Measuring Dark Matter in the Solar Neighborhood using Normalizing Flows and Gaia DR3

Follow-up to:
“Measuring Galactic dark matter through unsupervised machine learning”
Sung Hak Lim; May 8, 2023

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Introduction

Dark matter (DM) dominates visible matter. Only evidence for its existence is gravitational.

DM halo envelops the Milky Way (MW). Key features:

- Local density critical for direct detection
- Density profile reveals DM particle physics

<u>Stellar orbits trace the potential of our DM halo</u>

- Stellar rotation curves trace the total enclosed mass
- Stars → collisionless fluid. Statistical description of dynamics encoded in the **phase space *f***. Evolves in a potential under the **collisionless Boltzmann equation**

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{x}} + \vec{a} \cdot \frac{\partial}{\partial \vec{v}}\right] f(\vec{x}, \vec{v}) = 0 \qquad \vec{a} = -\vec{\nabla}\Phi$$

The Collisionless Boltzmann Equation

$$\left[\cancel{\frac{\partial}{\partial t}} + \vec{v}\cdot\frac{\partial}{\partial \vec{x}} + \vec{a}\cdot\frac{\partial}{\partial \vec{v}}\right] f(\vec{x},\vec{v}) = 0$$

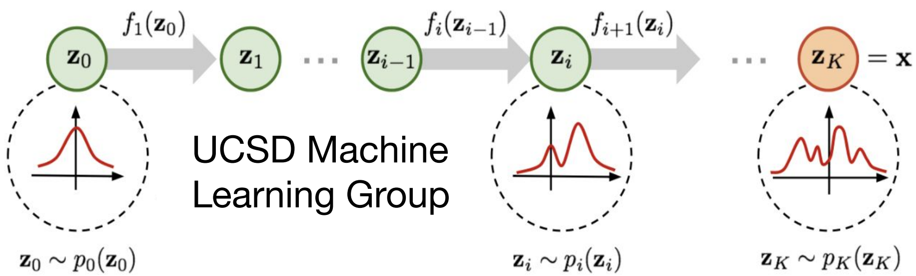
Infer acceleration field **a**(**x**) given f(**x**,**v**), assuming dynamic equilibrium.

Measurements of f(**x**,**v**) historically limited by low statistics. Instead, the CBE is usually solved by:

- The Jeans equations (moments of f(**x**,**v**))
- Distribution function modeling

Accuracy of **a**(**x**) limited by assumed dynamical models and symmetries. **There is another way!**

<u>Normalizing flows</u> are flexible high-dimensional density estimators that can learn f(**x**,**v**) directly from the data.



Previously:[1] Solving the CBE with Normalizing Flows in h277

arXiv:2205.01129 & https://doi.org/10.1093/mnras/stad843

Learned f(**x**,**v**) of a simulated galaxy (h277[2]) using normalizing flows.
Recovered the mass density field ρ(**x**) without models or symmetries.

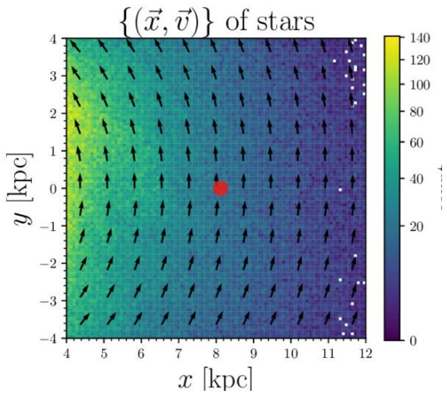
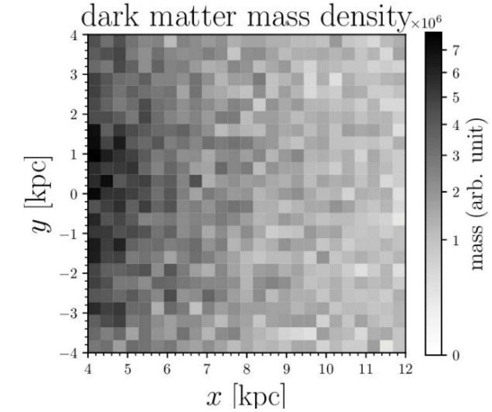
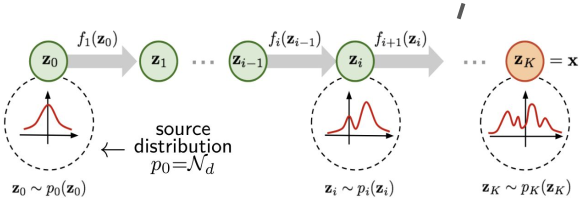


Figure credit:
S.H. Lim (2022)



$$\{(\vec{x},\vec{v})\} \longrightarrow f(\vec{x},\vec{v}) \longrightarrow \vec{a}(\vec{x}) \longrightarrow \rho(\vec{x})$$



Step 1: Normalizing Flows

$$\left[\cancel{\frac{\partial}{\partial t}} + \vec{v}\cdot\frac{\partial}{\partial \vec{x}} + \vec{a}\cdot\frac{\partial}{\partial \vec{v}}\right] f(\vec{x},\vec{v}) = 0$$

Step 2: Collisionless Boltzmann Equation

$$\rho(\vec{x}) \propto -\nabla\cdot\vec{a}$$

Step 3: Poisson Equation

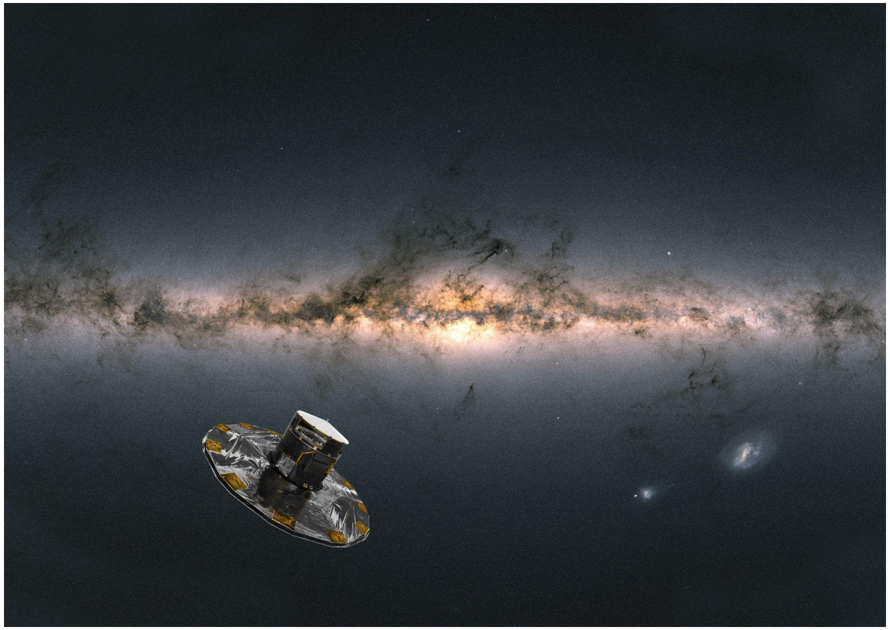
Gaia Data Release 3 (DR3)

Gaia measures **x**,**v**, and stellar properties of billions of sources in the MW.

DR3 includes ~1.8 billion sources with 5D kinematics. 31.5 million with RVS spectra.

- Spectra reveal <u>radial velocity.</u> Full 6D kinematics!

Need to learn f(**x**,**v**) of a complete, equilibrated stellar population…



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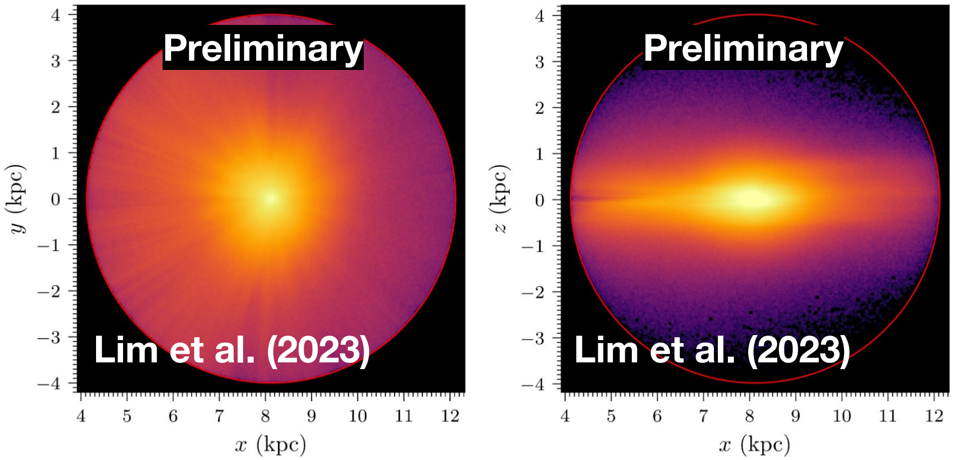


Tracer Selection

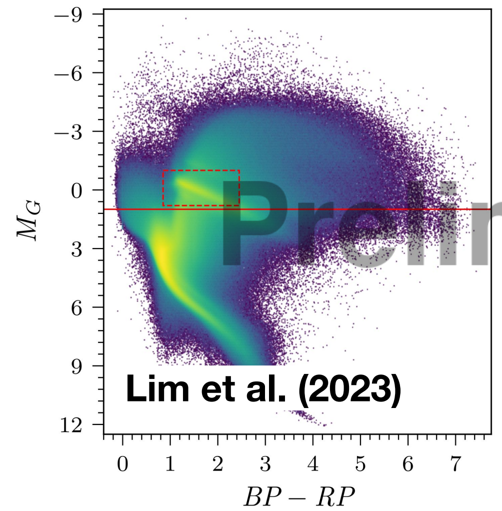
Select stars up to 4 kpc away.

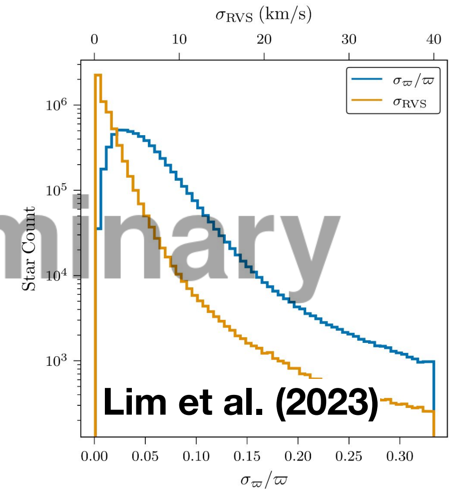
Completeness:

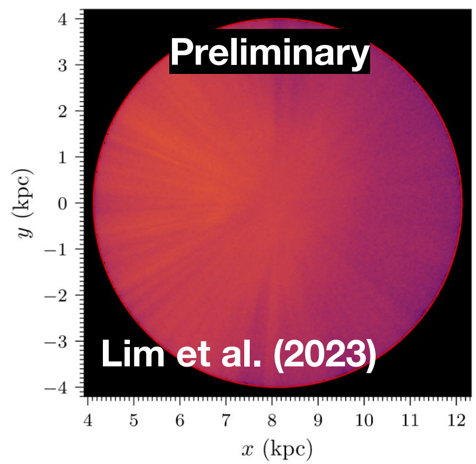
- Remove dim stars
- Dust extinction contaminates f(**x**,**v**) in the disk

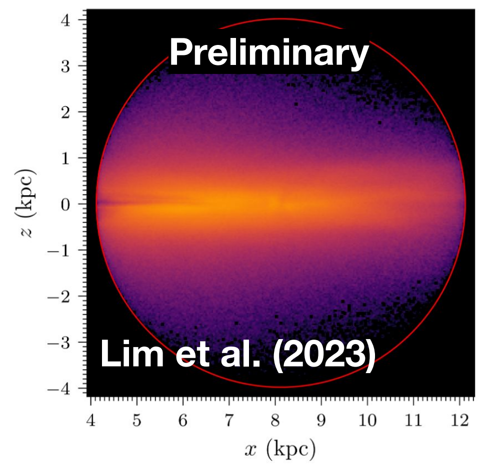
Left with <u>5.8 million</u> stars:

- Primarily Red Clump stars, ~2 billion yrs old (equilibrated)
- ~40x improved statistics vs h277



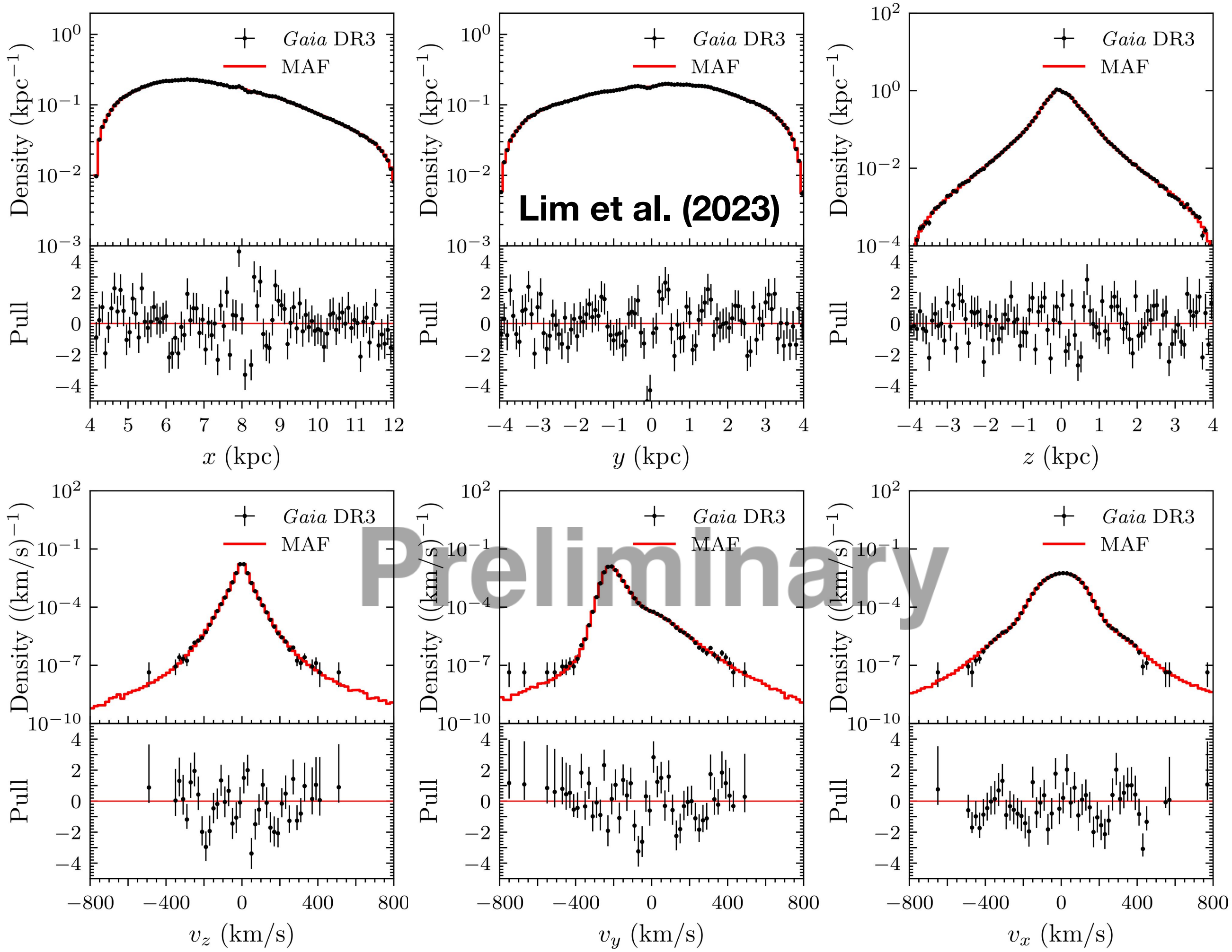






Results

Trained Normalizing Flows



(if curious, an animation of the MAF transformation (Gaussian ⟶ Galaxy) is in the backup slides)

Galactic Acceleration Field

Solve for **a**(**x**) using the flow model for f(**x**,**v**) via the CBE

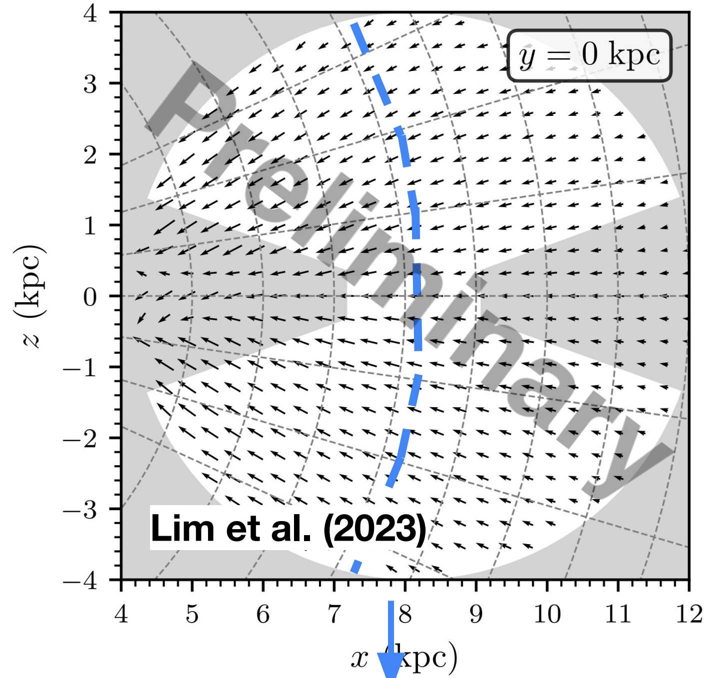
$$\left[\cancel{\frac{\partial}{\partial t}} + \vec{v}\cdot\frac{\partial}{\partial \vec{x}} + \vec{a}\cdot\frac{\partial}{\partial \vec{v}}\right] f(\vec{x},\vec{v}) = 0$$

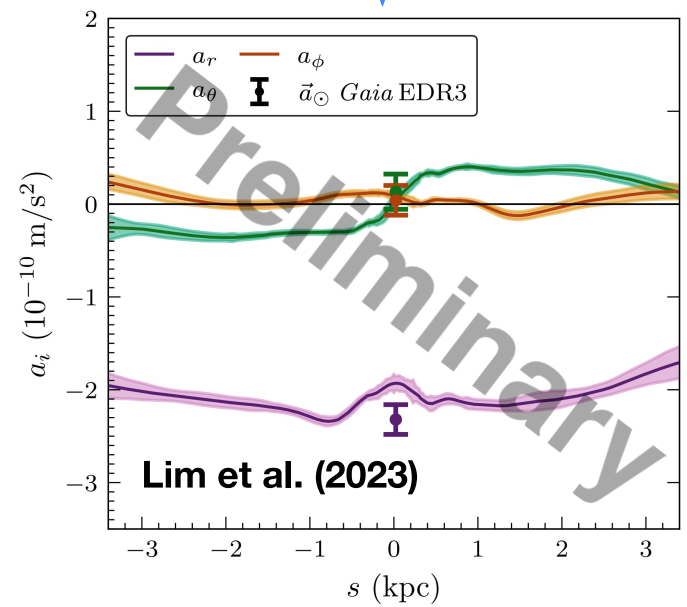
Measured solar acceleration with errors:

	Gaia EDR3	**This work**
a_x $(10^{-10}\mathrm{m/s^2})$	-2.32 ± 0.16	-1.94 ± 0.22
a_y $(10^{-10}\mathrm{m/s^2})$	0.04 ± 0.16	0.08 ± 0.08
a_z $(10^{-10}\mathrm{m/s^2})$	-0.14 ± 0.19	-0.06 ± 0.08
$\lvert\vec{a}\rvert$ $(10^{-10}\mathrm{m/s^2})$	2.32 ± 0.16	1.94 ± 0.22

Preliminary

Includes statistical and measurement uncertainties (likewise for mass densities), ask how at the end!

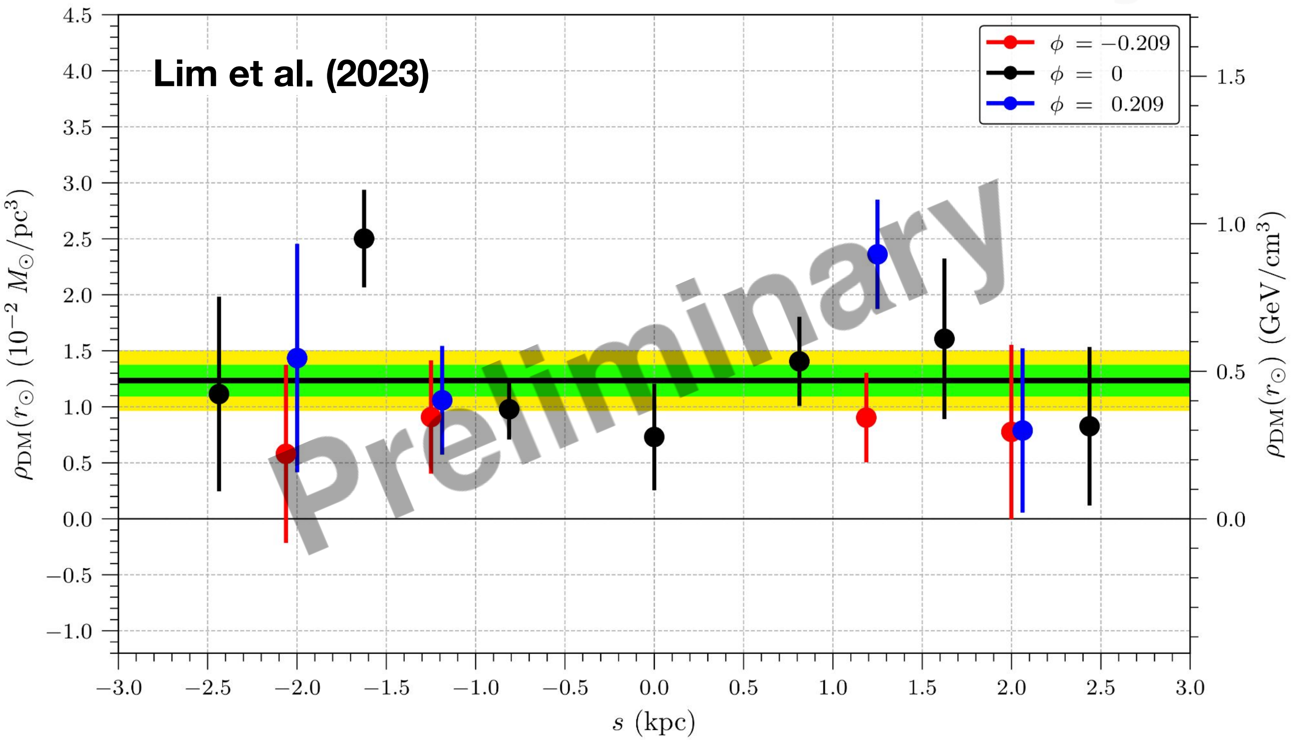


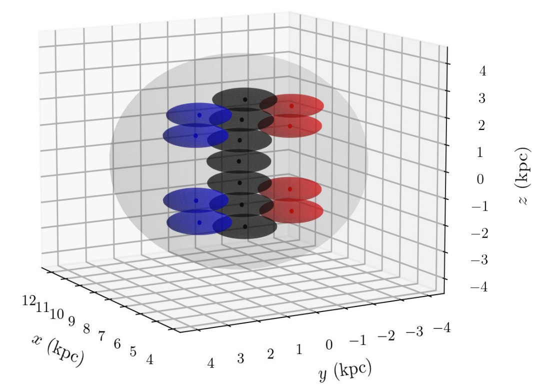


Local Dark Matter Density $\rho(\vec{x}) \propto -\nabla \cdot \vec{a}$

Evaluate ρ at the solar radius ~8.1 kpc. Subtract baryonic mass model[3,4] from ρ to obtain ρ_{DM}.

For a spherical halo: ρ_{DM} = 0.47 +/- 0.05 GeV/cm^3 at the solar location.

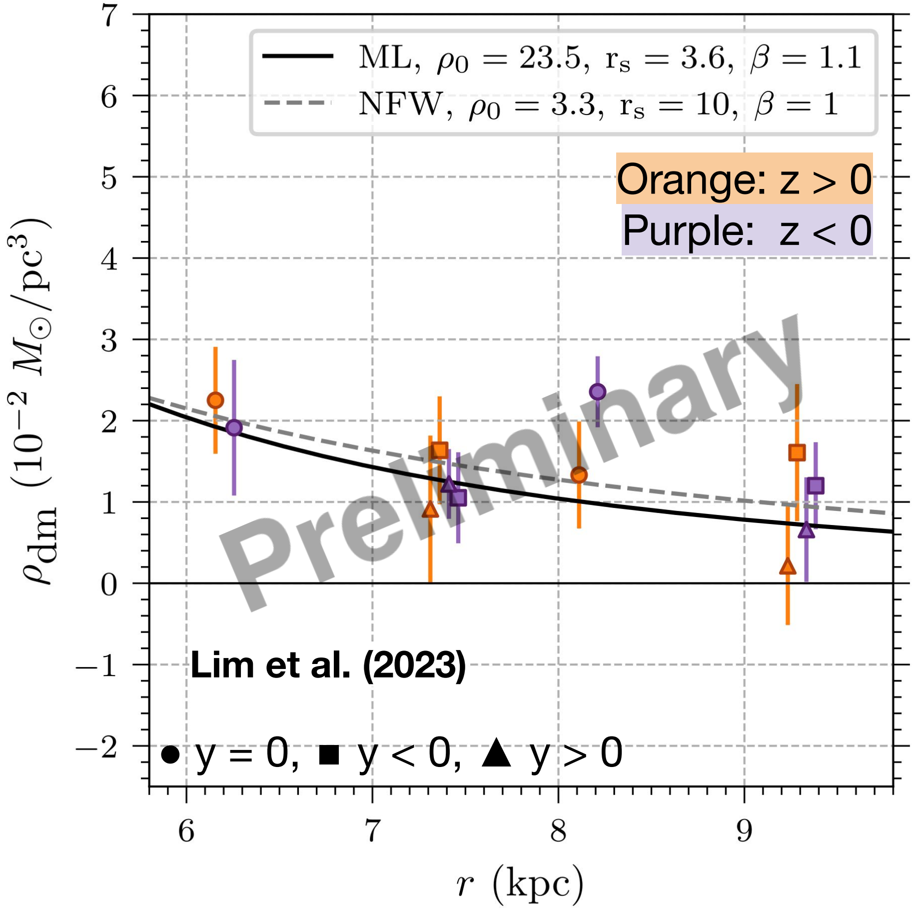


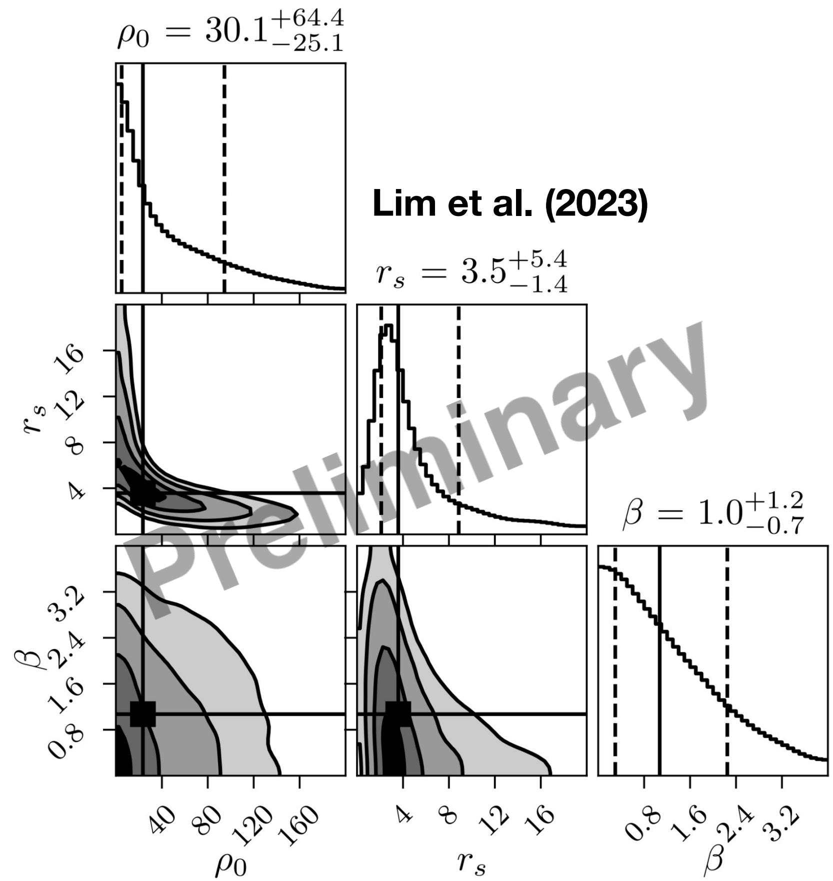


Bonus: Radial Profile of the Milky Way's Dark Matter Halo

Consistent with NFW. Can place broad constraints on halo parameters:

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)^{\beta}\left(1+\frac{r}{r_s}\right)^{3-\beta}}$$



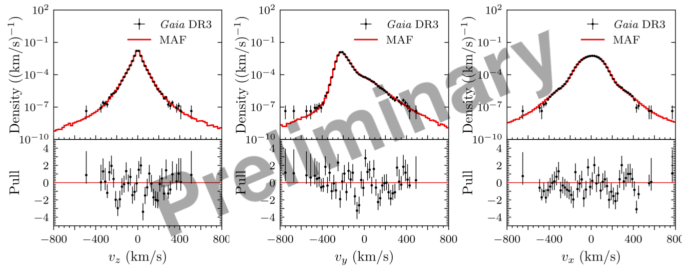


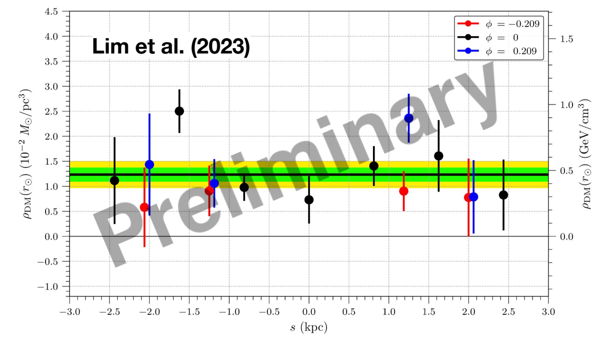
Conclusion

- Flows can learn the complex stellar phase space of the MW
- Measured local ρ_{DM} with minimal assumptions and realistic errors

Future Directions:

- Larger MAFs, CNFs, score-matching networks could improve fit stability
- Next *Gaia* data releases 4 and 5 will provide more complete data
- Use independent measures of **a** to estimate disequilibrium
- Mapping full *Gaia* dataset





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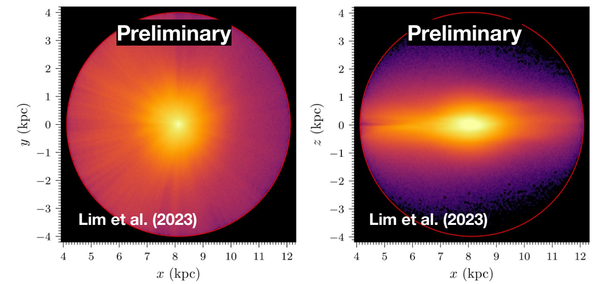


Thank you!

References:

1. M. R. Buckley, S. H. Lim, E. Putney, and D. Shih, arXiv e-prints arXiv:2205.01129 (2022), 2205.01129
2. A. Zolotov, A. M. Brooks, B. Willman, F. Governato, A. Pontzen, C. Christensen, A. Dekel, T. Quinn, S. Shen, and J. Wadsley, Astrophys. J. 761, 71 (2012), 1207.0007
3. C. F. McKee, A. Parravano, and D. J. Hollenbach, Astrophys. J. 814, 13 (2015), 1509.05334.
4. X. Ou, A.-C. Eilers, L. Necib, and A. Frebel, arXiv eprints arXiv:2303.12838 (2023), 2303.12838.

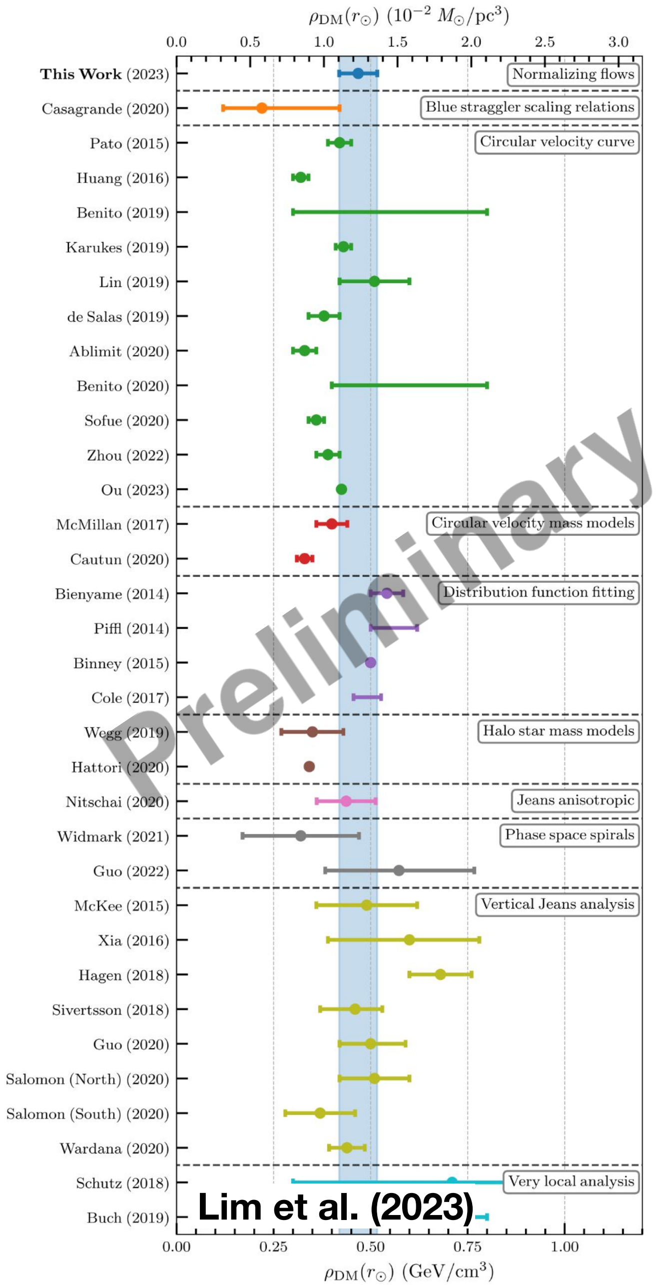
Additional Slides

Comparison to Recent Measurements

ρ_{DM} = 0.47 +/- 0.05 GeV/cm^3 is consistent with most previous measurements of the local dark matter density.

Large scatter visible between different measurements using similar procedures.

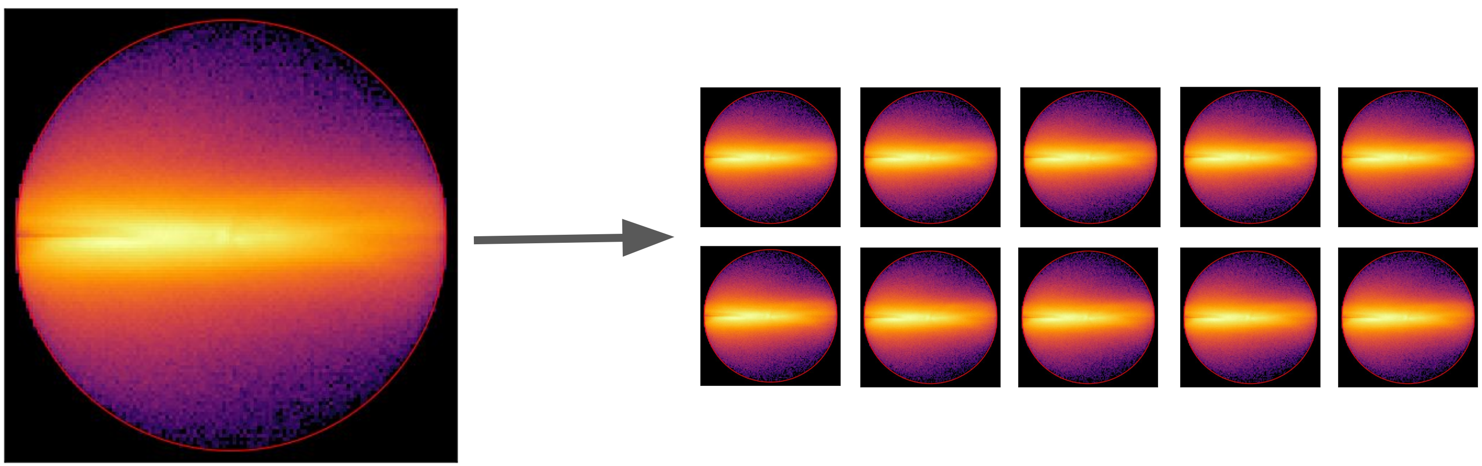
Many measurements report extremely precise values that are inconsistent with one another.



Propagating Statistical Errors

Includes low-statistics and training uncertainties.

- Non-parametric bootstrap i.e. re-sampling with replacement. Retrain networks on each resampled realization of the dataset. Estimates uncertainty from regions of f(**x**,**v**) with low statistics.

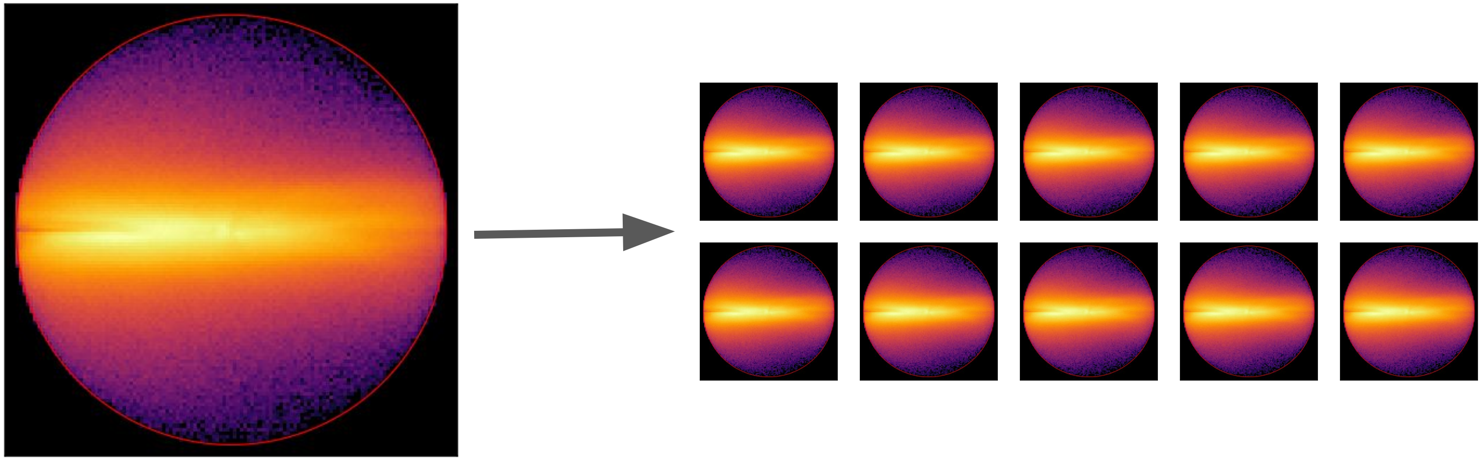


- Ensemble-average random initializations of the networks. We ensemble average 100 normalizing flows.

Propagating Measurement Errors

Perturb dataset by the *Gaia* DR3 error model. Retrain networks on each reperturbed realization of the dataset. Estimates f(**x**,**v**) uncertainty from measurement errors.

- Dominant uncertainties from radial velocity and parallax



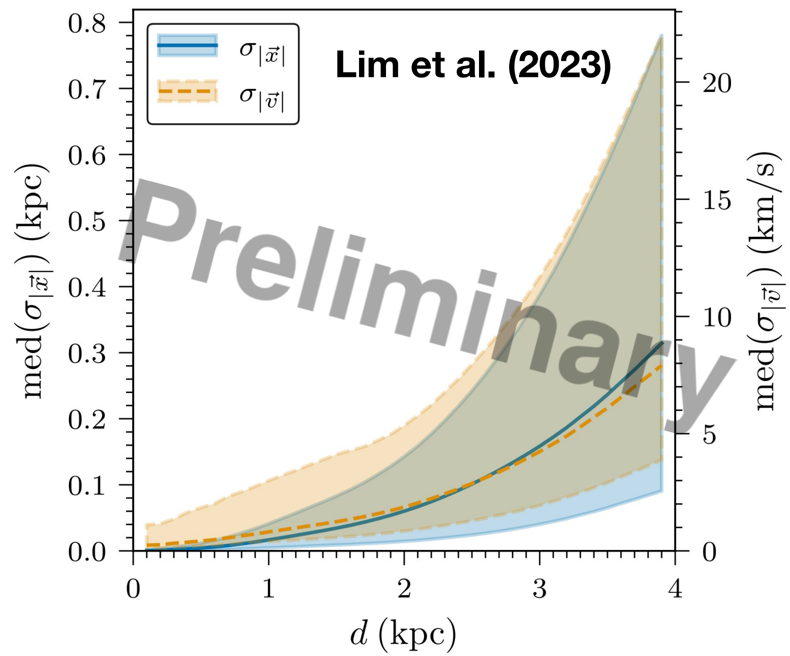
Parallax is not the most reliable distance measure past ~2 kpc. Relative error cut helps, but it is still imperfect. Future data releases will help.

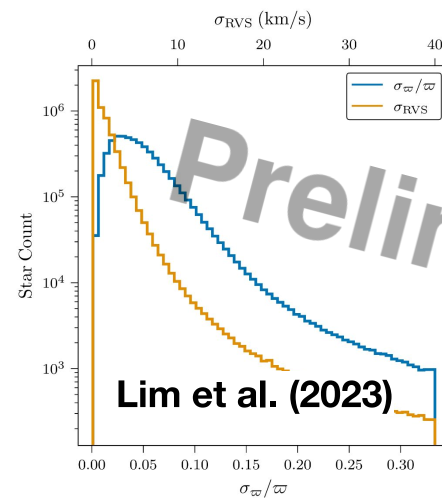
Gaia Measurement Errors

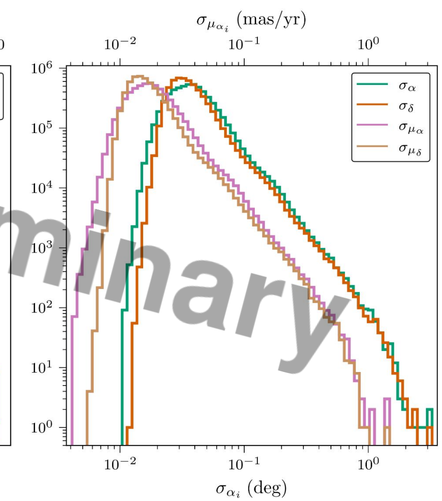
Astrometric errors and propagated from ICRS to Cartesian coordinates.

Primary uncertainties for stars are parallax and radial velocities.

- Parallax is correlated with proper motion, but large relative parallax errors are *extremely problematic*.
- Radial velocities are measured by the Gaia spectrometer. Difficult to measure at high precision.







Network architecture

Number density

$$f(\vec{x},\vec{v}) = p(\vec{x}) * p(\vec{v}|\vec{x})$$

Conditional velocity distribution

2 networks trained independently

- One learns p(**x**) only using measured positions.
- Another learns p(**v**|**x**) from both positions and velocities.

Network hyperparameters

- Gaussian base distributions
- MAFs constructed out of MADE blocks
  - Hidden layers have 48 input features
  - GELU activation used, ensures smooth second derivatives

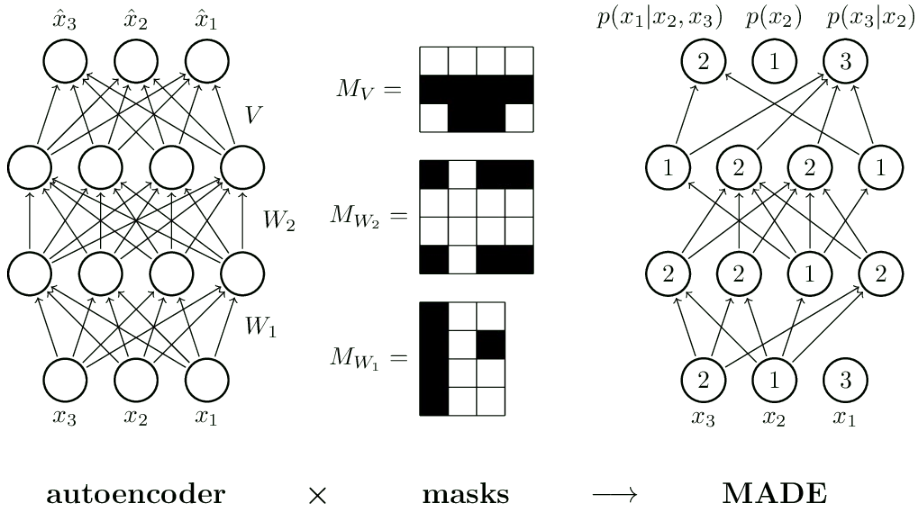
ML python packages:

- PyTorch
- nflows

Masked Autoregressive Flows

Autoregressive model: transformation modeled as a product of conditionals. MAFs enforce autoregressive property by masking network connections, allowing for complex non-linear transformations.

$$p(x_1, ..., x_n) = p(x_1)p(x_2|x_1)p(x_3|x_2, x_1)...p(x_n|x_{n-1}, ..., x_1)$$



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MAF Visualization

Visualizing the flow: Base distribution (3D Gaussian) mapped to Gaia DR3's p(**x**). (Viewed along 3 axes)

Lim et al. (2023)

Jeans Analyses

Moments of the CBE yield the Jeans equations:

$$\int d^3p(p_z * \mathrm{CBE}) \longrightarrow \quad a_z = \frac{1}{\nu}\frac{\partial(\nu\overline{v_z^2})}{\partial z} + \frac{1}{\nu R}\frac{\partial(\nu R\overline{v_R v_z})}{\partial R}$$

$$\int d^3p(p_R * \mathrm{CBE}) \longrightarrow a_R = \frac{1}{\nu}\frac{\partial(\nu\overline{v_R^2})}{\partial R} + \frac{1}{\nu}\frac{\partial(\nu\overline{v_R v_z})}{\partial z} + \frac{\overline{v_R^2} - \overline{v_\phi^2}}{R}$$

Key notes:

- Axisymmetry implicitly assumed when $a_\phi = 0$.
- Must model each velocity moment and number density
- Many terms are noisy. Calculation of ρ can involve high order derivatives of fits to noisy data.

Disequilibrium

Rotating system in dynamic equilibrium should be axisymmetric. We solve the equilibrium CBE, but we do not enforce equilibrium in our model of f(**x**,**v**).

We can perform a closure test of this assumption by measuring deviations from axisymmetry in **a**(**x**) and ρ(**x**)

In the future, we can use f(**x**,**v**) with independent measurements of **a**(**x**) to estimate df/dt.