# CU-Later Code Library

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May 21th 2024

#### **Templates** Contents **Templates** 1 Ken's template #include <bits/stdc++.h> 1 using namespace std; Kevin's Template Extended . . . . . . . . . . . . . . . . . #define all(v) (v).begin(), (v).end()typedef long long 11; typedef long double ld; Geometry 1 #define pb push\_back #define sz(x) (int)(x).size()Strings 3 #define fi first #define se second 9 Flows 3 #define endl '\n' $O(N^2M)$ , on unit networks $O(N^{1/2}M)$ . . . . . . 3 MCMF - maximize flow, then minimize its cost. Kevin's template 4 // paste Kaurov's Template, minus last line typedef vector<int> vi; 2 5 Graphs typedef vector<11> v11; Kuhn's algorithm for bipartite matching . . . . . . 5 typedef pair<int, int> pii; 6 typedef pair<11, 11> pl1; EULERIAN CYCLE DFS . . . . . . . . . . . . . . . . typedef pair<double, double> pdd; const ld PI = acosl(-1); Strongly Connected Components: Kosaraju's Algoconst $11 \mod 7 = 1e9 + 7$ ; 6 const 11 mod9 = 998244353; 6 10 const 11 INF = 2\*1024\*1024\*1023; 6 const char nl = '\n'; 11 #define form(i, n) for (int i = 0; i < int(n); i++) HLD ON EDGES DFS . . . . . . . . . . . . . . . . 6 ll k, n, m, u, v, w; 13 7 string s, t; 7 Math 16 bool multiTest = 1: void solve(int tt){ 7 17 18 7 Extended Euclidean Algorithm . . . . . . . . . . . 7 int main(){ 7 21 ios::sync\_with\_stdio(0);cin.tie(0);cout.tie(0); cout<<fixed<< setprecision(14);</pre> 8 22 23 8 int t = 1;8 25 if (multiTest) cin >> t; 26 forn(ii, t) solve(ii); 9 Data Structures 27 9 9 Suffix Array and LCP array . . . . . . . . . . . . . . . . Kevin's Template Extended 9 • to type after the start of the contest 10 10 #pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt") Persistent Segment Tree . . . . . . . . . . . . . . . . . 11 #include <ext/pb\_ds/assoc\_container.hpp> #include <ext/pb\_ds/tree\_policy.hpp> using namespace \_\_gnu\_pbds; Miscellaneous 11 template<class T> using ordered\_set = tree<T, null\_type, 11 less<T>, rb\_tree\_tag, tree\_order\_statistics\_node\_update>; 11 $vi d4x = \{1, 0, -1, 0\};$ $vi d4y = \{0, 1, 0, -1\};$ Setting Fixed D.P. Precision . . . . . . . . . . . . . . . vi $d8x = \{1, 0, -1, 0, 1, 1, -1, -1\};$ vi d8y = {0, 1, 0, -1, 1, -1, 1, -1}; rng(chrono::steady\_clock::now().time\_since\_epoch().count()); Geometry template<typename T> struct TPoint{ Тх, у; int id; static constexpr T eps = static\_cast<T>(1e-9); TPoint() : x(0), y(0), id(-1) {} TPoint(const $T\& x_-$ , const $T\& y_-$ ) : $x(x_-)$ , $y(y_-)$ , id(-1) {}

 $\rightarrow$  y(y\_), id(id\_) {}

 $\label{eq:total_total_total} TPoint(const \ T\& \ x\_, \ const \ T\& \ y\_, \ const \ int \ id\_) \ : \ x(x\_),$ 

TPoint operator + (const TPoint& rhs) const {

```
return TPoint(x + rhs.x, y + rhs.y);
                                                                            int sign(const T& x){
11
                                                                        82
                                                                              if (abs(x) <= TPoint<T>::eps) return 0;
12
                                                                        83
13
      TPoint operator - (const TPoint& rhs) const {
                                                                        84
                                                                              return x > 0? +1 : -1;
        return TPoint(x - rhs.x, y - rhs.y);
14
                                                                        85
                                                                            template<typename T>
      TPoint operator * (const T& rhs) const {
                                                                            T area(const vector<TPoint<T>>& pts){
16
                                                                        87
17
        return TPoint(x * rhs, y * rhs);
                                                                               int n = sz(pts);
                                                                        88
                                                                              T ans = 0;
18
                                                                        89
      TPoint operator / (const T& rhs) const {
                                                                              for (int i = 0; i < n; i++){
19
                                                                        90
        return TPoint(x / rhs, y / rhs);
20
                                                                                 ans += vmul(pts[i], pts[(i + 1) % n]);
21
                                                                        92
      TPoint ort() const {
22
                                                                        93
                                                                              return abs(ans) / 2;
23
        return TPoint(-y, x);
                                                                        94
                                                                            template<typename T>
24
                                                                        95
      T abs2() const {
                                                                            T dist_pp(const TPoint<T>& a, const TPoint<T>& b){
25
        return x * x + y * y;
                                                                              return sqrt(sq(a.x - b.x) + sq(a.y - b.y));
26
                                                                        97
27
                                                                        98
28
    };
                                                                        99
                                                                            template<typename T>
    template<typename T>
                                                                            TLine<T> perp_line(const TLine<T>& 1, const TPoint<T>& p){
29
                                                                       100
    bool operator< (TPoint<T>& A, TPoint<T>& B){
                                                                              T na = -1.b, nb = 1.a, nc = -na * p.x - nb * p.y;
30
                                                                       101
      return make_pair(A.x, A.y) < make_pair(B.x, B.y);</pre>
                                                                              return TLine<T>(na, nb, nc);
31
                                                                       102
32
                                                                       103
    template<typename T>
33
                                                                       104
                                                                            template<typename T>
    bool operator== (TPoint<T>& A, TPoint<T>& B){
                                                                            TPoint<T> projection(const TPoint<T>& p, const TLine<T>& 1){
      return abs(A.x - B.x) \leftarrow TPoint < T > :: eps && abs(A.y - B.y) <= 106
                                                                              return intersection(1, perp_line(1, p));
35
        TPoint<T>::eps;
                                                                       107
    7
                                                                             template<typename T>
36
                                                                       108
    template<typename T>
                                                                            T dist_pl(const TPoint<T>& p, const TLine<T>& 1){
37
                                                                       109
    struct TLine{
                                                                               return dist_pp(p, projection(p, 1));
      T a, b, c;
39
                                                                       111
      TLine(): a(0), b(0), c(0) {}
                                                                            template<typename T>
                                                                       112
40
      TLine(const T\& a_, const T\& b_, const T\& c_) : a(a_), b(b_), 113
                                                                            struct TRay{
                                                                              TLine<T> 1;
                                                                       114
42
      TLine(const TPoint<T>& p1, const TPoint<T>& p2){
                                                                              TPoint<T> start, dirvec;
        a = p1.y - p2.y;
43
                                                                       116
                                                                              TRay() : 1(), start(), dirvec() {}
        b = p2.x - p1.x;
                                                                       117
                                                                               TRay(const TPoint<T>& p1, const TPoint<T>& p2){
44
        c = -a * p1.x - b * p1.y;
45
                                                                       118
                                                                                1 = TLine < T > (p1, p2);
                                                                                start = p1, dirvec = p2 - p1;
46
                                                                       119
                                                                              }
47
    };
    template<typename T>
48
                                                                       121
                                                                            1:
    T det(const T& a11, const T& a12, const T& a21, const T& a22){ _{122}}
49
                                                                             template<typename T>
      return a11 * a22 - a12 * a21;
                                                                            bool is_on_line(const TPoint<T>& p, const TLine<T>& 1){
50
                                                                       123
                                                                              return abs(l.a * p.x + l.b * p.y + l.c) <= TPoint<T>::eps;
51
                                                                       124
    template<typename T>
52
                                                                       125
    T sq(const T& a){
                                                                            template<typename T>
53
                                                                       126
      return a * a;
                                                                            bool is_on_ray(const TPoint<T>& p, const TRay<T>& r){
54
                                                                       127
55
                                                                       128
                                                                               if (is_on_line(p, r.l)){
                                                                                return sign(smul(r.dirvec, TPoint<T>(p - r.start))) != -1;
    template<typename T>
56
                                                                       129
57
    T smul(const TPoint<T>& a, const TPoint<T>& b){
                                                                       130
      return a.x * b.x + a.y * b.y;
                                                                               else return false;
58
                                                                       131
59
                                                                       132
    template<typename T>
                                                                            template<typename T>
60
                                                                       133
    T vmul(const TPoint<T>& a, const TPoint<T>& b){
                                                                       134
                                                                            bool is_on_seg(const TPoint<T>& P, const TPoint<T>& A, const
62
      return det(a.x, a.y, b.x, b.y);
                                                                             \hookrightarrow TPoint<T>& B){
                                                                              return is_on_ray(P, TRay<T>(A, B)) && is_on_ray(P,
63
                                                                       135
    template<typename T>
                                                                             \hookrightarrow TRay<T>(B, A));
64
    bool parallel(const TLine<T>& 11, const TLine<T>& 12){
                                                                            }
65
                                                                       136
      return abs(vmul(TPoint<T>(11.a, 11.b), TPoint<T>(12.a,
                                                                            template<typename T>
     T dist_pr(const TPoint<T>& P, const TRay<T>& R){
                                                                       138
67
                                                                       139
                                                                               auto H = projection(P, R.1);
    template<typename T>
                                                                       140
                                                                              return is_on_ray(H, R)? dist_pp(P, H) : dist_pp(P, R.start);
68
    bool equivalent(const TLine<T>& 11, const TLine<T>& 12){
69
                                                                       141
      return parallel(11, 12) &&
                                                                            template<typename T>
      abs(det(11.b, 11.c, 12.b, 12.c)) <= TPoint<T>::eps &&
                                                                            T dist_ps(const TPoint<T>& P, const TPoint<T>& A, const
71
                                                                       143

→ TPoint<T>& B){
      abs(det(11.a, 11.c, 12.a, 12.c)) <= TPoint<T>::eps;
72
                                                                              auto H = projection(P, TLine<T>(A, B));
73
                                                                       144
    template<typename T>
                                                                               if (is_on_seg(H, A, B)) return dist_pp(P, H);
74
                                                                       145
    TPoint<T> intersection(const TLine<T>& 11, const TLine<T>&
                                                                       146
                                                                               else return min(dist_pp(P, A), dist_pp(P, B));

→ 12) {

                                                                       147
      return TPoint<T>(
                                                                             template<typename T>
                                                                       148
                                                                            bool acw(const TPoint<T>& A, const TPoint<T>& B){
        det(-11.c, 11.b, -12.c, 12.b) / det(11.a, 11.b, 12.a,
                                                                       149
                                                                       150
                                                                              T \text{ mul} = vmul(A, B);
        det(11.a, -11.c, 12.a, -12.c) / det(11.a, 11.b, 12.a,
                                                                               return mul > 0 || abs(mul) <= TPoint<T>::eps;
                                                                       151
        12.b)
                                                                       152
                                                                       153
                                                                             template<typename T>
                                                                            bool cw(const TPoint<T>& A, const TPoint<T>& B){
80
                                                                       154
    template<typename T>
                                                                               T mul = vmul(A, B);
                                                                       155
```

```
return mul < 0 || abs(mul) <= TPoint<T>::eps;
                                                                              template<typename T>
                                                                              vector<TPoint<T>> minkowski_sum(vector<TPoint<T>> P,
157
                                                                        228
158
     template<typename T>

    vector<TPoint<T>> Q){
     vector<TPoint<T>> convex_hull(vector<TPoint<T>> pts){
                                                                                minkowski_rotate(P);
159
                                                                        229
        sort(all(pts));
                                                                                minkowski_rotate(Q);
       pts.erase(unique(all(pts)), pts.end());
                                                                                P.pb(P[0]);
161
                                                                        231
162
        vector<TPoint<T>> up, down;
                                                                        232
                                                                                Q.pb(Q[0]);
       for (auto p : pts){
163
                                                                                vector<TPoint<T>> ans;
                                                                        233
         while (sz(up) > 1 \&\& acw(up.end()[-1] - up.end()[-2], p -
                                                                                int i = 0, j = 0;
                                                                        234
164
        up.end()[-2])) up.pop_back();
                                                                                while (i < sz(P) - 1 \mid | j < sz(Q) - 1){
         while (sz(down) > 1 \&\& cw(down.end()[-1] - down.end()[-2], 236
                                                                                  ans.pb(P[i] + Q[j]);
165
         p - down.end()[-2])) down.pop_back();
166
         up.pb(p), down.pb(p);
                                                                                  if (i == sz(P) - 1) curmul = -1;
                                                                                  else if (j == sz(Q) - 1) curmul = +1;
167
                                                                        239
       for (int i = sz(up) - 2; i >= 1; i--) down.pb(up[i]);
                                                                                  else curmul = vmul(P[i + 1] - P[i], Q[j + 1] - Q[j]);
168
                                                                        240
                                                                                  if (abs(curmul) < TPoint<T>::eps || curmul > 0) i++;
       return down:
                                                                        241
169
170
                                                                        242
                                                                                  if (abs(curmul) < TPoint<T>::eps || curmul < 0) j++;</pre>
171
                                                                        243
     template<typename T>
                                                                                return ans;
172
                                                                        244
     bool in_triangle(TPoint<T>& P, TPoint<T>& A, TPoint<T>& B,
                                                                             }
173
                                                                        245
      → TPoint<T>& C){
                                                                              using Point = TPoint<11>; using Line = TLine<11>; using Ray =
                                                                        246
       if (is_on_seg(P, A, B) || is_on_seg(P, B, C) || is_on_seg(P,
                                                                              \rightarrow TRay<11>; const ld PI = acos(-1);

⇔ C. A)) return true:

       return cw(P - A, B - A) == cw(P - B, C - B) &&
       cw(P - A, B - A) == cw(P - C, A - C);
                                                                              Strings
176
177
     template<typename T>
178
                                                                              vector<int> prefix_function(string s){
179
     void prep_convex_poly(vector<TPoint<T>>& pts){
                                                                               int n = sz(s);
       rotate(pts.begin(), min_element(all(pts)), pts.end());
180
                                                                                vector<int> pi(n);
181
                                                                                for (int i = 1; i < n; i++){
     // 0 - Outside, 1 - Exclusively Inside, 2 - On the Border
182
                                                                                  int k = pi[i - 1];
     template<typename T>
183
                                                                                  while (k > 0 \&\& s[i] != s[k]){
     int in_convex_poly(TPoint<T>& p, vector<TPoint<T>>& pts){
184
                                                                                    k = pi[k - 1];
        int n = sz(pts);
       if (!n) return 0;
186
                                                                                  pi[i] = k + (s[i] == s[k]);
        if (n <= 2) return is_on_seg(p, pts[0], pts.back());</pre>
187
                                                                         10
       int 1 = 1, r = n - 1;
188
                                                                         11
                                                                                return pi;
       while (r - 1 > 1){
189
                                                                             }
          int mid = (1 + r) / 2;
                                                                              vector<int> kmp(string s, string k){
                                                                         13
          if (acw(pts[mid] - pts[0], p - pts[0])) 1 = mid;
191
                                                                                string st = k + "#" + s;
                                                                         14
192
                                                                                vector<int> res;
193
                                                                                auto pi = pf(st);
                                                                         16
       if (!in_triangle(p, pts[0], pts[1], pts[1 + 1])) return 0;
194
                                                                                for (int i = 0; i < sz(st); i++){
        if (is_on_seg(p, pts[1], pts[1 + 1]) ||
195
                                                                                  if (pi[i] == sz(k)){
                                                                         18
          is_on_seg(p, pts[0], pts.back()) ||
196
                                                                         19
                                                                                    res.pb(i - 2 * sz(k));
          is_on_seg(p, pts[0], pts[1])
197
                                                                         20
       ) return 2;
198
                                                                                }
                                                                         21
       return 1:
199
                                                                                return res;
200
                                                                             }
                                                                         23
     // 0 - Outside, 1 - Exclusively Inside, 2 - On the Border
201
                                                                              vector<int> z_function(string s){
                                                                         24
202
     template<typename T>
                                                                               int n = sz(s):
                                                                         25
     int in_simple_poly(TPoint<T> p, vector<TPoint<T>>& pts){
203
                                                                                vector<int> z(n);
                                                                         26
       int n = sz(pts):
                                                                                int 1 = 0, r = 0;
205
       bool res = 0;
                                                                                for (int i = 1; i < n; i++){
                                                                         28
206
       for (int i = 0; i < n; i++){
                                                                                  if (r \ge i) z[i] = min(z[i - 1], r - i + 1);
          auto a = pts[i], b = pts[(i + 1) % n];
207
                                                                                  while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]]){
         if (is_on_seg(p, a, b)) return 2;
208
          if (((a.y > p.y) - (b.y > p.y)) * vmul(b - p, a - p) >
      \  \, \neg \  \, TPoint < T > : :eps) \{
                                                                                  if (i + z[i] - 1 > r){
                                                                         33
           res ^= 1;
210
                                                                                    l = i, r = i + z[i] - 1;
211
                                                                         35
212
                                                                         36
       return res;
213
                                                                         37
                                                                                return z;
214
     template<typename T>
215
     void minkowski_rotate(vector<TPoint<T>>& P){
216
        int pos = 0;
217
                                                                              Flows
218
       for (int i = 1; i < sz(P); i++){
          if (abs(P[i].y - P[pos].y) <= TPoint<T>::eps){
219
220
            if (P[i].x < P[pos].x) pos = i;
                                                                              O(N^2M), on unit networks O(N^{1/2}M)
221
          else if (P[i].y < P[pos].y) pos = i;</pre>
222
                                                                              struct FlowEdge {
223
                                                                                  int v, u;
       rotate(P.begin(), P.begin() + pos, P.end());
224
                                                                                  long long cap, flow = 0;
                                                                          3
225
                                                                                  FlowEdge(int v, int u, long long cap) : v(v), u(u),
     // P and Q are strictly convex, points given in
226
                                                                              \hookrightarrow cap(cap) {}
      }:
```

227

#### struct Dinic { const long long flow\_inf = 1e18; vector<FlowEdge> edges; vector<vector<int>> adj; 9 int n, m = 0;int s, t; 11 12 vector<int> level, ptr; queue<int> q; 13 Dinic(int n, int s, int t) : n(n), s(s), t(t) { 14 adj.resize(n); level.resize(n): 16 ptr.resize(n); 17 } 18 void add\_edge(int v, int u, long long cap) { 19 edges.emplace\_back(v, u, cap); 20 edges.emplace\_back(u, v, 0); 21 22 adj[v].push\_back(m); adj[u].push\_back(m + 1); 23 m += 2;24 7 25 bool bfs() { 26 while (!q.empty()) { 27 int v = q.front(); 28 q.pop(); for (int id : adj[v]) { 30 if (edges[id].cap - edges[id].flow < 1)</pre> 31 continue; if (level[edges[id].u] != -1) 33 continue; level[edges[id].u] = level[v] + 1; 35 q.push(edges[id].u); 36 } 37 } 38 39 return level[t] != -1; } 40 long long dfs(int v, long long pushed) { 41 if (pushed == 0) 42 return 0; 43 if (v == t) 44 return pushed; 45 for (int& cid = ptr[v]; cid < (int)adj[v].size();</pre> 46 cid++) { int id = adj[v][cid]; 47 int u = edges[id].u; 48 if (level[v] + 1 != level[u] || edges[id].cap -49 edges[id].flow < 1) 50 continue: long long tr = dfs(u, min(pushed, edges[id].cap -51 edges[id].flow)); if (tr == 0) 52 53 continue; edges[id].flow += tr; 54 edges[id ^ 1].flow -= tr; 56 return tr; } 57 return 0; 59 long long flow() { long long f = 0; 61 62 while (true) { fill(level.begin(), level.end(), -1); 63 level[s] = 0;64 q.push(s); if (!bfs()) 66 67 fill(ptr.begin(), ptr.end(), 0); 68 while (long long pushed = dfs(s, flow\_inf)) { 69 70 f += pushed; 71 } 73 return f: } 74 }; 75 // To recover flow through original edges: iterate over even 76 indices in edges.

# MCMF – maximize flow, then minimize its cost. O(Fmn).

```
#include <ext/pb_ds/priority_queue.hpp>
template <typename T, typename C>
class MCMF {
public:
   static constexpr T eps = (T) 1e-9;
   struct edge {
     int from;
     int to;
     T c:
     T f:
     C cost;
   };
   int n;
   vector<vector<int>> g;
   vector<edge> edges;
   vector<C> d;
   vector<C> pot;
   __gnu_pbds::priority_queue<pair<C, int>> q;
   vector<typename decltype(q)::point_iterator> its;
   vector<int> pe;
   const C INF_C = numeric_limits<C>::max() / 2;
   explicit MCMF(int n_{int} n_{int}) : n(n_{int}), g(n), d(n), pot(n, 0),
\rightarrow its(n), pe(n) {}
   int add(int from, int to, T forward_cap, C edge_cost, T

→ backward_cap = 0) {
     \texttt{assert(0} \mathrel{<=} \texttt{from } \&\& \texttt{ from } < \texttt{n} \&\& \texttt{ 0} \mathrel{<=} \texttt{to } \&\& \texttt{ to } < \texttt{n);}
     assert(forward_cap >= 0 && backward_cap >= 0);
     int id = static_cast<int>(edges.size());
     g[from].push_back(id);
     edges.push_back({from, to, forward_cap, 0, edge_cost});
     g[to].push_back(id + 1);
     edges.push_back({to, from, backward_cap, 0, -edge_cost});
     return id;
   void expath(int st) {
     fill(d.begin(), d.end(), INF_C);
     q.clear();
     fill(its.begin(), its.end(), q.end());
     its[st] = q.push({pot[st], st});
     d[st] = 0;
     while (!q.empty()) {
       int i = q.top().second;
       q.pop();
       its[i] = q.end();
       for (int id : g[i]) {
         const edge &e = edges[id];
         int j = e.to;
         if (e.c - e.f > eps \&\& d[i] + e.cost < d[j]) {
           d[j] = d[i] + e.cost;
           pe[j] = id;
            if (its[j] == q.end()) {
              its[j] = q.push({pot[j] - d[j], j});
              q.modify(its[j], {pot[j] - d[j], j});
         }
       }
     swap(d, pot);
   pair<T, C> max_flow(int st, int fin) {
     T flow = 0;
     C cost = 0;
     bool ok = true;
     for (auto& e : edges) {
       if (e.c - e.f > eps && e.cost + pot[e.from] - pot[e.to]
ok = false;
```

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67

69

70

```
break:
    if (ok) {
      expath(st);
    } else {
      vector<int> deg(n, 0);
      for (int i = 0; i < n; i++) {
        for (int eid : g[i]) {
          auto& e = edges[eid];
          if (e.c - e.f > eps) {
            deg[e.to] += 1;
        }
      }
      vector<int> que;
      for (int i = 0; i < n; i++) {
        if (deg[i] == 0) {
          que.push_back(i);
      for (int b = 0; b < (int) que.size(); b++) {</pre>
        for (int eid : g[que[b]]) {
          auto& e = edges[eid];
          if (e.c - e.f > eps) {
            deg[e.to] -= 1;
            if (deg[e.to] == 0) {
              que.push_back(e.to);
          }
      fill(pot.begin(), pot.end(), INF_C);
      pot[st] = 0;
      if (static_cast<int>(que.size()) == n) {
        for (int v : que) {
          if (pot[v] < INF_C) {</pre>
            for (int eid : g[v]) {
              auto& e = edges[eid];
              if (e.c - e.f > eps) \{
                if (pot[v] + e.cost < pot[e.to]) {</pre>
                  pot[e.to] = pot[v] + e.cost;
                  pe[e.to] = eid;
            }
          }
        }
      } else {
        que.assign(1, st);
        vector<bool> in_queue(n, false);
        in_queue[st] = true;
        for (int b = 0; b < (int) que.size(); b++) {</pre>
          int i = que[b];
          in_queue[i] = false;
          for (int id : g[i]) {
            const edge &e = edges[id];
            if (e.c - e.f > eps && pot[i] + e.cost <
→ pot[e.to]) {
              pot[e.to] = pot[i] + e.cost;
              pe[e.to] = id;
              if (!in_queue[e.to]) {
                que.push_back(e.to);
                in_queue[e.to] = true;
          }
        }
      }
    while (pot[fin] < INF_C) {
      T push = numeric_limits<T>::max();
      int v = fin;
      while (v != st) {
        const edge &e = edges[pe[v]];
        push = min(push, e.c - e.f);
        v = e.from;
```

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101 102 103

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121

122

123

 $\frac{125}{126}$ 

127

128

130

131

132

134

135

136

137

138

139

140

141

142

143

144

145

146

147

```
}
148
              v = fin;
149
              while (v != st) {
150
                edge &e = edges[pe[v]];
151
                 e.f += push;
                edge &back = edges[pe[v] ^ 1];
153
154
                back.f -= push;
155
                v = e.from;
156
              flow += push;
              cost += push * pot[fin];
158
              expath(st);
159
160
           return {flow, cost};
161
         }
162
     };
163
164
      // Examples: MCMF < int, int > g(n); g.add(u, v, c, w, 0);
165
       \hookrightarrow g.max_flow(s,t).
      \begin{tabular}{ll} /\!/ & \textit{To recover flow through original edges: iterate over even} \end{tabular}
```

# Graphs

# Kuhn's algorithm for bipartite matching

```
The graph is split into 2 halves of n1 and n2 vertices.
     Complexity: O(n1 * m). Usually runs much faster. MUCH
3

→ FASTER!!!

 4
     const int N = 305;
     vector < int > g[N]; // Stores edges from left half to right.
     bool used[N]; // Stores if vertex from left half is used.
     int\ mt[N];\ /\!/\ For\ every\ vertex\ in\ right\ half,\ stores\ to\ which

→ vertex in left half it's matched (-1 if not matched).

     bool try_dfs(int v){
11
12
       if (used[v]) return false;
       used[v] = 1;
13
       for (auto u : g[v]){
14
         if (mt[u] == -1 || try_dfs(mt[u])){
           mt[u] = v;
16
           return true;
17
         }
18
       }
19
20
       return false;
21
23
     int main(){
       for (int i = 1; i <= n2; i++) mt[i] = -1;
25
       for (int i = 1; i <= n1; i++) used[i] = 0;
26
       for (int i = 1; i <= n1; i++){
27
         if (try_dfs(i)){
28
29
           for (int j = 1; j <= n1; j++) used[j] = 0;</pre>
         }
30
31
32
       vector<pair<int, int>> ans;
33
       for (int i = 1; i <= n2; i++){
         if (mt[i] != -1) ans.pb({mt[i], i});
       }
35
    }
36
37
    // Finding maximal independent set: size = # of nodes - # of
38
      \leftrightarrow edges in matching.
     // To construct: launch Kuhn-like DFS from unmatched nodes in
39
      \hookrightarrow the left half.
    // Independent set = visited nodes in left half + unvisited in
      \hookrightarrow right half.
    // Finding minimal vertex cover: complement of maximal
      \hookrightarrow independent set.
```

### Dijkstra's Algorithm

```
priority_queue<pair<11, 11>, vector<pair<11, 11>>,

    greater<pair<11, 11>>> q;

    dist[start] = 0;
    q.push({0, start});
    while (!q.empty()){
        auto [d, v] = q.top();
        q.pop();
        if (d != dist[v]) continue;
        for (auto [u, w] : g[v]){
          if (dist[u] > dist[v] + w){
9
            dist[u] = dist[v] + w;
10
            q.push({dist[u], u});
11
        }
13
    }
14
```

#### **EULERIAN CYCLE DFS**

```
void dfs(int v){
while (!g[v].empty()){
int u = g[v].back();

g[v].pop_back();

dfs(u);

ans.pb(v);

}

}
```

# Strongly Connected Components: Kosaraju's Algorithm

```
vector<vector<int>>> adj, adj_rev;
    vector<bool> used:
    vector<int> order, component;
    void dfs1(int v) {
5
         used[v] = true;
         for (auto u : adj[v])
8
9
             if (!used[u])
                 dfs1(u);
10
11
         order.push_back(v);
12
13
    }
14
    void dfs2(int v) {
15
         used[v] = true;
16
17
         component.push_back(v);
19
         for (auto u : adj_rev[v])
             if (!used[u])
20
21
                 dfs2(u):
    }
22
23
    int main(){
24
25
         used.assign(n, false);
26
27
         for (int i = 0; i < n; i++)
             if (!used[i])
29
                 dfs1(i);
30
         used.assign(n, false);
31
         reverse(order.begin(), order.end());
32
         for (auto v : order)
33
             if (!used[v]) {
34
                 dfs2(v);
35
36
                 // process
                 component.clear();
37
             }
38
    }
39
```

## Finding Bridges

```
Bridges.
2
    Results are stored in a map "is_bridge".
    For each connected component, call "dfs(starting vertex,

    starting vertex)".

5
    const int N = 2e5 + 10; // Careful with the constant!
6
    vector<int> g[N];
    int tin[N], fup[N], timer;
    map<pair<int, int>, bool> is_bridge;
10
11
    void dfs(int v, int p){
12
      tin[v] = ++timer;
13
      fup[v] = tin[v];
14
15
      for (auto u : g[v]){
16
        if (!tin[u]){
17
           dfs(u, v);
           if (fup[u] > tin[v]){
18
             is_bridge[{u, v}] = is_bridge[{v, u}] = true;
20
21
           fup[v] = min(fup[v], fup[u]);
22
         else{
23
           if (u != p) fup[v] = min(fup[v], tin[u]);
24
25
26
    }
27
```

#### Virtual Tree

```
// order stores the nodes in the queried set
    sort(all(order), [&] (int u, int v){return tin[u] < tin[v];});</pre>
    int m = sz(order);
    for (int i = 1; i < m; i++){
4
5
         order.pb(lca(order[i], order[i - 1]));
6
    sort(all(order), [&] (int u, int v){return tin[u] < tin[v];});</pre>
    order.erase(unique(all(order)), order.end());
    vector<int> stk{order[0]};
    for (int i = 1; i < sz(order); i++){</pre>
10
         int v = order[i];
11
         while (tout[stk.back()] < tout[v]) stk.pop_back();</pre>
12
         int u = stk.back();
13
         vg[u].pb({v, dep[v] - dep[u]});
14
15
         stk.pb(v);
    }
```

#### HLD ON EDGES DFS

```
void dfs1(int v, int p, int d){
      par[v] = p;
2
3
      for (auto e : g[v]){
        if (e.fi == p){
4
          g[v].erase(find(all(g[v]), e));
        }
7
      }
      dep[v] = d;
9
       sz[v] = 1;
10
11
      for (auto [u, c] : g[v]){
        dfs1(u, v, d + 1);
12
         sz[v] += sz[u];
13
14
      if (!g[v].empty()) iter_swap(g[v].begin(),
15

→ max_element(all(g[v]), comp));
16
17
    void dfs2(int v, int rt, int c){
      pos[v] = sz(a);
18
      a.pb(c);
19
      root[v] = rt:
20
21
      for (int i = 0; i < sz(g[v]); i++){
        auto [u, c] = g[v][i];
22
        if (!i) dfs2(u, rt, c);
23
```

```
25
    }
26
                                                                           void sieve (int n){
                                                                             fill(is_composite, is_composite + n, false);
    int getans(int u, int v){
27
      int res = 0;
                                                                             phi[1] = 1;
      for (; root[u] != root[v]; v = par[root[v]]){
                                                                             for (int i = 2; i < n; ++i) {
29
        if (dep[root[u]] > dep[root[v]]) swap(u, v);
30
                                                                               if (!is_composite[i]) {
                                                                                 prime.push_back (i);
31
        res = max(res, rmq(0, 0, n - 1, pos[root[v]], pos[v]));
                                                                                 phi[i] = i - 1; //i is prime
32
                                                                      11
33
      if (pos[u] > pos[v]) swap(u, v);
                                                                             for (int j = 0; j < prime.size () && i * prime[j] < n; ++j)
      return max(res, rmq(0, 0, n - 1, pos[u] + 1, pos[v]));
34
                                                                               is_composite[i * prime[j]] = true;
                                                                       14
                                                                               if (i % prime[j] == 0) {
                                                                      15
    Centroid Decomposition
                                                                                 phi[i * prime[j]] = phi[i] * prime[j]; //prime[j]
                                                                            \hookrightarrow divides i
    vector<char> res(n), seen(n), sz(n);
                                                                                 break;
    function<int(int, int)> get_size = [&](int node, int fa) {
                                                                                 } else {
      sz[node] = 1;
                                                                                 phi[i * prime[j]] = phi[i] * phi[prime[j]]; //prime[j]
                                                                       19
      for (auto& ne : g[node]) {
                                                                               does not divide i
        if (ne == fa || seen[ne]) continue;
                                                                                 }
                                                                      20
        sz[node] += get_size(ne, node);
                                                                      21
                                                                             }
                                                                      22
      return sz[node];
                                                                           }
    }:
    function<int(int, int, int)> find_centroid = [&](int node, int
     \hookrightarrow fa, int t) {
                                                                           Gaussian Elimination
11
      for (auto& ne : g[node])
        if (ne != fa && !seen[ne] && sz[ne] > t / 2) return
                                                                           bool is_0(Z v) { return v.x == 0; }

    find_centroid(ne, node, t);

                                                                           Z abs(Z v) { return v; }
                                                                           bool is_0(double v) { return abs(v) < 1e-9; }</pre>
    }:
14
    function<void(int, char)> solve = [&](int node, char cur) {
                                                                           // 1 => unique solution, 0 => no solution, -1 => multiple
15
      get_size(node, -1); auto c = find_centroid(node, -1,

⇒ solutions

16
                                                                           template <typename T>
     ⇔ sz[node]);
17
      seen[c] = 1, res[c] = cur;
                                                                           int gaussian_elimination(vector<vector<T>>> &a, int limit) {
                                                                               if (a.empty() || a[0].empty()) return -1;
      for (auto& ne : g[c]) {
18
        if (seen[ne]) continue;
                                                                             int h = (int)a.size(), w = (int)a[0].size(), r = 0;
        solve(ne, char(cur + 1)); // we can pass c here to build
                                                                             for (int c = 0; c < limit; c++) {</pre>
20
                                                                       10
                                                                               int id = -1;
                                                                      11
21
      }
                                                                       12
                                                                               for (int i = r; i < h; i++) {
    };
                                                                                 if (!is_0(a[i][c]) \&\& (id == -1 || abs(a[id][c]) <
                                                                      13
                                                                               abs(a[i][c]))) {
                                                                                   id = i:
                                                                      14
    Math
                                                                      15
                                                                               }
                                                                      16
                                                                      17
                                                                               if (id == -1) continue;
    Binary exponentiation
                                                                               if (id > r) {
                                                                                 swap(a[r], a[id]);
                                                                      19
    11 power(ll a, ll b){
                                                                                 for (int j = c; j < w; j++) a[id][j] = -a[id][j];
      ll res = 1;
                                                                      21
      for (; b; a = a * a \% MOD, b >>= 1){
                                                                               vector<int> nonzero;
                                                                      22
        if (b & 1) res = res * a \% MOD;
                                                                               for (int j = c; j < w; j++) {
                                                                      23
                                                                                 if (!is_0(a[r][j])) nonzero.push_back(j);
                                                                      24
      return res;
                                                                      25
                                                                               T inv_a = 1 / a[r][c];
                                                                      26
                                                                               for (int i = r + 1; i < h; i++) {
                                                                                 if (is_0(a[i][c])) continue;
    Extended Euclidean Algorithm
                                                                                 T coeff = -a[i][c] * inv_a;
                                                                                 for (int j : nonzero) a[i][j] += coeff * a[r][j];
    // gives (x, y) for ax + by = g
                                                                               }
                                                                      31
    // solutions given (x0, y0): a(x0 + kb/g) + b(y0 - ka/g) = g
    int gcd(int a, int b, int& x, int& y) {
                                                                      33
      x = 1, y = 0; int sum1 = a;
                                                                             for (int row = h - 1; row >= 0; row--) {
                                                                      34
      int x2 = 0, y2 = 1, sum2 = b;
                                                                               for (int c = 0; c < limit; c++) {
                                                                      35
      while (sum2) {
                                                                                 if (!is_0(a[row][c])) {
                                                                      36
        int q = sum1 / sum2;
                                                                                   T inv_a = 1 / a[row][c];
        tie(x, x2) = make_tuple(x2, x - q * x2);
                                                                                   for (int i = row - 1; i >= 0; i--) {
        tie(y, y2) = make_tuple(y2, y - q * y2);
                                                                                     if (is_0(a[i][c])) continue;
        tie(sum1, sum2) = make_tuple(sum2, sum1 - q * sum2);
10
                                                                                     T coeff = -a[i][c] * inv_a;
                                                                      40
11
                                                                                     for (int j = c; j < w; j++) a[i][j] += coeff *
                                                                      41
12
      return sum1:
                                                                               a[row][j];
    }
                                                                                   }
                                                                      42
                                                                                   break;
                                                                      43
                                                                                 }
    Linear Sieve
                                                                      44
                                                                      45
                                                                             } // not-free variables: only it on its line
    vector<int> prime;
                                                                      46
    bool is_composite[MAX_N];
                                                                             for(int i = r; i < h; i++) if(!is_0(a[i][limit])) return 0;</pre>
```

int phi[MAX\_N];

3

else dfs2(u, u, c);

```
return (r == limit) ? 1 : -1;
                                                                              auto fft = [&](vector<complex<ld>>& p, int inv) {
48
                                                                       11
                                                                                for (int i = 0; i < len; i++)
49
                                                                       12
50
                                                                        13
                                                                                  if (i < rev[i]) swap(p[i], p[rev[i]]);</pre>
    template <typename T>
                                                                                for (int mid = 1; mid < len; mid *= 2) {</pre>
51
                                                                        14
    pair<int, vector<T>> solve_linear(vector<vector<T>> a, const
                                                                                   auto w1 = complex<ld>(cos(PI / mid), (inv ? -1 : 1) *

    vector<T> &b, int w) {

    sin(PI / mid));
      int h = (int)a.size();
                                                                                  for (int i = 0; i < len; i += mid * 2) {
53
                                                                        16
                                                                                    auto wk = complex<ld>(1, 0);
      for (int i = 0; i < h; i++) a[i].push_back(b[i]);</pre>
                                                                        17
54
      int sol = gaussian_elimination(a, w);
                                                                                    for (int j = 0; j < mid; j++, wk = wk * w1) {
55
                                                                       18
      if(!sol) return {0, vector<T>()};
                                                                                       auto x = p[i + j], y = wk * p[i + j + mid];
      vector<T> x(w, 0);
                                                                                      p[i + j] = x + y, p[i + j + mid] = x - y;
57
                                                                       20
      for (int i = 0; i < h; i++) {
58
                                                                       21
        for (int j = 0; j < w; j++) {
                                                                                  }
59
                                                                       22
          if (!is_0(a[i][j])) {
60
                                                                       23
            x[j] = a[i][w] / a[i][j];
                                                                                if (inv == 1) {
61
                                                                       ^{24}
                                                                                  for (int i = 0; i < len; i++) p[i].real(p[i].real() /</pre>
            break:
                                                                       25
62
63
        }
64
                                                                       26
                                                                                }
      }
                                                                              };
65
                                                                       27
      return {sol, x};
                                                                              fft(a, 0), fft(b, 0);
66
                                                                       28
                                                                              for (int i = 0; i < len; i++) a[i] = a[i] * b[i];
                                                                       29
                                                                              fft(a, 1);
                                                                              a.resize(n + m - 1);
                                                                       31
    NTT
                                                                              vector<ld> res(n + m - 1);
                                                                              for (int i = 0; i < n + m - 1; i++) res[i] = a[i].real();
                                                                       33
    void ntt(vector<ll>& a, int f) {
                                                                              return res:
                                                                       34
      int n = int(a.size());
                                                                       35
      vector<ll> w(n);
      vector<int> rev(n);
      for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] / 2) | ((i
                                                                            is_prime
     \leftrightarrow & 1) * (n / 2));
      for (int i = 0; i < n; i++) {
                                                                               • (Miller-Rabin primality test)
        if (i < rev[i]) swap(a[i], a[rev[i]]);</pre>
                                                                            i128 power(i128 a, i128 b, i128 MOD = 1, i128 res = 1) {
      11 \text{ wn} = power(f ? (MOD + 1) / 3 : 3, (MOD - 1) / n);
                                                                              for (; b; b /= 2, (a *= a) %= MOD)
9
                                                                                if (b & 1) (res *= a) %= MOD;
      w[0] = 1;
      for (int i = 1; i < n; i++) w[i] = w[i - 1] * wn % MOD;
                                                                              return res:
11
      for (int mid = 1; mid < n; mid *= 2) {
                                                                            }
        for (int i = 0; i < n; i += 2 * mid) {
13
14
           for (int j = 0; j < mid; j++) {
                                                                            bool is_prime(ll n) {
            ll x = a[i + j], y = a[i + j + mid] * w[n / (2 * mid)
                                                                              if (n < 2) return false;
       * j] % MOD;
                                                                              static constexpr int A[] = \{2, 3, 5, 7, 11, 13, 17, 19, 23\};
            a[i + j] = (x + y) \% MOD, a[i + j + mid] = (x + MOD - i)
                                                                              int s = __builtin_ctzll(n - 1);
                                                                              ll d = (n - 1) >> s;
        y) % MOD;
                                                                        11
                                                                              for (auto a : A) {
17
        }
                                                                                if (a == n) return true;
18
                                                                        13
                                                                                11 x = (11)power(a, d, n);
19
                                                                       14
      if (f) {
                                                                                if (x == 1 \mid \mid x == n - 1) continue;
20
                                                                       15
        ll iv = power(n, MOD - 2);
                                                                                bool ok = false;
21
                                                                       16
        for (auto& x : a) x = x * iv % MOD;
                                                                                for (int i = 0; i < s - 1; ++i) {
22
                                                                       17
                                                                                  x = 11((i128)x * x % n); // potential overflow!
23
    }
                                                                                  if (x == n - 1) {
24
25
    vector<ll> mul(vector<ll> a, vector<ll> b) {
                                                                       20
                                                                                    ok = true;
      int n = 1, m = (int)a.size() + (int)b.size() - 1;
                                                                                     break;
26
                                                                       21
                                                                                  }
      while (n < m) n *= 2;
28
      a.resize(n), b.resize(n):
                                                                       23
                                                                                if (!ok) return false;
      ntt(a, 0), ntt(b, 0); // if squaring, you can save one NTT
29
                                                                              7
                                                                       25
      for (int i = 0; i < n; i++) a[i] = a[i] * b[i] % MOD;
                                                                       26
                                                                              return true;
30
31
      ntt(a, 1);
                                                                       27
      a.resize(m):
32
                                                                            ll pollard_rho(ll x) {
33
      return a;
                                                                              ll s = 0, t = 0, c = rng() \% (x - 1) + 1;
                                                                        2
   }
                                                                              ll stp = 0, goal = 1, val = 1;
                                                                              for (goal = 1;; goal *= 2, s = t, val = 1) {
                                                                                for (stp = 1; stp <= goal; ++stp) {
    FFT
                                                                                  t = 11(((i128)t * t + c) % x);
                                                                                  val = 11((i128)val * abs(t - s) % x);
    const ld PI = acosl(-1);
                                                                                  if ((stp % 127) == 0) {
    auto mul = [&](const vector<ld>& aa, const vector<ld>& bb) {
                                                                        9
                                                                                    11 d = gcd(val, x);
      int n = (int)aa.size(), m = (int)bb.size(), bit = 1;
                                                                                     if (d > 1) return d;
                                                                        10
      while ((1 << bit) < n + m - 1) bit++;
                                                                        11
                                                                                  }
      int len = 1 << bit;</pre>
                                                                                }
      vector<complex<ld>>> a(len), b(len);
                                                                       12
                                                                                11 d = gcd(val, x);
                                                                        13
      vector<int> rev(len);
                                                                                if (d > 1) return d;
      for (int i = 0; i < n; i++) a[i].real(aa[i]);</pre>
                                                                       14
                                                                       15
      for (int i = 0; i < m; i++) b[i].real(bb[i]);</pre>
                                                                            }
                                                                       16
      for (int i = 0; i < len; i++) rev[i] = (rev[i >> 1] >> 1) |
```

```
11 get_max_factor(11 _x) {
18
       11 max_factor = 0;
19
       function \langle void(11) \rangle fac = [&](11 x) {
20
         if (x \le max_factor | | x < 2) return;
21
         if (is_prime(x)) {
           max_factor = max_factor > x ? max_factor : x;
23
24
         }
25
         11 p = x;
26
27
         while (p >= x) p = pollard_rho(x);
         while ((x \% p) == 0) x /= p;
28
29
         fac(x), fac(p);
30
      }:
31
      fac(x);
      return max_factor;
32
33
```

#### Data Structures

### Sparse Table

```
const int N = 2e5 + 10, LOG = 20; // Change the constant!
    template<typename T>
    struct SparseTable{
    int lg[N];
    T st[N][LOG];
    int n;
    // Change this function
    function\langle T(T, T) \rangle f = [\&] (T a, T b){
9
      return min(a, b);
11
12
    void build(vector<T>& a){
13
      n = sz(a);
14
15
      lg[1] = 0;
      for (int i = 2; i \le n; i++) lg[i] = lg[i / 2] + 1;
16
17
      for (int k = 0; k < LOG; k++){
18
        for (int i = 0; i < n; i++){
19
          if (!k) st[i][k] = a[i];
20
          else st[i][k] = f(st[i][k-1], st[min(n-1, i+(1 <<
21
        (k - 1)) [k - 1];
22
      }
23
    }
24
25
    T query(int 1, int r){
26
      int sz = r - 1 + 1;
27
      return f(st[1][lg[sz]], st[r - (1 << lg[sz]) + 1][lg[sz]]);
29
    };
```

## Suffix Array and LCP array

• (uses SparseTable above)

```
struct SuffixArray{
       vector<int> p, c, h;
      SparseTable<int> st;
      In the end, array c gives the position of each suffix in p
      using 1-based indexation!
      SuffixArray() {}
10
      SuffixArray(string s){
11
12
        buildArrav(s):
        buildLCP(s);
13
14
         buildSparse();
15
16
      void buildArray(string s){
17
        int n = sz(s) + 1;
```

```
p.resize(n), c.resize(n);
    for (int i = 0; i < n; i++) p[i] = i;
    sort(all(p), [&] (int a, int b){return s[a] < s[b];});</pre>
    c[p[0]] = 0;
    for (int i = 1; i < n; i++){
      c[p[i]] = c[p[i - 1]] + (s[p[i]] != s[p[i - 1]]);
    vector<int> p2(n), c2(n);
    // w is half-length of each string.
    for (int w = 1; w < n; w <<= 1){
      for (int i = 0; i < n; i++){
        p2[i] = (p[i] - w + n) \% n;
      vector<int> cnt(n);
      for (auto i : c) cnt[i]++;
      for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
      for (int i = n - 1; i \ge 0; i--){
        p[--cnt[c[p2[i]]]] = p2[i];
      c2[p[0]] = 0;
      for (int i = 1; i < n; i++){
        c2[p[i]] = c2[p[i - 1]] +
        (c[p[i]] != c[p[i - 1]] ||
        c[(p[i] + w) \% n] != c[(p[i - 1] + w) \% n]);
      c.swap(c2);
    p.erase(p.begin());
  void buildLCP(string s){
    // The algorithm assumes that suffix array is already
 \Rightarrow built on the same string.
    int n = sz(s);
    h.resize(n - 1);
    int k = 0;
    for (int i = 0; i < n; i++){
      if (c[i] == n){
        k = 0;
        continue;
      int j = p[c[i]];
      while (i + k < n \&\& j + k < n \&\& s[i + k] == s[j + k])
 \hookrightarrow k++;
      h[c[i] - 1] = k;
      if (k) k--;
    }
    /*
    Then an RMQ Sparse Table can be built on array h
    to calculate LCP of 2 non-consecutive suffixes.
  }
  void buildSparse(){
    st.build(h);
  // l and r must be in O-BASED INDEXATION
  int lcp(int 1, int r){
    1 = c[1] - 1, r = c[r] - 1;
    if (1 > r) swap(1, r);
    return st.query(1, r - 1);
  }
};
```

#### Aho Corasick Trie

• For each node in the trie, the suffix link points to the longest proper suffix of the represented string. The terminal-link tree has square-root height (can be constructed by DFS).

```
const int S = 26;

// Function converting char to int.
int ctoi(char c){
```

19

20

21

22

24

25

26

27

29

31

32

33

34 35

36 37

38

39

40

41

43

44

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47 48

49

51

52

53

54

55

57

59

61

63

64

65

66

70 71

72

73

75

76

77

```
return c - 'a':
5
6
    // To add terminal links, use DFS
    struct Node{
      vector<int> nxt:
10
11
       int link;
12
      bool terminal;
13
      Node() {
        nxt.assign(S, -1), link = 0, terminal = 0;
15
16
17
    }:
18
    vector<Node> trie(1);
19
20
21
     // add_string returns the terminal vertex.
    int add_string(string& s){
22
      int v = 0:
23
      for (auto c : s){
24
         int cur = ctoi(c);
25
         if (trie[v].nxt[cur] == -1){
26
          trie[v].nxt[cur] = sz(trie);
27
           trie.emplace_back();
29
           = trie[v].nxt[cur];
30
      }
31
      trie[v].terminal = 1;
32
    }
34
35
36
    Suffix links are compressed.
37
    This means that:
      If vertex v has a child by letter x, then:
39
         trie[v].nxt[x] points to that child.
40
       If vertex v doesn't have such child, then:
41
         trie[v].nxt[x] points to the suffix link of that child
42
         if we would actually have it.
43
44
    void add_links(){
45
46
      queue<int> q:
      q.push(0);
47
      while (!q.empty()){
48
        auto v = q.front();
49
         int u = trie[v].link;
50
         q.pop();
51
         for (int i = 0; i < S; i++){
52
53
          int& ch = trie[v].nxt[i];
           if (ch == -1){
54
55
             ch = v? trie[u].nxt[i] : 0;
           }
56
             trie[ch].link = v? trie[u].nxt[i] : 0;
58
59
             q.push(ch);
60
61
      }
62
63
64
65
    bool is_terminal(int v){
      return trie[v].terminal;
66
67
68
    int get_link(int v){
69
70
      return trie[v].link;
71
72
    int go(int v, char c){
73
      return trie[v].nxt[ctoi(c)];
```

### Convex Hull Trick

• Allows to insert a linear function to the hull in (1) and get the minimum/maximum value of the stored function

- at a point in  $O(\log n)$ .
- NOTE: The lines must be added in the order of decreasing/increasing gradients. CAREFULLY CHECK THE SETUP BEFORE USING!
- IMPORTANT: THE DEFAULT VERSION SURELY WORKS. IF MODIFIED VERSIONS DON'T WORK, TRY TRANSFORMING THEM TO THE DEFAULT ONE BY CHANGING SIGNS.

```
struct line{
       11 k. b:
2
       11 f(11 x){
         return k * x + b:
4
    };
     vector<line> hull;
     void add_line(line nl){
10
11
       if (!hull.empty() && hull.back().k == nl.k){
         nl.b = min(nl.b, hull.back().b); // Default: minimum. For
12
         maximum change "min" to "max".
         hull.pop_back();
13
       }
       while (sz(hull) > 1){
15
         auto& 11 = hull.end()[-2], 12 = hull.back();
16
         if ((nl.b - l1.b) * (l2.k - nl.k) >= (nl.b - l2.b) * (l1.k)

    - nl.k)) hull.pop_back(); // Default: decreasing gradient
     \,\,\hookrightarrow\,\,k.\ \textit{For increasing $k$ change the sign to}\,\,<=.
         else break:
18
19
20
       hull.pb(nl);
21
22
    11 get(11 x){
23
24
       int l = 0, r = sz(hull);
       while (r - 1 > 1){
25
         int mid = (1 + r) / 2;
26
         if (hull[mid - 1].f(x) >= hull[mid].f(x)) 1 = mid; //
27
     \hookrightarrow Default: minimum. For maximum change the sign to <=.
         else r = mid;
       }
29
       return hull[1].f(x);
30
31
```

#### Li-Chao Segment Tree

- allows to add linear functions in any order and query minimum/maximum value of those at a point, all in O(log n).
- Clear: clear()

```
const 11 INF = 1e18; // Change the constant!
    struct LiChaoTree{
       struct line{
        11 k, b;
         line(){
5
          k = b = 0;
         line(ll k_, ll b_){
9
           k = k_{,} b = b_{;}
10
         11 f(11 x){
11
          return k * x + b:
12
         };
13
14
       };
       int n;
15
16
       bool minimum, on_points;
       vector<11> pts;
17
       vector<line> t;
18
19
         for (auto& 1 : t) 1.k = 0, 1.b = minimum? INF : -INF;
```

```
23
      LiChaoTree(int n_, bool min_){ // This is a default
24
     \hookrightarrow constructor for numbers in range [0, n - 1].
         n = n_, minimum = min_, on_points = false;
25
         t.resize(4 * n);
         clear():
27
28
29
      LiChaoTree(vector<ll> pts_, bool min_){ // This constructor
30
     \leftrightarrow will build LCT on the set of points you pass. The points
     → may be in any order and contain duplicates.
31
         pts = pts_, minimum = min_;
32
         sort(all(pts));
         pts.erase(unique(all(pts)), pts.end());
33
         on_points = true;
         n = sz(pts):
35
         t.resize(4 * n);
37
         clear();
38
39
       void add_line(int v, int l, int r, line nl){
40
         // Adding on segment [l, r)
41
         int m = (1 + r) / 2;
42
         11 lval = on_points? pts[1] : 1, mval = on_points? pts[m]
         if ((minimum && nl.f(mval) < t[v].f(mval)) || (!minimum &&
44
     \rightarrow nl.f(mval) > t[v].f(mval))) swap(t[v], nl);
         if (r - 1 == 1) return;
45
         if ((minimum && nl.f(lval) < t[v].f(lval)) || (!minimum &&
     \rightarrow nl.f(lval) > t[v].f(lval))) add_line(2 * v + 1, 1, m, nl);
         else add_line(2 * v + 2, m, r, nl);
47
48
49
50
      11 get(int v, int l, int r, int x){
         int m = (1 + r) / 2;
51
         if (r - l == 1) return t[v].f(on_points? pts[x] : x);
52
53
         else{
          if (minimum) return min(t[v].f(on_points? pts[x] : x), x
54
     \leftrightarrow < m? get(2 * v + 1, 1, m, x) : get(2 * v + 2, m, r, x));
           else return max(t[v].f(on\_points? pts[x] : x), x < m?
55
         get(2 * v + 1, 1, m, x) : get(2 * v + 2, m, r, x));
56
      }
57
       void add_line(ll k, ll b){
59
         add_line(0, 0, n, line(k, b));
60
61
62
63
      11 get(11 x){
        return get(0, 0, n, on_points? lower_bound(all(pts), x) -
     → pts.begin() : x);
      }; // Always pass the actual value of x, even if LCT is on
     \hookrightarrow points.
    };
66
```

# Persistent Segment Tree

• for RSQ

```
struct Node {
        ll val;
        Node *1, *r;
         Node(ll x) : val(x), l(nullptr), r(nullptr) {}
         Node(Node *11, Node *rr) {
            1 = 11, r = rr;
            val = 0;
            if (1) val += 1->val;
10
            if (r) val += r->val;
11
12
         Node(Node *cp) : val(cp->val), l(cp->l), r(cp->r) {}
    }:
13
    const int N = 2e5 + 20;
14
    ll a[N]:
15
    Node *roots[N];
16
17
    int n, cnt = 1;
    Node *build(int 1 = 1, int r = n) {
```

```
if (1 == r) return new Node(a[1]);
19
        int mid = (1 + r) / 2;
20
        return new Node(build(1, mid), build(mid + 1, r));
21
22
    Node *update(Node *node, int val, int pos, int l = 1, int r =
     if (1 == r) return new Node(val);
24
        int mid = (1 + r) / 2;
25
        if (pos > mid)
26
            return new Node(node->1, update(node->r, val, pos, mid
        + 1, r));
        else return new Node(update(node->1, val, pos, 1, mid),
       node->r):
29
    11 query(Node *node, int a, int b, int l = 1, int r = n) {
30
        if (1 > b || r < a) return 0;
31
        if (1 >= a \&\& r <= b) return node->val;
        int mid = (1 + r) / 2;
33
        return query(node->1, a, b, 1, mid) + query(node->r, a, b,
34
       mid + 1, r);
35
```

# Miscellaneous

#### Ordered Set

## Measuring Execution Time

```
ld tic = clock();
// execute algo...
ld tac = clock();
// Time in milliseconds
cerr << (tac - tic) / CLOCKS_PER_SEC * 1000 << endl;
// No need to comment out the print because it's done to cerr.</pre>
```

#### Setting Fixed D.P. Precision

```
cout << setprecision(d) << fixed;
// Each number is rounded to d digits after the decimal point,
and truncated.</pre>
```