HOMEWORK PROBLEMS 1
ASTR 360

Incidentals and The Milky Way

Due date: Wednesday, 1/31/24 — 10pm

1. (a) Suppose that liquid hydrogen is roughly the same density as water, i.e. 1 g cm^{-3}. Estimate the mean separation of hydrogen atoms in this state.

(b) Compare this estimate to the Bohr radius a_0. Thereby obtain an estimate of the approximate excitation quantum number n at which the electron radius is about the value of the mean atomic separation.

(c) Use this value of n to determine an estimate for the approximate binding energy of electrons in the liquid hydrogen. Why should the binding energy be less than the Rydberg value?

[18 points credit]

2. (a) Use Newton's force law for gravity and the formula for centripetal acceleration to deduce that the effective radius (the event horizon, at which even light cannot escape) of a Schwarzschild (= non-rotating) black hole of mass M is of the order of GM/c^2, where c is the speed of light.

(b) Two G0 stars (like the sun) on the main sequence have apparent magnitudes that differ by 10. If the parallax of the nearer one is 10 arcsec, what is the parallax of the more distant one?

[15 points credit]

3. Carroll and Ostlie problem 24.9.

[10 points credit]

4. (a) Consider the gravitational field of the thick disk of the Milky Way in the neighborhood of (but outside) the thin disk. Treat the thin disk as being an infinite quasi-planar cylindrical region of small thickness d (and radius $R \to \infty$), and uniform mass density ρ of stars. Using the gravitational force law, or otherwise, show that the gravitational field outside the thin disk is uniform and perpendicular to the plane. Find an expression for the gravitational acceleration $\boldsymbol{g}$ in terms of ρ and d.

(b) The vertical structure (in the z direction) of the thick disk obeys the force balance **equation of hydrostatic equilibrium**, which takes the form

$$\frac{dP}{dz} = -\rho\, |\boldsymbol{g}| \quad \text{for} \quad P = \frac{\rho k T}{M} \quad . \tag{1}$$

Here $3kT/2$ represents the average kinetic energy of the disk's moving stars. If all stars in the thick disk also have mass M, and possess a 3D velocity dispersion $\sigma = \sqrt{\langle v^2 \rangle}$, show that these stars have an exponential distribution $\exp\{-z/h\}$ with distance z from the central plane of the thin disk, and find an expression for the scale height h.

(c) Now, by treating the thin disk as a cylinder of radius $R = 25\,\text{kpc}$ and thickness $d = 300\,\text{pc}$, with a mass of $M_d = 6 \times 10^{10} M_\odot$, data that is listed in Table 24.1 of C&O, use your results to evaluate your expression for h. You can assume that $\sigma \sim 35\,\text{km sec}^{-1}$ as in the book. How does your value for h compare with that noted in Table 24.1?

[30 points credit]

5. Consider a population of stars that is distributed spherically in the Milky Way, but with two components of two different radial scales r_i and r_o ($\gg r_i$). Suppose that they are standard candles of intrinsic luminosity L, i.e. have a delta function for their luminosity function. Assume that their space number density distribution (bispherical) is given by

$$n(r) \;=\; n_i \,\exp\biggl\{ -\frac{r}{r_i} \biggr\} + n_o \left(\frac{r}{r_o} \right)^2 \exp\biggl\{ -\frac{r}{r_o} \biggr\} \quad . \tag{2}$$

Here r is the radius out from the Galactic Center, and n_i and n_o are the population number scales of the inner and outer components respectively.

(a) Consider the perspective of an observer *at the Galactic center*. Determine a general mathematical form for the $\log N - \log S$ distribution (number of sources $N(> S)$ detected to exceed a given flux S for these sources as measured by the observer, assuming Euclidean space.

(b) Show that in the bright source limit of distances $d \ll r_i$, your result in (a) reproduces the standard $S^{-3/2}$ form for homogeneous sources distributed *ad infinitum* in flat spacetime, under certain assumptions to be stated.

(c) Consider the case where the radial extent of the bispherical distribution becomes influential: when $d \gg r_o$ ($\gg r_i$), what index α for $N(> S) \propto S^{-\alpha}$ is obtained?

(d) Use your assembled results to plot a schematic of the distribution you obtained in (a). Treat all representative cases for the ratio n_i/n_o.

[27 points credit]