Text as Data

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Regular Expressions (from Jurafsky Slides)

Systematic Searches

A language for searching texts:

- Count mentions of a person
- Calculate amount of money discussed
- Prepare texts for analysis: Identify where to "split" a document
- ...

Provide a quick introduction here, with some examples

Regular Expressions, Some Basics (from Jurafsky Slides)

- Disjunctions

RE	**Match**	**Example Patterns Matched**
`[mM]oney`	Money or money	"Money"
`[abc]`	'a', 'b', *or* 'c'	"Investing in Iran"
		"is dangerous business"
`[1234567890]`	any digit	"sitting on $7.5 billion dollars"
		"2005 and 2006, more than "
		"$150 million dollars"
`[\.]`	A period	" 'Run!', he screamed."

Regular Expressions, Some Basics (from Jurafsky Slides)

- Ranges

RE	Match	Example Patterns Matched
`[A-Z]`	an upper case letter	"Rep. Anthony Weiner (D-Brooklyn & Queens)"
`[a-z]`	a lower case letter	"ACORN's"
`[0-9]`	a single digit	"(9th CD) "

Regular Expressions, Some Basics (from Jurafsky Slides)

- Negations

RE	Match	Example Patterns Matched
[^A-Z]	not an upper case letter	"ACORN's"
[^Ss]	neither 'S' nor 's'	"ACORN's"
[^\.]	not a period	" 'Run!', he screamed."

Regular Expressions, Some Basics (from Jurafsky Slides)

- Optional Characters: `?`, `*`, `+`

RE	Match	Example Patterns Matched
`colou?r`	Words with `u` 0 or 1 times	"color" or "colour "
`oo*h!`	Words with `o` 0 or more times	"oh!" or "ooh!" or "oooh!"
`o+h!`	Words with `o` 1 or more times	"oh!" or "ooh!" or "oooooh!" or

Regular Expressions, Some Basics (from Jurafsky Slides)

- Wild Cards .

RE	**Match**	**Example Patterns Matched**
`beg.n`	Any word with "beg" then "n"	"begin" or
		"began" or
		"begun" or
		"beggn" (Poor grammar!)

Regular Expressions, Some Basics (from Jurafsky Slides)

- Start of the line anchor ^, end of the line anchor $

RE	Match	Example Patterns Match
`^[A-Z]`	Upper case start of line	"Palo Alto"
		"the town of Palo Alto"
`^[^A-Z]`	Not upper case start of line	"the town of Palo Alto"
		"Palo Alto"
`^.`	Start of line	"Palo Alto"
		"the town of Palo Alto"
`.$`	Identify character that ends a line	"Wait!"
		"This is the end."

Regular Expressions, Some Basics (from Jurafsky Slides)

- "Or" | statements, Useful short hand

RE	**Match**	**Example Patterns Matched**
yours|mine	Matches "yours" or "mine"	"it's either <u>yours</u> or <u>mine</u>"
\ d	Any digit	"<u>1</u>-Mississippi"
\ D	Any non-digit	"1<u>-Mississippi</u>"
\ s	Any whitespace character	"1,<u> </u>2"
\ S	Any non-whitespace character	"<u>1,</u> <u>2</u>"
\ w	Any alpha-numeric	"<u>1</u>-<u>Mississippi</u> "
\ W	Any non-alpha numeric	"1<u>-</u>Mississippi"

Regular Expressions, Some Basics (from Jurafsky Slides)

Quick Example to Illuminate Differences:
A “simple” example: identify all instances of `the`.

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- `[tT]he`

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  Returns words that are too long (`theocrat`, `theme`)

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  Misses the first "the" in a sentence

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- (^ | [^ a-zA-Z])[tT]he[^ a-zA-Z]

An Example: Searching for Tea Party Language

Grimmer, Westwood, and Messing (2014): Criticism and credit

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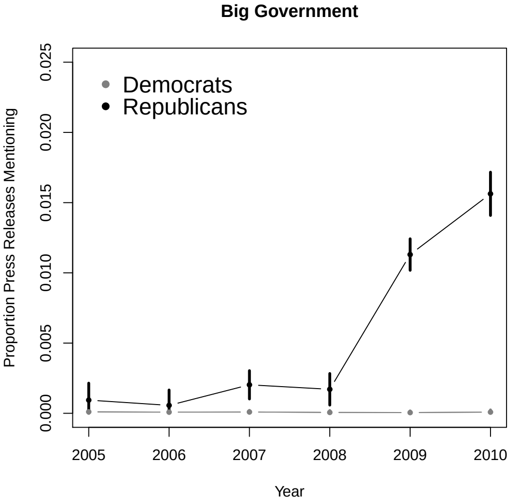

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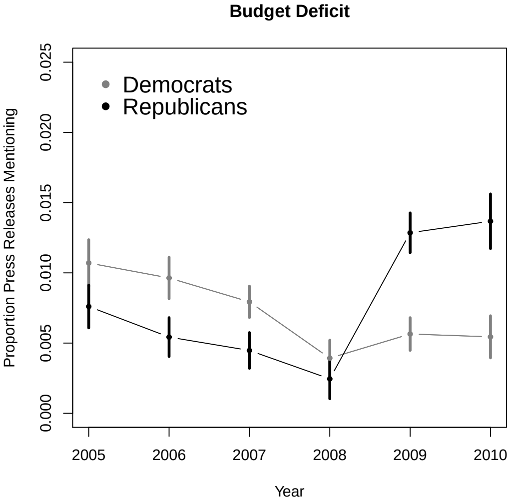

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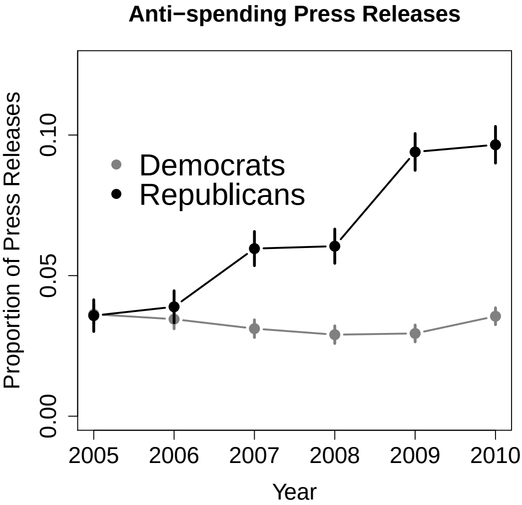

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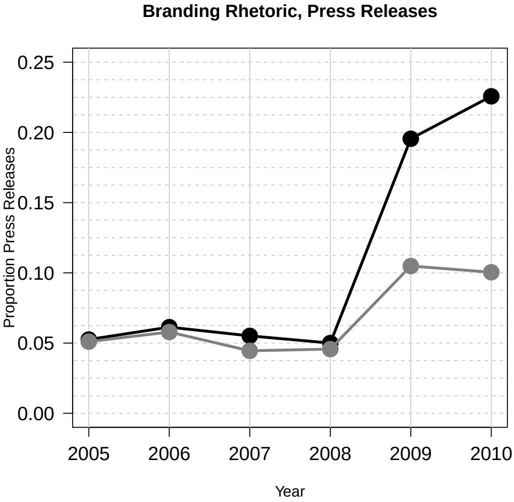
Goodman, Grimmer, Parker, Zlotnik (2015): Criticism



Regular Expressions on Steroids: Cheating Detection Software

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- Useful:
  - Media uptake
  - Joint Press Releases

Research process:

- Discovery
- Measurement
- Causal Inference (Prediction)

Gary King, Jen Pan, and Molly Roberts (2015)$\rightsquigarrow$ discovery

Texts and Geometry

Consider a document-term matrix

$$\boldsymbol{X} = \begin{pmatrix} 1 & 2 & 0 & \ldots & 0 \\ 0 & 0 & 3 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \ldots & 3 \end{pmatrix}$$

Texts and Geometry

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- Provides a geometry

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Suppose documents live in a space $\rightsquigarrow$ rich set of results from linear algebra

- Provides a geometry $\rightsquigarrow$ modify with word weighting
- Natural notions of distance
- Kernel Trick: richer comparisons of large feature spaces
- Building block for clustering, supervised learning, and scaling

Texts in Space

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$$\text{Doc1} \quad = \quad (1, 1, 3, \ldots, 5)$$

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$$\begin{aligned} \text{Doc1} &= (1, 1, 3, \ldots, 5) \\ \text{Doc2} &= (2, 0, 0, \ldots, 1) \end{aligned}$$

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\textbf{Doc1}, \textbf{Doc2} &\in \Re^{J}
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Inner Product between documents:

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\textbf{Doc1} \cdot \textbf{Doc2} &= (1, 1, 3, \ldots, 5)^{'}(2, 0, 0, \ldots, 1) \\
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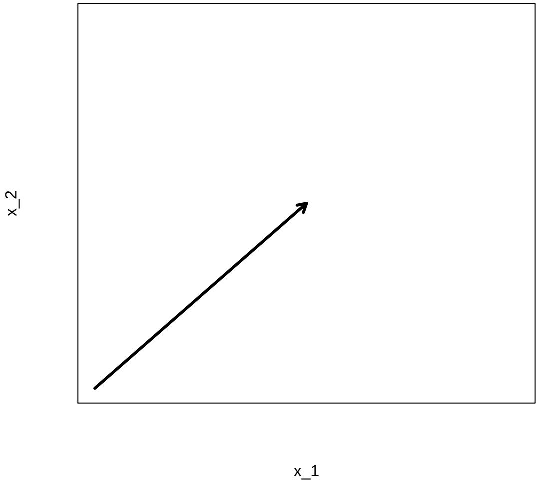
&= 7
\end{aligned}$$

Vector Length



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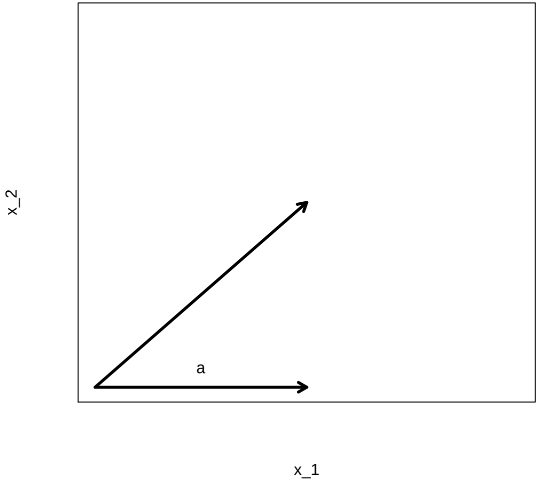
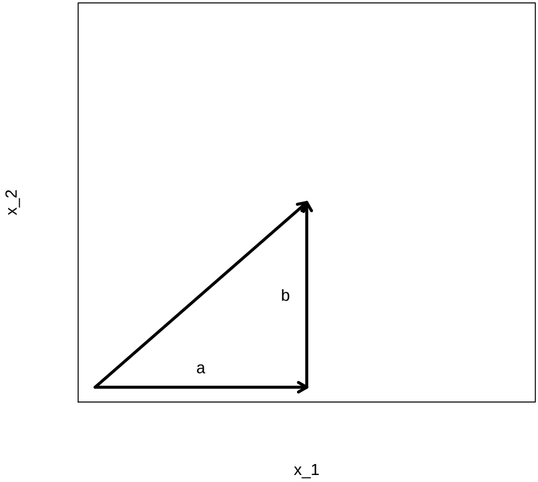


- Pythogorean Theorem: Side with length *a*

Vector Length



- Pythogorean Theorem: Side with length a
- Side with length b and right triangle

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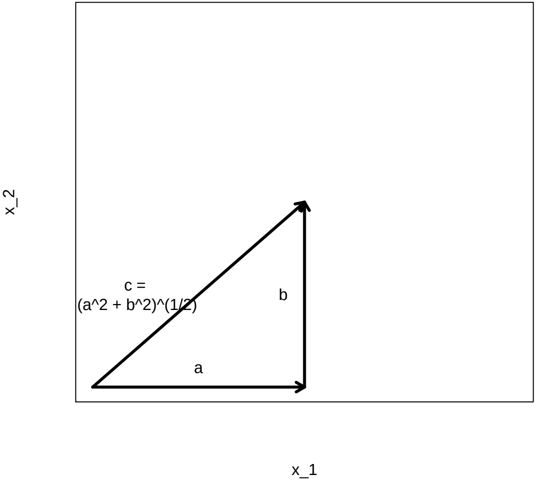

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- $c = \sqrt{a^2 + b^2}$

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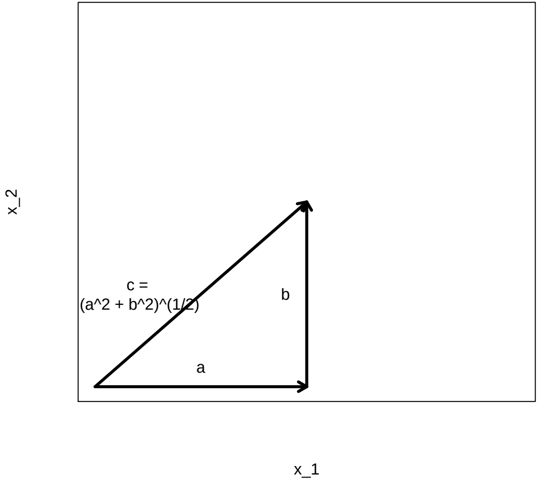

- Pythogorean Theorem: Side with length a
- Side with length b and right triangle
- $c = \sqrt{a^2 + b^2}$
- This is generally true

Vector (Euclidean) Length

Definition

Suppose $\boldsymbol{v} \in \Re^J$. *Then, we will define its length as*

$$\begin{aligned} ||\boldsymbol{v}|| &= (\boldsymbol{v} \cdot \boldsymbol{v})^{1/2} \\ &= (v_1^2 + v_2^2 + v_3^2 + \ldots + v_J^2)^{1/2} \end{aligned}$$

Measures of Dissimilarity

Initial guess$\rightsquigarrow$ Distance metrics
Properties of a metric: (distance function) $d(\cdot, \cdot)$. Consider arbitrary documents $\boldsymbol{X}_i$, $\boldsymbol{X}_j$, $\boldsymbol{X}_k$

$\rightsquigarrow$

Measures of Dissimilarity

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3) $d(\boldsymbol{X}_i, \boldsymbol{X}_j) = d(\boldsymbol{X}_j, \boldsymbol{X}_i)$
4) $d(\boldsymbol{X}_i, \boldsymbol{X}_k) \leq d(\boldsymbol{X}_i, \boldsymbol{X}_j) + d(\boldsymbol{X}_j, \boldsymbol{X}_k)$

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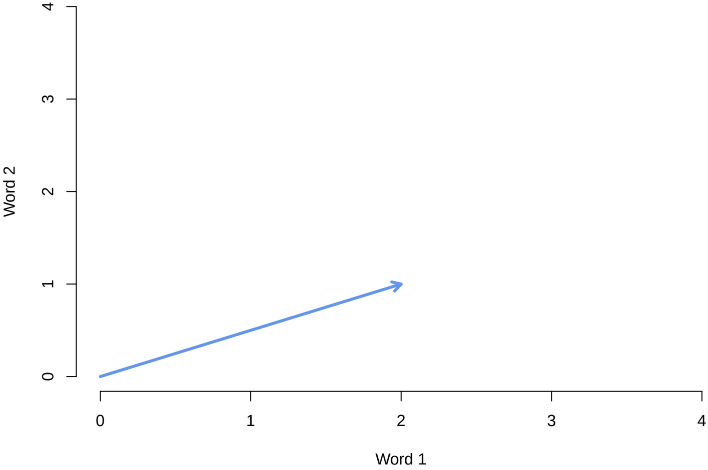
Explore distance functions to compare documents$\rightsquigarrow$Do we want additional assumptions/properties?

Measuring the Distance Between Documents

Euclidean Distance



Measuring the Distance Between Documents

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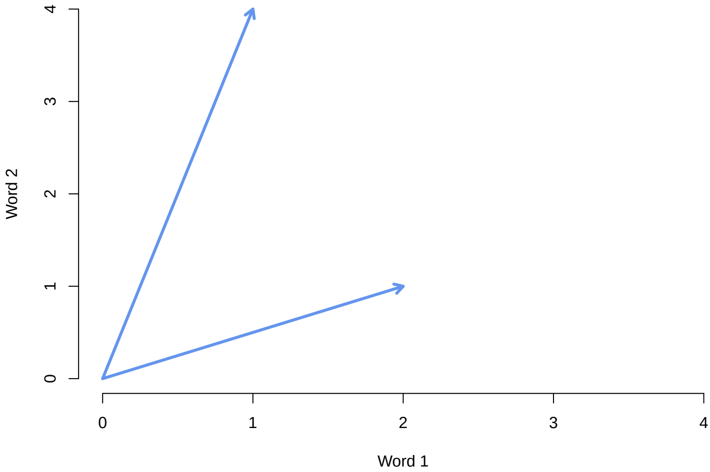

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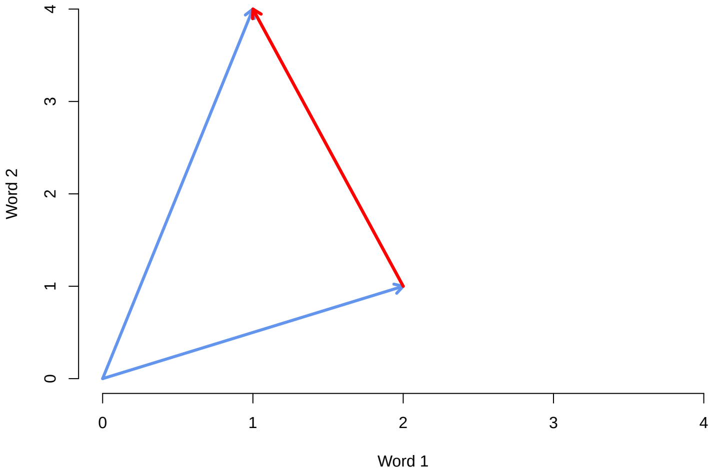

Measuring the Distance Between Documents

Definition

The Euclidean distance between documents $\boldsymbol{X}_i$ *and* $\boldsymbol{X}_j$ *as*

$$||\boldsymbol{X}_i - \boldsymbol{X}_j|| \quad = \quad \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

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$$||\boldsymbol{X}_i - \boldsymbol{X}_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

Suppose $\boldsymbol{X}_i = (1, 4)$ and $\boldsymbol{X}_j = (2, 1)$. The distance between the documents is:

$$\begin{aligned} ||(1,4) - (2,1)|| &= \sqrt{(1-2)^2 + (4-1)^2} \\ &= \sqrt{10} \end{aligned}$$

Measuring the Distance Between Documents

Many distance metrics

Measuring the Distance Between Documents

Many distance metrics Consider the Minkowski family

Measuring the Distance Between Documents

Many distance metrics Consider the Minkowski family

Definition

The Minkowski Distance between documents $\boldsymbol{X}_i$ *and* $\boldsymbol{X}_j$ *for value p is*

$$d_p(\mathbf{X}_i, \mathbf{X}_j) \quad = \quad \left(\sum_{m=1}^{J} |x_{im} - x_{jm}|^p \right)^{1/p}$$

Members of the Minkowski Family

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Manhattan metric

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Manhattan metric

$$d_1(\mathbf{X}_i, \mathbf{X}_j) = \sum_{m=1}^{J} |x_{im} - x_{jm}|$$

Members of the Minkowski Family

Manhattan metric

$$
\begin{aligned}
d_1(\mathbf{X}_i, \mathbf{X}_j) &= \sum_{m=1}^{J} |x_{im} - x_{jm}| \\
d_1((1,4),(2,1)) &= |1| + |3| = 4
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Minkowski (p) metric

$$
\begin{aligned}
d_p(\mathbf{X}_i, \mathbf{X}_j) &= \left(\sum_{m=1}^{J} |x_{im} - x_{jm}|^p \right)^{1/p} \\
d_p((1,4),(2,1)) &= (|1-2|^p + |4-1|^p)^{1/p}
\end{aligned}
$$

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Comparing the Metrics

Suppose $\boldsymbol{X}_i = (10, 4, 3)$, $\boldsymbol{X}_j = (0, 4, 3)$, and $\boldsymbol{X}_k = (0, 0, 0)$

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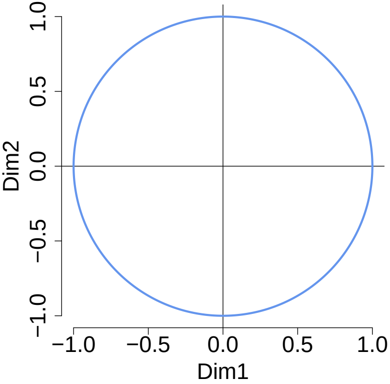
More generally: $\boldsymbol{\Sigma}$ could be symmetric and positive-definite
What does $\boldsymbol{\Sigma}$ do?

Some Intuition: The Unit Circle



$$\boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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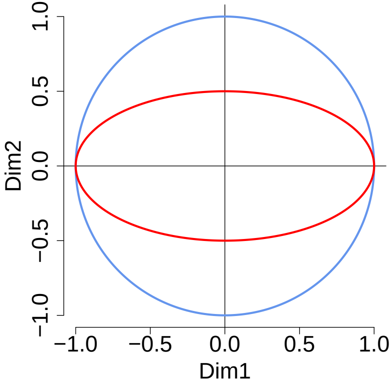

$$\boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix}$$

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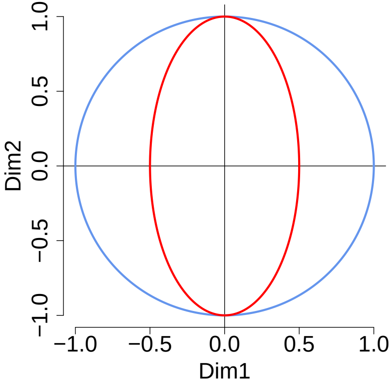

$$\boldsymbol{\Sigma} = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix}$$

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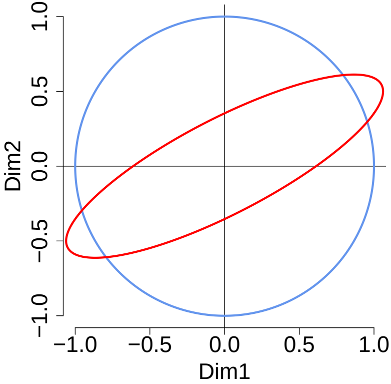

$$\boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0.3 \\ 0.3 & 0.5 \end{pmatrix}$$

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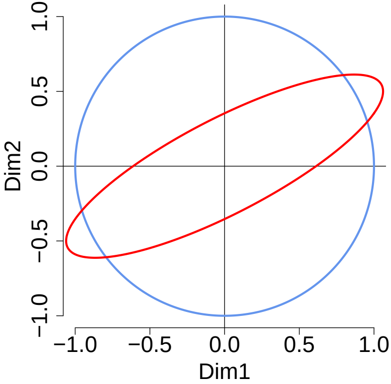

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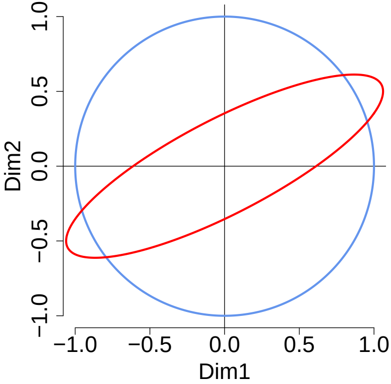

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Measuring Distance with Mahalanobis

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Then distance is Euclidean

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Special Case 2: Diagonal Matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & 0 & \ldots & 0 \\ 0 & \sigma_2^2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_J^2 \end{pmatrix}$$

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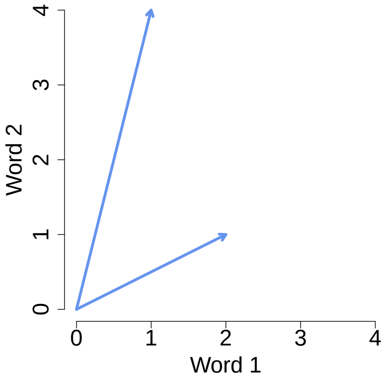
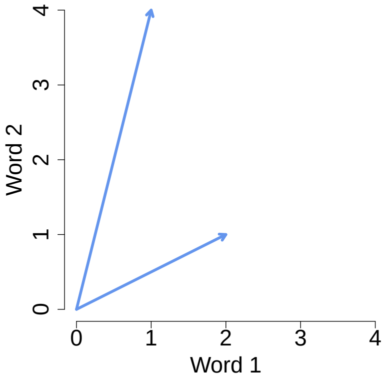
How should additional words be treated?

Measuring Similarity



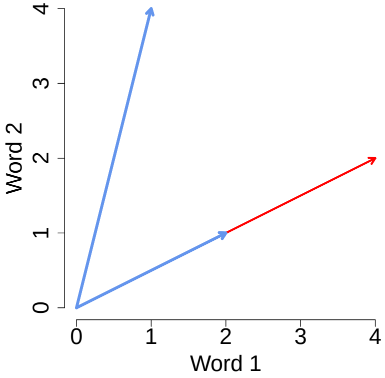
Measure 1: Inner product

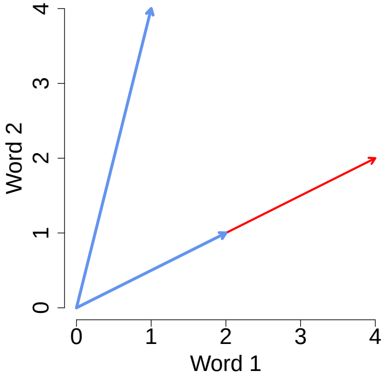
Measuring Similarity



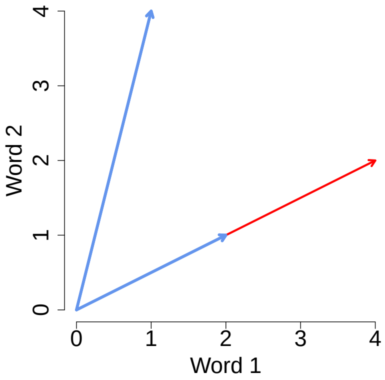
Measure 1: Inner product

$$(2,1)^{'} \cdot (1,4) \quad = \quad 6$$


Word 2
Word 1
0
1
2
3
4



Problem(?): length dependent



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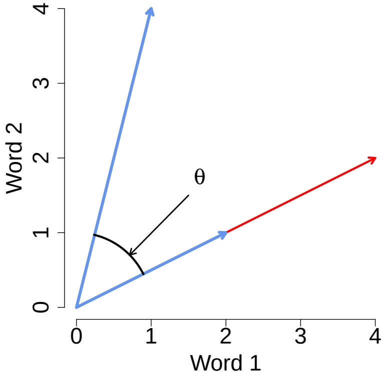
$$(4,2)^{'}(1,4) \quad = \quad 12$$



Problem(?): length dependent

$$\begin{aligned}(4,2)^{'}(1,4) &= 12 \\ a \cdot b &= ||a|| \times ||b|| \times \cos\theta\end{aligned}$$

Cosine Similarity

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(0.89, 0.45)^{'}(0.24, 0.97) &= 0.65
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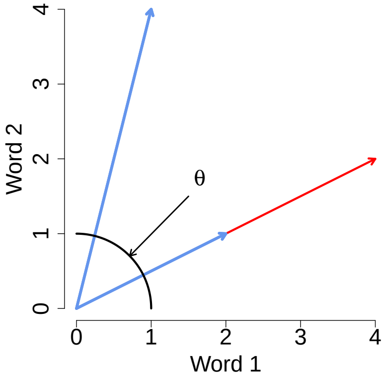

$\cos\theta$: removes document length from similarity measure

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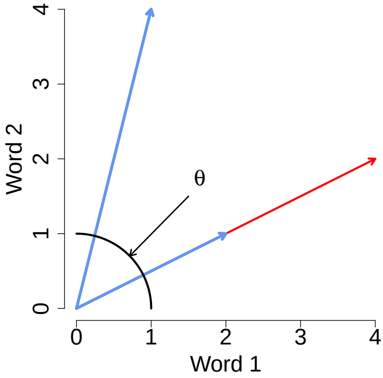

$\cos\theta$: removes document length from similarity measure
Projects texts to unit length representation$\rightsquigarrow$ onto sphere

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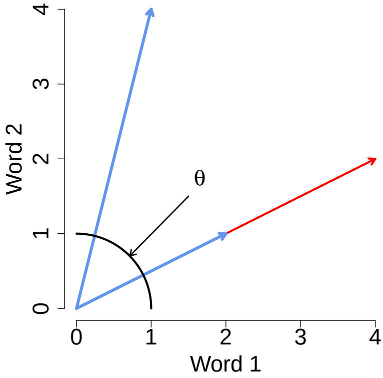

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Von Mises-Fisher Distribution

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- Useful for clustering, hierarchies of topics

Kernel Similarity

Definition

Suppose we have documents $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$. Define the Gaussian kernel as

$$k(\boldsymbol{X}_i, \boldsymbol{X}_j) = \exp\left(-\frac{||\boldsymbol{X}_i - \boldsymbol{X}_j||^2}{\sigma^2}\right)$$

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As $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ become more dissimilar, then $k(\boldsymbol{X}_i, \boldsymbol{X}_j) = 0$

Kernel Similarity

Definition

Suppose we have documents $\boldsymbol{X}_i$ *and* $\boldsymbol{X}_j$. *Define the Gaussian kernel as*

$$k(\boldsymbol{X}_i, \boldsymbol{X}_j) \quad = \quad \exp\left(-\frac{||\boldsymbol{X}_i - \boldsymbol{X}_j||^2}{\sigma^2}\right)$$

- Kernel of the Gaussian distribution
- $\sigma^2 =$ determines sensitivity of the kernel
- If $\boldsymbol{X}_i = \boldsymbol{X}_j$ then $k(\boldsymbol{X}_i, \boldsymbol{X}_j) = 1$
- As $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ become more dissimilar, then $k(\boldsymbol{X}_i, \boldsymbol{X}_j) = 0$
- Result$\leadsto$ often justify setting some kernel weights to zero

The Kernel Trick

Suppose all of our documents $\boldsymbol{X}_i \in \Re^J$

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- $\rightsquigarrow$ Kernels provide methods for capture wide array of transformations.
- Kernel Trick$\rightsquigarrow$ calculate inner products on untransformed data (Gaussian Kernel), implicitly use wide array of ϕ's.

Weighting Words

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How to generate weights?

- Assumptions about separating words
- Use training set to identify separating words (Monroe, Ideology measurement)

Weighting Words: TF-IDF Weighting

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Why log ?

- Maximum at $n_j = 1$
- Decreases at rate $\frac{1}{n_j} \Rightarrow$ diminishing "penalty" for more common use
- Other functional forms are fine, embed assumptions about penalization of common use

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$$\mathbf{X}_{i,\text{idf}} \equiv \underbrace{\mathbf{X}_i}_{\text{tf}} \times \mathbf{idf} \quad = \quad (X_{i1} \times \text{idf}_1, X_{i2} \times \text{idf}_2, \ldots, X_{iJ} \times \text{idf}_J)$$

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$$\begin{aligned}
\mathbf{X}_{i,\text{idf}} \cdot \mathbf{X}_{j,\text{idf}} &= (\mathbf{X}_i \times \textbf{idf})'(\mathbf{X}_j \times \textbf{idf}) \\
&= (\text{idf}_1^2 \times X_{i1} \times X_{j1}) + (\text{idf}_2^2 \times X_{i2} \times X_{j2}) + \\
&\quad \ldots + (\text{idf}_J^2 \times X_{iJ} \times X_{jJ})
\end{aligned}$$

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$$\boldsymbol{\Sigma} = \begin{pmatrix} \text{idf}_1^2 & 0 & 0 & \ldots & 0 \\ 0 & \text{idf}_2^2 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \text{idf}_J^2 \end{pmatrix}$$

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If we use tf-idf for our documents, then

$$\begin{aligned} d_2(\boldsymbol{X}_i, \boldsymbol{X}_j) &= \sqrt{\sum_{m=1}^{J} (x_{im,\text{idf}} - x_{jm,\text{idf}})^2} \\ &= \sqrt{(\boldsymbol{X}_i - \boldsymbol{X}_j)' \boldsymbol{\Sigma} (\boldsymbol{X}_i - \boldsymbol{X}_j)} \end{aligned}$$

Final Product

Applying some measure of distance, similarity (if symmetric) yields:

$$\mathbf{D} = \begin{pmatrix} 0 & d(1,2) & d(1,3) & \ldots & d(1,N) \\ d(2,1) & 0 & d(2,3) & \ldots & d(2,N) \\ d(3,1) & d(3,2) & 0 & \ldots & d(3,N) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d(N,1) & d(N,2) & d(N,3) & \ldots & 0 \end{pmatrix}$$

Lower Triangle contains unique information $N(N-1)/2$

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- Today: Text representation and similarity calculation
- Tuesday: Projecting to low dimensional space

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How do we preserve word order and semantic language?
After stemming, stopping, bag of wording:

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are identical.
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Similarity and Dissimilarity of Many Things

Throughout the course we'll measure similarity between documents
We'll also (implicitly) study similarity of probability distributions
Develop a measure of distribution dissimilarity

Similarity of Probability Distributions

Definition

Suppose P is a continuous random variable with density $p : \Re \rightarrow \Re$ and Q is a continuous random variable with density $q : \Re \rightarrow q$.
We can define the KL-Divergence between P and Q as

$$KL(P||Q) \quad = \quad \int_{-\infty}^{\infty} p(x) \log \frac{p(x)}{q(x)} dx$$

Assessing Similarity of Other Things

KL-divergence measures dissimilarity between two distributions.

Consider a function. $f(x) = -x^2$.

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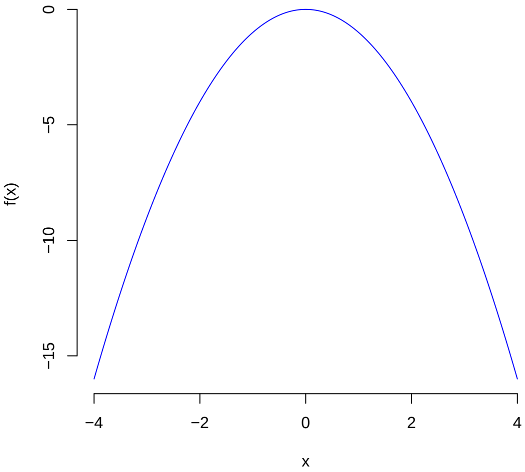
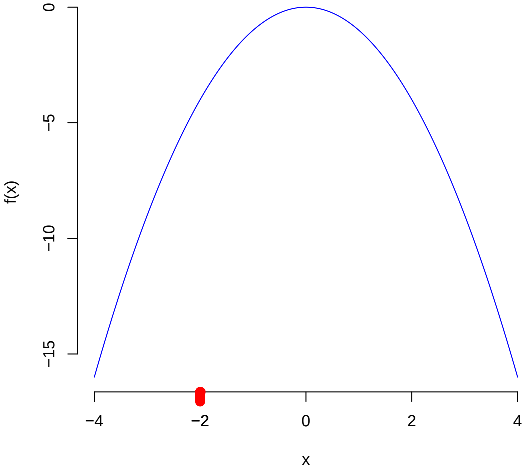
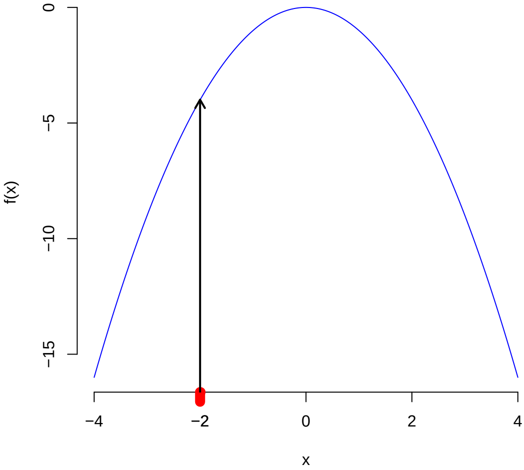


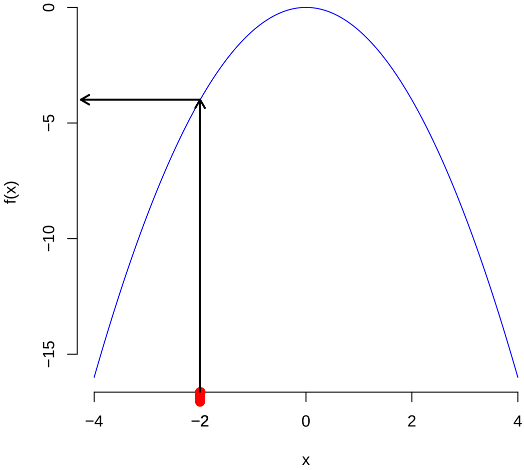
Take some input (-2 here)



Then obtain the value of $f(-2)$



Then obtain the value of $f(-2) = -4$



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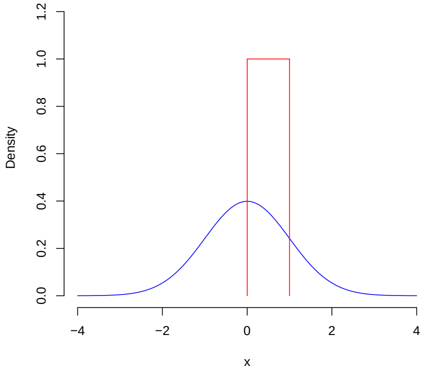
For example, we could set q = Uniform(0,1) and p = Normal(0, 1)



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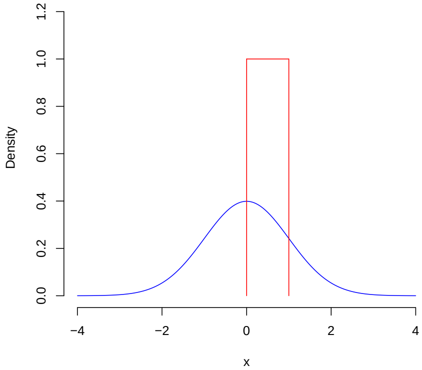
KL(Uniform(0,1)||Normal(0,1)) = 1.09



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Then make this approximation the best possible–minimize the KL-divergence.

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Approximate a Normal(0,1) with symmetric Uniform distribution, Uniform(-b, b).

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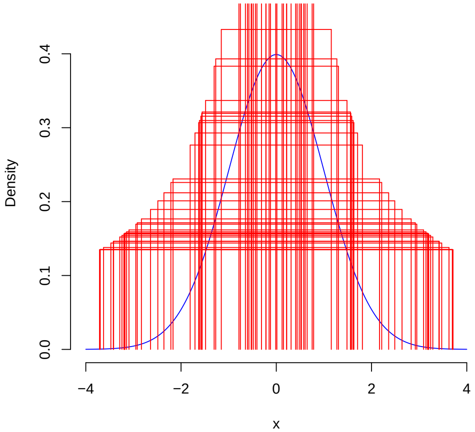

Answer:

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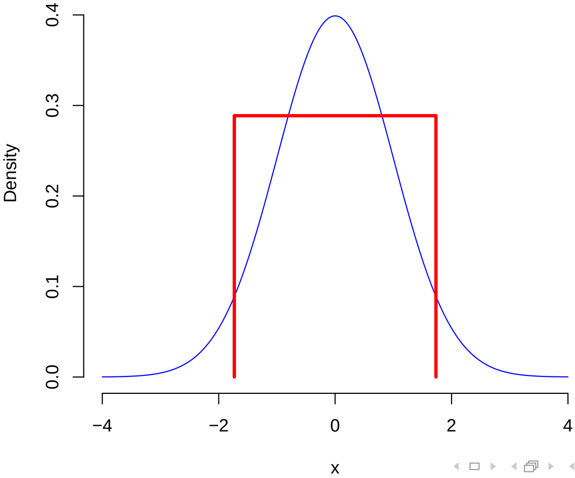

1) Documents in vector space $\rightsquigarrow$ geometry of texts
2) Many methods to measure similarity and dissimilarity