EE564 First Project II: TESLA Model S Induction Motor

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Specifications

In this project, design of the induction motor that is used in **Tesla Model S**. Normally it has different variations, to keep things simple; **RWD 85 Model** will be used that has the following specs:

- **Maximum Power:** 270 kW
- **Maximum Torque:** 441 Nm
- **Top Speed:** 225 km/h

Except for these given specs, these are also found from internet:

- **Number of poles:** 4
- Maximum RPM value of our motor is 21848 RPM. This value is calculated by considering Tesla Model S has 21' tires and 9.73 to 1 gear ratio.

If we assume average speed is 85 km/h. Then rated RPM value of motor will be 7960 RPM.

```
In this case supply frequency will be 265 Hz.
```

- **Number of phases:** 3
- **Line supply voltage:** 400 V
- **Rated Power:** 185 kW

Main Dimensions of Stator Core

Boldea's The Induction Machine Handbook is going to be used to determine parameters and dimensions of motor. In Chapter 15, it is explained that $D_{is}^2 L$ output constant concept will be used. For internal stator diameter formula below will be used:

$$D_{is}^3 = \frac{2pp_1 S_{gap}}{\pi \lambda f C_0}$$

To be able to calculate D_{is}, airgap power is needed.

At this point targeted efficiency is taken as 95 %.

Power factor is taken as 0.89

Another required parameter to be able to calculate airgap power is Ke that is defined as E1 to Vin ratio in equation 14.8.

$$K_E \approx 0.98 - 0.005 p_1$$

Now everything is ready for airgap apparent power:

$$S_{gap} = \frac{K_E P_n}{\eta_n \cos \Phi}$$

Airgap power is calculated as 212.2 KVA.

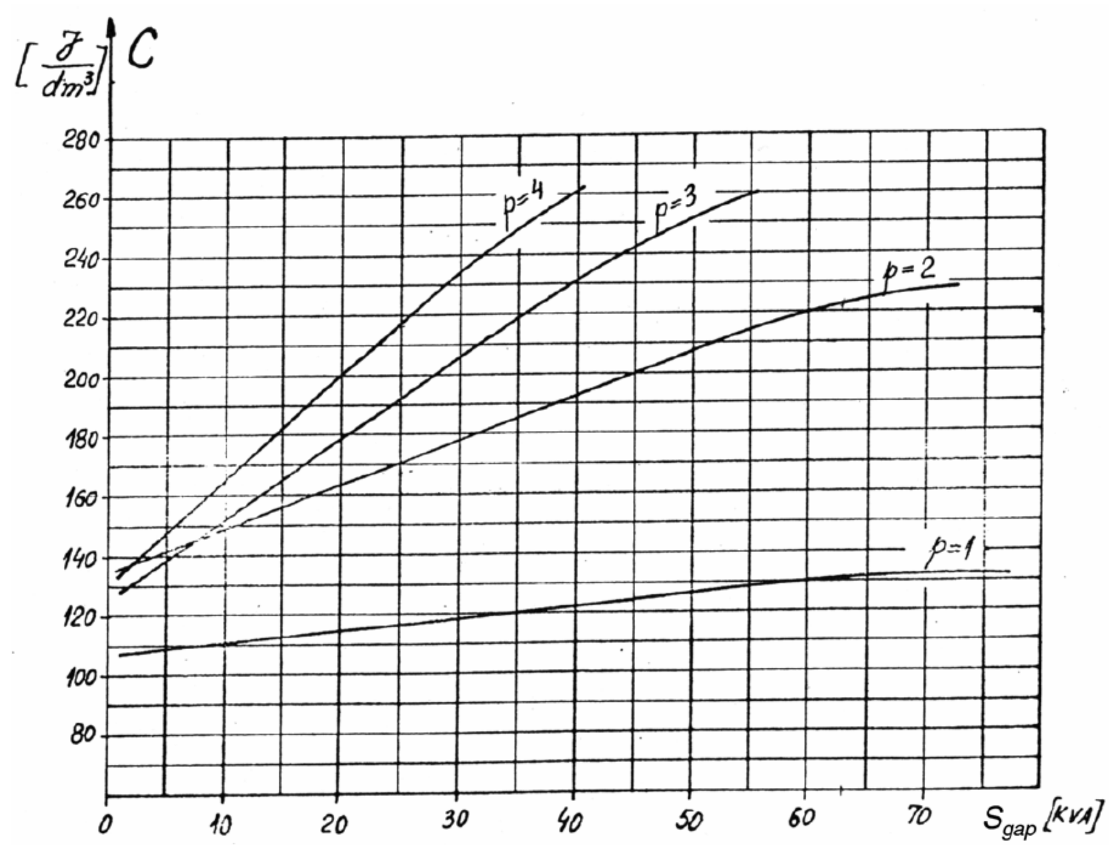
After this calculation, missing parameters are stack aspect ratio and Esson's constant Co. Stack aspect ratio is selected from table below:

Table 15.1. Stack aspect ratio λ

$2p_1$	2	4	6	8
λ	0.6 – 1.0	1.2 – 1.8	1.6 – 2.2	2 -3

It is selected as 1.5.

Esson's constant is selected using Figure 14.14.



Because our calculated airgap power is out of figure's range it isn't possible to read a certain value but for two pole-pairs after 60 kVA Esson's constant starts to saturate and for our airgap apparent power this value is taken as 240 J/dm^3.

Now we are ready to calculate internal stator diameter:

Internal diameter of stator is calculated as 17.827 cm.

For realistic dimensions it is going to be taken as 17.9 cm.

Now we can calculate stack length, deriving its formula from equation 15.2:

$$L = \frac{\lambda \pi D_{is}}{2p}$$

Stack length L is 21.09 cm.

For being realistic it is going to be taken as 21.1 cm.

By using equation 14.14 it is possible to calculate the pole pitch:

$$\tau = \frac{\pi D_{is}}{2p}$$

Pole pitch is 14.06 cm.

Next step is deciding external stator diameter. For its calculation, table below will be used.

Table 15.2. Inner/outer stator diameter ratio

$2p_1$	2	4	6	8
$\frac{D_{is}}{D_{out}}$	0.54 – 0.58	0.61 – 0.63	0.68 – 0.71	0.72 – 0.74

It gives us information about ratio of internal and external stator diameters. For 4 poles this ratio will be taken as 0.61.

External diameter of stator is calculated as 29.344 cm.

For realistic dimensions it is going to be taken as 29.4 cm.

For suitable airgap calculation book's equation of 14.38 may be used as well as the equation defined during the EE564 lecture of 6th April. Here it is important to remind that the minimum airgap is 0.2 mm.

Formula discussed in the lecture is as follows:

$$airgap = 0.18 + 0.006P^{0.4}mm$$

Book equation of 14.38 is

$$airgap = 0.1 + 0.012P^{\frac{1}{3}}mm$$

As known, too small airgap would produces large space airgap field harmonics and additional losses while a too large one would reduce the power factor and efficiency. Therefore, average of these two calculated airgap values will be used as actual airgap value.

Airgap is calculated as 0.8656 mm.

For being realistic it is going to be taken as 0.87 mm.

The Stator Winding

Following James Hendershot's lecture notes, for 4 poles and 185 kW of rated power Stator slot number will be selected. Our rated power is nearly 250 HP and from table below, it is advised to choose 58 stator slots for our case.

HP	2 POLE			4 POLE		
	STD	EM	XE	STD	EM	XE
1	32/24	32/24	32/24	28/36	45/36	45/36
1.5	32/24	32/24	32/24	28/36	45/36	45/36
2	32/24	32/24	32/24	28/36	45/36	45/36
3	32/24	32/24	32/2	28/36	45/36	45/36
5	32/24	32/24	32/24	28/36	45/36	45/36
7.5	28/36	28/36	28/36	28/36	45/36	45/36
10	28/36	28/36	28/36	28/36	45/36	45/36
15	28/36	28/36	28/36	40/48	40/48	40/48
20	28/36	20/36	28/36	40/48	40/48	40/48
25	28/36	28/36	28/36	40/48	40/48	40/48
30	28/36	28/36	28/36	40/48	40/48	40/48
40	28/36	28/36	28/36	40/48	40/48	40/48
50	28/36	28/36	28/36	40/48	40/48	40/48
60	38 or 40/48	38 or 40/48	38 or 40/48	47/60	47/60	47/60
75	38 or 40/48	38 or 40/48	38 or 40/48	47/60	47/60	47/60
100	38 or 40/48	38 or 40/48	38 or 40/48	73/60	47/60	47/60
125	38 or 40/48	38 or 40/48	38 or 40/48	58/72	58/72	58/72
150	38 or 40/48	38 or 40/48	38 or 40/48	58/72	58/72	58/72
200	38 or 40/48	38 or 40/48	38 or 40/48	58/72	58/72	56or58/72
250	38 or 40/48	38 or 40/48	38 or 40/48	56or58/72	56or58/72	56or58/72

Here we should remember that the total number of slots per stator should be divisible by the number of phases. So it should be a number that is multiple of 3.

$$q = \frac{N_s}{2pm}$$

If we think about its formula above (taken from book; 4.7), it is possible to see that choosing Ns/m integer doesn't guarantee that q is an integer. In fact it doesn't have to be an integer and may be selected as a fraction. But in most induction machines, q is an integer to provide complete (pole to pole) symmetry for the winding. So in our case Ns must be multiple of 2pm=12. Advised number is 58, so Ns would be taken as 60. It was tried and seen that number of conductors, flux density and other parameteres don't meet the expectations. So it is going to be taken as 48.

```
Number of slots per pole per phase is 4 .
```

Now we should decide pitch factor. It can be selected as 5/6 to reduce 5th harmonic and reduce 7th harmonics. So two layered winding with chorded coils will be used.

```
Selected pitch angle is 150 degree .
```

It is possible to calculate the electrical angle between emfs in neighboring slots α_{ec}

$$\alpha_{ec} = \frac{2\pi p}{N_s}$$

```
It is 0.262 radian means 15 degree.
```

Now we can calculate pitch factor. Due to chorded coils of stator,induced voltage will drop but by means of harmonics we will have better results.

$$k_p = sin(\frac{\lambda}{2})$$

```
Pitch factor is calculated as 0.97 .
```

Using formula below it is possible to calculate distribution factor.

$$k_d = \frac{sin(q\frac{\alpha}{2})}{qsin\frac{\alpha}{2}}$$

```
Distribution factor is calculated as 0.96 .
```

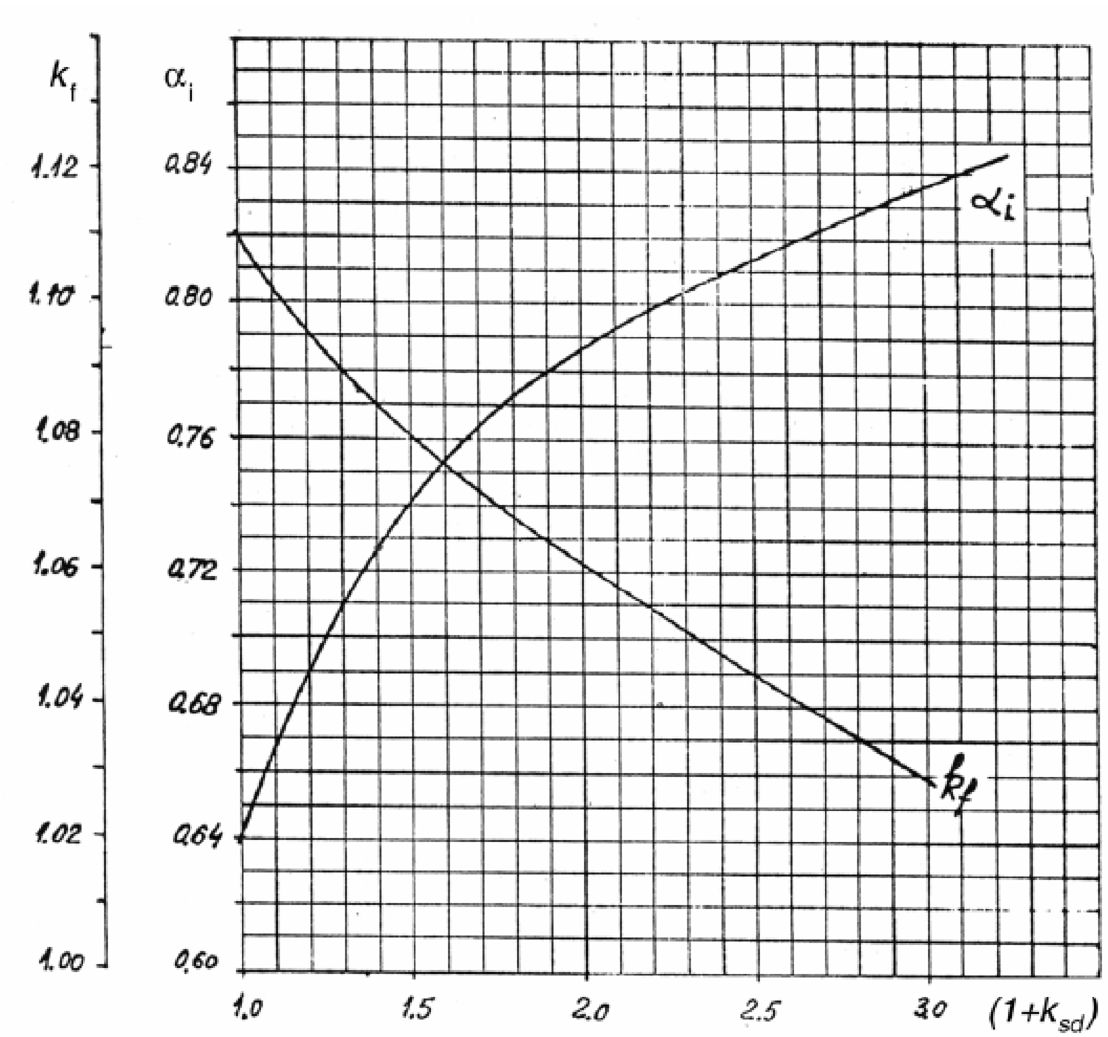
Multiplicaiton of distribution and pitch factors are called as winding factor.

```
Winding factor is calculated as 0.93 .
```

Using recommended intervals given in 15.11, it is possible to select the airgap flux density. For 4 poles suggested interval is 0.65 to 0.78 Tesla. To decrease iron losses minimum of this interval will be taken as airgap flux density.

The pole coefficient α_i and form factor Kf depend on the tooth saturation factor 1+Kst. If 1+Kst is taken as 1.4 than Kst is 0.4.

Using this value and graph below, it is possible to select form factor and flux density shape factor.



    Form factor is selected as 1.085 .

    Flux density shape factor is selected as 0.729 .

Using these coefficients it is possible to calculate pole flux.

$$\phi = \alpha_i \tau L B_g$$

    Pole flux is calculated as 15.137 mWb.

The number of per phase can be calculated using formula below given with (15.12).

$$N_{ph} = \frac{K_E V_{ph}}{4 K_f K_w f \phi}$$

    The number of turns per phase is calculated as 17.0 turns/phase.

The number of conductors per slot ns can be calculated using formula below:

$$n_s = \frac{a_1 N_{ph}}{p_1 q}$$

Here a1 ise the number of current paths in parallel and will be taken as 1 for our case.

```
It is calculated as 2.13 .
```

It should be an even number as there are two distinct coils per slot in a double layer winding. So ns is selected as 2.

If we turn back and recalculate the actual airgap flux density:

```
Recalculated airgap flux density is 0.745 T .
```

Now we can calculate rated current. 15.16 formula will be used:

$$I_{in} = \frac{P_n}{\eta cos\phi_n \sqrt{3} V_1}$$

```
Rated phase current is calculated as 315.8 A .
```

To be able to calculate wire cross section, current density will be selected first. Here, recommendation 15.17 will be followed and for 4 poles current density will be taken as 6 A/mm^2.

$$A_{co} = \frac{I_{in}}{J_{cos}}$$

```
Magnetic cross section area is calculated as 52.64 mm^2.
```

Using cross-sectional wire area information it is possible to calculate the diameter wire gauge.

$$d_{co} = \sqrt{\frac{4A_{co}}{\pi}}$$

```
Wire gauge diameter is 8.19 mm.
```

Because this diameter is not small, 30 conductors will be paralleled to decrase the diameter of each conductor.

```
New wire gauge diameter is 1.49 mm.
```

By using table 15.3, it is possible to jump from wire diameter to insulated wire diameter.

```
Insulated wire gauge diameter is 1.53 mm.
```

Stator Slot Sizing

Since we know wire diameter, number of conductors in parallel and number of turns per slot, it is possible for us to calculate the slot area. Only missing parameter is fill factor selection. For round wire and at our rated power level it is advised to be taken between 0.4 and 0.44. So it is selected as 0.44. For useful area calculation formula (15.21) will be used:

$$A_{su} = \frac{\pi d_{co}^2 a_p n_s}{4 K_{fill}}$$

```
Calculated useful slot area is 239.256 mm^2.
```

There are two recommended stator slot shapes in the book as follows:

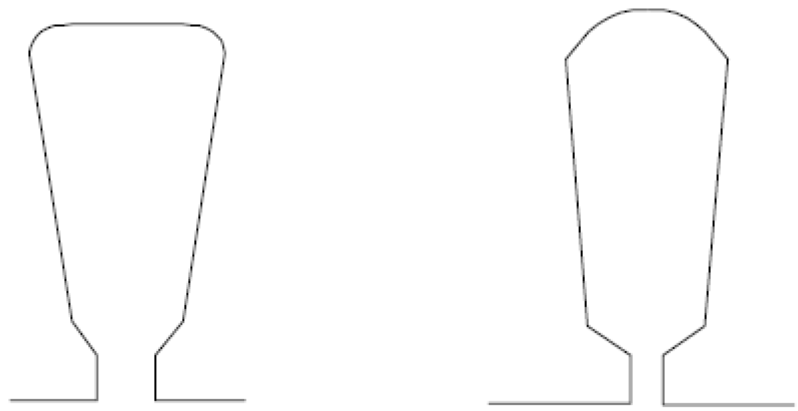


Figure 15.4 Recommended stator slot shapes

For ease of calculation and to be able to use explained slot geometry, left-hand side of stator slot shape is selected.

For this shape, explained lengths and are as in the figure below:

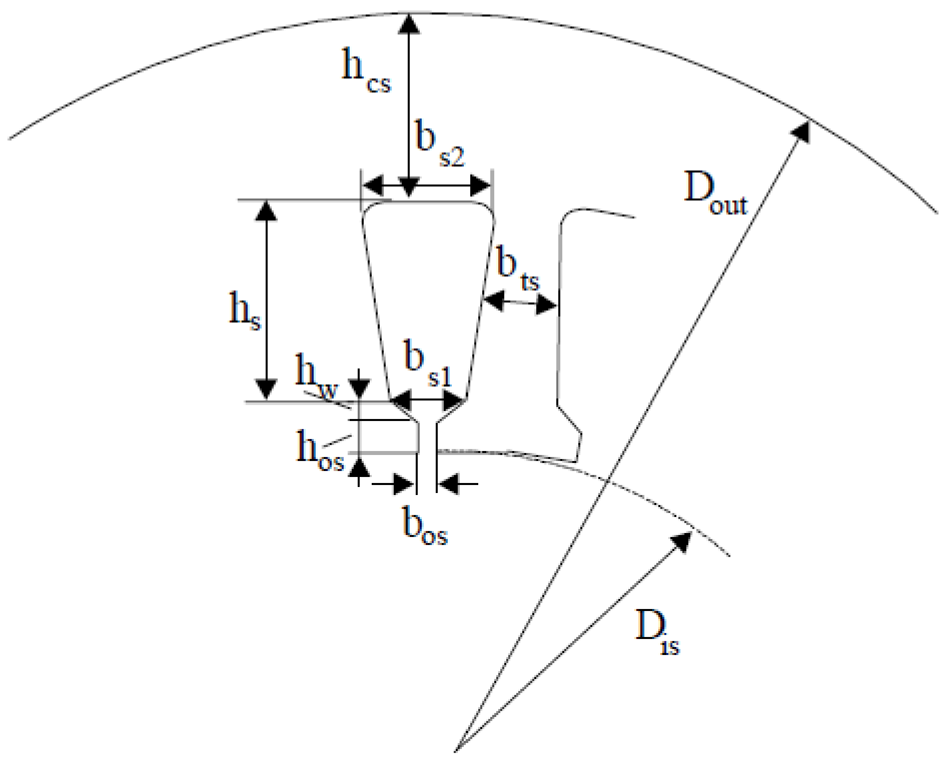


Figure 15.5 Stator slot geometry

Here, some parameters will be selected via following books' suggestions and some of them will be calculated.

Suggested variables are bos, hos and hw. bos can be defined as slot opening length and it is selected as 2 mm. hos is the height of slot opening and it is taken as 1 mm. hw is wedge height and it is selected as 3 mm.

Assuming that all the airgap flux passes through stator teeth:

$$B_g \tau_s L \approx B_{ts} b_{ts} L K_{Fe}$$

Here, Kfe is a constant to include lamination insulation's effect and suggested to be defined as 0.96. It is suggested to have a tooth flux density between 1.5 and 1.65 T. Let us take it as 1.6 T and determine bts:

Slot pitch isn't calculated yet, it is possible to use equation (15.3) for it.

$$\tau_s = \frac{\tau}{3q}$$

```
Slot pitch is 11.716 mm.

Tooth width is 5.685 mm.

It is better to take it as 5.6 mm.
```

For this value let us recalculate flux density of tooth:

```
Recalculated tooth-flux density is 1.62 Tesla.
```

This value is still inside the suggested range.

With the variables we know, using equation (15.23) it is possible to calculate the slot lower width:

$$b_{s1} = \frac{\pi(D_{is}+2h_{os}+2h_w)}{N_s} - b_{ts}$$

```
Lower slot width is 6.6 mm.
```

At this point missing variables are slot height and upper slow width. If slot area's round corners are ignored and area is taken as a trapezoid, its are would be;

$$A_{su} = h_s \frac{b_{s1}+b_{s2}}{2}$$

Also, we have(15.25) formula as follows:

$$b_{s2} \approx b_{s1} + 2h_s tan\frac{\pi}{N_s}$$

From these 2 equations;

$$b_{s2} = \sqrt{4A_{su} tan\frac{\pi}{N_s} + b_{s1}^2}$$

```
Upper slot width is 10.3 mm.
```

$$h_s = \frac{2A_{su}}{b_{s1}+b_{s2}}$$

Slot height is 28.3 mm.

Now we can proceed in calculating mmf of airgap and teeth. The airgap mmf is:

$$F_{mg} \approx K_c * g * \frac{B_g}{\mu_0}$$

Here, Kc is the Carter's cofficient and it helps us to consider airgap surface as smooth and make our calculations directly. It is expected that it is greater but close to 1; its formula is:

$$K_c = \frac{\tau_s}{\tau_s - b_e}$$

$$b_e = K b_{os}$$

$$K = \frac{\frac{b_{os}}{g}}{5 + \frac{b_{os}}{g}}$$

Carter coefficient is calculated as 1.06 .

Now airgap mmf may be calculated:

The airgap mmf is 545.336 Aturns.

Using tooth flux density and tooth's related heights we may calculate also mmf of stator tooth. Its formula is given with (15.30) and as follows:

$$F_{mts} = H_{ts}(h_s + h_{os} + h_w)$$

Only missing parameter is H of stator tooth and its value will be taken from table 15.4 of lamination magnetization curve.

H of stator tooth is selected as 2960 A/m for 1.62 Tesla.

So the stator tooth mmf is 95.608 Aturns.

Using formula given in (15.29) it is possible to calculate rotor tooth mmf since we take 1 + Kst value as 1.4 at the earlier part of design.

$$1 + K_{st} = 1 + \frac{F_{mts} + F_{mtr}}{F_{mg}}$$

Rotor tooth mmf is found as 122.526 Aturns.

As this value is only slightly larger than that of stator tooth, we may go on with the design process.

Only missing dimension for stator side is its back core and this value may be calculated with the formula below (15.32):

$$h_{cs} = \frac{D_{out} - (D_{is} + 2(h_s + h_{os} + h_w))}{2}$$

Back core is calculated as 2.5 cm.

Here, we should take a look at to the flux density to avoid a saturation.

$$B_{cs} = \frac{\phi}{2Lh_{cs}}$$

```
Back core flux density is 1.42 Tesla.
```

This is an acceptable value, therefore outer diameter won't be changed.

Rotor Slots

First step of designing rotor slots might be deciding the number of rotor slots. An arbitrary selection isn't possible because all stor and rotor slot combinations aren't possible. At this point there are so many resources that suggest the most adequate combination.

Once EE565 lecture notes are reviewed, suggestions are found as :

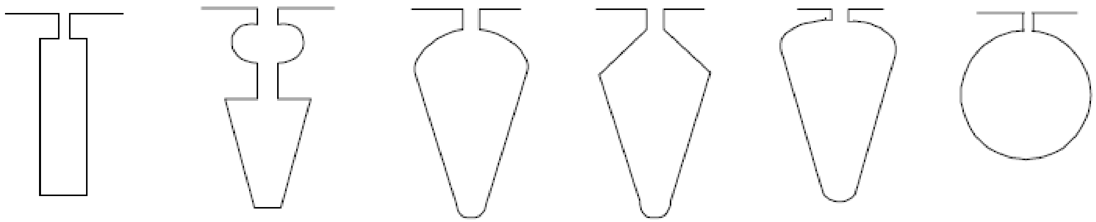
p	Q_s	Q_r
1	24	28, 16, 22
	36	24, 28, 48, 16
	48	40, 52
	60	48
2	36	24, 40, 42, 60, 30, 44
	48	60, 84, 56, 44
	60	72, 48, 84, 44

From the textbook, suggestions are as follows:

Table 15.5. Stator / rotor slot numbers

$2p_1$	N_s	N_r – skewed rotor slots
2	24	18, 20, 22, 28, 30, ,33,34
	36	25,27,28,29,30,43
	48	30,37,39,40,41
4	24	16,18,20,30,33,34,35,36
	36	28,30,32,34,45,48
	48	36,40,44,57,59
	72	42,48,54,56,60,61,62,68,76

For fair play, their intersection number 44 was selected as number of rotor slots at first. But then it couldn't be possible to design rotor slot for our rated power. So this number is selected as 56 in the second attempt. But that one also couldn't be enough. Then 84 is selected and it is OK.



a.) b.) c.) d.) e.) f.)

As we did for stator shape selection, for ease of area calculation textbook's selection will be followed and option c will be selected among suggested options.

We can go on the design with calculating the bar current:

$$I_b = K_I \frac{2mN_s k_w}{N_r} I_{in}$$

Here K# will be calculated with the formula below:

$$K_I \approx 0.8cos\phi_{In} + 0.2$$

This constant calculated as 0.912 .

Bar current is 324.2 A.

For high efficiency, the rotor current density in the rotor bar is selected as 3.42A/mm^2. Using this value, rotor slot area may be calculated.

$$A_b = \frac{I_b}{J_b}$$

The rotor slot area is 94.80 mm^2.

Since we have calculated bar current, it is possible to calculate also end ring current using formula below (15.37)

$$I_{er} = \frac{I_b}{2sin\frac{\pi p}{N_r}}$$

End ring current is 2169.2 A.

Current density of end ring is less than 75 to 80 % of the bar. The middle point will be selected as the multiplier.

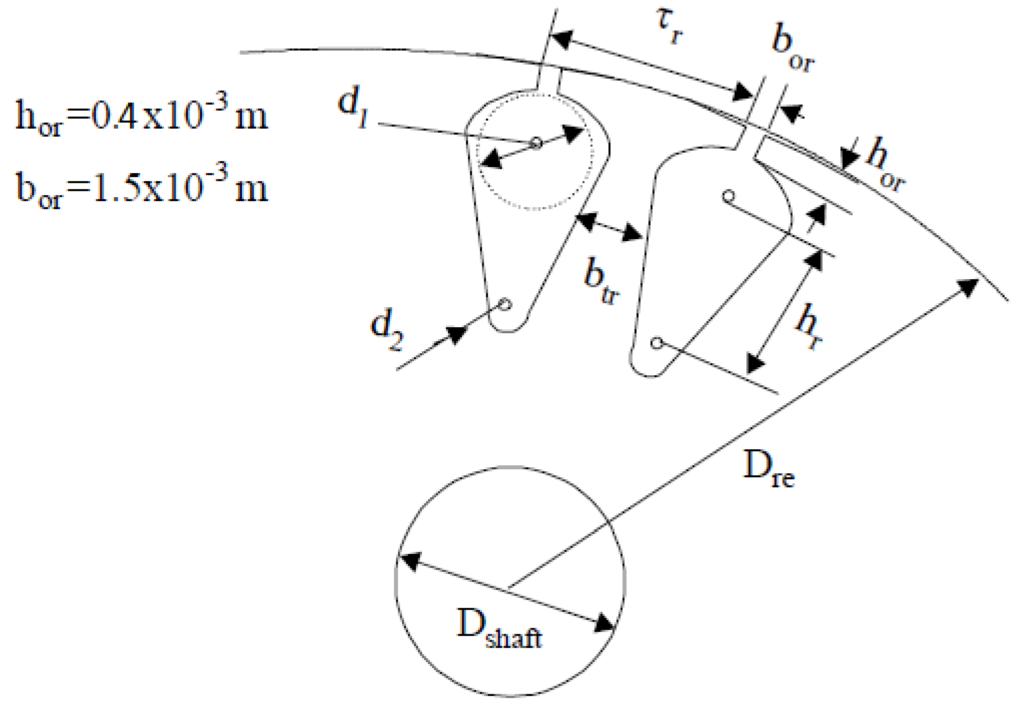
77,5 % of the bar current density is taken and it is 2.65 A/mm^2.

With this variables it is possible to calculate the end ring cross section area:

$$A_{er} = \frac{I_{er}}{J_{er}}$$

The rotor end ring area is 8.18 cm^2.

We may now proceed to rotor slot sizing based on the variables defined on figure below:



Let us calculate the rotor slot pitch using formula below (15.39)

$$\tau_r = \frac{\pi(D_{is}-2g)}{N_r}$$

The rotor slot pitch is 6.63 mm.

Rotor tooth flux density can be selected as 1.65 Tesla. Than it is possible to calculate the tooth width:

$$b_{tr} \approx \frac{B_g}{K_{Fe}B_{tr}}\tau_r$$

Rotor tooth width width is 3.1 mm.

Now d1 diameter may be calculated using formula (15.42)

$$d_1 = \frac{\pi(D_{re}-2h_{or})-N_r b_{tr}}{\pi+N_r}$$

This diameter is 3.5 mm.

To completely define the rotor slot geometry, it is needed to use slot area equations (15.43) and (15.44)

$$A_b = \frac{\pi}{8}(d_1^2+d_2^2)+\frac{(d_1+d_2)h_r}{2}$$

$$d_1 - d_2 = 2h_r tan\frac{\pi}{N_r}$$

From the second equation it is found that d2 is equal to 3.5-0.0374hr . It will be substated in the first equation.

By this way hr is found as 29.8 mm.

So d2 diameter is 1.3 mm.

Now let's verify the rotor teeth mmf for Btr is 1.65 Tesla and H of rotor tooth is 3460 A/m.

$$F_{mtr} = H_{tr}(h_r + h_{or} + \frac{d_1+d_2}{2})$$

Rotor tooth mmf is recalculated as 112.744 Aturns.

Previous calculation was 122.526 and it is acceptable.

Now we can calculate a rotor back core allowing a flux density between 1.4 and 1.7. It is taken as 1.7 T.

$$h_{cr} = \frac{\phi}{2LB_{cr}}$$

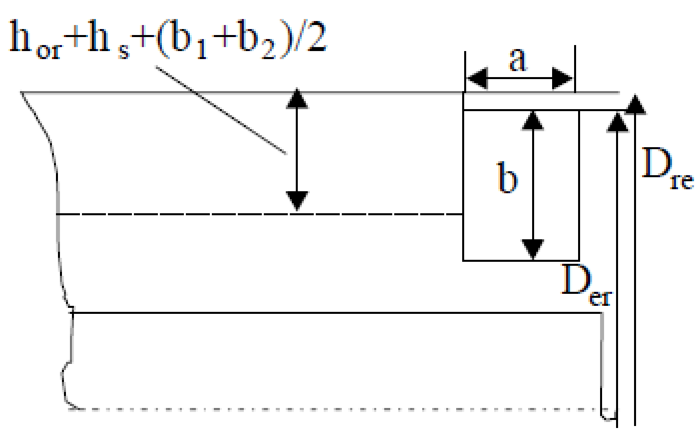
Rotor back core is 21.1 mm.

For rotor, missing part is shaft diameter and its details. The maximum shaft diameter is:

$$(D_{shaft})_{max} \leq D_{is} - 2g - 2(h_{or} + h_r + h_{cr} + \frac{d_1+d_2}{2})$$

So this maximum value is 69.9 mm.

For end ring detail, two extra lengths will be defined and their explanations are in the figure below:



$$b = 1.1(h_r + hor + \frac{d_1+d_2}{2})$$

a and b are calculated as 22.8 and 35.8 mm.

The Magnetization Current

The magnetization mmf Flm is

$$F_{lm} = 2(K_c g \frac{B_g}{\mu_0} + F_{mts} + F_{mtr} + F_{mcs} + F_{mcr})$$

To be able to calculate unknown mmfs in this equation, again Carter's coefficient wil be used. Its first multiplier was already calculated. Here its second part that is related with the rotor will be calculated.

Rotor slot opening is 1.5 mm.

$K_{c2} = \frac{\tau_r}{\tau_r - b_e}$

$b_e = K b_{or}$

$K = \frac{\frac{b_{or}}{g}}{5 + \frac{b_{or}}{g}}$

Carter coefficient was calculated as 1.06.

Now its total recalculated value is 1.14.

Since these values are close to each other, new value will be used in next calculations.

For calculating back core mmfs, another missing parameter is subunitary empirical coefficient that defines an average length of flux path in the back core. Formula (15.60) will be used for its calculation:

$C_{cs,r} \approx 0.88 e^{-0.4 B_{cs,r}{}^2}$

For back core calculations, formula below will be used:

$F_{mcs,r} = C_{cs,r} \frac{\pi (D_{out,shaft} - h_{cs,r})}{2p} H_{cs,r}$

Bcs and Bcr were 1.42 and 1.70 Tesla.

From table 15.4 corresponding H values are 905 and 4800

Back core mmfs are 232.5 and 257.0 Aturns

Now we can calculate magnetization mmf:

It is calculated as 2590 Aturns

For magnetization current formula (15.62) will be used:

$I_\mu = \frac{\pi p (f_{lm}/2)}{3\sqrt{2} N_{ph} k_w}$

The magnetization current is calculated as 121.7 A

Its (p.u.) value is 0.39

Resistances and Inductances

For stator phase resistance formula (15.63) will be used.

$R_s = \rho_{Co} \frac{l_c N_{ph}}{A_{co} a_1}$

Here coil length includes the active part 2L and the end connection part 2lend

$$l_c = 2(L + l_{end})$$

End connection length depends on the coil span, number of poles, shape of coils and number of layers in the winding. For its calculation formula (15.65) will be used.

$$l_{end} = 2y - 0.02$$

Here, y is the coil span and can be calculated as

$$y = \beta\tau$$

In coil span formula, beta is chording factor and it is already selected as 5/6.

The end connection length is 21.4 cm

So the coil length is 21.4 cm

Another consideration for stator resistance is the copper resistivity. At room temperature its value is $1.7810^{-8}\Omega m$ and at 115 degree it increases 37%. Because we don't know the rated temperature yet and this value will be taken for 90 degrees.

The stator resistance is 6.157 mohm

Now we can continue with rotor/end ring segment equivalent resistance. Its value will be calculated using formula (15.70)

$$R_{be} = \rho_{Al}\left[\frac{LK_R}{A_b} + \frac{l_{er}}{2A_{er}sin^2(\frac{\pi p}{N_r})}\right]$$

Here, end ring segment length is 214.3 mm

Here using formulas (15.72) and (15.73) skin effect resistance coefficient is calculated.

It is 5.47

If we consider the same temperature again;

Rbe is calculated as0.294 mohm

In the equivalent circuit we use referred rotor resistance. Therefore its referred value is going to be calculated using formula (15.74)

$$R_r' = \frac{4m}{N_r}(N_{ph}k_w)^2 R_{be}$$

Referred rotor resistance is 10.44 mohm

For calculating stator phase leakage reactance, slot differential and end ring connection coefficients are going to be calculated first.

The stator phase reactance is 114.05 mohm

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