Problem 1 (25.2) Eric Cai

a) $M_a = -21.51$

$\log(R_{25}) = -0.249 M_B - 4$

$R_{25} = 10^{0.249 \times 21.51 - 4}$

$= 22.7 \text{ kpc}$

b) Sc:

$M_B = -11 \log(V_{max}) + 3.31$

$10^{\left(\frac{M_B - 3.31}{-11}\right)} = V_{max}$

$V_{max} = 180 \text{ km/s}$

c) $P = \frac{2\pi R}{V}$

$w = \frac{2\pi}{P} = \frac{V}{R} = \frac{180}{22.7}$

$= 2.569742 \text{ rad/s}$

$= 8.1 \times 10^{-9} \text{ rad/y}$

$= 0.001673 \text{ ''/yr}$

d) $T = \frac{1}{w} = \frac{1}{0.001673} \approx 598 \text{ yr}$

The seperation in Hawali is 0.2'', if good 0.1''

which, might be possible, if all fortunes on the world is

in you.

Problem 2 (25.6)

$$L(R,z) = L_0 e^{-\frac{R}{h_R}} \operatorname{sech}^2(z/z_0)$$

$z=0$:

$$\operatorname{Log}\left[\frac{L(R)}{L_0}\right] = -\left(\frac{R}{h_R}\right)\log e = -0.434\left(\frac{R}{h_R}\right)$$

$$\log\left(\frac{L}{L_0}\right) = -0.434\left(\frac{R}{h_R}\right)$$

$$\frac{M_0 - M}{2.5} = -0.434\left(\frac{R}{h_R}\right)$$

$$\mu_0 - \mu = -1.09\left(\frac{R}{h_R}\right)$$

$$\mu = \mu_0 + 1.09\left(\frac{R}{h_R}\right)$$

Problem 3 (25.7)

a) eq 24.13: $\log\left(\frac{I}{I_e}\right) = -3.3307\left(\left(\frac{r}{r_e}\right)^{\frac{1}{4}} - 1\right)$

$$\frac{\ln\left(\frac{I}{I_e}\right)}{\ln(10)} = -3.3307\left(\left(\frac{r}{r_e}\right)^{\frac{1}{4}} - 1\right)$$

$$\frac{\ln\left(\frac{I}{I_e}\right)}{2.3} = -3.3307\left(\left(\frac{r}{r_e}\right)^{\frac{1}{4}} - 1\right)$$

$$\ln\left(\frac{I}{I_0}\right) = -7.66\left(\left(\frac{r}{r_e}\right)^{\frac{1}{4}}-1\right)$$

$$I = I_e\, e^{\overset{\substack{\text{(little off)}\\ \uparrow}}{-7.66}\left(\left(\frac{r}{r_e}\right)^{\frac{1}{4}}-1\right)}$$

b) $L_{tot} = \int_0^{\infty} 2\pi r\, I\, dr$

let $x = \left(\frac{r}{r_e}\right)^{\frac{1}{4}} = r^{\frac{1}{4}} r_e^{-\frac{1}{4}}$

$$\frac{dx}{dr} = \frac{1}{4} r^{-\frac{3}{4}} r_e^{-\frac{1}{4}}$$

$$dr = 4r^{\frac{3}{4}} r_e^{\frac{1}{4}}\, dx$$

$$= 4\frac{r^{\frac{3}{4}}}{r_e^{\frac{3}{4}}} r_e\, dx$$

$$= 4x^3 r_e\, dx$$

$$x^4 = \frac{r}{r_e}$$

$$r = x^4 r_e$$

$$\therefore L_{tot} = \int_0^{\infty} 2\pi x^4 r_e\, e^{-7.66x} I_e e^{7.66}\, 4x^3 r_e\, dx$$

$$= 8\pi r_e^2 e^{7.67} I_e \int_0^{\infty} e^{-7.67x} x^7\, dx$$

$$= 8!\,\pi r_e^2 \frac{e^{7.67}}{(7.66)^8} I_e \approx 7.22\pi r_e^2 I_e$$

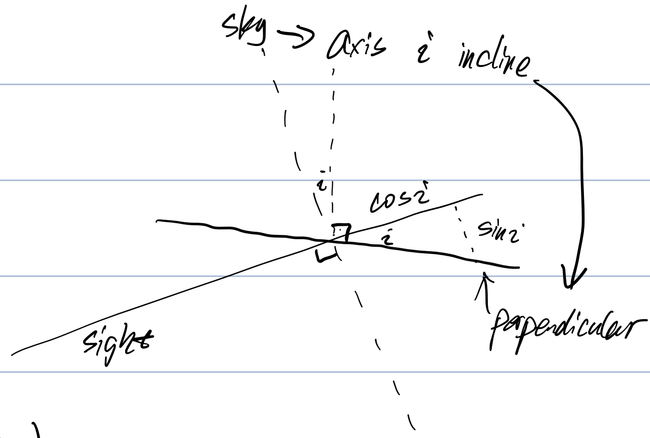
c) $r=0 \sim r=r_e$, let $x = 7.67\left(\frac{r}{r_e}\right)^{\frac{1}{4}}$ instead so no consider extra terms.

$$\int_0^{7.67} x^7 e^{-x} = 7!\; e^{-x}\sum_{i}^{7}(-1)^i \frac{x^{7-i}}{(7-i)!\,(-1)^{i+1}}\Bigg|_0^b = 0.5\,7! = \frac{1}{2}\int_0^{\infty} x^7 e^{-x}$$

$$= \frac{1}{2} L_{tot}$$

Problem 4

$H(R)$



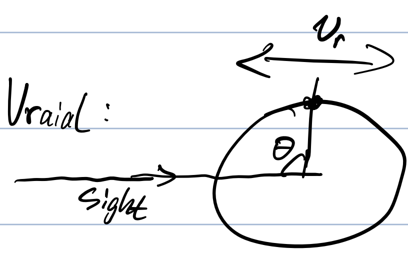
$v = H(R)$

Ca II $\quad F_\lambda \propto \delta(\lambda - \lambda_0)$

a) $\dfrac{\Delta\lambda}{\lambda_0} = \dfrac{V_r}{c} = z$

b) broaden: $F_{\lambda b} = \delta(\lambda - \lambda_{new})$

first we adjust the velocity to V_{raial}:



incline $\downarrow$ rotation component $\downarrow$

$V_r = V\cos(i) \times \sin(\theta)$

$= H(R)\cos(i)\sin(\theta)$

We want $F_b(\lambda, R)$

which for each displacement $\dfrac{\Delta\lambda}{\lambda_0} = \dfrac{V_r}{c}$, the new line is at $\lambda_{new} = \lambda_0 + \Delta\lambda$

$\lambda_{new} = \lambda_0 + \frac{V_r}{c}\lambda_0$

$= \left(1 + \frac{V_r}{c}\right)\lambda_0$

$\therefore\ F_b(\lambda, R) = \Sigma\,\delta\left(\lambda - \left(1 + \frac{V_r}{c}\right)\lambda_0\right)$

$= \displaystyle\int_{\theta=0}^{2\pi} \delta\left(\lambda - \left(1 + \frac{H(R)}{c}\cos(i)\sin(\theta)\right)\lambda_0\right) d\theta$

$x = \sin\theta$

$\dfrac{dx}{d\theta} = \cos\theta$ $\qquad dx = \cos\theta\, d\theta$

$1 - x^2$

$$F_b(\lambda R) = \int_{-1}^{1} \delta\left(\lambda - \left(1 + \frac{\Theta(R)}{c}\cos(i)x\right)\lambda_0\right)\frac{1}{\sqrt{1-x^2}}\,dx$$

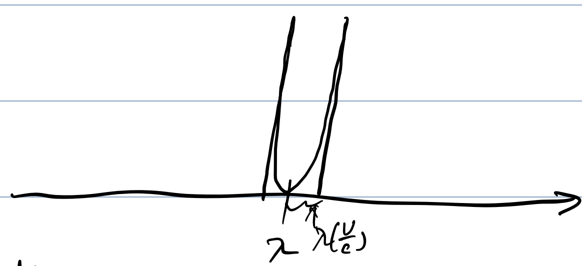
integrant > 0

$$\lambda = \left(1 + \frac{\Theta(R)}{c}\cos i \, x\right)\lambda_0$$

$$\left(\frac{\lambda}{\lambda_0} - 1\right) = \cos(i)\,\frac{\Theta}{c}\,x$$

$$\left(\frac{\lambda}{\lambda_0} - 1\right)\frac{c}{\Theta\cos(i)} = x$$

$$F_b(\lambda, R) = \lambda\left(1 + \frac{1}{\sqrt{1 - \left(\frac{\lambda}{\lambda_0} - 1\right)^2\left(\frac{c}{\Theta\cos(i)}\right)^2}}\right)$$

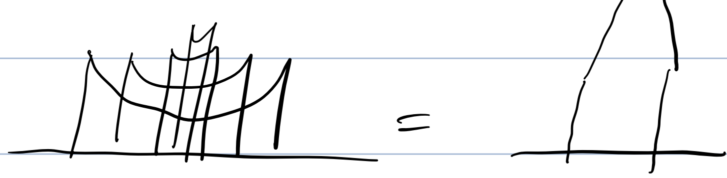


c) Consider opacity,

The more edge on, the more ~~light~~ blocked from smaller R.

Thus: more edge on:

less edge on:



Also, velocity dispersion adds a gaussian convolution to the shape, so will be blurry.

d) in such case, from the shape of lines it is possible to determin i, by how clear the two peaks of red and blue shifts are.

c) $E_K = \frac{3}{2}k_B T = \frac{1}{2}mv^2$, take m to be hydrogen

$m = 1.67 \times 10^{-27}$ kg

$k_B = 1.383 \times 10^{-23}$

$V = 220 \times 10^3$ m/s

$T = \frac{mv^2}{3k_B}$

$= 1.95$ MT, a little too high

∴ probably not dominate.