Question 8:

Proof: by the definition of a limit of a sequence,

$$(\forall \varepsilon > 0)(\exists n \in N)(\forall m > n)\left[|a_m - L| < \varepsilon\right]$$

Thus: ~~for sequence $\{Ma_n\}_{n=1}^{\infty}$, the limit is L.~~

~~$$(\forall \varepsilon > 0)(\exists n \in N)(\forall m > n)\left[|Ma_m - L| < \varepsilon\right]$$~~

this statement is also true for all $\frac{\varepsilon}{M}$ if $M > 0$.

thus:

$$(\forall \tfrac{\varepsilon}{M} > 0)(\exists n \in N)(\forall m > n)\left[|a_m - L| < \tfrac{\varepsilon}{M}\right]$$

$$\Rightarrow (\forall \varepsilon > 0)(\exists n \in N)(\forall m > n)\left[M|a_m - L| < \varepsilon\right]$$

by algebra, we know that this is equivalent to:

$$(\forall \varepsilon > 0)(\exists n \in N)(\forall m > n)\left[|Ma_m - ML| < \varepsilon\right]$$

This proves ML is the limit of sequence $\{Ma_n\}_{n=1}^{\infty}$