091M4041H - Assignment 2

Algorithm Design and Analysis

October 21, 2018

Notice:

1. Please submit your answer in hard copy AND submit a digital version to UCAS website http://sep.ucas.ac.cn.
2. Hard copy should be submitted before 9 am. November 2 and digital version should be submitted before 11 pm. November 2.
3. You can choose **three** from problems 1-5, and you should do at least the following things:
   (a) Describe the optimal substructure and DP equation;
   (b) Describe your algorithm in daily language or pseudo-code;
   (c) Prove the correctness of your algorithm;
   (d) Analyse the complexity of your algorithm.
4. You should finish problems 6-7 on Universal Online Judge before 11 pm. November 2.

1 Money robbing

A robber is planning to rob houses along a street. Each house has a certain amount of money stashed, the only constraint stopping you from robbing each of them is that adjacent houses have security system connected and it will automatically contact the police if two adjacent houses were broken into on the same night.

1. Given a list of non-negative integers representing the amount of money of each house, determine the maximum amount of money you can rob tonight without alerting the police.
2. What if all houses are arranged in a circle?

2 Node selection

You are given a binary tree, and each node in the tree has a positive integer weight. If you select a node, then its children and parent nodes cannot be selected. Your task is to find a set of nodes, which has the maximum sum of weight.

3 Decoding

A message containing letters from A-Z is being encoded to numbers using the following mapping:

$$\begin{aligned}&\text{A} : 1\\&\text{B} : 2\\&\dots\\&\text{Z} : 26\end{aligned}$$

Given an encoded message containing digits, determine the total number of ways to decode it.

For example, given encoded message "12", it could be decoded as "AB" (1 2) or "L" (12). The number of ways decoding "12" is 2.

4 Longest Consecutive Subsequence

You are given a sequence of integers L and an integer k, your task is to find the longest consecutive subsequence the sum of which is the multiple of k.

5 Maximum profit of transactions

You have an array for which the i-th element is the price of a given stock on day i.

Design an algorithm and implement it to find the maximum profit. You may complete at most two transactions.

Note: You may not engage in multiple transactions at the same time (ie, you must sell the stock before you buy again), and one transaction includes buying and selling.

6 Longest increasing subsequence

A numeric sequence of a_i is increasing if $a_1 < a_2 < \cdots < a_N$. Let the subsequence of the given numeric sequence $(a_1, a_2, \dots, a_N)$ be any sequence $(a_{i_1}, a_{i_2}, ..., a_{i_K})$, where $1 <= i_1 < i_2 < \cdots < i_K <= N$. For example, sequence $(1, 7, 3, 5, 9, 4, 8)$ has increasing subsequences, e. g., $(1, 7)$, $(3, 4, 8)$ and many others. The longest increasing subsequence is of length 4, e. g., $(1, 3, 5, 8)$.

Your program, when given the numeric sequence, must find the length of its longest increasing subsequence.

7 Top k Paths

A robot is located at the top-left corner of a $m \times n$ grid. The robot can only move either down or right at any point in time and it is trying to reach the bottom-right corner of the grid. There is a non-negative integer in each small grid which means the score that the robot get when it passes this grid. Now you need to calculate the top k total scores when the robot reach the destination.

Note: The top k total scores may contain same total score got from different paths.