upright position (angle is zero) and it does not move (angular velocity is zero).

Obviously this is the desired situation, and therefore we don't have to do anything (speed is zero).



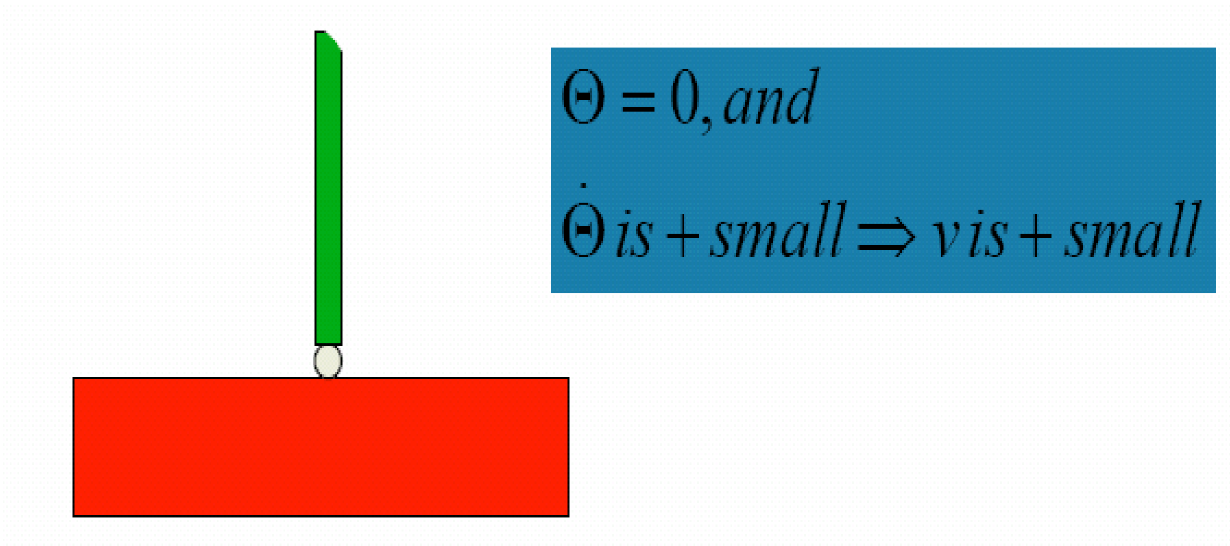
Rule 1

If ***angle*** is ***zero*** and ***angular velocity*** is ***zero***

Then ***speed*** is ***zero***

Let's consider another case: the pole is in upright position as before but is in motion at *low* velocity in *positive* direction.

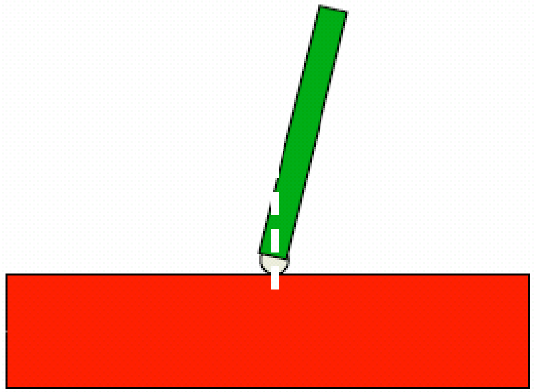
Naturally we would have to compensate for the pole's movement by moving the platform in the same direction at *low* speed.



Rule 2

If ***angle*** is ***zero*** and ***angular velocity*** is ***positive-low***

Then ***speed*** is ***positive-low***



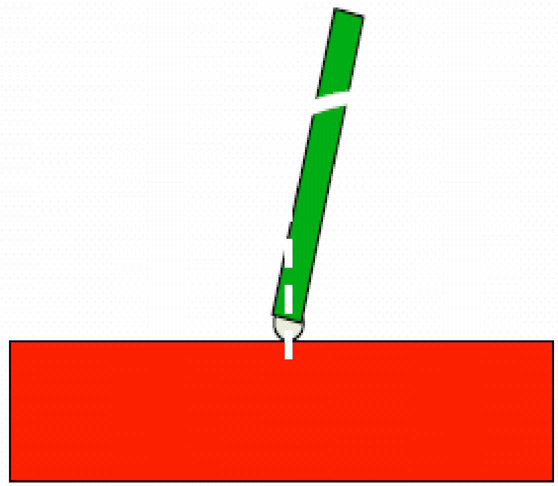
$$\Theta \; is + low \; and$$

$$\dot{\Theta} = 0, \Rightarrow v \; is + low$$

Rule 3

If ***angle*** is ***positive-low*** and ***angular velocity*** is ***zero***

Then ***speed*** is ***positive-low***



$$\Theta \, is + low \, and$$
$$\dot{\Theta} = -low, \Rightarrow v = 0$$

Rule 4

If ***angle*** is ***positive-low*** and ***angular velocity*** is ***negative-low***

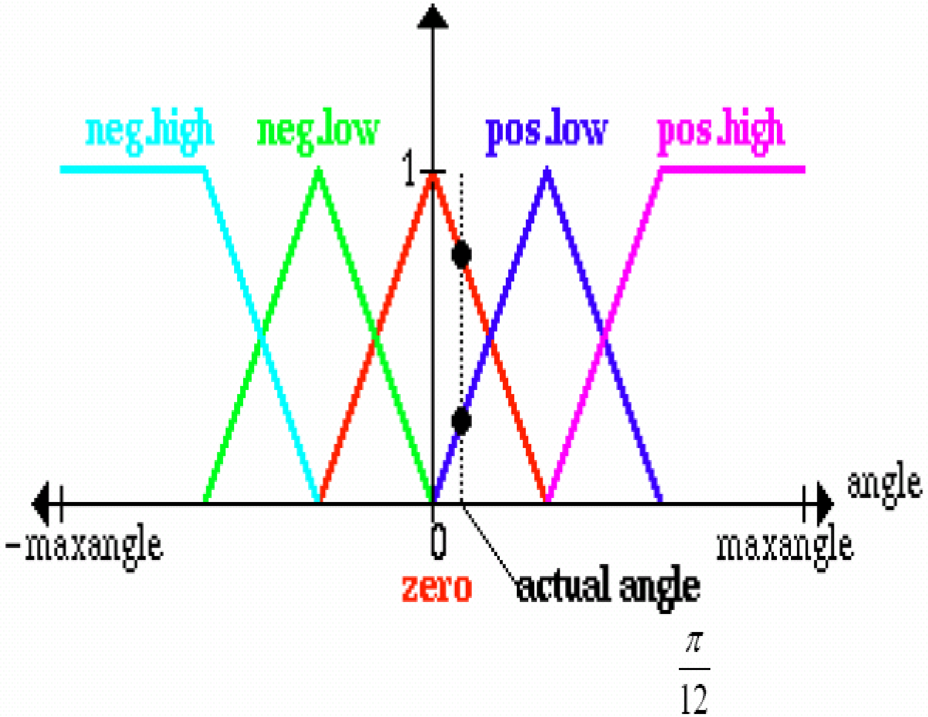
Then speed is zero

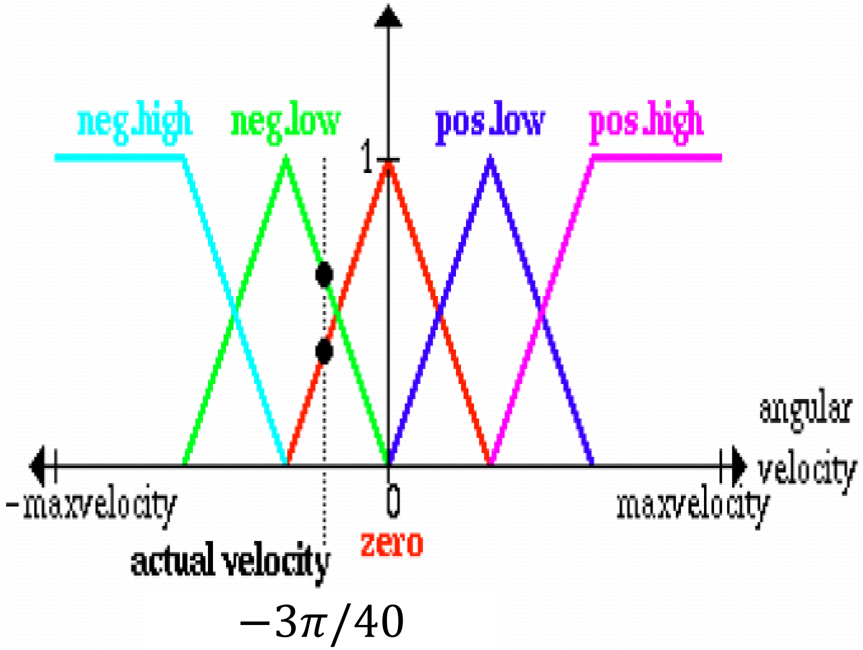
Etc.

	Angle				
speed	NH	NL	Z	PL	PH
NH			NH		
NL			NL	Z	
Z	NH	NL	Z	PL	PH
PL		Z	PL		
PH			PH		

For the input angle (x) = $\pi/12$ rads, and angular velocity (y) = $-3\pi/40$ rad/s, compute the control output.

Input Fuzzification

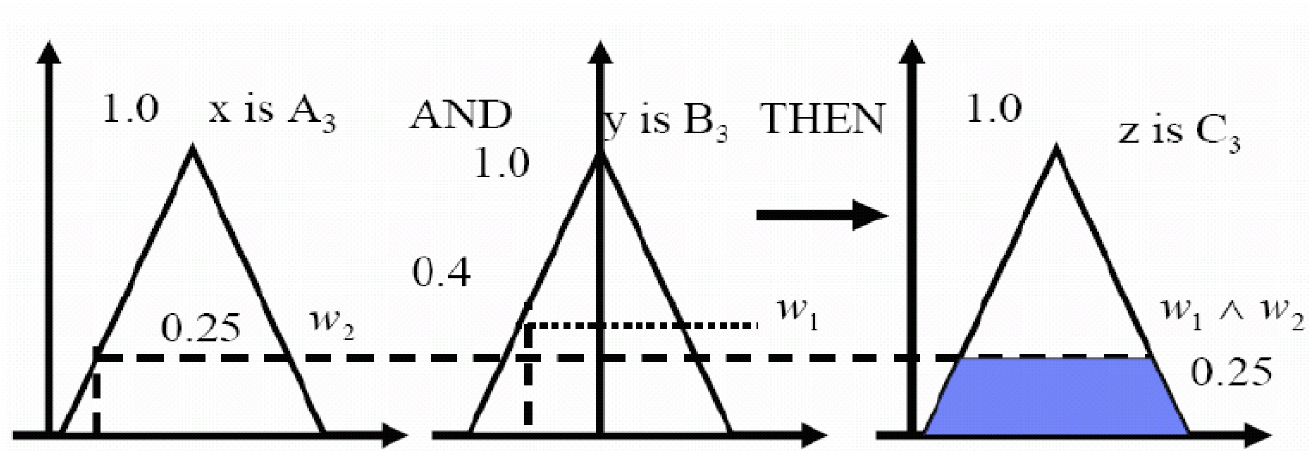


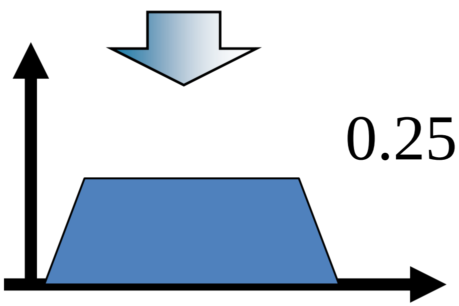


Consider the conclusion reached by **Rule 3**

*If **angle**(x) is **positive-low** (A_3) and **angular-velocity**(y) is **zero** (B_3)*

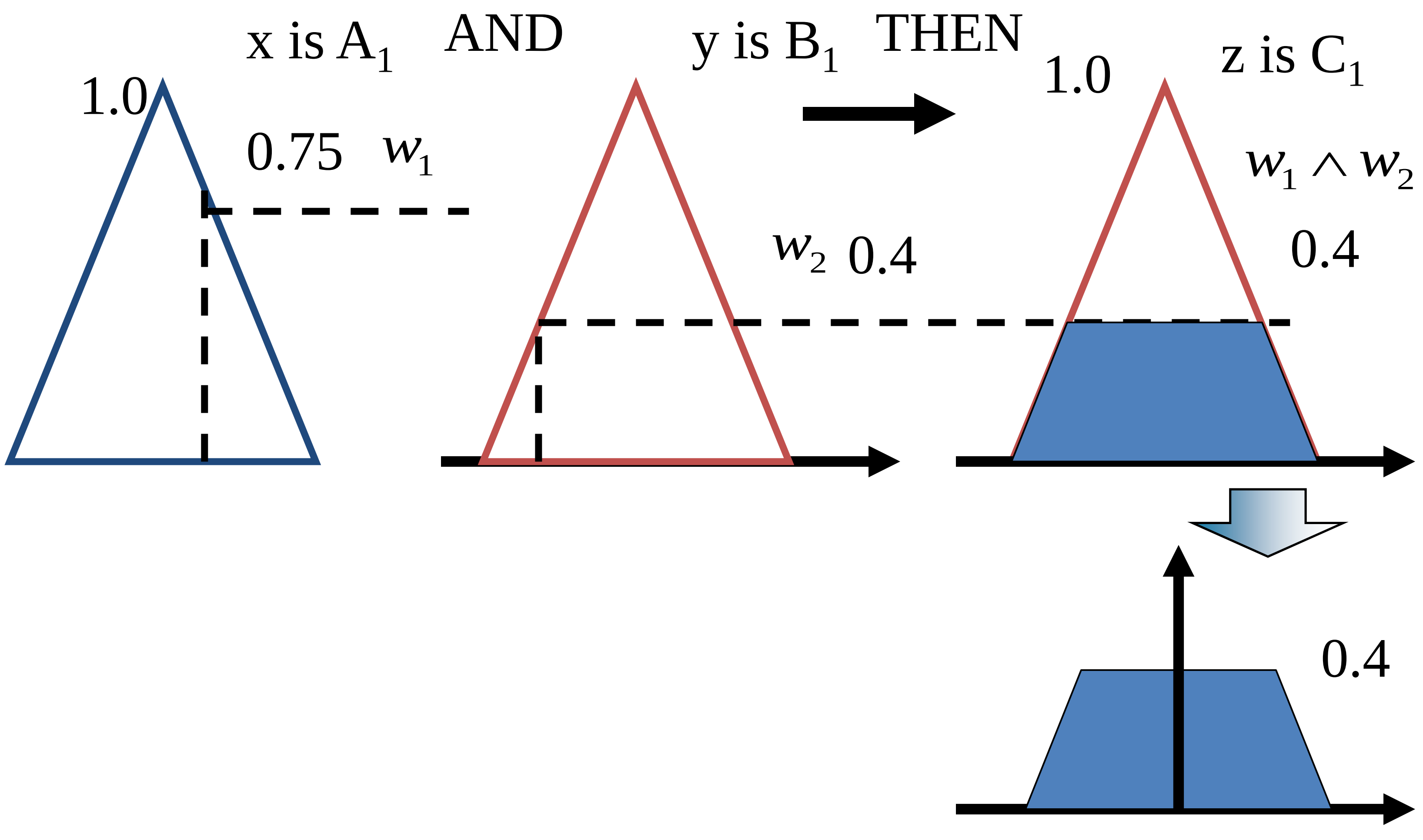
*Then **speed(z)** is **positive-low** (C_3),*



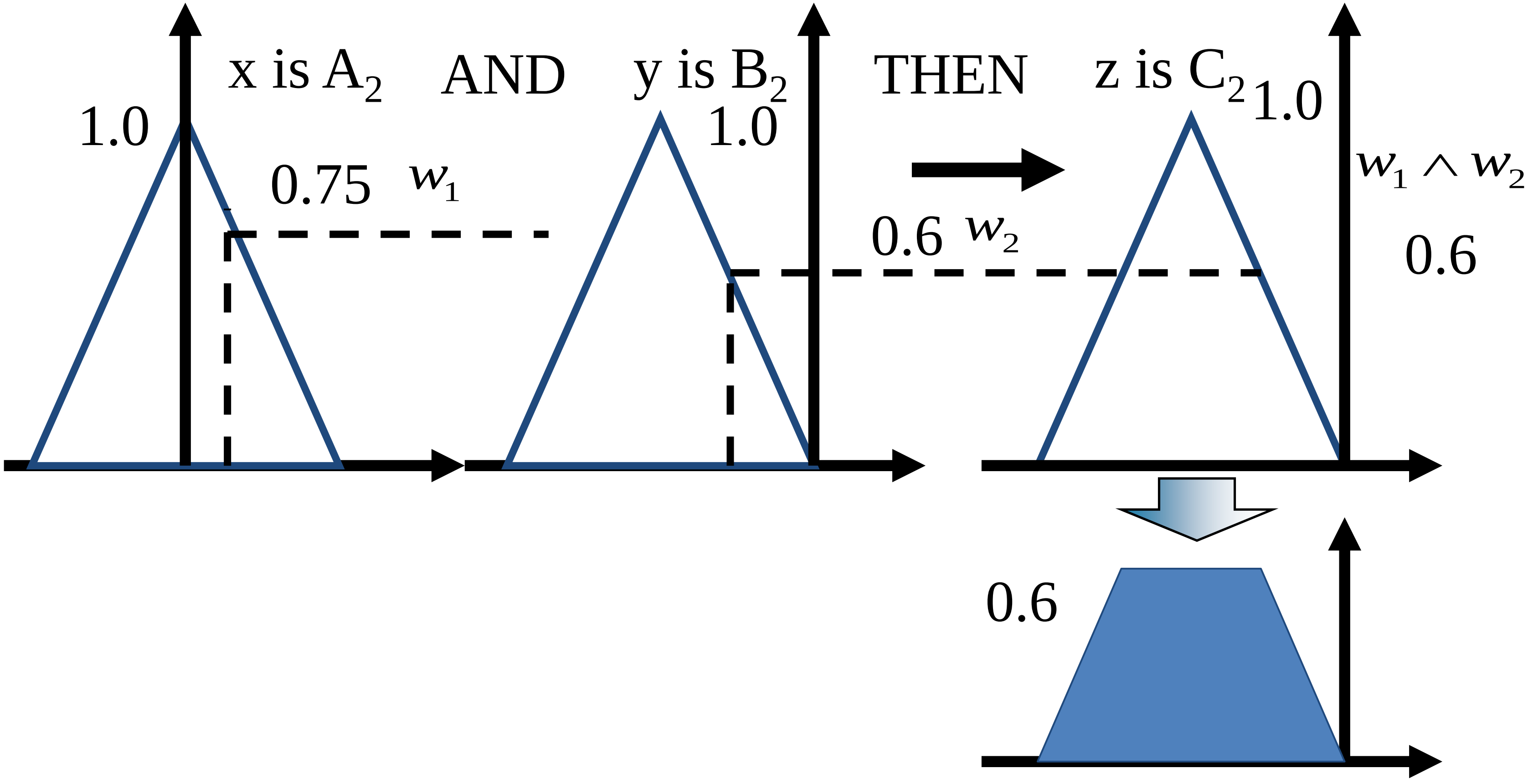


Similarly

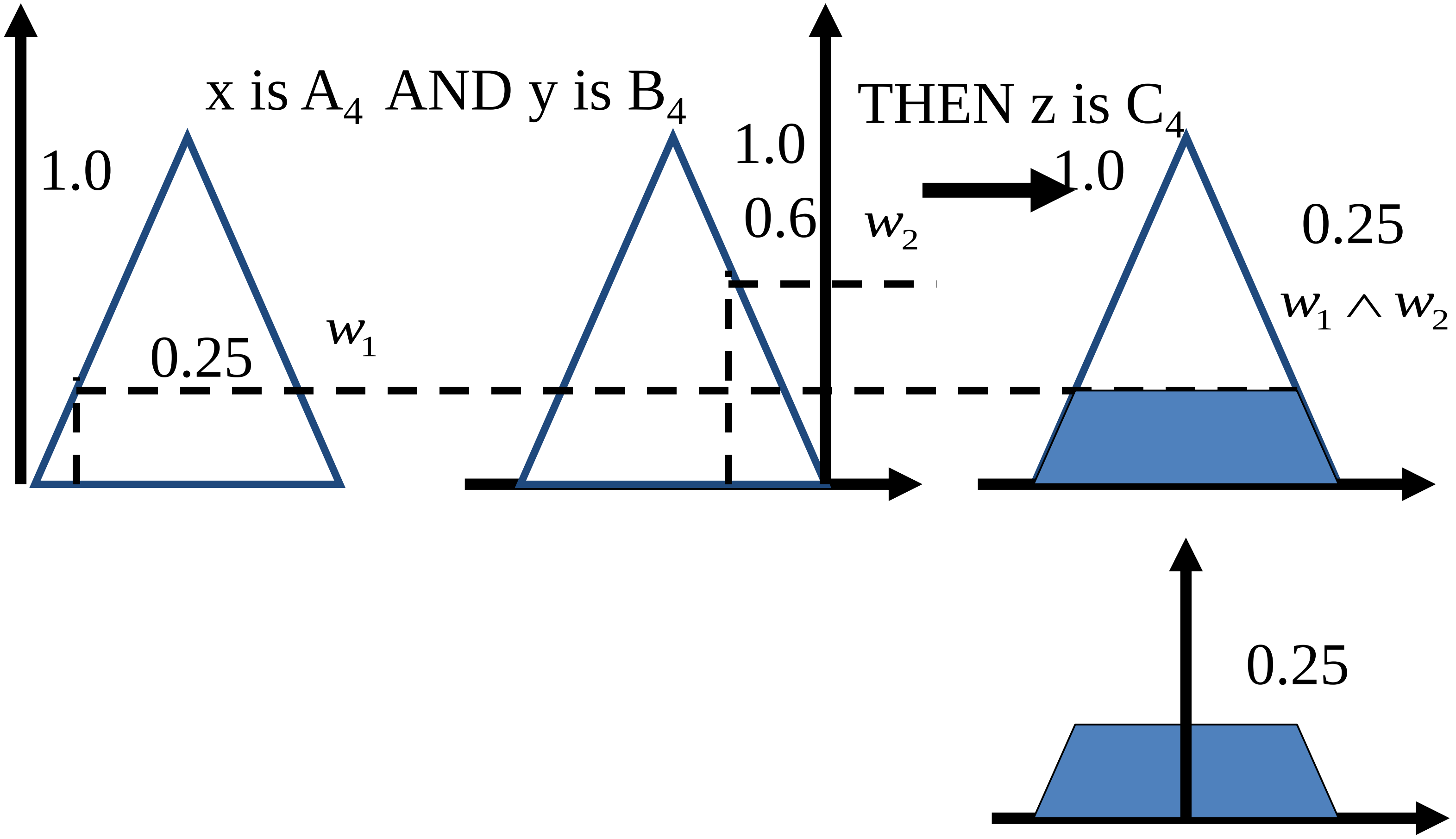
If ***angle*** is ***zero*** and ***angular velocity*** is ***zero*** then ***speed*** is ***zero*** (**Rule 1**)



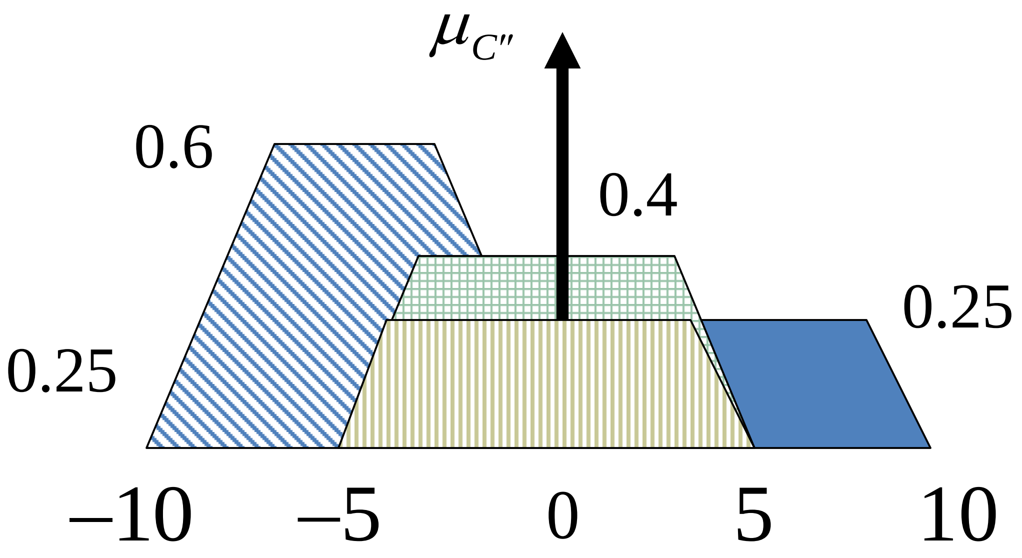
If **angle** is **zero** and **angular velocity** is **negative low** then **speed** is **negative low** (a rule from the table)



If **angle** is **positive low** and **angular velocity** is **negative low**
then **speed** is **zero**-(**Rule 4**)



Fuzzy Aggregation of the Four Rules



Speed is $C'' = C'_1 \cup C'_2 \cup C'_3 \cup C'_4$

COG Defuzzification

We have to choose a crisp value for the final output speed.

The Center Of Gravity (COG) for *continuous* case is computed as shown.

$$z_{COG} = \frac{\int_Z C''(z) \cdot z dz}{\int_Z C''(z) dz}$$

For the *discrete* case, if C'' is defined over a finite universal set $\{z_{1,} z_2, \ldots, z_n\}$, the formula is

$$z_{COG} = \frac{\sum_{k=1}^{n} C''(z_k) \cdot z_k}{\sum_{k=1}^{n} C''(z_k)}$$

Defuzzification

Since the fuzzy sets C'' is the union of the fuzzy sets C_1', C_2', C_3' and C_4', *a good approximation* to the COG for *symmetrical membership function* is to use the *weighted average method* where the *centres* of the individual fuzzy sets are weighted by the areas of their corresponding fuzzy sets

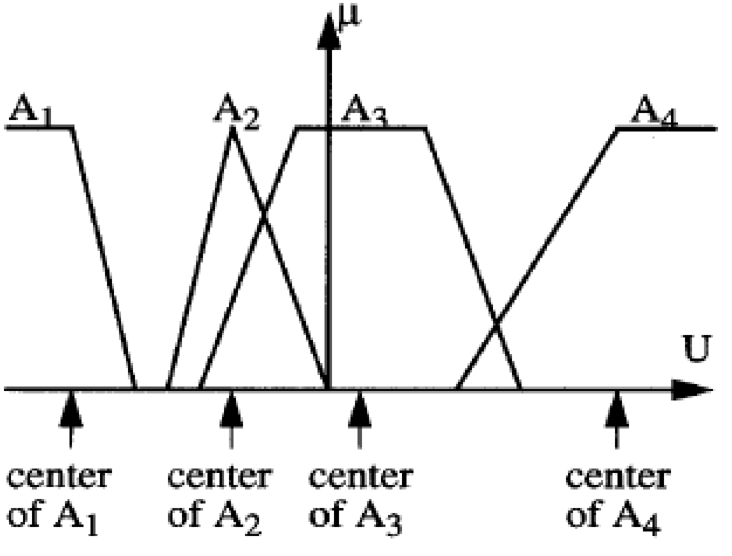
$$z_{COG} = \frac{\sum_{i=1}^{4} z_i a_i}{\sum_{i=1}^{4} a_i}$$

where, z_i, and a_i represent the center and area of the implied fuzzy sets C_1', C_2', C_3' and C_4'.

Centre of a Fuzzy Set

The centre of a fuzzy set is defined as follows:

1. If the mean value of all points at which the membership function of the fuzzy set achieves its maximum value is finite, then define this mean value as the centre of the fuzzy set
2. If the mean value equals positive (negative) infinite, then the centre is defined as the smallest (largest) among all points that achieve the maximum membership value.



Area of Trapezoids

For a trapezoid with a base of w, and a height of h, the area is:

$$area = w\left(h - \frac{h^2}{2}\right)$$

Hence

$$a_1 = 10\left(0.4 - \frac{0.4^2}{2}\right) = 3.200,$$

$$a_2 = 10\left(0.6 - \frac{0.6^2}{2}\right) = 4.200,$$

$$a_3 = 10\left(0.25 - \frac{0.25^2}{2}\right) = 2.188,$$

$$a_4 = 10\left(0.25 - \frac{0.25^2}{2}\right) = 2.188$$

Output Crisp Value

Hence, the input angle (*x*) = $\pi/12$ **rads, and angular velocity (y) =** $-3\pi/32$ **rad/s, the desired output speed is:**

$$speed_{COG}^{crisp} = \frac{(0)(3.200)+(-5)(4.200)+(5)(2.188)+0(2.188)}{(3.200)+(4.200)+(2.188)+(2.188)}$$

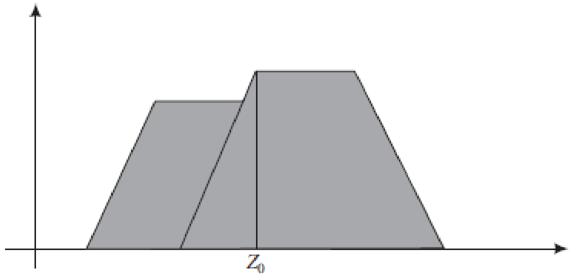
$$= -0.85$$

Other Defuzzification Methods

- First of Maxima

The defuzzified value of a fuzzy set $C(z)$ is the smallest maximizing element, i.e.

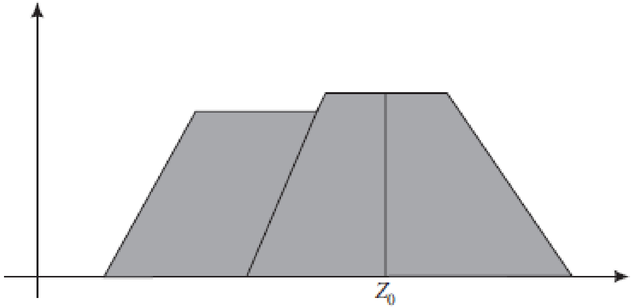
$$z_0 = \min\left\{ z \middle| C(z) = \max_w C(w) \right\}$$



- Middle-of-Maxima

The defuzzified value of a fuzzy set is defined as a mean of all values of the Universe of Discourse, having maximal membership values

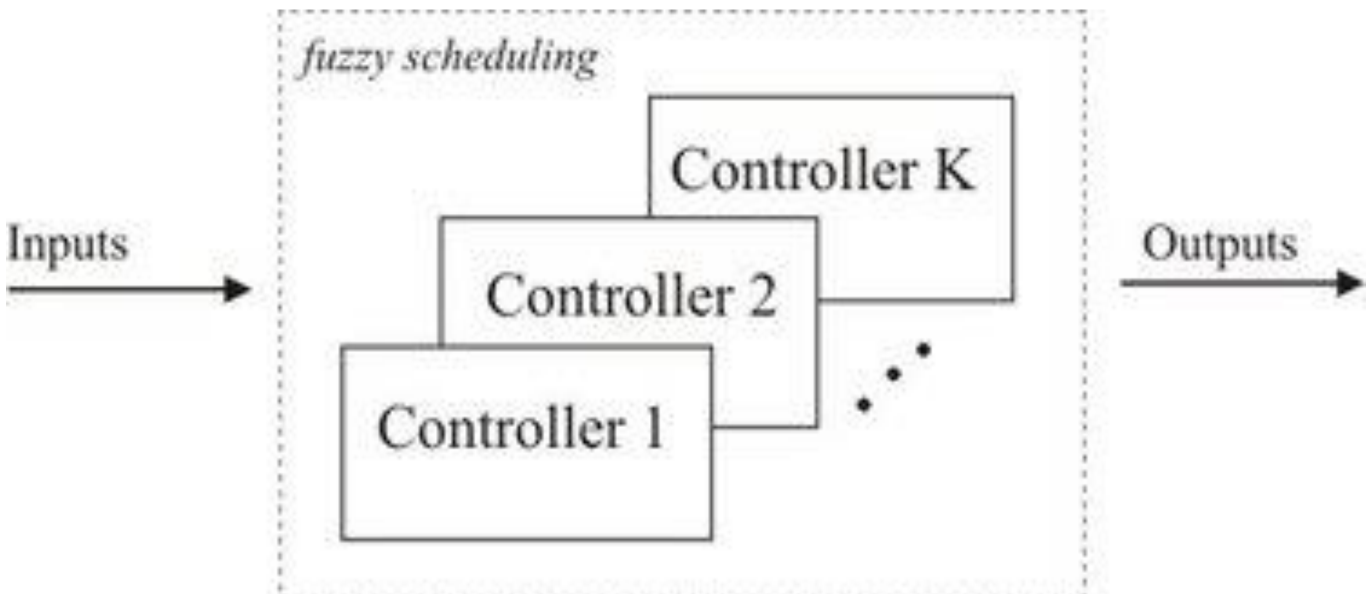
$$z_0 = \frac{1}{N}\sum_{i=1}^{N} z_i$$



where $\{z_1, \ldots, z_N\}$ is the set of discrete points in Z that attains maximum of $C(z)$

4.4 Takagi-Sugeno Controller

Takagi-Sugeno (TS) fuzzy controllers are similar to gain scheduling approaches. Several linear controllers are defined, each valid in one particular region of the controller's input space. The overall controller's output is obtained by interpolating the local linear controllers



An example of a TS control rule base is

$$\begin{aligned} R_1 &: \text{ If } r \text{ is } Low \text{ then } u_1 = P_{\mathrm{Low}} e + D_{\mathrm{Low}} \dot{e} \\ R_2 &: \text{ If } r \text{ is } High \text{ then } u_2 = P_{\mathrm{High}} e + D_{\mathrm{High}} \dot{e} \end{aligned}$$

The controller's output is therefore linear in e and ė, but the parameters of the linear mapping depend on the reference

$$\begin{aligned} u &= \frac{\mu_{\mathrm{Low}}(r) u_1 + \mu_{\mathrm{High}}(r) u_2}{\mu_{\mathrm{Low}}(r) + \mu_{\mathrm{High}}(r)} \\ &= \frac{\mu_{\mathrm{Low}}(r)(P_{\mathrm{Low}} e + D_{\mathrm{Low}} \dot{e}) + \mu_{\mathrm{High}}(r)\left(P_{\mathrm{High}} e + D_{\mathrm{High}} \dot{e}\right)}{\mu_{\mathrm{Low}}(r) + \mu_{\mathrm{High}}(r)} \end{aligned}$$

If the local controllers differ only in their parameters, the TS controller is a rule-based form of a gain-scheduling mechanism.

Another Example of T-S Controller

(*can use either product or min* for intersection operation)

fuzzy control is achieved in the Sugeno-Takagi model. For a simple example, consider a fuzzy rule base consisting of only two rules:

$$\begin{aligned} &\mathrm{R}_1\text{: If } x_1 \text{ is } A_1 \text{ and } x_2 \text{ is } B_1 \text{ then } y = f_1(\mathbf{x}) \\ &\mathrm{R}_2\text{: If } x_1 \text{ is } A_2 \text{ and } x_2 \text{ is } B_2 \text{ then } y = f_2(\mathbf{x}) \end{aligned}$$

where A_i and B_i are fuzzy sets and

$$\begin{aligned} f_1(\mathbf{x}) &= z_{11}x_1 + z_{12}x_2 + z_{13} \\ f_2(\mathbf{x}) &= z_{21}x_1 + z_{22}x_2 + z_{23} \end{aligned}$$

Recall that when numerical input $\mathbf{x} = (x_1, x_2)$ is presented, the inference mechanism will produce the numerical output

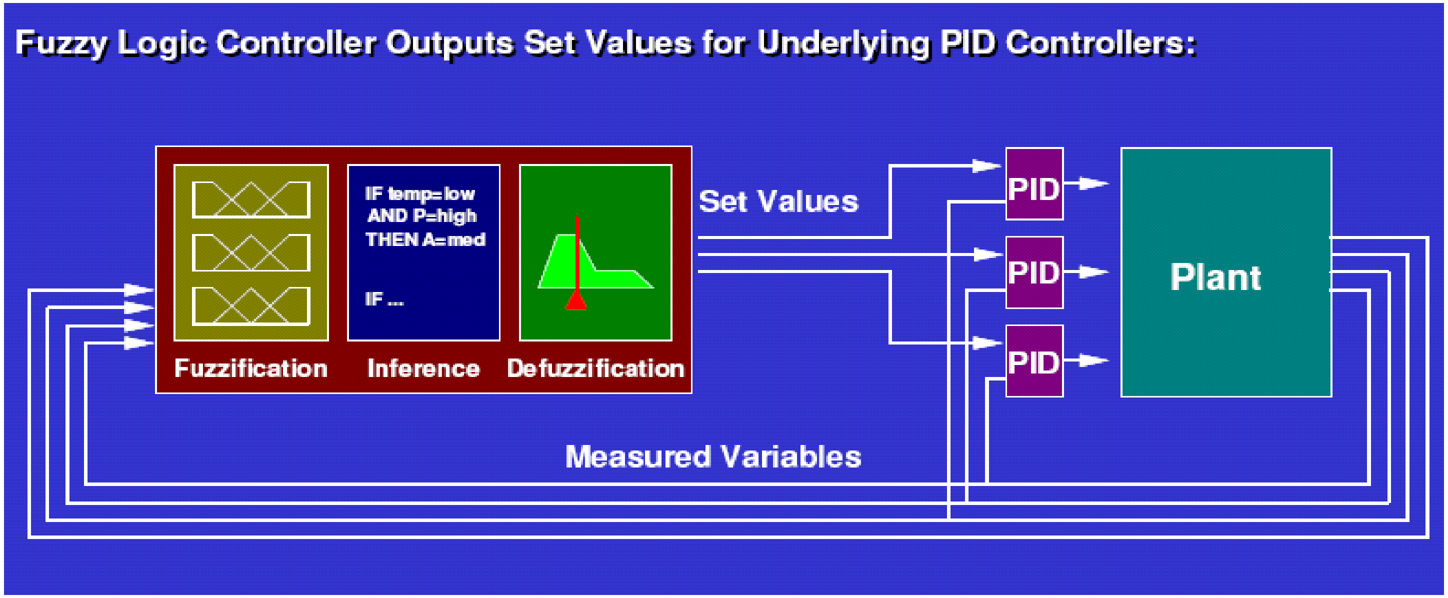
$$y^* = \frac{A_1(x_1) B_1(x_2) f_1(\mathbf{x}) + A_2(x_1) B_2(x_2) f_2(\mathbf{x})}{A_1(x_1) B_1(x_2) + A_2(x_1) B_2(x_2)}$$

4.5 Fuzzy Supervisory Control

A fuzzy inference system can also be applied at a higher, supervisory level of the control hierarchy.

A supervisory controller is a secondary controller which augments an existing controller so that the control objectives can be met which would not be possible without the supervision.

Supervising Set Point Values

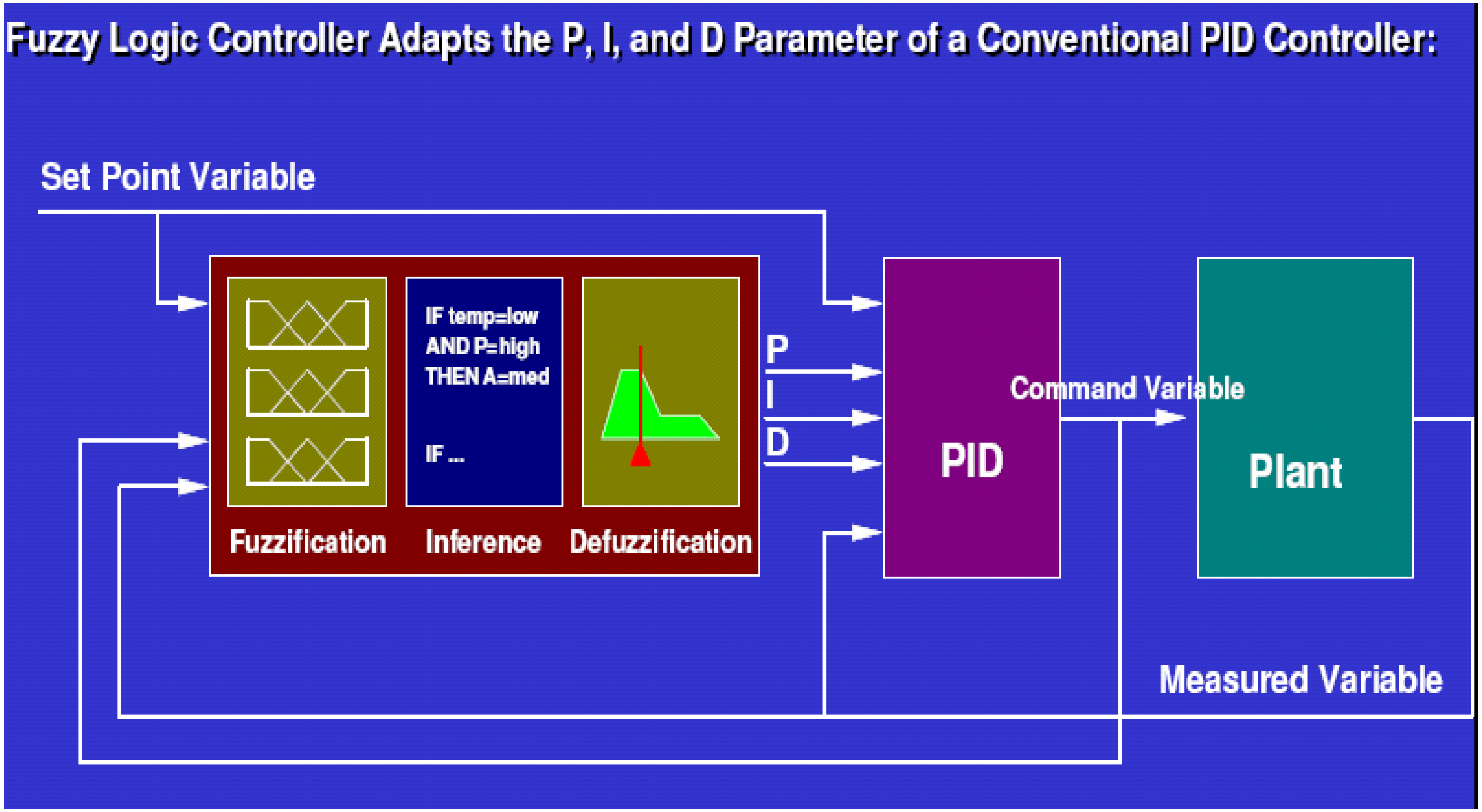


- Each single process variable is controlled by a PID controller, while the set point values for the PID controller come from the fuzzy logic system. Fuzzy logic lets engineers design supervisory set point values from operator experience and experimental results.

- This arrangement is typical for cases like control of several temperature zones of an oven or control of oxygen concentrations in different zones of a wastewater basin.

Adaptation of Controller Parameters

A supervisory controller can adjust the parameters of a low-level controller according to the process information, for example the tuning of the gains of PID controllers.



In this way, static or dynamic behavior of the low-level control system can be modified in order to cope with

- process nonlinearities, or
- changes in the operating point, or
- environmental conditions.

An advantage of a supervisory structure is that it can be added to already existing control systems. Hence, the original controllers can always be used as initial controllers for which the supervisory controller can be tuned for improving the performance.

Many processes in the industry are controlled by PID controllers. Despite their advantages, conventional PID controllers suffer from the fact that the controller must be re-tuned when the operating conditions change.

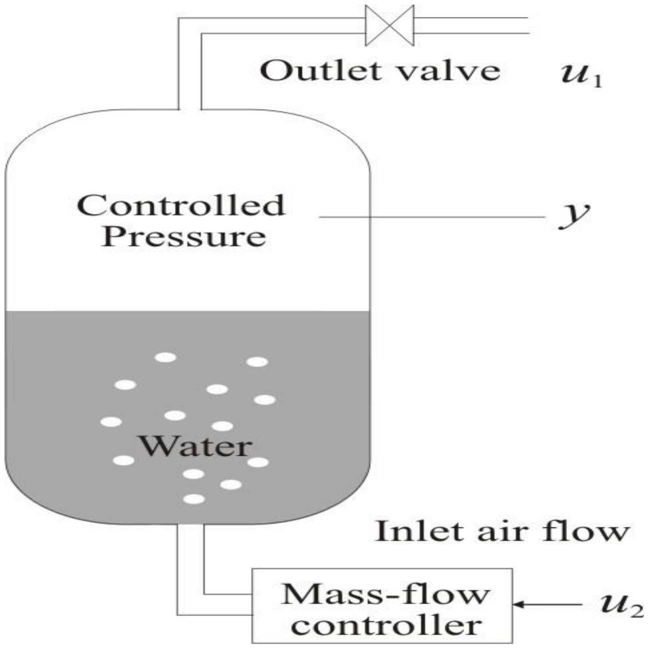
This disadvantage can be reduced by using a fuzzy supervisor for adjusting the parameters of the low-level controller.

As an example, a set of rules can be obtained from experts to adjust the gains *P* and *D* of a PD controller based on the current set-point *r*. The rules may look like

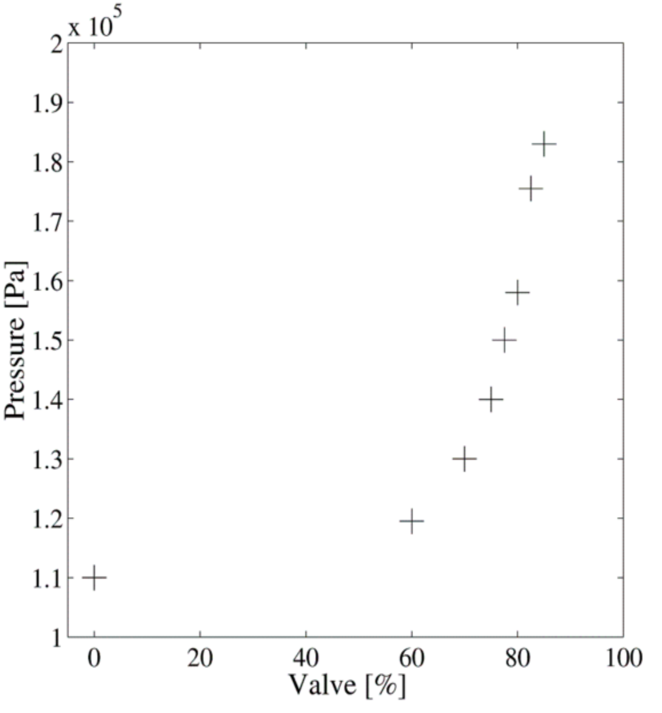
If process output is *High*
Then reduce proportional gain *Slightly*
and
increase derivative gain *Moderately*

Example: Pressure Control

A supervisory fuzzy controller has been applied to pressure control in a laboratory



Experimental Setup



Nonlinear steady-state characteristic

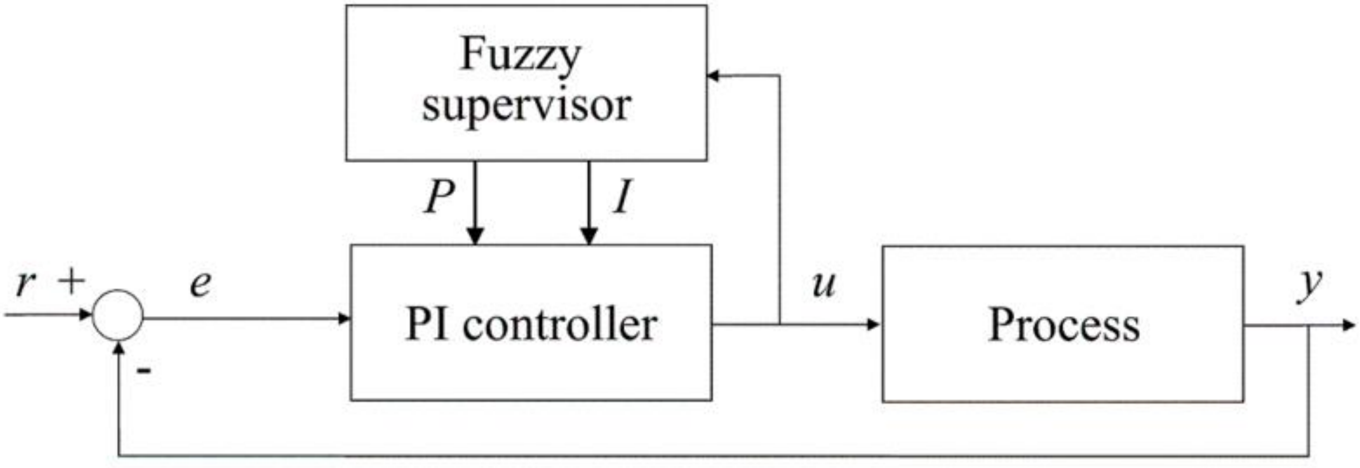
The volume of the tank is *40* liters, and normally it is filled with *25* liters of water. At the bottom of the tank, air is fed into the water at a constant flow-rate, kept constant by a local mass-flow controller.

The *air pressure* above the water level is *controlled* **by** *an outlet valve* at the top of the tank. With a constant input flow-rate, the system has *a single input*, the **valve position**, and *a single output*, the **air pressure**.

Because of the underlying physical mechanisms, and because of the nonlinear behavior of the control valve, the process has a nonlinear steady-state characteristic, as well as a nonlinear dynamic behavior.

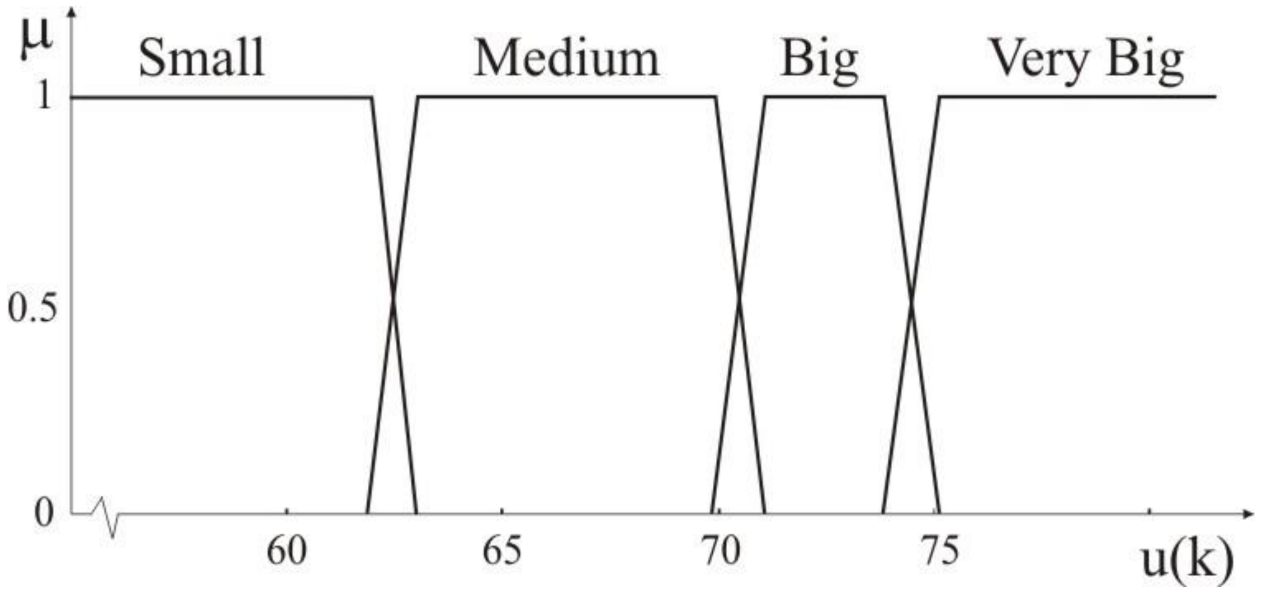
A single-input, two-output supervisor was designed. The input of the supervisor is the valve position *u(k)* and the outputs are the proportional and the integral gains of a conventional PI controller.

The supervisor updates the PI gains at each sample of the low-level control loop (*5* s).



The supervisory fuzzy control scheme

The domain of the valve position between 0 and 100 was modeled with *Small*, *Medium*, *Big* and *Very Big* as membership functions.



Membership functions for *u(k)*

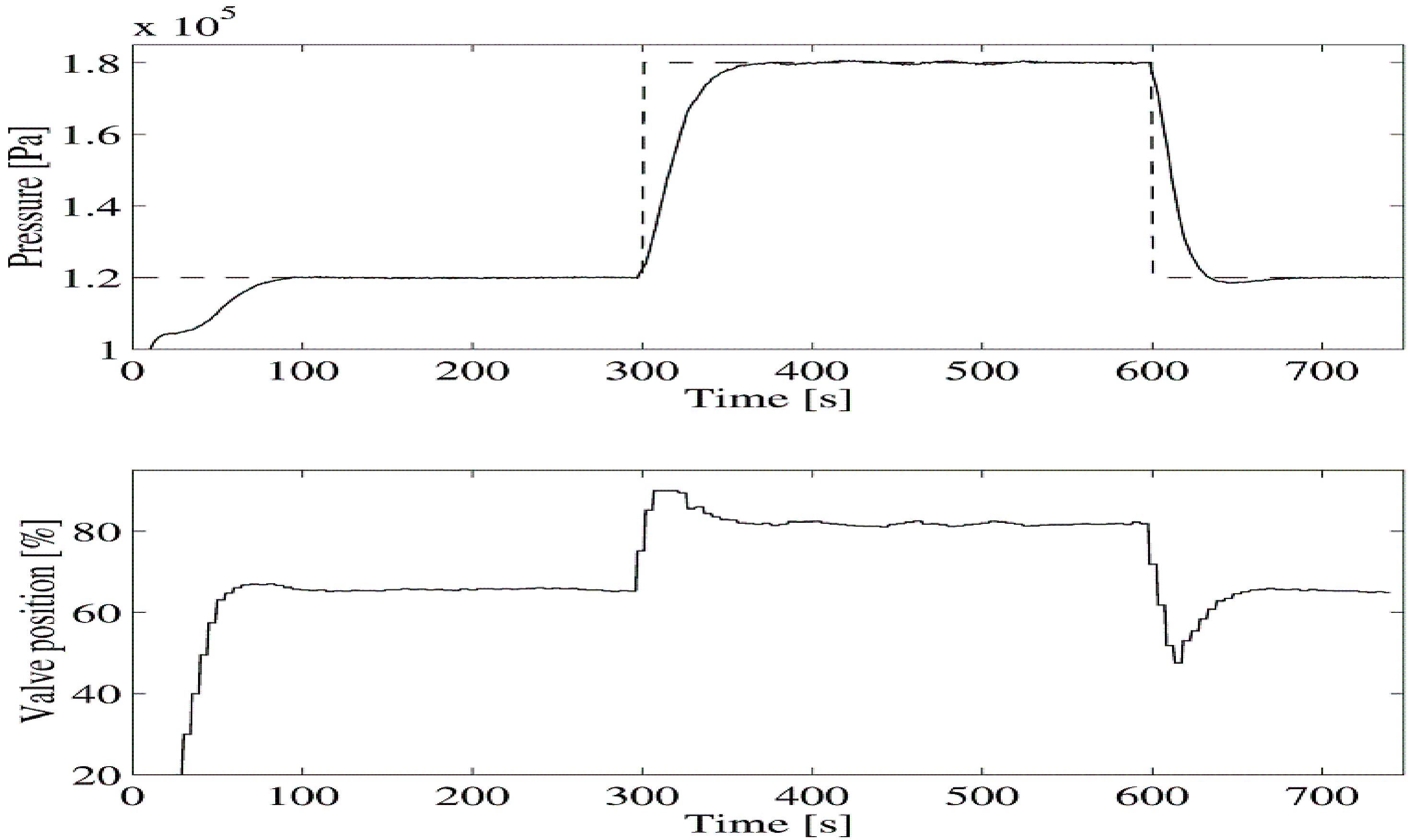
The PI gains *P* and *I* associated with each of the fuzzy sets are given as follows:

Gains*u(k)*	**Small**	**Medium**	**Big**	**Very big**
P	190	170	155	140
I	150	90	70	50

The *P* and *I* values were found through simulations in the respective regions of the valve positions.

The overall numerical *output* of the supervisor is computed by Takagi-Sugeno formulation in Slide 45 or 46.

The supervisory fuzzy controller, tested and tuned through simulations, was applied to the process directly (without further tuning), under the nominal conditions. The real-time control results are shown below



Adaptation of Control Laws

A supervisory structure can be used for implementing different control strategies in a single controller.

An example is choosing proportional control with a high gain, when the system is very far from the desired reference signal and switching to a PI-control in the neighborhood of the reference signal.

Because the parameters are changed during the dynamic response, supervisory controllers are in general nonlinear.