Problem 1 (24.15)

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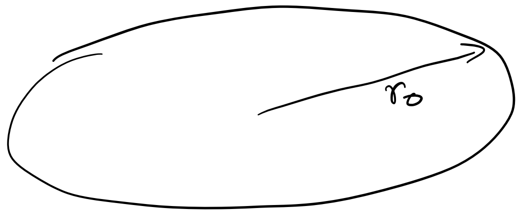
a)




for escape speed, $-\frac{GMm}{r} + \frac{1}{2}mv^2 = 0$ to equilibrium

$$v = \sqrt{\frac{2GM}{r_0}}$$

As of P908, $V_{esc} = 300\,km/s$

$R_0 = 8\ kpc$

$$\frac{v^2 r_0}{2G} = M = \frac{3\times10^2\times10^3\,m/s \times 8\times10^3 \times 206265 \times 1.5\times10^8}{2\times G}$$

$$= 8.36\times10^{10}\ M_\odot$$

Ex 24.3.1 gives $8.8\times10^{10}\ M_\odot$

Pretty close!

b) let $v = 500$

$$M = \frac{v^2 r_0}{2G} = 2.32\times10^{11}$$

the extra mass from DM

c) Since by observation mass exist at $r \gg R_0$,

which is not approximatable through solar surrounding.

Problem 2. (24.18)

a) $\frac{a^3}{P^2}=1$ $(a: AU,\ P: yr)$

$$\Theta \equiv R\frac{d\theta}{dt}$$

of 2.37 read out:

$$P = \frac{2\pi}{\sqrt{G(m_1+m_2)}} R^{\frac{3}{2}}$$

$$\Theta = R\frac{d\theta}{dt}$$

$$\Theta\, dt = R\, d\theta$$

$$\Theta P = R\, 2\pi$$

$$P = \frac{2\pi R}{\Theta}$$

$\therefore$ equate: $\frac{2\pi}{\sqrt{G(m_1+m_2)}} R^{\frac{3}{2}} = \frac{2\pi R}{\Theta(R)}$

$$\sqrt{\frac{R}{G(m_1+m_2)}} = \frac{1}{\Theta}$$

$$\Theta(R) = \sqrt{\frac{G(m_1+m_2)}{R}}$$

b) $A \equiv -\frac{1}{2}\left[\frac{d\Theta}{dR}\Big|_{R_0} - \frac{\Theta_0}{R_0}\right]$

$B \equiv -\frac{1}{2}\left[\frac{d\Theta}{dR}\Big|_{R_0} + \frac{\Theta_0}{R_0}\right]$

$$\frac{d}{dR}\Theta = \sqrt{G(m_1+m_2)}\left(-\frac{1}{2}R^{-\frac{3}{2}}\right)$$

$$= -\frac{1}{2}\frac{\sqrt{G(m_1+m_2)}}{\sqrt{R^3}} = -\frac{1}{2}\sqrt{G(m_1+m_2)}\ R^{-\frac{3}{2}}$$

$$\frac{d}{dR}(\Theta_{R_0}) = -\frac{1}{2}\frac{\sqrt{G(m_1+m_2)}}{R_0^{\frac{3}{2}}}$$

$$\frac{\Theta_{R_0}}{R_0} = \sqrt{\frac{G(M_1+M_2)}{R_0^3}}$$

$$A = -\frac{1}{2} \times \left(-\frac{3}{2}\sqrt{\frac{G(m_1+M_2)}{R_0^3}}\right)$$

$$= \frac{3}{4}\sqrt{\frac{G(m_1+M_2)}{R_0^3}}$$

$$B = -\frac{1}{2} \times \left(-\frac{1}{2}\sqrt{\frac{G(m_1+M_2)}{R_0^3}} + \sqrt{\frac{G(m_1+M_2)}{R_0^3}}\right)$$

$$= -\frac{1}{4}\sqrt{\frac{G(m_1+M_2)}{R_0^3}}$$

Or : $A = \frac{3}{4}\frac{\Theta_0}{R_0}$

$B = -\frac{1}{4}\frac{\Theta_0}{R_0}$

c) $R_0 = 8\ kpc$, $\Theta_0 = 220\ km/s$ $\quad$ ($km\ s^{-1}\ kpc^{-1}$

$$A = \frac{3}{4}\frac{220}{8} = 20.625\ km\ s^{-1} kpc^{-1}$$

$$B = -\frac{1}{4}\frac{220}{8} = -6.875\ km\ s^{-1}\ kpc^{-1}$$

d) No! This assume ① we are at edge of Galaxy

② no DM

Problem 3 (24.21)

use eq 24.5 estimate $\rho_{DM}$ locally in

$kg\,m^{-3}$, $M_\odot pc^{-3}$, $M_\odot AU^{-3}$

$r = R_0 = 8kpc$ $\qquad$ $\rho(r) = \dfrac{V^2}{4\pi G r^2}$

$V = \Theta_0 = 220\ km/s$

1 mm $\quad \dfrac{m}{mm}$

$$\therefore \rho(r) = \frac{\Theta_0^2}{4\pi G R_0^2}$$

$= 9.47\times10^{-22}\ kg/m^3$

$= 9.47e{-22}\ kg \times \dfrac{M_\odot}{kg} \times m^{-3} \times \left(\dfrac{pc}{m}\right)^{-3}$

$= 9.47e{-22} \times \overline{1.989e30}\, M_\odot \times 3.085\times10^{3\times16}\ pc^{-3}$

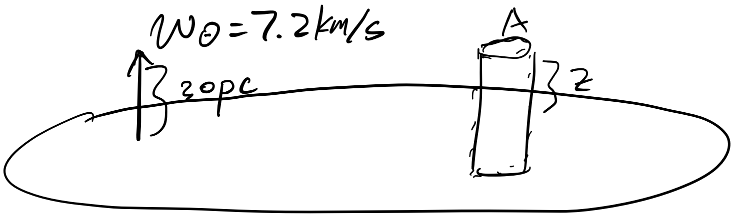
$= 0.1399\ M_\odot/pc$

$= 9.47e{-22} \times \dfrac{1}{1.989e30} M_\odot \times 1.5e11\ AU^{-3}$

$= 1.61\times10^{-18}\ M_\odot/AU^3$

Problem 4 24:27

a)




$$2Ag = -4\pi GM = -4\pi G \rho A 2z$$

$$g = -4\pi G \rho z$$

b) $g = \dfrac{d^2z}{dt^2} = -4\pi G \rho z$

$\therefore \dfrac{d^2z}{dt^2} + 4\pi G \rho z = 0$ , $k = 4\pi G\rho$

c) $w_0^2 = 4\pi G\rho$

$z = A\cos(\sqrt{4\pi G\rho}\, t + \phi_0)$

$w = \dfrac{d}{dt} z = -\sqrt{4\pi G\rho}\, A \sin(\sqrt{4\pi \rho G}\, t + \phi_0)$

d) $P = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{4\pi \rho G}} = 2.15 \times 10^{15}$ s

$= 6.82 \times 10^{8}$ yr

e) take $\phi_0 = 0$

$A\cos(\omega t_0) = 30\,\text{pc}$

$-\omega A \sin(\omega t_0) = 7.2$ km/s

$A \sin(\omega t_0) = -79.95$ pc

$A^2\cos^2(\omega t_0) + A^2\sin^2(\omega t_0) = A^2 = 7292.546$ pc

$A = 85.4$ pc

f) $\frac{2\pi R_0}{\Theta_0} = P_{orbit} = 7 \times 10^5$ s

updown $= \frac{P_{orbit}}{P} = \frac{7}{2.15} = 3.25$ repetitions