The Distribution of the Sum of Uniform Random Variables

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***Abstract*—In this paper, we will study the problem of the sum of uniform random variables.**

Cause central limit theorem, the sum of i.i.d random variables should obey normal distribution.

Simulations are provided to corroborate the proposed studies.

***Index Terms*—CLT,Random variables**

I. Problem description

In this article, we try to find out and verify the probability density function of a set of s by simulation with MATLAB software and Central Limit Theorem.

$$s = \sum_{i=1}^{100} x_i \tag{1}$$

where x_i is independent random variable with uniform distribution and its range is between $(-2, 2)$.

In probability theory, the central limit theorem establishes that, in some situations, when independent random variables are added, their properly normalized sum tends toward a normal distribution even if the original variables themselves are not normally distributed. [1]

In this problem, we can consider that there are 100 set of independent uniform variables are added, and because of the central limit theorem, it should obeys normal distribution.

II. Simulation Process and Results

MATLAB code is below here:

```
N=100;
nTimes=10^4;
for t=1:1:nTimes
    x=4*rand(N,1)-2;
    s=sum(x);
    nSum(t)=s;
end
ksdensity(nSum);
```

The code record a set of s with 10^4 elements and plot its distribution density function.

Cause x_i obeys uniform distribution, where $\mu = 0, \sigma = \frac{4}{3}$ when $s = \sum_{i=1}^{100} x_i$. By using central limit theorem, we know that the sum of s obeys $N\ (0, \frac{160000}{9})$. Meanwhile, we can plot the probability density function of $N\ (0, \frac{160000}{9})$.

The density function of normal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{2}$$

In this problem, its function is

$$f(x) = \frac{\sqrt{3}}{20\sqrt{2\pi}} exp\left(-\frac{3x^2}{800}\right) \tag{3}$$

III. Findings

By observing Fig.1, we have successfully test and verify that the sum of independent formal random variables obey normal distribution.

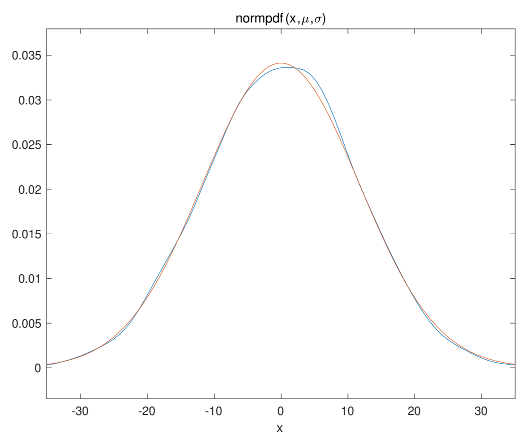

Fig. 1. Normal Distribution and read distribution.

References

[1] Wikipedia, “Central limit theorem”