CU-Later Code Library

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Templates Contents **Templates** 1 Ken's template #include <bits/stdc++.h> 1 using namespace std; Kevin's Template Extended #define all(v) (v).begin(), (v).end()typedef long long 11; typedef long double ld; Geometry 1 #define pb push_back #define sz(x) (int)(x).size()Strings 3 #define fi first #define se second 9 Flows 3 #define endl '\n' $O(N^2M)$, on unit networks $O(N^{1/2}M)$ 3 MCMF - maximize flow, then minimize its cost. Kevin's template 4 // paste Kaurov's Template, minus last line typedef vector<int> vi; 2 5 Graphs typedef vector<ll> vll; Kuhn's algorithm for bipartite matching 5 typedef pair<int, int> pii; Hungarian algorithm for Assignment Problem . . . 6 typedef pair<11, 11> pll; Dijkstra's Algorithm 6 typedef pair<double, double> pdd; const ld PI = acosl(-1); const $11 \mod 7 = 1e9 + 7$; 6 const 11 mod9 = 998244353; 6 10 const 11 INF = 2*1024*1024*1023; 7 const char nl = '\n'; 11 #define form(i, n) for (int i = 0; i < int(n); i++) ll k, n, m, u, v, w; 7 string s, t; 7 Math 16 bool multiTest = 1: void solve(int tt){ 7 17 18 7 Extended Euclidean Algorithm 7 int main(){ 8 21 ios::sync_with_stdio(0);cin.tie(0);cout.tie(0); cout<<fixed<< setprecision(14);</pre> 8 22 23 9 int t = 1;9 $is_prime \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ 25 if (multiTest) cin >> t; 26 forn(ii, t) solve(ii); 9 Data Structures 27 9 9 Suffix Array and LCP array Kevin's Template Extended 10 • to type after the start of the contest 11 11 #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt") Persistent Segment Tree 11 #include <ext/pb_ds/assoc_container.hpp> #include <ext/pb_ds/tree_policy.hpp> using namespace __gnu_pbds; 4 Miscellaneous 12 template<class T> using ordered_set = tree<T, null_type, 12 less<T>, rb_tree_tag, tree_order_statistics_node_update>; 12 $vi d4x = \{1, 0, -1, 0\};$ $vi d4y = \{0, 1, 0, -1\};$ 12 Setting Fixed D.P. Precision vi $d8x = \{1, 0, -1, 0, 1, 1, -1, -1\};$ Common Bugs and General Advice 12 vi d8y = $\{0, 1, 0, -1, 1, -1, 1, -1\};$ ¬ rng(chrono::steady_clock::now().time_since_epoch().count()); Geometry template<typename T> struct TPoint{ Тх, у; int id;

static constexpr T eps = static_cast<T>(1e-9);

TPoint operator + (const TPoint& rhs) const {

 $\label{eq:total_total_total} TPoint(const \ T\& \ x_, \ const \ T\& \ y_) : x(x_), \ y(y_), \ id(-1) \ \{\}$ $TPoint(const \ T\& \ x_, \ const \ T\& \ y_, \ const \ int \ id_) : x(x_),$

TPoint() : x(0), y(0), id(-1) {}

 \rightarrow y(y_), id(id_) {}

```
return TPoint(x + rhs.x, y + rhs.y);
                                                                            int sign(const T& x){
11
                                                                        82
                                                                              if (abs(x) <= TPoint<T>::eps) return 0;
12
                                                                        83
13
      TPoint operator - (const TPoint& rhs) const {
                                                                        84
                                                                              return x > 0? +1 : -1;
        return TPoint(x - rhs.x, y - rhs.y);
14
                                                                        85
                                                                            template<typename T>
      TPoint operator * (const T& rhs) const {
                                                                            T area(const vector<TPoint<T>>& pts){
16
                                                                        87
17
        return TPoint(x * rhs, y * rhs);
                                                                               int n = sz(pts);
                                                                        88
                                                                              T ans = 0;
18
                                                                        89
      TPoint operator / (const T& rhs) const {
                                                                              for (int i = 0; i < n; i++){
19
                                                                        90
        return TPoint(x / rhs, y / rhs);
20
                                                                                 ans += vmul(pts[i], pts[(i + 1) % n]);
21
                                                                        92
      TPoint ort() const {
22
                                                                        93
                                                                              return abs(ans) / 2;
23
        return TPoint(-y, x);
                                                                        94
                                                                            template<typename T>
24
                                                                        95
      T abs2() const {
                                                                            T dist_pp(const TPoint<T>& a, const TPoint<T>& b){
25
        return x * x + y * y;
                                                                              return sqrt(sq(a.x - b.x) + sq(a.y - b.y));
26
                                                                        97
27
                                                                        98
28
    };
                                                                        99
                                                                            template<typename T>
    template<typename T>
                                                                            TLine<T> perp_line(const TLine<T>& 1, const TPoint<T>& p){
29
                                                                       100
    bool operator< (TPoint<T>& A, TPoint<T>& B){
                                                                              T na = -1.b, nb = 1.a, nc = -na * p.x - nb * p.y;
30
                                                                       101
      return make_pair(A.x, A.y) < make_pair(B.x, B.y);</pre>
                                                                              return TLine<T>(na, nb, nc);
31
                                                                       102
32
                                                                       103
    template<typename T>
33
                                                                       104
                                                                            template<typename T>
    bool operator== (TPoint<T>& A, TPoint<T>& B){
                                                                            TPoint<T> projection(const TPoint<T>& p, const TLine<T>& 1){
      return abs(A.x - B.x) \leftarrow TPoint < T > :: eps && abs(A.y - B.y) <= 106
                                                                              return intersection(1, perp_line(1, p));
35
        TPoint<T>::eps;
                                                                       107
    7
                                                                             template<typename T>
36
                                                                       108
    template<typename T>
                                                                            T dist_pl(const TPoint<T>& p, const TLine<T>& 1){
37
                                                                       109
    struct TLine{
                                                                               return dist_pp(p, projection(p, 1));
      T a, b, c;
39
                                                                       111
      TLine(): a(0), b(0), c(0) {}
                                                                            template<typename T>
                                                                       112
40
      TLine(const T\& a_, const T\& b_, const T\& c_) : a(a_), b(b_), 113
                                                                            struct TRay{
                                                                              TLine<T> 1;
                                                                       114
42
      TLine(const TPoint<T>& p1, const TPoint<T>& p2){
                                                                              TPoint<T> start, dirvec;
        a = p1.y - p2.y;
43
                                                                       116
                                                                              TRay() : 1(), start(), dirvec() {}
        b = p2.x - p1.x;
                                                                       117
                                                                               TRay(const TPoint<T>& p1, const TPoint<T>& p2){
44
        c = -a * p1.x - b * p1.y;
45
                                                                       118
                                                                                1 = TLine < T > (p1, p2);
                                                                                start = p1, dirvec = p2 - p1;
46
                                                                       119
                                                                              }
47
    };
    template<typename T>
48
                                                                       121
                                                                            1:
    T det(const T& a11, const T& a12, const T& a21, const T& a22){ _{122}}
49
                                                                             template<typename T>
      return a11 * a22 - a12 * a21;
                                                                            bool is_on_line(const TPoint<T>& p, const TLine<T>& 1){
50
                                                                       123
                                                                              return abs(l.a * p.x + l.b * p.y + l.c) <= TPoint<T>::eps;
51
                                                                       124
    template<typename T>
52
                                                                       125
    T sq(const T& a){
                                                                            template<typename T>
53
                                                                       126
      return a * a;
                                                                            bool is_on_ray(const TPoint<T>& p, const TRay<T>& r){
54
                                                                       127
55
                                                                       128
                                                                               if (is_on_line(p, r.l)){
                                                                                return sign(smul(r.dirvec, TPoint<T>(p - r.start))) != -1;
    template<typename T>
56
                                                                       129
57
    T smul(const TPoint<T>& a, const TPoint<T>& b){
                                                                       130
      return a.x * b.x + a.y * b.y;
                                                                               else return false;
58
                                                                       131
59
                                                                       132
    template<typename T>
                                                                            template<typename T>
60
                                                                       133
    T vmul(const TPoint<T>& a, const TPoint<T>& b){
                                                                       134
                                                                            bool is_on_seg(const TPoint<T>& P, const TPoint<T>& A, const
62
      return det(a.x, a.y, b.x, b.y);
                                                                             \hookrightarrow TPoint<T>& B){
                                                                              return is_on_ray(P, TRay<T>(A, B)) && is_on_ray(P,
63
                                                                       135
    template<typename T>
                                                                             \hookrightarrow TRay<T>(B, A));
64
    bool parallel(const TLine<T>& 11, const TLine<T>& 12){
                                                                            }
65
                                                                       136
      return abs(vmul(TPoint<T>(11.a, 11.b), TPoint<T>(12.a,
                                                                            template<typename T>
     T dist_pr(const TPoint<T>& P, const TRay<T>& R){
                                                                       138
67
                                                                       139
                                                                               auto H = projection(P, R.1);
    template<typename T>
                                                                       140
                                                                              return is_on_ray(H, R)? dist_pp(P, H) : dist_pp(P, R.start);
68
    bool equivalent(const TLine<T>& 11, const TLine<T>& 12){
69
                                                                       141
      return parallel(11, 12) &&
                                                                            template<typename T>
      abs(det(11.b, 11.c, 12.b, 12.c)) <= TPoint<T>::eps &&
                                                                            T dist_ps(const TPoint<T>& P, const TPoint<T>& A, const
71
                                                                       143

→ TPoint<T>& B){
      abs(det(11.a, 11.c, 12.a, 12.c)) <= TPoint<T>::eps;
72
                                                                              auto H = projection(P, TLine<T>(A, B));
73
                                                                       144
    template<typename T>
                                                                               if (is_on_seg(H, A, B)) return dist_pp(P, H);
74
                                                                       145
    TPoint<T> intersection(const TLine<T>& 11, const TLine<T>&
                                                                       146
                                                                               else return min(dist_pp(P, A), dist_pp(P, B));

→ 12) {

                                                                       147
      return TPoint<T>(
                                                                             template<typename T>
                                                                       148
                                                                            bool acw(const TPoint<T>& A, const TPoint<T>& B){
        det(-11.c, 11.b, -12.c, 12.b) / det(11.a, 11.b, 12.a,
                                                                       149
                                                                       150
                                                                              T \text{ mul} = vmul(A, B);
        det(11.a, -11.c, 12.a, -12.c) / det(11.a, 11.b, 12.a,
                                                                               return mul > 0 || abs(mul) <= TPoint<T>::eps;
                                                                       151
        12.b)
                                                                       152
                                                                       153
                                                                             template<typename T>
                                                                            bool cw(const TPoint<T>& A, const TPoint<T>& B){
80
                                                                       154
    template<typename T>
                                                                               T mul = vmul(A, B);
                                                                       155
```

```
return mul < 0 || abs(mul) <= TPoint<T>::eps;
                                                                              template<typename T>
                                                                              vector<TPoint<T>> minkowski_sum(vector<TPoint<T>> P,
157
                                                                        228
158
     template<typename T>

    vector<TPoint<T>> Q){
     vector<TPoint<T>> convex_hull(vector<TPoint<T>> pts){
                                                                                minkowski_rotate(P);
159
                                                                        229
        sort(all(pts));
                                                                                minkowski_rotate(Q);
       pts.erase(unique(all(pts)), pts.end());
                                                                                P.pb(P[0]);
161
                                                                        231
162
        vector<TPoint<T>> up, down;
                                                                        232
                                                                                Q.pb(Q[0]);
       for (auto p : pts){
163
                                                                                vector<TPoint<T>> ans;
                                                                        233
         while (sz(up) > 1 \&\& acw(up.end()[-1] - up.end()[-2], p -
                                                                                int i = 0, j = 0;
                                                                        234
164
        up.end()[-2])) up.pop_back();
                                                                                while (i < sz(P) - 1 \mid | j < sz(Q) - 1){
         while (sz(down) > 1 \&\& cw(down.end()[-1] - down.end()[-2], 236
                                                                                  ans.pb(P[i] + Q[j]);
165
         p - down.end()[-2])) down.pop_back();
166
         up.pb(p), down.pb(p);
                                                                                  if (i == sz(P) - 1) curmul = -1;
                                                                                  else if (j == sz(Q) - 1) curmul = +1;
167
                                                                        239
       for (int i = sz(up) - 2; i >= 1; i--) down.pb(up[i]);
                                                                                  else curmul = vmul(P[i + 1] - P[i], Q[j + 1] - Q[j]);
168
                                                                        240
                                                                                  if (abs(curmul) < TPoint<T>::eps || curmul > 0) i++;
       return down:
                                                                        241
169
170
                                                                        242
                                                                                  if (abs(curmul) < TPoint<T>::eps || curmul < 0) j++;</pre>
171
                                                                        243
     template<typename T>
                                                                                return ans;
172
                                                                        244
     bool in_triangle(TPoint<T>& P, TPoint<T>& A, TPoint<T>& B,
                                                                             }
173
                                                                        245
      → TPoint<T>& C){
                                                                              using Point = TPoint<11>; using Line = TLine<11>; using Ray =
                                                                        246
       if (is_on_seg(P, A, B) || is_on_seg(P, B, C) || is_on_seg(P,
                                                                               \rightarrow TRay<11>; const ld PI = acos(-1);

⇔ C. A)) return true:

       return cw(P - A, B - A) == cw(P - B, C - B) &&
       cw(P - A, B - A) == cw(P - C, A - C);
                                                                              Strings
176
177
     template<typename T>
178
                                                                              vector<int> prefix_function(string s){
179
     void prep_convex_poly(vector<TPoint<T>>& pts){
                                                                               int n = sz(s);
       rotate(pts.begin(), min_element(all(pts)), pts.end());
180
                                                                                vector<int> pi(n);
181
                                                                                for (int i = 1; i < n; i++){
     // 0 - Outside, 1 - Exclusively Inside, 2 - On the Border
182
                                                                                  int k = pi[i - 1];
     template<typename T>
183
                                                                                  while (k > 0 \&\& s[i] != s[k]){
     int in_convex_poly(TPoint<T>& p, vector<TPoint<T>>& pts){
184
                                                                                    k = pi[k - 1];
        int n = sz(pts);
       if (!n) return 0;
186
                                                                                  pi[i] = k + (s[i] == s[k]);
        if (n <= 2) return is_on_seg(p, pts[0], pts.back());</pre>
187
                                                                         10
       int 1 = 1, r = n - 1;
188
                                                                         11
                                                                                return pi;
       while (r - 1 > 1){
189
                                                                             }
          int mid = (1 + r) / 2;
                                                                              vector<int> kmp(string s, string k){
                                                                         13
          if (acw(pts[mid] - pts[0], p - pts[0])) 1 = mid;
191
                                                                                string st = k + "#" + s;
                                                                         14
192
                                                                                vector<int> res;
193
                                                                                auto pi = pf(st);
                                                                         16
       if (!in_triangle(p, pts[0], pts[1], pts[1 + 1])) return 0;
194
                                                                                for (int i = 0; i < sz(st); i++){
        if (is_on_seg(p, pts[1], pts[1 + 1]) ||
195
                                                                                  if (pi[i] == sz(k)){
                                                                         18
          is_on_seg(p, pts[0], pts.back()) ||
196
                                                                         19
                                                                                    res.pb(i - 2 * sz(k));
          is_on_seg(p, pts[0], pts[1])
197
                                                                         20
       ) return 2;
198
                                                                                }
                                                                         21
       return 1:
199
                                                                                return res;
200
                                                                             }
                                                                         23
     // 0 - Outside, 1 - Exclusively Inside, 2 - On the Border
201
                                                                              vector<int> z_function(string s){
                                                                         ^{24}
202
     template<typename T>
                                                                               int n = sz(s):
                                                                         25
     int in_simple_poly(TPoint<T> p, vector<TPoint<T>>& pts){
203
                                                                                vector<int> z(n);
                                                                         26
       int n = sz(pts):
                                                                                int 1 = 0, r = 0;
205
       bool res = 0;
                                                                                for (int i = 1; i < n; i++){
                                                                         28
206
       for (int i = 0; i < n; i++){
                                                                                  if (r \ge i) z[i] = min(z[i - 1], r - i + 1);
          auto a = pts[i], b = pts[(i + 1) % n];
207
                                                                                  while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]]){
         if (is_on_seg(p, a, b)) return 2;
208
          if (((a.y > p.y) - (b.y > p.y)) * vmul(b - p, a - p) >
      \  \, \neg \  \, TPoint < T > : :eps) \{
                                                                                  if (i + z[i] - 1 > r){
                                                                         33
           res ^= 1;
210
                                                                                    l = i, r = i + z[i] - 1;
211
                                                                         35
212
                                                                         36
       return res;
213
                                                                         37
                                                                                return z;
214
     template<typename T>
215
     void minkowski_rotate(vector<TPoint<T>>& P){
216
        int pos = 0;
217
                                                                              Flows
218
       for (int i = 1; i < sz(P); i++){
          if (abs(P[i].y - P[pos].y) <= TPoint<T>::eps){
219
220
            if (P[i].x < P[pos].x) pos = i;
                                                                              O(N^2M), on unit networks O(N^{1/2}M)
221
          else if (P[i].y < P[pos].y) pos = i;</pre>
222
                                                                              struct FlowEdge {
223
                                                                                  int v, u;
       rotate(P.begin(), P.begin() + pos, P.end());
224
                                                                                  long long cap, flow = 0;
                                                                          3
225
                                                                                  FlowEdge(int v, int u, long long cap) : v(v), u(u),
     // P and Q are strictly convex, points given in
226
                                                                               \hookrightarrow cap(cap) {}
      }:
```

156

struct Dinic { const long long flow_inf = 1e18; vector<FlowEdge> edges; vector<vector<int>> adj; 9 int n, m = 0;int s, t; 11 12 vector<int> level, ptr; queue<int> q; 13 Dinic(int n, int s, int t) : n(n), s(s), t(t) { 14 adj.resize(n); level.resize(n): 16 ptr.resize(n); 17 } 18 void add_edge(int v, int u, long long cap) { 19 edges.emplace_back(v, u, cap); 20 edges.emplace_back(u, v, 0); 21 22 adj[v].push_back(m); adj[u].push_back(m + 1); 23 m += 2;24 7 25 bool bfs() { 26 while (!q.empty()) { 27 int v = q.front(); 28 q.pop(); for (int id : adj[v]) { 30 if (edges[id].cap - edges[id].flow < 1)</pre> 31 continue; if (level[edges[id].u] != -1) 33 continue; level[edges[id].u] = level[v] + 1; 35 q.push(edges[id].u); 36 } 37 } 38 39 return level[t] != -1; } 40 long long dfs(int v, long long pushed) { 41 if (pushed == 0) 42 return 0; 43 if (v == t) 44 return pushed; 45 for (int& cid = ptr[v]; cid < (int)adj[v].size();</pre> 46 cid++) { int id = adj[v][cid]; 47 int u = edges[id].u; 48 if (level[v] + 1 != level[u] || edges[id].cap -49 edges[id].flow < 1) 50 continue: long long tr = dfs(u, min(pushed, edges[id].cap -51 edges[id].flow)); if (tr == 0) 52 53 continue; edges[id].flow += tr; 54 edges[id ^ 1].flow -= tr; 56 return tr; } 57 return 0; 59 long long flow() { long long f = 0; 61 62 while (true) { fill(level.begin(), level.end(), -1); 63 level[s] = 0;64 q.push(s); if (!bfs()) 66 67 fill(ptr.begin(), ptr.end(), 0); 68 while (long long pushed = dfs(s, flow_inf)) { 69 70 f += pushed; 71 } 73 return f: } 74 }; 75 // To recover flow through original edges: iterate over even 76 indices in edges.

MCMF – maximize flow, then minimize its cost. O(Fmn).

```
#include <ext/pb_ds/priority_queue.hpp>
template <typename T, typename C>
class MCMF {
public:
   static constexpr T eps = (T) 1e-9;
   struct edge {
     int from;
     int to;
     T c:
     T f:
     C cost;
   };
   int n;
   vector<vector<int>> g;
   vector<edge> edges;
   vector<C> d;
   vector<C> pot;
   __gnu_pbds::priority_queue<pair<C, int>> q;
   vector<typename decltype(q)::point_iterator> its;
   vector<int> pe;
   const C INF_C = numeric_limits<C>::max() / 2;
   explicit MCMF(int n_{int} n_{int}) : n(n_{int}), g(n), d(n), pot(n, 0),
\rightarrow its(n), pe(n) {}
   int add(int from, int to, T forward_cap, C edge_cost, T

→ backward_cap = 0) {
     \texttt{assert(0} \mathrel{<=} \texttt{from } \&\& \texttt{ from } < \texttt{n} \&\& \texttt{ 0} \mathrel{<=} \texttt{to } \&\& \texttt{ to } < \texttt{n);}
     assert(forward_cap >= 0 && backward_cap >= 0);
     int id = static_cast<int>(edges.size());
     g[from].push_back(id);
     edges.push_back({from, to, forward_cap, 0, edge_cost});
     g[to].push_back(id + 1);
     edges.push_back({to, from, backward_cap, 0, -edge_cost});
     return id;
   void expath(int st) {
     fill(d.begin(), d.end(), INF_C);
     q.clear();
     fill(its.begin(), its.end(), q.end());
     its[st] = q.push({pot[st], st});
     d[st] = 0;
     while (!q.empty()) {
       int i = q.top().second;
       q.pop();
       its[i] = q.end();
       for (int id : g[i]) {
         const edge &e = edges[id];
         int j = e.to;
         if (e.c - e.f > eps \&\& d[i] + e.cost < d[j]) {
           d[j] = d[i] + e.cost;
           pe[j] = id;
            if (its[j] == q.end()) {
              its[j] = q.push({pot[j] - d[j], j});
              q.modify(its[j], {pot[j] - d[j], j});
         }
       }
     swap(d, pot);
   pair<T, C> max_flow(int st, int fin) {
     T flow = 0;
     C cost = 0;
     bool ok = true;
     for (auto& e : edges) {
       if (e.c - e.f > eps && e.cost + pot[e.from] - pot[e.to]
ok = false;
```

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```
break:
    if (ok) {
      expath(st);
    } else {
      vector<int> deg(n, 0);
      for (int i = 0; i < n; i++) {
        for (int eid : g[i]) {
          auto& e = edges[eid];
          if (e.c - e.f > eps) {
            deg[e.to] += 1;
        }
      }
      vector<int> que;
      for (int i = 0; i < n; i++) {
        if (deg[i] == 0) {
          que.push_back(i);
      for (int b = 0; b < (int) que.size(); b++) {</pre>
        for (int eid : g[que[b]]) {
          auto& e = edges[eid];
          if (e.c - e.f > eps) {
            deg[e.to] -= 1;
            if (deg[e.to] == 0) {
              que.push_back(e.to);
          }
      fill(pot.begin(), pot.end(), INF_C);
      pot[st] = 0;
      if (static_cast<int>(que.size()) == n) {
        for (int v : que) {
          if (pot[v] < INF_C) {</pre>
            for (int eid : g[v]) {
              auto& e = edges[eid];
              if (e.c - e.f > eps) \{
                if (pot[v] + e.cost < pot[e.to]) {</pre>
                  pot[e.to] = pot[v] + e.cost;
                  pe[e.to] = eid;
            }
          }
        }
      } else {
        que.assign(1, st);
        vector<bool> in_queue(n, false);
        in_queue[st] = true;
        for (int b = 0; b < (int) que.size(); b++) {</pre>
          int i = que[b];
          in_queue[i] = false;
          for (int id : g[i]) {
            const edge &e = edges[id];
            if (e.c - e.f > eps && pot[i] + e.cost <
→ pot[e.to]) {
              pot[e.to] = pot[i] + e.cost;
              pe[e.to] = id;
              if (!in_queue[e.to]) {
                que.push_back(e.to);
                in_queue[e.to] = true;
          }
        }
      }
    while (pot[fin] < INF_C) {
      T push = numeric_limits<T>::max();
      int v = fin;
      while (v != st) {
        const edge &e = edges[pe[v]];
        push = min(push, e.c - e.f);
        v = e.from;
```

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101 102 103

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123

 $\frac{125}{126}$

127

128

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140

141

142

143

144

145

146

147

```
}
148
              v = fin;
149
              while (v != st) {
150
                edge &e = edges[pe[v]];
151
                 e.f += push;
                edge &back = edges[pe[v] ^ 1];
153
154
                back.f -= push;
155
                v = e.from;
156
              flow += push;
              cost += push * pot[fin];
158
              expath(st);
159
160
           return {flow, cost};
161
         }
162
     };
163
164
      // Examples: MCMF < int, int > g(n); g.add(u, v, c, w, 0);
165
       \hookrightarrow g.max_flow(s,t).
      \begin{tabular}{ll} /\!/ & \textit{To recover flow through original edges: iterate over even} \end{tabular}
```

Graphs

Kuhn's algorithm for bipartite matching

```
The graph is split into 2 halves of n1 and n2 vertices.
     Complexity: O(n1 * m). Usually runs much faster. MUCH
3

→ FASTER!!!

 4
     const int N = 305;
     vector < int > g[N]; // Stores edges from left half to right.
     bool used[N]; // Stores if vertex from left half is used.
     int\ mt[N];\ /\!/\ For\ every\ vertex\ in\ right\ half,\ stores\ to\ which

→ vertex in left half it's matched (-1 if not matched).

     bool try_dfs(int v){
11
12
       if (used[v]) return false;
       used[v] = 1;
13
       for (auto u : g[v]){
14
         if (mt[u] == -1 || try_dfs(mt[u])){
           mt[u] = v;
16
           return true;
17
         }
18
       }
19
20
       return false;
21
23
     int main(){
       for (int i = 1; i <= n2; i++) mt[i] = -1;
25
       for (int i = 1; i <= n1; i++) used[i] = 0;
26
       for (int i = 1; i <= n1; i++){
27
         if (try_dfs(i)){
28
29
           for (int j = 1; j <= n1; j++) used[j] = 0;</pre>
         }
30
31
32
       vector<pair<int, int>> ans;
33
       for (int i = 1; i <= n2; i++){
         if (mt[i] != -1) ans.pb({mt[i], i});
       }
35
    }
36
37
    // Finding maximal independent set: size = # of nodes - # of
38
      \leftrightarrow edges in matching.
     // To construct: launch Kuhn-like DFS from unmatched nodes in
39
      \hookrightarrow the left half.
    // Independent set = visited nodes in left half + unvisited in
      \hookrightarrow right half.
    // Finding minimal vertex cover: complement of maximal
      \hookrightarrow independent set.
```

Hungarian algorithm for Assignment Problem

• Given a 1-indexed $(n \times m)$ matrix A, select a number in each row such that each column has at most 1 number selected, and the sum of the selected numbers is minimized.

```
int INF = 1e9; // constant greater than any number in the
      \hookrightarrow matrix
     \label{eq:vector} \texttt{vector} < \texttt{int} > \ \texttt{u(n+1)} \,, \ \ \texttt{v(m+1)} \,, \ \ \texttt{p(m+1)} \,, \ \ \texttt{way(m+1)} \,;
     for (int i=1; i<=n; ++i) {
         p[0] = i;
          int j0 = 0;
          vector<int> minv (m+1, INF);
          vector<bool> used (m+1, false);
              used[j0] = true;
 9
              int i0 = p[j0], delta = INF, j1;
10
11
              for (int j=1; j<=m; ++j)
12
                   if (!used[j]) {
                        int cur = A[i0][j]-u[i0]-v[j];
13
                        if (cur < minv[j])</pre>
14
                             minv[j] = cur, way[j] = j0;
                        if (minv[j] < delta)</pre>
16
                             delta = minv[j], j1 = j;
17
                   }
              for (int j=0; j \le m; ++j)
19
                   if (used[j])
                        u[p[j]] += delta, v[j] -= delta;
21
22
                        minv[j] -= delta;
23
              j0 = j1;
24
          } while (p[j0] != 0);
25
26
          do {
              int j1 = way[j0];
27
              p[j0] = p[j1];
28
29
              j0 = j1;
          } while (j0);
30
     }
31
     vector<int> ans (n+1); // ans[i] stores the column selected
32

    for row i

     for (int j=1; j<=m; ++j)
33
          ans[p[j]] = j;
     int cost = -v[0]; // the total cost of the matching
```

Dijkstra's Algorithm

```
priority_queue<pair<11, 11>, vector<pair<11, 11>>,

    greater<pair<11, 11>>> q;

    dist[start] = 0;
    q.push({0, start});
    while (!q.empty()){
        auto [d, v] = q.top();
6
         q.pop();
         if (d != dist[v]) continue;
         for (auto [u, w] : g[v]){
           if (dist[u] > dist[v] + w){
            dist[u] = dist[v] + w;
10
            q.push({dist[u], u});
11
12
13
```

Eulerian Cycle DFS

```
void dfs(int v){
while (!g[v].empty()){
    int u = g[v].back();
    g[v].pop_back();
    dfs(u);
    ans.pb(v);
}
```

}

SCC and 2-SAT

```
void scc(vector<vector<int>>& g, int* idx) {
      int n = g.size(), ct = 0;
       int out[n];
      vector<int> ginv[n];
      memset(out, -1, sizeof out);
      memset(idx, -1, n * sizeof(int));
      function<void(int)> dfs = [&](int cur) {
         out[cur] = INT_MAX;
         for(int v : g[cur]) {
9
           ginv[v].push_back(cur);
           if(out[v] == -1) dfs(v);
11
12
13
         ct++; out[cur] = ct;
      };
14
       vector<int> order;
      for(int i = 0; i < n; i++) {</pre>
16
17
         order.push_back(i);
18
         if(out[i] == -1) dfs(i);
19
       sort(order.begin(), order.end(), [&](int& u, int& v) {
        return out[u] > out[v];
21
22
      ct = 0;
23
24
      stack<int> s;
       auto dfs2 = [&](int start) {
25
         s.push(start):
26
         while(!s.empty()) {
27
          int cur = s.top();
28
           s.pop();
29
           idx[cur] = ct;
30
31
           for(int v : ginv[cur])
             if(idx[v] == -1) s.push(v);
32
        }
33
      };
34
      for(int v : order) {
35
         if(idx[v] == -1) {
36
37
           dfs2(v);
           ct++;
38
         }
      }
40
    }
41
42
    // 0 => impossible, 1 => possible
43
    pair<int, vector<int>> sat2(int n, vector<pair<int,int>>&

    clauses) {
      vector<int> ans(n);
45
      vector<vector<int>>> g(2*n + 1);
46
      for(auto [x, y] : clauses) {
47
         x = x < 0 ? -x + n : x;
         y = y < 0 ? -y + n : y;
49
         int nx = x <= n ? x + n : x - n;</pre>
50
         int ny = y <= n ? y + n : y - n;</pre>
51
         g[nx].push_back(y);
52
53
         g[ny].push_back(x);
54
55
       int idx[2*n + 1];
      scc(g, idx);
56
       for(int i = 1; i <= n; i++) {
57
         if(idx[i] == idx[i + n]) return {0, {}};
58
         ans[i - 1] = idx[i + n] < idx[i];
59
60
      return {1, ans};
61
```

Finding Bridges

```
int tin[N], fup[N], timer;
    map<pair<int, int>, bool> is_bridge;
10
11
                                                                           Centroid Decomposition
    void dfs(int v, int p){
12
      tin[v] = ++timer;
                                                                          vector<char> res(n), seen(n), sz(n);
      fup[v] = tin[v];
14
                                                                          function<int(int, int)> get_size = [&](int node, int fa) {
15
      for (auto u : g[v]){
                                                                             sz[node] = 1:
        if (!tin[u]){
16
                                                                             for (auto& ne : g[node]) {
          dfs(u, v);
17
                                                                              if (ne == fa || seen[ne]) continue;
          if (fup[u] > tin[v]){
                                                                               sz[node] += get_size(ne, node);
            is_bridge[{u, v}] = is_bridge[{v, u}] = true;
19
20
                                                                            return sz[node]:
21
          fup[v] = min(fup[v], fup[u]);
22
                                                                          function<int(int, int, int)> find_centroid = [&](int node, int
                                                                      10
        else{
23

  fa, int t) {
          if (u != p) fup[v] = min(fup[v], tin[u]);
24
                                                                      11
                                                                             for (auto& ne : g[node])
25
                                                                               if (ne != fa && !seen[ne] && sz[ne] > t / 2) return
                                                                      12
      }
26

    find_centroid(ne, node, t);

   }
27
                                                                      13
                                                                            return node:
                                                                      14
                                                                          function < void(int, char) > solve = [\&](int node, char cur) \{
    Virtual Tree
                                                                      15
                                                                            get_size(node, -1); auto c = find_centroid(node, -1,
                                                                      16
    // order stores the nodes in the queried set

    sz[node]);
                                                                             seen[c] = 1, res[c] = cur;
    sort(all(order), [&] (int u, int v){return tin[u] < tin[v];});</pre>
    int m = sz(order);
                                                                             for (auto& ne : g[c]) {
                                                                               if (seen[ne]) continue;
    for (int i = 1; i < m; i++){
                                                                               solve(ne, char(cur + 1)); // we can pass c here to build
        order.pb(lca(order[i], order[i - 1]));
6
                                                                            }
    sort(all(order), [&] (int u, int v){return tin[u] < tin[v];});</pre>
                                                                          };
    order.erase(unique(all(order)), order.end());
    vector<int> stk{order[0]};
    for (int i = 1; i < sz(order); i++){</pre>
10
        int v = order[i];
                                                                           Math
11
        while (tout[stk.back()] < tout[v]) stk.pop_back();</pre>
12
        int u = stk.back():
13
                                                                           Binary exponentiation
        vg[u].pb({v, dep[v] - dep[u]});
15
        stk.pb(v);
                                                                          ll power(ll a, ll b){
    }
16
                                                                            ll res = 1;
                                                                       2
                                                                            for (; b; a = a * a \% MOD, b >>= 1){
                                                                              if (b & 1) res = res * a % MOD;
    HLD on Edges DFS
                                                                       6
                                                                             return res;
    void dfs1(int v, int p, int d){
      par[v] = p;
      for (auto e : g[v]){
        if (e.fi == p){
                                                                           Extended Euclidean Algorithm
          g[v].erase(find(all(g[v]), e));
          break:
                                                                          // gives (x, y) for ax + by = g
        }
                                                                          // solutions given (x0, y0): a(x0 + kb/g) + b(y0 - ka/g) = g
      }
                                                                           int gcd(int a, int b, int& x, int& y) {
9
      dep[v] = d;
                                                                            x = 1, y = 0; int sum1 = a;
      sz[v] = 1;
10
                                                                             int x2 = 0, y2 = 1, sum2 = b;
      for (auto [u, c] : g[v]){
11
                                                                             while (sum2) {
        dfs1(u, v, d + 1);
                                                                              int q = sum1 / sum2;
        sz[v] += sz[u]:
13
                                                                               tie(x, x2) = make_tuple(x2, x - q * x2);
14
                                                                              tie(y, y2) = make_tuple(y2, y - q * y2);
                                                                       9
      if (!g[v].empty()) iter_swap(g[v].begin(),
15
                                                                               tie(sum1, sum2) = make_tuple(sum2, sum1 - q * sum2);
                                                                      10

→ max_element(all(g[v]), comp));
                                                                      11
    }
16
                                                                      12
                                                                            return sum1;
    void dfs2(int v, int rt, int c){
17
                                                                          }
                                                                      13
      pos[v] = sz(a);
18
      a.pb(c);
19
      root[v] = rt;
20
                                                                           Linear Sieve
21
      for (int i = 0; i < sz(g[v]); i++){
        auto [u, c] = g[v][i];
22
                                                                             • Mobius Function
        if (!i) dfs2(u, rt, c);
23
        else dfs2(u, u, c);
                                                                          vector<int> prime;
24
                                                                           bool is_composite[MAX_N];
25
    }
26
                                                                          int mu[MAX_N];
    int getans(int u, int v){
27
28
      int res = 0:
                                                                           void sieve(int n){
      for (; root[u] != root[v]; v = par[root[v]]){
                                                                            fill(is_composite, is_composite + n, 0);
29
30
        if (dep[root[u]] > dep[root[v]]) swap(u, v);
                                                                            for (int i = 2; i < n; i++){
        res = max(res, rmq(0, 0, n - 1, pos[root[v]], pos[v]));
31
32
                                                                               if (!is_composite[i]){
      if (pos[u] > pos[v]) swap(u, v);
                                                                                 prime.push_back(i);
33
                                                                      10
      return max(res, rmq(0, 0, n - 1, pos[u] + 1, pos[v]));
                                                                                mu[i] = -1; //i is prime
```

} 35

```
for (int j = 0; j < prime.size() && i * prime[j] < n; j++){
                                                                                for (int row = h - 1; row >= 0; row--) {
13
                                                                         34
         is_composite[i * prime[j]] = true;
                                                                                  for (int c = 0; c < limit; c++) {</pre>
14
         if (i % prime[j] == 0){
                                                                                    if (!is_0(a[row][c])) {
15
                                                                         36
           mu[i * prime[j]] = 0; //prime[j] divides i
                                                                                      T inv_a = 1 / a[row][c];
                                                                                      for (int i = row - 1; i >= 0; i--) {
           break:
17
                                                                         38
                                                                                        if (is_0(a[i][c])) continue;
18
                                                                         39
           mu[i * prime[j]] = -mu[i]; //prime[j] does not divide i
                                                                                        T coeff = -a[i][c] * inv_a;
19
                                                                         40
                                                                                        for (int j = c; j < w; j++) a[i][j] += coeff *
20
                                                                         41

    a[row][j];

21
      }
22
                                                                         42
                                                                                      }
    }
                                                                         43
                                                                                      break;
                                                                         44
                                                                                    7

    Euler's Totient Function

                                                                                  }
                                                                         45
                                                                                } // not-free variables: only it on its line
                                                                         46
    vector<int> prime;
                                                                                for(int i = r; i < h; i++) if(!is_0(a[i][limit])) return 0;</pre>
                                                                         47
    bool is_composite[MAX_N];
                                                                         48
                                                                                return (r == limit) ? 1 : -1;
    int phi[MAX_N];
                                                                         49
                                                                         50
    void sieve(int n){
                                                                              template <typename T>
                                                                         51
      fill(is_composite, is_composite + n, 0);
                                                                              pair<int, vector<T>> solve_linear(vector<vector<T>> a, const
                                                                         52
      phi[1] = 1;
                                                                              \hookrightarrow vector<T> &b, int w) {
      for (int i = 2; i < n; i++){
                                                                                int h = (int)a.size():
                                                                         53
         if (!is_composite[i]){
                                                                                for (int i = 0; i < h; i++) a[i].push_back(b[i]);</pre>
           prime.push_back (i);
10
                                                                                int sol = gaussian_elimination(a, w);
                                                                         55
           phi[i] = i - 1; //i is prime
11
                                                                                if(!sol) return {0, vector<T>()};
                                                                         56
12
                                                                                vector<T> x(w, 0);
      for (int j = 0; j < prime.size () && i * prime[j] < n; j++){
13
                                                                                for (int i = 0; i < h; i++) {
         is_composite[i * prime[j]] = true;
14
                                                                                  for (int j = 0; j < w; j++) {
         if (i % prime[j] == 0){
15
                                                                         60
                                                                                    if (!is_0(a[i][j])) {
          phi[i * prime[j]] = phi[i] * prime[j]; //prime[j]
16
                                                                                      x[j] = a[i][w] / a[i][j];
                                                                         61
     \hookrightarrow divides i
                                                                                      break;
                                                                         62
          break;
17
                                                                         63
18
                                                                                  }
19
           phi[i * prime[j]] = phi[i] * phi[prime[j]]; //prime[j]
                                                                                }
                                                                         65
         does not divide i
                                                                         66
                                                                                return {sol, x};
20
           }
                                                                         67
21
22
      }
    }
                                                                              NTT
23
                                                                             void ntt(vector<ll>& a, int f) {
     Gaussian Elimination
                                                                               int n = int(a.size());
                                                                               vector<ll> w(n);
    bool is_0(Z v) { return v.x == 0; }
                                                                                vector<int> rev(n);
    Z abs(Z v) { return v; }
                                                                                for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] / 2) | ((i
    bool is_0(double v) { return abs(v) < 1e-9; }</pre>
3
                                                                               \leftrightarrow & 1) * (n / 2));
                                                                                for (int i = 0; i < n; i++) {
                                                                                  if (i < rev[i]) swap(a[i], a[rev[i]]);</pre>
    // 1 => unique solution, 0 => no solution, -1 => multiple
     \hookrightarrow solutions
                                                                                11 wn = power(f ? (MOD + 1) / 3 : 3, (MOD - 1) / n);
    template <typename T>
     int gaussian_elimination(vector<vector<T>>> &a, int limit) {
                                                                                w[0] = 1;
                                                                         10
         if (a.empty() || a[0].empty()) return -1;
                                                                                for (int i = 1; i < n; i++) w[i] = w[i - 1] * wn % MOD;
                                                                         11
       int h = (int)a.size(), w = (int)a[0].size(), r = 0;
                                                                                for (int mid = 1; mid < n; mid *= 2) {</pre>
      for (int c = 0; c < limit; c++) {</pre>
                                                                                  for (int i = 0; i < n; i += 2 * mid) {
         int id = -1;
                                                                                    for (int j = 0; j < mid; j++) {
11
                                                                         14
12
         for (int i = r; i < h; i++) {
                                                                                      11 x = a[i + j], y = a[i + j + mid] * w[n / (2 * mid)
           if (!is_0(a[i][c]) && (id == -1 || abs(a[id][c]) <
                                                                                 * j] % MOD;
        abs(a[i][c]))) {
                                                                                      a[i + j] = (x + y) \% MOD, a[i + j + mid] = (x + MOD - i)
                                                                         16
                                                                                  y) % MOD;
14
             id = i;
           }
                                                                                    }
15
                                                                         17
         }
                                                                                  }
16
                                                                         18
         if (id == -1) continue;
                                                                                }
17
                                                                         19
         if (id > r) {
                                                                                if (f) {
                                                                         20
18
19
           swap(a[r], a[id]);
                                                                         21
                                                                                  ll iv = power(n, MOD - 2);
           for (int j = c; j < w; j++) a[id][j] = -a[id][j];
                                                                         22
                                                                                  for (auto& x : a) x = x * iv % MOD;
20
21
                                                                         23
                                                                              }
22
         vector<int> nonzero;
                                                                         24
                                                                              vector<ll> mul(vector<ll> a, vector<ll> b) {
         for (int j = c; j < w; j++) {
23
                                                                         25
           if (!is_0(a[r][j])) nonzero.push_back(j);
                                                                                int n = 1, m = (int)a.size() + (int)b.size() - 1;
24
                                                                         26
                                                                                while (n < m) n *= 2;
25
                                                                         27
26
         T inv_a = 1 / a[r][c];
                                                                         28
                                                                                a.resize(n), b.resize(n);
         for (int i = r + 1; i < h; i++) {
                                                                                ntt(a, 0), ntt(b, 0); // if squaring, you can save one NTT
27
                                                                         29
28
           if (is_0(a[i][c])) continue;
           T coeff = -a[i][c] * inv_a;
                                                                                for (int i = 0; i < n; i++) a[i] = a[i] * b[i] % MOD;</pre>
29
                                                                         30
           for (int j : nonzero) a[i][j] += coeff * a[r][j];
                                                                                ntt(a, 1);
30
                                                                         31
         }
                                                                                a.resize(m):
31
                                                                         32
```

12

++r:

32

33

return a:

```
FFT
    const ld PI = acosl(-1);
    auto mul = [&](const vector<ld>& aa, const vector<ld>& bb) {
      int n = (int)aa.size(), m = (int)bb.size(), bit = 1;
      while ((1 << bit) < n + m - 1) bit++;
      int len = 1 << bit;</pre>
      vector<complex<ld>>> a(len), b(len);
                                                                        11
      vector<int> rev(len);
      for (int i = 0; i < n; i++) a[i].real(aa[i]);</pre>
                                                                        13
      for (int i = 0; i < m; i++) b[i].real(bb[i]);</pre>
      for (int i = 0; i < len; i++) rev[i] = (rev[i >> 1] >> 1) |
     auto fft = [&](vector<complex<ld>>& p, int inv) {
11
        for (int i = 0; i < len; i++)</pre>
12
                                                                        18
13
           if (i < rev[i]) swap(p[i], p[rev[i]]);</pre>
                                                                        19
         for (int mid = 1; mid < len; mid *= 2) {
14
                                                                        20
          auto w1 = complex<ld>(cos(PI / mid), (inv ? -1 : 1) *
15
                                                                        21

    sin(PI / mid));
                                                                        22
          for (int i = 0; i < len; i += mid * 2) {
16
                                                                        23
             auto wk = complex<ld>(1, 0);
17
             for (int j = 0; j < mid; j++, wk = wk * w1) {
18
                                                                        25
               auto x = p[i + j], y = wk * p[i + j + mid];
19
                                                                        26
               p[i + j] = x + y, p[i + j + mid] = x - y;
20
                                                                        27
21
                                                                        28
23
                                                                        30
         if (inv == 1) {
24
                                                                        31
          for (int i = 0; i < len; i++) p[i].real(p[i].real() /</pre>
25
     → len):
                                                                        33
        }
26
                                                                        34
      };
27
      fft(a, 0), fft(b, 0);
29
      for (int i = 0; i < len; i++) a[i] = a[i] * b[i];
      fft(a, 1);
30
31
      a.resize(n + m - 1);
      vector < ld > res(n + m - 1);
32
      for (int i = 0; i < n + m - 1; i++) res[i] = a[i].real();
34
      return res;
35
    is_prime
       • (Miller-Rabin primality test)
    typedef __int128_t i128;
    i128 power(i128 a, i128 b, i128 MOD = 1, i128 res = 1) {
                                                                        10
      for (; b; b /= 2, (a *= a) %= MOD)
                                                                        11
         if (b & 1) (res *= a) \%= MOD;
5
                                                                        12
      return res;
                                                                        13
                                                                        14
                                                                        15
9
    bool is_prime(ll n) {
      if (n < 2) return false;
10
      static constexpr int A[] = {2, 3, 5, 7, 11, 13, 17, 19, 23};
11
      int s = __builtin_ctzll(n - 1);
12
      11 d = (n - 1) >> s;
                                                                        20
      for (auto a : A) {
14
                                                                        21
         if (a == n) return true;
15
        11 x = (11)power(a, d, n);
16
        if (x == 1 \mid \mid x == n - 1) continue;
17
                                                                        23
         bool ok = false;
                                                                        24
         for (int i = 0; i < s - 1; ++i) {
19
           x = 11((i128)x * x % n); // potential overflow!
20
                                                                        26
          if (x == n - 1) {
21
             ok = true;
22
                                                                        28
             break;
23
                                                                        29
24
                                                                        30
25
        if (!ok) return false;
26
27
28
      return true;
```

}

```
typedef __int128_t i128;
    ll pollard_rho(ll x) {
      11 s = 0, t = 0, c = rng() % (x - 1) + 1;
      ll stp = 0, goal = 1, val = 1;
      for (goal = 1;; goal *= 2, s = t, val = 1) {
         for (stp = 1; stp <= goal; ++stp) {</pre>
          t = 11(((i128)t * t + c) \% x);
           val = 11((i128)val * abs(t - s) % x);
           if ((stp \% 127) == 0) {
             11 d = gcd(val, x);
             if (d > 1) return d;
           7
         11 d = gcd(val, x);
         if (d > 1) return d;
16
    }
    11 get_max_factor(ll _x) {
      11 max_factor = 0;
      function \langle void(11) \rangle fac = [&](11 x) {
         if (x \le max_factor || x \le 2) return;
         if (is_prime(x)) {
           max_factor = max_factor > x ? max_factor : x;
         11 p = x;
         while (p >= x) p = pollard_rho(x);
         while ((x \% p) == 0) x /= p;
         fac(x), fac(p);
      fac(x);
       return max_factor;
35
```

Data Structures

Sparse Table

17

5

```
const int N = 2e5 + 10, LOG = 20; // Change the constant!
template<typename T>
struct SparseTable{
int lg[N];
T st[N][LOG];
// Change this function
function\langle T(T, T) \rangle f = [\&] (T a, T b) \{
  return min(a, b);
void build(vector<T>& a){
 n = sz(a);
  lg[1] = 0;
  for (int i = 2; i \le n; i++) lg[i] = lg[i / 2] + 1;
  for (int k = 0; k < LOG; k++){
    for (int i = 0; i < n; i++){
      if (!k) st[i][k] = a[i];
      else st[i][k] = f(st[i][k-1], st[min(n-1, i+(1 << 
    (k - 1)))][k - 1]);
  }
T query(int 1, int r){
  int sz = r - 1 + 1;
  return f(st[1][lg[sz]], st[r - (1 << lg[sz]) + 1][lg[sz]]);
};
```

Suffix Array and LCP array

• (uses SparseTable above)

```
struct SuffixArray{
  vector<int> p, c, h;
  SparseTable<int> st;
  In the end, array c gives the position of each suffix in p
  using 1-based indexation!
  SuffixArray() {}
  SuffixArray(string s){
    buildArray(s);
    buildLCP(s);
    buildSparse();
  }
  void buildArray(string s){
    int n = sz(s) + 1;
    p.resize(n), c.resize(n);
    for (int i = 0; i < n; i++) p[i] = i;
    sort(all(p), [&] (int a, int b){return s[a] < s[b];});</pre>
    c[p[0]] = 0;
    for (int i = 1; i < n; i++){
      c[p[i]] = c[p[i-1]] + (s[p[i]] != s[p[i-1]]);
    vector<int> p2(n), c2(n);
    // w is half-length of each string.
    for (int w = 1; w < n; w <<= 1){
      for (int i = 0; i < n; i++){
        p2[i] = (p[i] - w + n) \% n;
      vector<int> cnt(n);
      for (auto i : c) cnt[i]++;
      for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
      for (int i = n - 1; i \ge 0; i--){
        p[--cnt[c[p2[i]]]] = p2[i];
      c2[p[0]] = 0;
      for (int i = 1; i < n; i++){
        c2[p[i]] = c2[p[i - 1]] +
        (c[p[i]] != c[p[i - 1]] ||
        c[(p[i] + w) \% n] != c[(p[i - 1] + w) \% n]);
      c.swap(c2);
    p.erase(p.begin());
  void buildLCP(string s){
    // The algorithm assumes that suffix array is already
   built on the same string.
    int n = sz(s);
    h.resize(n - 1);
    int k = 0;
    for (int i = 0; i < n; i++){
      if (c[i] == n){
        k = 0;
        continue;
      }
      int j = p[c[i]];
      while (i + k < n \&\& j + k < n \&\& s[i + k] == s[j + k])
     h[c[i] - 1] = k;
      if (k) k--;
    }
    Then an RMQ Sparse Table can be built on array h
    to calculate LCP of 2 non-consecutive suffixes.
  }
  void buildSparse(){
    st.build(h);
  // l and r must be in O-BASED INDEXATION
  int lcp(int 1, int r){
```

9

11

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72

73

74

```
1 = c[1] - 1, r = c[r] - 1;
76
        if (1 > r) swap(1, r);
        return st.query(1, r - 1);
    };
```

77

78

79

Aho Corasick Trie

• For each node in the trie, the suffix link points to the longest proper suffix of the represented string. terminal-link tree has square-root height (can be constructed by DFS).

```
const int S = 26;
2
3
    // Function converting char to int.
    int ctoi(char c){
      return c - 'a';
    // To add terminal links, use DFS
    struct Node{
      vector<int> nxt;
10
      int link;
      bool terminal;
12
13
      Node() {
14
        nxt.assign(S, -1), link = 0, terminal = 0;
15
      }
    };
17
18
    vector<Node> trie(1);
19
20
    // add_string returns the terminal vertex.
21
    int add_string(string& s){
22
      int v = 0;
23
      for (auto c : s){
24
         int cur = ctoi(c);
25
         if (trie[v].nxt[cur] == -1){
26
           trie[v].nxt[cur] = sz(trie);
27
           trie.emplace_back();
28
29
         v = trie[v].nxt[cur];
30
      }
31
32
      trie[v].terminal = 1;
33
      return v;
    }
34
36
    Suffix links are compressed.
37
     This means that:
38
      If vertex v has a child by letter x, then:
39
         trie[v].nxt[x] points to that child.
40
       If vertex v doesn't have such child, then:
41
42
         trie[v].nxt[x] points to the suffix link of that child
43
         if we would actually have it.
44
    void add_links(){
45
      queue<int> q;
46
       q.push(0);
48
       while (!q.empty()){
         auto v = q.front();
49
50
         int u = trie[v].link;
         q.pop();
51
         for (int i = 0; i < S; i++){
          int& ch = trie[v].nxt[i];
53
           if (ch == -1){
54
             ch = v? trie[u].nxt[i] : 0;
55
56
57
           else{
             trie[ch].link = v? trie[u].nxt[i] : 0;
58
             q.push(ch);
59
60
61
      }
62
    }
63
```

```
64
    bool is_terminal(int v){
65
66
      return trie[v].terminal;
67
    int get_link(int v){
69
70
      return trie[v].link;
71
72
    int go(int v, char c){
      return trie[v].nxt[ctoi(c)];
74
```

Convex Hull Trick

- Allows to insert a linear function to the hull in (1) and get the minimum/maximum value of the stored function at a point in $O(\log n)$.
- NOTE: The lines must be added in the order of decreasing/increasing gradients. CAREFULLY CHECK THE SETUP BEFORE USING!
- IMPORTANT: THE DEFAULT VERSION SURELY WORKS. IF MODIFIED VERSIONS DON'T WORK, TRY TRANSFORMING THEM TO THE DEFAULT ONE BY CHANGING SIGNS.

```
struct line{
                                                                         31
                                                                         32
      ll k, b;
2
3
      11 f(11 x){
                                                                         33
4
         return k * x + b;
                                                                         34
      };
5
    }:
                                                                         36
6
                                                                         37
                                                                         38
8
    vector<line> hull;
                                                                         39
9
10
    void add_line(line nl){
      if (!hull.empty() && hull.back().k == nl.k){
                                                                         41
11
         nl.b = min(nl.b, hull.back().b); // Default: minimum. For
12
         maximum change "min" to "max".
                                                                         43
13
        hull.pop_back();
      }
14
      while (sz(hull) > 1){
15
         auto& 11 = hull.end()[-2], 12 = hull.back();
16
         if ((nl.b - l1.b) * (l2.k - nl.k) >= (nl.b - l2.b) * (l1.k)
                                                                         46
        - nl.k)) hull.pop_back(); // Default: decreasing gradient
        k. For increasing k change the sign to <=.
18
         else break;
19
      hull.pb(nl);
                                                                         50
20
    }
21
                                                                         52
    11 get(11 x){
23
      int 1 = 0, r = sz(hull);
24
      while (r - 1 > 1){
25
         int mid = (1 + r) / 2;
26
         if (hull[mid - 1].f(x) >= hull[mid].f(x)) 1 = mid; //
27
        Default: minimum. For maximum change the sign to <=.
         else r = mid;
28
29
      }
                                                                         58
      return hull[1].f(x);
30
                                                                         61
```

Li-Chao Segment Tree

- allows to add linear functions in any order and query minimum/maximum value of those at a point, all in O(log n).
- Clear: clear()

```
const 11 INF = 1e18; // Change the constant!
   struct LiChaoTree{
2
     struct line{
       11 k. b:
```

```
k = b = 0;
    line(ll k_, ll b_){
     k = k_{-}, b = b_{-};
    ን:
    11 f(11 x){
     return k * x + b;
  };
  int n:
  bool minimum, on_points;
  vector<ll> pts;
  vector<line> t;
  void clear(){
    for (auto& 1 : t) 1.k = 0, 1.b = minimum? INF : -INF;
 LiChaoTree(int n_, bool min_){ // This is a default
\leftrightarrow constructor for numbers in range [0, n - 1].
    n = n_, minimum = min_, on_points = false;
    t.resize(4 * n);
  };
 LiChaoTree(vector<ll> pts_, bool min_){ // This constructor
\leftrightarrow will build LCT on the set of points you pass. The points
→ may be in any order and contain duplicates.
    pts = pts_, minimum = min_;
    sort(all(pts));
    pts.erase(unique(all(pts)), pts.end());
    on_points = true;
    n = sz(pts);
   t.resize(4 * n):
    clear();
  void add_line(int v, int l, int r, line nl){
    // Adding on segment [l, r)
    int m = (1 + r) / 2;
    11 lval = on_points? pts[1] : 1, mval = on_points? pts[m]
    \rightarrow nl.f(mval) > t[v].f(mval))) swap(t[v], nl);
    if (r - l == 1) return;
    if ((minimum && nl.f(lval) < t[v].f(lval)) || (!minimum &&
 \rightarrow nl.f(lval) > t[v].f(lval))) add_line(2 * v + 1, 1, m, nl);
    else add_line(2 * v + 2, m, r, nl);
  11 get(int v, int l, int r, int x){
    int m = (1 + r) / 2;
    if (r - 1 == 1) return t[v].f(on\_points? pts[x] : x);
      if (minimum) return min(t[v].f(on_points? pts[x] : x), x
\leftrightarrow < m? get(2 * v + 1, 1, m, x) : get(2 * v + 2, m, r, x));
      else return max(t[v].f(on\_points? pts[x] : x), x < m?
    get(2 * v + 1, 1, m, x) : get(2 * v + 2, m, r, x));
 }
  void add_line(ll k, ll b){
    add_line(0, 0, n, line(k, b));
  11 get(11 x){
   return get(0, 0, n, on_points? lower_bound(all(pts), x) -

    pts.begin() : x);
 }; // Always pass the actual value of x, even if LCT is on

→ points.

};
```

Persistent Segment Tree

• for RSQ

line(){

5

6

7

10

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```
struct Node {
        ll val;
2
        Node *1. *r:
        Node(ll x) : val(x), l(nullptr), r(nullptr) {}
        Node(Node *11, Node *rr) {
6
            1 = 11, r = rr;
             val = 0;
             if (1) val += 1->val;
             if (r) val += r->val;
11
         Node(Node *cp) : val(cp->val), 1(cp->1), r(cp->r) {}
12
13
    };
    const int N = 2e5 + 20;
14
    ll a[N];
    Node *roots[N];
16
    int n, cnt = 1;
    Node *build(int l = 1, int r = n) {
18
         if (1 == r) return new Node(a[1]);
19
20
         int mid = (1 + r) / 2;
         return new Node(build(1, mid), build(mid + 1, r));
21
    }
22
    Node *update(Node *node, int val, int pos, int l = 1, int r =
23
     \hookrightarrow n) {
        if (l == r) return new Node(val);
24
         int mid = (1 + r) / 2;
25
        if (pos > mid)
26
            return new Node(node->1, update(node->r, val, pos, mid
27
        + 1, r));
        else return new Node(update(node->1, val, pos, 1, mid),
28
        node->r);
    }
29
    ll query(Node *node, int a, int b, int l = 1, int r = n) {
30
31
        if (1 > b || r < a) return 0;
        if (1 \ge a \&\& r \le b) return node->val;
32
         int mid = (1 + r) / 2;
33
        return query(node->1, a, b, 1, mid) + query(node->r, a, b,
34
        mid + 1, r);
    }
```

Miscellaneous

Ordered Set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;
```

Measuring Execution Time

```
1  ld tic = clock();
2  // execute algo...
3  ld tac = clock();
4  // Time in milliseconds
5  cerr << (tac - tic) / CLOCKS_PER_SEC * 1000 << endl;
6  // No need to comment out the print because it's done to cerr.</pre>
```

Setting Fixed D.P. Precision

```
cout << setprecision(d) << fixed;
// Each number is rounded to d digits after the decimal point,
and truncated.</pre>
```

Common Bugs and General Advice

- Check overflow, array bounds
- Check variable overloading
- Check special cases (n=1?)
- Do something instead of nothing, stay organized
- Write stuff down!
- Don't get stuck on one approach!