Problem 1

$z = 4.25$

$\Omega_{m,0} = 0.27 \pm 0.04$ $\quad H_0 = 100h = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$

$\Omega_{rel,0} = 8.24 \times 10^{-5}$ $\quad = 2.3 \times 10^{-18} / s$

$\Omega_{\Lambda,0} = 0.73 \pm 0.04$

$$R = \frac{1}{1+z} = 0.190476$$

a) $$t = \sqrt{\frac{3}{8\pi G}} \int_0^R \frac{R' dR'}{\sqrt{\rho_{m,0} R' + \rho_{rel,0} + \rho_{\Lambda,0} R'^4}}$$

$$\rho_{c,0}(t) = \frac{3H_0^2}{8\pi G}$$

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} \qquad \rho(t) = \Omega(t)\,\rho_c(t)$$

$$\therefore t = \frac{1}{H_0} \int_0^R \frac{R' dR'}{\sqrt{\Omega_{m,0} R' + \Omega_{rel,0} + \Omega_{\Lambda,0} R'^4}}$$

$= \frac{1}{H_0}$

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In[24]:= Integrate[R / Sqrt[0.27 * R + 8.24 * 10^-5 + 0.73 * R^4],
         积分              平方根
          {R, 0, Rt}]

Out[24]= 0.106085 + 4.44089 × 10^-16 i
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$= 4.6 \times 10^{16}$ s

$= 1.46 \times 10^{9}$ yr $= 0.106\ t_0$

b) 29.169

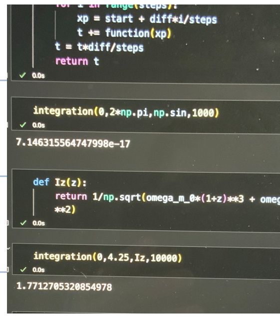
$$d_p = \frac{c}{H_0} I(z)$$

$$I(z) = \int_0^z \frac{dz'}{\sqrt{\Omega_{m,0}(1+z')^3 + \Omega_{rel}(1+z')^4 + \Omega_{\Lambda 0} + (1-\Omega_0)(1+z')^2}}$$

$$\Omega_0 = \sum \Omega_{i0}$$

$$z = 4.25$$

$$I(z) = 1.77$$



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        xp = start + diff*i/steps
        t += function(xp)
    t = t*diff/steps
    return t
✓ 0.0s

integration(0,2*np.pi,np.sin,1000)
✓ 0.0s
7.146315564747998e-17

def Iz(z):
    return 1/np.sqrt(omega_m_0*(1+z)**3 + omeg
**2)
✓ 0.0s

integration(0,4.25,Iz,10000)
✓ 0.0s
1.7712705320854978
```

$$d_p = \frac{c}{H_0} \cdot 1.77$$

$$= 2.3 \times 10^{26}\ m$$

$$= 7486\ Mpc$$

c) $$d_{p,t} = d_{p,0} \cdot R = \frac{d_{p,0}}{1+z}$$

$$= 1426\ Mpc$$

d) $$d_L(z) = \frac{c}{H_0}(1+z)\, S(z)$$

evaluate Ω_0: $\Omega_0 \approx 0.27 + 0.73 = 1$

$\therefore S(z) = I(z) = 1.77$

$$d_L(z) = \frac{c}{H_0}(1+z) \times 1.77$$

$$= 1.21 \times 10^{27} \text{ m}$$

$$= 3.93 \times 10^4 \text{ Mpc}$$

$$= 39.3 \text{ Gpc}$$

e) $$d_A = \frac{c}{H_0} \frac{S(z)}{1+z}$$

$$= d_{p,t}$$

$$= 1426 \text{ Mpc}$$

f) $$D = \frac{c}{H_0} \frac{S(z)\theta}{1+z}$$, $\theta = 5''$ $= \frac{5}{206265}$

$$D = d_A \theta$$

$$= 0.034 \text{ Mpc}$$

$$= 34 \text{ Kpc}$$

g) $I(1) = S(1) = 0.78563$

$$D = \frac{c}{H_0} \frac{S(1)}{1+1} \theta$$

$D = 1.24 \times 10^{21}\ m$

$= 0.04024\ Mpc$

$= 40.24\ Kpc$

Problem 2 29.32

a) $l = \frac{1}{n\sigma}$ $\sigma = 6.65 \times 10^{-25}$

$l = ct$

$\therefore\ ct = \frac{1}{n\sigma}$

$t = \frac{1}{cn\sigma}$

$n = \frac{\rho}{m} = \frac{\rho_{b,0}}{R^3} \frac{1}{m_H}$

$\therefore\ l = \frac{R^3 m_H}{c \rho_{b0} \sigma}$

b) from p29.26:

$$\tau = R^2\left[\frac{8}{3}\pi G\left[R\rho_{m,0} + \rho_{rel,0}\right]\right]^{-\frac{1}{2}}$$

$$= R^2 \frac{1}{H_0}\left[R\Omega_{m,0} + \Omega_{rel,0}\right]^{-\frac{1}{2}}$$

$$\therefore \quad \frac{R m_H}{c\sigma \rho_{b,0}} = \frac{1}{H_0}\left[R\Omega_{m,0} + \Omega_{rel,0}\right]^{-\frac{1}{2}}$$

$\sigma = 6.65 \times 10^{-25}\ cm^2$

$$\rho_{b,0} = \rho_{c,0}\Omega_{b,0} = \frac{3H_0^2}{8\pi G}\Omega_{b,0}$$

$$\frac{R m_H}{c\sigma}\ \frac{8\pi G}{3H_0^2\Omega_{b,0}} = \frac{1}{H_0}\left[R\Omega_{m,0} + \Omega_{rel,0}\right]^{-\frac{1}{2}}$$

$$R\ \frac{8\pi G m_H}{3c\sigma H_0\Omega_{b,0}} = \left[R\Omega_{m,0} + \Omega_{rel,0}\right]^{-\frac{1}{2}}$$

$$\frac{8\pi G m_H}{3c\sigma H_0\Omega_{b,0}} = 424$$

$$424R = (R\Omega_{m,0} + \Omega_{rel,0})^{-\frac{1}{2}}$$

$$R = 0.027 = \frac{1}{1+z}$$

$$\therefore \quad z = \frac{1}{0.027} - 1 = 35.6$$

Problem 3 30.7

$T_0 = 10^9 K$
$T = 2.75 K$

a) $RT = T_0 \rightarrow R = \frac{T_0}{T} = 2.725 \times 10^{-9}$

$\rho = \rho_0 (1+z)^3$

$= \rho_0 \frac{1}{R^3}$ $\quad \rho_0 = \rho_{c,0} \Omega_{b,0}$

$= \rho_0 \left(\frac{T}{T_0}\right)^3 = \frac{3H_0^2 \Omega_{b,0}}{8\pi G} \times (2.725 \times 10^{-9})^{-3} = 0.0224$

$d_h(z) = 2ct$

$= 1.07 \times 10^{11} m$

$M = \frac{4}{3}\pi \left(\frac{d_h(z)}{2}\right)^3 \times \rho_0$

$= 1.44 \times 10^{31} kg$

$= 7.23 M_\odot$

from fig 30.7, $M_{Jeans} = 200 M_\odot$

, two orders of mag larger.

b) $d_h = 2ct,$

$R(2c) = k_1\sqrt{t}$

$\therefore d_h \propto R^2$

$T = k_2 \frac{1}{\sqrt{t}}$

$RT = k_1 k_2 = K$

$\therefore R \propto T^{-1}$

$\therefore m_b \propto T^{-3}$ since $m_b = m_0 R^3$

S.T structure mass has same magnitude as Jeans in T dependence, before recombination.