INTERMEDIATE OR "MODERN" QUANTUM MECHANICS

QUANTUM PHYSICS:

- AT THE HEART OF ALL "CLASSICAL" ELECTRONICS
  ↳ MATERIALS PHYSICS — SEMICONDUCTORS (SI CHIPS, LEDS, ...)
- GOVERNS EXOTIC, POTENTIALLY TRANSFORMATIVE PHASES OF LOW-TEMP. MATTER
  - SUPERCONDUCTIVITY
  - TOPOLOGICAL MATTER
- QUANTUM COMPUTING
  - "ATOM-SCALE" APPROACHES (GUIDO PAGANO HERE @RICE; IonQ)
  - SUPERCONDUCTOR CIRCUITS (GOOGLE)
  - TOPOLOGICAL (MICROSOFT, STATION Q)
- "FUNDAMENTAL" PHYSICS
  - STANDARD MODEL OF ELEMENTARY PARTICLES: 3/4 FORCES (E+M, WEAK, STRONG Nuclear)
  - FRONTIER:
    - BEYOND STANDARD MODEL (MUONS ?!)
    - QUANTUM GRAVITY (STRINGS ?)

THIS COURSE: THOROUGH INTRODUCTION TO QUANTUM PHYSICS OF SMALL OR "SIMPLE" SYSTEMS

- A COUPLE "QUBITS"
- HYDROGEN ATOM
- QUANTUM WIRE

ETC.

QUANTUM PHYSICS: FORMALISM ⊕ INTERPRETATION

(A) • INTERPRETATION IS THE "FREAKY" PART. DEFIES CLASSICAL (MACROSCOPIC, HUMAN) INTUITION

• HARD TO UNDERSTAND, BUT NOT TO USE TO MAKE USEFUL PREDICTIONS (FEYNMAN: "SHUT UP AND CALCULATE")

(B) • FORMALISM: - LINEAR ALGEBRA IN "COMPLEX" VECTOR SPACES,
- DIFFERENTIAL EQUATIONS
→ UNIFYING NOTATION: DIRAC "BRA" $\langle\psi|$
"KET" $|\phi\rangle$

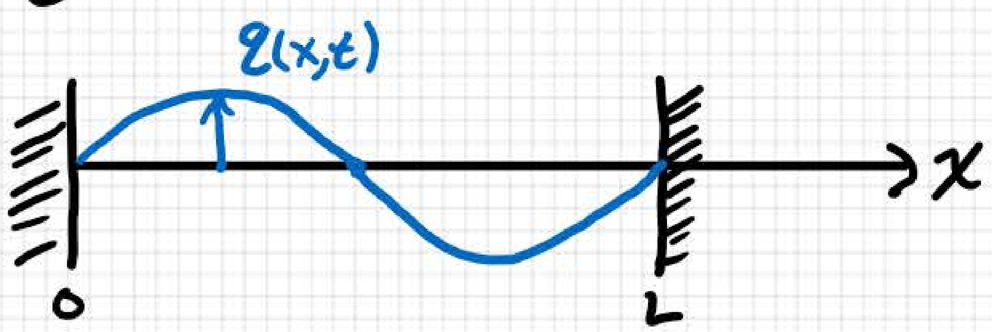
BOOK: TOWNSEND. STARTS WITH (A).
I PREFER TO START WITH (B), FOLLOWING CH. 1 OF SHANKAR.

- FORMALISM ISN'T HARD, BUT NOTATION IS NEW
- DON'T CONFUSE FORMALISM (MATH) WITH ACTUAL QUANTUM FREAKINESS (MEASUREMENT)
- AFTER A DETOUR THROUGH MATH, WE WILL RETURN TO QUANTUM PHYSICS OF SIMPLEST SYSTEMS: SINGLE SPINS. (TOWNSEND)

① MATHEMATICS OF QUANTUM MECHANICS

A "COLD OPEN": CLASSICAL BOUNDARY VALUE PROBLEM – WAVES ON A STRING

END OF 301: DERIVE WAVE EQN. FOR STRING STRETCHED BETWEEN ENDPOINTS



$q(x,t)$: TRANSVERSE DISPLACEMENT OF STRING AS FUNCTION OF POSITION x, TIME t

(1) WAVE EQN.: $$\frac{\partial^2 q}{\partial t^2} = v^2 \frac{\partial^2 q}{\partial x^2}$$; 301: $v = \sqrt{\frac{T}{\rho}}$

T: TENSION (FORCE)

ρ: MASS DENSITY

- UNITS: $\frac{1}{[t]^2}\cdot\cancel{[q]} = [v]^2\frac{\cancel{[q]}}{[L]^2}$ $\therefore$ $[v] = \frac{[L]}{[t]}$ SPEED

(2) BOUNDARY CONDITIONS: $q(0,t) = q(L,t) = 0$ CLAMPED ("DIRICHLET")

INITIAL CONDITIONS: 2 TIME DERIVATIVES (AS IN NEWTON'S 2^{ND})

$\longrightarrow$ NEED TWO INITIAL CONDITIONS ① "POSITION" $q(x,0)$

② "VELOCITY" $\left.\frac{\partial q}{\partial t}(x,t)\right|_{t=0}$

TRY SEPARATION OF VARIABLES (MORE SYSTEMATIC LATER – HERE AN "ANSATZ" (GUESS))

$$q(x,t) \equiv \phi(x)\,h(t) \;;\quad (1):\ \phi\ddot{h} = v^2\phi'' h \qquad \dot{a} \equiv \frac{\partial a}{\partial t};\ b' \equiv \frac{\partial b}{\partial x}$$

$$\Rightarrow \underbrace{\frac{\ddot{h}(t)}{h(t)}}_{\text{DEPENDS ONLY ON TIME}} = v^2 \underbrace{\frac{\phi''(x)}{\phi(x)}}_{\text{DEPENDS ONLY ON POSITION}} \equiv -\Omega^2$$

- WHY $-\Omega^2 \rightarrow \sin\Omega t,\ \cos\Omega t$
  $+\Omega^2 \rightarrow \cancel{e^{\Omega t}}, e^{-\Omega t}$ DOESN'T MAKE PHYSICAL SENSE
- Ω: $\frac{1}{[t]}$ FREQUENCY.

SPATIAL PART:

$$\phi'' + k^2\phi = 0 \,, \quad k \equiv \frac{\Omega}{v}$$

UNITS: $[k] = \frac{[\Omega]}{[v]} = \frac{1}{[t]}\frac{1}{[L]/[t]} = \frac{1}{[L]}$ "WAVENUMBER"

GEN. SOLUTION: $\phi(x) = A\sin kx + B\cos kx$

Boundary Conditions: $\phi(0) = 0 \rightarrow B = 0 \,;\ \phi(L) = A\sin kL = 0$

- $A = 0$ IS TRIVIAL SOLUTION (NO MOTION)

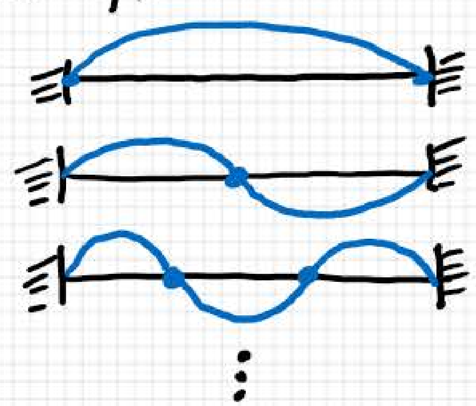
$$\therefore \quad kL = n\pi \quad \text{or} \quad k_n \equiv \frac{n\pi}{L} \,, \ n \in \mathbb{Z} \ \text{(INTEGERS)}$$

$$\therefore \quad \Omega_n = v k_n = \left(\frac{v}{L}\right) n\pi$$

QUANTIZED EIGENFREQUENCIES! (HARMONICS OF THE STRING)

$$\phi_n(x) = A_n \sin(k_n x)$$

EIGENFUNCTIONS OR MODES



h(t): $\ddot{h} + \Omega_n^2 h = 0$

$$\Rightarrow \quad h(t) = C_n \cos(\Omega_n t) + D_n \sin(\Omega_n t)$$

MOST GENERAL SOLUTION:

$$q(x,t) = \sum_{n=1}^{\infty} \left[C_n \cos(\Omega_n t) + D_n \sin(\Omega_n t)\right] \sin(k_n x)$$

INITIAL CONDITIONS: $q(x,0) = \sum_{n=1}^{\infty} C_n \sin(k_n x) \,; \quad \left.\frac{\partial q(x,t)}{\partial t}\right|_{t=0} = \sum_{n=1}^{\infty} D_n \Omega_n \sin(k_n x)$

EIGENFUNCTIONS $\phi_n(x) \equiv \sin\left(\frac{n\pi x}{L}\right)$, $n \in \{1,2,3,\ldots\}$

FORM A <u>COMPLETE</u>, <u>ORTHOGONAL</u> SET

<u>PROVISO</u>: ASSUME $g(x)$ IS <u>REAL-VALUED</u> $g(x) = g^{*}(x)$ ← COMPLEX CONJUGATE

<u>COMPLETE</u>: $\langle g|\phi_n\rangle \equiv \int_0^L g(x)\,\phi_n(x) = 0$ FOR <u>ALL</u> $n \Longrightarrow$ IMPLIES $g(x)=0$ (NOT PROVEN HERE)

$\langle g|\phi_n\rangle \equiv$ "INNER PRODUCT" OF "VECTORS" $g(x)$, $\phi_n(x)$

<u>ORTHOGONAL</u>: $\langle \phi_m|\phi_n\rangle = \int_0^L \phi_m(x)\,\phi_n(x)\,dx = \int_0^L dx\, \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)$ TRIG. IDENTITY

$$= \frac{1}{2}\int_0^L dx \left\{ \cos\left[\frac{(m-n)\pi x}{L}\right] - \cos\left[\frac{(m+n)\pi x}{L}\right]\right\}$$

$$= \begin{cases} \frac{L}{2}, & m=n \\ 0, & m\neq n \quad (m,n\geq 1) \end{cases}$$

ASSUME $\dot{q}(x,0) = 0 \Rightarrow D_n = 0$ FOR ALL n; $\quad q(x,0) \equiv f(x)$

$\therefore \quad f(x) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi x}{L}\right)$; "DOT" BOTH SIDES WITH $\phi_m(x)$:

$$\langle \phi_m|f\rangle = \int_0^L dx\, f(x)\sin\left(\frac{m\pi x}{L}\right) = \sum_{n=1}^{\infty} C_n \underbrace{\langle\phi_m|\phi_n\rangle}_{=\frac{L}{2}\delta_{m,n}} = C_m\frac{L}{2}$$

↑ KRONECKER DELTA FCN.

$$\boxed{\Longrightarrow \quad C_m = \frac{2}{L}\int_0^L dx\, f(x)\sin\left(\frac{m\pi x}{L}\right)}$$

SOLUTION IS COMPLETELY DETERMINED!

$$q(x,t) = \sum_{n=1}^{\infty} C_n \cos(\Omega_n t)\sin(K_n x); \quad K_n = \frac{n\pi}{L}$$

$$\Omega_n = v K_n$$