

Columbia University: CU Later Team Reference Document

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Templates

Ken's template

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 #define all(v) (v).begin(), (v).end()
4 typedef long long ll;
5 typedef long double ld;
6 #define pb push_back
7 #define sz(x) (int)(x).size()
8 #define fi first
9 #define se second
10 #define endl '\n'
```

Kevin's template

```
1 // paste Kaurov's Template, minus last line
2 typedef vector<int> vi;
3 typedef vector<ll> vll;
4 typedef pair<int, int> pii;
5 typedef pair<ll, ll> pll;
6 const char nl = '\n';
7 #define forn(i, n) for (int i = 0; i < int(n); i++)
8 ll k, n, m, u, v, w, x, y, z;
9 string s, t;
10
11 bool multiTest = 1;
12 void solve(int tt){
13 }
14
15 int main(){
16     ios::sync_with_stdio(0);cin.tie(0);cout.tie(0);
17     cout<<fixed<< setprecision(14);
18
19     int t = 1;
20     if (multiTest) cin >> t;
21     forn(ii, t) solve(ii);
22 }
```

Kevin's Template Extended

- to type after the start of the contest

```
1 typedef pair<double, double> pdd;
2 const ld PI = acosl(-1);
3 const ll mod7 = 1e9 + 7;
4 const ll mod9 = 998244353;
5 const ll INF = 2*1024*1024*1023;
6 #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
7 #include <ext/pb_ds/assoc_container.hpp>
8 #include <ext/pb_ds/tree_policy.hpp>
9 using namespace __gnu_pbds;
10 template<class T> using ordered_set = tree<T, null_type,
    ↳ less<T>, rb_tree_tag,
    ↳ tree_order_statistics_node_update>;
11 vi d4x = {1, 0, -1, 0};
```

```
12 vi d4y = {0, 1, 0, -1};
13 vi d8x = {1, 0, -1, 0, 1, 1, -1, -1};
14 vi d8y = {0, 1, 0, -1, 1, -1, 1, -1};
15 mt19937
    ↳ rng(chrono::steady_clock::now().time_since_epoch().count())
```

Geometry

- Basic stuff

```
1 template<typename T>
2 struct TPoint{
3     T x, y;
4     int id;
5     static constexpr T eps = static_cast<T>(1e-9);
6     TPoint() : x(0), y(0), id(-1) {}
7     TPoint(const T& x_, const T& y_) : x(x_), y(y_),
    ↳ id(-1) {}
8     TPoint(const T& x_, const T& y_, const int id_) :
    ↳ x(x_), y(y_), id(id_) {}
9
10     TPoint operator + (const TPoint& rhs) const {
11         return TPoint(x + rhs.x, y + rhs.y);
12     }
13     TPoint operator - (const TPoint& rhs) const {
14         return TPoint(x - rhs.x, y - rhs.y);
15     }
16     TPoint operator * (const T& rhs) const {
17         return TPoint(x * rhs, y * rhs);
18     }
19     TPoint operator / (const T& rhs) const {
20         return TPoint(x / rhs, y / rhs);
21     }
22     TPoint ort() const {
23         return TPoint(-y, x);
24     }
25     T abs2() const {
26         return x * x + y * y;
27     }
28     T len() const {
29         return sqrtl(abs2());
30     }
31     TPoint unit() const {
32         return TPoint(x, y) / len();
33     }
34 };
35 template<typename T>
36 bool operator< (TPoint<T>& A, TPoint<T>& B){
37     return make_pair(A.x, A.y) < make_pair(B.x, B.y);
38 }
39 template<typename T>
40 bool operator==(TPoint<T>& A, TPoint<T>& B){
41     return abs(A.x - B.x) <= TPoint<T>::eps && abs(A.y -
    ↳ B.y) <= TPoint<T>::eps;
42 }
43 template<typename T>
44 struct TLine{
```

```
45     T a, b, c;
46     TLine() : a(0), b(0), c(0) {}
47     TLine(const T& a_, const T& b_, const T& c_) : a(a_),
    ↳ b(b_), c(c_) {}
48     TLine(const TPoint<T>& p1, const TPoint<T>& p2){
49         a = p1.y - p2.y;
50         b = p2.x - p1.x;
51         c = -a * p1.x - b * p1.y;
52     }
53 };
54 template<typename T>
55 T det(const T& a11, const T& a12, const T& a21, const T&
    ↳ a22){
56     return a11 * a22 - a12 * a21;
57 }
58 template<typename T>
59 T sq(const T& a){
60     return a * a;
61 }
62 template<typename T>
63 T smul(const TPoint<T>& a, const TPoint<T>& b){
64     return a.x * b.x + a.y * b.y;
65 }
66 template<typename T>
67 T vmul(const TPoint<T>& a, const TPoint<T>& b){
68     return det(a.x, a.y, b.x, b.y);
69 }
70 template<typename T>
71 bool parallel(const TLine<T>& l1, const TLine<T>& l2){
72     return abs(vmul(TPoint<T>(l1.a, l1.b), TPoint<T>(l2.a,
    ↳ l2.b))) <= TPoint<T>::eps;
73 }
74 template<typename T>
75 bool equivalent(const TLine<T>& l1, const TLine<T>& l2){
76     return parallel(l1, l2) &&
77         abs(det(l1.b, l1.c, l2.b, l2.c)) <= TPoint<T>::eps &&
78         abs(det(l1.a, l1.c, l2.a, l2.c)) <= TPoint<T>::eps;
79 }
```

- Intersection

```
1 template<typename T>
2 TPoint<T> intersection(const TLine<T>& l1, const
    ↳ TLine<T>& l2){
3     return TPoint<T>{
4         det(-l1.c, l1.b, -l2.c, l2.b) / det(l1.a, l1.b,
    ↳ l2.a, l2.b),
5         det(l1.a, -l1.c, l2.a, -l2.c) / det(l1.a, l1.b,
    ↳ l2.a, l2.b)
6     };
7 }
8 template<typename T>
9 int sign(const T& x){
10     if (abs(x) <= TPoint<T>::eps) return 0;
11     return x > 0? +1 : -1;
12 }
```

- Area

```

1  template<typename T>
2  T area(const vector<TPoint<T>>& pts){
3      int n = sz(pts);
4      T ans = 0;
5      for (int i = 0; i < n; i++){
6          ans += vmul(pts[i], pts[(i + 1) % n]);
7      }
8      return abs(ans) / 2;
9  }
10 template<typename T>
11 T dist_pp(const TPoint<T>& a, const TPoint<T>& b){
12     return sqrt(sq(a.x - b.x) + sq(a.y - b.y));
13 }
14 template<typename T>
15 TLine<T> perp_line(const TLine<T>& l, const TPoint<T>&
    ↪ p){
16     T na = -l.b, nb = l.a, nc = - na * p.x - nb * p.y;
17     return TLine<T>(na, nb, nc);
18 }

```

• Projection

```

1  template<typename T>
2  TPoint<T> projection(const TPoint<T>& p, const TLine<T>&
    ↪ l){
3      return intersection(l, perp_line(l, p));
4  }
5  template<typename T>
6  T dist_pl(const TPoint<T>& p, const TLine<T>& l){
7      return dist_pp(p, projection(p, l));
8  }
9  template<typename T>
10 struct TRay{
11     TLine<T> l;
12     TPoint<T> start, dirvec;
13     TRay() : l(), start(), dirvec() {}
14     TRay(const TPoint<T>& p1, const TPoint<T>& p2){
15         l = TLine<T>(p1, p2);
16         start = p1, dirvec = p2 - p1;
17     }
18 };
19 template<typename T>
20 bool is_on_line(const TPoint<T>& p, const TLine<T>& l){
21     return abs(l.a * p.x + l.b * p.y + l.c) <=
    ↪ TPoint<T>::eps;
22 }
23 template<typename T>
24 bool is_on_ray(const TPoint<T>& p, const TRay<T>& r){
25     if (is_on_line(p, r.l)){
26         return sign(smul(r.dirvec, TPoint<T>(p - r.start)))
    ↪ != -1;
27     }
28     else return false;
29 }
30 template<typename T>
31 bool is_on_seg(const TPoint<T>& P, const TPoint<T>& A,
    ↪ const TPoint<T>& B){
32     return is_on_ray(P, TRay<T>(A, B)) && is_on_ray(P,
    ↪ TRay<T>(B, A));

```

```

33 }
34 template<typename T>
35 T dist_pr(const TPoint<T>& P, const TRay<T>& R){
36     auto H = projection(P, R.l);
37     return is_on_ray(H, R)? dist_pp(P, H) : dist_pp(P,
    ↪ R.start);
38 }
39 template<typename T>
40 T dist_ps(const TPoint<T>& P, const TPoint<T>& A, const
    ↪ TPoint<T>& B){
41     auto H = projection(P, TLine<T>(A, B));
42     if (is_on_seg(H, A, B)) return dist_pp(P, H);
43     else return min(dist_pp(P, A), dist_pp(P, B));
44 }

```

• acw

```

1  template<typename T>
2  bool acw(const TPoint<T>& A, const TPoint<T>& B){
3      T mul = vmul(A, B);
4      return mul > 0 || abs(mul) <= TPoint<T>::eps;
5  }

```

• CW

```

1  template<typename T>
2  bool cw(const TPoint<T>& A, const TPoint<T>& B){
3      T mul = vmul(A, B);
4      return mul < 0 || abs(mul) <= TPoint<T>::eps;
5  }

```

• Convex Hull

```

1  template<typename T>
2  vector<TPoint<T>> convex_hull(vector<TPoint<T>> pts){
3      sort(all(pts));
4      pts.erase(unique(all(pts)), pts.end());
5      vector<TPoint<T>> up, down;
6      for (auto p : pts){
7          while (sz(up) > 1 && acw(up.end()[-1] -
    ↪ up.end()[-2], p - up.end()[-2])) up.pop_back();
8          while (sz(down) > 1 && cw(down.end()[-1] -
    ↪ down.end()[-2], p - down.end()[-2]))
    ↪ down.pop_back();
9          up.pb(p), down.pb(p);
10     }
11     for (int i = sz(up) - 2; i >= 1; i--) down.pb(up[i]);
12     return down;
13 }

```

• in_triangle

```

1  template<typename T>
2  bool in_triangle(TPoint<T>& P, TPoint<T>& A, TPoint<T>&
    ↪ B, TPoint<T>& C){
3      if (is_on_seg(P, A, B) || is_on_seg(P, B, C) ||
    ↪ is_on_seg(P, C, A)) return true;
4      return cw(P - A, B - A) == cw(P - B, C - B) &&
    ↪ cw(P - A, B - A) == cw(P - C, A - C);
5  }

```

• prep_convex_poly

```

1  template<typename T>
2  void prep_convex_poly(vector<TPoint<T>>& pts){
3      rotate(pts.begin(), min_element(all(pts)), pts.end());
4  }

```

• in_convex_poly:

```

1  // 0 - Outside, 1 - Exclusively Inside, 2 - On the
    ↪ Border
2  template<typename T>
3  int in_convex_poly(TPoint<T>& p, vector<TPoint<T>>&
    ↪ pts){
4      int n = sz(pts);
5      if (!n) return 0;
6      if (n <= 2) return is_on_seg(p, pts[0], pts.back());
7      int l = 1, r = n - 1;
8      while (r - l > 1){
9          int mid = (l + r) / 2;
10         if (acw(pts[mid] - pts[0], p - pts[0])) l = mid;
11         else r = mid;
12     }
13     if (!in_triangle(p, pts[0], pts[l], pts[l + 1]))
    ↪ return 0;
14     if (is_on_seg(p, pts[l], pts[l + 1]) ||
15         is_on_seg(p, pts[0], pts.back()) ||
16         is_on_seg(p, pts[0], pts[l]))
17         return 2;
18     return 1;
19 }

```

• in_simple_poly

```

1  // 0 - Outside, 1 - Exclusively Inside, 2 - On the
    ↪ Border
2  template<typename T>
3  int in_simple_poly(TPoint<T> p, vector<TPoint<T>>& pts){
4      int n = sz(pts);
5      bool res = 0;
6      for (int i = 0; i < n; i++){
7          auto a = pts[i], b = pts[(i + 1) % n];
8          if (is_on_seg(p, a, b)) return 2;
9          if (((a.y > p.y) - (b.y > p.y)) * vmul(b - p, a - p)
    ↪ > TPoint<T>::eps){
10             res ^= 1;
11         }
12     }
13     return res;
14 }

```

• minkowski_rotate

```

1  template<typename T>
2  void minkowski_rotate(vector<TPoint<T>>& P){
3      int pos = 0;
4      for (int i = 1; i < sz(P); i++){
5          if (abs(P[i].y - P[pos].y) <= TPoint<T>::eps){

```

```

6         if (P[i].x < P[pos].x) pos = i;
7     }
8     else if (P[i].y < P[pos].y) pos = i;
9 }
10 rotate(P.begin(), P.begin() + pos, P.end());
11 }

    • minkowski_sum

1 // P and Q are strictly convex, points given in
  ↪ counterclockwise order
2 template<typename T>
3 vector<TPoint<T>> minkowski_sum(vector<TPoint<T>> P,
  ↪ vector<TPoint<T>> Q){
4     minkowski_rotate(P);
5     minkowski_rotate(Q);
6     P.pb(P[0]);
7     Q.pb(Q[0]);
8     vector<TPoint<T>> ans;
9     int i = 0, j = 0;
10    while (i < sz(P) - 1 || j < sz(Q) - 1){
11        ans.pb(P[i] + Q[j]);
12        T curmul;
13        if (i == sz(P) - 1) curmul = -1;
14        else if (j == sz(Q) - 1) curmul = +1;
15        else curmul = vmul(P[i + 1] - P[i], Q[j + 1] -
  ↪ Q[j]);
16        if (abs(curmul) < TPoint<T>::eps || curmul > 0) i++;
17        if (abs(curmul) < TPoint<T>::eps || curmul < 0) j++;
18    }
19    return ans;
20 }
21 using Point = TPoint<ll>; using Line = TLine<ll>; using
  ↪ Ray = TRay<ll>; const ld PI = acos(-1);

```

Strings

```

1 vector<int> prefix_function(string s){
2     int n = sz(s);
3     vector<int> pi(n);
4     for (int i = 1; i < n; i++){
5         int k = pi[i - 1];
6         while (k > 0 && s[i] != s[k]){
7             k = pi[k - 1];
8         }
9         pi[i] = k + (s[i] == s[k]);
10    }
11    return pi;
12 }
13 vector<int> kmp(string s, string k){
14     string st = k + "#" + s;
15     vector<int> res;
16     auto pi = pf(st);
17     for (int i = 0; i < sz(st); i++){
18         if (pi[i] == sz(k)){
19             res.pb(i - 2 * sz(k));
20         }

```

```

21     }
22     return res;
23 }
24 vector<int> z_function(string s){
25     int n = sz(s);
26     vector<int> z(n);
27     int l = 0, r = 0;
28     for (int i = 1; i < n; i++){
29         if (r >= i) z[i] = min(z[i - l], r - i + 1);
30         while (i + z[i] < n && s[z[i]] == s[i + z[i]]){
31             z[i]++;
32         }
33         if (i + z[i] - 1 > r){
34             l = i, r = i + z[i] - 1;
35         }
36     }
37     return z;
38 }

```

Manacher's algorithm

```

1 string longest_palindrome(string& s) {
2     // init "abc" -> "~$a#b#c$"
3     vector<char> t{'~', '#'};
4     for (char c : s) t.push_back(c), t.push_back('#');
5     t.push_back('$');
6     // manacher
7     int n = t.size(), r = 0, c = 0;
8     vector<int> p(n, 0);
9     for (int i = 1; i < n - 1; i++) {
10        if (i < r + c) p[i] = min(p[2 * c - i], r + c - i);
11        while (t[i + p[i] + 1] == t[i - p[i] - 1]) p[i]++;
12        if (i + p[i] > r + c) r = p[i], c = i;
13    }
14    // s[i] -> p[2 * i + 2] (even), p[2 * i + 2] (odd)
15    // output answer
16    int index = 0;
17    for (int i = 0; i < n; i++)
18        if (p[index] < p[i]) index = i;
19    return s.substr((index - p[index]) / 2, p[index]);
20 }

```

Flows

$O(N^2M)$, on unit networks $O(N^{1/2}M)$

```

1 struct FlowEdge {
2     int v, u;
3     ll cap, flow = 0;
4     FlowEdge(int v, int u, ll cap) : v(v), u(u),
  ↪ cap(cap) {}
5 };
6 struct Dinic {
7     const ll flow_inf = 1e18;
8     vector<FlowEdge> edges;
9     vector<vector<int>> adj;

```

```

10     int n, m = 0;
11     int s, t;
12     vector<int> level, ptr;
13     queue<int> q;
14     Dinic(int n, int s, int t) : n(n), s(s), t(t) {
15         adj.resize(n);
16         level.resize(n);
17         ptr.resize(n);
18     }
19     void add_edge(int v, int u, ll cap) {
20         edges.emplace_back(v, u, cap);
21         edges.emplace_back(u, v, 0);
22         adj[v].push_back(m);
23         adj[u].push_back(m + 1);
24         m += 2;
25     }
26     bool bfs() {
27         while (!q.empty()) {
28             int v = q.front();
29             q.pop();
30             for (int id : adj[v]) {
31                 if (edges[id].cap - edges[id].flow < 1)
32                     continue;
33                 if (level[edges[id].u] != -1)
34                     continue;
35                 level[edges[id].u] = level[v] + 1;
36                 q.push(edges[id].u);
37             }
38         }
39         return level[t] != -1;
40     }
41     ll dfs(int v, ll pushed) {
42         if (pushed == 0)
43             return 0;
44         if (v == t)
45             return pushed;
46         for (int& cid = ptr[v]; cid <
  ↪ (int)adj[v].size(); cid++) {
47             int id = adj[v][cid];
48             int u = edges[id].u;
49             if (level[v] + 1 != level[u] ||
  ↪ edges[id].cap - edges[id].flow < 1)
50                 continue;
51             ll tr = dfs(u, min(pushed, edges[id].cap -
  ↪ edges[id].flow));
52             if (tr == 0)
53                 continue;
54             edges[id].flow += tr;
55             edges[id ^ 1].flow -= tr;
56             return tr;
57         }
58         return 0;
59     }
60     ll flow() {
61         ll f = 0;
62         while (true) {
63             fill(level.begin(), level.end(), -1);

```

```

64         level[s] = 0;
65         q.push(s);
66         if (!bfs())
67             break;
68         fill(ptr.begin(), ptr.end(), 0);
69         while (ll pushed = dfs(s, flow_inf)) {
70             f += pushed;
71         }
72     }
73     return f;
74 }
75 };
76 // To recover flow through original edges: iterate over
    even indices in edges.

```

MCMF – maximize flow, then minimize its cost. $O(mn + Fm \log n)$.

```

1  #include <ext/pb_ds/priority_queue.hpp>
2  template <typename T, typename C>
3  class MCMF {
4  public:
5      static constexpr T eps = (T) 1e-9;
6
7      struct edge {
8          int from;
9          int to;
10         T c;
11         T f;
12         C cost;
13     };
14
15     int n;
16     vector<vector<int>> g;
17     vector<edge> edges;
18     vector<C> d;
19     vector<C> pot;
20     __gnu_pbds::priority_queue<pair<C, int>> q;
21     vector<typename decltype(q)::point_iterator> its;
22     vector<int> pe;
23     const C INF_C = numeric_limits<C>::max() / 2;
24
25     explicit MCMF(int n_) : n(n_), g(n), d(n), pot(n, 0),
    its(n), pe(n) {}
26
27     int add(int from, int to, T forward_cap, C edge_cost,
    T backward_cap = 0) {
28         assert(0 <= from && from < n && 0 <= to && to < n);
29         assert(forward_cap >= 0 && backward_cap >= 0);
30         int id = static_cast<int>(edges.size());
31         g[from].push_back(id);
32         edges.push_back({from, to, forward_cap, 0,
    edge_cost});
33         g[to].push_back(id + 1);
34         edges.push_back({to, from, backward_cap, 0,
    -edge_cost});
35         return id;

```

```

36     }
37
38     void expath(int st) {
39         fill(d.begin(), d.end(), INF_C);
40         q.clear();
41         fill(its.begin(), its.end(), q.end());
42         its[st] = q.push({pot[st], st});
43         d[st] = 0;
44         while (!q.empty()) {
45             int i = q.top().second;
46             q.pop();
47             its[i] = q.end();
48             for (int id : g[i]) {
49                 const edge &e = edges[id];
50                 int j = e.to;
51                 if (e.c - e.f > eps && d[i] + e.cost < d[j]) {
52                     d[j] = d[i] + e.cost;
53                     pe[j] = id;
54                     if (its[j] == q.end()) {
55                         its[j] = q.push({pot[j] - d[j], j});
56                     } else {
57                         q.modify(its[j], {pot[j] - d[j], j});
58                     }
59                 }
60             }
61         }
62         swap(d, pot);
63     }
64
65     pair<T, C> max_flow(int st, int fin) {
66         T flow = 0;
67         C cost = 0;
68         bool ok = true;
69         for (auto& e : edges) {
70             if (e.c - e.f > eps && e.cost + pot[e.from] -
    pot[e.to] < 0) {
71                 ok = false;
72                 break;
73             }
74         }
75         if (ok) {
76             expath(st);
77         } else {
78             vector<int> deg(n, 0);
79             for (int i = 0; i < n; i++) {
80                 for (int eid : g[i]) {
81                     auto& e = edges[eid];
82                     if (e.c - e.f > eps) {
83                         deg[e.to] += 1;
84                     }
85                 }
86             }
87             vector<int> que;
88             for (int i = 0; i < n; i++) {
89                 if (deg[i] == 0) {
90                     que.push_back(i);
91                 }
92             }

```

```

93         for (int b = 0; b < (int) que.size(); b++) {
94             for (int eid : g[que[b]]) {
95                 auto& e = edges[eid];
96                 if (e.c - e.f > eps) {
97                     deg[e.to] -= 1;
98                     if (deg[e.to] == 0) {
99                         que.push_back(e.to);
100                     }
101                 }
102             }
103         }
104         fill(pot.begin(), pot.end(), INF_C);
105         pot[st] = 0;
106         if (static_cast<int>(que.size()) == n) {
107             for (int v : que) {
108                 if (pot[v] < INF_C) {
109                     for (int eid : g[v]) {
110                         auto& e = edges[eid];
111                         if (e.c - e.f > eps) {
112                             if (pot[v] + e.cost < pot[e.to]) {
113                                 pot[e.to] = pot[v] + e.cost;
114                                 pe[e.to] = eid;
115                             }
116                         }
117                     }
118                 }
119             }
120         } else {
121             que.assign(1, st);
122             vector<bool> in_queue(n, false);
123             in_queue[st] = true;
124             for (int b = 0; b < (int) que.size(); b++) {
125                 int i = que[b];
126                 in_queue[i] = false;
127                 for (int id : g[i]) {
128                     const edge &e = edges[id];
129                     if (e.c - e.f > eps && pot[i] + e.cost <
    pot[e.to]) {
130                         pot[e.to] = pot[i] + e.cost;
131                         pe[e.to] = id;
132                         if (!in_queue[e.to]) {
133                             que.push_back(e.to);
134                             in_queue[e.to] = true;
135                         }
136                     }
137                 }
138             }
139         }
140     }
141     while (pot[fin] < INF_C) {
142         T push = numeric_limits<T>::max();
143         int v = fin;
144         while (v != st) {
145             const edge &e = edges[pe[v]];
146             push = min(push, e.c - e.f);
147             v = e.from;
148         }

```

```

149     v = fin;
150     while (v != st) {
151         edge &e = edges[pe[v]];
152         e.f += push;
153         edge &back = edges[pe[v] ^ 1];
154         back.f -= push;
155         v = e.from;
156     }
157     flow += push;
158     cost += push * pot[fin];
159     expath(st);
160 }
161 return {flow, cost};
162 }
163 };
164
165 // Examples: MCMF<int, int> g(n); g.add(u,v,c,w,0);
166 // To recover flow through original edges: iterate over
167 // even indices in edges.

```

Graphs

Kuhn's algorithm for bipartite matching

```

1  /*
2  The graph is split into 2 halves of n1 and n2 vertices.
3  Complexity: O(n1 * m). Usually runs much faster. MUCH
4  ↪ FASTER!!!
5  */
6
7  vector<int> g[N]; // Stores edges from left half to
8  ↪ right.
9  bool used[N]; // Stores if vertex from left half is
10 ↪ used.
11 int mt[N]; // For every vertex in right half, stores to
12 ↪ which vertex in left half it's matched (-1 if not
13 ↪ matched).
14
15 bool try_dfs(int v){
16     if (used[v]) return false;
17     used[v] = 1;
18     for (auto u : g[v]){
19         if (mt[u] == -1 || try_dfs(mt[u])){
20             mt[u] = v;
21             return true;
22         }
23     }
24     return false;
25 }
26
27 int main(){
28     // .....
29     for (int i = 1; i <= n2; i++) mt[i] = -1;
30     for (int i = 1; i <= n1; i++) used[i] = 0;
31     for (int i = 1; i <= n1; i++){

```

```

28     if (try_dfs(i)){
29         for (int j = 1; j <= n1; j++) used[j] = 0;
30     }
31 }
32 vector<pair<int, int>> ans;
33 for (int i = 1; i <= n2; i++){
34     if (mt[i] != -1) ans.pb({mt[i], i});
35 }
36 }
37
38 // Finding maximal independent set: size = # of nodes -
39 ↪ # of edges in matching.
40 // To construct: launch Kuhn-like DFS from unmatched
41 ↪ nodes in the left half.
42 // Independent set = visited nodes in left half +
43 ↪ unvisited in right half.
44 // Finding minimal vertex cover: complement of maximal
45 ↪ independent set.

```

Hungarian algorithm for Assignment Problem

- Given a 1-indexed $(n \times m)$ matrix A , select a number in each row such that each column has at most 1 number selected, and the sum of the selected numbers is minimized.

```

1 int INF = 1e9; // constant greater than any number in
2 ↪ the matrix
3 vector<int> u(n+1), v(m+1), p(m+1), way(m+1);
4 for (int i=1; i<=n; ++i) {
5     p[0] = i;
6     int j0 = 0;
7     vector<int> minv (m+1, INF);
8     vector<bool> used (m+1, false);
9     do {
10         used[j0] = true;
11         int i0 = p[j0], delta = INF, j1;
12         for (int j=1; j<=m; ++j)
13             if (!used[j]) {
14                 int cur = A[i0][j]-u[i0]-v[j];
15                 if (cur < minv[j])
16                     minv[j] = cur, way[j] = j0;
17                 if (minv[j] < delta)
18                     delta = minv[j], j1 = j;
19             }
20         for (int j=j0; j<=m; ++j)
21             if (used[j])
22                 u[p[j]] += delta, v[j] -= delta;
23             else
24                 minv[j] -= delta;
25         j0 = j1;
26     } while (p[j0] != 0);
27     do {
28         int j1 = way[j0];
29         p[j0] = p[j1];
30         j0 = j1;

```

```

30     } while (j0);
31 }
32 vector<int> ans (n+1); // ans[i] stores the column
33 ↪ selected for row i
34 for (int j=1; j<=m; ++j)
35     ans[p[j]] = j;
36 int cost = -v[0]; // the total cost of the matching

```

Dijkstra's Algorithm

```

1 priority_queue<pair<ll, ll>, vector<pair<ll, ll>>,
2     ↪ greater<pair<ll, ll>>> q;
3 dist[start] = 0;
4 q.push({0, start});
5 while (!q.empty()){
6     auto [d, v] = q.top();
7     q.pop();
8     if (d != dist[v]) continue;
9     for (auto [u, w] : g[v]){
10         if (dist[u] > dist[v] + w){
11             dist[u] = dist[v] + w;
12             q.push({dist[u], u});
13         }
14     }
15 }

```

Eulerian Cycle DFS

```

1 void dfs(int v){
2     while (!g[v].empty()){
3         int u = g[v].back();
4         g[v].pop_back();
5         dfs(u);
6         ans.pb(v);
7     }
8 }

```

SCC and 2-SAT

```

1 void scc(vector<vector<int>>& g, int* idx) {
2     int n = g.size(), ct = 0;
3     int out[n];
4     vector<int> ginv[n];
5     memset(out, -1, sizeof out);
6     memset(idx, -1, n * sizeof(int));
7     function<void(int)> dfs = [&](int cur) {
8         out[cur] = INT_MAX;
9         for(int v : g[cur]) {
10             ginv[v].push_back(cur);
11             if(out[v] == -1) dfs(v);
12         }
13         ct++; out[cur] = ct;
14     };
15     vector<int> order;
16     for(int i = 0; i < n; i++) {

```



```

17     order.push_back(i);
18     if(out[i] == -1) dfs(i);
19 }
20 sort(order.begin(), order.end(), [&](int& u, int& v) {
21     return out[u] > out[v];
22 });
23 ct = 0;
24 stack<int> s;
25 auto dfs2 = [&](int start) {
26     s.push(start);
27     while(!s.empty()) {
28         int cur = s.top();
29         s.pop();
30         idx[cur] = ct;
31         for(int v : ginv[cur])
32             if(idx[v] == -1) s.push(v);
33     }
34 };
35 for(int v : order) {
36     if(idx[v] == -1) {
37         dfs2(v);
38         ct++;
39     }
40 }
41 }
42
43 // 0 => impossible, 1 => possible
44 pair<int, vector<int>> sat2(int n, vector<pair<int, int>>&
45     ↪ clauses) {
46     vector<int> ans(n);
47     vector<vector<int>> g(2*n + 1);
48     for(auto [x, y] : clauses) {
49         x = x < 0 ? -x + n : x;
50         y = y < 0 ? -y + n : y;
51         int nx = x <= n ? x + n : x - n;
52         int ny = y <= n ? y + n : y - n;
53         g[nx].push_back(y);
54         g[ny].push_back(x);
55     }
56     int idx[2*n + 1];
57     scc(g, idx);
58     for(int i = 1; i <= n; i++) {
59         if(idx[i] == idx[i + n]) return {0, {}};
60         ans[i - 1] = idx[i + n] < idx[i];
61     }
62     return {1, ans};
63 }

```

Finding Bridges

```

1  /*
2  Bridges.
3  Results are stored in a map "is_bridge".
4  For each connected component, call "dfs(starting vertex,
5  ↪ starting vertex)".
6  */
7  const int N = 2e5 + 10; // Careful with the constant!

```

```

8  vector<int> g[N];
9  int tin[N], fup[N], timer;
10 map<pair<int, int>, bool> is_bridge;
11
12 void dfs(int v, int p){
13     tin[v] = ++timer;
14     fup[v] = tin[v];
15     for (auto u : g[v]){
16         if (!tin[u]){
17             dfs(u, v);
18             if (fup[u] > tin[v]){
19                 is_bridge[{v, u}] = is_bridge[{u, v}] = true;
20             }
21             fup[v] = min(fup[v], fup[u]);
22         }
23         else{
24             if (u != p) fup[v] = min(fup[v], tin[u]);
25         }
26     }
27 }

```

Virtual Tree

```

1  // order stores the nodes in the queried set
2  sort(all(order), [&](int u, int v){return tin[u] <
3  ↪ tin[v]});
4  int m = sz(order);
5  for (int i = 1; i < m; i++){
6      order.pb(lca(order[i], order[i - 1]));
7  }
8  sort(all(order), [&](int u, int v){return tin[u] <
9  ↪ tin[v]});
10 order.erase(unique(all(order)), order.end());
11 vector<int> stk{order[0]};
12 for (int i = 1; i < sz(order); i++){
13     int v = order[i];
14     while (tout[stk.back()] < tout[v]) stk.pop_back();
15     int u = stk.back();
16     vg[u].pb({v, dep[v] - dep[u]});
17     stk.pb(v);
18 }

```

HLD on Edges DFS

```

1  void dfs1(int v, int p, int d){
2      par[v] = p;
3      for (auto e : g[v]){
4          if (e.fi == p){
5              g[v].erase(find(all(g[v]), e));
6              break;
7          }
8      }
9      dep[v] = d;
10     sz[v] = 1;
11     for (auto [u, c] : g[v]){
12         dfs1(u, v, d + 1);
13         sz[v] += sz[u];
14     }
15 }

```

```

14 }
15 if (!g[v].empty()) iter_swap(g[v].begin(),
16 ↪ max_element(all(g[v]), comp));
17 }
18 void dfs2(int v, int rt, int c){
19     pos[v] = sz(a);
20     a.pb(c);
21     root[v] = rt;
22     for (int i = 0; i < sz(g[v]); i++){
23         auto [u, c] = g[v][i];
24         if (!i) dfs2(u, rt, c);
25         else dfs2(u, u, c);
26     }
27 }
28 int getans(int u, int v){
29     int res = 0;
30     for (; root[u] != root[v]; v = par[root[v]]){
31         if (dep[root[u]] > dep[root[v]]) swap(u, v);
32         res = max(res, rmq(0, 0, n - 1, pos[root[v]],
33 ↪ pos[v]));
34     }
35     if (pos[u] > pos[v]) swap(u, v);
36     return max(res, rmq(0, 0, n - 1, pos[u] + 1, pos[v]));
37 }

```

Centroid Decomposition

```

1  vector<char> res(n), seen(n), sz(n);
2  function<int(int, int)> get_size = [&](int node, int fa)
3  ↪ {
4      sz[node] = 1;
5      for (auto& ne : g[node]) {
6          if (ne == fa || seen[ne]) continue;
7          sz[node] += get_size(ne, node);
8      }
9      return sz[node];
10 };
11 function<int(int, int, int)> find_centroid = [&](int
12 ↪ node, int fa, int t) {
13     for (auto& ne : g[node])
14         if (ne != fa && !seen[ne] && sz[ne] > t / 2) return
15         ↪ find_centroid(ne, node, t);
16     return node;
17 };
18 function<void(int, char)> solve = [&](int node, char
19 ↪ cur) {
20     get_size(node, -1); auto c = find_centroid(node, -1,
21 ↪ sz[node]);
22     seen[c] = 1, res[c] = cur;
23     for (auto& ne : g[c]) {
24         if (seen[ne]) continue;
25         solve(ne, char(cur + 1)); // we can pass c here to
26 ↪ build tree
27     }
28 };

```


Math

Binary exponentiation

```
1 ll power(ll a, ll b){
2     ll res = 1;
3     for (; b; a = a * a % MOD, b >>= 1){
4         if (b & 1) res = res * a % MOD;
5     }
6     return res;
7 }
```

Matrix Exponentiation: $O(n^3 \log b)$

```
1 const int N = 100, MOD = 1e9 + 7;
2
3 struct matrix{
4     ll m[N][N];
5     int n;
6     matrix(){
7         n = N;
8         memset(m, 0, sizeof(m));
9     };
10    matrix(int n){
11        n = n;
12        memset(m, 0, sizeof(m));
13    };
14    matrix(int n_, ll val){
15        n = n_;
16        memset(m, 0, sizeof(m));
17        for (int i = 0; i < n; i++) m[i][i] = val;
18    };
19
20    matrix operator* (matrix oth){
21        matrix res(n);
22        for (int i = 0; i < n; i++){
23            for (int j = 0; j < n; j++){
24                for (int k = 0; k < n; k++){
25                    res.m[i][j] = (res.m[i][j] + m[i][k] *
26                    oth.m[k][j]) % MOD;
27                }
28            }
29            return res;
30        }
31    };
32
33    matrix power(matrix a, ll b){
34        matrix res(a.n, 1);
35        for (; b; a = a * a, b >>= 1){
36            if (b & 1) res = res * a;
37        }
38        return res;
39    }
```

Extended Euclidean Algorithm

```
1 // gives (x, y) for ax + by = g
2 // solutions given (x0, y0): a(x0 + kb/g) + b(y0 - ka/g) = g
3 int gcd(int a, int b, int& x, int& y) {
4     x = 1, y = 0; int sum1 = a;
5     int x2 = 0, y2 = 1, sum2 = b;
6     while (sum2) {
7         int q = sum1 / sum2;
8         tie(x, x2) = make_tuple(x2, x - q * x2);
9         tie(y, y2) = make_tuple(y2, y - q * y2);
10        tie(sum1, sum2) = make_tuple(sum2, sum1 - q * sum2);
11    }
12    return sum1;
13 }
```

Linear Sieve

• Mobius Function

```
1 vector<int> prime;
2 bool is_composite[MAX_N];
3 int mu[MAX_N];
4
5 void sieve(int n){
6     fill(is_composite, is_composite + n, 0);
7     mu[1] = 1;
8     for (int i = 2; i < n; i++){
9         if (!is_composite[i]){
10            prime.push_back(i);
11            mu[i] = -1; //i is prime
12        }
13        for (int j = 0; j < prime.size() && i * prime[j] < n; j++){
14            is_composite[i * prime[j]] = true;
15            if (i % prime[j] == 0){
16                mu[i * prime[j]] = 0; //prime[j] divides i
17                break;
18            } else {
19                mu[i * prime[j]] = -mu[i]; //prime[j] does not
20                divide i
21            }
22        }
23    }
```

• Euler's Totient Function

```
1 vector<int> prime;
2 bool is_composite[MAX_N];
3 int phi[MAX_N];
4
5 void sieve(int n){
6     fill(is_composite, is_composite + n, 0);
7     phi[1] = 1;
8     for (int i = 2; i < n; i++){
9         if (!is_composite[i]){
```

```
        prime.push_back(i);
        phi[i] = i - 1; //i is prime
    }
    for (int j = 0; j < prime.size() && i * prime[j] < n; j++){
        is_composite[i * prime[j]] = true;
        if (i % prime[j] == 0){
            phi[i * prime[j]] = phi[i] * prime[j]; //prime[j]
            divides i
            break;
        } else {
            phi[i * prime[j]] = phi[i] * phi[prime[j]];
            //prime[j] does not divide i
        }
    }
}
```

Gaussian Elimination

```
1 bool is_0(Z v) { return v.x == 0; }
2 Z abs(Z v) { return v; }
3 bool is_0(double v) { return abs(v) < 1e-9; }
4
5 // 1 => unique solution, 0 => no solution, -1 =>
6 // multiple solutions
7 template <typename T>
8 int gaussian_elimination(vector<vector<T>> &a, int
9 limit) {
10    if (a.empty() || a[0].empty()) return -1;
11    int h = (int)a.size(), w = (int)a[0].size(), r = 0;
12    for (int c = 0; c < limit; c++) {
13        int id = -1;
14        for (int i = r; i < h; i++) {
15            if (!is_0(a[i][c]) && (id == -1 || abs(a[id][c]) <
16            abs(a[i][c]))) {
17                id = i;
18            }
19        }
20        if (id == -1) continue;
21        if (id > r) {
22            swap(a[r], a[id]);
23            for (int j = c; j < w; j++) a[id][j] = -a[id][j];
24        }
25        vector<int> nonzero;
26        for (int j = c; j < w; j++) {
27            if (!is_0(a[r][j])) nonzero.push_back(j);
28        }
29        T inv_a = 1 / a[r][c];
30        for (int i = r + 1; i < h; i++) {
31            if (is_0(a[i][c])) continue;
32            T coeff = -a[i][c] * inv_a;
33            for (int j : nonzero) a[i][j] += coeff * a[r][j];
34        }
35        ++r;
36    }
37    for (int row = h - 1; row >= 0; row--) {
38        for (int c = 0; c < limit; c++) {
```

```

36     if (!is_0(a[row][c])) {
37         T inv_a = 1 / a[row][c];
38         for (int i = row - 1; i >= 0; i--) {
39             if (is_0(a[i][c])) continue;
40             T coeff = -a[i][c] * inv_a;
41             for (int j = c; j < w; j++) a[i][j] += coeff * a[row][j];
42         }
43         break;
44     }
45 } // not-free variables: only it on its line
46 for(int i = r; i < h; i++) if(!is_0(a[i][limit]))
47     return 0;
48 return (r == limit) ? 1 : -1;
49 }
50
51 template <typename T>
52 pair<int, vector<T>> solve_linear(vector<vector<T>> a,
53     const vector<T> &b, int w) {
54     int h = (int)a.size();
55     for (int i = 0; i < h; i++) a[i].push_back(b[i]);
56     int sol = gaussian_elimination(a, w);
57     vector<T> x(w, 0);
58     for (int i = 0; i < h; i++) {
59         for (int j = 0; j < w; j++) {
60             if (!is_0(a[i][j])) {
61                 x[j] = a[i][w] / a[i][j];
62                 break;
63             }
64         }
65     }
66     return {sol, x};
67 }

```

is_prime

- (Miller–Rabin primality test)

```

1 typedef __int128_t i128;
2
3 i128 power(i128 a, i128 b, i128 MOD = 1, i128 res = 1) {
4     for (; b; b /= 2, (a *= a) %= MOD)
5         if (b & 1) (res *= a) %= MOD;
6     return res;
7 }
8
9 bool is_prime(ll n) {
10     if (n < 2) return false;
11     static constexpr int A[] = {2, 3, 5, 7, 11, 13, 17,
12     19, 23};
13     int s = __builtin_ctzll(n - 1);
14     ll d = (n - 1) >> s;
15     for (auto a : A) {
16         if (a == n) return true;
17         ll x = (ll)power(a, d, n);
18         if (x == 1 || x == n - 1) continue;

```

```

18     bool ok = false;
19     for (int i = 0; i < s - 1; ++i) {
20         x = ll((i128)x * x % n); // potential overflow!
21         if (x == n - 1) {
22             ok = true;
23             break;
24         }
25     }
26     if (!ok) return false;
27 }
28 return true;
29 }
30
31 typedef __int128_t i128;
32
33 ll pollard_rho(ll x) {
34     ll s = 0, t = 0, c = rng() % (x - 1) + 1;
35     ll stp = 0, goal = 1, val = 1;
36     for (goal = 1; goal <= 2, s = t, val = 1) {
37         for (stp = 1; stp <= goal; ++stp) {
38             t = ll(((i128)t * t + c) % x);
39             val = ll(((i128)val * abs(t - s) % x);
40             if ((stp % 127) == 0) {
41                 ll d = gcd(val, x);
42                 if (d > 1) return d;
43             }
44         }
45         ll d = gcd(val, x);
46         if (d > 1) return d;
47     }
48 }
49
50 ll get_max_factor(ll _x) {
51     ll max_factor = 0;
52     function<void(ll)> fac = [&](ll x) {
53         if (x <= max_factor || x < 2) return;
54         if (is_prime(x)) {
55             max_factor = max_factor > x ? max_factor : x;
56             return;
57         }
58         ll p = x;
59         while (p >= x) p = pollard_rho(x);
60         while ((x % p) == 0) x /= p;
61         fac(x), fac(p);
62     };
63     fac(_x);
64     return max_factor;
65 }

```

Berlekamp-Massey

- Recovers any n -order linear recurrence relation from the first $2n$ terms of the sequence.
- Input s is the sequence to be analyzed.
- Output c is the shortest sequence c_1, \dots, c_n , such

that

$$s_m = \sum_{i=1}^n c_i \cdot s_{m-i}, \text{ for all } m \geq n.$$

- Be careful since c is returned in 0-based indexation.
- Complexity: $O(N^2)$

```

1 vector<ll> berlekamp_massey(vector<ll> s) {
2     int n = sz(s), l = 0, m = 1;
3     vector<ll> b(n), c(n);
4     ll ldd = b[0] = c[0] = 1;
5     for (int i = 0; i < n; i++, m++) {
6         ll d = s[i];
7         for (int j = 1; j <= l; j++) d = (d + c[j] * s[i -
8     j]) % MOD;
9         if (d == 0) continue;
10        vector<ll> temp = c;
11        ll coef = d * power(ldd, MOD - 2) % MOD;
12        for (int j = m; j < n; j++){
13            c[j] = (c[j] + MOD - coef * b[j - m]) % MOD;
14            if (c[j] < 0) c[j] += MOD;
15        }
16        if (2 * l <= i) {
17            l = i + 1 - l;
18            b = temp;
19            ldd = d;
20            m = 0;
21        }
22    }
23    c.resize(l + 1);
24    c.erase(c.begin());
25    for (ll &x : c)
26        x = (MOD - x) % MOD;
27    return c;
28 }

```

Calculating k-th term of a linear recurrence

- Given the first n terms s_0, s_1, \dots, s_{n-1} and the sequence c_1, c_2, \dots, c_n such that

$$s_m = \sum_{i=1}^n c_i \cdot s_{m-i}, \text{ for all } m \geq n,$$

the function `calc_kth` computes s_k .

- Complexity: $O(n^2 \log k)$

```

vector<ll> poly_mult_mod(vector<ll> p, vector<ll> q,
    vector<ll> &c) {
    vector<ll> ans(sz(p) + sz(q) - 1);
    for (int i = 0; i < sz(p); i++){
        for (int j = 0; j < sz(q); j++){

```

```

5     ans[i + j] = (ans[i + j] + p[i] * q[j]) % MOD;
6 }
7 }
8 int n = sz(ans), m = sz(c);
9 for (int i = n - 1; i >= m; i--){
10     for (int j = 0; j < m; j++){
11         ans[i - 1 - j] = (ans[i - 1 - j] + c[j] * ans[i]) % MOD;
12     }
13 }
14 ans.resize(m);
15 return ans;
16 }

17 ll calc_kth(vector<ll> s, vector<ll> c, ll k){
18     assert(sz(s) >= sz(c)); // size of s can be greater
19     // than c, but not less
20     if (k < sz(s)) return s[k];
21     vector<ll> res{1};
22     for (vector<ll> poly = {0, 1}; k >= 1){
23         poly_mult_mod(poly, poly, c, k >= 1){
24             if (k & 1) res = poly_mult_mod(res, poly, c);
25         }
26         ll ans = 0;
27         for (int i = 0; i < min(sz(res), sz(c)); i++) ans =
28             (ans + s[i] * res[i]) % MOD;
29         return ans;
30 }

```

Partition Function

- Returns number of partitions of n in $O(n^{1.5})$

```

1 int partition(int n) {
2     int dp[n + 1];
3     dp[0] = 1;
4     for (int i = 1; i <= n; i++) {
5         dp[i] = 0;
6         for (int j = 1, r = 1; i - (3 * j * j - j) / 2 >= 0;
7             ++j, r += -1) {
8             dp[i] += dp[i - (3 * j * j - j) / 2] * r;
9             if (i - (3 * j * j + j) / 2 >= 0) dp[i] += dp[i -
10                 (3 * j * j + j) / 2] * r;
11         }
12     }
13     return dp[n];
14 }

```

NTT

```

1 void ntt(vector<ll>& a, int f) {
2     int n = int(a.size());
3     vector<ll> w(n);
4     vector<int> rev(n);
5     for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] / 2) +
6         ((i & 1) * (n / 2));
7     for (int i = 0; i < n; i++) {

```

```

8         if (i < rev[i]) swap(a[i], a[rev[i]]);
9     }
10     ll wn = power(f ? (MOD + 1) / 3 : 3, (MOD - 1) / n);
11     w[0] = 1;
12     for (int i = 1; i < n; i++) w[i] = w[i - 1] * wn % MOD;
13     for (int mid = 1; mid < n; mid *= 2) {
14         for (int i = 0; i < n; i += 2 * mid) {
15             for (int j = 0; j < mid; j++) {
16                 ll x = a[i + j], y = a[i + j + mid] * w[n / (2 *
17                     mid) * j] % MOD;
18                 a[i + j] = (x + y) % MOD, a[i + j + mid] = (x -
19                     y) % MOD;
20             }
21         }
22     }
23     if (f) {
24         ll iv = power(n, MOD - 2);
25         for (auto& x : a) x = x * iv % MOD;
26     }
27 }
28 vector<ll> mul(vector<ll> a, vector<ll> b) {
29     int n = 1, m = (int)a.size() + (int)b.size() - 1;
30     while (n < m) n *= 2;
31     a.resize(n), b.resize(n);
32     ntt(a, 0), ntt(b, 0); // if squaring, you can save one
33     // NTT here
34     for (int i = 0; i < n; i++) a[i] = a[i] * b[i] % MOD;
35     ntt(a, 1);
36     a.resize(m);
37     return a;
38 }

```

FFT

```

1 const ld PI = acos(-1);
2 auto mul = [&](const vector<ld>& aa, const vector<ld>& bb) {
3     int n = (int)aa.size(), m = (int)bb.size(), bit = 1;
4     while ((1 << bit) < n + m - 1) bit++;
5     int len = 1 << bit;
6     vector<complex<ld>> a(len), b(len);
7     vector<int> rev(len);
8     for (int i = 0; i < n; i++) a[i].real(aa[i]);
9     for (int i = 0; i < m; i++) b[i].real(bb[i]);
10    for (int i = 0; i < len; i++) rev[i] = (rev[i >> 1] >
11        1) | ((i & 1) << (bit - 1));
12    auto fft = [&](vector<complex<ld>>& p, int inv) {
13        for (int i = 0; i < len; i++)
14            if (i < rev[i]) swap(p[i], p[rev[i]]);
15        for (int mid = 1; mid < len; mid *= 2) {
16            auto w1 = complex<ld>(cos(PI / mid), (inv ? -1 :
17                1) * sin(PI / mid));
18            for (int i = 0; i < len; i += mid * 2) {
19                auto wk = complex<ld>(1, 0);
20                for (int j = 0; j < mid; j++, wk = wk * w1) {
21                    auto x = p[i + j], y = wk * p[i + j + mid];
22                    p[i + j] = x + y, p[i + j + mid] = x - y;
23                }
24            }
25        }
26    }
27 }

```

```

28 }
29 }
30 }
31 if (inv == 1) {
32     for (int i = 0; i < len; i++)
33         p[i].real(p[i].real() / len);
34 }
35 };
36 fft(a, 0), fft(b, 0);
37 for (int i = 0; i < len; i++) a[i] = a[i] * b[i];
38 fft(a, 1);
39 a.resize(n + m - 1);
40 vector<ld> res(n + m - 1);
41 for (int i = 0; i < n + m - 1; i++) res[i] =
42     a[i].real();
43 return res;
44 };

```

MIT's FFT/NTT, Polynomial mod/log/exp Template

- For integers rounding works if $(|a| + |b|) \max(a, b) < \sim 10^9$, or in theory maybe 10^6
- $\frac{1}{P(x)}$ in $O(n \log n)$, $e^{P(x)}$ in $O(n \log n)$, $\ln(P(x))$ in $O(n \log n)$, $P(x)^k$ in $O(n \log n)$, Evaluates $P(x_1), \dots, P(x_n)$ in $O(n \log^2 n)$, Lagrange Interpolation in $O(n \log^2 n)$

```

1 // use #define FFT 1 to use FFT instead of NTT (default)
2 // Examples:
3 // poly a(n+1); // constructs degree n poly
4 // a[0].v = 10; // assigns constant term a_0 = 10
5 // poly b = exp(a);
6 // poly is vector<num>
7 // for NTT, num stores just one int named v
8 // for FFT, num stores two doubles named x (real), y
9 // (imag)
10
11 #define sz(x) ((int)x.size())
12 #define rep(i, j, k) for (int i = int(j); i < int(k); i++)
13 #define trav(a, x) for (auto& a : x)
14 #define per(i, a, b) for (int i = (b)-1; i >= (a); --i)
15 using ll = long long;
16 using vi = vector<int>;
17
18 namespace fft {
19     #if FFT
20     // FFT
21     using dbl = double;
22     struct num {
23         dbl x, y;
24         num(dbl x_ = 0, dbl y_ = 0): x(x_), y(y_) {}
25     };
26 }

```

```

25 inline num operator+(num a, num b) {
26     return num(a.x + b.x, a.y + b.y);
27 }
28 inline num operator-(num a, num b) {
29     return num(a.x - b.x, a.y - b.y);
30 }
31 inline num operator*(num a, num b) {
32     return num(a.x * b.x - a.y * b.y, a.x * b.y + a.y *
    ↪ b.x);
33 }
34 inline num conj(num a) { return num(a.x, -a.y); }
35 inline num inv(num a) {
36     dbl n = (a.x * a.x + a.y * a.y);
37     return num(a.x / n, -a.y / n);
38 }
39
40 #else
41 // NTT
42 const int mod = 998244353, g = 3;
43 // For  $p < 2^{30}$  there is also  $(5 \ll 25, 3)$ ,  $(7 \ll 26,$ 
    ↪  $3)$ ,
44 //  $(479 \ll 21, 3)$  and  $(483 \ll 21, 5)$ . Last two are  $>$ 
    ↪  $10^9$ .
45 struct num {
46     int v;
47     num(ll v_ = 0): v(int(v_ % mod)) {
48         if (v < 0) v += mod;
49     }
50     explicit operator int() const { return v; }
51 };
52 inline num operator+(num a, num b) { return num(a.v +
    ↪ b.v); }
53 inline num operator-(num a, num b) {
54     return num(a.v + mod - b.v);
55 }
56 inline num operator*(num a, num b) {
57     return num(1ll * a.v * b.v);
58 }
59 inline num pow(num a, int b) {
60     num r = 1;
61     do {
62         if (b & 1) r = r * a;
63         a = a * a;
64     } while (b >= 1);
65     return r;
66 }
67 inline num inv(num a) { return pow(a, mod - 2); }
68
69 #endif
70 using vn = vector<num>;
71 vi rev({0, 1});
72 vn rt(2, num(1)), fa, fb;
73 inline void init(int n) {
74     if (n <= sz(rt)) return;
75     rev.resize(n);
76     rep(i, 0, n) rev[i] = (rev[i >> 1] | ((i & 1) * n))
    ↪ 1;
77     rt.reserve(n);
78     for (int k = sz(rt); k < n; k *= 2) {
79         rt.resize(2 * k);
80         #if FFT
81             double a = M_PI / k;
82             num z(cos(a), sin(a)); // FFT
83         #else
84             num z = pow(num(g), (mod - 1) / (2 * k)); // NTT
85         #endif
86         rep(i, k / 2, k) rt[2 * i] = rt[i],
87             rt[2 * i + 1] = rt[i] * z;
88     }
89 }
90 inline void fft(vector<num>& a, int n) {
91     init(n);
92     int s = __builtin_ctz(sz(rev)) / n;
93     rep(i, 0, n) if (i < rev[i] >> s) swap(a[i], a[rev[i]
    ↪ >> s]);
94     for (int k = 1; k < n; k *= 2)
95         for (int i = 0; i < n; i += 2 * k) rep(j, 0, k) {
96             num t = rt[j + k] * a[i + j + k];
97             a[i + j + k] = a[i + j] - t;
98             a[i + j] = a[i + j] + t;
99         }
100 }
101 // Complex/NTT
102 vn multiply(vn a, vn b) {
103     int s = sz(a) + sz(b) - 1;
104     if (s <= 0) return {};
105     int L = s > 1 ? 32 - __builtin_clz(s - 1) : 0, n =
    ↪ << L;
106     a.resize(n), b.resize(n);
107     fft(a, n);
108     fft(b, n);
109     num d = inv(num(n));
110     rep(i, 0, n) a[i] = a[i] * b[i] * d;
111     reverse(a.begin() + 1, a.end());
112     fft(a, n);
113     a.resize(s);
114     return a;
115 }
116 // Complex/NTT power-series inverse
117 // Doubles  $b$  as  $b[:n] = (2 - a[:n] * b[:n/2]) * b[:n/2]$ 
118 vn inverse(const vn& a) {
119     if (a.empty()) return {};
120     vn b({inv(a[0])});
121     b.reserve(2 * a.size());
122     while (sz(b) < sz(a)) {
123         int n = 2 * sz(b);
124         b.resize(2 * n, 0);
125         if (sz(fa) < 2 * n) fa.resize(2 * n);
126         fill(fa.begin(), fa.begin() + 2 * n, 0);
127         copy(a.begin(), a.begin() + min(n, sz(a)),
    ↪ fa.begin());
128         fft(b, 2 * n);
129         fft(fa, 2 * n);
130         num d = inv(num(2 * n));
131         rep(i, 0, 2 * n) b[i] = b[i] * (2 - fa[i] * b[i])
    ↪ d;
132         reverse(b.begin() + 1, b.end());
133         fft(b, 2 * n);
134         b.resize(n);
135     }
136     b.resize(a.size());
137     return b;
138 }
139 #if FFT
140 // Double multiply (num = complex)
141 using vd = vector<double>;
142 vd multiply(const vd& a, const vd& b) {
143     int s = sz(a) + sz(b) - 1;
144     if (s <= 0) return {};
145     int L = s > 1 ? 32 - __builtin_clz(s - 1) : 0, n =
    ↪ << L;
146     if (sz(fa) < n) fa.resize(n);
147     if (sz(fb) < n) fb.resize(n);
148     fill(fa.begin(), fa.begin() + n, 0);
149     rep(i, 0, sz(a)) fa[i].x = a[i];
150     rep(i, 0, sz(b)) fa[i].y = b[i];
151     fft(fa, n);
152     trav(x, fa) x = x * x;
153     rep(i, 0, n) fb[i] = fa[(n - i) & (n - 1)] -
    ↪ conj(fa[i]);
154     fft(fb, n);
155     vd r(s);
156     rep(i, 0, s) r[i] = fb[i].y / (4 * n);
157     return r;
158 }
159 // Integer multiply mod  $m$  (num = complex)
160 vi multiply_mod(const vi& a, const vi& b, int m) {
161     int s = sz(a) + sz(b) - 1;
162     if (s <= 0) return {};
163     int L = s > 1 ? 32 - __builtin_clz(s - 1) : 0, n =
    ↪ << L;
164     if (sz(fa) < n) fa.resize(n);
165     if (sz(fb) < n) fb.resize(n);
166     rep(i, 0, sz(a)) fa[i] =
167         num(a[i] & ((1 << 15) - 1), a[i] >> 15);
168     fill(fa.begin() + sz(a), fa.begin() + n, 0);
169     rep(i, 0, sz(b)) fb[i] =
170         num(b[i] & ((1 << 15) - 1), b[i] >> 15);
171     fill(fb.begin() + sz(b), fb.begin() + n, 0);
172     fft(fa, n);
173     fft(fb, n);
174     double r0 = 0.5 / n; //  $1/2n$ 
175     rep(i, 0, n / 2 + 1) {
176         int j = (n - i) & (n - 1);
177         num g0 = (fb[i] + conj(fb[j])) * r0;
178         num g1 = (fb[i] - conj(fb[j])) * r0;
179         swap(g1.x, g1.y);
180         g1.y *= -1;
181         if (j != i) {
182             swap(fa[j], fa[i]);
183             fb[j] = fa[j] * g1;
184             fa[j] = fa[j] * g0;
185         }
186     }

```

```

186     fb[i] = fa[i] * conj(g1);
187     fa[i] = fa[i] * conj(g0);
188 }
189 fft(fa, n);
190 fft(fb, n);
191 vi r(s);
192 rep(i, 0, s) r[i] =
193     int((ll(fa[i].x + 0.5) + (ll(fa[i].y + 0.5) % m < 15) +
194         (ll(fb[i].x + 0.5) % m < 15) +
195         (ll(fb[i].y + 0.5) % m < 30)) %
196         m);
197     return r;
198 }
199 #endif
200 } // namespace fft
201 // For multiply_mod, use num = modnum, poly =
202     vector<num>
203 using fft::num;
204 using poly = fft::vn;
205 using fft::multiply;
206 using fft::inverse;
207
208 poly& operator+=(poly& a, const poly& b) {
209     if (sz(a) < sz(b)) a.resize(b.size());
210     rep(i, 0, sz(b)) a[i] = a[i] + b[i];
211     return a;
212 }
213
214 poly operator+(const poly& a, const poly& b) {
215     poly r = a;
216     r += b;
217     return r;
218 }
219
220 poly& operator-=(poly& a, const poly& b) {
221     if (sz(a) < sz(b)) a.resize(b.size());
222     rep(i, 0, sz(b)) a[i] = a[i] - b[i];
223     return a;
224 }
225
226 poly operator-(const poly& a, const poly& b) {
227     poly r = a;
228     r -= b;
229     return r;
230 }
231
232 poly& operator*(const poly& a, const poly& b) {
233     return multiply(a, b);
234 }
235
236 poly& operator*=(poly& a, const poly& b) { return a =
237     a * b; }
238
239 poly& operator*=(poly& a, const num& b) { // Optional
240     trav(x, a) x = x * b;
241     return a;
242 }
243
244 poly operator*(const poly& a, const num& b) {
245     poly r = a;
246     r *= b;
247     return r;
248 }
249
250 // Polynomial floor division; no leading 0's please
251 poly operator/(poly a, poly b) {
252     if (sz(a) < sz(b)) return {};
253     int s = sz(a) - sz(b) + 1;
254     reverse(a.begin(), a.end());
255     reverse(b.begin(), b.end());
256     a.resize(s);
257     b.resize(s);
258     a = a * inverse(move(b));
259     a.resize(s);
260     reverse(a.begin(), a.end());
261     return a;
262 }
263
264 poly& operator/=(poly& a, const poly& b) { return a = a
265     / b; }
266
267 poly& operator%=(poly& a, const poly& b) {
268     if (sz(a) >= sz(b)) {
269         poly c = (a / b) * b;
270         a.resize(sz(b) - 1);
271         rep(i, 0, sz(a)) a[i] = a[i] - c[i];
272     }
273     return a;
274 }
275
276 poly operator%(const poly& a, const poly& b) {
277     poly r = a;
278     r %= b;
279     return r;
280 }
281
282 // Log/exp/pow
283 poly deriv(const poly& a) {
284     if (a.empty()) return {};
285     poly b(sz(a) - 1);
286     rep(i, 1, sz(a)) b[i - 1] = a[i] * i;
287     return b;
288 }
289
290 poly integ(const poly& a) {
291     poly b(sz(a) + 1);
292     b[1] = 1; // mod p
293     rep(i, 2, sz(b)) b[i] =
294         b[fft::mod % i] * (-fft::mod / i); // mod p
295     rep(i, 1, sz(b)) b[i] = a[i - 1] * b[i]; // mod p
296     //rep(i, 1, sz(b)) b[i] = a[i - 1] * inv(num(i)); // else
297     return b;
298 }
299
300 poly log(const poly& a) { // MUST have a[0] == 1
301     poly b = integ(deriv(a) * inverse(a));
302     b.resize(a.size());
303     return b;
304 }
305
306 poly exp(const poly& a) { // MUST have a[0] == 0
307     poly b(1, num(1));
308     if (a.empty()) return b;
309     while (sz(b) < sz(a)) {
310         int n = min(sz(b) * 2, sz(a));
311         b.resize(n);
312         poly v = poly(a.begin(), a.begin() + n) - log(b);
313         v[0] = v[0] + num(1);
314         b *= v;
315     }
316     return b;
317 }
318
319 b.resize(n);
320 }
321 return b;
322 }
323
324 poly pow(const poly& a, int m) { // m >= 0
325     poly b(a.size());
326     if (!m) {
327         b[0] = 1;
328         return b;
329     }
330     int p = 0;
331     while (p < sz(a) && a[p].v == 0) ++p;
332     if (ll * m * p >= sz(a)) return b;
333     num mu = pow(a[p], m), di = inv(a[p]);
334     poly c(sz(a) - m * p);
335     rep(i, 0, sz(c)) c[i] = a[i + p] * di;
336     c = log(c);
337     trav(v, c) v = v * m;
338     c = exp(c);
339     rep(i, 0, sz(c)) b[i + m * p] = c[i] * mu;
340     return b;
341 }
342
343 // Multipoint evaluation/interpolation
344
345 vector<num> eval(const poly& a, const vector<num>& x) {
346     int n = sz(x);
347     if (!n) return {};
348     vector<poly> up(2 * n);
349     rep(i, 0, n) up[i + n] = poly({0 - x[i], 1});
350     per(i, 1, n) up[i] = up[2 * i] * up[2 * i + 1];
351     vector<poly> down(2 * n);
352     down[1] = a % up[1];
353     rep(i, 2, 2 * n) down[i] = down[i / 2] % up[i];
354     vector<num> y(n);
355     rep(i, 0, n) y[i] = down[i + n][0];
356     return y;
357 }
358
359 poly interp(const vector<num>& x, const vector<num>& y)
360     {
361         int n = sz(x);
362         assert(n);
363         vector<poly> up(n * 2);
364         rep(i, 0, n) up[i + n] = poly({0 - x[i], 1});
365         per(i, 1, n) up[i] = up[2 * i] * up[2 * i + 1];
366         vector<num> a = eval(deriv(up[1]), x);
367         vector<poly> down(2 * n);
368         rep(i, 0, n) down[i + n] = poly({y[i] * inv(a[i])});
369         per(i, 1, n) down[i] =
370             down[i * 2] * up[i * 2 + 1] + down[i * 2 + 1] * up[i
371             * 2];
372         return down[1];
373     }
374 }

```

Data Structures

Fenwick Tree

```
1 ll sum(int r) {
2     ll ret = 0;
3     for (; r >= 0; r = (r & r + 1) - 1) ret += bit[r];
4     return ret;
5 }
6 void add(int idx, ll delta) {
7     for (; idx < n; idx |= idx + 1) bit[idx] += delta;
8 }
```

Lazy Propagation SegTree

```
1 // Clear: clear() or build()
2 const int N = 2e5 + 10; // Change the constant!
3 template<typename T>
4 struct LazySegTree{
5     T t[4 * N];
6     T lazy[4 * N];
7     int n;
8
9     // Change these functions, default return, and lazy
10    ↪ mark.
11    T default_return = 0, lazy_mark =
12    ↪ numeric_limits<T>::min();
13    // Lazy mark is how the algorithm will identify that
14    ↪ no propagation is needed.
15    function<T(T, T)> f = [&] (T a, T b){
16        return a + b;
17    };
18    // f_on_seg calculates the function f, knowing the
19    ↪ lazy value on segment,
20    // segment's size and the previous value.
21    // The default is segment modification for RSQ. For
22    ↪ increments change to:
23    // return cur_seg_val + seg_size * lazy_val;
24    // For RMQ. Modification: return lazy_val;
25    ↪ Increments: return cur_seg_val + lazy_val;
26    function<T(T, int, T)> f_on_seg = [&] (T cur_seg_val,
27    ↪ int seg_size, T lazy_val){
28        return cur_seg_val + lazy_val * seg_size;
29    };
30    // upd_lazy updates the value to be propagated to
31    ↪ child segments.
32    // Default: modification. For increments change to:
33    // lazy[v] = (lazy[v] == lazy_mark? val : lazy[v]
34    ↪ + val);
35    function<void(int, T)> upd_lazy = [&] (int v, T val){
36        lazy[v] = val;
37    };
38    // Tip: for "get element on single index" queries, use
39    ↪ max() on segment: no overflows.
40
41    LazySegTree(int n_) : n(n_) {
42        clear(n);
43    }
```

```
34 }
35 void build(int v, int tl, int tr, vector<T>& a){
36     if (tl == tr) {
37         t[v] = a[tl];
38         return;
39     }
40     int tm = (tl + tr) / 2;
41     // left child: [tl, tm]
42     // right child: [tm + 1, tr]
43     build(2 * v + 1, tl, tm, a);
44     build(2 * v + 2, tm + 1, tr, a);
45     t[v] = f(t[2 * v + 1], t[2 * v + 2]);
46 }
47
48 LazySegTree(vector<T>& a){
49     build(a);
50 }
51
52 void push(int v, int tl, int tr){
53     if (lazy[v] == lazy_mark) return;
54     int tm = (tl + tr) / 2;
55     t[2 * v + 1] = f_on_seg(t[2 * v + 1], tm - tl + 1,
56    ↪ lazy[v]);
57     t[2 * v + 2] = f_on_seg(t[2 * v + 2], tr - tm,
58    ↪ lazy[v]);
59     upd_lazy(2 * v + 1, lazy[v]), upd_lazy(2 * v + 2,
60    ↪ lazy[v]);
61     lazy[v] = lazy_mark;
62 }
63
64 void modify(int v, int tl, int tr, int l, int r, T
65    ↪ val){
66     if (l > r) return;
67     if (tl == l && tr == r){
68         t[v] = f_on_seg(t[v], tr - tl + 1, val);
69         upd_lazy(v, val);
70         return;
71     }
72     push(v, tl, tr);
73     int tm = (tl + tr) / 2;
74     modify(2 * v + 1, tl, tm, l, min(r, tm), val);
75     modify(2 * v + 2, tm + 1, tr, max(l, tm + 1), r,
76    ↪ val);
77     t[v] = f(t[2 * v + 1], t[2 * v + 2]);
78 }
79
80 T query(int v, int tl, int tr, int l, int r) {
81     if (l > r) return default_return;
82     if (tl == l && tr == r) return t[v];
83     push(v, tl, tr);
84     int tm = (tl + tr) / 2;
85     return f(
86         query(2 * v + 1, tl, tm, l, min(r, tm)),
87         query(2 * v + 2, tm + 1, tr, max(l, tm + 1), r)
88     );
89 }
```

```
86 void modify(int l, int r, T val){
87     modify(0, 0, n - 1, l, r, val);
88 }
89
90 T query(int l, int r){
91     return query(0, 0, n - 1, l, r);
92 }
93
94 T get(int pos){
95     return query(pos, pos);
96 }
97
98 // Change clear() function to t.clear() if using
99 ↪ unordered_map for SegTree!!!
100 void clear(int n_){
101     n = n_;
102     for (int i = 0; i < 4 * n; i++) t[i] = 0, lazy[i] =
103    ↪ lazy_mark;
104 }
105
106 void build(vector<T>& a){
107     n = sz(a);
108     clear(n);
109     build(0, 0, n - 1, a);
110 };
```

Sparse Table

```
1 const int N = 2e5 + 10, LOG = 20; // Change the
2 ↪ constant!
3 template<typename T>
4 struct SparseTable{
5     int lg[N];
6     T st[N][LOG];
7     int n;
8
9     // Change this function
10    function<T(T, T)> f = [&] (T a, T b){
11        return min(a, b);
12    };
13
14    void build(vector<T>& a){
15        n = sz(a);
16        lg[1] = 0;
17        for (int i = 2; i <= n; i++) lg[i] = lg[i / 2] + 1;
18
19        for (int k = 0; k < LOG; k++){
20            for (int i = 0; i < n; i++){
21                if (!k) st[i][k] = a[i];
22                else st[i][k] = f(st[i][k - 1], st[min(n - 1, i +
23    ↪ (1 << (k - 1)))][k - 1]);
24            }
25        }
26
27        T query(int l, int r){
```



```

27     int sz = r - 1 + 1;
28     return f(st[l][lg[sz]], st[r - (1 << lg[sz]) +
    ↪ 1][lg[sz]]);
29 }
30 };

```

Suffix Array and LCP array

- (uses SparseTable above)

```

1 struct SuffixArray{
2     vector<int> p, c, h;
3     SparseTable<int> st;
4     /*
5     In the end, array c gives the position of each suffix
    ↪ in p
6     using 1-based indexation!
7     */
9     SuffixArray() {}
10
11     SuffixArray(string s){
12         buildArray(s);
13         buildLCP(s);
14         buildSparse();
15     }
16
17     void buildArray(string s){
18         int n = sz(s) + 1;
19         p.resize(n), c.resize(n);
20         for (int i = 0; i < n; i++) p[i] = i;
21         sort(all(p), [&] (int a, int b){return s[a] <
    ↪ s[b];});
22         c[p[0]] = 0;
23         for (int i = 1; i < n; i++){
24             c[p[i]] = c[p[i - 1]] + (s[p[i]] != s[p[i - 1]]);
25         }
26         vector<int> p2(n), c2(n);
27         // w is half-length of each string.
28         for (int w = 1; w < n; w <= 1){
29             for (int i = 0; i < n; i++){
30                 p2[i] = (p[i] - w + n) % n;
31             }
32             vector<int> cnt(n);
33             for (auto i : c) cnt[i]++;
34             for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
35             for (int i = n - 1; i >= 0; i--){
36                 p[--cnt[c[p2[i]]]] = p2[i];
37             }
38             c2[p[0]] = 0;
39             for (int i = 1; i < n; i++){
40                 c2[p[i]] = c2[p[i - 1]] +
41                 (c[p[i]] != c[p[i - 1]] ||
42                 c[(p[i] + w) % n] != c[(p[i - 1] + w) % n]);
43             }
44             c.swap(c2);
45         }
46         p.erase(p.begin());

```

```

47     }
48
49     void buildLCP(string s){
50         // The algorithm assumes that suffix array is
    ↪ already built on the same string.
51         int n = sz(s);
52         h.resize(n - 1);
53         int k = 0;
54         for (int i = 0; i < n; i++){
55             if (c[i] == n){
56                 k = 0;
57                 continue;
58             }
59             int j = p[c[i]];
60             while (i + k < n && j + k < n && s[i + k] == s[j +
    ↪ k]) k++;
61             h[c[i] - 1] = k;
62             if (k) k--;
63         }
64         /*
65         Then an RMQ Sparse Table can be built on array h
66         to calculate LCP of 2 non-consecutive suffixes.
67         */
68     }
69
70     void buildSparse(){
71         st.build(h);
72     }
73
74     // l and r must be in 0-BASED INDEXATION
75     int lcp(int l, int r){
76         l = c[l] - 1, r = c[r] - 1;
77         if (l > r) swap(l, r);
78         return st.query(l, r - 1);
79     }
80 }

```

Aho Corasick Trie

- For each node in the trie, the suffix link points to the longest proper suffix of the represented string. The terminal-link tree has square-root height (can be constructed by DFS).

```

1 const int S = 26;
2
3 // Function converting char to int.
4 int ctoi(char c){
5     return c - 'a';
6 }
7
8 // To add terminal links, use DFS
9 struct Node{
10     vector<int> nxt;
11     int link;
12     bool terminal;
13

```

```

14     Node() {
15         nxt.assign(S, -1), link = 0, terminal = 0;
16     }
17 };
18
19 vector<Node> trie(1);
20
21 // add_string returns the terminal vertex.
22 int add_string(string& s){
23     int v = 0;
24     for (auto c : s){
25         int cur = ctoi(c);
26         if (trie[v].nxt[cur] == -1){
27             trie[v].nxt[cur] = sz(trie);
28             trie.emplace_back();
29         }
30         v = trie[v].nxt[cur];
31     }
32     trie[v].terminal = 1;
33     return v;
34 }
35
36 /*
37 Suffix links are compressed.
38 This means that:
39 If vertex v has a child by letter x, then:
40     trie[v].nxt[x] points to that child.
41 If vertex v doesn't have such child, then:
42     trie[v].nxt[x] points to the suffix link of that
    ↪ child
43     if we would actually have it.
44 */
45 void add_links(){
46     queue<int> q;
47     q.push(0);
48     while (!q.empty()){
49         auto v = q.front();
50         int u = trie[v].link;
51         q.pop();
52         for (int i = 0; i < S; i++){
53             int& ch = trie[v].nxt[i];
54             if (ch == -1){
55                 ch = v? trie[u].nxt[i] : 0;
56             }
57             else{
58                 trie[ch].link = v? trie[u].nxt[i] : 0;
59                 q.push(ch);
60             }
61         }
62     }
63 }
64
65 bool is_terminal(int v){
66     return trie[v].terminal;
67 }
68
69 int get_link(int v){

```



```

70     return trie[v].link;
71 }
72
73 int go(int v, char c){
74     return trie[v].nxt[ctoi(c)];
75 }

```

Convex Hull Trick

- Allows to insert a linear function to the hull in (1) and get the minimum/maximum value of the stored function at a point in $O(\log n)$.
- NOTE: The lines must be added in the order of decreasing/increasing gradients. CAREFULLY CHECK THE SETUP BEFORE USING!
- IMPORTANT: THE DEFAULT VERSION SURELY WORKS. IF MODIFIED VERSIONS DON'T WORK, TRY TRANSFORMING THEM TO THE DEFAULT ONE BY CHANGING SIGNS.

```

1  struct line{
2      ll k, b;
3      ll f(ll x){
4          return k * x + b;
5      };
6  };
7
8  vector<line> hull;
9
10 void add_line(line nl){
11     if (!hull.empty() && hull.back().k == nl.k){
12         nl.b = min(nl.b, hull.back().b); // Default:
13         ↪ minimum. For maximum change "min" to "max".
14         hull.pop_back();
15     }
16     while (sz(hull) > 1){
17         auto& l1 = hull.end()[-2], l2 = hull.back();
18         if ((nl.b - l1.b) * (l2.k - nl.k) >= (nl.b - l2.b) *
19         ↪ (l1.k - nl.k)) hull.pop_back(); // Default:
20         ↪ decreasing gradient k. For increasing k change the
21         ↪ sign to <=.
22         else break;
23     }
24     hull.pb(nl);
25 }
26
27 ll get(ll x){
28     int l = 0, r = sz(hull);
29     while (r - l > 1){
30         int mid = (l + r) / 2;
31         if (hull[mid - 1].f(x) >= hull[mid].f(x)) l = mid;
32         ↪ // Default: minimum. For maximum change the sign to
33         ↪ <=.
34         else r = mid;
35     }
36 }

```

```

29 }
30 return hull[l].f(x);
31 }

```

Li-Chao Segment Tree

- allows to add linear functions in any order and query minimum/maximum value of those at a point, all in $O(\log n)$.
- Clear: clear()

```

const ll INF = 1e18; // Change the constant!
struct LiChaoTree{
    struct line{
        ll k, b;
        line(){
            k = b = 0;
        };
        line(ll k_, ll b_){
            k = k_, b = b_;
        };
        ll f(ll x){
            return k * x + b;
        };
    };
    int n;
    bool minimum, on_points;
    vector<ll> pts;
    vector<line> t;

    void clear(){
        for (auto& l : t) l.k = 0, l.b = minimum? INF :
        ↪ -INF;
    }

    LiChaoTree(int n_, bool min_){ // This is a default
        ↪ constructor for numbers in range [0, n - 1].
        n = n_, minimum = min_, on_points = false;
        t.resize(4 * n);
        clear();
    };

    LiChaoTree(vector<ll> pts_, bool min_){ // This
        ↪ constructor will build LCT on the set of points you
        ↪ pass. The points may be in any order and contain
        ↪ duplicates.
        pts = pts_, minimum = min_;
        sort(all(pts));
        pts.erase(unique(all(pts)), pts.end());
        on_points = true;
        n = sz(pts);
        t.resize(4 * n);
        clear();
    };

    void add_line(int v, int l, int r, line nl){
        // Adding on segment [l, r)

```

```

42     int m = (l + r) / 2;
43     ll lval = on_points? pts[l] : 1, mval = on_points?
44     ↪ pts[m] : m;
45     if ((minimum && nl.f(mval) < t[v].f(mval)) ||
46     ↪ (!minimum && nl.f(mval) > t[v].f(mval))) swap(t[v],
47     ↪ nl);
48     if (r - l == 1) return;
49     if ((minimum && nl.f(lval) < t[v].f(lval)) ||
50     ↪ (!minimum && nl.f(lval) > t[v].f(lval))) add_line(2
51     ↪ * v + 1, l, m, nl);
52     else add_line(2 * v + 2, m, r, nl);
53 }
54
55 ll get(int v, int l, int r, int x){
56     int m = (l + r) / 2;
57     if (r - l == 1) return t[v].f(on_points? pts[x] :
58     ↪ x);
59     else{
60         if (minimum) return min(t[v].f(on_points? pts[x] :
61         ↪ x), x < m? get(2 * v + 1, l, m, x) : get(2 * v + 2,
62         ↪ m, r, x));
63         else return max(t[v].f(on_points? pts[x] : x), x <
64         ↪ m? get(2 * v + 1, l, m, x) : get(2 * v + 2, m, r,
65         ↪ x));
66     }
67 }

void add_line(ll k, ll b){
    add_line(0, 0, n, line(k, b));
}

ll get(ll x){
    return get(0, 0, n, on_points? lower_bound(all(pts),
    ↪ x) - pts.begin() : x);
    }; // Always pass the actual value of x, even if LCT
    ↪ is on points.
}

```

Persistent Segment Tree

- for RSQ

```

struct Node {
    ll val;
    Node *l, *r;

    Node(ll x) : val(x), l(nullptr), r(nullptr) {}
    Node(Node *ll, Node *rr) {
        l = ll, r = rr;
        val = 0;
        if (l) val += l->val;
        if (r) val += r->val;
    }
    Node(Node *cp) : val(cp->val), l(cp->l), r(cp->r) {}
};

const int N = 2e5 + 20;

```

```

15 ll a[N];
16 Node *roots[N];
17 int n, cnt = 1;
18 Node *build(int l = 1, int r = n) {
19     if (l == r) return new Node(a[l]);
20     int mid = (l + r) / 2;
21     return new Node(build(l, mid), build(mid + 1, r));
22 }
23 Node *update(Node *node, int val, int pos, int l = 1,
    ↪ int r = n) {
24     if (l == r) return new Node(val);
25     int mid = (l + r) / 2;
26     if (pos > mid)
27         return new Node(node->l, update(node->r, val,
    ↪ pos, mid + 1, r));
28     else return new Node(update(node->l, val, pos, l,
    ↪ mid), node->r);
29 }
30 ll query(Node *node, int a, int b, int l = 1, int r = n)
    ↪ {
31     if (l > b || r < a) return 0;
32     if (l >= a && r <= b) return node->val;
33     int mid = (l + r) / 2;
34     return query(node->l, a, b, l, mid) + query(node->r,
    ↪ a, b, mid + 1, r);
35 }

```

Miscellaneous

Ordered Set

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4 typedef tree<int, null_type, less<int>, rb_tree_tag,
    ↪ tree_order_statistics_node_update> ordered_set;

```

Measuring Execution Time

```

1 ld tic = clock();
2 // execute algo...
3 ld tac = clock();
4 // Time in milliseconds
5 cerr << (tac - tic) / CLOCKS_PER_SEC * 1000 << endl;
6 // No need to comment out the print because it's done to
    ↪ cerr.

```

Setting Fixed D.P. Precision

```

1 cout << setprecision(d) << fixed;
2 // Each number is rounded to d digits after the decimal
    ↪ point, and truncated.

```

Common Bugs and General Advice

- Check overflow, array bounds
- Check variable overloading
- Check special cases (n=1?)
- Do something instead of nothing, stay organized
- Write stuff down!
- Don't get stuck on one approach!