Problem 1 (4.1 P169)

$$P = \alpha E = \oint q r' dr$$

$$= -q \times 0 + q \times d \quad \text{(Center of elect cloud as 0)}$$

$$= qd$$

$\alpha E = qd$

$-\vec{\nabla} V = \vec{E}$

$E = 500/1mm$

$= 5 \times 10^5$

$q = e = 1.6 \times 10^{-19}$

$\alpha = 0.667 \times 10^{-30} \times 4\pi\varepsilon_0$

$$d = \frac{\alpha E}{q} = \frac{0.667 \times 5 \times 10^5 \times 10^{-30}}{1.6 \times 10^{-19}} \times 4\pi\varepsilon_0$$

$= 2.318 \times 10^{-16} m$

$a_0 = 0.5A = 5 \times 10^{-11} m$

$$\frac{d}{a_0} = \frac{2.318}{5} \times 10^{-5}$$

$= 4.636 \times 10^{-6}$

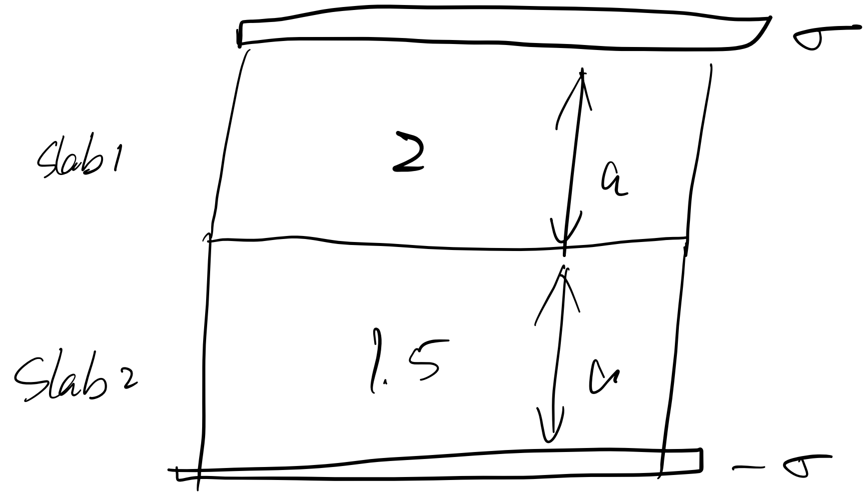
For ionization, we want $a_0 = d$

$$E = \frac{a_0 q}{\alpha} = 1.078 \times 10^{11}$$

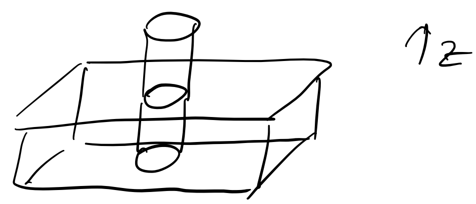
$V = E \times 1mm = 1.078 \times 10^8 V$

$= 1.078 GV$

Problem 2 : (4.18) P191



a) $\oint \vec{D} \cdot da = Q_{free}$



$\vec{D} \cdot 2\pi r^2 = -2\pi r^2 \sigma \hat{z}$

$\therefore \vec{D} = -\sigma \hat{z}$ (↓ direction)

Same for bottom

$\therefore \vec{D} = -\sigma \hat{z}$

b) since B the same,

$\vec{D} = \varepsilon \vec{E}$,

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r} = \begin{cases} \dfrac{-\sigma}{2\varepsilon_0} \hat{z} & 1 \\ \dfrac{-\sigma}{1.5\varepsilon_0} \hat{z} & 2 \end{cases}$$

c) $P = \varepsilon_0 \chi_e E$ $\qquad \varepsilon_0 \varepsilon_r = \varepsilon_0(1+\chi_e)$

$= \varepsilon_0(\varepsilon_r - 1)E$ $\qquad \varepsilon_r = 1+\chi_e, \quad \chi_e = \varepsilon_r - 1$

$$= \begin{cases} -\frac{\sigma}{2\varepsilon_0}\varepsilon_0(2-1)\hat{z} & 1 \\ -\frac{2\sigma}{3\varepsilon_0}\varepsilon_0(1.5-1)\hat{z} & 2 \end{cases}$$

$$= \begin{cases} -\frac{\sigma}{2}\hat{z} & 1 \\ -\frac{\sigma}{3}\hat{z} & 2 \end{cases}$$

d) $V = -\int \vec{E}\cdot dl$

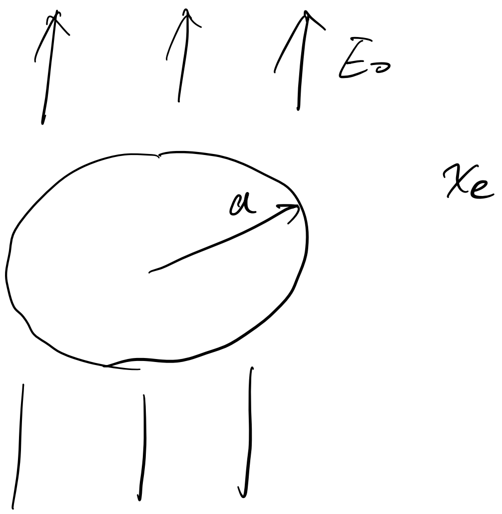
$= \left(\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{3\varepsilon_0}\right) a$

$= \frac{7\sigma}{6\varepsilon_0} a$, 0 point at bottom

e) $\rho_b = P \cdot \hat{n}$

$$= \begin{cases} -\frac{\sigma}{2} & \text{(at upper surface of 1)} \\ \frac{\sigma}{3} & \text{(at lower surface of 2)} \\ \frac{\sigma}{2} & \text{(bottom of 1)} \\ -\frac{\sigma}{3} & \text{(surface of 2)} \end{cases}$$

Problem 3 (4.22 P196)



$$V(s,\phi,z) = a_0 + b_0 \ln s + \sum_{k=1}^{\infty}\left[s^k(a_k \cos k\phi + b_k \sin k\phi) + s^{-k}(c_k \cos k\phi + d_k \sin k\phi)\right]$$

$$\begin{cases} V_{in} = V_{out} \\ \varepsilon \dfrac{\partial V_{in}}{\partial r} = \varepsilon_0 \dfrac{\partial V_{out}}{\partial r} \\ V_{out} \to -E_0 r\cos\theta \end{cases}$$

IC: $V(s=0) = 0$ (Defined, donce ask)

$V_{in} = \sum_{1}^{\infty} s^k (a_k \cos k\phi + b_k \sin k\phi)$ doh

$V_{out} = \sum_{k=1}^{\infty} s^{-k}(c_k \cos k\phi + d_k \sin k\phi) - E_0 s\cos\theta$

BC ①: $V_{in}|_a = V_{out}|_a$

$\therefore \int V_{in} \cos k\phi = \int V_{out} \cos k\phi$

$s^k a_k = s^{-k} c_k$, $b_k s^k = d_k s^{-k}$

$\therefore \quad a^{2k} a_k = c_k$, $\quad a^{1} a_1 = a^{-1} c_1 - E_0 a$

$a^{2k} b_k = d_k$ $\quad a(a_1 + E_0) = \frac{1}{a} c_1$

$a^2(a_1 + E_0) = c_1$

BC ②: $\varepsilon \frac{\partial}{\partial r} V_{in} = \varepsilon \sum_{k=1}^{\infty} k s^{k-1} (a_k \cos k\phi + b_k \sin k\phi)$

$\varepsilon_0 \frac{\partial}{\partial r} V_{out} = \varepsilon_0 \sum_{k=1}^{\infty} -k s^{-k-1} (c_k \cos k\phi + d_k \sin k\phi) - E_0 \cos\theta$

$\int LHS \cos k\phi \Big|_a = \pi k a^{k-1} a_k \varepsilon$

$\int RHS \cos k\phi \Big|_a = -k\pi a^{-k-1} c_k \varepsilon_0$

$\therefore \varepsilon a^{k-1} a_k = -a^{-k-1} c_k \varepsilon_0$

$\varepsilon_r a^k a_k = -a^{-k} c_k$

$* \; \varepsilon_r a^{2k} a_k = -c_k$

Similarly $* \; \varepsilon_r a^{2k} b_k = -d_k$

$\therefore \begin{cases} \varepsilon_r a^{2k} a_k = -a_k \\ \varepsilon_r a^{2k} b_k = -b_k \end{cases}$ $\quad a_k = 0 = c_k$, $\quad b_k = 0 = d_k$

$\therefore$ All VANISHES!

or do they?

$k=1$:

$LHS = (a_1 \cos\phi + b_1 \sin\phi)\,\varepsilon$

$RHS = (-a^{-2}(c_1 \cos\phi + d_1 \sin\phi) - E_0 \cos\phi)\,\varepsilon_0$

do FM with $\cos\phi$:

$\varepsilon a_1 = (-\frac{1}{a^2} c_1 - E_0)\,\varepsilon_0$

$\varepsilon_r a_1 = -\frac{1}{a^2} a^{2k} a_k - E_0$, $k=1$

$$(\varepsilon_r + 1) a_1 = -E_0$$

$$a_1 = -\frac{E_0}{\varepsilon_r + 1}$$

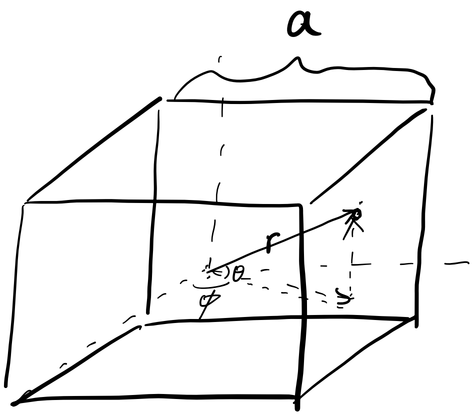
$$C_1 = a^2 (a_1 + E_0)$$
$$= a^2 \left(\frac{E_0}{\varepsilon_r + 1} + E_0 \right)$$

$$\therefore \quad V_{in} = -s \frac{E_0}{\varepsilon_r + 1} \cos(\phi)$$
$$V_{out} = -\frac{1}{s} a^2 \left(\frac{E_0}{\varepsilon_r + 1} + E_0 \right) \cos(\phi) - E_0 s \cos(\phi)$$

now $\varepsilon_r = 1 + \chi_e$

$$\therefore \begin{cases} V_{in}(s, \phi, z) = -s \dfrac{E_0}{2 + \chi_e} \cos(\phi) \\ V_{out}(s, \phi, z) = -\dfrac{1}{s} a^2 \left(\dfrac{E_0}{2 + \chi_e} + E_0 \right) \cos(\phi) - E_0 s \cos(\phi) \end{cases}$$

Problem 4 (4.34 P_{205})



$$\vec{P} = k\vec{r}$$

$$r^2 = x^2 + y^2 + z^2$$

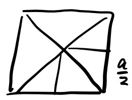
$$\sigma_b \equiv \vec{P}\cdot\hat{n}, \quad \rho_b \equiv -\nabla\cdot\vec{P}$$

$$\therefore \ \sigma_b = k(\hat{x}x + \hat{y}y + \hat{z}z)\cdot\hat{n}, \quad \rho_b = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr)$$

$$= \begin{cases} \frac{ka}{2} \ (+x), & -\frac{ka}{2} \ (-x) \\ \frac{ka}{2} \ (+y), & -\frac{ka}{2} \ (-y) \\ \frac{ka}{2} \ (+z), & -\frac{ka}{2} \ (-z) \end{cases} \qquad = -\frac{1}{r^2} 3r^2 k$$

$$= -3k \text{, uniform}$$

$\theta =$



Now we add up:

$$E_{tot\ Surface} = \oint \sigma_b \, dA$$

$$= \frac{6ka}{2} \times a^2$$

$$= 3ka^3$$

$$E_{tot\ In} = -3k \times a^3$$

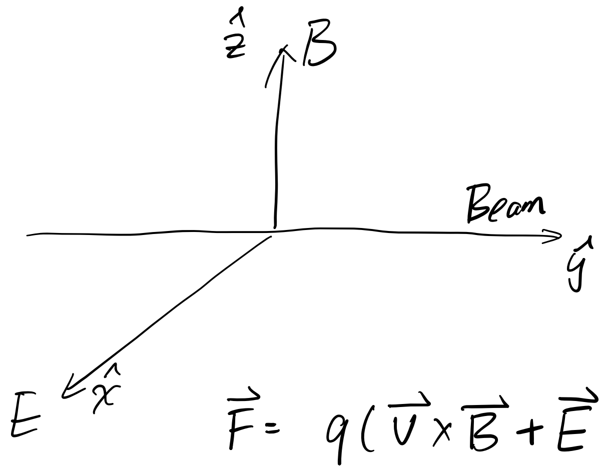
$$= -3ka^3$$

$$\therefore E_{tot} = 3ka^3 - 3ka^3$$

$$= 0$$

Problem 5 (5.3)

a)



$$\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$$

$$F_x = q(v_y B_z + E_x) = 0$$

$$v_y = -\frac{E_x}{B_z}$$

$$v = -\frac{E}{B}\hat{y} = -\frac{|\vec{E}|}{|\vec{B}|}\hat{y}$$

b) $$|a| = \frac{v^2}{R} = \frac{|qvB|}{m}$$

$$\frac{v}{R} m = qB$$

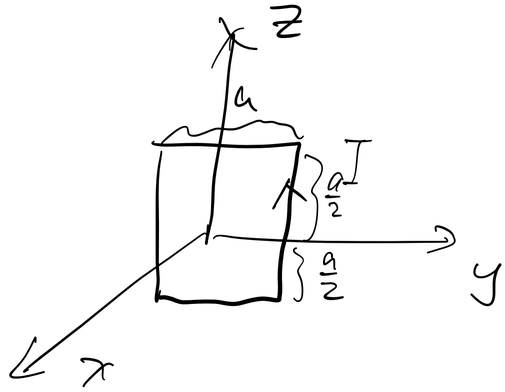
$$\frac{v}{R}\frac{1}{B} = \frac{q}{m}$$

$$\frac{q}{m} = \frac{1}{BR}\cdot\frac{-E}{B}$$

$$= -\frac{E}{B^2 R}$$

Problem 6 (5.4 P221)

$\vec{B} = kz\hat{x}$



$F = I\int(\vec{dl} \times \vec{B})$

$= I\left(k\frac{a}{2} \times a + k\frac{a}{2} \times a\right)$

$= Ika^2$