1. Linear-inequality feasibility

① 首先一个线性规划问题通常是最大化或最小化的优化问题。其约束通常是等式或不等式约束。

那么将线性不等式求解问题写成线性规划问题可以表示成如下：

$$\max \quad 0$$

$$\text{s.t.} \quad Ax \le b \quad \text{线性不等式.}$$

利用线性规划算法求解出来的 x 则为线性不等式的可行解。常用的椭圆法中，不断缩小椭圆，使得它完全落在可行解范围内或产生冲突则退出循环，因此线性规划问题的解一定是不等式的可行解。

2. Airplane Landing Problem

线性规划模型：

$$\max \quad d$$

$$\text{s.t.} \quad 0 < s_1 < t_1 < s_2 < t_2 < \cdots < s_n < t_n$$

$$x_j - x_{j-1} \ge d \quad \text{其中 } j = 2, 3, \cdots n$$

$$s_i \le x_i \le t_i \quad \text{其中 } i = 1, 2, \cdots n$$

模型中各常数说明如下：

d 为连续两架飞机的着陆时间间隔值

s_i 为第 i 架飞机的窗口起始时间

t_i 为第 i 架飞机的窗口降落时间

x_i 为第 i 架飞机的实际着陆时间

n 为飞机降落的总数量

我们假设 4 个飞机降落，其窗口时间分别为

$[s_1, t_1] = [9:00, 9:30]$

$[s_2, t_2] = [10:00, 11:00]$

$[s_3, t_3] = [11:15, 11:30]$

$[s_4, t_4] = [12:00, 12:15]$

```
var x1;
var x2;
var x3;
var x4;
var d;

maximize z: d;

s.t. con1: x2 - x1 >= d;
s.t. con2: x3 - x2 >= d;
s.t. con3: x4 - x3 >= d;
s.t. con4: 9.0 <= x1 <= 9.5;
s.t. con5: 10.0 <= x2 <= 11.0;
s.t. con6: 11.25 <= x3 <= 11.5;
s.t. con7: 12.0 <= x4 <= 12.25;

end;
```

线性规划方程

```
Problem:    airplane_landing_ex
Rows:       8
Columns:    5
Non-zeros:  14
Status:     OPTIMAL
Objective:  z = 1 (MAXimum)

   No.   Row name   St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------- ------------- ------------- -------------
     1 z            B              1
     2 con1         B              0            -0
     3 con2         B           0.25            -0
     4 con3         NL             0            -0                          -1
     5 con4         NL             9             9           9.5         < eps
     6 con5         NL            10            10            11         < eps
     7 con6         NL         11.25         11.25         11.5            -1
     8 con7         NU         12.25            12         12.25             1

   No. Column name  St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------- ------------- ------------- -------------
     1 x1           B              9
     2 x2           B             10
     3 x3           B          11.25
     4 x4           B          12.25
     5 d            B              1

Karush-Kuhn-Tucker optimality conditions:

KKT.PE: max.abs.err = 0.00e+00 on row 0
        max.rel.err = 0.00e+00 on row 0
        High quality

KKT.PB: max.abs.err = 0.00e+00 on row 0
        max.rel.err = 0.00e+00 on row 0
        High quality

KKT.DE: max.abs.err = 0.00e+00 on column 0
        max.rel.err = 0.00e+00 on column 0
        High quality

KKT.DB: max.abs.err = 0.00e+00 on row 0
        max.rel.err = 0.00e+00 on row 0
        High quality

End of output
```

GLPK 求解结果

7. Dual Simplex Algorithm

$$\min \quad -7x_1+7x_2-2x_3-x_4-6x_5$$

$$\text{s.t.} \quad \begin{aligned} 3x_1 - x_2 + x_3 - 2x_4 &= -3 \\ 2x_1 + x_2 + x_4 + x_5 &= 4 \\ -x_1 + 3x_2 - 3x_4 + x_6 &= 12 \\ x_1, x_2, x_3, x_4, x_5, x_6 &\geq 0 \end{aligned}$$

预处理:

$$x_3 = -3 - 3x_1 + x_2 + 2x_4$$

$$x_5 = 4 - 2x_1 - x_2 - x_4$$

∴ 原目标函数为

$$-7x_1 + 7x_2 - 2(-3 - 3x_1 + x_2 + 2x_4) - x_4 - 6(4 - 2x_1 - x_2 - x_4)$$

$$= 11x_1 + 11x_2 + x_4 - 18$$

$$\therefore C = \begin{bmatrix} 11 \\ 11 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad A = \begin{bmatrix} 3 & -1 & 1 & -2 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 & 0 \\ -1 & 3 & 0 & -3 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} -3 \\ 4 \\ 12 \end{bmatrix}$$

代码见后面一页

```
var x1 >= 0;
var x2 >= 0;
var x3 >= 0;
var x4 >= 0;
var x5 >= 0;
var x6 >= 0;

minimize z: -7*x1 + 7*x2 - 2*x3 - x4 - 6*x5;

s.t. con1: 3*x1 - x2 + x3 - 2*x4 = -3;
s.t. con2: 2*x1 + x2 + x4 + x5 = 4;
s.t. con3: -x1 + 3*x2 -3*x4 + x6 = 12;


end;
```

GLPK 源代码

```
Problem:    dual_simplex
Rows:       4
Columns:    6
Non-zeros:  17
Status:     OPTIMAL
Objective:  z = -16.5 (MINimum)

   No.   Row name   St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------- ------------- ------------- -------------
     1 z            B          -16.5
     2 con1         NS            -3            -3             =          -2.5
     3 con2         NS             4             4             =            -6
     4 con3         NS            12            12             =         < eps

   No. Column name  St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------- ------------- ------------- -------------
     1 x1           NL             0             0                        12.5
     2 x2           NL             0             0                        10.5
     3 x3           NL             0             0                         0.5
     4 x4           B            1.5             0
     5 x5           B            2.5             0
     6 x6           B           16.5             0

Karush-Kuhn-Tucker optimality conditions:

KKT.PE: max.abs.err = 0.00e+00 on row 0
        max.rel.err = 0.00e+00 on row 0
        High quality

KKT.PB: max.abs.err = 0.00e+00 on row 0
        max.rel.err = 0.00e+00 on row 0
        High quality

KKT.DE: max.abs.err = 0.00e+00 on column 0
        max.rel.err = 0.00e+00 on column 0
        High quality

KKT.DB: max.abs.err = 0.00e+00 on row 0
        max.rel.err = 0.00e+00 on row 0
        High quality

End of output
```

GLPK 结果