Text as Data

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Discovery

Search for new ways to organize text

- Complement, Not Replace, Organizations of Text
- There is No Ground Truth Conceptualization
- Once you have a conceptualization it is yours

Clustering: partition of documents

- Discover categories
- Assign documents to categories

Fully Automated Clustering

1) Notion of distance
2) Definition of "good" clustering
3) Optimization method

K-Means$\rightsquigarrow$ Objective Function

N documents $\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iJ})$ (normalized)

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   Hard Assignment

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Assume squared euclidean distance

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    - $\boldsymbol{\theta}_1 =$ Average across documents

K-Means $\rightsquigarrow$ Optimization

Coordinate descent

K-Means $\rightsquigarrow$ Optimization

Coordinate descent $\rightsquigarrow$ iterate between labels and centers.

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$$\text{Change} = f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta}^t) - f(\boldsymbol{X}, \boldsymbol{T}^{t-1}, \boldsymbol{\Theta}^{t-1})$$

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$$\tau_{im}^t = \begin{cases} 1 \text{ if } m = \arg\min_k \sum_{j=1}^J (x_{ij} - \theta_{kj}^t)^2 \\ 0 \text{ otherwise }, \end{cases} \quad .$$

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In words: Assign each document $\boldsymbol{x}_i$ to the closest center $\boldsymbol{\theta}_m^t$

K-Means$\rightsquigarrow$ Optimization

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3) Choose $\Theta^t \rightsquigarrow$ Focus on the center for cluster k

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3) Choose Θ^t $\rightsquigarrow$ Focus on the center for cluster k

$$f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k = \sum_{i=1}^{N} \tau_{ik}^t \left(\sum_{j=1}^{J} (x_{ij} - \theta_{jk})^2 \right)$$

K-Means$\rightsquigarrow$ Optimization

3) Choose $\Theta^t \rightsquigarrow$ Focus on the center for cluster k

$$
\begin{aligned}
f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k &= \sum_{i=1}^{N} \tau_{ik}^t \left(\sum_{j=1}^{J} (x_{ij} - \theta_{jk})^2 \right) \\
\frac{\partial f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k}{\partial \theta_{kj}} &= -2 \sum_{i=1}^{N} \tau_{ij}^t \left(x_{ij} - \theta_{jk} \right)
\end{aligned}
$$

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0 &= -2 \sum_{i=1}^{N} \tau_{ij}^t \left(x_{ij} - \theta_{jk}^*\right)
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\frac{\sum_{i=1}^{N} \tau_{ik}^t x_{ij}}{\sum_{i=1}^{N} \tau_{ik}^t} &= \theta_{jk}^*
\end{aligned}
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K-Means⇝ Optimization

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_i}{\sum_{i=1}^{N} \tau_{ik}}$$

K-Means $\rightsquigarrow$ Optimization

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_i}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_i$$

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In words: $\boldsymbol{\theta}^{t+1}$ is the average of the documents assigned to k.

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- Do until converged:

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  - For each document, find closest center$\rightsquigarrow$ $\boldsymbol{\tau}_i^t$
  - For each center, take average of assigned documents$\rightsquigarrow$ $\boldsymbol{\theta}_k^t$
  - Update change $f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta}^t) - f(\boldsymbol{X}, \boldsymbol{T}^{t-1}, \boldsymbol{\Theta}^{t-1})$

Visual Example

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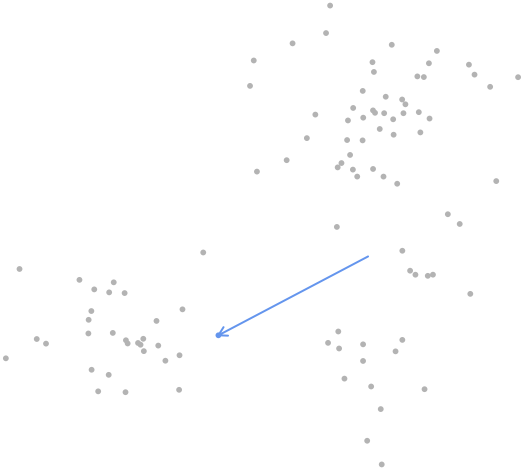

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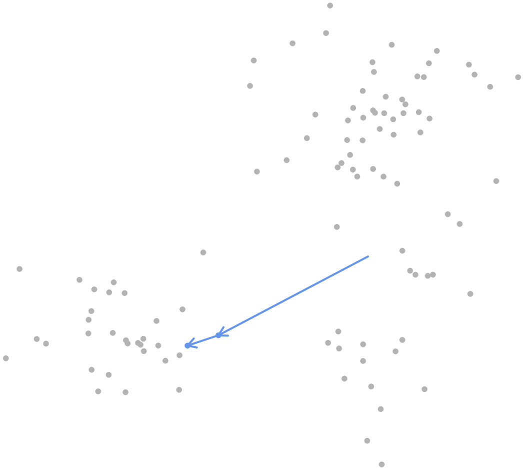

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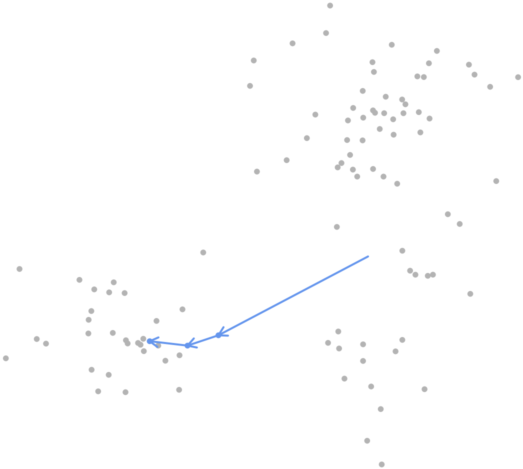

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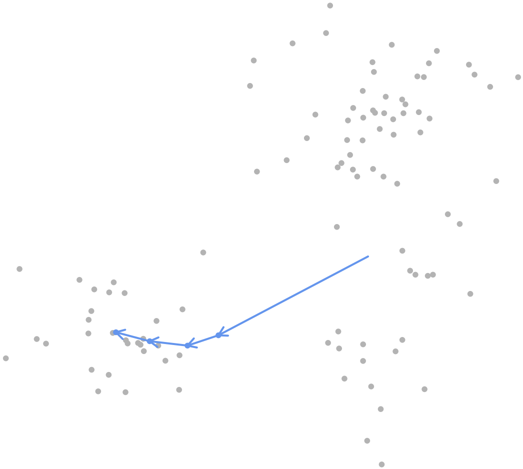

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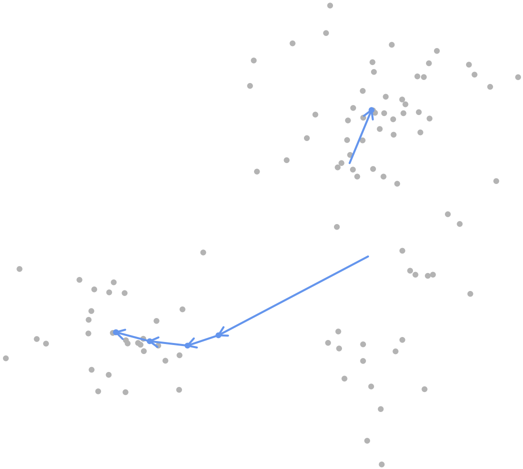

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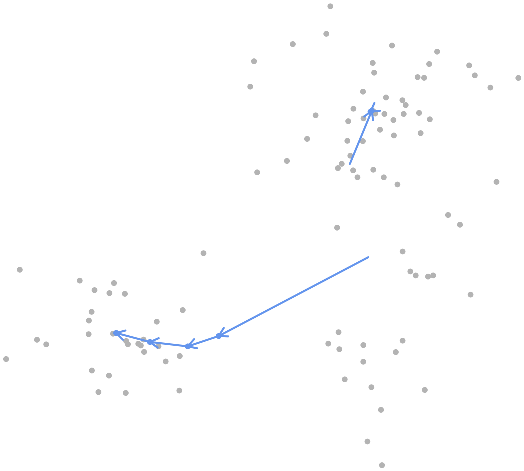

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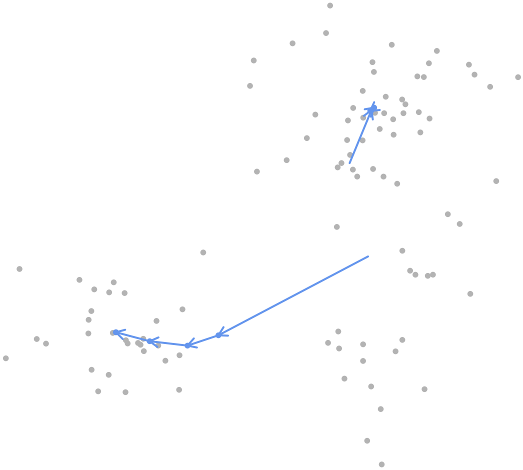

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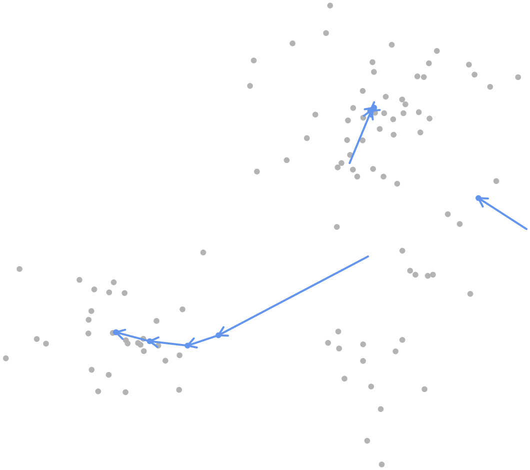

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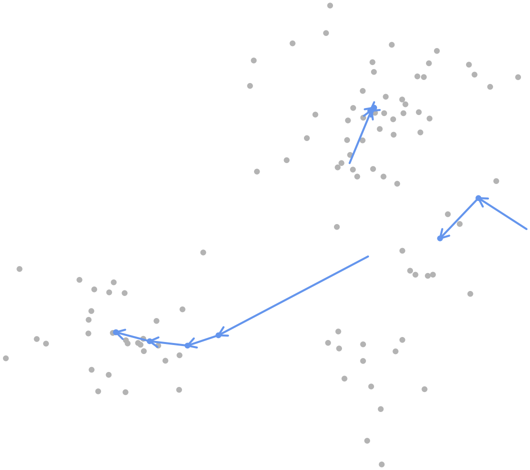

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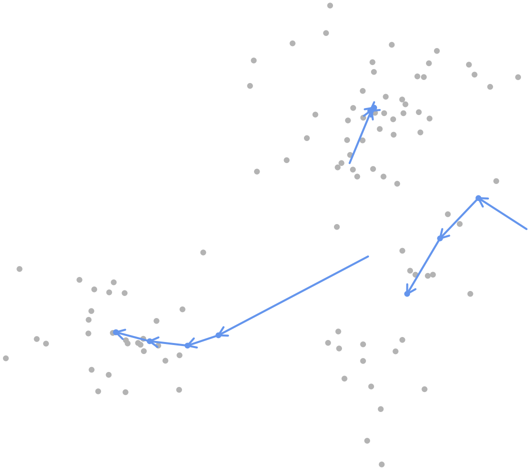

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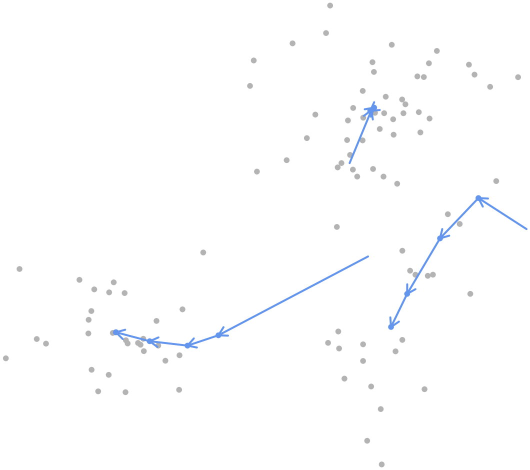

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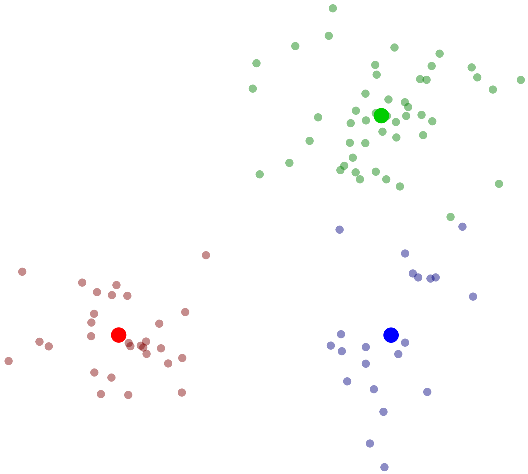

An Example: Jeff Flake

`To the R Code!`

Interpreting Cluster Components

Unsupervised methods

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Unsupervised methods $\rightsquigarrow$ low startup costs, high post-model costs

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- Apply clustering methods, we have groups of documents

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- How to interpret the groups?

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- How to interpret the groups?
- Two (broad) methods:

Interpreting Cluster Components

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`back to the R code!`

How Do We Choose K?

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- No one statistic captures how you want to use your data

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Mixture of Unigram Models (Mixture of Multinomials)

Mixture models $\rightsquigarrow$ wide range of applications

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- Draw a cluster label

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In words:

- Draw a cluster label
- Given distribution, draw realization

Mixture of Unigram Models (Mixture of Multinomials)

A mixture of unigram-language models

$$
\begin{aligned}
\boldsymbol{\pi} &\sim \text{Dirichlet}(\mathbf{1}) \\
\boldsymbol{\theta} &\sim \text{Dirichlet}(\mathbf{1}) \\
\boldsymbol{\tau}_i|\boldsymbol{\pi} &\sim \text{Multinomial}(1, \boldsymbol{\pi}) \\
\boldsymbol{x}_i|\tau_{ik} = 1, \boldsymbol{\theta}_k &\sim \text{Multinomial}(N_i, \boldsymbol{\theta}_k)
\end{aligned}
$$

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This implies the following posterior distribution:

$$p(\boldsymbol{T}, \boldsymbol{\Theta}, \boldsymbol{\pi} | \boldsymbol{X})$$

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&\propto \prod_{i=1}^{N} \prod_{k=1}^{K} \left[\pi_k \prod_{j=1}^{J} \theta_{jk}^{x_{ik}} \right]^{\tau_{ik}}
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Obtain MAP estimates $\rightsquigarrow$ EM Algorithm

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Obtain $\boldsymbol{\Theta}^{t+1}$, $\boldsymbol{\pi}^{t+1}$

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Our update steps will be strikingly similar to the K-Means algorithm

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&= \frac{\pi_k^t \prod_{j=1}^{J} (\theta_{jk}^t)^{x_{ij}}}{\sum_{m=1}^{K} \left(\pi_m^t \prod_{j=1}^{J} (\theta_{jm}^t)^{x_{ij}} \right)}
\end{aligned}
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Define:

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$$

Define:

$$
r_{ik}^t \equiv \frac{\pi_k^t \prod_{j=1}^{J}(\theta_{jk}^t)^{x_{ij}}}{\sum_{m=1}^{K}\left(\pi_m^t \prod_{j=1}^{J}(\theta_{jm}^t)^{x_{ij}}\right)}
$$

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\end{aligned}
$$

Define: Avoid underflow

$$
r_{ik}^t = \left[1 + \sum_{k' \neq k} \frac{\pi_{k'} \prod_{j=1}^{J}(\theta_{jk'}^t)^{x_{ij}}}{\pi_k \prod_{j=1}^{J}(\theta_{jk}^t)^{x_{ij}}}\right]^{-1}
$$

Mixture of Unigram Models (Mixture of Multinomials)

3) M-Step:

Mixture of Unigram Models (Mixture of Multinomials)

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$$\text{E}[\log \text{Complete data}|\boldsymbol{\theta}, \boldsymbol{\pi}] = \sum_{i=1}^{N}\sum_{k=1}^{K} E[\tau_{ik}] \log \left(\pi_k \prod_{j=1}^{J} \theta_{jk}^{x_{ik}} \right)$$

Mixture of Unigram Models (Mixture of Multinomials)

3) M-Step:

$$
\begin{aligned}
\text{E}[\log \text{Complete data}|\boldsymbol{\theta}, \boldsymbol{\pi}] &= \sum_{i=1}^{N} \sum_{k=1}^{K} E[\tau_{ik}] \log \left(\pi_k \prod_{j=1}^{J} \theta_{jk}^{x_{ik}} \right) \\
&= \sum_{i=1}^{N} \sum_{k=1}^{K} r_{ik}^{t} \log \pi_k + \sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{j=1}^{J} r_{ik}^{t} x_{ij} \log \theta_{jk}
\end{aligned}
$$

Mixture of Unigram Models (Mixture of Multinomials)

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\end{aligned}
$$

Example: Jeff Flake Again!

`To the R Code!`

Fully Automated Clustering

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- Notion of similarity and "good" partition $\rightsquigarrow$ clustering

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Appendix: Why EM Works

Goal:

$$\text{argmax}_{\boldsymbol{\theta}} p(\boldsymbol{X}|\theta) \quad = \quad \sum_{\boldsymbol{T}} p(\boldsymbol{X}, \boldsymbol{T}|\theta)$$

Define:

$$\begin{aligned}
\mathcal{L}(q, \boldsymbol{\theta}) &= \sum_{\boldsymbol{T}} q(\boldsymbol{T}) \log \left[\frac{p(\boldsymbol{X}, \boldsymbol{T}|\theta)}{q(\boldsymbol{T})} \right] \\
K(q||p) &= -\sum_{\boldsymbol{T}} q(\boldsymbol{T}) \log \left[\frac{p(\boldsymbol{T}|\boldsymbol{X}, \theta)}{q(\boldsymbol{T})} \right]
\end{aligned}$$

Then:

$$\log p(\boldsymbol{X}|\theta) \quad = \quad \mathcal{L}(q, \boldsymbol{\theta}) + K(q||p)$$

Appendix: Why EM Works

$$
\begin{aligned}
\log p(\boldsymbol{X}|\boldsymbol{\theta}) &= \mathcal{L}(q, \boldsymbol{\theta}) + K(q||p) \\
&= \sum_{\boldsymbol{T}} q(\boldsymbol{T}) \log \left[\frac{p(\boldsymbol{X}, \boldsymbol{T}|\boldsymbol{\theta})}{q(\boldsymbol{T})}\right] - \sum_{\boldsymbol{T}} q(\boldsymbol{T}) \log \left[\frac{p(\boldsymbol{T}|\boldsymbol{X}, \boldsymbol{\theta})}{q(\boldsymbol{T})}\right] \\
&= \sum_{\boldsymbol{T}} q(\boldsymbol{T}) \log(p(\boldsymbol{X}|\boldsymbol{\theta})) + \sum_{\boldsymbol{T}} q(\boldsymbol{T}) \log(p(\boldsymbol{T}|\boldsymbol{X}, \boldsymbol{\theta})) \\
&\quad - \sum_{\boldsymbol{T}} q(\boldsymbol{T}) \log q(\boldsymbol{T}) - \sum_{\boldsymbol{T}} q(\boldsymbol{T}) \log p(\boldsymbol{T}|\boldsymbol{X}, \boldsymbol{\theta}) + \sum_{\boldsymbol{T}} q(\boldsymbol{T}) \log q(\boldsymbol{T})
\end{aligned}
$$

Collect terms that cancel and recognize $\sum_{\boldsymbol{T}} q(\boldsymbol{T}) = 1$ and we see equivalence

Appendix: Why EM Works

$K(q||p) \geq 0$ with $K(q||p) = 0$ only if $q = p$. So, $\mathcal{L}(q, \boldsymbol{\theta})$ is a lower-bound on the log-likelihood.
E-step

$$\log p(\boldsymbol{X}|\boldsymbol{\theta}) - K(q||p) \quad = \quad \mathcal{L}(q, \boldsymbol{\theta})$$

$\mathcal{L}(q, \boldsymbol{\theta}) \rightsquigarrow$ biggest when $K(q||p) = 0$, so set

$$q(\boldsymbol{T}) \quad = \quad p(\boldsymbol{T}|\boldsymbol{X}, \boldsymbol{\theta})$$

M-step:
Given the new value of q, maximize parameters (expectation of the log complete data likelihood)
Change in log-likelihood will be greater$\rightsquigarrow$ because new maximum induces non-zero KL-divergence. Changes in log-likelihood are greater than changes in lower bound.