Logo, company name

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**COMP9334 - Capacity Planning of Computer Systems and Networks**

**T1 2022**

**Assignment 1**

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**Question 1 (3 Marks):**

Table

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1. ***Determine the service demands of disk-1, disk-2, disk-3 and the CPU.***

To get the Service demand of Disk-1, Disk-2, disk-3, and the CPU, we need to use the Service Demand Law:

The U(i) refer to the Utilization of devices, which mean that we need to calculate the Utilization of U(Disk-1), U(Disk-2), U(Disk-3) and U(CPU) and the Throughput of the System. Meanwhile we will convert the Monitor time from minute to second to match with the provided Busy time.

After calculate the Utilization of each devices, we need to get the Throughput of the System X(0).

Since we retrieve the X(0) and the Utilization of each devices, so we can calculate the Service Demand of each device of the System.

1. ***Use bottleneck analysis to determine the asymptotic bound on the system throughput when there are 4 interactive users, and the think time is 20 seconds.***

Bottleneck Analysis:

The first throughput bond will be limited by the Maximum Service demand of a device within the System. The service demand from highest to lowest: *D(Disk1) > D(Disk3) > D(CPU) > Disk (Disk2).* So, the first throughput bound value will be:

The Second bond, N is the number of Interactive Users and sums the service demand of all devices. Additionally, the throughput bound will also affect by the Thinking time:

Thus, by using the Bottleneck Analysis to get the Asymptotic bound:

**Question 2 (7 Marks)**

Diagram

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1. ***Formulate a continuous-time Markov chain for the part of the call centre consisting of Staff 1 and their three waiting slots***

The continuous-time Markov Chain for the part of the Call centre Consisting of Staff one described as Below:

A diagram of a diagram

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Brief explanation of the term:

* : The probability of Call that assign to Staff 1 from Dispatcher.
* : The Centre receives on average queries per hour.
* 1: Staff 1 complete average queries per hour.
* : \* in result can calculate the queries that assign to Staff 1.

**Define the States:**

* State 0: Staff 1 is Idle and waiting for calls.
* State 1: Staff 1 receives a call and serve the query right away.
* State 2: Staff 1 is serving a call, one query in waiting slots.
* State 3: Staff 1 is serving a call, two queries in waiting slots.
* State 4: Staff 1 is serving a call, three queries in waiting slots and the slots are full. A further query is assigned to Staff 1 will be rejected.

1. ***Write down the balance equations for the continuous-time Markov chain that you have formulated.***

Brief Explanation of the Terms:

* i: Probability in State i.

Balance Equation List:

* 0 = 11
* 1 = 12
* 2 = 13
* 3 = 14
* 0 + 12 = ( 1 + )1
* 1 + 13 = ( 1 + )2
* 2 + 14 = ( 1 + )3

1. ***Derive the expressions for the steady state probabilities of the continuous-time Markov***

***chain that you have formulated.***

0 + 1 + 2 + 3 + 4 = 1

**Steady State for P0:**

|  |  |  |  |
| --- | --- | --- | --- |
| 0 = 11  => 1 = 0 | 1 = 12  => 2 = 1  => 2 = 0 | 2 = 13  => 3 = 2  => 3 = 0 | 3 = 14  => 4 = 3  => 2 = 0 |

0 + \* 0 + 0 + 0 + 0 = 1

0 () = 1

0 =

**Steady State for P1:**

|  |  |  |  |
| --- | --- | --- | --- |
| 0 = 11  => 0 = 1 | 1 = 12  => 2 = 1 | 2 = 13  => 3 = 2  => 3 = 1 | 3 = 14  => 4 = 3  => 2 = 1 |

1 + 1 + 1 + 1 + 1 = 1

1 ( ) = 1

1 =

1 =

**Steady State for P2:**

|  |  |  |  |
| --- | --- | --- | --- |
| 0 = 11  => 0 = 1  => 0 = 2 | 1 = 12  => 1 = 2 | 2 = 13  => 3 = 2 | 3 = 14  => 4 = 3  => 2 = 2 |

2 + 2 + 2 + 2 + 2 = 1

2 (

2 =

2 =

**Steady State for P3:**

|  |  |  |  |
| --- | --- | --- | --- |
| 0 = 11  => 0 = 1  => 0 = 3 | 1 = 12  => 1 = 3 | 2 = 13  => 2 = ­3 | 3 = 14  => 4 = 3  => 4 = 3 |

3 + 3 + ­3 + 3 + 3 = 1

3­ () = 1  
 3 =

3 =

**Steady State for P4:**

|  |  |  |  |
| --- | --- | --- | --- |
| 0 = 11  => 0 = 1  => 0 = 4 | 1 = 12  => 1 = 4 | 2 = 13  => 2 = ­4 | 3 = 14  => 3 = 4  => 3 = 4 |

4 + 4 + ­4 + 4 + 4 = 1

4 =

4 =

1. ***Assuming that = 0.4, = 5.7 and 1 = 6.1. Determine the probability that a query that is dispatched to Staff 1 will be rejected.***

Query from Dispatched to Staff 1 get rejected mean that Staff 1 must be in State 4 as other state will have Waiting slot for query.

4 =

1. ***Assuming that = 0.4, = 5.7, 1 = 6.1 and 2 = 6.5, determine the mean waiting time of the queries that have not been rejected by the call centre. Note that Part (d) considers only queries that have been dispatched to Sta\_ 1 but Part (e) considers the whole call centre.***

**Staff 1 (Steady State Calculation):**

|  |  |
| --- | --- |
| **P0 =** | **P1 =** |
| **P2 =** | **P3 =** |
| **P4 =** | |

Since any request that arrives at stage 4 will be rejected, so we exclude it for the mean number of jobs calculation.

N1=k **=**  0.51084

**According to Little’s Law: Mean number of Job = Throughput x Response Time**

Response Time (Staff 1) = N1/ = 0.51084/ (0.4 \* 5.7) = 0.22405

**Mean waiting time = Mean Response Time – Mean Service Time**

Waiting time (Staff 1) = 0.22405 – 1/6.5= 0.07020

**Staff 2 (Steady State Calculation):**

|  |  |
| --- | --- |
| **P0 =** | **P1 =** |
| **P2 =** | **P3 =** |
| **P4 =** | |

N1=k **=**  0.74893

Response Time (Staff 2) = N1/ = 0. 74893/ (0.6 \* 5.7) = 0.21898

Waiting time (Staff 2) = 0.21898 – 1/6.1 = 0.06011

Mean Waiting time (Whole Call Centre & No Reject Call)

= Mean Waiting time (Staff 1) \* + Mean Waiting time (Staff 2) \*

= 0.07020\*0.4 + 0.06011 \* 0.6

= 0.064146 (hour)

**Question 3:**

Graphical user interface, text

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Chart

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Graphical user interface, text, Word

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**The Following Question uses a Python Program for calculation.**

1. ***Assuming that = 4, formulate a continuous-time Markov chain for the system using the state definition given earlier. You can answer this question by drawing a state transition diagram with all the states and transitions. You can express the transition rates in terms of 1, 2, 1, 2 and*** *.*

w1: Workload of Class 1 query.

w2: Workload of Class 2 query.

: The Constant processing rate of a server (Hourly).

The time that used to Complete the workload.

is the processing rate work workload per hour.

A picture containing text, wall, indoor

Description automatically generated

* There are 4 Servers, which means this model is M/M/m.
* We treat Class 1 Request as a one-person team and Class 2 Request as a Two-people team. So, when a Request finishes, we return a probability of a Class team finish.
* When State (2,1) to (1,1). There are two Class 1 Requests, and we need to consider the probability of either one of the Class 1 requests finishing. So, we need to µ/w1 \* 2.

1. ***Assuming that n = 4, 1 = 2.7, 2 = 1.5, 1 = 10.4, 2 = 15.3 and = 70. Answer the following questions.***
2. What are the steady state probabilities of the states for the continuous-time Markov chain?

|  |  |
| --- | --- |
| P (0,0): 0.4918992967968496  P (1,0): 0.19732188934365058  P (0,1): 0.16127269802125266  P (2,0): 0.03957713323406925  P (1,1): 0.06469339086338233 | P (0,2): 0.026437202997055195  P (3,0): 0.005292028101012529  P (2,1): 0.012975645824598293  P (4,0): 0.0005307148181300137 |

Please Check “Question3.py” and look for Q3\_b\_i

1. ***Determine the probability that an arriving Class 1 request will be rejected.***

This Situation only happens when all servers are occupied.

P [Class 1 will be rejected]: = P(4,0) + P(2,1) + P(0,2)

= 0.039943563639783505

Please Check “Question3.py” and look for Q3\_b\_ii

1. ***Determine the probability that an arriving Class 2 request will be rejected.***

This Situation only happens when less than 1 (Include 1) server is busy.

P [Class 2 will be rejected]: = P(1,1) + P(0,2) + P(3,0) + P(2,1) + P(4,0)

= 0.10992898260417834

Please Check “Question3.py” and look for Q3\_b\_iii

1. **Determine the probability that an arriving request will be rejected. Note that the hint in Question 2 is applicable.**

In order to calculate the Arriving request will be rejected. I can calculate probability of Class 1 income Request ( 1) of an overall income Request ( 1 + 2). The we can use the it calculate the probability of Class 1 arriving request will be rejected, same method apply to Class 2.

P [Class 1 Arriving Request rate] = = 0.642857

P [Arriving Request of Class 1 will be rejected] = (P(4,0) + P(2,1) + P(0,2))

P [Arriving Request of Class 2 will be rejected] = (P(1,1) + P(0,2) + P(3,0) + P(2,1) + P(4,0))

P [Arriving Request will be rejected] =

P [Arriving Request of Class 1 will be rejected] + P [Arriving Request of Class 2 will be rejected]

= 0.06493835612706737

Please Check “Question3.py” and look for Q3\_b\_iv

1. **Assuming that 1 = 2,7, 2 = 1,5, w1 = 10.4, w2 = 15.3 and = 70. What is the smallest value of n that can reduce the probability of rejecting an arriving request to a level lower than 0.05?**

The overall request is being rejected is 0.0649 when there are 4 servers which are slightly higher than 0.05. We increase the number of servers to 5.

Diagram, schematic

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|  |  |
| --- | --- |
| P (0,0): 0.48588275502151895  P (1,0): 0.19490839658577508  P (0,1): 0.1593001318249121  P (2,0): 0.03909305554377544  P (1,1): 0.06390211002347906  P (0,2): 0.026113843038440866 | P (3,0): 0.005227299998424833  P (2,1): 0.012816937496137826  P (1,2): 0.010475381607420349  P (4,0): 0.0005242235141277194  P (3,1): 0.0017138076423407625  P (5,0): 4.20577036477519e-05 |

P (Class 1 will be Rejected) = 0.012231246953408863

P (Class 2 will be Rejected) = 0.051686251002115276

P [Class 1 Income Request] = = 0.642857

P [Arriving Request of Class 1 will be rejected] = P (Class 1 will be Rejected) \* P [Class 1 Income Request]

P [Arriving Request of Class 2 will be rejected] = P (Class 2 will be Rejected) \* (1 - P [Class 1 Income Request])

P [Arriving Request will be rejected]

= P [Arriving Request of Class 1 will be rejected] + P [Arriving Request of Class 2 will be rejected]

= 0.026322319827946868

So, we can conclude that when number of Server is 5, the probability of request will be rejected less than 0.05.

Please Check “Question3.py” and look for Q3\_c