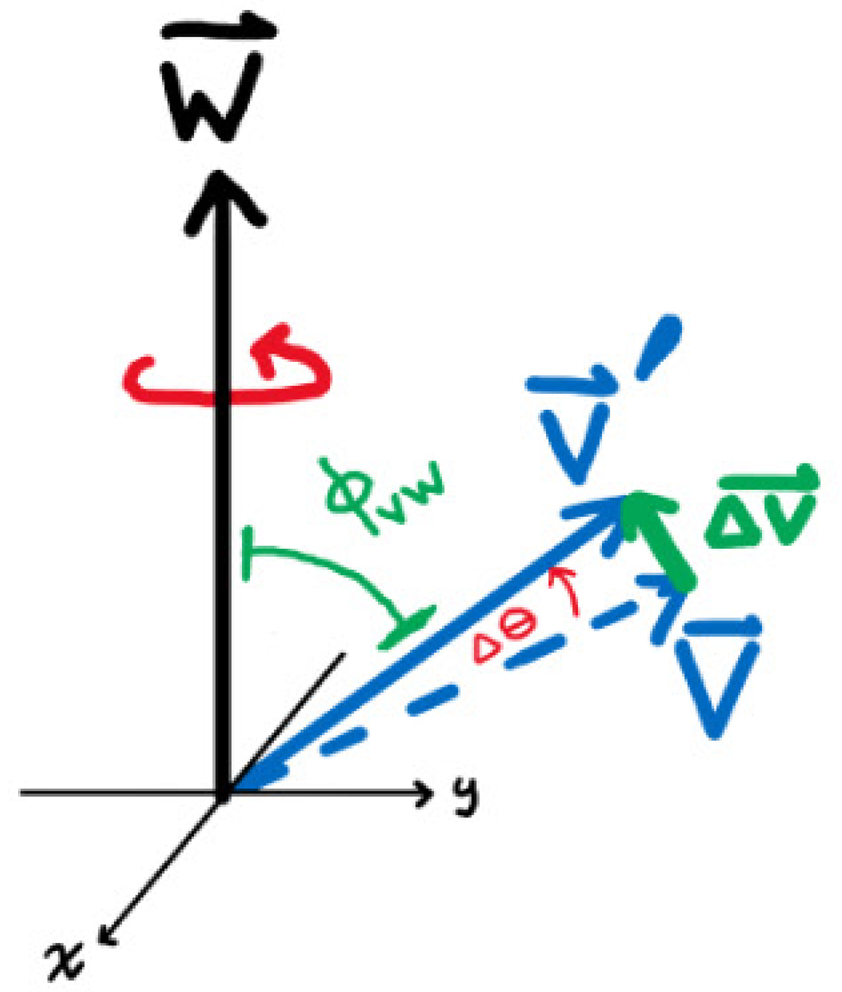


$\mathbb{V}^3(\mathbb{R})$:

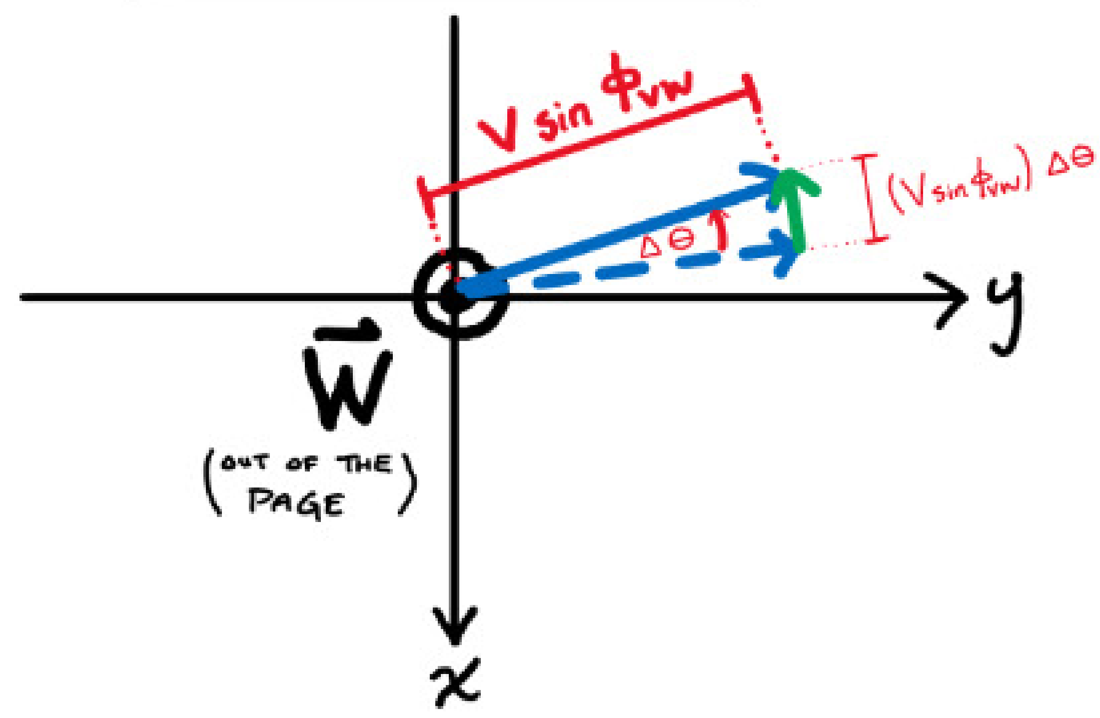
- Two Vectors $\vec{V}, \vec{W}$
- Consider a small Active rotation of $\vec{V}$ around $\vec{W}$, by a CCW angle $\Delta\theta$ $(0 \le \Delta\theta \ll 1)$
- Vector After Rotation $\equiv \vec{V}'$
- $\vec{V}' = \vec{V} + \vec{\Delta V}$; $\vec{\Delta V} = (\vec{n}_W \times \vec{V})\,\Delta\theta$; $\vec{n}_W \equiv \frac{\vec{W}}{|\vec{W}|}$

① $\vec{\Delta V}$ is $\perp$ to both $\vec{W}, \vec{V}$; preserves Norm

$$\vec{V}' \cdot \vec{V}' = (\vec{V} + (\vec{n}_W \times \vec{V})\Delta\theta)\cdot(\vec{V} + (\vec{n}_W \times \vec{V})\Delta\theta)$$
$$= V^2 + \underbrace{\mathcal{O}(\Delta\theta)^2}_{\text{higher order in } \Delta\theta}$$

② $|\vec{\Delta V}| = \underbrace{(V \sin\phi_{vw})}_{\text{Projection of } \vec{V} \text{ in plane } \perp \text{ to } \vec{W}} \cdot \Delta\theta$;

Top View of Rotation:



Rewrite using Levi-Civita:

- Assume orthonormal basis $\{\vec{n}_a\}$; $\vec{n}_a \cdot \vec{n}_b = \delta_{ab}$

$$\vec{V} = \sum_{a=1}^{3} V_a \vec{n}_a \; ; \quad \vec{W} = \sum_{b=1}^{3} W_b \vec{n}_b$$

- Let us define $\vec{\Delta\theta} \equiv \Delta\theta\, \vec{n}_W$; $\vec{V}' = \vec{V} + \vec{\Delta\theta} \times \vec{V}$

- using ϵ_{abc}:

$$\vec{V}' = \vec{V} + \epsilon_{bcd}\, \underbrace{\vec{n}_b}_{\text{Basis Vector}}\, (\vec{\Delta\theta})_c\, (\vec{V})_d$$

Basis Vector; Which Basis Vector: NOT a component label; Component label; Component label

Einstein Sum on b, c, d

- Suppose $\vec{V} \equiv \vec{n}_a$, a particular Basis Vector

$$\therefore \quad \vec{n}_a' = \vec{n}_a + \vec{\Delta\theta} \times \vec{n}_a = \vec{n}_a + \epsilon_{bcd}\, \vec{n}_b\, (\vec{\Delta\theta})_c\, (\vec{n}_a)_d$$

But: $(\vec{n}_a)_d = \delta_{a,d}$

$$\therefore \quad \vec{n}_a' = \vec{n}_a + \epsilon_{bca}\, \vec{n}_b\, (\vec{\Delta\theta})_c$$

Ket Notation:

$$|\vec{n}_a'\rangle = |\vec{n}_a\rangle + \epsilon_{bca}\, (\vec{\Delta\theta})_c\, |\vec{n}_b\rangle \qquad \text{Einstein sum } b, c$$
$$= |\vec{n}_a\rangle + (\vec{\Delta\theta} \times \vec{n}_a)_b\, |\vec{n}_b\rangle \qquad \text{Einstein sum } b$$
$$= |\vec{n}_a\rangle + |\vec{\Delta\theta} \times \vec{n}_a\rangle$$

- GENERATOR VERSION (LEC. 10, P. 3,4)

$$|\vec{n}_a'\rangle = e^{\vec{\Delta\theta}\cdot\hat{\vec{G}}}|\vec{n}_a\rangle \simeq (\hat{\mathbb{I}} + \vec{\Delta\theta}\cdot\hat{\vec{G}})|\vec{n}_a\rangle$$

$$\therefore \quad \vec{\Delta\theta}\cdot\hat{\vec{G}}\,|\vec{n}_a\rangle = |\vec{\Delta\theta}\times\vec{n}_a\rangle = (\vec{\Delta\theta}\times\vec{n}_a)_b\,|\vec{n}_b\rangle$$

$$= \epsilon_{bcd}\,(\vec{\Delta\theta})_c\,(\vec{n}_a)_d\,|\vec{n}_b\rangle$$

$$= \epsilon_{bca}\,(\vec{\Delta\theta})_c\,|\vec{n}_b\rangle$$