091M4041H - Assignment 2

Algorithm Design and Analysis

October 7, 2016

Notice:

1. Please submit your answer in hard copy AND submit a digital version to UCAS website http://sep.ucas.ac.cn.
2. Hard copy should be submitted before 9 am. October 21 and digital version should be submitted before 12 pm. October 21.
3. You can choose **three** from problems 1-5, and choose **one** from problems 6-7.
4. For problems 1-5, you should do at least the following things:
   (a) Describe the optimal substructure and DP equation;
   (b) Describe your algorithm in daily language or pseudo-code;
   (c) Prove the correctness of your algorithm;
   (d) Analyse the complexity of your algorithm.
5. For problems 6-7, you should implement your algorithm in C/C++/Java/Python with good comments.

1 Largest Divisible Subset

Given a set of distinct positive integers, find the largest subset such that every pair (S_i, S_j) of elements in this subset satisfies:$S_i \% S_j = 0$ or $S_j \% S_i = 0$.

2 Money robbing

A robber is planning to rob houses along a street. Each house has a certain amount of money stashed, the only constraint stopping you from robbing each of them is that adjacent houses have security system connected and it will automatically contact the police if two adjacent houses were broken into on the same night.

1. Given a list of non-negative integers representing the amount of money of each house, determine the maximum amount of money you can rob tonight without alerting the police.
2. What if all houses are arranged in a circle?

3 Partition

Given a string s, partition s such that every substring of the partition is a palindrome. Return the minimum cuts needed for a palindrome partitioning of s.

For example, given $s =$ "aab", return 1 since the palindrome partitioning ["aa", "b"] could be produced using 1 cut.

4 Decoding

A message containing letters from A-Z is being encoded to numbers using the following mapping:

$$\begin{aligned} &\text{A} : 1 \\ &\text{B} : 2 \\ &\ldots \\ &\text{Z} : 26 \end{aligned}$$

Given an encoded message containing digits, determine the total number of ways to decode it.

For example, given encoded message "12", it could be decoded as "AB" (1 2) or "L" (12). The number of ways decoding "12" is 2.

5 Frog Jump

A frog is crossing a river. The river is divided into x units and at each unit there may or may not exist a stone. The frog can jump on a stone, but it must not jump into the water.

If the frog's last jump was k units, then its next jump must be either $k-1, k$, or $k+1$ units. Note that the frog can only jump in the forward direction.

Given a list of stones' positions (in units) in sorted ascending order, determine if the frog is able to cross the river by landing on the last stone. Initially, the frog is on the first stone and assume the first jump must be 1 unit.

6 Maximum profit of transactions

You have an array for which the i-th element is the price of a given stock on day i.

Design an algorithm and implement it to find the maximum profit. You may complete at most two transactions.

Note: You may not engage in multiple transactions at the same time (ie, you must sell the stock before you buy again).

7 Maximum length

Given a sequence of n real numbers $a_1, \ldots, a_n$, determine a subsequence (not necessarily contiguous) of maximum length in which the values in the subsequence form a strictly increasing sequence.