Problem 1

a) Separation $d = \sqrt[3]{\frac{V}{n}}$

$\frac{m}{V} = 1 \text{ g/cm}^{-3}$

$\frac{n}{V} = 1 \text{ g/cm}^{-3} \div 1.008 \text{ g/mol}$

$= 0.99206 \text{ mol/cm}^{-3}$

take $N_A = 6.022 \times 10^{23} \text{ /mol}$

$\frac{N}{V} = \frac{n N_A}{V} = 6.022 \times 10^{23} \times 0.99206 \text{ /cm}^{-3}$

$= 5.9742 \times 10^{23} \text{ /cm}^{-3}$

$\frac{V}{N} = \frac{1}{5.9742e23} = 1.67386e{-24} \text{ cm}^3$

$d = \sqrt[3]{\frac{V}{N}} = 1.1873 \times 10^{-8} \text{ cm} = 1.1873 \times 10^{-10} \text{ m}$

$= 1.1873 \text{ Å}$

b) $a_0 = 5.29 \times 10^{-9} \text{ cm}$, which is about half of d

since $d = a_0 n^2$

$n^2 = \frac{d}{a_0} = 2.24$

$\sqrt{n} \approx 1.5$

but since n is quantized, it should be a distribution with $\mu_n = 1.5$,

so I would say $n=1$ \& $n=2$ are approximate quantum numbers.

Since talk about excited, then $n=2$

c) $E = -13.6\,\text{eV}\left(\frac{1}{1} - \frac{1}{4}\right)$

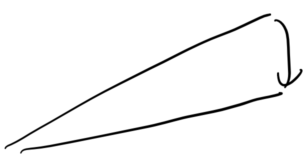
$= -13.6 \times 0.75\,\text{eV}$

$= -3.4\,\text{eV}$

the energy (absolute) need to be smaller since Rydberg gives ground E which has to be maximum.

Problem 2.

a)



$a = \frac{v^2}{r} = \omega^2 r = \frac{GM}{r^2}$

let $v = c$

$\frac{c^2}{r} = \frac{GM}{r^2}$

$r = \frac{GM}{c^2}$

b) $m_1 - m_2 = -2.5 \log\left(\frac{F_1}{F_2}\right) = 10$

$\log\left(\frac{F_1}{F_2}\right) = -4$

$\frac{F_1}{F_2} = 10^{-4}$

Since both Co, assume same luminosity L

$\therefore F_1 = \frac{L}{4\pi d_1^2}$

$F_2 = \frac{L}{4\pi d_2^2}$

$$\frac{F_1}{F_2} = \frac{d_2^2}{d_1^2} = 10^{-4}$$

$$\frac{d_2}{d_1} = 10^{-2}$$

$$d_2 = 100 d_1$$

if $d_1 = \frac{1}{10}$ pc, then $d_2 = 10$ pc

or, in the wording of the question, if nearer one has parallex 10", then the distant one has parallex 0.1"

Problem 3 (24.9, P935)

a) $T = 15K$, $r = 8kpc$, $h = 160p$

$\langle E_H \rangle = \frac{3}{2} k_B T_H = 3.105 \times 10^{-22}$ J

$N = \frac{M_G}{M_H} = \frac{0.5 \times 10^{10} M_\odot}{1.67e^{-27}} = 6 \times 10^{66}$ (Hydrogen molecules)

$= 3 \times 10^{66}$

$$u = \frac{N \times \langle E_H \rangle}{V} = \frac{3 \times 10^{66} \times 3.105 \times 10^{-22} J}{\pi r^2 \times h} = 5.781 \times 10^{41} \text{ erg/pc}^3$$

$= 2 \times 10^{-15}$

b) $u_m = \frac{B^2}{2\mu_0}$

$B \approx 4\,nT = 4\times10^{-9}$ $\qquad c^2 = \frac{1}{\varepsilon_0 \mu_0}$

$\mu_0 = 1.2566\times10^{-6}$

$u_m = 6.366\times10^{-12}$

$\frac{u}{u_m} = \frac{2}{6.366}\times10^{3} = 314$

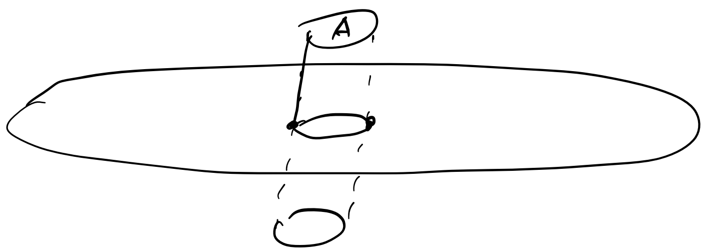
within 3 mag, possibly impact on sth

Problem 4

a) use Gauss's law from maxwell

$\oint E = \frac{Q_{enc}}{\varepsilon_0}$, here $E = g$, $Q_{enc} = \rho A d$, $\frac{1}{4\pi\varepsilon_0} = G$

$\varepsilon_0 = \frac{1}{4\pi G}$



$\therefore$ $LHS = 2gA$

$RHS = \rho A d \, 4\pi G$

$\therefore$ $2g\cancel{A} = \rho \cancel{A} d \, 4\pi G$

$g = 2\pi G \rho d \, \hat{z}$

b) $\frac{dP}{dz} = -\rho |g|$

$P = \frac{\rho k T}{M}$

$\langle E_k \rangle = \frac{3kT}{2}$ $\sigma = \sqrt{\langle v^2 \rangle}$

$= \frac{1}{2} M \langle v^2 \rangle$

$= \frac{1}{2} M \sigma^2$

$\therefore$ $kT = \frac{1}{3} M \sigma^2$

$P = \frac{1}{3} \rho \sigma^2$

$\nabla P = -\rho |g|$

$\frac{d}{dz} \frac{1}{3} \rho(z) \sigma^2 = -\rho(z) 2\pi G \bar{\rho} d$

$\rho'(z) = -\frac{6\pi G \bar{\rho} d}{\sigma^2} \rho(z)$

$\rho = C e^{-\frac{6\pi G \bar{\rho} d}{\sigma^2} z}$ $h = \frac{\sigma^2}{6\pi G \bar{\rho} d} = \frac{\sigma^2}{3|g|}$

c) $R = 25\,kpc$, $d = 300\,pc$ $\quad\quad 1\,pc = 3.086\times10^{16}\,m$

$M_d = 6e10\,M_\odot$ $\quad\quad 1M_\odot = 2\times10^{30}\,kg$

$$\rho_0 = \frac{M_d}{\pi R^2 d} = 0.1\ M_\odot/pc^3$$

$$g = 2.68946\times10^{-11}\ kg/m^3$$

$$h = \frac{\sigma^2}{3|g|} = \frac{(35\times10^3)^2}{3\times2.68946\times10^{-11}} \times \frac{\text{Parsec}}{\text{meters}}$$

$$= 492\ pc$$

the book gives $h(\text{thick disk}) \approx 1\,kpc$, which is within the same magnitude.

Problem 5.

$$n(r) = n_i \exp\left\{-\frac{r}{r_i}\right\} + n_o \left(\frac{r}{r_o}\right)^2 \exp\left\{-\frac{r}{r_o}\right\}$$

① The number of stars within radius r is given by

$$N(r) = \iint\int_{r'=0}^{r} n(r)\, r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \iint\int_{r'=0}^{r} \left(n_i e^{-\frac{r'}{r_i}} + n_o \left(\frac{r'}{r_o}\right)^2 e^{-\frac{r'}{r_o}}\right) r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= 2\pi \left(48 n_o r_o^3 - \frac{1}{r_o} 2\exp\left\{-\frac{r}{r_o}\right\} n_o \left(r^4 + 4r^3 r_o + 12 r^2 r_o^2 + 24 r r_o^3 + 24 r_o^4\right) + 4 n_i r_i^3 - 2\exp\left\{-\frac{r}{r_i}\right\} n_i r_i \left(r^2 + 2 r r_i + 2 r_i^2\right)\right)$$

Flux at r

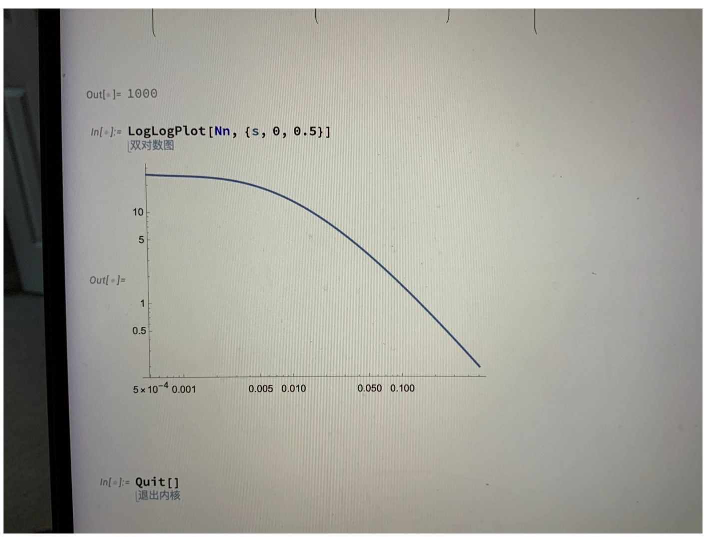
$$S(r) = \frac{L}{4\pi r^2}$$

Now we plot $\log N$ on $\log S$, so we parametrize

$$r = \sqrt{\frac{L}{4\pi S}}$$

$N(r) = N\left(\sqrt{\frac{L}{4\pi S}}\right)$, where $N(r)$ is described above & $S > 0$

Thus a plot should yield the following result:



b) in the bright source, $r_0 >> r_i >> d$ (in my representation $r >> r_0$)

then we shell see $r_0 \to \infty$, $r_i \to \infty$

then all $\frac{r^a}{r_0}$ \$ $\frac{r^a}{r_i} \to 0$, $\exp\left(-\frac{r}{r_0}\right) = \exp\left(-\frac{r}{r_i}\right) = 1$

$\therefore$ $N(r) = C\, r^3$ is only thing left, C is constant

$\therefore$ $\log N(r) = 3\log(r) + \log C$

$\log N\left(\sqrt{\frac{L}{4\pi S}}\right) = 3\log(S^{-\frac{1}{2}}) + 3\log\left(\sqrt{\frac{L}{4\pi}}\right) + \log C$

$= -\frac{3}{2}\log(S) + C_1$

So we recover the $-\frac{3}{2}$ relation ship, $N \propto S^{-\frac{3}{2}}$

c) Now we are taking $r_0 \to 0$, $r_i \to 0$, then

what really is left is $\frac{r^4}{r_0} e^{-\left(\frac{r}{r_0}\right)} \to \infty$, then

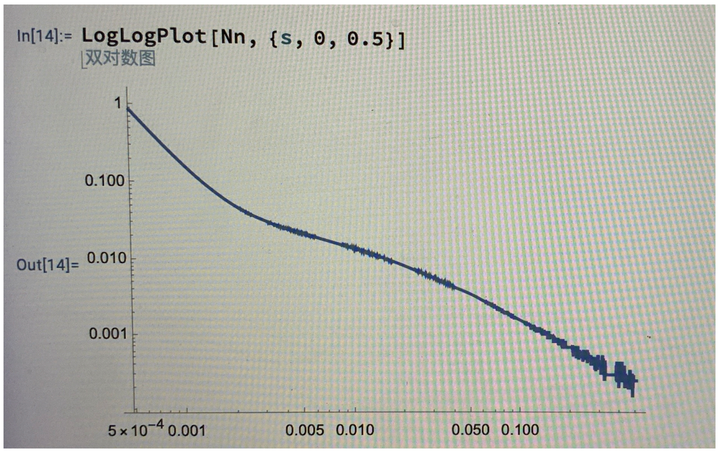
$\log(N) = 4\log(r) + C$

$= -2\log(S) + C_1$

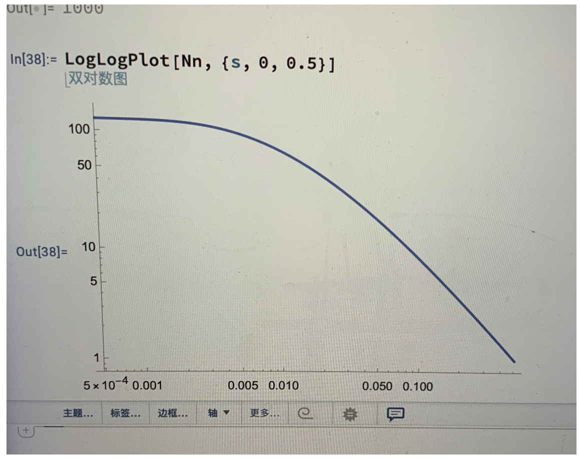
$N \propto S^{-2}$

d)

$n_i \ll n_o$

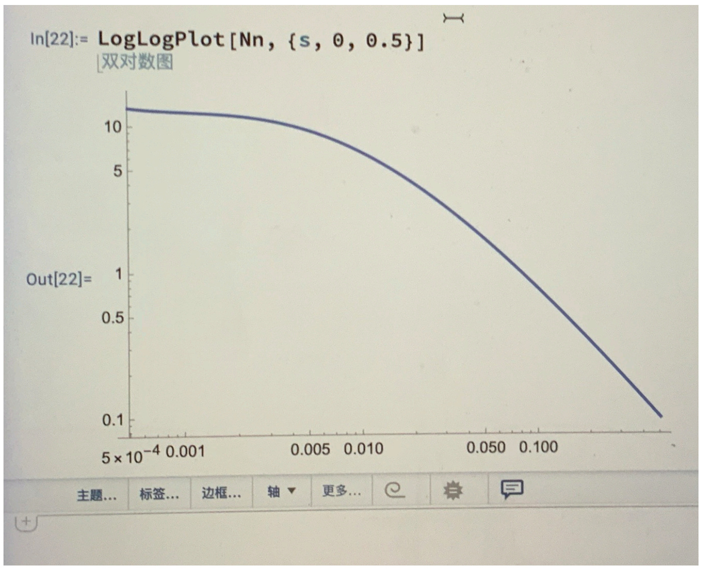


$n_i > n_o$

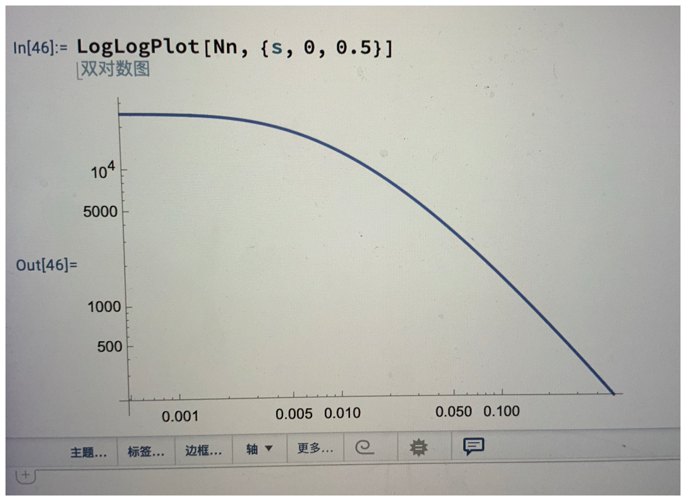


The bright part is stipper due to lack of closer stars that are in n_i.

$n_i < n_o$



$n_i \gg n_o$



$n_o = n_i$