6 Summary of Useful Cosmology Equations

- The full Friedmann Equation for FRW universes is

$$\boxed{(\dot{R})^2 - \frac{8\pi}{3}\,G\left[\rho + \frac{U_{\rm rad}}{c^2}\right]R^2 \;=\; -kc^2 + \frac{\Lambda}{3}\,R^2 \quad .} \tag{45}$$

Setting $\rho = \Omega_m\rho_c\,,\ U_{\rm rad}/c^2 = \Omega_{\rm rad}\rho_c$ for $\rho_c = 3H_0^2/(8\pi G)$, it integrates to give the **age function** for the universe:

C & O, pp. 1192–3

$$t \;=\; \frac{1}{H_0}\int_0^a \frac{d\Gamma}{\sqrt{\Omega_m/\Gamma + \Omega_{\rm rad}/\Gamma^2 + \Omega_\Lambda\Gamma^2 + (1-\Omega)}} \quad , \tag{46}$$

where $\Omega = \Omega_m + \Omega_{\rm rad} + \Omega_\Lambda$. Often $\Omega_\Lambda = \Lambda/(3H_0^2)$ is written Ω_v for the vacuum contribution.

- In terms of redshift z, this age function can be written

$$t(z) \;=\; \frac{1}{H_0}\int_z^\infty \frac{dz'}{(1+z')\,E(z')} \quad , \tag{47}$$

where

$$E(z) \;=\; \sqrt{\Omega_m(1+z)^3 + \Omega_{\rm rad}(1+z)^4 + \Omega_\Lambda + (1-\Omega)\,(1+z)^2} \quad . \tag{48}$$

- The **luminosity distance** d_L and the **angular diameter distance** d_A satisfy

$$d_L \;=\; \frac{c\,(1+z)}{H_0\sqrt{|\Omega-1|}}\,S_k(\Theta) \;\equiv\; (1+z)^2\,d_A \quad , \tag{49}$$

for

$$S_k(\Theta) \;=\; \begin{cases} \sin\Theta, & k=1, \\ \Theta, & k=0, \\ \sinh\Theta, & k=-1 \ . \end{cases} \tag{50}$$

and a **development angle**

$$\Theta \;=\; \sqrt{|\Omega-1|}\,\int_0^z \frac{dz'}{E(z')} \quad . \tag{51}$$