2.1 Acoustic Oscillations: the Robust Cosmic Ruler

• An essential beauty of the CMB fluctuations is that they provide a precise ruler that can be used to calibrate the cosmology of the universe. The principal reason for this is that at the recombination epoch, the photons contribute significantly to the total equation of state. Acoustic oscillations therefore propagate roughly at the sound speed c_s for a relativistic gas with adiabatic index $\gamma = 4/3$:

C & O, pp. 1263-7

$$c_s^2 \; = \; \frac{\partial P}{\partial \rho} \; = \; \frac{P}{3\rho} \; = \; \frac{c^2}{3} \quad \Rightarrow \quad c_s \; = \; \frac{c}{\sqrt{3}} \quad . \tag{10}$$

The physical distance on the horizon for self-gravitating density fluctuations (and therefore temperature fluctuations due to virialization) is then

$$D \; = \; c_s \, t_{\rm rec}(z) \quad . \tag{11}$$

This provides a robust ruler on the sky at this epoch – it is in practice a much tighter standard than the diameters of galaxy clusters provide.

• At recombination (atomic coupling), the universe's age is

$$t_{\rm rec}(z) \; = \; \frac{1}{H_0} \int_z^{\infty} \frac{dz'}{(1+z')\, E(z')} \quad , \tag{12}$$

where

$$E(z) \; = \; \sqrt{\Omega_m (1+z)^3 + \Omega_{\rm rad} (1+z)^4 + \Omega_\Lambda + (1-\Omega)\,(1+z)^2} \quad . \tag{13}$$

We can safely neglect the Ω_Λ and $(1-\Omega)$ terms in $E(z)$ at $t_{\rm rec}$. At recombination, the radiation term is only moderately influential, so we neglect it and arrive at the matter-dominated result

$$t_{\rm rec}(z) \; \approx \; \frac{2}{3(1+z)^{3/2}} \, \frac{1}{H_0 \sqrt{\Omega_m}} \quad , \tag{14}$$

which yields the acoustic fluctuation scale

$$D \; \approx \; \frac{2}{3\sqrt{3}} \, \frac{1}{(1+z)^{3/2}} \, \frac{c}{H_0 \sqrt{\Omega_m}} \quad . \tag{15}$$

Using $\Omega_m \sim 0.27$, including both baryonic and dark matter, at $z_{\rm rec} \sim 1550$ this yields $D \sim 55\,{\rm kpc}$. The universe was a much smaller place then!

Now, the angular diameter distance gives a corresponding angular scale

$$\theta \;=\; \frac{D}{d_A} \quad \text{with} \quad d_A \;\equiv\; \frac{c}{H_0(1+z)} \int_0^z \frac{dz'}{E(z')} \tag{16}$$

in the approximation that $\Omega \approx 1$, so that $S_k(\Theta) \to \Theta$. The integral is evaluated using only the Ω_m term, since the low z contributions dominate, yielding approximately $2/\sqrt{\Omega_m}$. Combining with Eq. (15), it follows that

$$\theta \;\approx\; \frac{1}{3\sqrt{3}}\,\frac{1}{\sqrt{1+z}} \;\sim\; 0.4^\circ \tag{17}$$

is the approximate largest angular scale (fundamental) for acoustic fluctuations in the recombination epoch.

- Better angular resolution than COBE affords was required to explore the principal and harmonics of such fluctuations $\Rightarrow$ WMAP and Planck.

2.2 Baryonic Seed Timescale

A principal implication of the COBE results is that the observed $\Delta T/T$ is too low (i.e. $\delta\rho/\rho$ is too low) for structure formation to be seeded by the observed baryonic matter under the action of gravity only. Here we identify this constraint/problem. This adds weight to arguments for the existence of hot or cold dark matter.

- The gravitational collapse timescale is

$$t_g \;\sim\; \sqrt{\frac{3}{8\pi G\rho}} \quad , \quad \rho \;=\; (1+z)^3\rho_0 \tag{18}$$

with $\rho_0 = \Omega_b\rho_c = (3H_0^2)/(8\pi G)\,\Omega_b$. Then

$$t_g(z) \;\sim\; \frac{1}{\sqrt{\Omega_b H_0^2}}\,\frac{1}{(1+z)^{3/2}} \quad . \tag{19}$$

Hence gravitational collapse is probable at redshift z if $t_g(z) < t_H(z) \sim 1/[H(z)]$. For the purposes of an estimate in a matter-dominated cosmology,

$$a(t) \;\propto\; t^{2/3} \quad \Rightarrow \quad H(z) \;=\; \frac{\dot{a}}{a} \;\propto\; \frac{1}{t} \;\propto\; \frac{1}{a^{3/2}} \;\propto\; (1+z)^{3/2} \tag{20}$$

Therefore,

$$H(z) \;\approx\; H_0\,(1+z)^{3/2} \quad\Rightarrow\quad \frac{t_g(z)}{t_H(z)} \sim \frac{1}{\sqrt{\Omega_b}} \quad . \tag{21}$$

It follows that if $\Omega_b \sim 0.04$, as is demanded by primordial nucleosynthesis constraints, $t_g(z)/t_H(z)$ is clearly greater than unity.

• Hence the observed baryonic matter cannot seed collapse at any z in a matter-cominated cosmology. This argument applies to other $a(t)$ choices.

$*$ Accordingly, the COBE $\Delta T/T$ implies the need for the existence of dark matter at at least the $\Omega_m \gtrsim 0.2$ level.

2.3 WMAP: the Era of Precision Cosmology

The launch of WMAP in June, 2001 ushered in a new era for cosmology, when much higher angular resolution relative to COBE emerged. This was immediately obvious from the WMAP skymap of CMB fluctuations, and enabled precision determination of key cosmological density parameters.

$\boxed{\textbf{Plot:}}$ WMAP Skymap

• Detailed models of CMB fluctuations and their evolution use **spherical harmonics** Y_{lm} to describe the departures from uniformity on the sky. The temperature fluctuations can be written via the series expansion

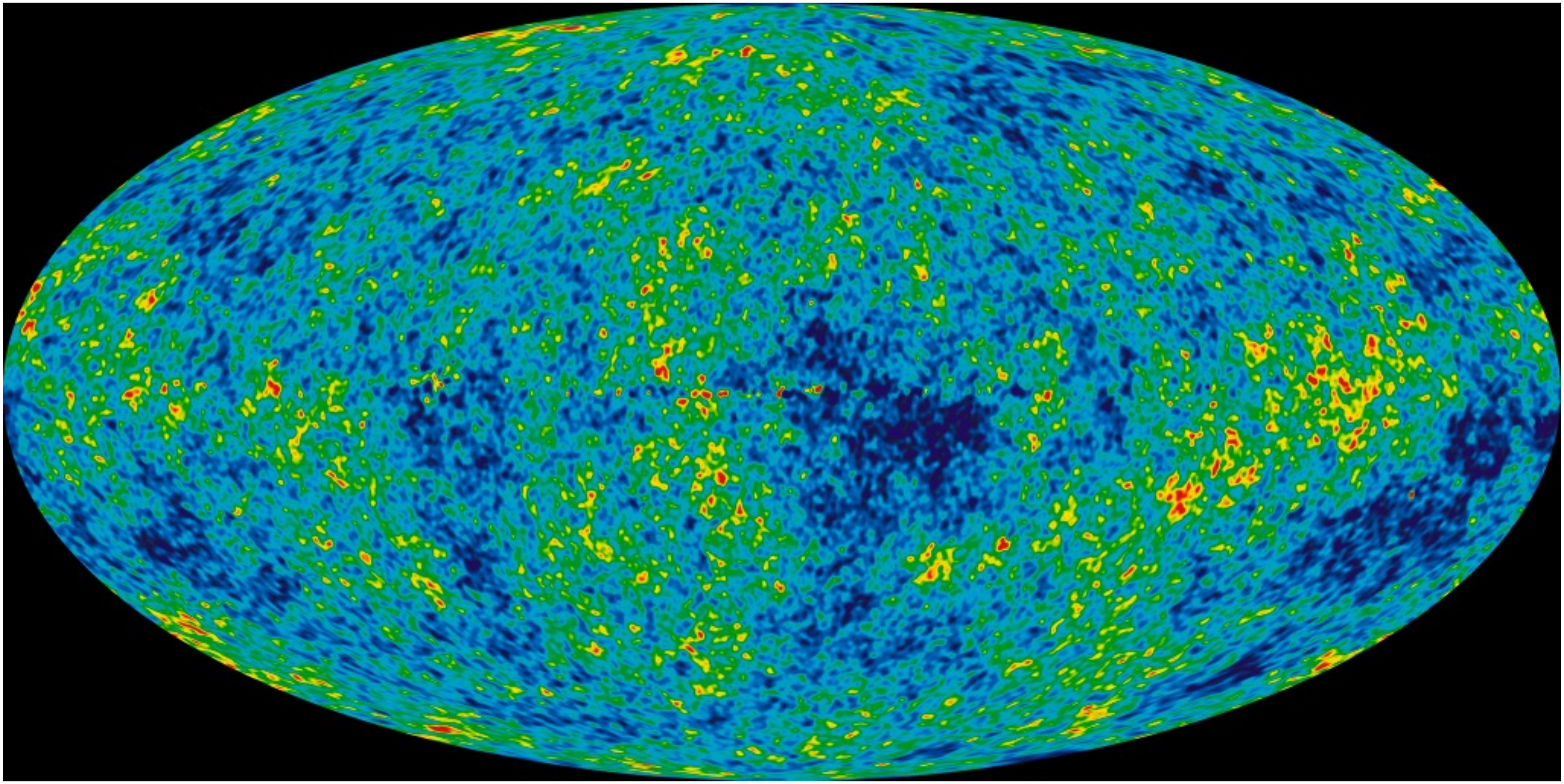
$$\frac{\Delta T}{T}(\theta,\,\phi) \;\equiv\; \frac{T(\theta,\,\phi)-T_0}{T_0} \;=\; \sum_{l=0}^{\infty}\sum_{m=-l}^{m=l} a_{lm} Y_{lm}(\theta,\,\phi) \quad . \tag{22}$$

where the spherical harmonics (of hydrogen atom fame) are given by

$$Y_{lm}(\theta,\,\phi) \;=\; \left[\frac{2l+1}{4\pi}\,\frac{(l-|m|)!}{(l+|m|)!}\right]^{1/2} P_{lm}(\cos\theta)\,e^{im\phi}\begin{cases}(-1)^m\;, & m\geq 0,\\ 1\quad , & m<0,\end{cases} \tag{23}$$

with $P_{lm}(\cos\theta)$ being the associated Legendre polynomials.

WMAP Skymap



- Improved WMAP angular resolution over COBE yielded a change (reduction) in the fundamental angular scale of CMB fluctuations!

• The azimuthal index m can be summed over so that there is a distinct correlation between angle θ and the **harmonic index** l,

$$\theta \sim \frac{\pi}{l} \quad ; \tag{24}$$

high harmonics l correspond to perturbations on small angular scales. The orthogonality relation for the Y_{lm} quickly yields the identity

$$a_{lm} = \int \frac{\Delta T}{T}(\theta,\, \phi)\, Y^*_{lm}\, d\Omega \quad . \tag{25}$$

For random phases ϕ, corresponding to so-called **Gaussian fluctuations** (which are consistent with the random phase assumptions of inflation), the power spectrum about each point in the sky is circularly symmetric and can be described by the coefficient

$$C_l = \frac{1}{2l+1} \sum_m a_{lm} a^*_{lm} = \langle |a_{lm}|^2 \rangle \quad . \tag{26}$$

This is the standard for model depiction and data interpretation.

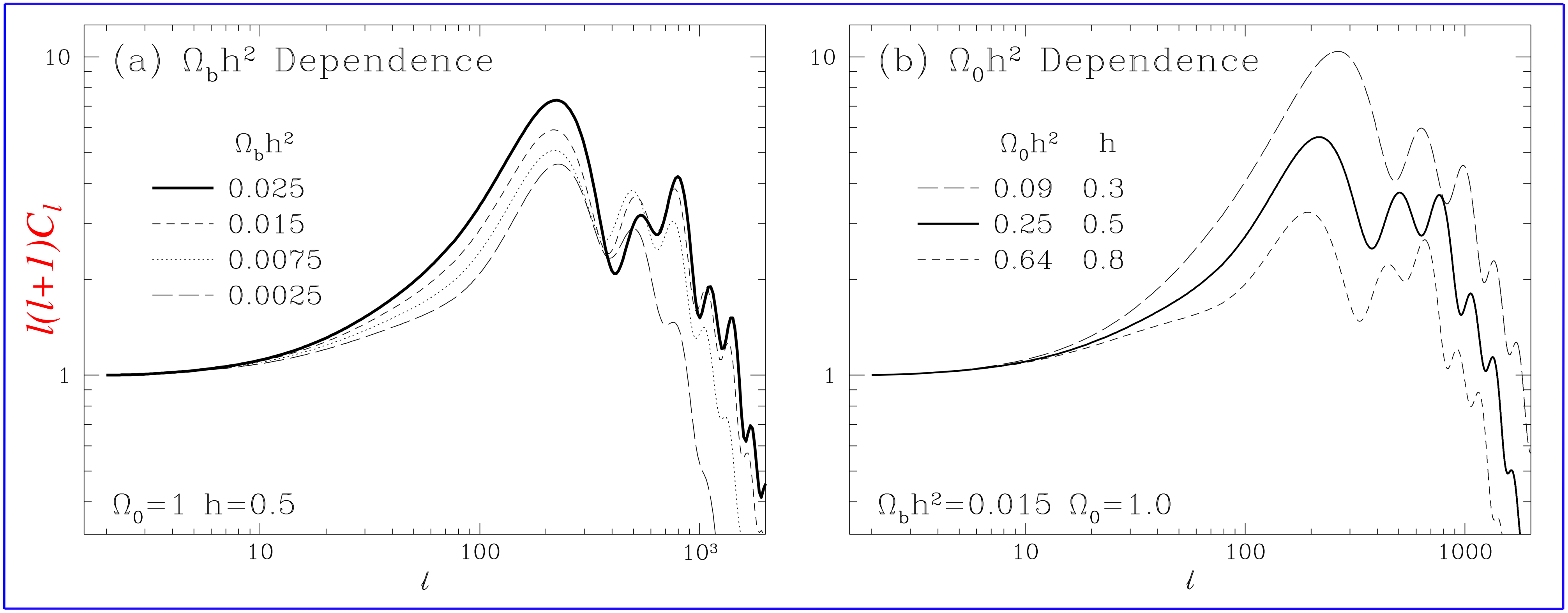
Plot: CMB Power Spectrum for Inflationary ΛCDM Cosmology

N.B. Non-Gaussian features such as abrupt temperature discontinuities, intense hot spots and linear structures are predicted in theories where large scale structures are seeded by topological defects or cosmic strings.

• For a Harrison-Zeldovich spectrum of initial density perturbations $P(k) \propto k^n$ for $n = 1$ (where $k = 2\pi/\lambda$ is the wavenumber), it is found that $C_l \propto [l(l+1)]^{-1}$. This corresponds to equal power on all lengthscales. Hence, the power spectrum is usually represented by plotting $l(l+1)\, C_l$ versus l.

• The detailed shape of the power spectrum depends critically on the cosmological parameters Ω_m, Ω_b, Ω_Λ, etc. For example, more baryonic matter provides greater potential for gravitational clumping, and so enhances fluctuations on smaller angular scales, i.e. at higher harmonic number l. Accordingly, the strength of the third peak is sensitive to Ω_b.

Sensitivity of CMB Power Spectra to Ω_b and H_0



- Variation of power spectrum with Ω_b (*left panel*) and H_0 (right panel) for adiabatic Harrison-Zeldovich models (n=1) of perturbation development. Increasing Ω_b increases asymmetry between odd and even peaks. Decreasing H_0 for fixed Ω_0 renders recombination and equipartition epochs more contemporaneous (i.e., higher T_b).
- From Wu (1996, *Lecture Notes in Physics* 470, 207), [arXiv/astro-ph: 951113]

This sensitivity enables precision cosmology by WMAP, which can probe a range of angular scales $\gtrsim 0.1^\circ$ scales.

Plot: Power Spectrum (Unpolarized) from WMAP

Details of the WMAP results and images can be found at the WMAP Web page at `http://map.gsfc.nasa.gov`. The values of key parameters are well constrained just by the unpolarized power spectrum.

Plot: Cosmological Parameters extracted from WMAP Power Spectrum

The power spectrum drops off dramatically above $l \gtrsim 10^3$. This is due to **Silk damping**, where efficient diffusion of photons out of localized, deeper gravitational potential wells smears out structure in the CMB. This arises because the recombination era is prior to the redshift of last scattering.

• WMAP also measured the polarization content of the CMB to a limited extent. Polarization arises due to Thomson scattering by free electrons. The electric field direction of an E/M wave that is radiated by an electron that is jostled by a linearly-polarized incoming E/M wave is dependent on the outgoing direction of the radiated wave. [*sketch a diagram*]. Backscatterings generate more polarization than forward ones.

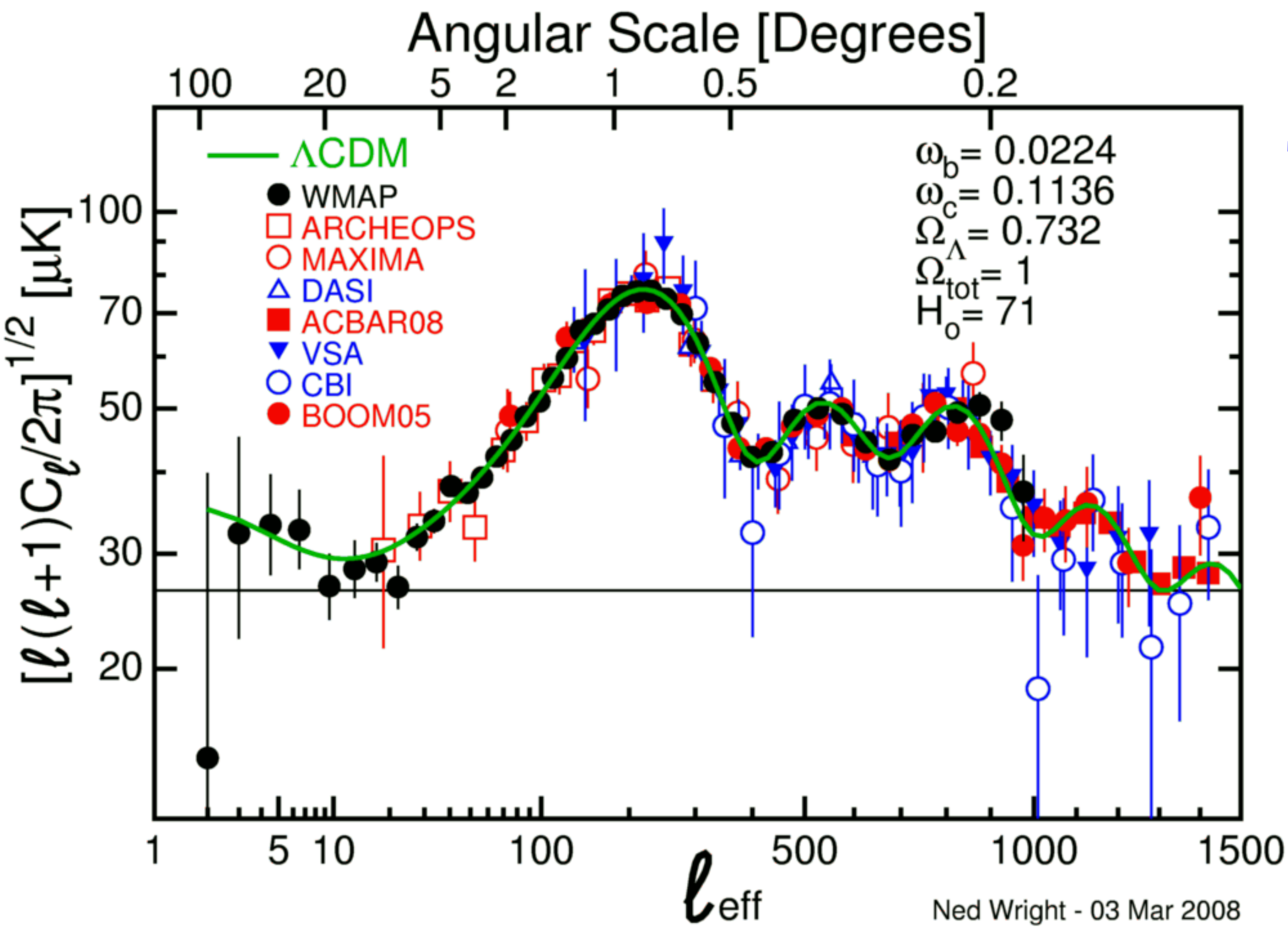
For a truly isotropic radiation field in an electron's rest frame, the net polarization in Thomson scatterings is zero. In the fluctuating densities/thermal motions associated with the recombination era, the photons and electrons are slightly anisotropic. Head-on $e\gamma$ collisions are more frequent than tail-on ones, and so the net polarization is at around the 1% level. *Planck* improved upon WMAP, yielding precision polarization measurements.

Plot: Planck CMB Power Spectra and Polarizations

∗ Anisotropies are greatest for photons transiting under-dense regions, so the polarization signal (EE) is anti-correlated with the intensity power (TT).

∗ Polarization can also be generated by primordial gravitational radiation (e.g. from the inflationary era) with more subtle quadrupolar signatures, so-called B-modes; detected these is the focus of next generation CMB facilities such as CMB S-4 at the South Pole and in Chile [`https://cmb-s4.org`].

CMB Power Spectrum in WMAP Era



TT

- CMB power spectrum from WMAP and other experiments, exhibiting clearly the principal peak at 0.8° and higher "harmonics" that constrain cosmological parameters.
- WMAP data is 5 year release. Plot due to Ned Wright.

WMAP Parameters

WMAP Cosmological Parameters Model: lcdm Data: wmap9			
$10^9\Delta^2_{\mathcal{R}}$	2.41 ± 0.10	H_0	70.0 ± 2.2 km/s/Mpc
$\ell(\ell+1)C_{220}/(2\pi)$	$5746 \pm 35\ \mu\mathrm{K}^2$	$d_A(z_{\rm eq})$	14194 ± 117 Mpc
$d_A(z_*)$	14029 ± 119 Mpc	$D_v(z=0.57)/r_s(z_d)$	13.28 ± 0.31
η	$(6.19 \pm 0.14) \times 10^{-10}$	$k_{\rm eq}$	0.00996 ± 0.00032
$\ell_{\rm eq}$	139.7 ± 3.5	ℓ_*	302.35 ± 0.65
n_b	$(2.542 \pm 0.056) \times 10^{-7}\ \mathrm{cm}^{-3}$	n_s	0.972 ± 0.013
Ω_b	0.0463 ± 0.0024	$\Omega_b h^2$	0.02264 ± 0.00050
Ω_c	0.233 ± 0.023	$\Omega_c h^2$	0.1138 ± 0.0045
Ω_Λ	0.721 ± 0.025	Ω_m	0.279 ± 0.025
$\Omega_m h^2$	0.1364 ± 0.0044	$r_s(z_d)$	152.3 ± 1.3 Mpc
$r_s(z_d)/D_v(z=0.106)$	0.346 ± 0.012	$r_s(z_d)/D_v(z=0.2)$	0.1889 ± 0.0060
$r_s(z_d)/D_v(z=0.35)$	0.1135 ± 0.0032	$r_s(z_d)/D_v(z=0.44)$	0.0932 ± 0.0024
$r_s(z_d)/D_v(z=0.54)$	0.0787 ± 0.0019	$r_s(z_d)/D_v(z=0.57)$	$0.0753^{+0.0017}_{-0.0018}$
$r_s(z_d)/D_v(z=0.6)$	0.0724 ± 0.0016	$r_s(z_d)/D_v(z=0.73)$	0.0624 ± 0.0013
$r_s(z_*)$	145.8 ± 1.2	R	1.728 ± 0.016
σ_8	0.821 ± 0.023	$\sigma_8\Omega_m^{0.5}$	0.434 ± 0.029
$\sigma_8\Omega_m^{0.6}$	0.382 ± 0.029	$A_{\rm SZ}$	< 2.0 (95% CL)
t_0	13.74 ± 0.11 Gyr	τ	0.089 ± 0.014
θ_*	0.010391 ± 0.000022	θ_*	$0.5953 \pm 0.0013\ ^\circ$
$\tau_{\rm rec}$	283.9 ± 2.4	$t_{\rm reion}$	453^{+63}_{-64} Myr
t_*	376371^{+4115}_{-4111} yr	z_d	1020.7 ± 1.1
$z_{\rm eq}$	3265^{+106}_{-105}	$z_{\rm rec}$	1088.16 ± 0.79
$z_{\rm reion}$	10.6 ± 1.1	z_*	$1090.97^{+0.85}_{-0.86}$

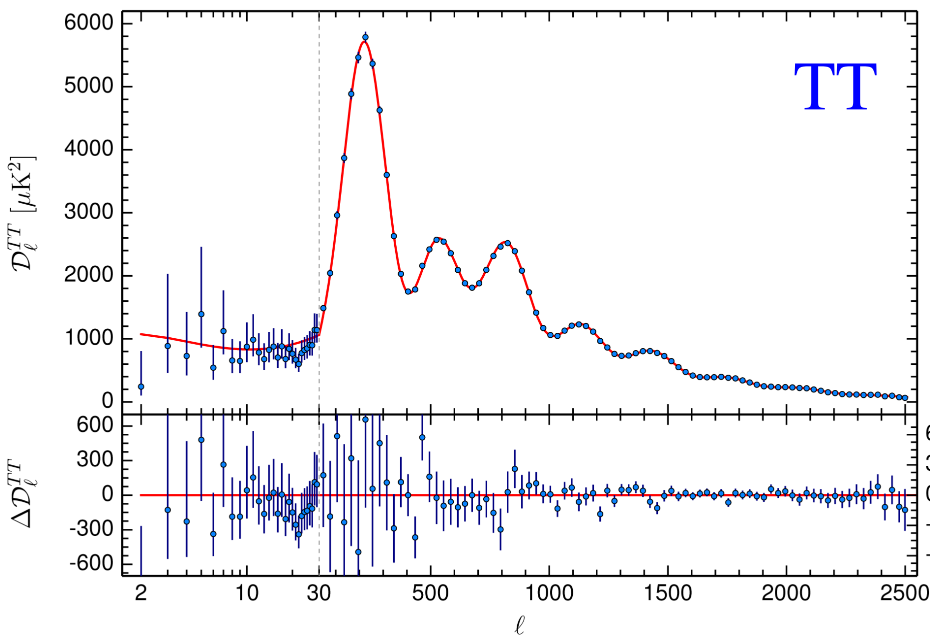
ΛCDM

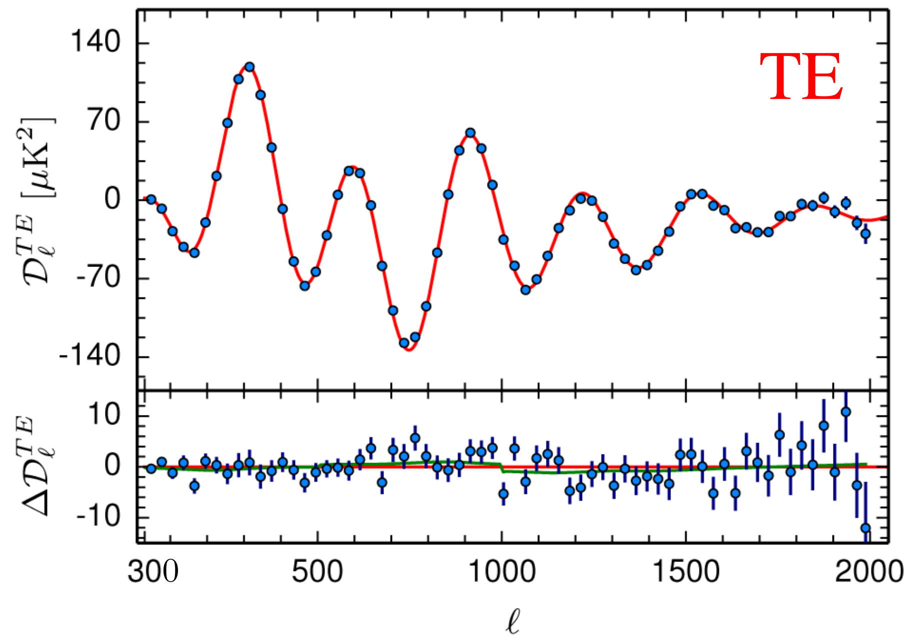
WMAP 9 year data release "vanilla" parameter set: with minimal Lambda-CDM modeling

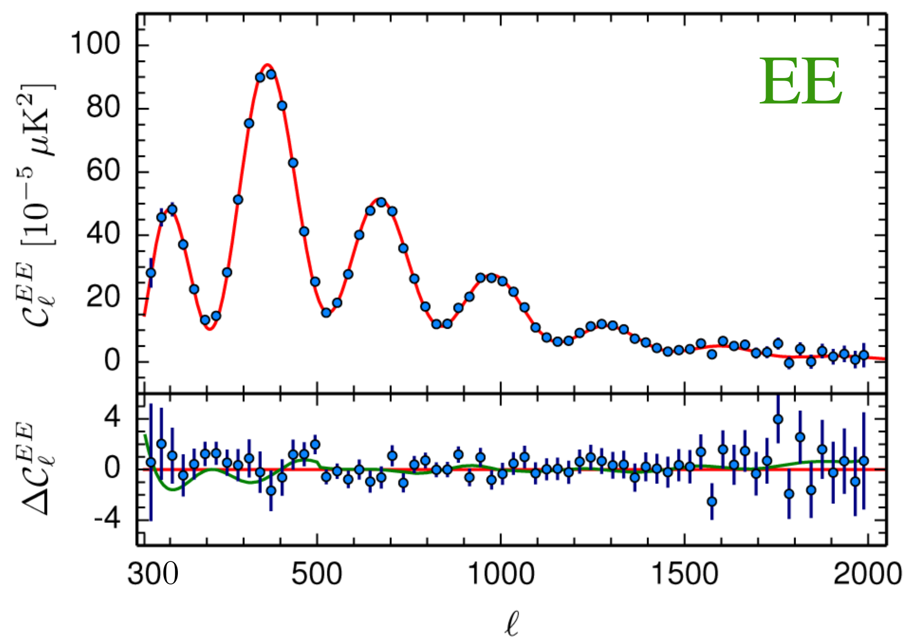
(see *http://lambda.gsfc.nasa.gov/product/map/dr5/parameters.cfm*)

- Minimal modeling case: parameters change slightly with extension and tweaking of models.

Planck CMB Power Spectra







- Planck 2015 data release CMB power spectra.
- *Above*: TT temperature fluctuations.
- *Top right*: TE spectrum
- *Right*: EE polarization spectrum

Planck PLA 2015 Wiki web page