

Columbia University: CU Later Team Reference Document

Kevin Yang, Innokentiy Kaurov, Eric Yuang Shao

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Templates

Ken's template

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 #define all(v) (v).begin(), (v).end()
4 typedef long long ll;
5 typedef long double ld;
6 #define pb push_back
7 #define sz(x) (int)(x).size()
8 #define fi first
9 #define se second
10 #define endl '\n'
```

Kevin's template

```
1 // paste Kaurov's Template, minus last line
2 typedef vector<int> vi;
3 typedef vector<ll> vll;
4 typedef pair<int, int> pii;
5 typedef pair<ll, ll> pll;
6 const char nl = '\n';
7 #define forn(i, n) for (int i = 0; i < int(n); i++)
8 ll k, n, m, u, v, w, x, y, z;
9 string s;
10
11 bool multiTest = 1;
12 void solve(int tt){
13 }
14
15 int main(){
16     ios::sync_with_stdio(0);cin.tie(0);cout.tie(0);
17     cout<<fixed<< setprecision(14);
18
19     int t = 1;
20     if (multiTest) cin >> t;
21     forn(ii, t) solve(ii);
22 }
```

Kevin's Template Extended

- to type after the start of the contest

```
1 typedef pair<double, double> pdd;
2 const ld PI = acos(-1);
3 const ll mod7 = 1e9 + 7;
4 const ll mod9 = 998244353;
5 const ll INF = 2*1024*1024*1023;
6 #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
7 #include <ext/pb_ds/assoc_container.hpp>
8 #include <ext/pb_ds/tree_policy.hpp>
9 using namespace __gnu_pbds;
10 template<class T> using ordered_set = tree<T, null_type,
11     ↪ less<T>, rb_tree_tag, tree_order_statistics_node_update>;
12 vi d4x = {1, 0, -1, 0};
13 vi d4y = {0, 1, 0, -1};
14 vi d8x = {1, 0, -1, 0, 1, 1, -1, -1};
15 vi d8y = {0, 1, 0, -1, 1, -1, 1, -1};
16 mt19937
17     ↪ rng(chrono::steady_clock::now().time_since_epoch().count());
```

Geometry

Point basics

```
1 const ld EPS = 1e-9;
2
3 struct point{
4     ld x, y;
5     point() : x(0), y(0) {}
6     point(ld x_, ld y_) : x(x_), y(y_) {}
7
8     point operator+ (point rhs) const{
```

```
9         return point(x + rhs.x, y + rhs.y);
10    }
11    point operator- (point rhs) const{
12        return point(x - rhs.x, y - rhs.y);
13    }
14    point operator* (ld rhs) const{
15        return point(x * rhs, y * rhs);
16    }
17    point operator/ (ld rhs) const{
18        return point(x / rhs, y / rhs);
19    }
20    point ort() const{
21        return point(-y, x);
22    }
23    ld abs2() const{
24        return x * x + y * y;
25    }
26    ld len() const{
27        return sqrt(abs2());
28    }
29    point unit() const{
30        return point(x, y) / len();
31    }
32    point rotate(ld a) const{
33        return point(x * cosl(a) - y * sinl(a), x * sinl(a) + y *
34    ↪ cosl(a));
35    }
36    friend ostream& operator<<(ostream& os, point p){
37        return os << "(" << p.x << "," << p.y << ")";
38    }
39
40    bool operator< (point rhs) const{
41        return make_pair(x, y) < make_pair(rhs.x, rhs.y);
42    }
43    bool operator== (point rhs) const{
44        return abs(x - rhs.x) < EPS && abs(y - rhs.y) < EPS;
45    };
46
47    ld sq(ld a){
48        return a * a;
49    }
50    ld smul(point a, point b){
51        return a.x * b.x + a.y * b.y;
52    }
53    ld vmul(point a, point b){
54        return a.x * b.y - a.y * b.x;
55    }
56    ld dist(point a, point b){
57        return (a - b).len();
58    }
59    bool acw(point a, point b){
60        return vmul(a, b) > -EPS;
61    }
62    bool cw(point a, point b){
63        return vmul(a, b) < EPS;
64    }
65    int sgn(ld x){
66        return (x > EPS) - (x < EPS);
67    }
```

Line basics

```
1 struct line{
2     ld a, b, c;
3     line() : a(0), b(0), c(0) {}
4     line(ld a_, ld b_, ld c_) : a(a_), b(b_), c(c_) {}
5     line(point p1, point p2){
6         a = p1.y - p2.y;
7         b = p2.x - p1.x;
8         c = -a * p1.x - b * p1.y;
9     }
10 };
11
12 ld det(ld a11, ld a12, ld a21, ld a22){
13     return a11 * a22 - a12 * a21;
```

```

14 }
15 bool parallel(line l1, line l2){
16     return abs(vmul(point(l1.a, l1.b), point(l2.a, l2.b))) <
        ↳ EPS;
17 }
18 bool operator==(line l1, line l2){
19     return parallel(l1, l2) &&
20     abs(det(l1.b, l1.c, l2.b, l2.c)) < EPS &&
21     abs(det(l1.a, l1.c, l2.a, l2.c)) < EPS;
22 }

```

Line and segment intersections

```

1 // {p, 0} - unique intersection, {p, 1} - infinite, {p, 2} -
    ↳ none
2 pair<point, int> line_inter(line l1, line l2){
3     if (parallel(l1, l2)){
4         return {point(), l1 == l2? 1 : 2};
5     }
6     return {point(
7         det(-l1.c, l1.b, -l2.c, l2.b) / det(l1.a, l1.b, l2.a,
        ↳ l2.b),
8         det(l1.a, -l1.c, l2.a, -l2.c) / det(l1.a, l1.b, l2.a,
        ↳ l2.b)
9     ), 0};
10 }
11
12 // Checks if p lies on ab
13 bool is_on_seg(point p, point a, point b){
14     return abs(vmul(p - a, p - b)) < EPS && smul(p - a, p - b) <
        ↳ EPS;
15 }
16
17 /*
18 If a unique intersection point between the line segments going
    ↳ from a to b and from c to d exists then it is returned.
19 If no intersection point exists an empty vector is returned.
20 If infinitely many exist a vector with 2 elements is returned,
    ↳ containing the endpoints of the common line segment.
21 */
22 vector<point> segment_inter(point a, point b, point c, point
    ↳ d) {
23     auto oa = vmul(d - c, a - c), ob = vmul(d - c, b - c), oc =
        ↳ vmul(b - a, c - a), od = vmul(b - a, d - a);
24     if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0) return
        ↳ {(a * ob - b * oa) / (ob - oa)};
25     set<point> s;
26     if (is_on_seg(a, c, d)) s.insert(a);
27     if (is_on_seg(b, c, d)) s.insert(b);
28     if (is_on_seg(c, a, b)) s.insert(c);
29     if (is_on_seg(d, a, b)) s.insert(d);
30     return {all(s)};
31 }
32

```

Distances from a point to line and segment

```

1 // Distance from p to line ab
2 ld line_dist(point p, point a, point b){
3     return vmul(b - a, p - a) / (b - a).len();
4 }
5
6 // Distance from p to segment ab
7 ld segment_dist(point p, point a, point b){
8     if (a == b) return (p - a).len();
9     auto d = (a - b).abs2(), t = min(d, max((ld)0, smul(p - a, b
        ↳ - a)));
10    return ((p - a) * d - (b - a) * t).len() / d;
11 }

```

Polygon area

```

1 ld area(vector<point> pts){
2     int n = sz(pts);
3     ld ans = 0;
4     for (int i = 0; i < n; i++){
5         ans += vmul(pts[i], pts[(i + 1) % n]);
6     }
7     return abs(ans) / 2;
8 }

```

Convex hull

- Complexity: $O(n \log n)$.

```

1 vector<point> convex_hull(vector<point> pts){
2     sort(all(pts));
3     pts.erase(unique(all(pts)), pts.end());
4     vector<point> up, down;
5     for (auto p : pts){
6         while (sz(up) > 1 && acw(up.end()[-1] - up.end()[-2], p -
        ↳ up.end()[-2])) up.pop_back();
7         while (sz(down) > 1 && cw(down.end()[-1] - down.end()[-2],
        ↳ p - down.end()[-2])) down.pop_back();
8         up.pb(p), down.pb(p);
9     }
10    for (int i = sz(up) - 2; i >= 1; i--) down.pb(up[i]);
11    return down;
12 }

```

Point location in a convex polygon

- Complexity: $O(n)$ precalculation and $O(\log n)$ query.

```

1 void prep_convex_poly(vector<point>& pts){
2     rotate(pts.begin(), min_element(all(pts)), pts.end());
3 }
4
5 // 0 - Outside, 1 - Exclusively Inside, 2 - On the Border
6 int in_convex_poly(point p, vector<point>& pts){
7     int n = sz(pts);
8     if (!n) return 0;
9     if (n <= 2) return is_on_seg(p, pts[0], pts.back());
10    int l = 1, r = n - 1;
11    while (r - l > 1){
12        int mid = (l + r) / 2;
13        if (acw(pts[mid] - pts[0], p - pts[0])) l = mid;
14        else r = mid;
15    }
16    if (!in_triangle(p, pts[0], pts[l], pts[l + 1])) return 0;
17    if (is_on_seg(p, pts[l], pts[l + 1]) ||
18        is_on_seg(p, pts[0], pts.back()) ||
19        is_on_seg(p, pts[0], pts[l]))
20        return 2;
21    return 1;
22 }

```

Point location in a simple polygon

- Complexity: $O(n)$.

```

1 // 0 - Outside, 1 - Exclusively Inside, 2 - On the Border
2 int in_simple_poly(point p, vector<point>& pts){
3     int n = sz(pts);
4     bool res = 0;
5     for (int i = 0; i < n; i++){
6         auto a = pts[i], b = pts[(i + 1) % n];
7         if (is_on_seg(p, a, b)) return 2;
8         if (((a.y > p.y) - (b.y > p.y)) * vmul(b - p, a - p) >
        ↳ EPS){
9             res ^= 1;
10        }
11    }

```

```

12     return res;
13 }

```

Minkowski Sum

- For two convex polygons P and Q , returns the set of points $(p + q)$, where $p \in P, q \in Q$.
- This set is also a convex polygon.
- Complexity: $O(n)$.

```

1 void minkowski_rotate(vector<point>& P){
2     int pos = 0;
3     for (int i = 1; i < sz(P); i++){
4         if (abs(P[i].y - P[pos].y) <= EPS){
5             if (P[i].x < P[pos].x) pos = i;
6         }
7         else if (P[i].y < P[pos].y) pos = i;
8     }
9     rotate(P.begin(), P.begin() + pos, P.end());
10 }
11 // P and Q are strictly convex, points given in
12 // counterclockwise order.
13 vector<point> minkowski_sum(vector<point> P, vector<point> Q){
14     minkowski_rotate(P);
15     minkowski_rotate(Q);
16     P.pb(P[0]);
17     Q.pb(Q[0]);
18     vector<point> ans;
19     int i = 0, j = 0;
20     while (i < sz(P) - 1 || j < sz(Q) - 1){
21         ans.pb(P[i] + Q[j]);
22         ld curmul;
23         if (i == sz(P) - 1) curmul = -1;
24         else if (j == sz(Q) - 1) curmul = +1;
25         else curmul = vmul(P[i + 1] - P[i], Q[j + 1] - Q[j]);
26         if (abs(curmul) < EPS || curmul > 0) i++;
27         if (abs(curmul) < EPS || curmul < 0) j++;
28     }
29     return ans;
30 }

```

Half-plane intersection

- Given N half-plane conditions in the form of a ray, computes the vertices of their intersection polygon.
- Complexity: $O(N \log N)$.
- A ray is defined by a point p and direction vector dp . The half-plane is to the **left** of the direction vector.

```

1 // Extra functions needed: point operations, smul, vmul
2 const ld EPS = 1e-9;
3
4 int sgn(ld a){
5     return (a > EPS) - (a < -EPS);
6 }
7 int half(point p){
8     return p.y != 0? sgn(p.y) : -sgn(p.x);
9 }
10 bool angle_comp(point a, point b){
11     int A = half(a), B = half(b);
12     return A == B? vmul(a, b) > 0 : A < B;
13 }
14 struct ray{
15     point p, dp; // origin, direction
16     ray(point p_, point dp_){
17         p = p_, dp = dp_;
18     }
19     point isect(ray l){
20         return p + dp * (vmul(l.dp, l.p - p) / vmul(l.dp, dp));
21     }
22     bool operator<(ray l){
23         return angle_comp(dp, l.dp);
24     }
25 }

```

```

25 };
26 vector<point> half_plane_isect(vector<ray> rays, ld DX = 1e9,
27 // constrain the area to [0, DX] x [0, DY]
28 rays.pb({point(0, 0), point(1, 0)});
29 rays.pb({point(DX, 0), point(0, 1)});
30 rays.pb({point(DX, DY), point(-1, 0)});
31 rays.pb({point(0, DY), point(0, -1)});
32 sort(all(rays));
33 {
34     vector<ray> nrays;
35     for (auto t : rays){
36         if (nrays.empty() || vmul(nrays.back().dp, t.dp) > EPS){
37             nrays.pb(t);
38             continue;
39         }
40         if (vmul(t.dp, t.p - nrays.back().p) > 0) nrays.back() =
41 // t;
42     }
43     swap(rays, nrays);
44 }
45 auto bad = [&] (ray a, ray b, ray c){
46     point p1 = a.isect(b), p2 = b.isect(c);
47     if (smul(p2 - p1, b.dp) <= EPS){
48         if (vmul(a.dp, c.dp) <= 0) return 2;
49         return 1;
50     }
51     return 0;
52 };
53 #define reduce(t) \
54     while (sz(poly) > 1){ \
55         int b = bad(poly[sz(poly) - 2], poly.back(), t); \
56         if (b == 2) return {}; \
57         if (b == 1) poly.pop_back(); \
58         else break; \
59     }
60 deque<ray> poly;
61 for (auto t : rays){
62     reduce(t);
63     poly.pb(t);
64 }
65 for (; poly.pop_front()){
66     reduce(poly[0]);
67     if (!bad(poly.back(), poly[0], poly[1])) break;
68 }
69 assert(sz(poly) >= 3); // expect nonzero area
70 vector<point> poly_points;
71 for (int i = 0; i < sz(poly); i++){
72     poly_points.pb(poly[i].isect(poly[(i + 1) % sz(poly)]));
73 }
74 return poly_points;
75 }

```

Strings

```

1 vector<int> prefix_function(string s){
2     int n = sz(s);
3     vector<int> pi(n);
4     for (int i = 1; i < n; i++){
5         int k = pi[i - 1];
6         while (k > 0 && s[i] != s[k]){
7             k = pi[k - 1];
8         }
9         pi[i] = k + (s[i] == s[k]);
10    }
11    return pi;
12 }
13 // Returns the positions of the first character
14 vector<int> kmp(string s, string k){
15     string st = k + "#" + s;
16     vector<int> res;
17     auto pi = prefix_function(st);
18     for (int i = 0; i < sz(st); i++){
19         if (pi[i] == sz(k)){
20             res.pb(i - 2 * sz(k));
21         }
22     }
23 }

```

```

22     }
23     return res;
24 }
25 vector<int> z_function(string s){
26     int n = sz(s);
27     vector<int> z(n);
28     int l = 0, r = 0;
29     for (int i = 1; i < n; i++){
30         if (r >= i) z[i] = min(z[i - l], r - i + 1);
31         while (i + z[i] < n && s[z[i]] == s[i + z[i]]){
32             z[i]++;
33         }
34         if (i + z[i] - 1 > r){
35             l = i, r = i + z[i] - 1;
36         }
37     }
38     return z;
39 }

```

Manacher's algorithm

```

1  /*
2  Finds longest palindromes centered at each index
3  even[i] = d --> [i - d, i + d - 1] is a max-palindrome
4  odd[i] = d --> [i - d, i + d] is a max-palindrome
5  */
6  pair<vector<int>, vector<int>> manacher(string s) {
7      vector<char> t{'^', '#'};
8      for (char c : s) t.push_back(c), t.push_back('#');
9      t.push_back('$');
10     int n = t.size(), r = 0, c = 0;
11     vector<int> p(n, 0);
12     for (int i = 1; i < n - 1; i++) {
13         if (i < r + c) p[i] = min(p[2 * c - i], r + c - i);
14         while (t[i + p[i] + 1] == t[i - p[i] - 1]) p[i]++;
15         if (i + p[i] > r + c) r = p[i], c = i;
16     }
17     vector<int> even(sz(s)), odd(sz(s));
18     for (int i = 0; i < sz(s); i++){
19         even[i] = p[2 * i + 1] / 2, odd[i] = p[2 * i + 2] / 2;
20     }
21     return {even, odd};
22 }

```

Aho-Corasick Trie

- Given a set of strings, constructs a trie with suffix links.
- For a particular node, *link* points to the longest proper suffix of this node that's contained in the trie.
- nxt* encodes suffix links in a compressed format:
 - If vertex *v* has a child by letter *x*, then *trie[v].nxt[x]* points to that child.
 - If vertex *v* doesn't have such child, then *trie[v].nxt[x]* points to the suffix link of that child if we would actually have it.
- Facts:** suffix link graph can be seen as a tree; terminal link tree has height $O(\sqrt{N})$, where *N* is the sum of strings' lengths.
- Usage:** add all strings, then call *add_links()*.

```

1  const int S = 26;
2
3  // Function converting char to int.
4  int ctoi(char c){
5      return c - 'a';
6  }
7
8  // To add terminal links, use DFS
9  struct Node{
10     vector<int> nxt;
11     int link;
12     bool terminal;
13 }

```

```

14 Node() {
15     nxt.assign(S, -1), link = 0, terminal = 0;
16 }
17 };
18
19 vector<Node> trie(1);
20
21 // add_string returns the terminal vertex.
22 int add_string(string& s){
23     int v = 0;
24     for (auto c : s){
25         int cur = ctoi(c);
26         if (trie[v].nxt[cur] == -1){
27             trie[v].nxt[cur] = sz(trie);
28             trie.emplace_back();
29         }
30         v = trie[v].nxt[cur];
31     }
32     trie[v].terminal = 1;
33     return v;
34 }
35
36 void add_links(){
37     queue<int> q;
38     q.push(0);
39     while (!q.empty()){
40         auto v = q.front();
41         int u = trie[v].link;
42         q.pop();
43         for (int i = 0; i < S; i++){
44             int& ch = trie[v].nxt[i];
45             if (ch == -1){
46                 ch = v? trie[u].nxt[i] : 0;
47             }
48             else{
49                 trie[ch].link = v? trie[u].nxt[i] : 0;
50                 q.push(ch);
51             }
52         }
53     }
54 }
55
56 bool is_terminal(int v){
57     return trie[v].terminal;
58 }
59
60 int get_link(int v){
61     return trie[v].link;
62 }
63
64 int go(int v, char c){
65     return trie[v].nxt[ctoi(c)];
66 }

```

Flows

$O(N^2M)$, on unit networks $O(N^{1/2}M)$

```

1  struct FlowEdge {
2      int from, to;
3      ll cap, flow = 0;
4      FlowEdge(int u, int v, ll cap) : from(u), to(v), cap(cap)
5      ⇐ {}
6  };
7  struct Dinic {
8      const ll flow_inf = 1e18;
9      vector<FlowEdge> edges;
10     vector<vector<int>> adj;
11     int n, m = 0;
12     int s, t;
13     vector<int> level, ptr;
14     vector<bool> used;
15     queue<int> q;
16     Dinic(int n, int s, int t) : n(n), s(s), t(t) {
17         adj.resize(n);

```

```

17     level.resize(n);
18     ptr.resize(n);
19 }
20 void add_edge(int u, int v, ll cap) {
21     edges.emplace_back(u, v, cap);
22     edges.emplace_back(v, u, 0);
23     adj[u].push_back(m);
24     adj[v].push_back(m + 1);
25     m += 2;
26 }
27 bool bfs() {
28     while (!q.empty()) {
29         int v = q.front();
30         q.pop();
31         for (int id : adj[v]) {
32             if (edges[id].cap - edges[id].flow < 1)
33                 continue;
34             if (level[edges[id].to] != -1)
35                 continue;
36             level[edges[id].to] = level[v] + 1;
37             q.push(edges[id].to);
38         }
39     }
40     return level[t] != -1;
41 }
42 ll dfs(int v, ll pushed) {
43     if (pushed == 0)
44         return 0;
45     if (v == t)
46         return pushed;
47     for (int& cid = ptr[v]; cid < (int)adj[v].size();
48 ↪ cid++) {
49         int id = adj[v][cid];
50         int u = edges[id].to;
51         if (level[v] + 1 != level[u] || edges[id].cap -
52 ↪ edges[id].flow < 1)
53             continue;
54         ll tr = dfs(u, min(pushed, edges[id].cap -
55 ↪ edges[id].flow));
56         if (tr == 0)
57             continue;
58         edges[id].flow += tr;
59         edges[id ^ 1].flow -= tr;
60         return tr;
61     }
62     return 0;
63 }
64 ll flow() {
65     ll f = 0;
66     while (true) {
67         fill(level.begin(), level.end(), -1);
68         level[s] = 0;
69         q.push(s);
70         if (!bfs())
71             break;
72         fill(ptr.begin(), ptr.end(), 0);
73         while (ll pushed = dfs(s, flow_inf)) {
74             f += pushed;
75         }
76     }
77     return f;
78 }
79 void cut_dfs(int v){
80     used[v] = 1;
81     for (auto i : adj[v]){
82         if (edges[i].flow < edges[i].cap &&
83 ↪ !used[edges[i].to]){
84             cut_dfs(edges[i].to);
85         }
86     }
87 }
88 // Assumes that max flow is already calculated
89 // true -> vertex is in S, false -> vertex is in T
90 vector<bool> min_cut(){
91     used = vector<bool>(n);

```

```

90     cut_dfs(s);
91     return used;
92 }
93 };
94 // To recover flow through original edges: iterate over even
95 ↪ indices in edges.

MCMF – maximize flow, then minimize its
cost.  $O(mn + Fm \log n)$ .

1  #include <ext/pb_ds/priority_queue.hpp>
2  template <typename T, typename C>
3  class MCMF {
4      public:
5          static constexpr T eps = (T) 1e-9;
6
7          struct edge {
8              int from;
9              int to;
10             T c;
11             T f;
12             C cost;
13         };
14
15         int n;
16         vector<vector<int>>> g;
17         vector<edge> edges;
18         vector<C> d;
19         vector<C> pot;
20         __gnu_pbds::priority_queue<pair<C, int>> q;
21         vector<typename decltype(q)::point_iterator> its;
22         vector<int> pe;
23         const C INF_C = numeric_limits<C>::max() / 2;
24
25         explicit MCMF(int n_) : n(n_), g(n), d(n), pot(n, 0),
26 ↪ its(n), pe(n) {}
27
28         int add(int from, int to, T forward_cap, C edge_cost, T
29 ↪ backward_cap = 0) {
30             assert(0 <= from && from < n && 0 <= to && to < n);
31             assert(forward_cap >= 0 && backward_cap >= 0);
32             int id = static_cast<int>(edges.size());
33             g[from].push_back(id);
34             edges.push_back({from, to, forward_cap, 0, edge_cost});
35             g[to].push_back(id + 1);
36             edges.push_back({to, from, backward_cap, 0, -edge_cost});
37             return id;
38         }
39
40         void expath(int st) {
41             fill(d.begin(), d.end(), INF_C);
42             q.clear();
43             fill(its.begin(), its.end(), q.end());
44             its[st] = q.push({pot[st], st});
45             d[st] = 0;
46             while (!q.empty()) {
47                 int i = q.top().second;
48                 q.pop();
49                 its[i] = q.end();
50                 for (int id : g[i]) {
51                     const edge &e = edges[id];
52                     int j = e.to;
53                     if (e.c - e.f > eps && d[i] + e.cost < d[j]) {
54                         d[j] = d[i] + e.cost;
55                         pe[j] = id;
56                         if (its[j] == q.end()) {
57                             its[j] = q.push({pot[j] - d[j], j});
58                         } else {
59                             q.modify(its[j], {pot[j] - d[j], j});
60                         }
61                     }
62                 }
63             }
64             swap(d, pot);
65         }

```

```

65 pair<T, C> max_flow(int st, int fin) {
66     T flow = 0;
67     C cost = 0;
68     bool ok = true;
69     for (auto& e : edges) {
70         if (e.c - e.f > eps && e.cost + pot[e.from] - pot[e.to]
↪ < 0) {
71             ok = false;
72             break;
73         }
74     }
75     if (ok) {
76         expath(st);
77     } else {
78         vector<int> deg(n, 0);
79         for (int i = 0; i < n; i++) {
80             for (int eid : g[i]) {
81                 auto& e = edges[eid];
82                 if (e.c - e.f > eps) {
83                     deg[e.to] += 1;
84                 }
85             }
86         }
87         vector<int> que;
88         for (int i = 0; i < n; i++) {
89             if (deg[i] == 0) {
90                 que.push_back(i);
91             }
92         }
93         for (int b = 0; b < (int) que.size(); b++) {
94             for (int eid : g[que[b]]) {
95                 auto& e = edges[eid];
96                 if (e.c - e.f > eps) {
97                     deg[e.to] -= 1;
98                     if (deg[e.to] == 0) {
99                         que.push_back(e.to);
100                     }
101                 }
102             }
103         }
104         fill(pot.begin(), pot.end(), INF_C);
105         pot[st] = 0;
106         if (static_cast<int>(que.size()) == n) {
107             for (int v : que) {
108                 if (pot[v] < INF_C) {
109                     for (int eid : g[v]) {
110                         auto& e = edges[eid];
111                         if (e.c - e.f > eps) {
112                             if (pot[v] + e.cost < pot[e.to]) {
113                                 pot[e.to] = pot[v] + e.cost;
114                                 pe[e.to] = eid;
115                             }
116                         }
117                     }
118                 }
119             }
120         } else {
121             que.assign(1, st);
122             vector<bool> in_queue(n, false);
123             in_queue[st] = true;
124             for (int b = 0; b < (int) que.size(); b++) {
125                 int i = que[b];
126                 in_queue[i] = false;
127                 for (int id : g[i]) {
128                     const edge &e = edges[id];
129                     if (e.c - e.f > eps && pot[i] + e.cost <
↪ pot[e.to]) {
130                         pot[e.to] = pot[i] + e.cost;
131                         pe[e.to] = id;
132                         if (!in_queue[e.to]) {
133                             que.push_back(e.to);
134                             in_queue[e.to] = true;
135                         }
136                     }
137                 }
138             }
139         }

```

```

140     }
141     while (pot[fin] < INF_C) {
142         T push = numeric_limits<T>::max();
143         int v = fin;
144         while (v != st) {
145             const edge &e = edges[pe[v]];
146             push = min(push, e.c - e.f);
147             v = e.from;
148         }
149         v = fin;
150         while (v != st) {
151             edge &e = edges[pe[v]];
152             e.f += push;
153             edge &back = edges[pe[v] ^ 1];
154             back.f -= push;
155             v = e.from;
156         }
157         flow += push;
158         cost += push * pot[fin];
159         expath(st);
160     }
161     return {flow, cost};
162 }
163 };
164
165 // Examples: MCMF<int, int> g(n); g.add(u,v,c,w,0);
↪ g.max_flow(s,t).
166 // To recover flow through original edges: iterate over even
↪ indices in edges.

```

Graphs

Kuhn's algorithm for bipartite matching

```

1  /*
2  The graph is split into 2 halves of n1 and n2 vertices.
3  Complexity: O(n1 * m). Usually runs much faster. MUCH
↪ FASTER!!!
4  */
5  const int N = 305;
6
7  vector<int> g[N]; // Stores edges from left half to right.
8  bool used[N]; // Stores if vertex from left half is used.
9  int mt[N]; // For every vertex in right half, stores to which
↪ vertex in left half it's matched (-1 if not matched).
10
11 bool try_dfs(int v){
12     if (used[v]) return false;
13     used[v] = 1;
14     for (auto u : g[v]){
15         if (mt[u] == -1 || try_dfs(mt[u])){
16             mt[u] = v;
17             return true;
18         }
19     }
20     return false;
21 }
22
23 int main(){
24     // .....
25     for (int i = 1; i <= n2; i++) mt[i] = -1;
26     for (int i = 1; i <= n1; i++) used[i] = 0;
27     for (int i = 1; i <= n1; i++){
28         if (try_dfs(i)){
29             for (int j = 1; j <= n1; j++) used[j] = 0;
30         }
31     }
32     vector<pair<int, int>> ans;
33     for (int i = 1; i <= n2; i++){
34         if (mt[i] != -1) ans.pb({mt[i], i});
35     }
36 }
37
38 // Finding maximal independent set: size = # of nodes - # of
↪ edges in matching.

```



```

39 // To construct: launch Kuhn-like DFS from unmatched nodes in
   ↳ the left half.
40 // Independent set = visited nodes in left half + unvisited in
   ↳ right half.
41 // Finding minimal vertex cover: complement of maximal
   ↳ independent set.

```

```

3     int u = g[v].back();
4     g[v].pop_back();
5     dfs(u);
6     ans.pb(v);
7 }
8 }

```

Hungarian algorithm for Assignment Problem

- Given a 1-indexed $(n \times m)$ matrix A , select a number in each row such that each column has at most 1 number selected, and the sum of the selected numbers is minimized.

```

1 int INF = 1e9; // constant greater than any number in the
   ↳ matrix
2 vector<int> u(n+1), v(m+1), p(m+1), way(m+1);
3 for (int i=1; i<=n; ++i) {
4     p[0] = i;
5     int j0 = 0;
6     vector<int> minv (m+1, INF);
7     vector<bool> used (m+1, false);
8     do {
9         used[j0] = true;
10        int i0 = p[j0], delta = INF, j1;
11        for (int j=1; j<=m; ++j)
12            if (!used[j]) {
13                int cur = A[i0][j]-u[i0]-v[j];
14                if (cur < minv[j])
15                    minv[j] = cur, way[j] = j0;
16                if (minv[j] < delta)
17                    delta = minv[j], j1 = j;
18            }
19        for (int j=0; j<=m; ++j)
20            if (used[j])
21                u[p[j]] += delta, v[j] -= delta;
22            else
23                minv[j] -= delta;
24        j0 = j1;
25    } while (p[j0] != 0);
26    do {
27        int j1 = way[j0];
28        p[j0] = p[j1];
29        j0 = j1;
30    } while (j0);
31 }
32 vector<int> ans (n+1); // ans[i] stores the column selected
   ↳ for row i
33 for (int j=1; j<=m; ++j)
34     ans[p[j]] = j;
35 int cost = -v[0]; // the total cost of the matching

```

Dijkstra's Algorithm

```

1 priority_queue<pair<ll, ll>, vector<pair<ll, ll>>,
   ↳ greater<pair<ll, ll>>> q;
2 dist[start] = 0;
3 q.push({0, start});
4 while (!q.empty()){
5     auto [d, v] = q.top();
6     q.pop();
7     if (d != dist[v]) continue;
8     for (auto [u, w] : g[v]){
9         if (dist[u] > dist[v] + w){
10             dist[u] = dist[v] + w;
11             q.push({dist[u], u});
12         }
13     }
14 }

```

Eulerian Cycle DFS

```

1 void dfs(int v){
2     while (!g[v].empty()){

```

SCC and 2-SAT

```

1 void scc(vector<vector<int>>& g, int* idx) {
2     int n = g.size(), ct = 0;
3     int out[n];
4     vector<int> ginv[n];
5     memset(out, -1, sizeof out);
6     memset(idx, -1, n * sizeof(int));
7     function<void(int)> dfs = [&](int cur) {
8         out[cur] = INT_MAX;
9         for(int v : g[cur]) {
10             ginv[v].push_back(cur);
11             if(out[v] == -1) dfs(v);
12         }
13         ct++; out[cur] = ct;
14     };
15     vector<int> order;
16     for(int i = 0; i < n; i++) {
17         order.push_back(i);
18         if(out[i] == -1) dfs(i);
19     }
20     sort(order.begin(), order.end(), [&](int& u, int& v) {
21         return out[u] > out[v];
22     });
23     ct = 0;
24     stack<int> s;
25     auto dfs2 = [&](int start) {
26         s.push(start);
27         while(!s.empty()) {
28             int cur = s.top();
29             s.pop();
30             idx[cur] = ct;
31             for(int v : ginv[cur])
32                 if(idx[v] == -1) s.push(v);
33         }
34     };
35     for(int v : order) {
36         if(idx[v] == -1) {
37             dfs2(v);
38             ct++;
39         }
40     }
41 }
42
43 // 0 => impossible, 1 => possible
44 pair<int, vector<int>> sat2(int n, vector<pair<int, int>>&
   ↳ clauses) {
45     vector<int> ans(n);
46     vector<vector<int>> g(2*n + 1);
47     for(auto [x, y] : clauses) {
48         x = x < 0 ? -x + n : x;
49         y = y < 0 ? -y + n : y;
50         int nx = x <= n ? x + n : x - n;
51         int ny = y <= n ? y + n : y - n;
52         g[nx].push_back(y);
53         g[ny].push_back(x);
54     }
55     int idx[2*n + 1];
56     scc(g, idx);
57     for(int i = 1; i <= n; i++) {
58         if(idx[i] == idx[i + n]) return {0, {}};
59         ans[i - 1] = idx[i + n] < idx[i];
60     }
61     return {1, ans};
62 }

```

Finding Bridges

```
1  /*
2  Bridges.
3  Results are stored in a map "is_bridge".
4  For each connected component, call "dfs(starting vertex,
   ↪ starting vertex)".
5  */
6  const int N = 2e5 + 10; // Careful with the constant!
7
8  vector<int> g[N];
9  int tin[N], fup[N], timer;
10 map<pair<int, int>, bool> is_bridge;
11
12 void dfs(int v, int p){
13     tin[v] = ++timer;
14     fup[v] = tin[v];
15     for (auto u : g[v]){
16         if (!tin[u]){
17             dfs(u, v);
18             if (fup[u] > tin[v]){
19                 is_bridge[{v, u}] = is_bridge[{u, v}] = true;
20             }
21             fup[v] = min(fup[v], fup[u]);
22         }
23         else{
24             if (u != p) fup[v] = min(fup[v], tin[u]);
25         }
26     }
27 }
```

Virtual Tree

```
1  // order stores the nodes in the queried set
2  sort(all(order), [&] (int u, int v){return tin[u] < tin[v];});
3  int m = sz(order);
4  for (int i = 1; i < m; i++){
5      order.pb(lca(order[i], order[i - 1]));
6  }
7  sort(all(order), [&] (int u, int v){return tin[u] < tin[v];});
8  order.erase(unique(all(order)), order.end());
9  vector<int> stk{order[0]};
10 for (int i = 1; i < sz(order); i++){
11     int v = order[i];
12     while (tout[stk.back()] < tout[v]) stk.pop_back();
13     int u = stk.back();
14     vg[u].pb({v, dep[v] - dep[u]});
15     stk.pb(v);
16 }
```

HLD on Edges DFS

```
1  void dfs1(int v, int p, int d){
2      par[v] = p;
3      for (auto e : g[v]){
4          if (e.fi == p){
5              g[v].erase(find(all(g[v]), e));
6              break;
7          }
8      }
9      dep[v] = d;
10     sz[v] = 1;
11     for (auto [u, c] : g[v]){
12         dfs1(u, v, d + 1);
13         sz[v] += sz[u];
14     }
15     if (!g[v].empty()) iter_swap(g[v].begin(),
   ↪ max_element(all(g[v]), comp));
16 }
17 void dfs2(int v, int rt, int c){
18     pos[v] = sz[a];
19     a.pb(c);
20     root[v] = rt;
21     for (int i = 0; i < sz(g[v]); i++){
22         auto [u, c] = g[v][i];
23         if (!i) dfs2(u, rt, c);
```

```
24         else dfs2(u, u, c);
25     }
26 }
27 int getans(int u, int v){
28     int res = 0;
29     for (; root[u] != root[v]; v = par[root[v]]){
30         if (dep[root[u]] > dep[root[v]]) swap(u, v);
31         res = max(res, rmq(0, 0, n - 1, pos[root[v]], pos[v]));
32     }
33     if (pos[u] > pos[v]) swap(u, v);
34     return max(res, rmq(0, 0, n - 1, pos[u] + 1, pos[v]));
35 }
```

Centroid Decomposition

```
1  vector<char> res(n), seen(n), sz(n);
2  function<int(int, int)> get_size = [&](int node, int fa) {
3      sz[node] = 1;
4      for (auto& ne : g[node]) {
5          if (ne == fa || seen[ne]) continue;
6          sz[node] += get_size(ne, node);
7      }
8      return sz[node];
9  };
10 function<int(int, int, int)> find_centroid = [&](int node, int
   ↪ fa, int t) {
11     for (auto& ne : g[node])
12         if (ne != fa && !seen[ne] && sz[ne] > t / 2) return
   ↪ find_centroid(ne, node, t);
13     return node;
14 };
15 function<void(int, char)> solve = [&](int node, char cur) {
16     get_size(node, -1); auto c = find_centroid(node, -1,
   ↪ sz[node]);
17     seen[c] = 1, res[c] = cur;
18     for (auto& ne : g[c]) {
19         if (seen[ne]) continue;
20         solve(ne, char(cur + 1)); // we can pass c here to build
   ↪ tree
21     }
22 };
```

Math

Binary exponentiation

```
1  ll power(ll a, ll b){
2      ll res = 1;
3      for (; b; a = a * a % MOD, b >>= 1){
4          if (b & 1) res = res * a % MOD;
5      }
6      return res;
7  }
```

Matrix Exponentiation: $O(n^3 \log b)$

```
1  const int N = 100, MOD = 1e9 + 7;
2
3  struct matrix{
4      ll m[N][N];
5      int n;
6      matrix(){
7          n = N;
8          memset(m, 0, sizeof(m));
9      };
10     matrix(int n_){
11         n = n_;
12         memset(m, 0, sizeof(m));
13     };
14     matrix(int n_, ll val){
15         n = n_;
16         memset(m, 0, sizeof(m));
17         for (int i = 0; i < n; i++) m[i][i] = val;
18     };
```

```

19 matrix operator* (matrix oth){
20     matrix res(n);
21     for (int i = 0; i < n; i++){
22         for (int j = 0; j < n; j++){
23             for (int k = 0; k < n; k++){
24                 res.m[i][j] = (res.m[i][j] + m[i][k] * oth.m[k][j])
25                 ↪ % MOD;
26             }
27         }
28     }
29     return res;
30 }
31 };
32
33 matrix power(matrix a, ll b){
34     matrix res(a.n, 1);
35     for (; b; a = a * a, b >>= 1){
36         if (b & 1) res = res * a;
37     }
38     return res;
39 }

```

Extended Euclidean Algorithm

- $O(\max(\log a, \log b))$
- Finds solution (x, y) to $ax + by = \gcd(a, b)$
- Can find all solutions given $(x_0, y_0) : \forall k, a(x_0 + kb/g) + b(y_0 - ka/g) = \gcd(a, b)$.

```

1 ll euclid(ll a, ll b, ll &x, ll &y) {
2     if (!b) return x = 1, y = 0, a;
3     ll d = euclid(b, a % b, y, x);
4     return y -= a/b * x, d;
5 }

```

CRT

- $crt(a, m, b, n)$ computes x such that $x \equiv a \pmod{m}$, $x \equiv b \pmod{n}$
- If $|a| < m$ and $|b| < n$, x will obey $0 \leq x < \text{lcm}(m, n)$.
- Assumes $mn < 2^{62}$.
- $O(\max(\log m, \log n))$

```

1 ll crt(ll a, ll m, ll b, ll n) {
2     if (n > m) swap(a, b), swap(m, n);
3     ll x, y, g = euclid(m, n, x, y);
4     assert((a - b) % g == 0); // else no solution
5     // can replace assert with whatever needed
6     x = (b - a) % n * x % n / g * m + a;
7     return x < 0 ? x + m*n/g : x;
8 }

```

Linear Sieve

- Mobius Function

```

1 vector<int> prime;
2 bool is_composite[MAX_N];
3 int mu[MAX_N];
4
5 void sieve(int n){
6     fill(is_composite, is_composite + n, 0);
7     mu[1] = 1;
8     for (int i = 2; i < n; i++){
9         if (!is_composite[i]){
10             prime.push_back(i);
11             mu[i] = -1; //i is prime
12         }
13         for (int j = 0; j < prime.size() && i * prime[j] < n; j++){
14             is_composite[i * prime[j]] = true;
15             if (i % prime[j] == 0){
16                 mu[i * prime[j]] = 0; //prime[j] divides i
17                 break;

```

```

18         } else {
19             mu[i * prime[j]] = -mu[i]; //prime[j] does not divide i
20         }
21     }
22 }
23 }

```

• Euler's Totient Function

```

1 vector<int> prime;
2 bool is_composite[MAX_N];
3 int phi[MAX_N];
4
5 void sieve(int n){
6     fill(is_composite, is_composite + n, 0);
7     phi[1] = 1;
8     for (int i = 2; i < n; i++){
9         if (!is_composite[i]){
10             prime.push_back(i);
11             phi[i] = i - 1; //i is prime
12         }
13         for (int j = 0; j < prime.size() && i * prime[j] < n; j++){
14             is_composite[i * prime[j]] = true;
15             if (i % prime[j] == 0){
16                 phi[i * prime[j]] = phi[i] * prime[j]; //prime[j]
17                 ↪ divides i
18                 break;
19             } else {
20                 phi[i * prime[j]] = phi[i] * phi[prime[j]]; //prime[j]
21                 ↪ does not divide i
22             }
23         }
24     }
25 }

```

Gaussian Elimination

```

1 bool is_0(Z v) { return v.x == 0; }
2 Z abs(Z v) { return v; }
3 bool is_0(double v) { return abs(v) < 1e-9; }
4
5 // 1 => unique solution, 0 => no solution, -1 => multiple
6 ↪ solutions
7 template <typename T>
8 int gaussian_elimination(vector<vector<T>> &a, int limit) {
9     if (a.empty() || a[0].empty()) return -1;
10     int h = (int)a.size(), w = (int)a[0].size(), r = 0;
11     for (int c = 0; c < limit; c++) {
12         int id = -1;
13         for (int i = r; i < h; i++) {
14             if (!is_0(a[i][c]) && (id == -1 || abs(a[id][c]) <
15                 ↪ abs(a[i][c]))) {
16                 id = i;
17             }
18         }
19         if (id == -1) continue;
20         if (id > r) {
21             swap(a[r], a[id]);
22             for (int j = c; j < w; j++) a[id][j] = -a[id][j];
23         }
24         vector<int> nonzero;
25         for (int j = c; j < w; j++) {
26             if (!is_0(a[r][j])) nonzero.push_back(j);
27         }
28         T inv_a = 1 / a[r][c];
29         for (int i = r + 1; i < h; i++) {
30             if (is_0(a[i][c])) continue;
31             T coeff = -a[i][c] * inv_a;
32             for (int j : nonzero) a[i][j] += coeff * a[r][j];
33         }
34         ++r;
35     }
36     for (int row = h - 1; row >= 0; row--) {
37         for (int c = 0; c < limit; c++) {
38             if (!is_0(a[row][c])) {
39                 T inv_a = 1 / a[row][c];
40                 for (int i = row - 1; i >= 0; i--) {

```

```

39         if (is_0(a[i][c])) continue;
40         T coeff = -a[i][c] * inv_a;
41         for (int j = c; j < w; j++) a[i][j] += coeff *
↪ a[row][j];
42     }
43     break;
44 }
45 }
46 } // not-free variables: only it on its line
47 for(int i = r; i < h; i++) if(!is_0(a[i][limit])) return 0;
48 return (r == limit) ? 1 : -1;
49 }
50
51 template <typename T>
52 pair<int, vector<T>> solve_linear(vector<vector<T>> a, const
↪ vector<T> &b, int w) {
53     int h = (int)a.size();
54     for (int i = 0; i < h; i++) a[i].push_back(b[i]);
55     int sol = gaussian_elimination(a, w);
56     if(!sol) return {0, vector<T>()};
57     vector<T> x(w, 0);
58     for (int i = 0; i < h; i++) {
59         for (int j = 0; j < w; j++) {
60             if (!is_0(a[i][j])) {
61                 x[j] = a[i][w] / a[i][j];
62                 break;
63             }
64         }
65     }
66     return {sol, x};
67 }

```

is_prime

- (Miller–Rabin primality test)

```

1 typedef __int128_t i128;
2
3 i128 power(i128 a, i128 b, i128 MOD = 1, i128 res = 1) {
4     for (; b; b /= 2, (a *= a) %= MOD)
5         if (b & 1) (res *= a) %= MOD;
6     return res;
7 }
8
9 bool is_prime(ll n) {
10     if (n < 2) return false;
11     static constexpr int A[] = {2, 3, 5, 7, 11, 13, 17, 19, 23};
12     int s = __builtin_ctzll(n - 1);
13     ll d = (n - 1) >> s;
14     for (auto a : A) {
15         if (a == n) return true;
16         ll x = (ll)power(a, d, n);
17         if (x == 1 || x == n - 1) continue;
18         bool ok = false;
19         for (int i = 0; i < s - 1; ++i) {
20             x = ll((i128)x * x % n); // potential overflow!
21             if (x == n - 1) {
22                 ok = true;
23                 break;
24             }
25         }
26         if (!ok) return false;
27     }
28     return true;
29 }
30
31 typedef __int128_t i128;
32
33 ll pollard_rho(ll x) {
34     ll s = 0, t = 0, c = rng() % (x - 1) + 1;
35     ll stp = 0, goal = 1, val = 1;
36     for (goal = 1;; goal *= 2, s = t, val = 1) {
37         for (stp = 1; stp <= goal; ++stp) {
38             t = ll(((i128)t * t + c) % x);
39             val = ll(((i128)val * abs(t - s) % x);
40             if ((stp % 127) == 0) {
41                 ll d = gcd(val, x);

```

```

12         if (d > 1) return d;
13     }
14 }
15 ll d = gcd(val, x);
16 if (d > 1) return d;
17 }
18 }
19
20 ll get_max_factor(ll _x) {
21     ll max_factor = 0;
22     function<void(ll)> fac = [&](ll x) {
23         if (x <= max_factor || x < 2) return;
24         if (is_prime(x)) {
25             max_factor = max_factor > x ? max_factor : x;
26             return;
27         }
28         ll p = x;
29         while (p >= x) p = pollard_rho(x);
30         while ((x % p) == 0) x /= p;
31         fac(x), fac(p);
32     };
33     fac(_x);
34     return max_factor;
35 }

```

Berlekamp-Massey

- Recovers any n -order linear recurrence relation from the first $2n$ terms of the sequence.
- Input s is the sequence to be analyzed.
- Output c is the shortest sequence c_1, \dots, c_n , such that

$$s_m = \sum_{i=1}^n c_i \cdot s_{m-i}, \text{ for all } m \geq n.$$

- Be careful since c is returned in 0-based indexation.
- Complexity: $O(N^2)$

```

1 vector<ll> berlekamp_massey(vector<ll> s) {
2     int n = sz(s), l = 0, m = 1;
3     vector<ll> b(n), c(n);
4     ll ldd = b[0] = c[0] = 1;
5     for (int i = 0; i < n; i++, m++) {
6         ll d = s[i];
7         for (int j = 1; j <= l; j++) d = (d + c[j] * s[i - j]) %
↪ MOD;
8         if (d == 0) continue;
9         vector<ll> temp = c;
10        ll coef = d * power(ldd, MOD - 2) % MOD;
11        for (int j = m; j < n; j++){
12            c[j] = (c[j] + MOD - coef * b[j - m]) % MOD;
13            if (c[j] < 0) c[j] += MOD;
14        }
15        if (2 * l <= i) {
16            l = i + 1 - l;
17            b = temp;
18            ldd = d;
19            m = 0;
20        }
21    }
22    c.resize(l + 1);
23    c.erase(c.begin());
24    for (ll &x : c)
25        x = (MOD - x) % MOD;
26    return c;
27 }

```

Calculating k-th term of a linear recurrence

- Given the first n terms s_0, s_1, \dots, s_{n-1} and the sequence c_1, c_2, \dots, c_n such that

$$s_m = \sum_{i=1}^n c_i \cdot s_{m-i}, \text{ for all } m \geq n,$$

the function `calc_kth` computes s_k .

- Complexity: $O(n^2 \log k)$

```

1 vector<ll> poly_mult_mod(vector<ll> p, vector<ll> q,
  ↳ vector<ll>& c){
2     vector<ll> ans(sz(p) + sz(q) - 1);
3     for (int i = 0; i < sz(p); i++){
4         for (int j = 0; j < sz(q); j++){
5             ans[i + j] = (ans[i + j] + p[i] * q[j]) % MOD;
6         }
7     }
8     int n = sz(ans), m = sz(c);
9     for (int i = n - 1; i >= m; i--){
10        for (int j = 0; j < m; j++){
11            ans[i - 1 - j] = (ans[i - 1 - j] + c[j] * ans[i]) % MOD;
12        }
13    }
14    ans.resize(m);
15    return ans;
16 }

17 ll calc_kth(vector<ll> s, vector<ll> c, ll k){
18     assert(sz(s) >= sz(c)); // size of s can be greater than c,
  ↳ but not less
19     if (k < sz(s)) return s[k];
20     vector<ll> res{1};
21     for (vector<ll> poly = {0, 1}; k; poly = poly_mult_mod(poly,
  ↳ poly, c), k >= 1){
22         if (k & 1) res = poly_mult_mod(res, poly, c);
23     }
24     ll ans = 0;
25     for (int i = 0; i < min(sz(res), sz(c)); i++) ans = (ans +
  ↳ s[i] * res[i]) % MOD;
26     return ans;
27 }

```

Partition Function

- Returns number of partitions of n in $O(n^{1.5})$

```

1 int partition(int n) {
2     int dp[n + 1];
3     dp[0] = 1;
4     for (int i = 1; i <= n; i++) {
5         dp[i] = 0;
6         for (int j = 1, r = 1; i - (3 * j * j - j) / 2 >= 0; ++j,
  ↳ r += -1) {
7             dp[i] += dp[i - (3 * j * j - j) / 2] * r;
8             if (i - (3 * j * j + j) / 2 >= 0) dp[i] += dp[i - (3 * j
  ↳ * j + j) / 2] * r;
9         }
10    }
11    return dp[n];
12 }

```

NTT

```

1 void ntt(vector<ll>& a, int f) {
2     int n = int(a.size());
3     vector<ll> w(n);
4     vector<int> rev(n);
5     for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] / 2) | ((i
  ↳ & 1) * (n / 2));
6     for (int i = 0; i < n; i++) {
7         if (i < rev[i]) swap(a[i], a[rev[i]]);
8     }
9     ll wn = power(f ? (MOD + 1) / 3 : 3, (MOD - 1) / n);
10    w[0] = 1;
11    for (int i = 1; i < n; i++) w[i] = w[i - 1] * wn % MOD;
12    for (int mid = 1; mid < n; mid *= 2) {
13        for (int i = 0; i < n; i += 2 * mid) {
14            for (int j = 0; j < mid; j++) {
15                ll x = a[i + j], y = a[i + j + mid] * w[n / (2 * mid)
  ↳ * j] % MOD;
16                a[i + j] = (x + y) % MOD, a[i + j + mid] = (x + MOD -
  ↳ y) % MOD;

```

```

17    }
18    }
19 }
20 if (f) {
21     ll iv = power(n, MOD - 2);
22     for (auto& x : a) x = x * iv % MOD;
23 }
24 }
25 vector<ll> mul(vector<ll> a, vector<ll> b) {
26     int n = 1, m = (int)a.size() + (int)b.size() - 1;
27     while (n < m) n *= 2;
28     a.resize(n), b.resize(n);
29     ntt(a, 0), ntt(b, 0); // if squaring, you can save one NTT
  ↳ here
30     for (int i = 0; i < n; i++) a[i] = a[i] * b[i] % MOD;
31     ntt(a, 1);
32     a.resize(m);
33     return a;
34 }

```

FFT

```

1 const ld PI = acosl(-1);
2 auto mul = [&](const vector<ld>& aa, const vector<ld>& bb) {
3     int n = (int)aa.size(), m = (int)bb.size(), bit = 1;
4     while ((1 << bit) < n + m - 1) bit++;
5     int len = 1 << bit;
6     vector<complex<ld>> a(len), b(len);
7     vector<int> rev(len);
8     for (int i = 0; i < n; i++) a[i].real(aa[i]);
9     for (int i = 0; i < m; i++) b[i].real(bb[i]);
10    for (int i = 0; i < len; i++) rev[i] = (rev[i >> 1] >> 1) |
  ↳ ((i & 1) << (bit - 1));
11    auto fft = [&](vector<complex<ld>>& p, int inv) {
12        for (int i = 0; i < len; i++)
13            if (i < rev[i]) swap(p[i], p[rev[i]]);
14        for (int mid = 1; mid < len; mid *= 2) {
15            auto w1 = complex<ld>(cos(PI / mid), (inv ? -1 : 1) *
  ↳ sin(PI / mid));
16            for (int i = 0; i < len; i += mid * 2) {
17                auto wk = complex<ld>(1, 0);
18                for (int j = 0; j < mid; j++, wk = wk * w1) {
19                    auto x = p[i + j], y = wk * p[i + j + mid];
20                    p[i + j] = x + y, p[i + j + mid] = x - y;
21                }
22            }
23        }
24        if (inv == 1) {
25            for (int i = 0; i < len; i++) p[i].real(p[i].real() /
  ↳ len);
26        }
27    };
28    fft(a, 0), fft(b, 0);
29    for (int i = 0; i < len; i++) a[i] = a[i] * b[i];
30    fft(a, 1);
31    a.resize(n + m - 1);
32    vector<ld> res(n + m - 1);
33    for (int i = 0; i < n + m - 1; i++) res[i] = a[i].real();
34    return res;
35 };

```

MIT's FFT/NTT, Polynomial mod/log/exp Template

- For integers rounding works if $(|a| + |b|) \max(a, b) < \sim 10^9$, or in theory maybe 10^6
- $\frac{1}{P(x)}$ in $O(n \log n)$, $e^{P(x)}$ in $O(n \log n)$, $\ln(P(x))$ in $O(n \log n)$, $P(x)^k$ in $O(n \log n)$, Evaluates $P(x_1), \dots, P(x_n)$ in $O(n \log^2 n)$, Lagrange Interpolation in $O(n \log^2 n)$

```

1 // use #define FFT 1 to use FFT instead of NTT (default)
2 // Examples:
3 // poly a(n+1); // constructs degree n poly

```

```

4 // a[0].v = 10; // assigns constant term a_0 = 10
5 // poly b = exp(a);
6 // poly is vector<num>
7 // for NTT, num stores just one int named v
8 // for FFT, num stores two doubles named x (real), y (imag)
9
10 #define sz(x) ((int)x.size())
11 #define rep(i, j, k) for (int i = int(j); i < int(k); i++)
12 #define trav(a, x) for (auto &a : x)
13 #define per(i, a, b) for (int i = (b)-1; i >= (a); --i)
14 using ll = long long;
15 using vi = vector<int>;
16
17 namespace fft {
18 #if FFT
19 // FFT
20 using dbl = double;
21 struct num {
22     dbl x, y;
23     num(dbl x_ = 0, dbl y_ = 0): x(x_), y(y_) {}
24 };
25 inline num operator+(num a, num b) {
26     return num(a.x + b.x, a.y + b.y);
27 }
28 inline num operator-(num a, num b) {
29     return num(a.x - b.x, a.y - b.y);
30 }
31 inline num operator*(num a, num b) {
32     return num(a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x);
33 }
34 inline num conj(num a) { return num(a.x, -a.y); }
35 inline num inv(num a) {
36     dbl n = (a.x * a.x + a.y * a.y);
37     return num(a.x / n, -a.y / n);
38 }
39
40 #else
41 // NTT
42 const int mod = 998244353, g = 3;
43 // For p < 2^30 there is also (5 << 25, 3), (7 << 26, 3),
44 // (479 << 21, 3) and (483 << 21, 5). Last two are > 10^9.
45 struct num {
46     int v;
47     num(ll v_ = 0): v(int(v_ % mod)) {
48         if (v < 0) v += mod;
49     }
50     explicit operator int() const { return v; }
51 };
52 inline num operator+(num a, num b) { return num(a.v + b.v); }
53 inline num operator-(num a, num b) {
54     return num(a.v + mod - b.v);
55 }
56 inline num operator*(num a, num b) {
57     return num(1ll * a.v * b.v);
58 }
59 inline num pow(num a, int b) {
60     num r = 1;
61     do {
62         if (b & 1) r = r * a;
63         a = a * a;
64     } while (b >>= 1);
65     return r;
66 }
67 inline num inv(num a) { return pow(a, mod - 2); }
68
69 #endif
70 using vn = vector<num>;
71 vi rev({0, 1});
72 vn rt(2, num(1)), fa, fb;
73 inline void init(int n) {
74     if (n <= sz(rt)) return;
75     rev.resize(n);
76     rep(i, 0, n) rev[i] = (rev[i >> 1] | ((i & 1) * n)) >> 1;
77     rt.reserve(n);
78     for (int k = sz(rt); k < n; k *= 2) {
79         rt.resize(2 * k);
80 #if FFT

```

```

81         double a = M_PI / k;
82         num z(cos(a), sin(a)); // FFT
83     #else
84         num z = pow(num(g), (mod - 1) / (2 * k)); // NTT
85     #endif
86     rep(i, k / 2, k) rt[2 * i] = rt[i],
87         rt[2 * i + 1] = rt[i] * z;
88 }
89 }
90 inline void fft(vector<num>& a, int n) {
91     init(n);
92     int s = __builtin_ctz(sz(rev) / n);
93     rep(i, 0, n) if (i < rev[i] >> s) swap(a[i], a[rev[i] >>
94         s]);
95     for (int k = 1; k < n; k *= 2)
96         for (int i = 0; i < n; i += 2 * k) rep(j, 0, k) {
97             num t = rt[j + k] * a[i + j + k];
98             a[i + j + k] = a[i + j] - t;
99             a[i + j] = a[i + j] + t;
100         }
101 // Complex/NTT
102 vn multiply(vn a, vn b) {
103     int s = sz(a) + sz(b) - 1;
104     if (s <= 0) return {};
105     int L = s > 1 ? 32 - __builtin_clz(s - 1) : 0, n = 1 << L;
106     a.resize(n), b.resize(n);
107     fft(a, n);
108     fft(b, n);
109     num d = inv(num(n));
110     rep(i, 0, n) a[i] = a[i] * b[i] * d;
111     reverse(a.begin() + 1, a.end());
112     fft(a, n);
113     a.resize(s);
114     return a;
115 }
116 // Complex/NTT power-series inverse
117 // Doubles b as b[:n] = (2 - a[:n] * b[:n/2]) * b[:n/2]
118 vn inverse(const vn& a) {
119     if (a.empty()) return {};
120     vn b({inv(a[0])});
121     b.reserve(2 * a.size());
122     while (sz(b) < sz(a)) {
123         int n = 2 * sz(b);
124         b.resize(2 * n, 0);
125         if (sz(fa) < 2 * n) fa.resize(2 * n);
126         fill(fa.begin(), fa.begin() + 2 * n, 0);
127         copy(a.begin(), a.begin() + min(n, sz(a)), fa.begin());
128         fft(b, 2 * n);
129         fft(fa, 2 * n);
130         num d = inv(num(2 * n));
131         rep(i, 0, 2 * n) b[i] = b[i] * (2 - fa[i] * b[i]) * d;
132         reverse(b.begin() + 1, b.end());
133         fft(b, 2 * n);
134         b.resize(n);
135     }
136     b.resize(a.size());
137     return b;
138 }
139 #if FFT
140 // Double multiply (num = complex)
141 using vd = vector<double>;
142 vd multiply(const vd& a, const vd& b) {
143     int s = sz(a) + sz(b) - 1;
144     if (s <= 0) return {};
145     int L = s > 1 ? 32 - __builtin_clz(s - 1) : 0, n = 1 << L;
146     if (sz(fa) < n) fa.resize(n);
147     if (sz(fb) < n) fb.resize(n);
148     fill(fa.begin(), fa.begin() + n, 0);
149     rep(i, 0, sz(a)) fa[i].x = a[i];
150     rep(i, 0, sz(b)) fa[i].y = b[i];
151     fft(fa, n);
152     trav(x, fa) x = x * x;
153     rep(i, 0, n) fb[i] = fa[(n - i) & (n - 1)] - conj(fa[i]);
154     fft(fb, n);
155     vd r(s);
156     rep(i, 0, s) r[i] = fb[i].y / (4 * n);

```



```

157     return r;
158 }
159 // Integer multiply mod m (num = complex)
160 vi multiply_mod(const vi& a, const vi& b, int m) {
161     int s = sz(a) + sz(b) - 1;
162     if (s <= 0) return {};
163     int L = s > 1 ? 32 - __builtin_clz(s - 1) : 0, n = 1 << L;
164     if (sz(fa) < n) fa.resize(n);
165     if (sz(fb) < n) fb.resize(n);
166     rep(i, 0, sz(a)) fa[i] =
167         num(a[i] & ((1 << 15) - 1), a[i] >> 15);
168     fill(fa.begin() + sz(a), fa.begin() + n, 0);
169     rep(i, 0, sz(b)) fb[i] =
170         num(b[i] & ((1 << 15) - 1), b[i] >> 15);
171     fill(fb.begin() + sz(b), fb.begin() + n, 0);
172     fft(fa, n);
173     fft(fb, n);
174     double r0 = 0.5 / n; // 1/2n
175     rep(i, 0, n / 2 + 1) {
176         int j = (n - i) & (n - 1);
177         num g0 = (fb[i] + conj(fb[j])) * r0;
178         num g1 = (fb[i] - conj(fb[j])) * r0;
179         swap(g1.x, g1.y);
180         g1.y *= -1;
181         if (j != i) {
182             swap(fa[j], fa[i]);
183             fb[j] = fa[j] * g1;
184             fa[j] = fa[j] * g0;
185         }
186         fb[i] = fa[i] * conj(g1);
187         fa[i] = fa[i] * conj(g0);
188     }
189     fft(fa, n);
190     fft(fb, n);
191     vi r(s);
192     rep(i, 0, s) r[i] =
193         int((ll(fa[i].x + 0.5) + (ll(fa[i].y + 0.5) % m << 15) +
194             (ll(fb[i].x + 0.5) % m << 15) +
195             (ll(fb[i].y + 0.5) % m << 30)) %
196             m);
197     return r;
198 }
199 #endif
200 } // namespace fft
201 // For multiply_mod, use num = modnum, poly = vector<num>
202 using fft::num;
203 using poly = fft::vn;
204 using fft::multiply;
205 using fft::inverse;
206
207 poly& operator+=(poly& a, const poly& b) {
208     if (sz(a) < sz(b)) a.resize(b.size());
209     rep(i, 0, sz(b)) a[i] = a[i] + b[i];
210     return a;
211 }
212 poly operator+(const poly& a, const poly& b) {
213     poly r = a;
214     r += b;
215     return r;
216 }
217 poly& operator-=(poly& a, const poly& b) {
218     if (sz(a) < sz(b)) a.resize(b.size());
219     rep(i, 0, sz(b)) a[i] = a[i] - b[i];
220     return a;
221 }
222 poly operator-(const poly& a, const poly& b) {
223     poly r = a;
224     r -= b;
225     return r;
226 }
227 poly operator*(const poly& a, const poly& b) {
228     return multiply(a, b);
229 }
230 poly& operator*=(poly& a, const poly& b) { return a = a * b; }
231
232 poly& operator*=(poly& a, const num& b) { // Optional
233     trav(x, a) x = x * b;

```

```

234     return a;
235 }
236 poly operator*(const poly& a, const num& b) {
237     poly r = a;
238     r *= b;
239     return r;
240 }
241 // Polynomial floor division; no leading 0's please
242 poly operator/(poly a, poly b) {
243     if (sz(a) < sz(b)) return {};
244     int s = sz(a) - sz(b) + 1;
245     reverse(a.begin(), a.end());
246     reverse(b.begin(), b.end());
247     a.resize(s);
248     b.resize(s);
249     a = a * inverse(move(b));
250     a.resize(s);
251     reverse(a.begin(), a.end());
252     return a;
253 }
254 poly& operator/=(poly& a, const poly& b) { return a = a / b; }
255 poly& operator%=(poly& a, const poly& b) {
256     if (sz(a) >= sz(b)) {
257         poly c = (a / b) * b;
258         a.resize(sz(b) - 1);
259         rep(i, 0, sz(a)) a[i] = a[i] - c[i];
260     }
261     return a;
262 }
263 poly operator%(const poly& a, const poly& b) {
264     poly r = a;
265     r %= b;
266     return r;
267 }
268 // Log/exp/pow
269 poly deriv(const poly& a) {
270     if (a.empty()) return {};
271     poly b(sz(a) - 1);
272     rep(i, 1, sz(a)) b[i - 1] = a[i] * i;
273     return b;
274 }
275 poly integ(const poly& a) {
276     poly b(sz(a) + 1);
277     b[1] = 1; // mod p
278     rep(i, 2, sz(b)) b[i] =
279         b[fft::mod % i] * (-fft::mod / i); // mod p
280     rep(i, 1, sz(b)) b[i] = a[i - 1] * b[i]; // mod p
281     // rep(i, 1, sz(b)) b[i] = a[i - 1] * inv(num(i)); // else
282     return b;
283 }
284 poly log(const poly& a) { // MUST have a[0] == 1
285     poly b = integ(deriv(a) * inverse(a));
286     b.resize(a.size());
287     return b;
288 }
289 poly exp(const poly& a) { // MUST have a[0] == 0
290     poly b(1, num(1));
291     if (a.empty()) return b;
292     while (sz(b) < sz(a)) {
293         int n = min(sz(b) * 2, sz(a));
294         b.resize(n);
295         poly v = poly(a.begin(), a.begin() + n) - log(b);
296         v[0] = v[0] + num(1);
297         b *= v;
298         b.resize(n);
299     }
300     return b;
301 }
302 poly pow(const poly& a, int m) { // m >= 0
303     poly b(a.size());
304     if (!m) {
305         b[0] = 1;
306         return b;
307     }
308     int p = 0;
309     while (p < sz(a) && a[p].v == 0) ++p;
310     if (1ll * m * p >= sz(a)) return b;

```

```

311 num mu = pow(a[p], m), di = inv(a[p]);
312 poly c(sz(a) - m * p);
313 rep(i, 0, sz(c)) c[i] = a[i + p] * di;
314 c = log(c);
315 trav(v, c) v = v * m;
316 c = exp(c);
317 rep(i, 0, sz(c)) b[i + m * p] = c[i] * mu;
318 return b;
319 }
320 // Multipoint evaluation/interpolation
321
322 vector<num> eval(const poly& a, const vector<num>& x) {
323     int n = sz(x);
324     if (!n) return {};
325     vector<poly> up(2 * n);
326     rep(i, 0, n) up[i + n] = poly({0 - x[i], 1});
327     per(i, 1, n) up[i] = up[2 * i] * up[2 * i + 1];
328     vector<poly> down(2 * n);
329     down[1] = a % up[1];
330     rep(i, 2, 2 * n) down[i] = down[i / 2] % up[i];
331     vector<num> y(n);
332     rep(i, 0, n) y[i] = down[i + n][0];
333     return y;
334 }
335
336 poly interp(const vector<num>& x, const vector<num>& y) {
337     int n = sz(x);
338     assert(n);
339     vector<poly> up(n * 2);
340     rep(i, 0, n) up[i + n] = poly({0 - x[i], 1});
341     per(i, 1, n) up[i] = up[2 * i] * up[2 * i + 1];
342     vector<num> a = eval(deriv(up[1]), x);
343     vector<poly> down(2 * n);
344     rep(i, 0, n) down[i + n] = poly({y[i] * inv(a[i])});
345     per(i, 1, n) down[i] =
346         down[i * 2] * up[i * 2 + 1] + down[i * 2 + 1] * up[i * 2];
347     return down[1];
348 }

```

Simplex method for linear programs

- Maximize $c^T x$ subject to $Ax \leq b, x \geq 0$.
- Returns $-\infty$ if there is no solution, $+\infty$ if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The (arbitrary) input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
- Complexity: $O(NM \cdot \text{pivots})$. $O(2^n)$ in general (very hard to achieve).

```

1 typedef double T; // might be much slower with long doubles
2 typedef vector<T> vd;
3 typedef vector<vd> vvd;
4 const T eps = 1e-8, inf = 1/.0;
5 #define MP make_pair
6 #define ltj(X) if(s == -1 || MP(X[j], N[j]) < MP(X[s], N[s]))
7     ↪ s=j
8 #define rep(i, a, b) for(int i = a; i < (b); ++i)
9
10 struct LPSolver {
11     int m, n;
12     vector<int> N, B;
13     vvd D;
14     LPSolver(const vvd& A, const vd& b, const vd& c) : m(sz(b)),
15     ↪ n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)){
16         rep(i, 0, m) rep(j, 0, n) D[i][j] = A[i][j];
17         rep(i, 0, m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
18         ↪ rep(j, 0, n) { N[j] = j; D[m][j] = -c[j]; }
19         N[n] = -1; D[m+1][n] = 1;
20     };
21     void pivot(int r, int s){
22         T *a = D[r].data(), inv = 1 / a[s];

```

```

20     rep(i, 0, m+2) if (i != r && abs(D[i][s]) > eps) {
21         T *b = D[i].data(), inv2 = b[s] * inv;
22         rep(j, 0, n+2) b[j] -= a[j] * inv2;
23         b[s] = a[s] * inv2;
24     }
25     rep(j, 0, n+2) if (j != s) D[r][j] *= inv;
26     rep(i, 0, m+2) if (i != r) D[i][s] *= -inv;
27     D[r][s] = inv;
28     swap(B[r], N[s]);
29 }
30 bool simplex(int phase){
31     int x = m + phase - 1;
32     for (;;) {
33         int s = -1;
34         rep(j, 0, n+1) if (N[j] != -phase) ltj(D[x]); if (D[x][s]
35     ↪ >= -eps) return true;
36         int r = -1;
37         rep(i, 0, m) {
38             if (D[i][s] <= eps) continue;
39             if (r == -1 || MP(D[i][n+1] / D[i][s], B[i]) <
40     ↪ MP(D[r][n+1] / D[r][s], B[r])) r = i;
41         }
42         if (r == -1) return false;
43         pivot(r, s);
44     }
45 }
46 T solve(vd &x){
47     int r = 0;
48     rep(i, 1, m) if (D[i][n+1] < D[r][n+1]) r = i;
49     if (D[r][n+1] < -eps) {
50         pivot(r, n);
51         if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
52         rep(i, 0, m) if (B[i] == -1) {
53             int s = 0;
54             rep(j, 1, n+1) ltj(D[i]);
55             pivot(i, s);
56         }
57     }
58     bool ok = simplex(1); x = vd(n);
59     rep(i, 0, m) if (B[i] < n) x[B[i]] = D[i][n+1];
60     return ok ? D[m][n+1] : inf;
61 }

```

Data Structures

Fenwick Tree

```

1 ll sum(int r) {
2     ll ret = 0;
3     for (; r >= 0; r = (r & r + 1) - 1) ret += bit[r];
4     return ret;
5 }
6 void add(int idx, ll delta) {
7     for (; idx < n; idx |= idx + 1) bit[idx] += delta;
8 }

```

Lazy Propagation SegTree

```

1 // Clear: clear() or build()
2 const int N = 2e5 + 10; // Change the constant!
3 template<typename T>
4 struct LazySegTree{
5     T t[4 * N];
6     T lazy[4 * N];
7     int n;
8
9     // Change these functions, default return, and lazy mark.
10    T default_return = 0, lazy_mark = numeric_limits<T>::min();
11    // Lazy mark is how the algorithm will identify that no
12    ↪ propagation is needed.
13    function<T(T, T)> f = [&] (T a, T b){
14        return a + b;
15    };

```



```

15 // f_on_seg calculates the function f, knowing the lazy
16 ↪ value on segment,
17 // segment's size and the previous value.
18 // The default is segment modification for RSQ. For
19 ↪ increments change to:
20 // return cur_seg_val + seg_size * lazy_val;
21 // For RMQ. Modification: return lazy_val; Increments:
22 ↪ return cur_seg_val + lazy_val;
23 function<T(T, int, T)> f_on_seg = [&] (T cur_seg_val, int
24 ↪ seg_size, T lazy_val){
25     return seg_size * lazy_val;
26 };
27 // upd_lazy updates the value to be propagated to child
28 ↪ segments.
29 // Default: modification. For increments change to:
30 // lazy[v] = (lazy[v] == lazy_mark? val : lazy[v] +
31 ↪ val);
32 function<void(int, T)> upd_lazy = [&] (int v, T val){
33     lazy[v] = val;
34 };
35 // Tip: for "get element on single index" queries, use max()
36 ↪ on segment: no overflows.
37
38 LazySegTree(int n_) : n(n_) {
39     clear(n);
40 }
41
42 void build(int v, int tl, int tr, vector<T>& a){
43     if (tl == tr) {
44         t[v] = a[tl];
45         return;
46     }
47     int tm = (tl + tr) / 2;
48     // left child: [tl, tm]
49     // right child: [tm + 1, tr]
50     build(2 * v + 1, tl, tm, a);
51     build(2 * v + 2, tm + 1, tr, a);
52     t[v] = f(t[2 * v + 1], t[2 * v + 2]);
53 }
54
55 LazySegTree(vector<T>& a){
56     build(a);
57 }
58
59 void push(int v, int tl, int tr){
60     if (lazy[v] == lazy_mark) return;
61     int tm = (tl + tr) / 2;
62     t[2 * v + 1] = f_on_seg(t[2 * v + 1], tm - tl + 1,
63 ↪ lazy[v]);
64     t[2 * v + 2] = f_on_seg(t[2 * v + 2], tr - tm, lazy[v]);
65     upd_lazy(2 * v + 1, lazy[v]), upd_lazy(2 * v + 2,
66 ↪ lazy[v]);
67     lazy[v] = lazy_mark;
68 }
69
70 void modify(int v, int tl, int tr, int l, int r, T val){
71     if (l > r) return;
72     if (tl == l && tr == r){
73         t[v] = f_on_seg(t[v], tr - tl + 1, val);
74         upd_lazy(v, val);
75         return;
76     }
77     push(v, tl, tr);
78     int tm = (tl + tr) / 2;
79     modify(2 * v + 1, tl, tm, l, min(r, tm), val);
80     modify(2 * v + 2, tm + 1, tr, max(l, tm + 1), r, val);
81     t[v] = f(t[2 * v + 1], t[2 * v + 2]);
82 }
83
84 T query(int v, int tl, int tr, int l, int r) {
85     if (l > r) return default_return;
86     if (tl == l && tr == r) return t[v];
87     push(v, tl, tr);
88     int tm = (tl + tr) / 2;
89     return f(
90         query(2 * v + 1, tl, tm, l, min(r, tm)),
91         query(2 * v + 2, tm + 1, tr, max(l, tm + 1), r)
92     );
93 }

```

```

83     );
84 }
85
86 void modify(int l, int r, T val){
87     modify(0, 0, n - 1, l, r, val);
88 }
89
90 T query(int l, int r){
91     return query(0, 0, n - 1, l, r);
92 }
93
94 T get(int pos){
95     return query(pos, pos);
96 }
97
98 // Change clear() function to t.clear() if using
99 ↪ unordered_map for SegTree!!!
100 void clear(int n_){
101     n = n_;
102     for (int i = 0; i < 4 * n; i++) t[i] = 0, lazy[i] =
103 ↪ lazy_mark;
104 }
105
106 void build(vector<T>& a){
107     n = sz(a);
108     clear(n);
109     build(0, 0, n - 1, a);
110 }
111 };

```

Sparse Table

```

1 const int N = 2e5 + 10, LOG = 20; // Change the constant!
2 template<typename T>
3 struct SparseTable{
4     int lg[N];
5     T st[N][LOG];
6     int n;
7
8     // Change this function
9     function<T(T, T)> f = [&] (T a, T b){
10         return min(a, b);
11     };
12
13 void build(vector<T>& a){
14     n = sz(a);
15     lg[1] = 0;
16     for (int i = 2; i <= n; i++) lg[i] = lg[i / 2] + 1;
17
18     for (int k = 0; k < LOG; k++){
19         for (int i = 0; i < n; i++){
20             if (!k) st[i][k] = a[i];
21             else st[i][k] = f(st[i][k - 1], st[min(n - 1, i + (1 <<
22 ↪ (k - 1))))[k - 1]);
23         }
24     }
25
26 T query(int l, int r){
27     int sz = r - l + 1;
28     return f(st[l][lg[sz]], st[r - (1 << lg[sz]) + 1][lg[sz]]);
29 }
30 };

```

Suffix Array and LCP array

- (uses SparseTable above)

```

1 struct SuffixArray{
2     vector<int> p, c, h;
3     SparseTable<int> st;
4     /*
5     In the end, array c gives the position of each suffix in p
6     using 1-based indexation!
7     */
8 }

```

Aho Corasick Trie

- For each node in the trie, the suffix link points to the longest proper suffix of the represented string. The terminal-link tree has square-root height (can be constructed by DFS).

```

1  const int S = 26;
2
3  // Function converting char to int.
4  int ctoi(char c){
5      return c - 'a';
6  }
7
8  // To add terminal links, use DFS
9  struct Node{
10     vector<int> nxt;
11     int link;
12     bool terminal;
13
14     Node() {
15         nxt.assign(S, -1), link = 0, terminal = 0;
16     }
17 };
18
19 vector<Node> trie(1);
20
21 // add_string returns the terminal vertex.
22 int add_string(string& s){
23     int v = 0;
24     for (auto c : s){
25         int cur = ctoi(c);
26         if (trie[v].nxt[cur] == -1){
27             trie[v].nxt[cur] = sz(trie);
28             trie.emplace_back();
29         }
30         v = trie[v].nxt[cur];
31     }
32     trie[v].terminal = 1;
33     return v;
34 }
35
36 /*
37 Suffix links are compressed.
38 This means that:
39 If vertex v has a child by letter x, then:
40     trie[v].nxt[x] points to that child.
41 If vertex v doesn't have such child, then:
42     trie[v].nxt[x] points to the suffix link of that child
43     if we would actually have it.
44 */
45 void add_links(){
46     queue<int> q;
47     q.push(0);
48     while (!q.empty()){
49         auto v = q.front();
50         int u = trie[v].link;
51         q.pop();
52         for (int i = 0; i < S; i++){
53             int& ch = trie[v].nxt[i];
54             if (ch == -1){
55                 ch = v? trie[u].nxt[i] : 0;
56             }
57             else{
58                 trie[ch].link = v? trie[u].nxt[i] : 0;
59                 q.push(ch);
60             }
61         }
62     }
63 }
64
65 bool is_terminal(int v){
66     return trie[v].terminal;
67 }
68
69 int get_link(int v){
70     return trie[v].link;

```

```

9  SuffixArray() {}
10
11 SuffixArray(string s){
12     buildArray(s);
13     buildLCP(s);
14     buildSparse();
15 }
16
17 void buildArray(string s){
18     int n = sz(s) + 1;
19     p.resize(n), c.resize(n);
20     for (int i = 0; i < n; i++) p[i] = i;
21     sort(all(p), [&] (int a, int b){return s[a] < s[b];});
22     c[p[0]] = 0;
23     for (int i = 1; i < n; i++){
24         c[p[i]] = c[p[i - 1]] + (s[p[i]] != s[p[i - 1]]);
25     }
26     vector<int> p2(n), c2(n);
27     // w is half-length of each string.
28     for (int w = 1; w < n; w <= 1){
29         for (int i = 0; i < n; i++){
30             p2[i] = (p[i] - w + n) % n;
31         }
32         vector<int> cnt(n);
33         for (auto i : c) cnt[i]++;
34         for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
35         for (int i = n - 1; i >= 0; i--){
36             p[--cnt[c[p2[i]]]] = p2[i];
37         }
38         c2[p[0]] = 0;
39         for (int i = 1; i < n; i++){
40             c2[p[i]] = c2[p[i - 1]] +
41                 (c[p[i]] != c[p[i - 1]] ||
42                  c[(p[i] + w) % n] != c[(p[i - 1] + w) % n]);
43         }
44         c.swap(c2);
45     }
46     p.erase(p.begin());
47 }
48
49 void buildLCP(string s){
50     // The algorithm assumes that suffix array is already
51     // built on the same string.
52     int n = sz(s);
53     h.resize(n - 1);
54     int k = 0;
55     for (int i = 0; i < n; i++){
56         if (c[i] == n){
57             k = 0;
58             continue;
59         }
60         int j = p[c[i]];
61         while (i + k < n && j + k < n && s[i + k] == s[j + k])
62             k++;
63         h[c[i] - 1] = k;
64         if (k) k--;
65     }
66     /*
67     Then an RMQ Sparse Table can be built on array h
68     to calculate LCP of 2 non-consecutive suffixes.
69     */
70 }
71
72 void buildSparse(){
73     st.build(h);
74 }
75
76 // l and r must be in 0-BASED INDEXATION
77 int lcp(int l, int r){
78     l = c[l] - 1, r = c[r] - 1;
79     if (l > r) swap(l, r);
80     return st.query(l, r - 1);

```

```

71 }
72
73 int go(int v, char c){
74     return trie[v].nxt[ctoi(c)];
75 }

```

Convex Hull Trick

- Allows to insert a linear function to the hull in $O(1)$ and get the minimum/maximum value of the stored function at a point in $O(\log n)$.
- NOTE: The lines must be added in the order of decreasing/increasing gradients. CAREFULLY CHECK THE SETUP BEFORE USING!
- IMPORTANT: THE DEFAULT VERSION SURELY WORKS. IF MODIFIED VERSIONS DON'T WORK, TRY TRANSFORMING THEM TO THE DEFAULT ONE BY CHANGING SIGNS.

```

1 struct line{
2     ll k, b;
3     ll f(ll x){
4         return k * x + b;
5     };
6 };
7
8 vector<line> hull;
9
10 void add_line(line nl){
11     if (!hull.empty() && hull.back().k == nl.k){
12         nl.b = min(nl.b, hull.back().b); // Default: minimum. For
        ↪ maximum change "min" to "max".
13         hull.pop_back();
14     }
15     while (sz(hull) > 1){
16         auto& l1 = hull.end()[-2], l2 = hull.back();
17         if ((nl.b - l1.b) * (l2.k - nl.k) >= (nl.b - l2.b) * (l1.k
        ↪ - nl.k)) hull.pop_back(); // Default: decreasing gradient
        ↪ k. For increasing k change the sign to <=.
18         else break;
19     }
20     hull.pb(nl);
21 }
22
23 ll get(ll x){
24     int l = 0, r = sz(hull);
25     while (r - l > 1){
26         int mid = (l + r) / 2;
27         if (hull[mid - 1].f(x) >= hull[mid].f(x)) l = mid; //
        ↪ Default: minimum. For maximum change the sign to <=.
28         else r = mid;
29     }
30     return hull[l].f(x);
31 }

```

Li-Chao Segment Tree

- allows to add linear functions in any order and query minimum/maximum value of those at a point, all in $O(\log n)$.
- Clear: clear()

```

1 const ll INF = 1e18; // Change the constant!
2 struct LiChaoTree{
3     struct line{
4         ll k, b;
5         line(){
6             k = b = 0;
7         };
8         line(ll k_, ll b_){
9             k = k_, b = b_;
10        };
11        ll f(ll x){

```

```

12         return k * x + b;
13     };
14 };
15 int n;
16 bool minimum, on_points;
17 vector<ll> pts;
18 vector<line> t;
19
20 void clear(){
21     for (auto& l : t) l.k = 0, l.b = minimum? INF : -INF;
22 }
23
24 LiChaoTree(int n_, bool min_){ // This is a default
        ↪ constructor for numbers in range [0, n - 1].
25     n = n_, minimum = min_, on_points = false;
26     t.resize(4 * n);
27     clear();
28 };
29
30 LiChaoTree(vector<ll> pts_, bool min_){ // This constructor
        ↪ will build LCT on the set of points you pass. The points
        ↪ may be in any order and contain duplicates.
31     pts = pts_, minimum = min_;
32     sort(all(pts));
33     pts.erase(unique(all(pts)), pts.end());
34     on_points = true;
35     n = sz(pts);
36     t.resize(4 * n);
37     clear();
38 };
39
40 void add_line(int v, int l, int r, line nl){
41     // Adding on segment [l, r)
42     int m = (l + r) / 2;
43     ll lval = on_points? pts[l] : l, mval = on_points? pts[m]
        ↪ : m;
44     if ((minimum && nl.f(mval) < t[v].f(mval)) || (!minimum &&
        ↪ nl.f(mval) > t[v].f(mval))) swap(t[v], nl);
45     if (r - l == 1) return;
46     if ((minimum && nl.f(lval) < t[v].f(lval)) || (!minimum &&
        ↪ nl.f(lval) > t[v].f(lval))) add_line(2 * v + 1, l, m, nl);
47     else add_line(2 * v + 2, m, r, nl);
48 }
49
50 ll get(int v, int l, int r, int x){
51     int m = (l + r) / 2;
52     if (r - l == 1) return t[v].f(on_points? pts[x] : x);
53     else{
54         if (minimum) return min(t[v].f(on_points? pts[x] : x), x
        ↪ < m? get(2 * v + 1, l, m, x) : get(2 * v + 2, m, r, x));
55         else return max(t[v].f(on_points? pts[x] : x), x < m?
        ↪ get(2 * v + 1, l, m, x) : get(2 * v + 2, m, r, x));
56     }
57 }
58
59 void add_line(ll k, ll b){
60     add_line(0, 0, n, line(k, b));
61 }
62
63 ll get(ll x){
64     return get(0, 0, n, on_points? lower_bound(all(pts), x) -
        ↪ pts.begin() : x);
65 }; // Always pass the actual value of x, even if LCT is on
        ↪ points.
66 };

```

Persistent Segment Tree

- for RSQ

```

1 struct Node {
2     ll val;
3     Node *l, *r;
4
5     Node(ll x) : val(x), l(nullptr), r(nullptr) {}
6     Node(Node *ll, Node *rr) {
7         l = ll, r = rr;

```

```

8         val = 0;
9         if (l) val += l->val;
10        if (r) val += r->val;
11    }
12    Node(Node *cp) : val(cp->val), l(cp->l), r(cp->r) {}
13 };
14 const int N = 2e5 + 20;
15 ll a[N];
16 Node *roots[N];
17 int n, cnt = 1;
18 Node *build(int l = 1, int r = n) {
19     if (l == r) return new Node(a[l]);
20     int mid = (l + r) / 2;
21     return new Node(build(l, mid), build(mid + 1, r));
22 }
23 Node *update(Node *node, int val, int pos, int l = 1, int r =
    ↪ n) {
24     if (l == r) return new Node(val);
25     int mid = (l + r) / 2;
26     if (pos > mid)
27         return new Node(node->l, update(node->r, val, pos, mid
    ↪ + 1, r));
28     else return new Node(update(node->l, val, pos, l, mid),
    ↪ node->r);
29 }
30 ll query(Node *node, int a, int b, int l = 1, int r = n) {
31     if (l > b || r < a) return 0;
32     if (l >= a && r <= b) return node->val;
33     int mid = (l + r) / 2;
34     return query(node->l, a, b, l, mid) + query(node->r, a, b,
    ↪ mid + 1, r);
35 }

```

Dynamic Programming

Sum over Subset DP

- Computes $f[A] = \sum_{B \subseteq A} a[B]$.
- Complexity: $O(2^n \cdot n)$.

```

1 for (int i = 0; i < (1 << n); i++) f[i] = a[i];
2 for (int i = 0; i < n; i++) for (int mask = 0; mask < (1 <<
    ↪ n); mask++) if ((mask >> i) & 1){
3     f[mask] += f[mask ^ (1 << i)];
4 }

```

Divide and Conquer DP

- Helps to compute 2D DP of the form:
- $dp[i][j] = \min_{0 \leq k \leq j-1} (dp[i-1][k] + cost(k+1, j))$
- **Necessary condition:** let $opt(i, j)$ be the optimal k for the state (i, j) . Then, $opt(i, j) \leq opt(i, j+1)$.
- **Sufficient condition:** $cost(a, d) + cost(b, c) \geq cost(a, c) + cost(b, d)$ where $a < b < c < d$.
- Complexity: $O(M \cdot N \cdot \log N)$ for computing $dp[M][N]$.

```

1 vector<ll> dp_old(N), dp_new(N);
2
3 void rec(int l, int r, int optl, int optr){
4     if (l > r) return;
5     int mid = (l + r) / 2;
6     pair<ll, int> best = {INF, optl};
7     for (int i = optl; i <= min(mid - 1, optr); i++){ // If k
    ↪ can be j, change to "i <= min(mid, optr)".
8         ll cur = dp_old[i] + cost(i + 1, mid);
9         if (cur < best.fi) best = {cur, i};
10    }
11    dp_new[mid] = best.fi;
12
13    rec(l, mid - 1, optl, best.se);
14    rec(mid + 1, r, best.se, optr);
15 }
16

```

```

17 // Computes the DP "by layers"
18 fill(all(dp_old), INF);
19 dp_old[0] = 0;
20 while (layers--){
21     rec(0, n, 0, n);
22     dp_old = dp_new;
23 }

```

Knuth's DP Optimization

- Computes DP of the form
- $dp[i][j] = \min_{i \leq k \leq j-1} (dp[i][k] + dp[k+1][j] + cost(i, j))$
- **Necessary Condition:** $opt(i, j-1) \leq opt(i, j) \leq opt(i+1, j)$
- **Sufficient Condition:** For $a \leq b \leq c \leq d$, $cost(b, c) \leq cost(a, d)$ AND $cost(a, d) + cost(b, c) \geq cost(a, c) + cost(b, d)$
- Complexity: $O(n^2)$

```

1 int N;
2 int dp[N][N], opt[N][N];
3 auto C = [&](int i, int j) {
4     // Implement cost function C.
5 };
6 for (int i = 0; i < N; i++) {
7     opt[i][i] = i;
8     // Initialize dp[i][i] according to the problem
9 }
10 for (int i = N-2; i >= 0; i--) {
11     for (int j = i+1; j < N; j++) {
12         int mn = INT_MAX;
13         int cost = C(i, j);
14         for (int k = opt[i][j-1]; k <= min(j-1, opt[i+1][j]); k++){
15             ↪ {
16                 if (mn >= dp[i][k] + dp[k+1][j] + cost) {
17                     opt[i][j] = k;
18                     mn = dp[i][k] + dp[k+1][j] + cost;
19                 }
20                 dp[i][j] = mn;
21             }
22 }

```

Miscellaneous

Ordered Set

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4 typedef tree<int, null_type, less<int>, rb_tree_tag,
    ↪ tree_order_statistics_node_update> ordered_set;

```

Measuring Execution Time

```

1 ld tic = clock();
2 // execute algo...
3 ld tac = clock();
4 // Time in milliseconds
5 cerr << (tac - tic) / CLOCKS_PER_SEC * 1000 << endl;
6 // No need to comment out the print because it's done to cerr.

```

Setting Fixed D.P. Precision

```

1 cout << setprecision(d) << fixed;
2 // Each number is rounded to d digits after the decimal point,
    ↪ and truncated.

```

Common Bugs and General Advice

- Check overflow, array bounds

- Check variable overloading
- Check special cases ($n=1?$)
- Do something instead of nothing, stay organized
- Write stuff down!
- Don't get stuck on one approach!