

Closeness centrality of urban networks: Bologna case study

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1 Introduction

Urban studies are a multidisciplinary field that examines the social, economic, cultural, and physical aspects of cities and urban areas. It seeks to understand the complexities of urban environments, including the dynamics of population, land use, transportation, infrastructure, and social interactions. One crucial aspect of urban studies involves analyzing the spatial structure and connectivity within cities, and this is where network analysis plays a significant role, particularly in assessing the centrality of roads.

When talking about these studies Gabriele Dupuy must be mentioned. Dupuy has been instrumental in applying real network analysis to study the spatial structure and connectivity within cities. By adopting a network perspective, he has explored how elements such as roads, transportation systems, and social interactions relate to and influence the overall dynamics of urban areas. Through his work, Dupuy has demonstrated how network analysis can be used to assess the centrality of roads within urban landscapes. In this report we want to focus on this field of study calculating the closeness centrality of strategic nodes in the road network of the city of Bologna. We aim to examine how these measures can be influenced by various factors.

2 Problem and Motivation

In studies of urban networks, centrality plays a key role: "Urban design has brought its contribution by means of the 'space syntax' methodology. All these approaches - though under different terms like 'accessibility', 'proximity', 'integration', 'connectivity', 'cost', or 'effort' - focus on the idea that some places (or streets) are more important than others because they are more central" (Porta et al. 2006, p. 705).

However, recent studies (Porta et al. 2006, Gil 2017, Hillier & Penn 2004, Okabe & Sugihara 2012) applying network analysis to urban planning, have highlighted the limitations of centrality measures in urban networks.

2.1 Measures of centrality in catchment areas

The urban network system is a intricate spatial structure so in order to conduct spatial network analysis it becomes crucial to establish an artificial boundary for the network model. These con-

fined networks, often referred to as *catchment area*, represent localized sub-networks within the broader global network system. According to the studies, the centrality of nodes is influenced by the node's position relative to the reference catchment area. The centrality measure is more distorted the further the node's position is from the geometric center of the reference area. "These boundary determination problems, which have been termed the *edge effect* (Gil 2017, Porta et al. 2006) or the *boundary effect* (Okabe & Sugihara 2012), have significant impact on the studies focusing on the structural properties of the network models"(Chen & Dietrich 2023, p. 2).

If the value of the closeness centrality of a chosen node is the highest among all the nodes, one cannot be sure whether it is because this chosen node happens to be placed in the center of the catchment area or it really is the most important node in the entire street network(Chen & Dietrich 2023, p. 11).

Research on this phenomenon led us to an article by two researchers from Braunschweig and Hamburg: "Normalized closeness centrality of urban networks: impact of the location of the catchment area and evaluation based on an idealized network" by Hsiao-Hui Chen and Udo Dietrich (2023).¹ The article proposes a study and evaluation of the placement effect on closeness centrality using The Plaza Luceros in Alicante as a study sample. In particular, the researchers selected eight progressive catchment areas of $3000 \times 3000m^2$: "We have chosen the area size that is larger than the acceptable walking distance for the pedestrian because this allows more space to move the chosen node further away from the center in order to investigate the placement effect. One of our targets is to foster pedestrians in cities and to help to develop walkability, visibility, and accessibility of points of interest"(Chen & Dietrich 2023, p. 4). Through the catchment areas, the variation in the centrality measure of the same node is compared as the node moves away from the center of the section. According to the researchers, the results show significant repercussions of the border effect on calculations, establishing 100m as the threshold beyond which the border effect becomes significant. The first implication of these results is that a direct comparison of the closeness centrality between different nodes in the same catchment area is only possible if these nodes are less than 100m away from each other. Secondly, when comparing two nodes that are further than the threshold distance, which is 100m in the case of Alicante, it is better to create two separate catchment areas where these two nodes are the respective center points. The researchers clarify that the 100m threshold applies only to their specific case study and encourage conducting similar measurements on other real networks.

2.2 Our contribution

Our study aims to embrace that proposal by following the same research steps but using the city of Bologna as a reference. We have considered five different areas: the city center in square "Piazza di Porta Ravagnana", the city center in square "Piazza Malpighi", the area near "Porta Santo Stefano" and finally the western and south western suburbs (Borgo Panigale and Savena). We chose to isolate these nodes to have a more comprehensive overview of the city of Bologna. Our goal is to evaluate the border effect in these nodes separately, and from their comparison, decide whether it makes sense to calculate a single threshold for the entire city. The considered areas, in fact, differ from each other: while some zones have a strongly self-organized character, others are more of a planned nature. According to our calculations, the way

¹<https://doi.org/10.1007/s41109-023-00585-0>

the border effect influences different areas shows a difference significant enough to distinguish the downtown areas from the others. If the centrality variation in Piazza di Porta Ravagnana and Piazza Malpighi follows a similar trend, the same cannot be said for the zones in the two outskirts, which are also quite distinct from each other. As for the liminal area of Porta Santo Stefano, on the other hand, it can, according to our calculations, be classified within the group of downtown areas. In conclusion, our study has established that it is challenging to determine a single threshold value for the entire city.

3 Datasets

To initiate our research and comparative analysis based on the road organization of the city of Bologna, we sought the necessary data to undertake our calculations. Thanks to the *Open Data Bologna*² project, where the municipal government shares an updated list of data for the benefit of the citizens, we were able to explore the contents of various intriguing datasets. The portal is public and aims to share data with the residents of Bologna (and beyond) for research and proposals for reuse. In addition to the updated dataset archive, the exploration of the *Storie di dati*³ section is noteworthy, where some graphically intuitive analyses of research based on open data are made available, covering topics such as health, education, gender equality, and ecology, among others. The datasets that captured our interest include those related to road nodes, where each node represents a road intersection, and unoriented road edges, where the edges are the roads connecting the intersections. The dataset of road nodes⁴ provides information about the names of the nodes (each node has a corresponding ID) and their geographical coordinates. The dataset of road edges⁵ offers details such as the starting and ending nodes for each segment, the street name, and its length in meters. We downloaded the data in CSV format and processed and organized it using Python code with the *Pandas* library. Subsequently, we constructed a graph using *NetworkX*, incorporating the nodes and edges from the datasets. Moving to the initial visualization phase, we utilized the capabilities of *Matplotlib* and *Bokeh* to create visual representations.

To compare the measurements obtained by Chen & Dietrich (2023), we have defined five heterogeneous areas of $3000 \times 3000m^2$ in the city of Bologna. Some areas are located within the city center, in the oldest part of the urban network, while others are in peripheral or neighborhood spaces, outside the central zone and more recent in terms of urban planning. The five selected nodes, that correspond to the central points of the catchment areas are: Piazza Malpighi (44.49583448793263N, 11.337015936927742E), road intersection under the Two Towers - Piazza di Porta Ravagnana (44.49425927760075N, 11.346428140931987E), Porta Santo Stefano (44.48432911721366N, 11.356145824460073E), West suburb Borgo Panigale - Viale Palmiro Togliatti (44.506573158986185N, 11.285141903090834E), South suburb Savena - Via Domenico Scarlatti (44.467809198300095N, 11.374600820904973E). As can be seen in Figure 1, the five nodes are distributed across the urban area of Bologna, capturing different examples of spatial construction. This variation ranges from self-organised historical environments to others, that present a more grid-like and modern structure.

²Official link to the Open data Bologna website: <https://opendata.comune.bologna.it/pages/home/> (last visited 09/01/2024).

³Storie di Dati website: <https://www.comune.bologna.it/dati> (last visited 09/01/2024).

⁴Road nodes dataset: https://opendata.comune.bologna.it/explore/dataset/riffter_nodi_pt/information/ (last visited 09/01/2024).

⁵Road edges dataset: https://opendata.comune.bologna.it/explore/dataset/riffter_arcstra_li/information/ (last visited 09/01/2024).



Figure 1: Bologna network graph with our four chosen nodes.

4 Validity and Reliability

The datasets of the city of Bologna are public and published under the CC BY 4.0 license. We know that the data are collected by the municipality of Bologna, however, the documentation on the municipality’s website is not clear regarding the sources of the dataset or the data processing methods. Nevertheless, the data, which are updated daily, can be compared with other data map platforms to verify their validity (as for example *OpenStreetMap* or *Google Maps*). The cleanliness of the dataset substantially enhances the usability and reliability of the information. Data are stored in various file formats to ensure interoperability. For our research, we used open-source libraries to build graphs and analyze data. Therefore, the work is entirely reproducible and editable⁶.

5 Measures and Results

As observed in the *Problem and Motivation*, the objective of our research is to reproduce the study proposed by Chen & Dietrich (2023), by applying the same approach to five different nodes coming from the same street network, in our case the city of Bologna. The goal is to identify if the border effect emerges in the city of Bologna and how it behaves in it; to analyse the deviation of the values of closeness centrality for each node, and to evaluate the thresholds of each area while trying to determine if it is possible to define one.

Therefore, this section will be divided into two subsection: in the first one we will discuss the results coming from the application of the measure of *closeness centrality*; in the second one we will observe how this measure of centrality varies as percentage according to the position in the relative catchment area.

⁶Open GitHub repository of our project: https://github.com/EricaAndreose/network_analysis

5.1 Closeness centrality

The measure on which this section will focus is the *closeness centrality*. In Chen & Dietrich (2023) it is defined as “the reciprocal of the sum of the shortest distance between the chosen node v and all the other nodes in the catchment area”. It is described using the formula:

$$C_c(v) = \frac{1}{\sum_{u=1}^N S(v,u)}$$

This value is then multiplied by the number of connections between one node and the other (in the following formula the number of reachable nodes $N - 1$).

$$C_N(v) = (N - 1) \times C_c(v)$$

Although, comparing these measures with the rest of the literature (Porta et al. 2006, Freeman 1978) it has been noticed that $C_N(v)$ corresponds to the closeness centrality as defined elsewhere. It can be defined as the inverse of the mean of the shortest paths, or the ratio between the number of reachable nodes and the shortest path connecting it to all the others,

$$C_c(v) = \frac{N - 1}{\sum_{u=1}^N S(v,u)}$$

where N represents the number of nodes (so that $N - 1$ represents the number of reachable nodes), and $S(v,u)$ the shortest distance between v and u . The edges can be weighted as in our case, where the weight of the edge corresponds to the length of the street connecting two nodes (in meters), or not (in that case all edges have distance equal to 1).

Therefore, since the reference article always uses $C_N(v)$ from its measurements, from now on, we will refer to the last formula when talking about closeness centrality C_c .

Said so, as proposed by Chen & Dietrich (2023), we have defined a $3000 \times 3000m^2$ catchment area around each of the five nodes of interest. Then, we have calculated eight catchment areas moving the centre of them progressively further from the initial node by 50m, 100m, 200m, 300m, 500m, 1000m, 1500m. In such a way we can measure the variation of the value of C_c according to the position of the node in respect of the centre of each catchment area. The results of this analysis are reported in the Table 1-5 (it must be specified that the catchment area moves from west to east for the first four nodes, while in the last one from south to north). We cannot include here due to space reason all the table showing also the graph as it varies in each catchment areas. We insert here as an example Table 7. The other graphs may be found in the GitHub repository cited above.

Unit	Distance between red chosen node and center node (m)	Closeness centrality of the red chosen node 1/km	Average distance from all nodes to chosen node	Total nodes number	Normalized closeness centrality 1/km
Catchment area 1	0	0.000999169689876203	997.8374875373878	1003	1.0021671990575831
Catchment area 2	50	0.000987617341298798	1002.3448959365709	1009	0.9976605897370487
Catchment area 3	100	0.000994969434538971	1000.0557213930348	1005	0.9999442817116659
Catchment area 4	200	0.000984763427956877	1004.6256206554121	1007	0.9953956771952576
Catchment area 5	300	0.000981596055544104	1013.6805970149254	1005	0.9865040358321825
Catchment area 6	500	0.000943955475508131	1048.8831683168316	1010	0.9533950302632125
Catchment area 7	1000	0.0008042923473996022	1200.124513745174	1036	0.8332468719059879
Catchment area 8	1500	0.0007139832913630156	1397.7974051896208	1002	0.715411257947417

Table 1: Piazza Malpighi catchment area table of results.

Unit	Distance between red chosen node and center node (m)	Closeness centrality of the red chosen node 1/km	Average distance from all nodes to chosen node	Total nodes number	Normalized closeness centrality 1/km
Catchment area 1	0	0.0010032233566449	976.285014691479	1021	1.0242910471344429
Catchment area 2	50	0.0010155294767585923	973.0316205533596	1012	1.0277158304796954
Catchment area 3	100	0.001005321164925953	980.0068965517241	1015	1.0204009823998423
Catchment area 4	200	0.0010213889050608798	978.08091908909191	1001	1.0224102939659407
Catchment area 5	300	0.0009871424693368872	997.0718503937007	1016	1.0029367488462773
Catchment area 6	500	0.0009967197951541477	1009.3470824949699	994	0.9907394763832228
Catchment area 7	1000	0.000939764833248128	1162.9464480874317	915	0.8598848224220371
Catchment area 8	1500	0.0007587449145122104	1557.8794326241134	846	0.64189819767733

Table 2: Due Torri catchment area table of results.

Unit	Distance between red chosen node and center node (m)	Closeness centrality of the red chosen node 1/km	Average distance from all nodes to chosen node	Total nodes number	Normalized closeness centrality 1/km
Catchment area 1	0	0.0010507192698761937	1221.7317073170732	779	0.8185103112335549
Catchment area 2	50	0.0010654608491083692	1217.3294422827496	771	0.8214703146625526
Catchment area 3	100	0.001052247231800595	1224.673969702165	776	0.8165438518772618
Catchment area 4	200	0.0010360504100687521	1240.6221079691518	778	0.8060472190334892
Catchment area 5	300	0.001044773780357208	1247.9074315514993	767	0.80113414895339785
Catchment area 6	500	0.0009972873783309398	1314.1808650065532	763	0.7609302696665071
Catchment area 7	1000	0.0008837607200175338	1496.7301587301588	756	0.6681231043332555
Catchment area 8	1500	0.0007360312312772056	1717.620733249052	791	0.5822007039402696

Table 3: Porta Santo Stefano catchment area table of results.

Unit	Distance between red chosen node and center node (m)	Closeness centrality of the red chosen node 1/km	Average distance from all nodes to chosen node	Total nodes number	Normalized closeness centrality 1/km
Catchment area 1	0	0.0013051456673079283	1532.396	500	0.6525728336539641
Catchment area 2	50	0.001269367372687995	1541.6712328767123	511	0.6486467274439766
Catchment area 3	100	0.0012646989637056692	1526.4517374517375	518	0.6551140631995366
Catchment area 4	200	0.0012508537076554747	1528.5927342256214	523	0.6541964891038132
Catchment area 5	300	0.0012241550269926184	1529.7565543071162	534	0.6536987844104583
Catchment area 6	500	0.001241810430835596	1530.3505747126437	522	0.6534450448961812
Catchment area 7	1000	0.0012401716397549421	1593.557312529645	506	0.6275268497160007
Catchment area 8	1500	0.0009657991215091191	2175.235294117647	476	0.4597203818383407

Table 4: West suburb - Borgo Panigale catchment area table of results.

Unit	Distance between red chosen node and center node (m)	Closeness centrality of the red chosen node 1/km	Average distance from all nodes to chosen node	Total nodes number	Normalized closeness centrality 1/km
Catchment area 1	0	0.00129526789377898	1303.93760539629	593	0.7669078611288935
Catchment area 2	50	0.00124962198934822	1320.531351353136	606	0.7572709255450226
Catchment area 3	100	0.001254685388123412	1335.5974025974026	616	0.7487286199084022
Catchment area 4	200	0.001238657385034153	1381.6552795031057	644	0.7237695355961994
Catchment area 5	300	0.0010976382118595418	1403.7704160246533	649	0.7123671994968426
Catchment area 6	500	0.0010084650556773556	1466.8727810650887	676	0.6817223776378923
Catchment area 7	1000	0.0008476264340779732	1666.334745762712	708	0.600119515327205
Catchment area 8	1500	0.0005786571566291831	2201.4509554140127	785	0.45424586795390876

Table 5: South suburb - Savena catchment area table of results.

We should precise that *number of nodes* does not represent the total number of nodes present in the graph, but the total number of nodes directly connected with the interested node, since the shortest path can be calculated only among connected nodes.

Here we plot the C_c as function of the distance of the studied node from the centre of the catchment area.

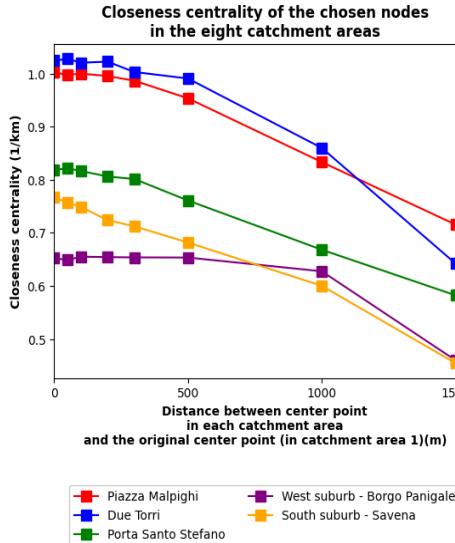


Figure 2: Variation of C_c of the five nodes in the different catchment areas.

As we expected starting from the results obtained in Chen & Dietrich (2023), the C_c , as shown by Figure 2, is influenced by the border effect, since in all the five cases the closeness centrality tends to lower down. Also, the C_c seems to be influenced by the position of the node in the network, in fact, the nodes in the central part of the city appear to show higher values of C_c compared to those in the periphery, at the same distance of the node from the centre of the catchment area.

5.2 Percentage Deviation of C_c

While Figure 2 allows to understand the relative variation of C_c for each node, it says little about the variation of it in the single areas. Because of that, following the reasoning of Chen & Dietrich (2023), in this section we study the variation of C_c with respect to its value in the centre of the graph. These values are calculated dividing the C_c measured translating the catchment area with the one measured at 0m. This process was applied to the five nodes object of our research. We then convert the values in percentage to facilitate comparison.

One of the objectives of this research is to identify a distance between the chosen node and the central point of the catchment area that can be considered a significant threshold after which the value of C_c varies from the one measured when the node is in the centre of the catchment area; if the distance between another node and the one studied is major than the value identified the C_c of the two points cannot be compared due to the border effect. To establish this value, (Chen & Dietrich 2023) proposes to study the percentage deviation of the C_c in the different catchment areas establishing a level of significance of deviation above which the variation of C_c is acceptable.

This acceptable percentage deviation is assumed at 2% in Chen & Dietrich (2023), but it is chosen a priori by the authors of the paper. Here we propose a possible approach to define it based on the data collected.

Since the C_c can be defined as the inverse of the mean of the shortest paths, it is related to its *standard deviation std*. Because of that, we decided to consider the percentage variation of *std* of the node in each catchment area. We calculate the median of the values obtained for each node. We adopted the median since we analysed few cases and the result could have been too sensible to outliers. Finally, through this approach we have obtained a value for each area which corresponds to the mean of the absolute value of the medians. These results are shown in Table 6. Moreover, we identified a common percentage deviation value as the mean of the percentage deviation of the medians, which results to be 4.980850, rounded at 5%. Since we are dealing with deviation values, differently from Chen & Dietrich (2023), which considers only values between 100% and their threshold we believe that this percentage deviation should be accepted both above and below the 100%. Because of that we consider the values included between 95% and 105%.

Unit	Piazza Malpighi	Due Torri	Porta Santo Stefano	West periphery	South periphery	Mean of each row
Catchment area 1	100.000000	100.000000	100.000000	100.000000	100.000000	100.000000
Catchment area 2	100.726821	100.404869	99.645142	100.024627	101.284883	100.417268
Catchment area 3	100.622096	102.052376	100.758892	96.594654	102.928345	100.591272
Catchment area 4	101.449542	102.998299	103.861679	96.944613	108.415046	102.733836
Catchment area 5	103.775327	105.555464	105.944208	95.970272	111.911473	104.631349
Catchment area 6	111.088793	109.110384	121.455663	97.020877	120.240250	111.783193
Catchment area 7	136.410554	132.056677	137.578798	97.081667	144.661352	129.557810
Catchment area 8	157.492155	144.853927	147.691738	133.854616	186.590632	154.096613
Median	102.612435	104.276881	104.902944	97.051272	110.163259	103.801358
Deviation of absolute values	2Unut.612435	4.276881	4.902944	2.948728	10.163259	4.980850

Table 6: Percentage deviation for each node with the medians of them and the mean of the medians

Figure 3 shows the percentage deviation of each node and its value for each node, while Figure 4 shows the percentage deviation in relation to the average deviation.

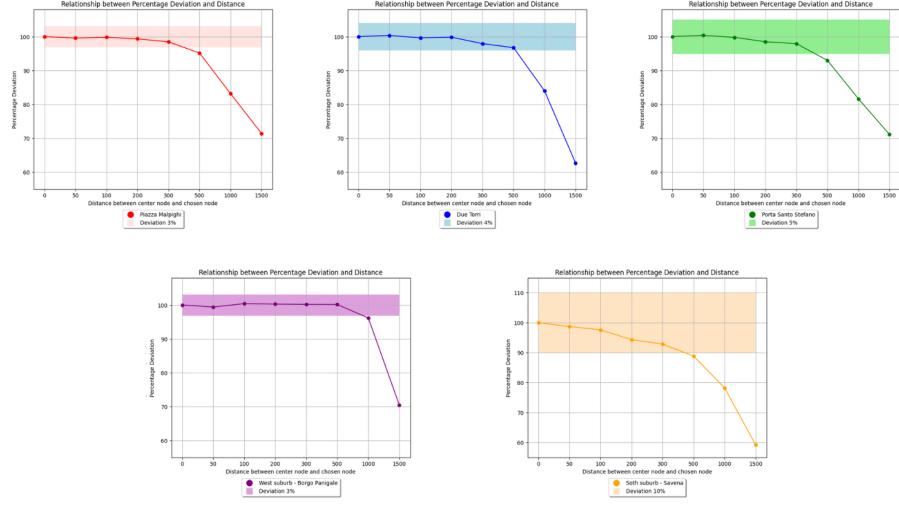


Figure 3: Plots with the relationship between percentage deviation and distance for the different catchment areas.

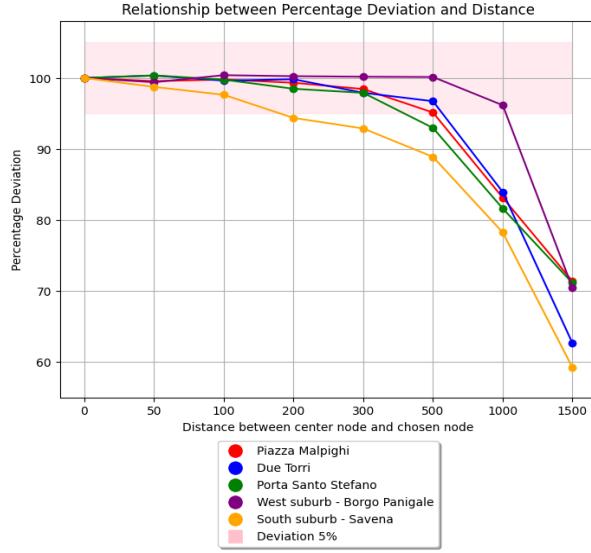


Figure 4: Plot with the relationship between percentage deviation and distance for all the considered catchment areas.

For what concern the percentage deviation of C_c , what emerges is that, based on our approach, the significance value for each node can be identified between 300m and 500m, when calculated for the single node. On the other hand, once we apply a common value we see that the significant point varies for each node, ranges between 100m (south-east periphery) and 1000m (west periphery).

6 Conclusion

We have noticed that the *closeness centrality* tends to be greater in the central part of the city, while it decreases as we move towards the periphery. We can explain this phenomenon because

of the street network structure in the two parts of the city. In the city centre, streets tend to be shorter with more intersections. On the other hand, in the external part, the presence of longer streets connecting a major number of nodes more distant between each other, lower down the C_c value.

As shown in the reference article, the border effects seems to always lower down C_c . We believe that this changing is due to the increasing of distances between the node and the other nodes. An interesting result is the one obtained about the percentage deviation of the C_c . It seems to us that the different results obtained are mostly dependent on the geographical topology of the street network. For example, in the south periphery, the urban structure changes a lot between the first catchment area (less dense and with longer edges) and the last one. It cannot be defined a singular distance valid for all five nodes.

Moreover, as can be seen in Figure 4, the three nodes nearby the centre of the city show similar percentage deviation, since the surrounding sub-graphs are drastically different. On the other hand, the two peripheral areas behave in very different ways. We think that this result is driven by the different topology of the areas of the city. In fact, while the three nodes in the centre are close between each other, so the topology of the network does not change a lot, the two peripheral nodes are far away, being extracted from two completely different sub-networks, which results in very different behaviours. Consequently, we can affirm that the periphery of the city cannot be analyzed as a whole, but distinctions should be made accordingly to the different areas of it.

7 Critique

Our report does not solve completely the problem proposed, but it seems to us, that it suggests a valid approach to apply to similar studies.

First of all, we believe that using a broader term such as closeness centrality, instead of specifically referencing what Chen & Dietrich (2023) defines as normalized closeness centrality, enables a more straightforward application of this method in other studies, avoiding potential misunderstandings.

Secondly, compared to the article from which we took inspiration, the application of the *std* to define the significance of variation allows to define the value according to the data collected instead of using an *a priori* value. On the other hand, comparing different areas of the same city allowed us to highlight how the deviation of C_c varies between different areas of a single city, but it did not lead to any significant value.

A possible improvement of this study could be applying the same measures on more nodes of the city and at the same time, increasing the number of measurements in the same catching area (every 50m for example). Such an approach could help to better define the deviation value (maybe also using other statistics), studying the behaviour of the decrement of the C_c , and trying to define a statistical value of the distance at which the deviation is significant.

Furthermore, it could be interesting to connect the results of our study with the measures applied in Cardillo et al. (2006). In fact, in this paper it is shown that self-organized cities and planned ones behave differently. Since the centre and the periphery can follow respectively in the former and the latter group, seeing the relationship between measures such as the relative efficiency, the meshedness coefficient, and the number of cycles of different length and the C_c could lead to a better understanding the correlation between the structure of the street graph and the variation of C_c in different areas of the city.

Finally, not only the closeness centrality is sensitive to the border effect, but all the path-based

measures(Chen & Dietrich 2023, p. 2). Starting from this observation, an interesting development of this research could be to apply the same methodology to the betweenness centrality.

Unit	Image	Distance between chosen node and center node (m)	Cc(node) 1/km	Average distance all nodes to chosen node	Average edges length	Total nodes number	Normalized Centrality 1/km	std	Percentage Deviation CN(v)	Percentage Deviation std	Percentage Deviation edges length
area 1		0	0.001003	976.285015	73.011580	1021	1.024291	494.536764	100.000000	100.000000	100.000000
area 2		50	0.001016	973.031621	73.078929	1012	1.027716	496.538991	100.334356	100.404869	100.092244
area 3		100	0.001005	980.006897	73.777930	1015	1.020401	504.686516	99.620219	102.052376	101.049628
area 4		200	0.001021	978.080919	73.615972	1001	1.022410	509.364455	99.816385	102.998299	100.827803
area 5		300	0.000987	997.071850	74.123448	1016	1.002937	522.010576	97.915212	105.555464	101.522865
area 6		500	0.000997	1009.347082	74.900844	994	0.990739	539.590963	96.724410	109.110384	102.587622
area 7		1000	0.000940	1162.946448	78.097918	915	0.859885	653.068820	83.949267	132.056677	106.966481
area 8		1500	0.000759	1557.879433	85.046769	846	0.641898	716.355924	62.667559	144.853927	116.483944

Table 7: Catchment areas 2 Due Torri - Piazza di Porta Ravagnana with different center nodes and indicator values of (blue) node under investigation in the selected catchment areas.

References

- Cardillo, A., Scellato, S., Latora, V. & Porta, S. (2006), ‘Structural properties of planar graphs of urban street patterns’, *Phys. Rev. E Stat. Nonlin. Soft Matter Phys.* **73**(6 Pt 2), 066107.
URL: <https://doi.org/10.1103/PhysRevE.73.066107>
- Chen, H.-H. & Dietrich, U. (2023), ‘Normalized closeness centrality of urban networks: Impact of the location of the catchment area and evaluation based on an idealized network’, *Appl. Netw. Sci.* **8**(1).
URL: <https://doi.org/10.1007/s41109-023-00585-0>
- Freeman, L. C. (1978), ‘Centrality in social networks conceptual clarification’, *Soc. Networks* **1**(3), 215–239.
URL: [https://doi.org/10.1016/0378-8733\(78\)90021-7](https://doi.org/10.1016/0378-8733(78)90021-7)
- Gil, J. (2017), ‘Street network analysis “edge effects”: Examining the sensitivity of centrality measures to boundary conditions’, *Environ. Plan. B Urban Anal. City Sci.* **44**(5), 819–836.
URL: <https://doi.org/10.1177/026581351665067>
- Hillier, B. & Penn, A. (2004), ‘Rejoinder to carlo ratti’, *Environ. Plann. B Plann. Des.* **31**(4), 501–511.
URL: <https://doi.org/10.1068/b3019a>
- Okabe, A. & Sugihara, K. (2012), *Spatial Analysis along Networks: Statistical and Computational Methods*, John Wiley & Sons.
- Porta, S., Crucitti, P. & Latora, V. (2006), ‘The network analysis of urban streets: A primal approach’, *Environment and Planning B: Planning and Design* **33**(5), 705–725.
URL: <https://doi.org/10.1068/b32045>