CU-Later Code Library

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Templates Contents **Templates** 1 Ken's template #include <bits/stdc++.h> 1 using namespace std; Kevin's Template Extended #define all(v) (v).begin(), (v).end()typedef long long 11; typedef long double ld; Geometry 1 #define pb push_back $\#define \ sz(x) \ (int)(x).size()$ 3 Strings #define fi first 3 #define se second #define endl '\n' 4 $O(N^2M)$, on unit networks $O(N^{1/2}M)$ 4 Kevin's template MCMF - maximize flow, then minimize its cost. // paste Kaurov's Template, minus last line 4 typedef vector<int> vi; typedef vector<ll> vll; Graphs 5 typedef pair<int, int> pii; Kuhn's algorithm for bipartite matching 5 typedef pair<11, 11> pl1; Hungarian algorithm for Assignment Problem . . . 6 typedef pair<double, double> pdd; const ld PI = acosl(-1); 6 const 11 mod7 = 1e9 + 7; 6 const 11 mod9 = 998244353; 6 10 const 11 INF = 2*1024*1024*1023; 7 const char nl = '\n'; 11 #define form(i, n) for (int i = 0; i < int(n); i++) ll k, n, m, u, v, w; 7 string s, t; Centroid Decomposition bool multiTest = 1: 16 7 void solve(int tt){ Math 17 18 7 Extended Euclidean Algorithm 7 20 int main(){ 8 21 ios::sync_with_stdio(0);cin.tie(0);cout.tie(0); cout<<fixed<< setprecision(14);</pre> Gaussian Elimination 8 22 23 8 int t = 1;9 25 if (multiTest) cin >> t; 9 26 forn(ii, t) solve(ii); 27 **Data Structures** 9 9 Kevin's Template Extended Lazy Propagation SegTree 10 • to type after the start of the contest 10 Suffix Array and LCP array 11 #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt") 11 #include <ext/pb_ds/assoc_container.hpp> #include <ext/pb_ds/tree_policy.hpp> 12 using namespace __gnu_pbds; 12 template<class T> using ordered_set = tree<T, null_type, Persistent Segment Tree 13 less<T>, rb_tree_tag, tree_order_statistics_node_update>; $vi d4x = \{1, 0, -1, 0\};$ $vi d4y = \{0, 1, 0, -1\};$ Miscellaneous **13** vi $d8x = \{1, 0, -1, 0, 1, 1, -1, -1\};$ 13 vi d8y = $\{0, 1, 0, -1, 1, -1, 1, -1\};$ Measuring Execution Time 13 Setting Fixed D.P. Precision 13 ¬ rng(chrono::steady_clock::now().time_since_epoch().count()); Common Bugs and General Advice 13 Geometry template<typename T> struct TPoint{ Тх, у; int id; static constexpr T eps = static_cast<T>(1e-9); TPoint() : x(0), y(0), id(-1) {} TPoint(const $T\& x_-$, const $T\& y_-$) : $x(x_-)$, $y(y_-)$, id(-1) {} $\label{eq:total_total_total} TPoint(const \ T\& \ x_, \ const \ T\& \ y_, \ const \ int \ id_) \ : \ x(x_),$ \rightarrow y(y_), id(id_) {}

TPoint operator + (const TPoint& rhs) const {

```
return TPoint(x + rhs.x, y + rhs.y);
                                                                            int sign(const T& x){
11
                                                                        82
                                                                              if (abs(x) <= TPoint<T>::eps) return 0;
12
                                                                        83
13
      TPoint operator - (const TPoint& rhs) const {
                                                                        84
                                                                              return x > 0? +1 : -1;
        return TPoint(x - rhs.x, y - rhs.y);
14
                                                                        85
                                                                            template<typename T>
      TPoint operator * (const T& rhs) const {
                                                                            T area(const vector<TPoint<T>>& pts){
16
                                                                        87
17
        return TPoint(x * rhs, y * rhs);
                                                                               int n = sz(pts);
                                                                        88
                                                                              T ans = 0;
18
                                                                        89
      TPoint operator / (const T& rhs) const {
                                                                              for (int i = 0; i < n; i++){
19
                                                                        90
        return TPoint(x / rhs, y / rhs);
20
                                                                                 ans += vmul(pts[i], pts[(i + 1) % n]);
21
                                                                        92
      TPoint ort() const {
22
                                                                        93
                                                                              return abs(ans) / 2;
23
        return TPoint(-y, x);
                                                                        94
                                                                            template<typename T>
24
                                                                        95
      T abs2() const {
                                                                            T dist_pp(const TPoint<T>& a, const TPoint<T>& b){
25
        return x * x + y * y;
                                                                              return sqrt(sq(a.x - b.x) + sq(a.y - b.y));
26
                                                                        97
27
                                                                        98
28
    };
                                                                        99
                                                                            template<typename T>
    template<typename T>
                                                                            TLine<T> perp_line(const TLine<T>& 1, const TPoint<T>& p){
29
                                                                       100
    bool operator< (TPoint<T>& A, TPoint<T>& B){
                                                                              T na = -1.b, nb = 1.a, nc = -na * p.x - nb * p.y;
30
                                                                       101
      return make_pair(A.x, A.y) < make_pair(B.x, B.y);</pre>
                                                                              return TLine<T>(na, nb, nc);
31
                                                                       102
32
                                                                       103
    template<typename T>
33
                                                                       104
                                                                            template<typename T>
    bool operator== (TPoint<T>& A, TPoint<T>& B){
                                                                            TPoint<T> projection(const TPoint<T>& p, const TLine<T>& 1){
      return abs(A.x - B.x) \leftarrow TPoint < T > :: eps && abs(A.y - B.y) <= 106
                                                                              return intersection(l, perp_line(l, p));
35
        TPoint<T>::eps;
                                                                       107
    7
                                                                             template<typename T>
36
                                                                       108
    template<typename T>
                                                                            T dist_pl(const TPoint<T>& p, const TLine<T>& 1){
37
                                                                       109
    struct TLine{
                                                                               return dist_pp(p, projection(p, 1));
      T a, b, c;
39
                                                                       111
      TLine(): a(0), b(0), c(0) {}
                                                                            template<typename T>
                                                                       112
40
      TLine(const T\& a_, const T\& b_, const T\& c_) : a(a_), b(b_), 113
                                                                            struct TRay{
                                                                              TLine<T> 1;
                                                                       114
42
      TLine(const TPoint<T>& p1, const TPoint<T>& p2){
                                                                              TPoint<T> start, dirvec;
        a = p1.y - p2.y;
43
                                                                       116
                                                                              TRay() : 1(), start(), dirvec() {}
        b = p2.x - p1.x;
                                                                       117
                                                                               TRay(const TPoint<T>& p1, const TPoint<T>& p2){
44
        c = -a * p1.x - b * p1.y;
45
                                                                       118
                                                                                1 = TLine < T > (p1, p2);
                                                                                start = p1, dirvec = p2 - p1;
46
                                                                       119
                                                                              }
47
    };
    template<typename T>
48
                                                                       121
                                                                            1:
    T det(const T& a11, const T& a12, const T& a21, const T& a22){ _{122}}
49
                                                                             template<typename T>
      return a11 * a22 - a12 * a21;
                                                                            bool is_on_line(const TPoint<T>& p, const TLine<T>& 1){
50
                                                                       123
                                                                              return abs(l.a * p.x + l.b * p.y + l.c) <= TPoint<T>::eps;
51
                                                                       124
    template<typename T>
52
                                                                       125
    T sq(const T& a){
                                                                            template<typename T>
53
                                                                       126
      return a * a;
                                                                            bool is_on_ray(const TPoint<T>& p, const TRay<T>& r){
54
                                                                       127
55
                                                                       128
                                                                               if (is_on_line(p, r.l)){
                                                                                return sign(smul(r.dirvec, TPoint<T>(p - r.start))) != -1;
    template<typename T>
56
                                                                       129
57
    T smul(const TPoint<T>& a, const TPoint<T>& b){
                                                                       130
      return a.x * b.x + a.y * b.y;
                                                                               else return false;
58
                                                                       131
59
                                                                       132
    template<typename T>
                                                                            template<typename T>
60
                                                                       133
    T vmul(const TPoint<T>& a, const TPoint<T>& b){
                                                                       134
                                                                            bool is_on_seg(const TPoint<T>& P, const TPoint<T>& A, const
62
      return det(a.x, a.y, b.x, b.y);
                                                                             \hookrightarrow TPoint<T>& B){
                                                                              return is_on_ray(P, TRay<T>(A, B)) && is_on_ray(P,
63
                                                                       135
    template<typename T>
                                                                             \hookrightarrow TRay<T>(B, A));
64
    bool parallel(const TLine<T>& 11, const TLine<T>& 12){
                                                                            }
65
                                                                       136
      return abs(vmul(TPoint<T>(11.a, 11.b), TPoint<T>(12.a,
                                                                            template<typename T>
     T dist_pr(const TPoint<T>& P, const TRay<T>& R){
                                                                       138
67
                                                                       139
                                                                               auto H = projection(P, R.1);
    template<typename T>
                                                                       140
                                                                              return is_on_ray(H, R)? dist_pp(P, H) : dist_pp(P, R.start);
68
    bool equivalent(const TLine<T>& 11, const TLine<T>& 12){
69
                                                                       141
      return parallel(11, 12) &&
                                                                            template<typename T>
      abs(det(11.b, 11.c, 12.b, 12.c)) <= TPoint<T>::eps &&
                                                                            T dist_ps(const TPoint<T>& P, const TPoint<T>& A, const
71
                                                                       143

→ TPoint<T>& B){
      abs(det(11.a, 11.c, 12.a, 12.c)) <= TPoint<T>::eps;
72
                                                                              auto H = projection(P, TLine<T>(A, B));
73
                                                                       144
    template<typename T>
                                                                               if (is_on_seg(H, A, B)) return dist_pp(P, H);
74
                                                                       145
    TPoint<T> intersection(const TLine<T>& 11, const TLine<T>&
                                                                       146
                                                                               else return min(dist_pp(P, A), dist_pp(P, B));

→ 12) {

                                                                       147
      return TPoint<T>(
                                                                             template<typename T>
                                                                       148
                                                                            bool acw(const TPoint<T>& A, const TPoint<T>& B){
        det(-11.c, 11.b, -12.c, 12.b) / det(11.a, 11.b, 12.a,
                                                                       149
                                                                       150
                                                                              T \text{ mul} = vmul(A, B);
        det(11.a, -11.c, 12.a, -12.c) / det(11.a, 11.b, 12.a,
                                                                               return mul > 0 || abs(mul) <= TPoint<T>::eps;
                                                                       151
        12.b)
                                                                       152
                                                                       153
                                                                             template<typename T>
                                                                            bool cw(const TPoint<T>& A, const TPoint<T>& B){
80
                                                                       154
    template<typename T>
                                                                               T mul = vmul(A, B);
                                                                       155
```

```
return mul < 0 || abs(mul) <= TPoint<T>::eps;
                                                                              template<typename T>
156
                                                                        227
                                                                              vector<TPoint<T>> minkowski_sum(vector<TPoint<T>> P,
157
                                                                        228
158
     template<typename T>

    vector<TPoint<T>> Q){
     vector<TPoint<T>> convex_hull(vector<TPoint<T>> pts){
                                                                                minkowski_rotate(P);
159
                                                                        229
        sort(all(pts));
                                                                                minkowski_rotate(Q);
       pts.erase(unique(all(pts)), pts.end());
                                                                                P.pb(P[0]);
161
                                                                        231
162
        vector<TPoint<T>> up, down;
                                                                        232
                                                                                Q.pb(Q[0]);
       for (auto p : pts){
163
                                                                        233
                                                                                vector<TPoint<T>> ans;
         while (sz(up) > 1 \&\& acw(up.end()[-1] - up.end()[-2], p -
                                                                                int i = 0, j = 0;
                                                                        234
164
         up.end()[-2])) up.pop_back();
                                                                                while (i < sz(P) - 1 \mid | j < sz(Q) - 1){
         while (sz(down) > 1 \&\& cw(down.end()[-1] - down.end()[-2], 236
                                                                                  ans.pb(P[i] + Q[j]);
165
         p - down.end()[-2])) down.pop_back();
                                                                                  T curmul;
166
         up.pb(p), down.pb(p);
                                                                        238
                                                                                  if (i == sz(P) - 1) curmul = -1;
                                                                                  else if (j == sz(Q) - 1) curmul = +1;
167
                                                                        239
       for (int i = sz(up) - 2; i >= 1; i--) down.pb(up[i]);
                                                                                  else curmul = vmul(P[i + 1] - P[i], Q[j + 1] - Q[j]);
168
                                                                        240
                                                                                  if (abs(curmul) < TPoint<T>::eps || curmul > 0) i++;
       return down:
169
                                                                        241
170
     }
                                                                        242
                                                                                  if (abs(curmul) < TPoint<T>::eps || curmul < 0) j++;</pre>
171
                                                                        243
     template<typename T>
                                                                                return ans;
172
                                                                        244
     bool in_triangle(TPoint<T>& P, TPoint<T>& A, TPoint<T>& B,
                                                                             }
173
                                                                        245
      → TPoint<T>& C){
                                                                              using Point = TPoint<11>; using Line = TLine<11>; using Ray =
                                                                        246
       if (is_on_seg(P, A, B) || is_on_seg(P, B, C) || is_on_seg(P,
                                                                               \rightarrow TRay<11>; const ld PI = acos(-1);

⇔ C. A)) return true:

       return cw(P - A, B - A) == cw(P - B, C - B) &&
       cw(P - A, B - A) == cw(P - C, A - C);
176
                                                                              Strings
177
     template<typename T>
178
                                                                              vector<int> prefix_function(string s){
179
     void prep_convex_poly(vector<TPoint<T>>& pts){
                                                                               int n = sz(s);
       rotate(pts.begin(), min_element(all(pts)), pts.end());
180
                                                                                vector<int> pi(n);
181
                                                                                for (int i = 1; i < n; i++){
     // 0 - Outside, 1 - Exclusively Inside, 2 - On the Border
182
                                                                                  int k = pi[i - 1];
     template<typename T>
183
                                                                                  while (k > 0 \&\& s[i] != s[k]){
     int in_convex_poly(TPoint<T>& p, vector<TPoint<T>>& pts){
184
                                                                                    k = pi[k - 1];
185
        int n = sz(pts);
       if (!n) return 0;
186
                                                                                  pi[i] = k + (s[i] == s[k]);
        if (n <= 2) return is_on_seg(p, pts[0], pts.back());</pre>
187
                                                                         10
       int 1 = 1, r = n - 1;
188
                                                                         11
                                                                                return pi;
       while (r - 1 > 1){
189
                                                                             7
                                                                         12
          int mid = (1 + r) / 2;
                                                                              vector<int> kmp(string s, string k){
          if (acw(pts[mid] - pts[0], p - pts[0])) 1 = mid;
191
                                                                                string st = k + "#" + s;
                                                                         14
192
                                                                                vector<int> res:
                                                                         15
193
                                                                                auto pi = pf(st);
       if (!in_triangle(p, pts[0], pts[1], pts[1 + 1])) return 0;
194
                                                                                for (int i = 0; i < sz(st); i++){
                                                                         17
        if (is_on_seg(p, pts[l], pts[l + 1]) ||
195
                                                                                  if (pi[i] == sz(k)){
          is_on_seg(p, pts[0], pts.back()) ||
196
                                                                                    res.pb(i - 2 * sz(k));
                                                                         19
          is_on_seg(p, pts[0], pts[1])
197
                                                                         20
       ) return 2;
198
                                                                                }
                                                                         21
       return 1:
199
                                                                                return res;
                                                                         22
200
                                                                             }
     // 0 - Outside, 1 - Exclusively Inside, 2 - On the Border
201
                                                                              vector<int> z function(string s){
                                                                         24
202
     template<typename T>
                                                                                int n = sz(s);
                                                                         25
     int in_simple_poly(TPoint<T> p, vector<TPoint<T>>& pts){
203
                                                                         26
                                                                                vector<int> z(n):
204
       int n = sz(pts):
                                                                                int 1 = 0, r = 0;
                                                                         27
205
       bool res = 0;
                                                                                for (int i = 1; i < n; i++){
206
       for (int i = 0; i < n; i++){
                                                                                  if (r >= i) z[i] = min(z[i - 1], r - i + 1);
                                                                         29
          auto a = pts[i], b = pts[(i + 1) % n];
207
                                                                                  while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]]){
          if (is_on_seg(p, a, b)) return 2;
208
                                                                                    z[i]++;
          if (((a.y > p.y) - (b.y > p.y)) * vmul(b - p, a - p) >
                                                                                  }
      \  \, \neg \  \, TPoint < T > : :eps) \{
                                                                                  if (i + z[i] - 1 > r){
                                                                         33
           res ^= 1;
210
                                                                                    1 = i, r = i + z[i] - 1;
                                                                         34
211
         }
                                                                         35
212
                                                                                }
                                                                         36
       return res;
213
                                                                         37
                                                                                return z;
214
     }
     template<typename T>
215
     void minkowski_rotate(vector<TPoint<T>>& P){
216
        int pos = 0;
217
                                                                              Manacher's algorithm
218
       for (int i = 1; i < sz(P); i++){
          if (abs(P[i].y - P[pos].y) <= TPoint<T>::eps){
219
                                                                              string longest_palindrome(string& s) {
220
            if (P[i].x < P[pos].x) pos = i;
                                                                                // init "abc" -> "^$a#b#c$"
221
                                                                                vector<char> t{'^', '#'};
222
          else if (P[i].y < P[pos].y) pos = i;</pre>
                                                                                for (char c : s) t.push_back(c), t.push_back('#');
223
                                                                                t.push_back('$');
       rotate(P.begin(), P.begin() + pos, P.end());
224
                                                                                // manacher
225
                                                                                int n = t.size(), r = 0, c = 0;
     // P and Q are strictly convex, points given in
226
                                                                                vector<int> p(n, 0);
      for (int i = 1; i < n - 1; i++) {
```

```
if (i < r + c) p[i] = min(p[2 * c - i], r + c - i);
10
                                                                         56
                                                                                          return tr;
         while (t[i + p[i] + 1] == t[i - p[i] - 1]) p[i]++;
                                                                                      }
                                                                         57
11
         if (i + p[i] > r + c) r = p[i], c = i;
12
                                                                         58
                                                                                      return 0;
13
                                                                         59
         // s[i] \rightarrow p[2 * i + 2] (even), p[2 * i + 2] (odd)
                                                                                  long long flow() {
       // output answer
                                                                                      long long f = 0;
15
                                                                         61
16
       int index = 0;
                                                                         62
                                                                                      while (true) {
      for (int i = 0; i < n; i++)
                                                                                          fill(level.begin(), level.end(), -1);
17
                                                                         63
         if (p[index] < p[i]) index = i;</pre>
                                                                                          level[s] = 0;
18
                                                                         64
19
      return s.substr((index - p[index]) / 2, p[index]);
                                                                                          q.push(s);
                                                                                          if (!bfs())
20
                                                                         66
                                                                         67
                                                                         68
                                                                                          fill(ptr.begin(), ptr.end(), 0);
                                                                                          while (long long pushed = dfs(s, flow_inf)) {
                                                                         69
    Flows
                                                                                              f += pushed;
                                                                         71
                                                                         72
                                                                                      }
    O(N^2M), on unit networks O(N^{1/2}M)
                                                                         73
                                                                                      return f;
                                                                         74
    struct FlowEdge {
                                                                             };
                                                                         75
         int v, u;
                                                                             // To recover flow through original edges: iterate over even
         long long cap, flow = 0;
3
                                                                              \hookrightarrow indices in edges.
         FlowEdge(int v, int u, long long cap) : v(v), u(u),
    };
                                                                             MCMF – maximize flow, then minimize its
    struct Dinic {
                                                                             cost. O(Fmn).
         const long long flow_inf = 1e18;
8
         vector<FlowEdge> edges;
         vector<vector<int>> adj;
                                                                             #include <ext/pb_ds/priority_queue.hpp>
9
         int n, m = 0;
                                                                             template <typename T, typename C>
10
         int s, t;
                                                                             class MCMF {
         vector<int> level, ptr;
                                                                              public:
12
                                                                                 static constexpr T eps = (T) 1e-9;
13
         queue<int> q;
         Dinic(int n, int s, int t) : n(n), s(s), t(t) {
14
             adj.resize(n);
                                                                                 struct edge {
15
             level.resize(n);
                                                                                  int from:
16
             ptr.resize(n);
                                                                                   int to;
17
                                                                                  Tc;
                                                                                  T f:
         void add_edge(int v, int u, long long cap) {
19
                                                                         11
             edges.emplace_back(v, u, cap);
                                                                                  C cost;
20
                                                                         12
21
             edges.emplace_back(u, v, 0);
                                                                         13
                                                                                 };
             adj[v].push_back(m);
                                                                         14
22
             adj[u].push_back(m + 1);
23
                                                                         15
             m += 2;
                                                                                vector<vector<int>> g;
24
                                                                         16
25
                                                                         17
                                                                                 vector<edge> edges;
         bool bfs() {
                                                                                 vector<C> d:
26
                                                                         18
27
             while (!q.empty()) {
                                                                         19
                                                                                 vector<C> pot;
                 int v = q.front();
                                                                                 __gnu_pbds::priority_queue<pair<C, int>> q;
                                                                         20
                 q.pop();
                                                                                 vector<typename decltype(q)::point_iterator> its;
29
                                                                         21
                 for (int id : adj[v]) {
                                                                                 vector<int> pe;
                                                                                 const C INF_C = numeric_limits<C>::max() / 2;
                     if (edges[id].cap - edges[id].flow < 1)</pre>
31
                                                                         23
                         continue;
                                                                         24
32
                     if (level[edges[id].u] != -1)
                                                                                 explicit MCMF(int n_) : n(n_), g(n), d(n), pot(n, 0),
                                                                         25
                                                                              \hookrightarrow its(n), pe(n) {}
                         continue:
34
                     level[edges[id].u] = level[v] + 1;
                                                                         26
                                                                                 int add(int from, int to, T forward_cap, C edge_cost, T
                     q.push(edges[id].u);
36
                                                                         27
37

    backward_cap = 0) {
             }
                                                                                   \verb"assert(0 <= from && from < n && 0 <= to && to < n);
38
                                                                         28
             return level[t] != -1;
                                                                                   assert(forward_cap >= 0 && backward_cap >= 0);
                                                                         29
39
         }
                                                                                   int id = static_cast<int>(edges.size());
40
                                                                         30
         long long dfs(int v, long long pushed) {
                                                                                   g[from].push_back(id);
41
                                                                         31
             if (pushed == 0)
                                                                                   edges.push_back({from, to, forward_cap, 0, edge_cost});
42
                                                                                   g[to].push_back(id + 1);
43
                 return 0;
                                                                         33
             if (v == t)
                                                                                   edges.push_back({to, from, backward_cap, 0, -edge_cost});
                                                                         34
44
45
                 return pushed;
                                                                         35
                                                                                  return id;
             for (int& cid = ptr[v]; cid < (int)adj[v].size();</pre>
46
                                                                         36
        cid++) {
                 int id = adj[v][cid];
                                                                                 void expath(int st) {
47
                                                                         38
                 int u = edges[id].u;
                                                                                   fill(d.begin(), d.end(), INF_C);
                                                                         39
48
                 if (level[v] + 1 != level[u] || edges[id].cap -
                                                                                   q.clear();
49
                                                                         40
     ⇔ edges[id].flow < 1)</pre>
                                                                                   fill(its.begin(), its.end(), q.end());
                                                                         41
50
                     continue;
                                                                         42
                                                                                   its[st] = q.push({pot[st], st});
                 long long tr = dfs(u, min(pushed, edges[id].cap -
                                                                                   d[st] = 0:
51
                                                                         43
         edges[id].flow));
                                                                                   while (!q.empty()) {
                                                                         44
52
                 if (tr == 0)
                                                                         45
                                                                                     int i = q.top().second;
                     continue;
                                                                                     q.pop();
53
                                                                         46
                 edges[id].flow += tr;
                                                                         47
                                                                                     its[i] = q.end();
                 edges[id ^ 1].flow -= tr;
                                                                                     for (int id : g[i]) {
                                                                         48
```

```
const edge &e = edges[id];
49
                                                                            125
               int j = e.to;
50
                                                                            126
               if (e.c - e.f > eps \&\& d[i] + e.cost < d[j]) {
51
                                                                            127
                 d[j] = d[i] + e.cost;
52
                                                                            128
                 pe[j] = id;
                 if (its[j] == q.end()) {
54
55
                    its[j] = q.push({pot[j] - d[j], j});
                                                                            130
                 } else {
                                                                            131
 56
                    q.modify(its[j], {pot[j] - d[j], j});
57
                                                                            132
                 }
               }
59
                                                                            134
             }
60
                                                                            135
61
           }
                                                                            136
           swap(d, pot);
62
                                                                            137
63
64
                                                                            139
65
         pair<T, C> max_flow(int st, int fin) {
                                                                            140
66
           T flow = 0;
                                                                            141
           C cost = 0;
67
                                                                            142
68
           bool ok = true;
                                                                            143
           for (auto& e : edges) {
69
                                                                            144
             if (e.c - e.f > eps && e.cost + pot[e.from] - pot[e.to]
         < 0) {
                                                                            146
               ok = false;
72
               break;
                                                                            148
             }
73
                                                                            149
           }
74
                                                                            150
75
           if (ok) {
                                                                            151
             expath(st);
77
           } else {
                                                                            153
             vector<int> deg(n, 0);
78
                                                                            154
79
             for (int i = 0; i < n; i++) {
                                                                            155
               for (int eid : g[i]) {
80
                                                                            156
                 auto& e = edges[eid];
                 if (e.c - e.f > eps) {
82
                                                                            158
                    deg[e.to] += 1;
 83
                                                                            159
 84
                                                                            160
               }
85
                                                                            161
             }
                                                                            162
             vector<int> que;
87
                                                                            163
             for (int i = 0; i < n; i++) {
88
                                                                            164
               if (deg[i] == 0) {
89
                                                                            165
                  que.push_back(i);
90
               }
91
             }
92
             for (int b = 0; b < (int) que.size(); b++) {</pre>
               for (int eid : g[que[b]]) {
94
                 auto& e = edges[eid];
95
96
                 if (e.c - e.f > eps) {
97
                    deg[e.to] -= 1;
98
                    if (deg[e.to] == 0) {
99
                      que.push_back(e.to);
100
                 }
101
               }
102
             }
103
             fill(pot.begin(), pot.end(), INF_C);
104
             pot[st] = 0;
             if (static_cast<int>(que.size()) == n) {
106
107
               for (int v : que) {
                 if (pot[v] < INF_C) {</pre>
108
                    for (int eid : g[v]) {
109
                      auto& e = edges[eid];
111
                      if (e.c - e.f > eps) {
                        if (pot[v] + e.cost < pot[e.to]) {</pre>
112
                          pot[e.to] = pot[v] + e.cost;
113
                          pe[e.to] = eid;
114
                        }
                      }
116
                    }
117
                 }
118
               }
119
             } else {
120
               que.assign(1, st);
121
               vector<bool> in_queue(n, false);
122
               in_queue[st] = true;
123
               for (int b = 0; b < (int) que.size(); b++) {</pre>
```

```
int i = que[b];
            in_queue[i] = false;
            for (int id : g[i]) {
               const edge &e = edges[id];
               if (e.c - e.f > eps && pot[i] + e.cost <
 → pot[e.to]) {
                 pot[e.to] = pot[i] + e.cost;
                 pe[e.to] = id;
                 if (!in_queue[e.to]) {
                   que.push_back(e.to);
                   in_queue[e.to] = true;
          }
        }
     }
      while (pot[fin] < INF_C) {</pre>
        T push = numeric_limits<T>::max();
        int v = fin;
        while (v != st) {
          const edge &e = edges[pe[v]];
          push = min(push, e.c - e.f);
          v = e.from;
        }
        v = fin;
        while (v != st) {
          edge &e = edges[pe[v]];
          e.f += push;
          edge &back = edges[pe[v] ^ 1];
          back.f -= push;
          v = e.from;
        }
        flow += push;
        cost += push * pot[fin];
        expath(st);
     return {flow, cost};
   }
}:
// Examples: MCMF < int, int > g(n); g.add(u, v, c, w, 0);
 \hookrightarrow g.max_flow(s,t).
\begin{tabular}{ll} /\!/ & \textit{To recover flow through original edges: iterate over even} \end{tabular}

    indices in edges.
```

Graphs

Kuhn's algorithm for bipartite matching

```
The graph is split into 2 halves of n1 and n2 vertices.
     Complexity: O(n1 * m). Usually runs much faster. MUCH
     → FASTER!!!
4
5
     const int N = 305;
    vector<int> g[N]; // Stores edges from left half to right.
     {\bf bool\ used[N];\ /\!/\ Stores\ if\ vertex\ from\ left\ half\ is\ used.}
     int mt[N]; // For every vertex in right half, stores to which
     \,\,\hookrightarrow\,\, vertex in left half it's matched (-1 if not matched).
10
11
    bool try_dfs(int v){
       if (used[v]) return false;
12
       used[v] = 1;
14
       for (auto u : g[v]){
         if (mt[u] == -1 || try_dfs(mt[u])){
15
          mt[u] = v;
16
           return true;
17
18
         }
      }
19
       return false;
20
21
22
23
    int main(){
    // .....
```

```
for (int i = 1; i \le n2; i++) mt[i] = -1;
25
       for (int i = 1; i <= n1; i++) used[i] = 0;
26
      for (int i = 1; i <= n1; i++){
27
         if (try_dfs(i)){
28
           for (int j = 1; j \le n1; j++) used[j] = 0;
29
        }
30
31
32
      vector<pair<int, int>> ans;
      for (int i = 1; i <= n2; i++){
33
         if (mt[i] != -1) ans.pb({mt[i], i});
35
    }
36
37
    // Finding maximal independent set: size = # of nodes - # of
38
     \leftrightarrow edges in matching.
    // To construct: launch Kuhn-like DFS from unmatched nodes in
        the left half.
    // Independent set = visited nodes in left half + unvisited in
     \hookrightarrow right half.
    // Finding minimal vertex cover: complement of maximal

    independent set.
```

Hungarian algorithm for Assignment Problem

• Given a 1-indexed $(n \times m)$ matrix A, select a number in each row such that each column has at most 1 number selected, and the sum of the selected numbers is minimized.

```
int INF = 1e9; // constant greater than any number in the
     \rightarrow matrix
     vector\langle int \rangle u(n+1), v(m+1), p(m+1), way(m+1);
    for (int i=1; i<=n; ++i) {
         p[0] = i;
         int j0 = 0;
         vector<int> minv (m+1, INF);
6
         vector<bool> used (m+1, false);
         do {
             used[j0] = true;
             int i0 = p[j0], delta = INF, j1;
10
             for (int j=1; j<=m; ++j)</pre>
11
12
                  if (!used[j]) {
                      int cur = A[i0][j]-u[i0]-v[j];
13
                      if (cur < minv[j])</pre>
15
                          minv[j] = cur, way[j] = j0;
                      if (minv[j] < delta)</pre>
16
17
                          delta = minv[j], j1 = j;
                 }
18
             for (int j=0; j<=m; ++j)
                 if (used[i])
20
21
                      u[p[j]] += delta, v[j] -= delta;
22
                  else
                      minv[j] -= delta;
23
             j0 = j1;
24
         } while (p[j0] != 0);
25
26
             int j1 = way[j0];
27
             p[j0] = p[j1];
28
29
             j0 = j1;
         } while (j0);
30
    }
31
    vector<int> ans (n+1); // ans[i] stores the column selected
32
     \hookrightarrow for row i
33
    for (int j=1; j<=m; ++j)
         ans[p[j]] = j;
34
    int cost = -v[0]; // the total cost of the matching
```

Dijkstra's Algorithm

Eulerian Cycle DFS

```
void dfs(int v){
  while (!g[v].empty()){
    int u = g[v].back();
    g[v].pop_back();
    dfs(u);
    ans.pb(v);
}
```

SCC and 2-SAT

```
void scc(vector<vector<int>>& g, int* idx) {
       int n = g.size(), ct = 0;
       int out[n];
       vector<int> ginv[n];
       memset(out, -1, sizeof out);
       memset(idx, -1, n * sizeof(int));
       function<void(int)> dfs = [&](int cur) {
        out[cur] = INT_MAX;
         for(int v : g[cur]) {
           ginv[v].push_back(cur);
           if(out[v] == -1) dfs(v);
11
12
         ct++; out[cur] = ct;
13
14
       vector<int> order;
15
       for(int i = 0; i < n; i++) {</pre>
16
17
         order.push_back(i);
         if(out[i] == -1) dfs(i);
18
19
       sort(order.begin(), order.end(), [&](int& u, int& v) {
20
21
         return out[u] > out[v];
       });
^{22}
       ct = 0;
23
       stack<int> s;
       auto dfs2 = [&](int start) {
25
        s.push(start);
26
27
         while(!s.empty()) {
          int cur = s.top();
28
           s.pop();
           idx[cur] = ct;
30
31
           for(int v : ginv[cur])
32
             if(idx[v] == -1) s.push(v);
33
        }
      };
34
      for(int v : order) {
35
        if(idx[v] == -1) {
37
           dfs2(v);
           ct++;
39
         }
      }
40
41
42
    // 0 => impossible, 1 => possible
43
     pair<int, vector<int>> sat2(int n, vector<pair<int,int>>&

    clauses) {
      vector<int> ans(n);
      vector<vector<int>> g(2*n + 1);
46
47
      for(auto [x, y] : clauses) {
        x = x < 0 ? -x + n : x;
48
        y = y < 0 ? -y + n : y;
49
         int nx = x <= n ? x + n : x - n;</pre>
50
         int ny = y <= n ? y + n : y - n;</pre>
51
```

```
g[nx].push_back(y);
                                                                               dfs1(u, v, d + 1);
52
                                                                      12
        g[ny].push_back(x);
                                                                               sz[v] += sz[u];
                                                                      13
53
54
                                                                      14
      int idx[2*n + 1];
                                                                             if (!g[v].empty()) iter_swap(g[v].begin(),
55
                                                                      15
      scc(g, idx);
                                                                              max_element(all(g[v]), comp));
      for(int i = 1; i <= n; i++) {
                                                                          }
57
                                                                      16
        if(idx[i] == idx[i + n]) return {0, {}};
                                                                           void dfs2(int v, int rt, int c){
58
                                                                      17
        ans[i - 1] = idx[i + n] < idx[i];
                                                                             pos[v] = sz(a);
59
                                                                      18
                                                                             a.pb(c);
60
                                                                      19
      return {1, ans};
                                                                             root[v] = rt;
                                                                             for (int i = 0; i < sz(g[v]); i++){</pre>
62
                                                                      21
                                                                               auto [u, c] = g[v][i];
                                                                               if (!i) dfs2(u, rt, c);
                                                                      23
    Finding Bridges
                                                                               else dfs2(u, u, c);
                                                                      24
                                                                             }
                                                                      25
                                                                          }
                                                                      26
    Bridges.
                                                                      27
                                                                           int getans(int u, int v){
    Results are stored in a map "is_bridge".
                                                                      28
                                                                             int res = 0;
    For each connected component, call "dfs(starting vertex,
                                                                             for (; root[u] != root[v]; v = par[root[v]]){
                                                                      29

    starting vertex)".

                                                                               if (dep[root[u]] > dep[root[v]]) swap(u, v);
                                                                      30
                                                                               res = max(res, rmq(0, 0, n - 1, pos[root[v]], pos[v]));
                                                                      31
    const int N = 2e5 + 10; // Careful with the constant!
                                                                      32
                                                                             if (pos[u] > pos[v]) swap(u, v);
                                                                      33
    vector<int> g[N];
                                                                      34
                                                                             return max(res, rmq(0, 0, n - 1, pos[u] + 1, pos[v]));
9
    int tin[N], fup[N], timer;
                                                                      35
    map<pair<int, int>, bool> is_bridge;
10
11
    void dfs(int v, int p){
                                                                           Centroid Decomposition
      tin[v] = ++timer;
13
14
      fup[v] = tin[v];
                                                                           vector<char> res(n), seen(n), sz(n);
                                                                           function<int(int, int)> get_size = [&](int node, int fa) {
15
      for (auto u : g[v]){
        if (!tin[u]){
                                                                             sz[node] = 1;
16
          dfs(u, v);
                                                                             for (auto& ne : g[node]) {
                                                                               if (ne == fa || seen[ne]) continue;
          if (fup[u] > tin[v]){
18
            is_bridge[{u, v}] = is_bridge[{v, u}] = true;
                                                                               sz[node] += get_size(ne, node);
19
                                                                             }
20
          fup[v] = min(fup[v], fup[u]);
                                                                             return sz[node];
21
22
                                                                       9
                                                                          };
                                                                           function<int(int, int, int)> find_centroid = [&](int node, int
        else{
23
          if (u != p) fup[v] = min(fup[v], tin[u]);

    fa, int t) {
24
                                                                             for (auto& ne : g[node])
25
                                                                      11
                                                                               if (ne != fa && !seen[ne] && sz[ne] > t / 2) return
26
    }

    find_centroid(ne, node, t);

27
                                                                             return node;
                                                                      13
                                                                      14
    Virtual Tree
                                                                           function<void(int, char)> solve = [&](int node, char cur) {
                                                                      15
                                                                             get_size(node, -1); auto c = find_centroid(node, -1,
    // order stores the nodes in the queried set
                                                                            ⇔ sz[node]):
    sort(all(order), [&] (int u, int v){return tin[u] < tin[v];});</pre>
                                                                             seen[c] = 1, res[c] = cur;
    int m = sz(order);
                                                                             for (auto& ne : g[c]) {
    for (int i = 1; i < m; i++){
                                                                               if (seen[ne]) continue;
                                                                      19
5
        order.pb(lca(order[i], order[i - 1]));
                                                                               solve(ne, char(cur + 1)); // we can pass c here to build
6
    sort(all(order), [&] (int u, int v){return tin[u] < tin[v];});</pre>
    order.erase(unique(all(order)), order.end());
                                                                          };
    vector<int> stk{order[0]};
    for (int i = 1; i < sz(order); i++){</pre>
10
        int v = order[i];
                                                                           Math
        while (tout[stk.back()] < tout[v]) stk.pop_back();</pre>
12
        int u = stk.back();
13
        vg[u].pb({v, dep[v] - dep[u]});
                                                                           Binary exponentiation
        stk.pb(v);
15
    }
                                                                          ll power(ll a, ll b){
                                                                            11 res = 1;
                                                                       2
                                                                             for (; b; a = a * a \% MOD, b >>= 1){
    HLD on Edges DFS
                                                                               if (b & 1) res = res * a % MOD;
                                                                       5
    void dfs1(int v, int p, int d){
                                                                             return res;
                                                                       6
      par[v] = p;
      for (auto e : g[v]){
        if (e.fi == p){
          g[v].erase(find(all(g[v]), e));
                                                                           Extended Euclidean Algorithm
          break:
        }
                                                                          // gives (x, y) for ax + by = g
                                                                          // solutions given (x0, y0): a(x0 + kb/g) + b(y0 - ka/g) = g
      }
      dep[v] = d;
                                                                           int gcd(int a, int b, int& x, int& y) {
9
                                                                       3
      sz[v] = 1;
                                                                            x = 1, y = 0; int sum1 = a;
                                                                             int x2 = 0, y2 = 1, sum2 = b;
      for (auto [u, c] : g[v]){
```

```
for (int c = 0; c < limit; c++) {</pre>
      while (sum2) {
                                                                        10
         int q = sum1 / sum2;
                                                                                 int id = -1;
                                                                        11
                                                                                 for (int i = r; i < h; i++) {
         tie(x, x2) = make_tuple(x2, x - q * x2);
                                                                        12
         tie(y, y2) = make_tuple(y2, y - q * y2);
                                                                                  if (!is_0(a[i][c]) && (id == -1 || abs(a[id][c]) <
         tie(sum1, sum2) = make_tuple(sum2, sum1 - q * sum2);
                                                                                 abs(a[i][c]))) {
11
                                                                        14
                                                                                    id = i:
12
      return sum1;
                                                                        15
    }
                                                                                 }
                                                                        16
13
                                                                                 if (id == -1) continue;
                                                                        17
                                                                                 if (id > r) {
    Linear Sieve
                                                                                   swap(a[r], a[id]);
                                                                        19
                                                                                   for (int j = c; j < w; j++) a[id][j] = -a[id][j];
                                                                        20

    Mobius Function

                                                                        21
                                                                                 vector<int> nonzero;
                                                                        22
    vector<int> prime;
                                                                                 for (int j = c; j < w; j++) {
                                                                        23
    bool is_composite[MAX_N];
                                                                                  if (!is_0(a[r][j])) nonzero.push_back(j);
                                                                        24
    int mu[MAX_N];
3
                                                                        25
                                                                                 T inv_a = 1 / a[r][c];
                                                                        26
    void sieve(int n){
                                                                                 for (int i = r + 1; i < h; i++) {
                                                                        27
      fill(is_composite, is_composite + n, 0);
                                                                                   if (is_0(a[i][c])) continue;
                                                                        28
      mu[1] = 1:
                                                                                   T coeff = -a[i][c] * inv_a;
                                                                        29
      for (int i = 2; i < n; i++){
                                                                                   for (int j : nonzero) a[i][j] += coeff * a[r][j];
                                                                        30
9
        if (!is_composite[i]){
                                                                                 }
                                                                        31
          prime.push_back(i);
10
          mu[i] = -1; //i is prime
                                                                              }
                                                                        33
12
                                                                               for (int row = h - 1; row >= 0; row--) {
                                                                        34
13
       for (int j = 0; j < prime.size() && i * prime[j] < n; j++){
                                                                                 for (int c = 0; c < limit; c++) {</pre>
         is_composite[i * prime[j]] = true;
14
                                                                                  if (!is_0(a[row][c])) {
                                                                        36
         if (i % prime[j] == 0){
15
                                                                                     T inv_a = 1 / a[row][c];
          mu[i * prime[j]] = 0; //prime[j] divides i
                                                                                     for (int i = row - 1; i >= 0; i--) {
                                                                        38
           break;
17
                                                                                       if (is_0(a[i][c])) continue;
                                                                        39
          } else {
                                                                                       T coeff = -a[i][c] * inv_a;
                                                                        40
           mu[i * prime[j]] = -mu[i]; //prime[j] does not divide i
19
                                                                                       for (int j = c; j < w; j++) a[i][j] += coeff *
                                                                        41
20

    a[row][j];

21
                                                                        42
                                                                                    }
      }
22
                                                                        43
                                                                                     break;
    }
                                                                                   }
                                                                        44
                                                                        45

    Euler's Totient Function

                                                                              } // not-free variables: only it on its line
                                                                        46
                                                                              for(int i = r; i < h; i++) if(!is_0(a[i][limit])) return 0;</pre>
    vector<int> prime;
                                                                        47
                                                                              return (r == limit) ? 1 : -1;
    bool is_composite[MAX_N];
                                                                        48
    int phi[MAX N];
                                                                        49
                                                                        50
    void sieve(int n){
                                                                            template <typename T>
                                                                        51
                                                                            pair<int,vector<T>> solve_linear(vector<vector<T>> a, const
      fill(is_composite, is_composite + n, 0);
                                                                        52
                                                                              → vector<T> &b, int w) {
      phi[1] = 1:
                                                                              int h = (int)a.size();
      for (int i = 2; i < n; i++){
                                                                        53
                                                                              for (int i = 0; i < h; i++) a[i].push_back(b[i]);</pre>
         if (!is_composite[i]){
                                                                        54
          prime.push_back (i);
                                                                              int sol = gaussian_elimination(a, w);
                                                                              if(!sol) return {0, vector<T>()};
          phi[i] = i - 1; //i is prime
                                                                        56
11
12
                                                                               vector<T> x(w, 0);
      for (int j = 0; j < prime.size () && i * prime[j] < n; j++){
                                                                              for (int i = 0; i < h; i++) {
13
         is_composite[i * prime[j]] = true;
                                                                                 for (int j = 0; j < w; j++) {
14
                                                                                   if (!is_0(a[i][j])) {
         if (i % prime[j] == 0){
15
                                                                        60
          phi[i * prime[j]] = phi[i] * prime[j]; //prime[j]
                                                                        61
                                                                                     x[j] = a[i][w] / a[i][j];
16
       divides i
                                                                                     break;
                                                                        62
          break:
17
                                                                        63
                                                                                }
18
          phi[i * prime[j]] = phi[i] * phi[prime[j]]; //prime[j]
                                                                        65
19
        does not divide i
                                                                        66
                                                                              return {sol, x};
          }
                                                                        67
        }
21
      }
22
                                                                            NTT
                                                                            void ntt(vector<ll>& a, int f) {
                                                                        1
                                                                              int n = int(a.size());
     Gaussian Elimination
                                                                              vector<11> w(n):
    bool is_0(Z v) { return v.x == 0; }
                                                                              vector<int> rev(n);
    Z abs(Z v) { return v; }
                                                                              for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] / 2) | ((i
    bool is_0(double v) { return abs(v) < 1e-9; }</pre>
                                                                             \leftrightarrow & 1) * (n / 2));
                                                                              for (int i = 0; i < n; i++) {
    // 1 => unique solution, 0 => no solution, -1 => multiple
                                                                                if (i < rev[i]) swap(a[i], a[rev[i]]);</pre>

⇒ solutions

                                                                              11 wn = power(f ? (MOD + 1) / 3 : 3, (MOD - 1) / n);
    template <typename T>
    int gaussian_elimination(vector<vector<T>>> &a, int limit) {
                                                                        10
         if (a.empty() || a[0].empty()) return -1;
                                                                              for (int i = 1; i < n; i++) w[i] = w[i - 1] * wn % MOD;
                                                                        11
       int h = (int)a.size(), w = (int)a[0].size(), r = 0;
                                                                              for (int mid = 1; mid < n; mid *= 2) {</pre>
```

```
for (int i = 0; i < n; i += 2 * mid) {
13
                                                                              return res:
          for (int j = 0; j < mid; j++) {
14
            ll x = a[i + j], y = a[i + j + mid] * w[n / (2 * mid)
         * j] % MOD;
                                                                             bool is_prime(ll n) {
            a[i + j] = (x + y) \% MOD, a[i + j + mid] = (x + MOD - i)
                                                                               if (n < 2) return false;
       y) % MOD;
                                                                               static constexpr int A[] = \{2, 3, 5, 7, 11, 13, 17, 19, 23\};
                                                                        11
                                                                               int s = __builtin_ctzll(n - 1);
17
                                                                        12
        }
                                                                               11 d = (n - 1) >> s;
18
                                                                        13
      }
                                                                               for (auto a : A) {
19
                                                                        14
20
      if (f) {
                                                                                 if (a == n) return true;
                                                                                 11 x = (11)power(a, d, n);
        11 iv = power(n, MOD - 2);
21
                                                                        16
                                                                                 if (x == 1 \mid \mid x == n - 1) continue;
22
        for (auto& x : a) x = x * iv % MOD;
                                                                        17
                                                                                 bool ok = false;
23
                                                                        18
                                                                                 for (int i = 0; i < s - 1; ++i) {
24
                                                                        19
    vector<ll> mul(vector<ll> a, vector<ll> b) {
                                                                                   x = 11((i128)x * x % n); // potential overflow!
25
      int n = 1, m = (int)a.size() + (int)b.size() - 1;
                                                                                   if (x == n - 1) {
26
                                                                        21
      while (n < m) n *= 2;
                                                                                     ok = true;
      a.resize(n), b.resize(n);
28
                                                                        23
                                                                                     break;
      ntt(a, 0), ntt(b, 0); // if squaring, you can save one NTT
                                                                                7
                                                                        25
      for (int i = 0; i < n; i++) a[i] = a[i] * b[i] % MOD;
                                                                                 if (!ok) return false;
30
                                                                        26
31
      ntt(a, 1);
                                                                        27
      a.resize(m):
32
                                                                        28
                                                                              return true;
33
      return a:
    }
34
                                                                             typedef __int128_t i128;
                                                                        1
    FFT
                                                                            11 pollard_rho(ll x) {
                                                                              11 s = 0, t = 0, c = rng() \% (x - 1) + 1;
    const ld PI = acosl(-1);
                                                                               ll stp = 0, goal = 1, val = 1;
    auto mul = [&](const vector<ld>& aa. const vector<ld>& bb) {
                                                                              for (goal = 1;; goal *= 2, s = t, val = 1) {
      int n = (int)aa.size(), m = (int)bb.size(), bit = 1;
                                                                                 for (stp = 1; stp <= goal; ++stp) {</pre>
      while ((1 << bit) < n + m - 1) bit++;
                                                                                   t = 11(((i128)t * t + c) \% x);
      int len = 1 << bit;</pre>
                                                                        9
                                                                                   val = 11((i128)val * abs(t - s) % x);
      vector<complex<ld>>> a(len), b(len);
                                                                                   if ((stp % 127) == 0) {
      vector<int> rev(len);
                                                                                    11 d = gcd(val, x);
                                                                        11
      for (int i = 0; i < n; i++) a[i].real(aa[i]);</pre>
                                                                                     if (d > 1) return d;
      for (int i = 0; i < m; i++) b[i].real(bb[i]);</pre>
                                                                                   }
                                                                        13
      for (int i = 0; i < len; i++) rev[i] = (rev[i >> 1] >> 1) |
10
                                                                        14
     \rightarrow ((i & 1) << (bit - 1));
                                                                                 ll d = gcd(val, x);
                                                                        15
      auto fft = [&](vector<complex<ld>>& p, int inv) {
11
                                                                                 if (d > 1) return d;
                                                                        16
         for (int i = 0; i < len; i++)
          if (i < rev[i]) swap(p[i], p[rev[i]]);</pre>
13
                                                                            }
                                                                        18
        for (int mid = 1; mid < len; mid *= 2) {</pre>
14
                                                                        19
15
          auto w1 = complex<ld>(cos(PI / mid), (inv ? -1 : 1) *
                                                                        20
                                                                            11 get_max_factor(ll _x) {

    sin(PI / mid));
                                                                              11 max_factor = 0;
                                                                        21
          for (int i = 0; i < len; i += mid * 2) {
16
                                                                               function < void(11) > fac = [&](11 x) {
            auto wk = complex<ld>(1, 0);
17
                                                                                 if (x <= max_factor || x < 2) return;</pre>
                                                                        23
            for (int j = 0; j < mid; j++, wk = wk * w1) {
18
                                                                        24
                                                                                 if (is_prime(x)) {
               auto x = p[i + j], y = wk * p[i + j + mid];
19
                                                                                  max_factor = max_factor > x ? max_factor : x;
                                                                        25
               p[i + j] = x + y, p[i + j + mid] = x - y;
20
                                                                        26
                                                                                   return;
21
                                                                                 }
                                                                        27
          }
22
                                                                                 11 p = x;
                                                                        28
23
                                                                                 while (p >= x) p = pollard_rho(x);
        if (inv == 1) {
24
                                                                                 while ((x \% p) == 0) x /= p;
          for (int i = 0; i < len; i++) p[i].real(p[i].real() /</pre>
25
                                                                                 fac(x), fac(p);
        len);
                                                                              };
        }
26
                                                                              fac(x):
                                                                        33
27
                                                                        34
                                                                              return max_factor;
      fft(a, 0), fft(b, 0);
28
                                                                        35
      for (int i = 0; i < len; i++) a[i] = a[i] * b[i];
29
      fft(a, 1);
30
      a.resize(n + m - 1);
31
      vector < ld > res(n + m - 1);
32
      for (int i = 0; i < n + m - 1; i++) res[i] = a[i].real();
33
                                                                             Data Structures
35
   };
                                                                             Fenwick Tree
    is_prime
                                                                            11 sum(int r) {
                                                                                 11 ret = 0;
```

(Miller–Rabin primality test)

for (; b; b /= 2, (a *= a) %= MOD) if (b & 1) (res *= a) %= MOD;

i128 power(i128 a, i128 b, i128 MOD = 1, i128 res = 1) {

typedef __int128_t i128;

3

3

return ret;

void add(int idx, ll delta) {

for (; $r \ge 0$; r = (r & r + 1) - 1) ret += bit[r];

for (; idx < n; idx |= idx + 1) bit[idx] += delta;

Lazy Propagation SegTree 67 68 push(v, tl, tr); // Clear: clear() or build() int tm = (tl + tr) / 2;69 const int N = 2e5 + 10; // Change the constant! modify(2 * v + 1, tl, tm, l, min(r, tm), val);70 template<typename T> modify(2 * v + 2, tm + 1, tr, max(1, tm + 1), r, val);71 struct LazySegTree{ t[v] = f(t[2 * v + 1], t[2 * v + 2]);72 T t[4 * N];73 T lazy[4 * N];74 int n: T query(int v, int tl, int tr, int l, int r) { if (1 > r) return default_return; 76 // Change these functions, default return, and lazy mark. if (tl == 1 && tr == r) return t[v]; 77 T default_return = 0, lazy_mark = numeric_limits<T>::min(); 10 78 push(v, tl, tr); // Lazy mark is how the algorithm will identify that no 11 int tm = (tl + tr) / 2;79 → propagation is needed. return f(function $T(T, T) > f = [\&] (T a, T b){$ 12 query(2 * v + 1, tl, tm, l, min(r, tm)),81 return a + b: 13 82 query(2 * v + 2, tm + 1, tr, max(1, tm + 1), r)14 83 // f_on_seg calculates the function f, knowing the lazy } 84 → value on seament. 85 // segment's size and the previous value. void modify(int 1, int r, T val){ 86 // The default is segment modification for RSQ. For modify(0, 0, n - 1, 1, r, val); 87 \rightarrow increments change to: 88 // return cur_seg_val + seg_size * lazy_val; // For RMQ. Modification: return lazy_val; Increments: 19 T query(int 1, int r){ 90 → return cur_seg_val + lazy_val; return query(0, 0, n - 1, 1, r); 91 function<T(T, int, T)> f_on_seg = [&] (T cur_seg_val, int 20 seg_size, T lazy_val){ 93 21 return seg_size * lazy_val; T get(int pos){ 22 95 return query(pos, pos); // upd_lazy updates the value to be propagated to child 96 ⇒ seaments. 97 // Default: modification. For increments change to: // Change clear() function to t.clear() if using // $lazy[v] = (lazy[v] == lazy_mark? val : lazy[v] +$ 25 unordered_map for SegTree!!! void clear(int n_){ 99 function<void(int, T)> upd_lazy = [&] (int v, T val){ 26 $n = n_{;}$ 100 lazy[v] = val; 27 for (int i = 0; i < 4 * n; i++) t[i] = 0, lazy[i] = 101 → lazy mark; // Tip: for "get element on single index" queries, use max() 29 } \hookrightarrow on segment: no overflows. 103 30 void build(vector<T>& a){ 104 LazySegTree(int n_) : n(n_) { 31 n = sz(a): 105 clear(n); 32 clear(n); 106 33 107 build(0, 0, n - 1, a); 34 108 void build(int v, int tl, int tr, vector<T>& a){ 35 109 36 if (tl == tr) { t[v] = a[t1]; 37 return: Sparse Table 38 7 const int N = 2e5 + 10, LOG = 20; // Change the constant! int tm = (tl + tr) / 2;40 template<typename T> // left child: [tl, tm] 41 struct SparseTable{ // right child: [tm + 1, tr]42 build(2 * v + 1, tl, tm, a);int lg[N]; 43 build(2 * v + 2, tm + 1, tr, a); T st[N][LOG]; 44 t[v] = f(t[2 * v + 1], t[2 * v + 2]);6 45 46 47 // Change this function LazySegTree(vector<T>& a){ functionT(T, T) > f = [&] (T a, T b)48 49 build(a); return min(a, b); 10 }; 50 11 51 void build(vector<T>& a){ void push(int v, int tl, int tr){ 52 13 if (lazy[v] == lazy_mark) return; 53 n = sz(a): 14 54 int tm = (tl + tr) / 2;lg[1] = 0; $t[2 * v + 1] = f_on_seg(t[2 * v + 1], tm - tl + 1,$ for (int i = 2; $i \le n$; i++) lg[i] = lg[i / 2] + 1; 55 16 $t[2 * v + 2] = f_on_seg(t[2 * v + 2], tr - tm, lazy[v]);$ for (int k = 0; k < LOG; k++){ 56 18 $upd_{lazy}(2 * v + 1, lazy[v]), upd_{lazy}(2 * v + 2,$ for (int i = 0; i < n; i++){ 57 19 \rightarrow lazy[v]); 20 if (!k) st[i][k] = a[i]; else st[i][k] = f(st[i][k-1], st[min(n-1, i+(1 <<lazy[v] = lazy mark; 21 58 59 (k - 1))[k - 1]); } 60 22 61 void modify(int v, int tl, int tr, int l, int r, T val){ } 23 if (1 > r) return; } 62 24 63 if (tl == 1 && tr == r){ 25 t[v] = f_on_seg(t[v], tr - tl + 1, val); 26 T query(int 1, int r){ 64 upd_lazy(v, val); int sz = r - 1 + 1;

66

return:

```
return f(st[1][lg[sz]], st[r - (1 << lg[sz]) + 1][lg[sz]]);
                                                                              }
28
29
    };
                                                                              void buildSparse(){
30
                                                                       70
                                                                               st.build(h);
                                                                       71
                                                                       72
    Suffix Array and LCP array
                                                                       73
                                                                              // l and r must be in O-BASED INDEXATION
                                                                       74
       • (uses SparseTable above)
                                                                              int lcp(int 1, int r){
                                                                       75
                                                                                1 = c[1] - 1, r = c[r] - 1;
                                                                       76
    struct SuffixArray{
                                                                       77
                                                                                if (1 > r) swap(1, r);
      vector<int> p, c, h;
                                                                                return st.query(1, r - 1);
                                                                       78
      SparseTable<int> st;
                                                                       79
                                                                        80
                                                                            }:
       In the end, array c gives the position of each suffix in p
       using 1-based indexation!
                                                                            Aho Corasick Trie
9
      SuffixArray() {}
                                                                               • For each node in the trie, the suffix link points to the
                                                                                 longest proper suffix of the represented string. The
      SuffixArray(string s){
11
         buildArray(s);
                                                                                 terminal-link tree has square-root height (can be con-
12
        buildLCP(s):
13
                                                                                 structed by DFS).
        buildSparse();
14
                                                                            const int S = 26;
15
16
17
      void buildArray(string s){
                                                                            // Function converting char to int.
                                                                            int ctoi(char c){
        int n = sz(s) + 1;
18
19
         p.resize(n), c.resize(n);
                                                                              return c - 'a';
20
        for (int i = 0; i < n; i++) p[i] = i;
         sort(all(p), [&] (int a, int b){return s[a] < s[b];});</pre>
21
         c[p[0]] = 0;
                                                                            // To add terminal links, use DFS
         for (int i = 1; i < n; i++){
                                                                            struct Node{
23
          c[p[i]] = c[p[i-1]] + (s[p[i]] != s[p[i-1]]);
                                                                              vector<int> nxt;
                                                                        10
                                                                              int link:
25
                                                                       11
         vector<int> p2(n), c2(n);
                                                                              bool terminal;
26
27
         // w is half-length of each string.
                                                                       13
         for (int w = 1; w < n; w <<= 1){
                                                                              Node() {
28
                                                                       14
           for (int i = 0; i < n; i++){
                                                                                nxt.assign(S, -1), link = 0, terminal = 0;
29
                                                                       15
            p2[i] = (p[i] - w + n) \% n;
30
                                                                       16
31
                                                                       17
                                                                            };
32
          vector<int> cnt(n);
           for (auto i : c) cnt[i]++;
                                                                            vector<Node> trie(1);
33
                                                                       19
           for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
34
                                                                       20
           for (int i = n - 1; i >= 0; i--){
                                                                            // add_string returns the terminal vertex.
35
                                                                       21
            p[--cnt[c[p2[i]]]] = p2[i];
36
                                                                       22
                                                                            int add_string(string& s){
                                                                              int v = 0:
37
                                                                       23
38
           c2[p[0]] = 0;
                                                                       24
                                                                              for (auto c : s){
           for (int i = 1; i < n; i++){
                                                                                int cur = ctoi(c);
            c2[p[i]] = c2[p[i - 1]] +
                                                                                if (trie[v].nxt[cur] == -1){
40
                                                                       26
             (c[p[i]] != c[p[i - 1]] ||
                                                                                  trie[v].nxt[cur] = sz(trie);
            c[(p[i] + w) \% n] != c[(p[i - 1] + w) \% n]);
                                                                                  trie.emplace_back();
42
                                                                       28
43
                                                                       29
           c.swap(c2);
                                                                                v = trie[v].nxt[cur];
44
                                                                       30
45
                                                                       31
        p.erase(p.begin());
                                                                              trie[v].terminal = 1;
46
                                                                       32
47
                                                                              return v:
                                                                       33
48
                                                                       34
49
      void buildLCP(string s){
                                                                       35
        // The algorithm assumes that suffix array is already
50
                                                                       36
     \hookrightarrow built on the same string.
                                                                            Suffix links are compressed.
                                                                       37
        int n = sz(s);
                                                                            This means that:
51
                                                                       38
        h.resize(n - 1);
                                                                              If vertex v has a child by letter x, then:
52
        int k = 0;
                                                                                trie[v].nxt[x] points to that child.
53
                                                                       40
         for (int i = 0; i < n; i++){
                                                                              If vertex v doesn't have such child, then:
                                                                       41
54
55
          if (c[i] == n){
                                                                       42
                                                                                trie[v].nxt[x] points to the suffix link of that child
                                                                                if we would actually have it.
            k = 0;
                                                                       43
56
            continue;
57
                                                                       44
                                                                            void add_links(){
58
                                                                       45
          int j = p[c[i]];
                                                                              queue<int> q;
59
                                                                       46
          while (i + k < n \&\& j + k < n \&\& s[i + k] == s[j + k])
                                                                              q.push(0);
60
                                                                       47
                                                                              while (!q.empty()){
     48
61
          h[c[i] - 1] = k;
                                                                       49
                                                                                auto v = q.front();
                                                                                int u = trie[v].link;
          if (k) k--;
62
                                                                       50
        }
                                                                                q.pop();
63
                                                                       51
         /*
                                                                                for (int i = 0; i < S; i++){
64
                                                                       52
         Then an RMQ Sparse Table can be built on array h
                                                                                  int& ch = trie[v].nxt[i];
65
                                                                       53
```

54

to calculate LCP of 2 non-consecutive suffixes.

66

if (ch == -1){

ch = v? trie[u].nxt[i] : 0;

```
}
56
           elsef
57
             trie[ch].link = v? trie[u].nxt[i] : 0;
58
             q.push(ch);
59
60
        }
61
62
    }
63
64
    bool is_terminal(int v){
      return trie[v].terminal;
66
67
68
    int get_link(int v){
69
      return trie[v].link;
70
71
72
    int go(int v, char c){
73
      return trie[v].nxt[ctoi(c)];
74
```

Convex Hull Trick

- Allows to insert a linear function to the hull in (1) and get the minimum/maximum value of the stored function at a point in O(log n).
- NOTE: The lines must be added in the order of decreasing/increasing gradients. CAREFULLY CHECK THE SETUP BEFORE USING!
- IMPORTANT: THE DEFAULT VERSION SURELY WORKS. IF MODIFIED VERSIONS DON'T WORK, TRY TRANSFORMING THEM TO THE DEFAULT ONE BY CHANGING SIGNS.

```
struct line{
1
      11 k, b;
      11 f(11 x){
        return k * x + b;
      };
5
6
    vector<line> hull;
    void add_line(line nl){
10
      if (!hull.empty() && hull.back().k == nl.k){
11
        nl.b = min(nl.b, hull.back().b); // Default: minimum. For
       maximum change "min" to "max".
        hull.pop_back();
      }
14
      while (sz(hull) > 1){
15
        auto& 11 = hull.end()[-2], 12 = hull.back();
16
        if ((nl.b - l1.b) * (l2.k - nl.k) >= (nl.b - l2.b) * (l1.k)
17
        - nl.k)) hull.pop_back(); // Default: decreasing gradient
       k. For increasing k change the sign to <=.
         else break;
18
      }
19
      hull.pb(nl);
20
    }
21
22
    11 get(11 x){
23
      int 1 = 0, r = sz(hull);
24
      while (r - 1 > 1){
25
        int mid = (1 + r) / 2;
26
         if (hull[mid - 1].f(x) >= hull[mid].f(x)) 1 = mid; //
27
        Default: minimum. For maximum change the sign to <=.
        else r = mid:
28
      }
29
30
      return hull[1].f(x);
31
```

Li-Chao Segment Tree

- allows to add linear functions in any order and query minimum/maximum value of those at a point, all in O(log n).
- Clear: clear()

2

3

9

10

11

12

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46

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52

53

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56

57

58

59

60

61

62

```
const 11 INF = 1e18; // Change the constant!
struct LiChaoTree{
  struct line{
    11 k. b:
    line(){
     k = b = 0:
    line(ll k_, ll b_){
      k = k_{,} b = b_{;}
    11 f(11 x){
     return k * x + b;
    };
  };
  int n;
  bool minimum, on_points;
  vector<11> pts;
  vector<line> t;
  void clear(){
    for (auto& 1 : t) 1.k = 0, 1.b = minimum? INF : -INF;
 \leftrightarrow constructor for numbers in range [0, n - 1].
    n = n_, minimum = min_, on_points = false;
    t.resize(4 * n);
    clear():
  };
 LiChaoTree(vector<ll> pts_, bool min_){ // This constructor
 → will build LCT on the set of points you pass. The points
 → may be in any order and contain duplicates.
    pts = pts_, minimum = min_;
    sort(all(pts));
    pts.erase(unique(all(pts)), pts.end());
    on_points = true;
    n = sz(pts);
    t.resize(4 * n);
    clear();
  void add_line(int v, int l, int r, line nl){
    // Adding on segment [l, r)
    int m = (1 + r) / 2;
    11 lval = on_points? pts[1] : 1, mval = on_points? pts[m]
    if ((minimum && nl.f(mval) < t[v].f(mval)) || (!minimum &&
    nl.f(mval) > t[v].f(mval))) swap(t[v], nl);
    if (r - l == 1) return;
    if ((minimum && nl.f(lval) < t[v].f(lval)) || (!minimum &&
 \leftrightarrow nl.f(lval) > t[v].f(lval))) add_line(2 * v + 1, 1, m, nl);
    else add_line(2 * v + 2, m, r, nl);
  11 get(int v, int 1, int r, int x){
    int m = (1 + r) / 2:
    if (r - l == 1) return t[v].f(on_points? pts[x] : x);
      if (minimum) return min(t[v].f(on_points? pts[x] : x), x
   < m? get(2 * v + 1, 1, m, x) : get(2 * v + 2, m, r, x));
      else return max(t[v].f(on\_points? pts[x] : x), x < m?
    get(2 * v + 1, 1, m, x) : get(2 * v + 2, m, r, x));
  }
  void add_line(ll k, ll b){
    add_line(0, 0, n, line(k, b));
```

Persistent Segment Tree

• for RSQ

```
struct Node {
        ll val:
        Node *1, *r;
         Node(ll x) : val(x), l(nullptr), r(nullptr) {}
         Node(Node *11, Node *rr) {
             1 = 11, r = rr;
             val = 0;
             if (1) val += 1->val;
9
             if (r) val += r->val;
10
11
         Node(Node *cp) : val(cp->val), l(cp->l), r(cp->r) {}
12
    };
    const int N = 2e5 + 20;
14
15
    Node *roots[N];
16
    int n, cnt = 1;
17
    Node *build(int l = 1, int r = n) {
        if (1 == r) return new Node(a[1]);
19
         int mid = (1 + r) / 2;
         return new Node(build(1, mid), build(mid + 1, r));
21
22
23
    Node *update(Node *node, int val, int pos, int l = 1, int r =
     \hookrightarrow n) {
        if (1 == r) return new Node(val);
        int mid = (1 + r) / 2;
25
         if (pos > mid)
26
            return new Node(node->1, update(node->r, val, pos, mid
27
        + 1, r));
        else return new Node(update(node->1, val, pos, 1, mid),
        node->r):
    }
29
    11 query(Node *node, int a, int b, int l = 1, int r = n) {
30
         if (1 > b || r < a) return 0;
31
         if (1 \ge a \&\& r \le b) return node->val;
32
         int mid = (1 + r) / 2;
33
        return query(node->1, a, b, 1, mid) + query(node->r, a, b,
        mid + 1, r);
    }
35
```

Miscellaneous

Ordered Set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
tree_order_statistics_node_update> ordered_set;
```

Measuring Execution Time

```
1  ld tic = clock();
2  // execute algo...
3  ld tac = clock();
4  // Time in milliseconds
5  cerr << (tac - tic) / CLOCKS_PER_SEC * 1000 << endl;
6  // No need to comment out the print because it's done to cerr.</pre>
```

Setting Fixed D.P. Precision

```
cout << setprecision(d) << fixed;
// Each number is rounded to d digits after the decimal point,

→ and truncated.</pre>
```

Common Bugs and General Advice

- Check overflow, array bounds
- Check variable overloading
- Check special cases (n=1?)
- Do something instead of nothing, stay organized
- Write stuff down!
- Don't get stuck on one approach!