The Distribution of the Sum of Independent and Identically Distributed Random Variables

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***Abstract*—In this paper, the problem of the sum of independent and identically distributed random variables are studied, which should obey normal distribution by using Central Limit Theorem. Simulations are provided to corroborate the proposed studies.**

***Index Terms*—Central Limit Theorem, Random Variables, Independent and Identically Distribution, Simulation**

I. Problem Description

A. Central Limit Theorem [1]

In probability theory, the central limit theorem establishes that, in some situations, when independent random variables are added, their properly normalized sum tends toward a normal distribution even if the original variables themselves are not normally distributed. The theorem is a key concept in probability theory because it implies that probabilistic and statistical methods that works for normal distributions can be applicable to many problems involving other types of distribution.

Mathematically, if $X_1, X_2, ..., X_n$ is a random sample of size n taken from a population with mean μ and finite variance σ^2 and if $\bar{X}$ is the sample mean, the limiting form of the distribution of $Z = \frac{\bar{X}_n - \mu}{\frac{\sigma}{\sqrt{n}}}$ as $n \to \infty$, is the standard normal distribution.

B. Our Work

We produce 100 sets of independent and identically uniform variables $x_1, x_2, ..., x_{100}$ from a population of uniform distribution variables whose range is between $(-2, 2)$. Then we calculate $\sum_{i=1}^{100} x_i$ as s. We do this procedure for 10^4 times and we write a MATLAB code to simulate it.Then we test and find that s obeys normal distribution by using Central Limit Theorem.

II. Simulation Process and Results

We produce $x_1, x_2, ..., x_{100}$ and adding them up then do it for 10^4 times, and drawing its histogram.

Cause x_i obeys uniform distribution, where $\mu = 0, \sigma = \frac{4}{3}$ when $s = \sum_{i=1}^{100} x_i$. By using central limit theorem, we know that the sum of s obeys $N\ (0, \frac{160000}{9})$. Meanwhile, we can plot the probability density function of $N\ (0, \frac{160000}{9})$.

The density function of normal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{1}$$

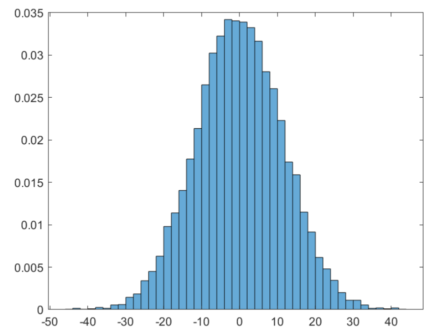

Fig. 1. Histogram of s

In this problem, the density function of normal distribution is

$$f(x) = \frac{\sqrt{3}}{20\sqrt{2\pi}} exp\left(-\frac{3x^2}{800}\right) \tag{2}$$

III. Findings

Comparing the distribution of s and a standard normal distribution, we find the distribution of s may obey normal distribution. Then by the calculation of Central Limit Theorem, we verify the sum of independent and identical variables obey distribution.

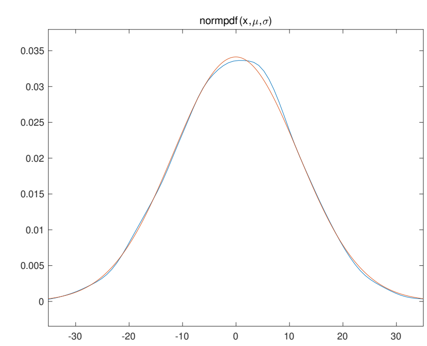

Fig. 2. Normal Distribution and simulated distribution.

References

[1] Montgomery, Douglas C.; Runger, George C. (2014). “Applied Statistics and Probability for Engineers (6th ed.)”. Wiley. p. 241. ISBN 9781118539712.