Data Structure　Homework 3　 2024

每題10分，最多100分，繳交至4/28 23:59

1. The inorder and preorder traversal of a binary tree are **d b e a f c g** and **a b d e c f g**, respectively. The postorder traversal of the binary tree is : ___?

   ANS: d e b f g c a

   [Step1]: Inorder traversal (d b e **a** f c g) visits nodes in the order: left subtree, root, right subtree. The root **a** splits the inorder sequence into: left subtree's inorder sequence (d,b,e) and right subtree's inorder sequence (f,c,g)

   [Step2]: Taking the next element in preorder **b** the root for the next subtree, it splits the left sequence (d,**b**,e) by **b**, with **d** on the left and **e** on the right. The same is done for the right subtree with c as the next root, splitting sequence (f,**c**,g) into **f** (left of c) and **g** (right of c).

2. For a binary tree, (a) if the number of nodes is 123456, what's the minimum level of the tree? (b) if the number of leaf nodes is 123456, what's the minimum level of the tree? (Assume that root is in level 1.) (5 points for each)

答:

(a)在第1層到第 n 層最多有2^{n+1}-1個 nodes，又2^{15+1}-1<123456<2^{16+1}-1，因此最少有16層

(b)第 n 層最多有2n 個 leaf nodes，又2^{16}<123456<2^{17}，因此最少有17層

第二題答案計算時是以root那層為level 0計算，而不是level 1，
另外b部分答案寫得‘第 n 層最多有2n 個 leaf nodes…… ‘，
修正為‘第 n 層最多有2 ˆ n 個 leaf nodes……’ ，

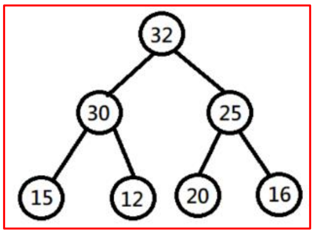
3. There are n numbers in a binary max heap. What’s the time complexity for finding the smallest number in the heap?

   ANS: O(n) Need to check at least all leaves.

   The largest number is always at the root, accessible in O(1) time, but the smallest number could be at any position at the bottom level. The nature of a binary max heap does not guarantee any order beyond the largest value. Since a binary heap is a complete binary tree, in the worst case, leaf nodes approximately make up half of the total nodes, meaning nearly n/2 might need to be checked. Constants are ignored in Big O notation, so the worst-case time complexity is O(n).

4. The elements **32, 15, 20, 30, 12, 25, 16** are inserted one by one in the given order into a max heap. **Please draw seven diagrams to depict the max heap after each insertion**, starting from an empty heap and showing the changes in the heap's structure after each element is added. This requires a total of seven diagrams to represent the step-by-step construction of the max heap.

   ANS:



5. We have an array A as shown in the following table which is the implicit array representation of a binary tree.

A[i]	A[0]	A[1]	A[2]	A[3]	A[4]	A[5]
value	2	9	7	31	13	8

Is it a max heap or a min heap ? Why ?

Ans：min heap

所有的父節點的值都比子節點的值小

6. Please identify the correctness of the following options and prove your answer.

   a. Every disconnected graph has an isolated vertex.

答:否，存在反例 A-B C-D

   b. There exists a simple graph with 6 vertices whose degrees are 1,2,3,4,5,6

答: 否，只有六個 vertices 若要有其中一個 vertex 的 degree 為6，根據鴿籠原理必然會有 loop 或 multiple edges 產生。

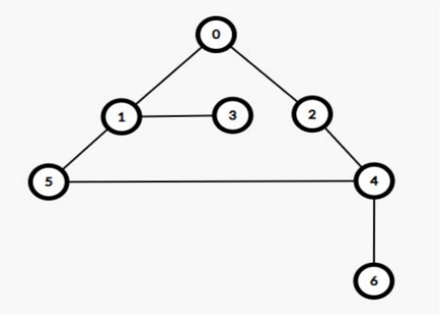
   c. Every simple graph with n vertices and k edges has at least n-k components.

答:是，一個有 n 個 vertices 的圖，在沒有 edge 時有 n 個 components，每添加一個 edge，會使得 components 數量減0或1，因此添加 k 個 edges 的圖最少有 n-k 個 components

   d. Every graph with n edges has total degree sum 2n.

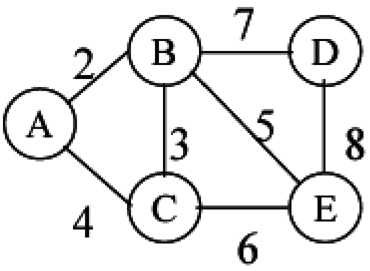
答:是，因為每個 edge 都有兩個端點(就算是 loop)，因此加總所有 vertices 的 degrees 時會算到每一個 edge 兩次，故為真。

7. Given a graph G as follow, now performing BFS on G (with the starting point vertex 0), please list the order in which each vertex is discovered (When encountering multiple possible directions,, choose the one with the smaller number.).

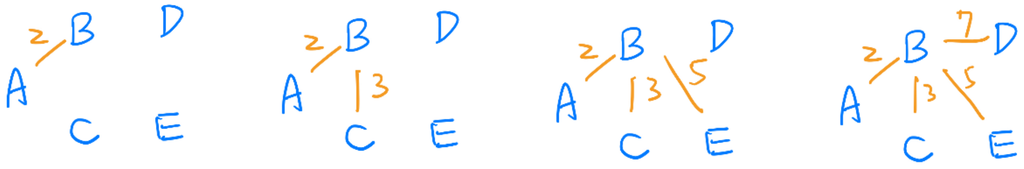


答: 0123546

8. Please draw the minimum cost spanning tree of the graph below using Kruskal algorithm. (need to draw your steps)



Ans:

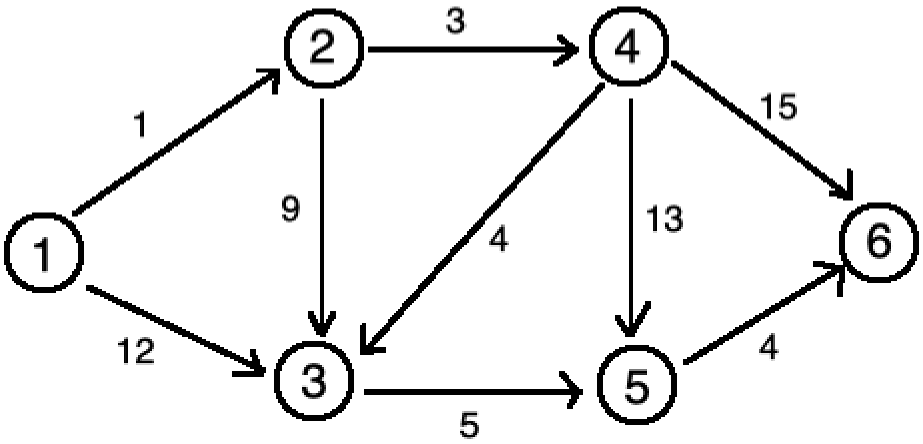


9. Please describe at least three distinctions between DFS and BFS.

Ans：

v.s	BFS	DFS
FIFO or LIFO	FIFO (First In First Out)	LIFO (Last In First Out)
Data Structure	Queue	Stack
Speed (slower/faster)	Generally slower than DFS	Generally faster than BFS
Memory (more/less)	Requires more memory space	Requires less memory space
Backtracking Needed	No	Yes (May be required)

10. Please find the shortest path from point 1 to point 6 using Dijkstra Algorithm. (need to write down your steps)



Ans: 124356

	1	2	3	4	5	6
dis	0	∞	∞	∞	∞	∞
1	0	1	12	∞	∞	∞
2	0	1	10	4	∞	∞
4	0	1	8	4	17	19
3	0	1	8	4	13	19
5	0	1	8	4	13	17