

1. Five yes-no questions are included in this problem. Please answer each question with either 'True' or 'False'.
- a. In C++ code, `A = new char[100];`
`cout<<A[10];` and `cout<<*(A+10);` get the same output. (T)
 - b. Array allow for accessing any element directly using its index, an array A of length n, accessing A[100] needs more time than to accessing A[1]. (F)
 - c. Array is the most fundamental data structure. We can use it to implement any data structure, even the user define structure. (T)
 - d. Elements of an array are stored sequentially in memory. (T)

2. Write down the time complexity using big O.

(1) $T(n) = T(n/2) + 1$

Ans. $O(\log n)$

(2) $\sum_{i=0}^n i^2$

Solution: $\frac{n(n+1)(2n+1)}{6} = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$

Ans: $O(n^3)$

(3) $2n^2 + 5^n$

Ans. $O(5^n)$

(4) $n^7 + 1.5^n$

Ans. $O(1.5^n)$

3. Sort the following options with the time complexity(O) from low to high.

(a) $(n-1)!$ (b) $\log(n!)$ (c) n^5 (d) n^n (e) $\log^2 n$ (f) $\sqrt{2}^{\log n}$

Ans: $e < f < b < c < a < d$

(a) $(n-1)!$ (b) $\log(n!)$ (c) n^5 (d) n^n (e) $\log^2 n$ (f) $\sqrt{2}^{\log n}$

(a) $(n-1)!$ 階層等級

(b) $\log(n!) = O(n \log n)$ 多項式等級

(c) n^5 多項式等級

常數 < 對數 < 多項式 < 指數 < 階層 < 指數的指數

(d) n^n 指數的指數

$e < f < b < c < a < d$

(e) $\log^2 n$ 對數等級

(f) $\sqrt{2}^{\log n} = \sqrt{n}$ 多項式等級

4. Please derive the corresponding time complexity (Big-O) for each of the following program segments.

(a) $k = 0;$

for ($i=0; i < n; i++$)

$k++;$

Ans: $O(n)$

(b) $k = 0;$

for ($i=0; i < n; i++$)

for ($j=0; j < 5*n; j++$)

$k++;$

Ans: $O(n^2)$

(c) $k = 0;$

for ($i=0; i < n; i++$)

for ($j=0; j \leq i*i; j++$)

$k++;$

Ans: $O(n^3)$

5. Assume that each int element of an array occupies 4 units of storage and each double element of an array occupies 8 units of storage.

Suppose that the first element of array A is A[0][0] and its address is 120. Please give the address of the indicated element in each of the following cases.

(a) int A[5][3] with column-major order, please find the address of element A[4][1]

Ans: 156

(b) double A[4][6] with row-major order, please find the address of element A[2][3]

Ans: 240

6. Show that if $T(n) = \sqrt{n}T(\sqrt{n}) + n$, $T(m) = k$ and $m = n^{\frac{1}{2^i}}$, then

$$T(n) = kn^{(2^i-1)/2^i} + in.$$

Handwritten derivation of the recurrence relation:

$$\begin{aligned}
 T(n) &= \sqrt{n}T(\sqrt{n}) + n \\
 &= n^{\frac{1}{2}}T(n^{\frac{1}{2}}) + n \\
 &= n^{\frac{1}{2}}(n^{\frac{1}{4}}T(n^{\frac{1}{4}}) + n^{\frac{1}{2}}) + n \\
 &= n^{\frac{3}{4}}T(n^{\frac{1}{4}}) + 2n \\
 &= n^{\frac{7}{8}}T(n^{\frac{1}{8}}) + 3n \\
 &= n^{\frac{2^i-1}{2^i}}T(n^{\frac{1}{2^i}}) + in, \text{ 又 } T(m) = T(n^{\frac{1}{2^i}}) = k \\
 &= k \cdot n^{(2^i-1)/2^i} + in
 \end{aligned}$$

7. First, the elements 1, 2, 3, 4, and 5 are sequentially added to a stack, beginning with 1. Next, the stack is popped four times, transferring each popped element into a queue. After that, the first two elements from the queue are moved back onto the stack. Now which item would be popped from the stack?

Ans:4

8. Five pieces of data (A, B, C, D, E) are entered into a stack. If we only use PUSH and POP operations, which output sequences are impossible?
- a. C, E, D, B, A
 - b. E, D, C, B, A
 - c. A, D, B, C, E
 - d. B, C, E, D, A
9. Consider the following postfix expression, where % denotes the remainder operator.

20 5 3 + 6 * 8 3 - % 1 + /

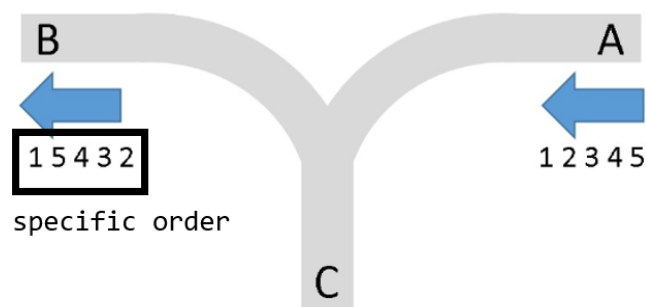
show its evaluation steps (Please draw a stack to show it).

ANS: 5

10. Rail problem (Hint : stack)

Suppose N cars are entering from direction A. Its number is fixed to (1,2, 3, ...N). Your task is to judge if these cars can leave to the direction B in a specific order.

For example, according to the above rules, this picture shows N = 5. If specific order = (1,2,3,4,5) and (1,5,4,3,2) are all feasible departure orders (true). If specific order = (5,4,1,2,3) is not feasible (false).



(1)According to the following message to answer questions. Can the train leave to direction B in a specific order?

If the answer is true: Write "true".

If the answer is false: Write "false".

(a) $N = 5$, specific order = (1,4,2,5,3)

Ans. false: after 1,4 , the next must be 3 or 5.

(b) $N = 7$, specific order = (1,6,2,5,4,3,7)

Ans. false: after 1,6 , the next must be 5 or 7.

(c) $N = 9$, specific order = (1,9,2,8,3,7,4,6,5)

Ans. false: after 1,9 , the next must be 8.

(2) $N = 4$. Write all specific orders that can leave from direction B.

Ans.

(1, 2, 3, 4)

(1, 2, 4, 3)

(1, 3, 2, 4)

(1, 3, 4, 2)

(1, 4, 3, 2)

(2, 1, 3, 4)

(2, 1, 4, 3)

(2, 3, 1, 4)

(2, 3, 4, 1)

(2, 4, 3, 1)

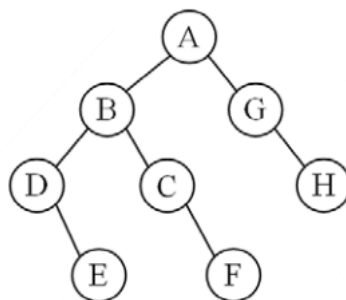
(3, 2, 1, 4)

(3, 2, 4, 1)

(3, 4, 2, 1)

(4, 3, 2, 1)

11. What is the order of "Postorder" in Figure below this question?



Ans: EDFCBHGA