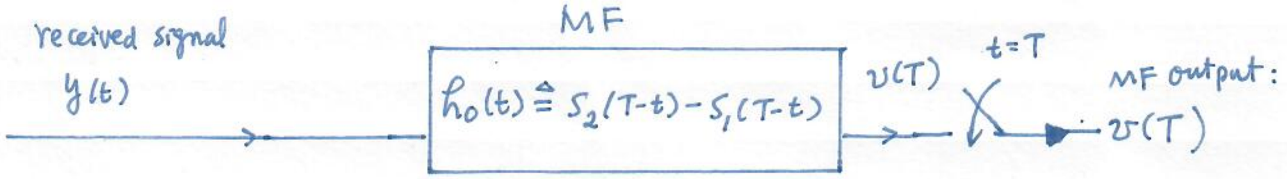
1. Theory (Summary from text)



- $y(t) = \begin{cases} s_1(t) + \omega(t) & \cdots \text{bit } 0 \\ s_2(t) + \omega(t) & \cdots \text{bit } 1 \end{cases}$

where $0 \le t < T$

$T \triangleq$ symbol period

$E_1 \triangleq$ energy of $s_1(t)$

$E_2 \triangleq$ energy of $s_2(t)$

$E \triangleq \dfrac{E_1 + E_2}{2}$: averge signal energy per bit.

- $v(T) = \begin{cases} s_{01}(T) + N \\ s_{02}(T) + N \end{cases}$

where

$$s_{01}(T) \triangleq \int_{-\infty}^{\infty} s_1(t) \cdot h_0(T-t)\, dt$$

$$s_{02}(T) \triangleq \int_{-\infty}^{\infty} s_2(t) \cdot h_0(T-t)\, dt$$

$$N \triangleq \int_{-\infty}^{\infty} \omega(t) \cdot h_0(T-t)\, dt$$

$$\Rightarrow \begin{cases} E[N] = 0 \\ Var[N] \triangleq \sigma_0^2 \end{cases}$$

$$\varsigma \triangleq \frac{s_{02}(T) - s_{01}(T)}{\sigma_0}$$

- $P_E = Q\left(\dfrac{\varsigma}{2}\right)$

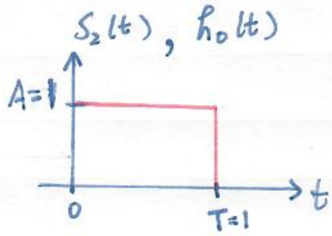
- Goal: Simulate P_E.

• Example : OOK

bit 0 : $s_1(t) = 0 , \quad 0 \le t < T$

bit 1 : $s_2(t) = \begin{cases} A, & 0 \le t < T \\ 0, & \text{otherwise} \end{cases}$

where $A = 1$ and $T = 1$ sec.



$\Rightarrow \begin{cases} E_1 = 0 \\ E_2 = 1 \\ E = \frac{0+1}{2} = \frac{1}{2} \end{cases}$

$\Rightarrow$ $s_{01}(T) = 0$

(1) $H_0(f) \triangleq \mathcal{F}[h_0(t)]$

(2) Parseval's theorem

$$\begin{cases} s_{02}(T) = 1 . \\ \sigma_0^2 = \frac{N_0}{2} \cdot \left[\int_{-\infty}^{\infty} |H_0(f)|^2 df \right] = \frac{N_0}{2} \cdot \left(\int_{-\infty}^{\infty} |h_0(t)|^2 dt \right) = \frac{N_0}{2} \\ \zeta = \frac{1}{\left(\sqrt{\frac{N_0}{2}} \right)} \end{cases}$$

2. Computer Simulation

- received samples

$$\tilde{y}(n) \triangleq y(t)\Big|_{t=nT_s} \quad \text{where} \quad \begin{aligned} T_s &\triangleq \text{sampling period} \\ F_s &\triangleq \text{sampling freq. (Hz).} \\ &= \frac{1}{T_s} \end{aligned}$$

Note : "tilde" notation: sampled version

- Simulation Method :

digital MF

$$\tilde{y}(n) \longrightarrow \boxed{\tilde{h}_0[n] \triangleq \tilde{s}_2[M-1-n] - \tilde{s}_1[M-1-n]} \longrightarrow \overset{n=M-1}{\diagup} \longrightarrow \tilde{v}$$

$n=M-1$ where $M \triangleq \frac{T}{T_s}$ (assuming M is an integer)

(digital MF output)

where

$$\tilde{v} \triangleq \begin{cases} \tilde{s}_{01}[M-1] + \tilde{N} \text{ ---- bit 0} \\ \tilde{s}_{02}[M-1] + \tilde{N} \text{ ---- bit 1} \end{cases}$$

$$\tilde{s}_{01}[M-1] = \sum_{n=0}^{M-1} \tilde{s}_1[n] \cdot \tilde{h}_0[M-1-n] \quad \text{---- convolution sum !}$$

$$\tilde{s}_{02}[M-1] = \sum_{n=0}^{M-1} \tilde{s}_2[n] \cdot \tilde{h}_0[M-1-n]$$

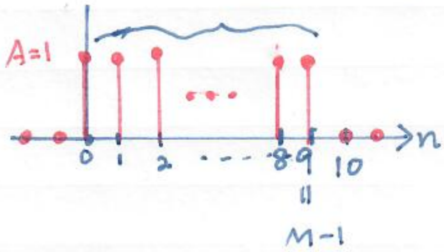
$$\tilde{N} = \sum_{n=0}^{M-1} \omega[n] \cdot \tilde{h}_0[M-1-n] \quad \text{其中} \begin{cases} \omega[n] \triangleq \omega(t)\Big|_{t=nT_s} \\ E[\tilde{N}] = 0 \\ \text{Var}[\tilde{N}] \triangleq \tilde{\sigma}_0^2 \end{cases}$$

$$\tilde{\xi} \triangleq \frac{\tilde{s}_{02}[M-1] - \tilde{s}_{01}[M-1]}{\tilde{\sigma}_0}$$

$$\tilde{P}_E \triangleq Q\left(\frac{\tilde{\xi}}{2}\right)$$

- Back to the OOK example with $T_s = \frac{1}{10}$ sec (i.e., $F_s = 10$ Hz) $\Rightarrow M=10$

Then $\tilde{S}_2[n]$ $M=10$ samples $= 1$ bit period



$\Longrightarrow$ this is also the plot of $\tilde{h}_0[n]$!

$\Rightarrow$
- $\tilde{S}_{01}[M-1] = 0$
- $\tilde{S}_{02}[M-1] = 10$

$*$ $\tilde{\sigma}_0^2 \triangleq Var[\tilde{N}] = Var\left[\sum_{n=0}^{M-1} w[n]\cdot\tilde{h}_0[M-1-n]\right]$

Q: How to compute $\tilde{\sigma}_0^2$?

A: It's typical to assume $w[n] \sim \mathcal{N}(0, \tilde{\sigma}^2_{sample})$ and $w[n]$'s are i.i.d.

*** $\tilde{\sigma}^2_{sample}$ 代表每個 noise sample $w[n]$ 的 variance (即 power). $\Rightarrow$ 模擬時根據它來產生 noise samples $w[n]$

Then,
$$\tilde{\sigma}_0^2 = \tilde{\sigma}^2_{sample}\cdot\left[\sum_{n=0}^{M-1}\tilde{h}_0[n]\right]$$
.... review 机统/隨机程序

$\therefore$ For this OOK example, $\tilde{\sigma}_0^2 = 10\cdot\tilde{\sigma}^2_{sample}$ (see the plot of $\tilde{h}_0[n]$ above.)

*** <u>Key question:</u> How to set $\tilde{\sigma}^2_{sample}$ in computer simulation?

<u>Hint</u>:

∵ 模擬的目的是使得 $\underbrace{\tilde{P}_E}_{\text{實際模擬產生之BER}} \simeq \underbrace{P_E}_{\text{理論之BER}}$

∴ 應設定 $\tilde{\xi} = \xi$ （但 $\tilde{\xi}$ 又是由 $\tilde{\sigma}^2_{sample}$ 決定）

→ 故可求出所需之 $\tilde{\sigma}_{sample}$