

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 1$$

$$\int_{-\infty}^{\infty} \underbrace{e^{-x^2/2}}_{\text{even}} dx = \sqrt{2\pi}$$

$$2 \int_0^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}$$

$$\frac{x}{\sqrt{2}} = z$$

$$dx = \sqrt{2} dz$$

$$I = 2 \int_0^{\infty} e^{-z^2} dz = \sqrt{\pi}$$

$$I^2 = \pi \int_0^{\infty} e^{-z^2} dz \cdot \int_0^{\infty} e^{-w^2} dw = 4 \int_0^{\infty} \int_0^{\infty} e^{-(z^2+w^2)} dz dw$$

$$= \left[ \begin{array}{l} w = zs \\ dw = z ds \end{array} \right] = 4 \int_0^{\infty} \left[ \int_0^{\infty} e^{-z^2 - z^2 s^2} z dz \right] ds$$

$$= 4 \int_0^{\infty} \left[ \int_0^{\infty} z e^{-z^2(1+s^2)} dz \right] ds$$

$$= 4 \int_0^{\infty} \left[ -\frac{1}{2(1+s^2)} e^{-z^2(1+s^2)} \right]_0^{\infty} ds$$

$$= 2 \int_0^{\infty} \frac{1}{1+s^2} ds = 2 \left[ \tan^{-1} s \right]_0^{\infty}$$

$$= 2 \cdot \frac{\pi}{2} = \pi$$



