Weak Signal Detection Based on Differential Oscillator

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Abstract—**The fault characteristic signals of rolling bearings are in low frequency band, and the useful signals are often buried in heavy noise and difficult to be detected. So a new weak signal detection method based on differential oscillator is proposed. Differential oscillator is a kind of signal detection method based on two-dimensional linear differential equations. When measured signals contain the frequency component- f_d, the phase diagram converges to the polar ring, otherwise it converges to the poles. The early system fault can be visually detected according to the change of the phase diagram. Based on the introduction of the setup principle of oscillator system parameter, this paper recommends and first analyzes the influence of noise on the oscillator phase diagram emphatically. The simulation signal and the incipient outer ring of the deep grove ball bearing are precise detected by this method.**

Keywords- ***differential oscillator; Weak signal detection;early fault; visual detection***

I. INTRODUCTION

Weak signal contains two meanings: one is that the signal itself is very weak; the other is that the signal itself is very weak compared to the strong environmental noise. In the engineering measurement, the main research content of weak signal processing technology is how to detect the weak signal in the strong noise environment. The traditional weak signal detection methods[1-3] are based on noise suppression. If the signal frequency is overlaped or similar with noise band, the useful signal must be damaged when suppressing noise. With the continuous development of nonlinear theory, the very low signal-noise-ratio signals have been detected successfully by adopting chaos oscillator method[4-6]and stochastic resonance method[7-9]. At present, there is little study on differential oscillator[10,11]. So this paper is based on the differential oscillator for detecting weak signals.

In this paper, some recommendations concerning how to select the differential oscillator parameters are briefly introduced, then it recommends and first analyzes the influence of noise on the oscillator phase diagram. Its conclusion shows that the noise makes the system track become rougher, roughness is determined by the disturbance variance. This method is successfully applied to the simulation signal and the outer ring fault of the deep grove ball bearing.

II. THE BASIC PRINCIPLE OF DIFFERENCE OSCILLATOR

Differential oscillator is sensitive to weak signals. When the detected signal contains the frequency component- f_d, thus the phase trajectory of difference oscillator converges to the polar ring . Thereby, we achieve the detection of weak periodic signal.

A. Differential oscillator detection model

Differential oscillator is a kind of signal detection method based on linear differential equations, the mathematical model is as following:

$$\begin{cases} x_{k+1} = ax_k + by_k \\ y_{k+1} = cx_k + dy_k + p\cos(2k\pi f_e + 2k\pi f_d / f_s) \cdot T(k) \end{cases} \tag{1}$$

Where p is the magnification, f_e is the system excitation frequency, f_d is to be detected frequency, f_s is the sampling frequency of the input signal, $T(k)$is the input signal sequence and a, b c, d are the differential oscillator system parameters.

We can solve these equations by means of eigenvector method. In order to discover the resonance property of this system, let

$$\begin{aligned} & f(k) = b \cdot p \cdot \cos(2\pi f_e k + \frac{2\pi f_d k}{f_s}) \\ & \alpha = -(a+d) \\ & \beta = ad - bc \end{aligned} \tag{2}$$

Then Eq.(1) can be transformed to the following form:

$$x_{k+2} + \alpha \cdot x_{k+1} + \beta \cdot x_k = f(k) \cdot T(k) \tag{3}$$

As can be seen, the differential oscillator detector is a second order system. The natural frequency of the system can be estimated by the following formula.

$$\omega_0 = \arccos(-\frac{\alpha(1+\beta)}{4\beta}) \tag{4}$$

$$\beta > 0 \,; \left|\frac{\alpha(1+\beta)}{4\beta}\right| < 1 \tag{5}$$

Use "(1)", when the test signal is not input, fix the value of α, change the value of β observe changes in the phase diagram, which we choose the most suitable α, β to construct the differential oscillator.

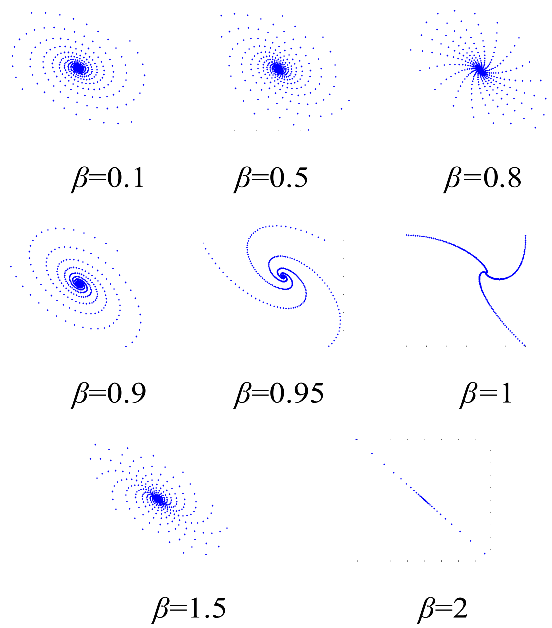


Figure 1. the phase diagram present different states with different β

Similarly, fix β and change α will get the same effect, the figure shows that when β belongs to [0.95 1], the differential equation's solution converges the fastest. Therefore, when α, β belong to [0.95 1], it is the best to construct the differential oscillator.

B. The impact of noise on the phase diagram

For the study of the impact of noise on the differential oscillator, we can transform the Eq. (2) into the form of differential equations, the results are as follows:

$$\ddot{x}(t) + A\cdot\dot{x}(t) + B\cdot x(t) = f(t)\cdot T(t) \tag{6}$$

Where $A = \alpha + 2$, $B = 1 + \alpha + \beta$

select Δx denotes the small perturbation to solution *x(t) by* Gaussian white noise *n(t)* of zero mean and σ^2 variance.

$$\Delta\ddot{x} + A\cdot\Delta\dot{x} + B\cdot\Delta x = f(t)\cdot n(t) \tag{7}$$

Make The Eq. (6) written in form of a vector differential equation:

$$\Delta\dot{X} = Z\Delta X(t) + N(t) \tag{8}$$

where

$$\Delta X = \begin{bmatrix}\Delta x\\ \Delta\dot{x}\end{bmatrix}, Z = \begin{bmatrix}0 & 1\\ -B & -A\end{bmatrix}, N(t) = \begin{bmatrix}0\\ f(t)\cdot n(t)\end{bmatrix}$$

According to the existence and uniqueness theorem, the equation has the unique solution which meets an initial condition that can be expressed as following:

$$\Delta X(t) = \phi(t,t_0)\Delta X_0 + \int_{t_0}^{t}\phi(t,\mu)N(\mu)d\mu \quad . \tag{9}$$

Where $\Phi(t,t_0)$ is a state transfer matrix of the system.

$$\frac{d\phi(t,t_0)}{dt} = Z\phi(t,t_0) \tag{10}$$

The Eq. (8) consists of two parts: the first part $\Phi(t,t_0)$ is the zero-input solution of the equation. It has nothing to do with the input signal and decay to zero rapidly, just reflecting the characteristics of the system itself . The Eq.(8) can be transformed as the following when consider steady-state statistical properties only.

$$\Delta X(t) = \int_{t_0}^{t}\phi(t,\mu)N(\mu)d\mu \tag{11}$$

$$E(\Delta X(t)) = \int_{t_0}^{t}\phi(t,\mu)E(N(\mu))d\mu = 0 \tag{12}$$

Its mean value can't explain the impact of noise on the output of the differential oscillator. So the variance need to be solved .

$$R(\Delta X(t), \Delta X(t)) = R_{\Delta x}(t,t) \tag{13}$$

$$= \int_{-\infty}^{t}\int_{-\infty}^{t}\phi(t,\mu)R\big(N(\mu),N(\nu)\big)\phi^*(t,\mu)d\mu d\nu$$

$$= \int_{-\infty}^{t}\int_{-\infty}^{t}\phi(t,\mu)E\left[\begin{pmatrix}0\\ f(u)n(\mu)\end{pmatrix}(0, f(u)n(\nu))\right]\phi^*(t,\mu)d\mu d\nu$$

$$= \int_{-\infty}^{t}\phi(t,\mu)L\phi^*(t,\mu)du$$

Where

$$L=\begin{bmatrix}0 & 0\\ 0 & \sigma^2\end{bmatrix}\bullet\frac{\mathrm{bp}}{2}\bullet \mathrm{f(t)}$$

The above analysis shows that the noise makes the system track becomes rough. roughness is determined by the disturbance variance. The noise only affect the local movement stability of the differential resonator. It does not affect the phase diagram convergence and divergence of the differential resonator.

C. The influence of the amplitude on the phase diagram

Given signal $C(k)$, provided the solutions of the corresponding differential equation as $d(k)$, Use "(3)":

$$d_{k+2}+\alpha\cdot d_{k+1}+\beta\cdot d_k=f(k)\cdot C(k) \tag{14}$$

Given another signal C(k), let $S(k)=k\cdot C(k)$

Assume that the input signal $S(k)$,the solution of equation for $y(k)$ .Use "(3)":

$$y_{k+2}+\alpha\cdot y_{k+1}+\beta\cdot y_k=f(k)\cdot S(k) \tag{15}$$

$$y_{k+2}+\alpha\cdot y_{k+1}+\beta\cdot y_k=f(k)\cdot k\cdot C(k) \tag{16}$$

Then make formula(14) with the times k:

$$k\cdot d_{k+2}+\alpha(k\cdot d_{k+1})+\beta(k\cdot d_k)=f(k)\cdot k\cdot C(k) \tag{17}$$

Compare the formula (13) and (14)，get:

$$y_k=k\cdot d_k \tag{18}$$

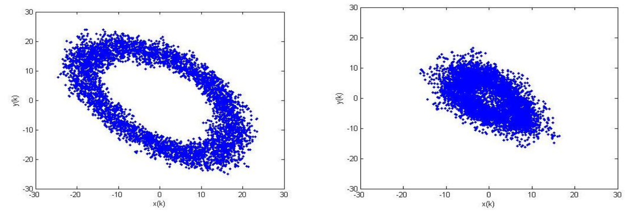
When the input signal amplitude increases, then differential oscillator equation's solution also correspondingly increases, the differential oscillator phase diagram area will enlarge. So differential oscillator phase diagram area is proportional to signal amplitude.

III. DIGITAL SIMULATION

The following simulation experiments is used to verify the above conclusion.First we construct a simulation function

$$s(k)=A\cdot\sin(2k\pi\cdot 17)+0.05\cdot\cos(2k\pi\cdot 50)+D\cdot \mathrm{noise} \tag{19}$$

where α=0.97,β=0.97; excitation frequency f_e =0.3319; magnification p=1, the system's initial value $x(1)=6$,$y(1)=6$; the signal amplitude A=0.1,noise is random gaussian white noise, D is the noise strength, we select D=0.5,when The detected frequency f_d=17Hz from the Eq.(13) we know the measured signal contains the f_d component, the phase diagram converges to the polar ring, when f_d=20Hz, the measured signal does not contain the f_d component, The phase diagram converges to the poles, As shown in Figure.2



(a) (b)

Figure 2. The differential resonator phase diagram (a) with the testing frequency f_d=17Hz (b) f_d=20Hz

In order to verify the effect of noise on the differential resonator, increasing the noise intensity D from the beginning 0.7, using step (step 0.1) while other parameters remain unchanged. We get a series of Phase diagrams As shown in Figure.3.

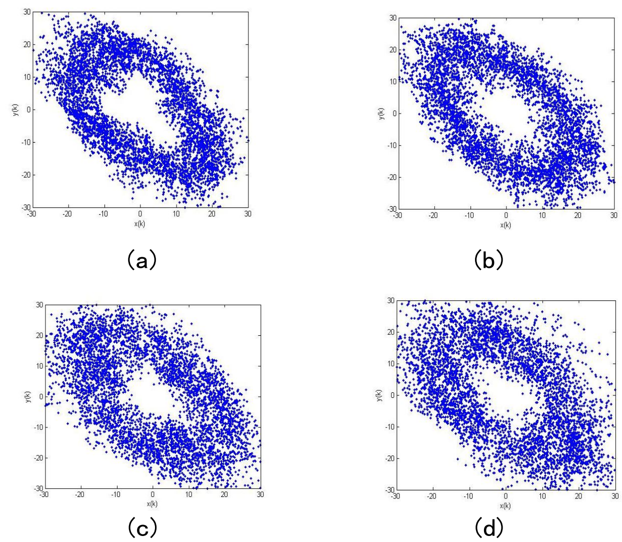
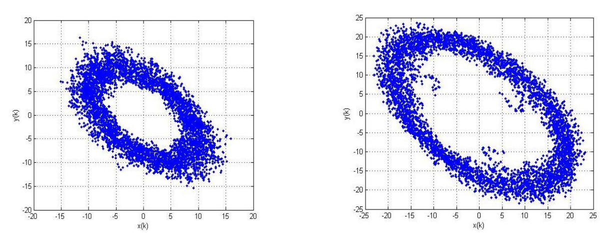
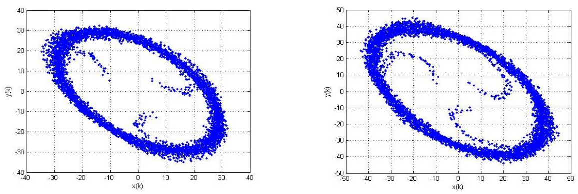


Figure 3. The differential oscillator phase diagram (a)with intensity of noise D=0.7 (d) D=0.8 (e) D=0.9 (f) D=1.0

Change the values of A, Fig.4shows the portraits with the values of signal amplitude A. When signal amplitude every increases 0.5, phase diagram area will increase by 100.



(a) (b)

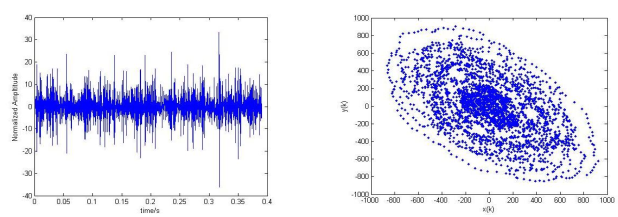


(c) (d)

Figure 4. The differential oscillator phase diagram (a)with signal amplitude A=0.7 (b) A=0.8 (c) A=0.9 (d) A=1.0

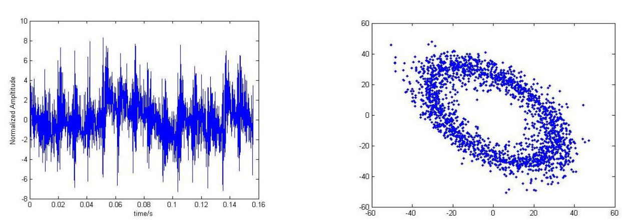
IV. APPLICATION

This experiment uses bearing vibration measuring instrument BVT-5 series produced by a research center, through the acceleration sensors to collect signal. The sampling frequency is 5120Hz. Bearing type is 6311. The bearing outer ring fault is processed through SG dual color metal lettering machine under the shaft rotating speed 1800r/min. Bearing parameters is d=20.638mm,D=87.5mm, $\alpha=0^0$，fr=31Hz. According to the structure parameters and the current bearing speed, we get bearing fault frequency f_0=94.75Hz. Two rolling bearings are tested in the test, among them one is normal and another is the bearing outer ring fault. The difference resonator parameters we select is α=0.97,β=0.999 ； Excitation frequency fe =0.3306, magnification P=2,the waveforms of the vibration signals of the two bearings are shown in Figure.4 (a)and(c) respectively. When testing frequency f_d =94.75， the phase plane of the difference resonator after introducing the two vibration signals are shown in Figure.4(b) and （d） respectively. From the picture, it is possible to distinguish the faulty bearing from the normal one and ascertain the type of the fault.



(a) Time domain wave form Of the bearing vibration signal

(b) The difference resonator phase diagram



(c) Time domain wave form Of the bearing vibration signal

(d) The difference resonator phase diagram

Figure 5. Use the differential resonator to detect the incipient outer ring of the deep grove ball bearing

V. CONCLUSION

In order to get the desired results, we should select the appropriate differential oscillator parameters to construct a differential oscillator model. Differential oscillator has a good anti-noise performance and the theoretical analysis and simulation experiments have verified this conclusion. The differential oscillator is applied to low speed and heavy rotating machinery of deep groove ball bearing fault, also obtained very good effect. It will provide a new method under the strong noise background for weak signals detection.

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