Problem 1:

推導：直線：$y = mx + k$.

$\because (x_1, y_1)$ 和 (x_2, y_2) 在 L 上 $\therefore m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\rightarrow y = \dfrac{y_2 - y_1}{x_2 - x_1} \cdot x + k$ — ①

將 $(0, a)$ 代入①，$a = k$.

$\rightarrow y = \dfrac{y_2 - y_1}{x_2 - x_1} \cdot x + a$ — ②

將 (x_1, y_1) 代入②求 a.

$\rightarrow a = y_1 - \dfrac{y_2 - y_1}{x_2 - x_1} \cdot x_1$ — ③ ※

將 (r, b) 代入②求 b.

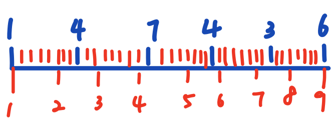
$b = \dfrac{y_2 - y_1}{x_2 - x_1} \cdot r + a \overset{③}{=\!=} \dfrac{y_2 - y_1}{x_2 - x_1} \cdot (r - x_1) + y_1$

$\rightarrow b = \dfrac{y_2 - y_1}{x_2 - x_1} \cdot (r - x_1) + y_1$ ※

Problem 2

a. $\ell = 9$

a. 近鄰：

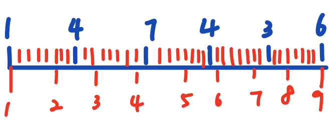


$(5,8) = 40$

取左邊值 1:1 2:4 3:4 4:7 5:7 6:4 7:3 8:3 9:6

Ans: [1, 4, 4, 7, 7, 4, 3, 3, 6] ※

b. 線性：



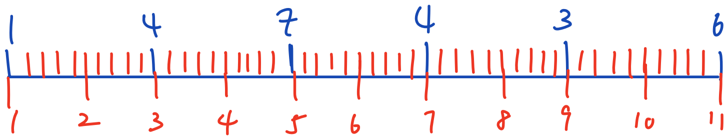
1 : 1 2 : $\dfrac{3\times1 + 5\times4}{8} = \dfrac{23}{8}$ 3 : $\dfrac{2\times7 + 6\times4}{8} = \dfrac{38}{8}$ 4 : $\dfrac{1\times4 + 6\times7}{8} = \dfrac{46}{8}$

5 : $\dfrac{7+4}{2} = \dfrac{44}{8}$ 6 : $\dfrac{1\times3 + 7\times4}{8} = \dfrac{31}{8}$ 7 : $\dfrac{2\times4 + 6\times3}{8} = \dfrac{26}{8}$ 8 : $\dfrac{3\times6 + 5\times3}{8} = \dfrac{33}{8}$ 9 : 6

Ans: $\left[1, \dfrac{23}{8}, \dfrac{38}{8}, \dfrac{46}{8}, \dfrac{44}{8}, \dfrac{31}{8}, \dfrac{26}{8}, \dfrac{33}{8}, 6\right]$ ※

b. length = 11

(a) 近鄰:



(5, 10) = 50

取左邊值, Ans = [1, 1, 4, 4, 7, 7, 4, 4, 3, 3, 6] #

(b) 線性:

Ans: $\left[1, \frac{5}{2}, 4, \frac{11}{2}, 7, \frac{11}{2}, 4, \frac{7}{2}, 3, \frac{9}{2}, 6\right]$ #

C. length = 2.

(a) 近鄰: Ans: [1, 6] #

(b) 线性: Ans: [1, 6] #

Problem 3.

当 $N=4$, $F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & (-i)^2 & (-i)^3 \\ 1 & (-i)^2 & (-i)^4 & (-i)^6 \\ 1 & (-i)^3 & (-i)^6 & (-i)^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$

(a)

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 14 \\ -2+2i \\ -2 \\ -2-2i \end{bmatrix}$$ #

(b)

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}\begin{bmatrix} 2 \\ -3 \\ 4 \\ -5 \end{bmatrix} = \begin{bmatrix} -2 \\ -2-2i \\ 14 \\ -2+2i \end{bmatrix}$$ #

(c)

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}\begin{bmatrix} -9 \\ -8 \\ -7 \\ -6 \end{bmatrix} = \begin{bmatrix} -30 \\ -2+2i \\ -2 \\ -2-2i \end{bmatrix}$$ #

(d)

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}\begin{bmatrix} -9 \\ 8 \\ -7 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -2-2i \\ -30 \\ -2+2i \end{bmatrix}$$ #