

Forecasting

Supply Chain Management, Chapter 7

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Outline

- Introduction
- Time-series forecasting
 - Static forecasting
 - Adaptive forecasting
 - Estimating the level, trend and seasonal factors
- Spreadsheet application
 - Using StatTools for linear regression
 - Using StatTools for adaptive methods
- Measures of forecast error
- Conclusion

Introduction: Business Case for Forecasting

"Over the years, ... while we've increased our business, we haven't had to increase our inventory or staff. We've just gotten better at forecasting product demand. As a result, we're able to achieve very high customer service levels without having to make unrealistically high investments in inventory."

-- David Robertson, Consumables Materials Manager, Alcon.



"Because I have good demand planning with low errors, I can better plan my logistics and the industrial production. So I can have better provisioning of the warehouse, fewer forklifts in operation and more efficient scheduling of resources in the factories. Plus, I will produce the right amount of products for the lowest possible costs for each region of the country."

-- Tiago Rino, Demand Planning Specialist, AmBev.

Companhia de Bebidas das Américas



Introduction to Forecasting

- What is forecasting?
 - Predict something unknown, usually in the future
 - In a supply chain setting, it is often the demand
 - What else?
 - Which strategy can benefit from forecasting?
- Why are we interested
 - Affects the decisions we make
 - What kinds of decisions?
- **The key assumption**
 - The future will be like the past

Characteristics of Forecasts

- They are usually wrong!
- Long-term forecasts are usually less accurate than short-term forecasts
 - Why?
- Aggregate forecasts are usually more accurate than disaggregate forecasts.
 - Intuition: Which is easier to forecast, the number of cans of _ Kroger will sell?
 - A. All soup
 - B. Campbell's™
 - C. All 'Italian Wedding' Soups
 - D. Campbell's™ Italian Wedding Soup

Introduction: Forecasting Methods

- Qualitative methods
Customer surveys, expert opinions, Delphi method
- Causal methods
Effects of the weather, the economy, etc.
- Time series
Estimate the future based on historical data
- Simulation
Combination of factors

Time-Series Forecasting: Example

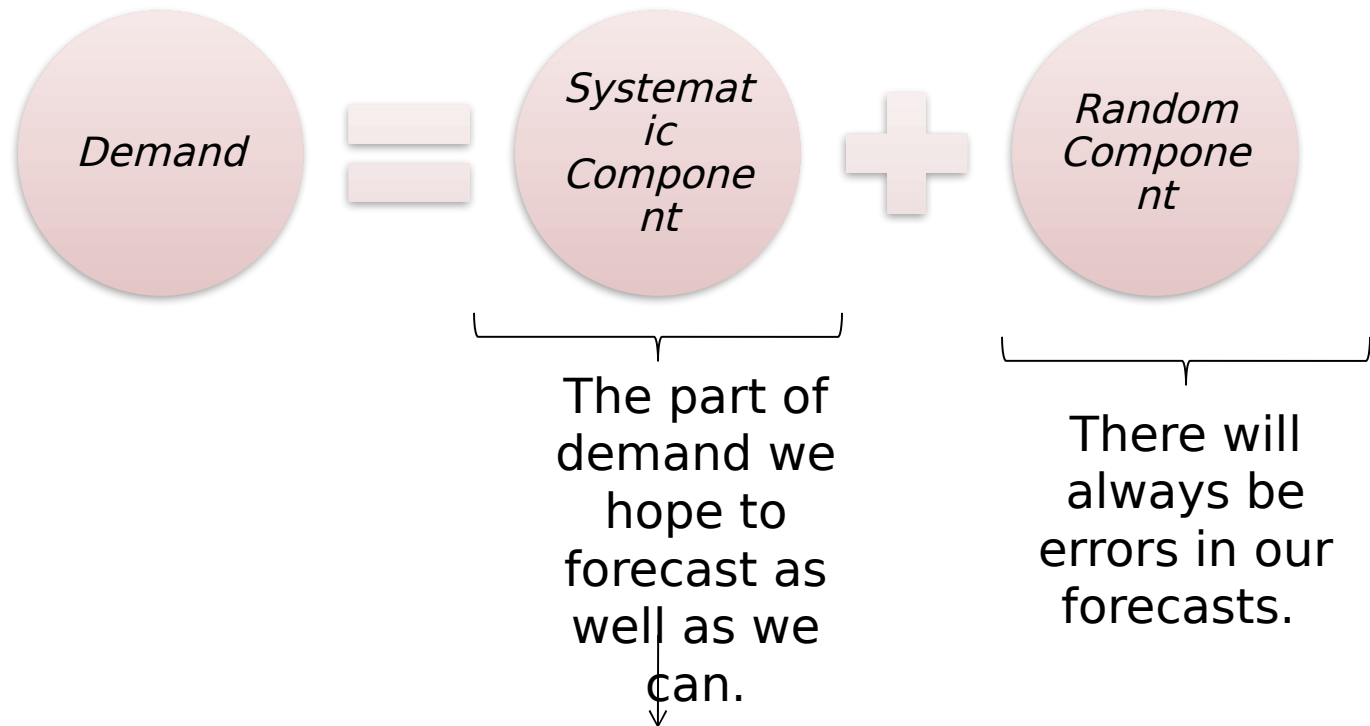
- Red Girl is a local business that makes chocolates. One of their products is an assorted chocolate gift box. They sell this product through the store, but most of the demand comes from the mail order business as their customers send these chocolates to friends and relatives during the holidays. The monthly demand data for this product over the last three years is shown on the next slide.

Time-Series Forecasting: Example

YearMonth			Dema	YearMonth			Dema	YearMonth			Dema
			nd				nd				nd
1	Jan		27	2	Jan		18	3	Jan		14
1	Feb		63	2	Feb		58	3	Feb		76
1	Mar		25	2	Mar		14	3	Mar		16
1	Apr		26	2	Apr		13	3	Apr		27
1	May		71	2	May		73	3	May		81
1	Jun		19	2	Jun		10	3	Jun		22
1	Jul		19	2	Jul		12	3	Jul		24
1	Aug		15	2	Aug		15	3	Aug		29
1	Sep		38	2	Sep		51	3	Sep		56
1	Oct		138	2	Oct		140	3	Oct		158
1	Nov		233	2	Nov		261	3	Nov		295
1	Dec		325	2	Dec		365	3	Dec		353

Given this data, what do you think the demand will be in October, November and December of next year?

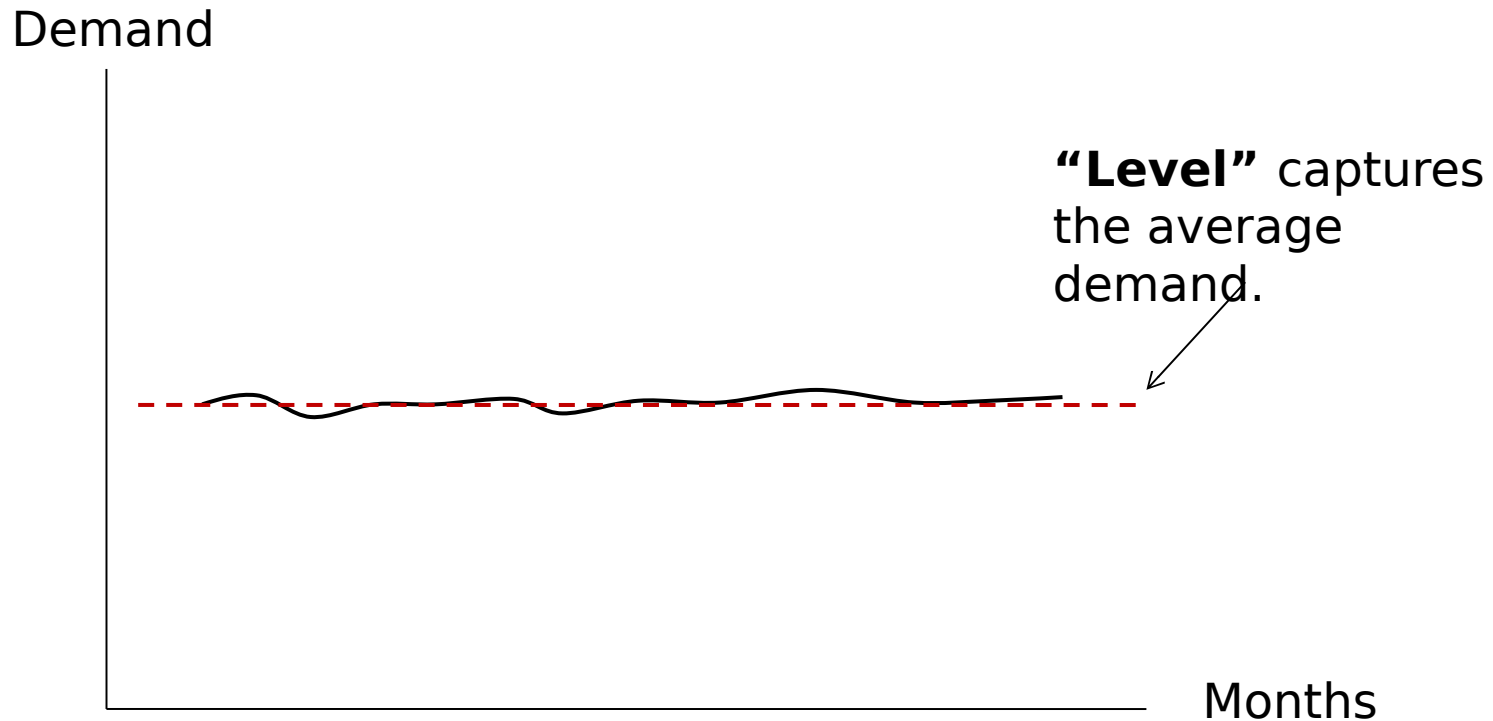
Time-Series Components



Relies on three estimates: The **level** of demand, the **trend** of demand, and the **seasonal factors** of demand.

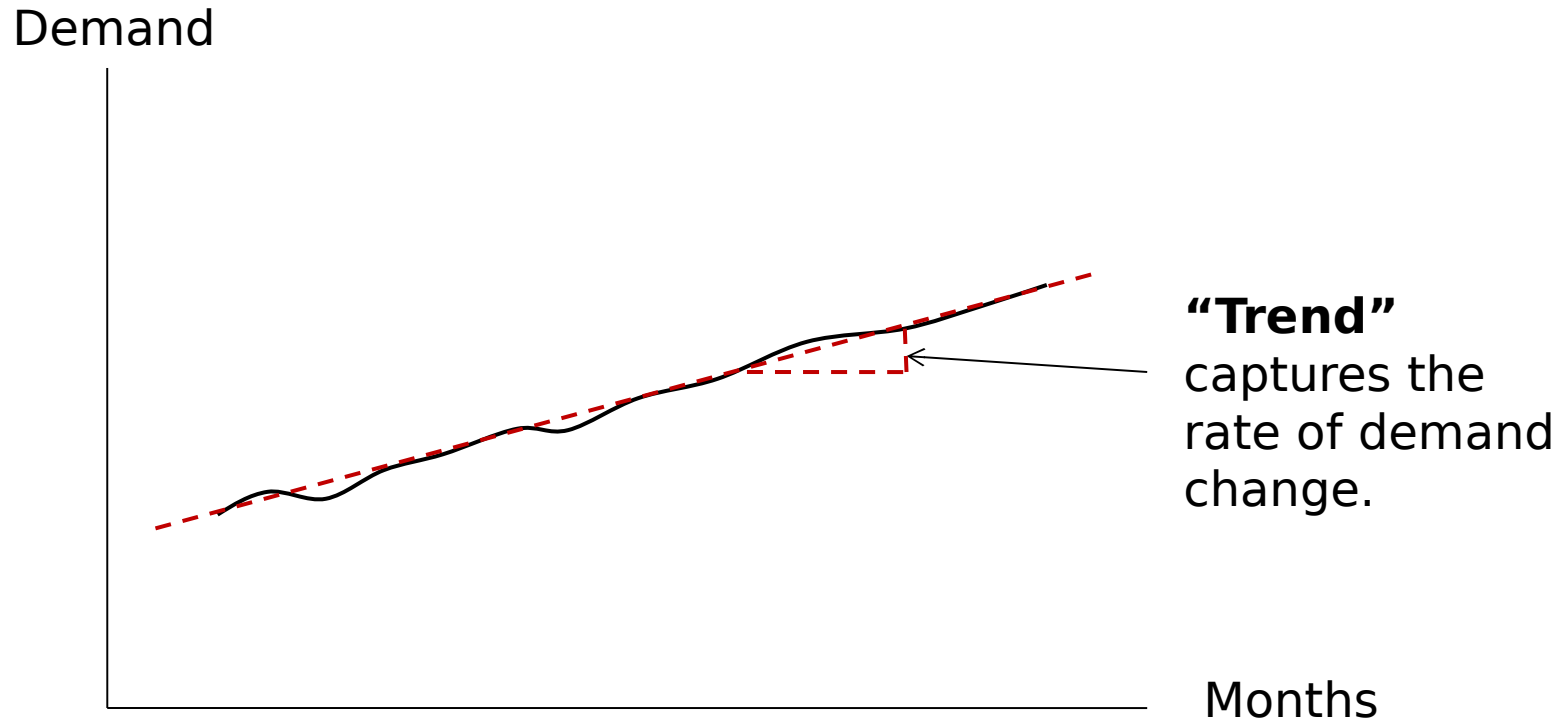
Time-Series Components

Consider the monthly demand for diapers at a grocery store.



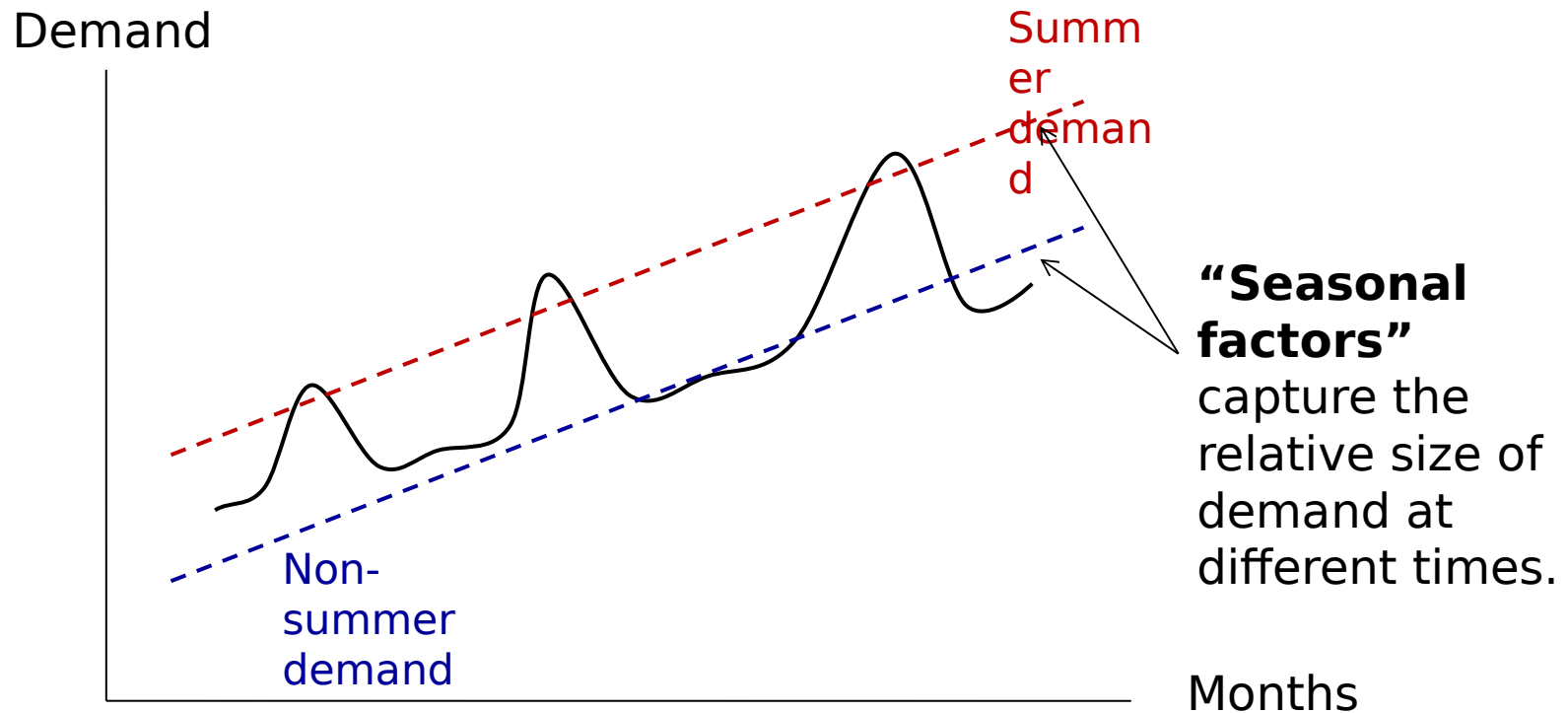
Time-Series Components

Imagine this diaper uses a new technology so the demand for it is growing.



Time-Series Components

Imagine this store is in a resort town, and the demand is much higher in the summer months.



Time-Series Components

- Demand must have a level (this is the amount of demand).
- **Trend** and **seasonal factor** may or may not be present.
- Example: Toothpaste
 - Expect level only
- Example: Pool toys
 - Expect level and seasonal factor; might have trend.
- Example: New video game
 - Expect level and trend; might have seasonal factor.

Time-Series Forecasting Methods: Classification

Time-series forecasting methods



Static

1. Estimate the level, the trend and the seasonal factors of demand.
2. Use those estimates to forecast future demand.
3. Stop.

Adaptive

Do the same as static, but update the level, trend and seasonal factor estimates every time new demand is observed.

Time-series forecasting methods



Multiplicative

Forecast = Level \times Trend^T \times Seasonal Factor

Additive

Forecast = Level + T \times Trend + Seasonal Factor

Mixed

Forecast = (Level + T \times Trend) \times Seasonal Factor

Static Forecasting



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Static Forecast

- Obtaining a **static forecast** gives us estimates of the level, trend, and seasonal factor.
 - A **static forecast** is used as **input** for the adaptive forecast.
1. Deseasonalize the demand data.
 2. Obtain level and trend by applying linear regression to deseasonalized demand data.
 3. Obtain estimates of the seasonal factors, using deseasonalized demand data and the estimates of trend and level.

Estimating level, trend, and seasonal factors

Example 1

- “Introduction to the Principles of Big Lebowski” is a course taught by the English department at IU (not true, for the record). The course has been taught in Fall, Spring and Summer for the last two years. The enrollments are shown below.

Year	Period	Enrollment
1	Fall	30
1	Spring	80
1	Summer	15
2	Fall	40
2	Spring	90
2	Summer	30

Step 1: Deseasonalize the demand data *Example 1*

- Assume there are 3 types of periods.
- Key idea:** each average should contain **1** of each type of period.
- In this example,

Year	Period	Time	Enrollment	Deseasonalized Demand
1	Fall	1	30	41.7
1	Spring	2	80	
1	Summer	3	15	
2	Fall	4	40	48.3
2	Spring	5	90	53.3
2	Summer	6	30	

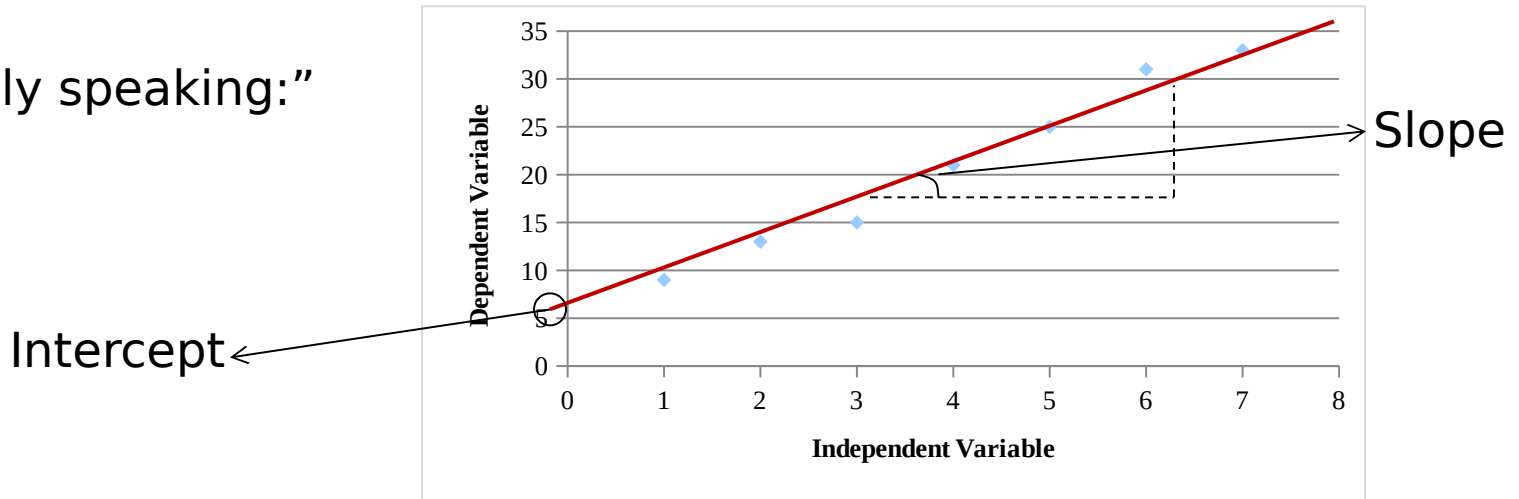
- If n is **odd**, take the moving average of the n periods centered around the period of interest (above).
- If n is **even**, we cannot center our moving average around the period of interest; we would have to take two moving averages of order $n/2$ and take **their** average.

Step 2: Estimate the level and trend *Example 1*

- Recall that linear regression fits a line to data in the following format:

$$\begin{array}{c} \text{Dependent} \\ \text{Variable} \end{array} = \begin{array}{c} \text{Intercep} \\ t \end{array} + \text{Slope} \times \begin{array}{c} \text{Independent} \\ \text{Variable} \end{array}$$

“Visually speaking:”



Given several values of the independent variable and the corresponding values of the dependent variable, linear regression finds the value of intercept and slope that match the data best (for least squares, by minimizing the squared error).

Step 2: Estimate the level and trend *Example 1*

- When we apply linear regression to deseasonalized demand data:

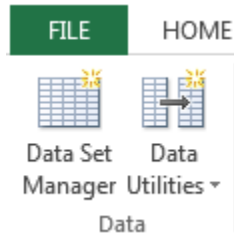
$$\text{Demand} = \underbrace{\text{Intercept}}_{\text{Level}} + \underbrace{\text{Slope Time}}_{\text{Trend}}$$

"Time" (Indep. Var.)	Deseasonalized demand (Dependent Var.)
2	41.7
3	45.0
4	48.3
5	53.5

- We can use the **Analysis Toolpak** or the more powerful add-in **StatTools** to do linear regression.

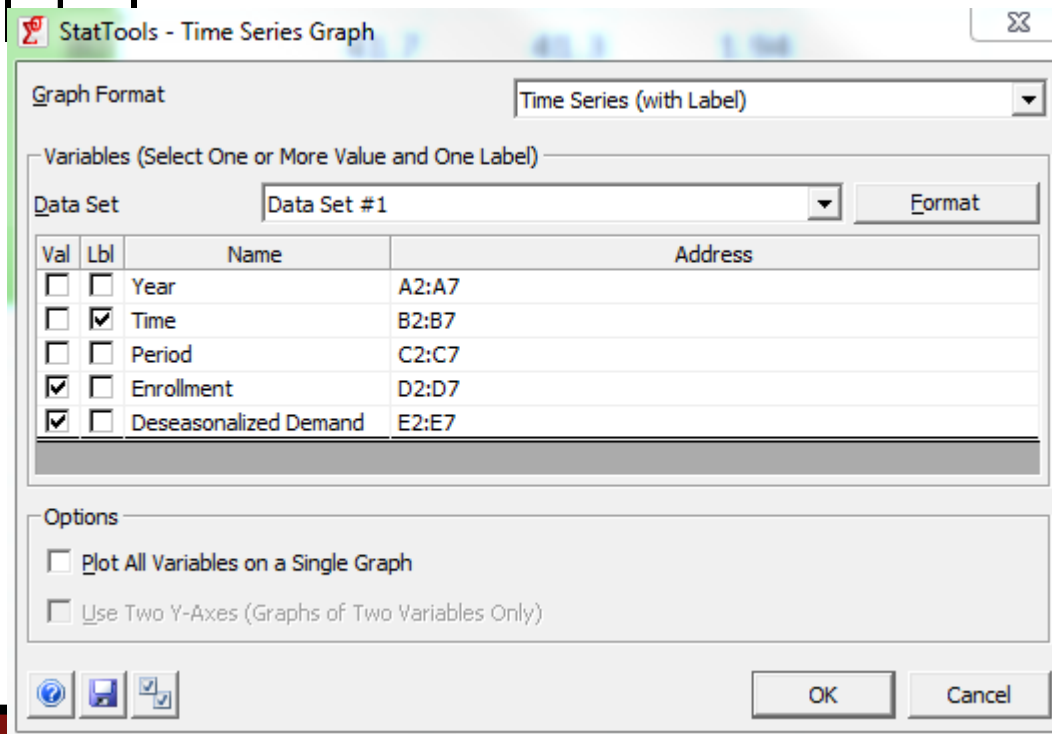
Step 2: Estimate the level and trend *Example 1*

- Use StatTools to define a data set.



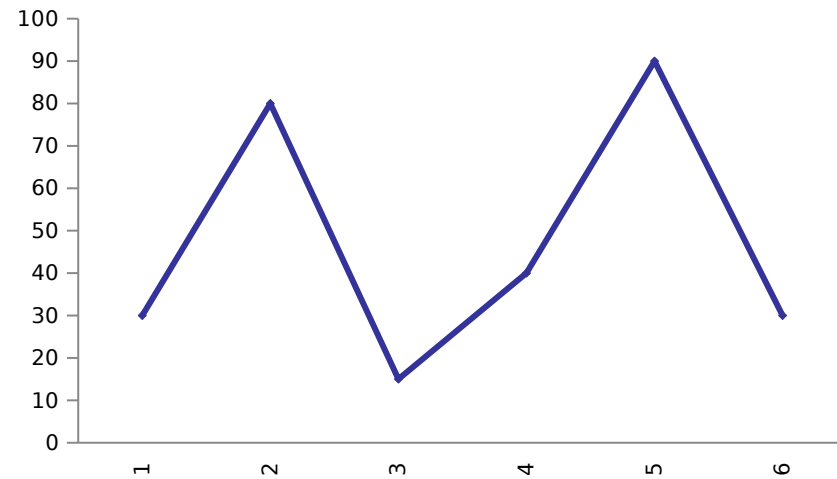
Step 2: Estimate the level and trend *Example 1*

- Before running **ANY** analysis, always plot the data!
- Use the “Time Series Graph” function in StatTools.
- Select (with label).
- Plot both **demand** and **deseasonalized demand** with **time** as

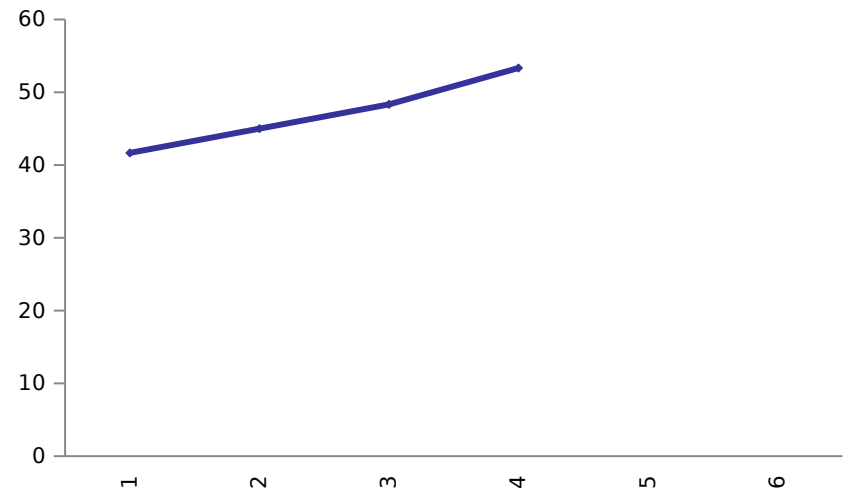


Step 2: Estimate the level and trend *Example 1*

Time Series of Enrollment / Data Set #1

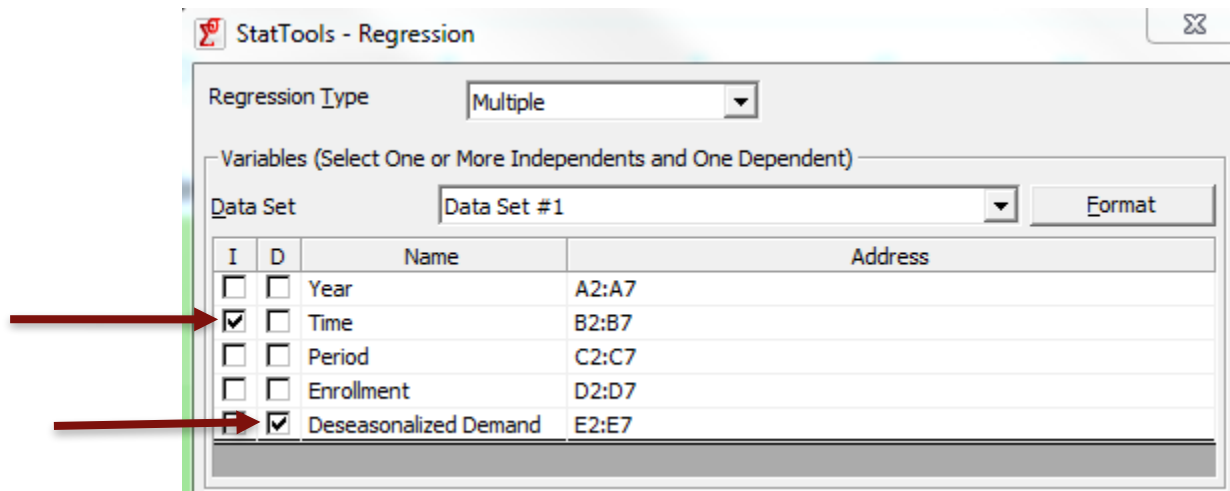


Time Series of Deseasonalized Demand / Data Set #1



Step 2: Estimate the level and trend *Example 1*

- Run a regression with **time** as the independent variable and **deseasonalized demand** as the dependent variable.



- You must have a **line** to use **linear regression**. Do not run linear regression directly on data that has any clear seasonal pattern!

Step 2: Estimate the level and trend Example 1

- In the resulting report, we obtain two coefficients: **constant** and **time**.
- The **constant** coefficient is the level at time **0**.
- The **time** coefficient is the trend.

<i>Multiple Regression for Deseasonalized Demand</i>					
<i>Summary</i>	Multiple R	R-Square	Adjusted R-Square	StErr of Estimate	
	0.9944	0.9888	0.9832	0.645497224	
<i>ANOVA Table</i>	Degrees of Freedom	Sum of Squares	Mean of Squares	F-Ratio	p-Value
Explained	1	73.47222222	73.47222222	176.3333	0.0056
Unexplained	2	0.833333333	0.416666667		
<i>Regression Table</i>	Coefficient	Standard Error	t-Value	p-Value	Confidence Interval 95%
					Lower Upper
Constant	33.66666667	1.060660172	31.7412	0.0010	29.10301428 38.23031905
Time	3.833333333	0.288675135	13.2791	0.0056	2.591264477 5.075402189

Step 3: Estimate the seasonal factor *Example 1*

Deseasonalized demand (Level) = $33.7 + 3.8 \text{ Time}$

Year	Period	"Time"	Enrollm ent	Deseasonal ized Data	Level	Seasonal Factor Estimate
1	Fall	1	30		$33.7 + 3.8 * 1 = 37.5$	$30 / 37.5 = 0.80$
1	Spring	2	80	41.7	41.3	1.94
1	Summ er	3	15	45.0	45.2	0.33
2	Fall	4	40	48.3	49.0	0.82
2	Spring	5	90	53.3	52.8	1.70
2	Summer	6	30		56.7	0.53

Average the seasonal factor estimates for each period:

Period	Seasonal Factor
Fall	0.81
Spring	1.82
Summer	0.43

Forecasting the future demand *Example 1*

Deseasonalized demand (Level) = $33.7 + 3.8 \text{ Time}$

Period	Seasonal Factor
Fall	0.81
Spring	1.82
Summer	0.43

- **Predict** the demand for enrollments in Year 3 Fall.
 - This is time period **7**
 - Fall seasonal factor is 0.81.
 - The predicted demand is $0.81 * (33.7 + 3.8 * 7) = 48.89$

Static Forecast

Example 2

- The demand for XYZ's product shows an annual pattern, where the demand grows from the first quarter to the next, then decreases in the third and fourth quarters. The demand data for the last two years is shown below.

Year	Quarter	Demand
1	1	500
1	2	1000
1	3	400
1	4	300
2	1	600
2	2	1200
2	3	700
2	4	400

- Estimate the level, trend, and seasonal factors.

Estimating level, trend, and seasonal factors *Example 2*

- Notice that , so deseasonalizing the demand is more complicated.

Year	Quarter	"Time"	Demand	Deseasonalized Demand
1	1	1	500	
1	2	2	1000	
1	3	3	400	????
1	4	4	300	
2	1	5	600	
2	2	6	1200	
2	3	7	700	
2	4	8	400	

- To calculate deseasonalized demand for time 3, we must use the demands from time 2, 3, 4, and the **average** of times 1 and 5.
- In other words, times 1 and 5 each get $\frac{1}{2}$ the weight of the others.

Estimating level, trend, and seasonal factors

Example 2

- Steps 2 and 3 are similar to Example 1.

Year	Quarter	"Time"	Demand	Deseasonalized Demand
1	1	1	500	$\times 1/2$
1	2	2	1000	
1	3	3	400	
1	4	4	300	
2	1	5	600	$\times 1/2$
2	2	6	1200	
2	3	7	700	
2	4	8	400	

Linear Regression Diagnostics

Example 3

- In our time-series analysis, X =time and Y =demand.
- *Constant*: Y -value when $X=0$. In other words, this is the initial level of the demand.
- *Time Coefficient*: Change in Y for every unit change in X . In other words, this is the trend of the demand.
- *R-squared*: Tells how well the fitted line explains the change in Y .
 - $R\text{-squared} = 0.60$ means 60% of the change in Y (demand) is explained by change in X (time).
 - The rest is either explained by some other factor (such as...)
or is unexplainable noise.
 - **Every data set has noise.**

Linear Regression Diagnostics

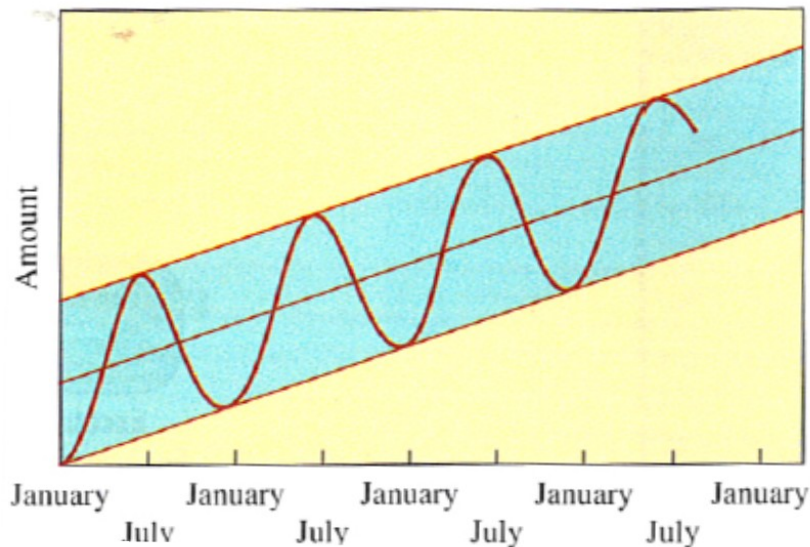
- *P-value*: Tells us the significance of a coefficient.
- Residual plot: shows the differences between the data and the predicted line.
 - No pattern: this is ideal, means the original data had a linear pattern.
 - Seasonal pattern in the residuals: this is bad, means the data was non-linear. **If there is a pattern in the residuals, linear regression is NOT appropriate to use on the data set.**
- See *Example 3*. Linear Regression Diagnostics.



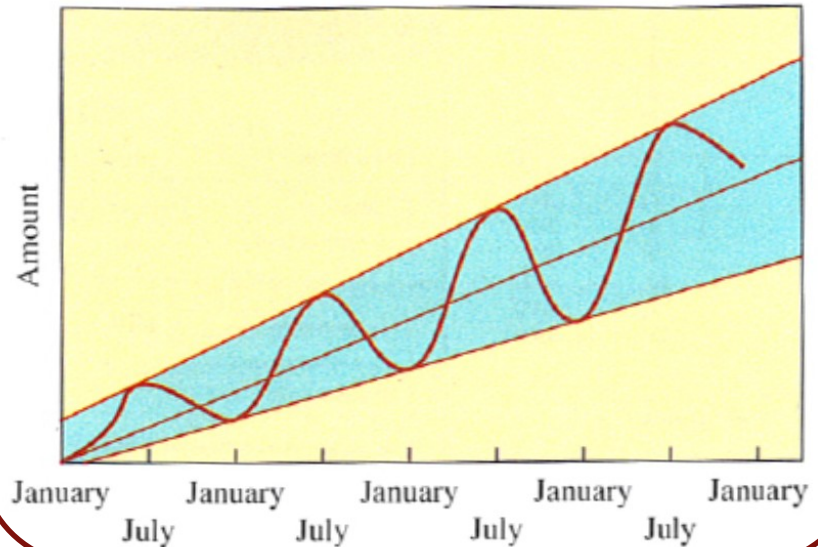
Types of Models: Mixed

- Mixed: $Demand = (Level + T \times Trend) \times Seasonal Factor$
 - Trend is **additive**: add one more “Trend” every time period.
 - Seasonal factor is **multiplicative**: seasons result in a % increase/decrease in demand.

A. Additive Seasonal



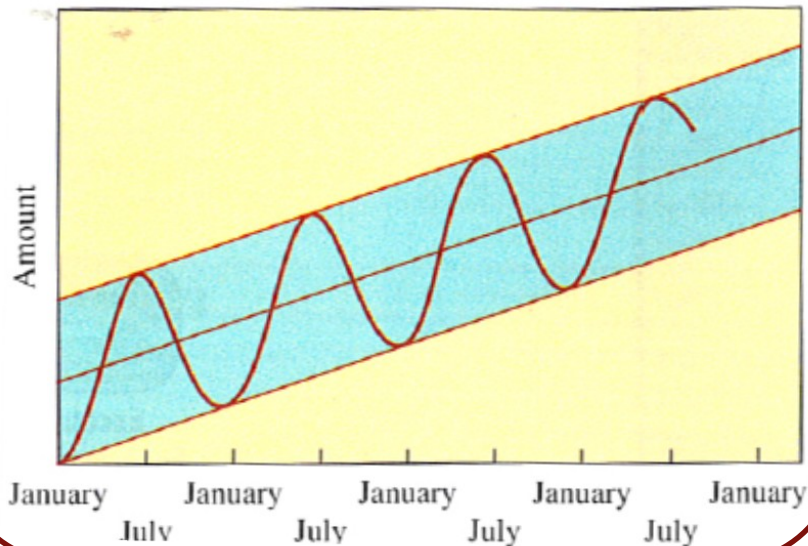
B. Multiplicative Seasonal



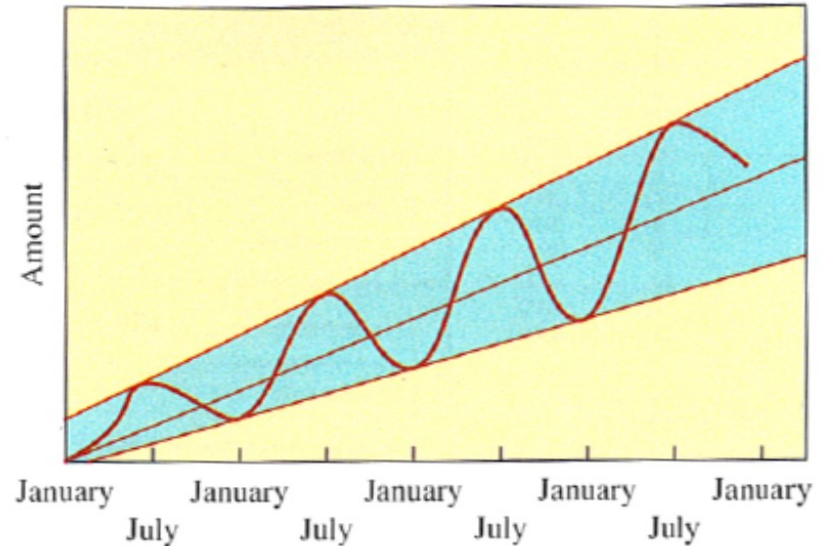
Types of Models: Additive

- Additive: $Demand = Level + \cancel{T} \times Trend + Seasonal Factor$
 - Trend is **additive**: add one more “Trend” every time period.
 - Seasonal factor is **additive**: seasons result in a **fixed** increase/decrease in demand, regardless of underlying

A. Additive Seasonal



B. Multiplicative Seasonal

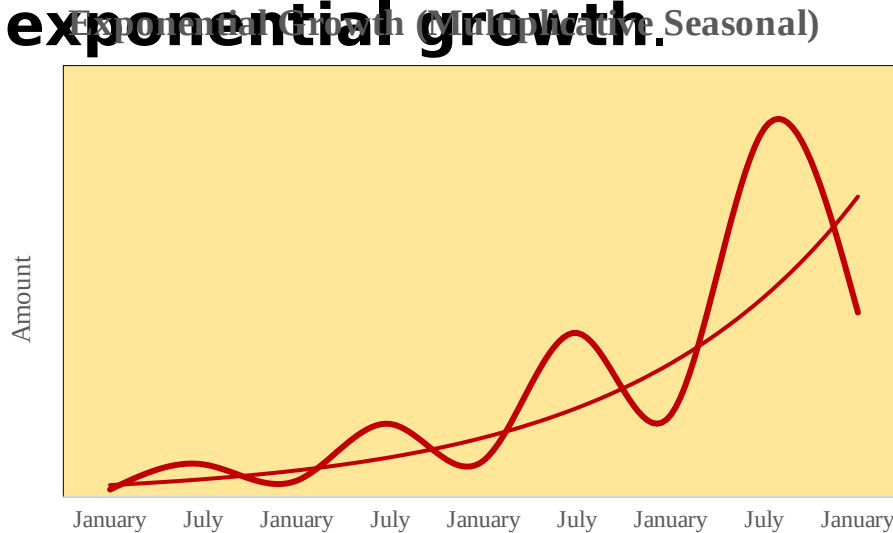


Types of Models: Additive

- Additive: $Demand = Level + T \times Trend + Seasonal\ Factor$
 - Trend is **additive**: add one more “Trend” every time period.
 - Seasonal factor is **additive**: seasons result in a **fixed** increase/decrease in demand, regardless of underlying demand.
- Modify calculation of seasonal factor
 - When seasonal factor is multiplicative, estimate by $SF = demand / level$
 - When seasonal factor is additive, estimate by $SF = demand - level$

Types of Models: Multiplicative

- Multiplicative: $Demand = Level \times Trend^T \times Seasonal\ Factor$
 - Trend is **multiplicative**: demand increases by same % each period.
 - Seasonal factor is **multiplicative**: seasons result in a % increase/decrease in demand.
- Also called **exponential growth**.



Types of Models: Multiplicative

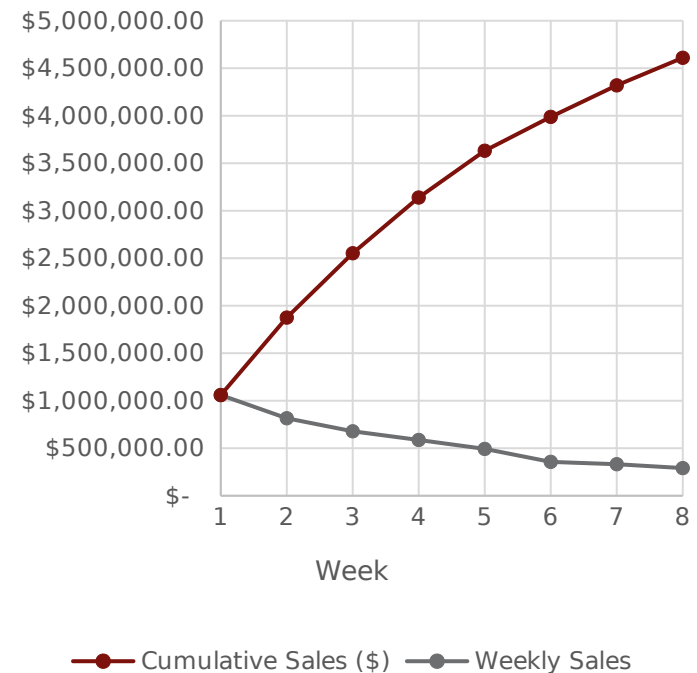
- Multiplicative: $Demand = Level \times Trend^T \times Seasonal\ Factor$
- We handle this model by taking the **log** of both sides:
 - $Log(Demand) = Log(Level \times Trend^T \times Seasonal\ Factor)$
 - Use the rules $Log(a \times b) = Log(a) + Log(b)$ and $Log(a^b) = b \times Log(a)$
- $Log(Demand) = Log(Level) + T \times Log(Trend) + Log(Seasonal\ Factor)$
- Thus, we use a **log transform** of the data and then handle the data as if it is an **additive** model!

Types of Models: Multiplicative Example 4

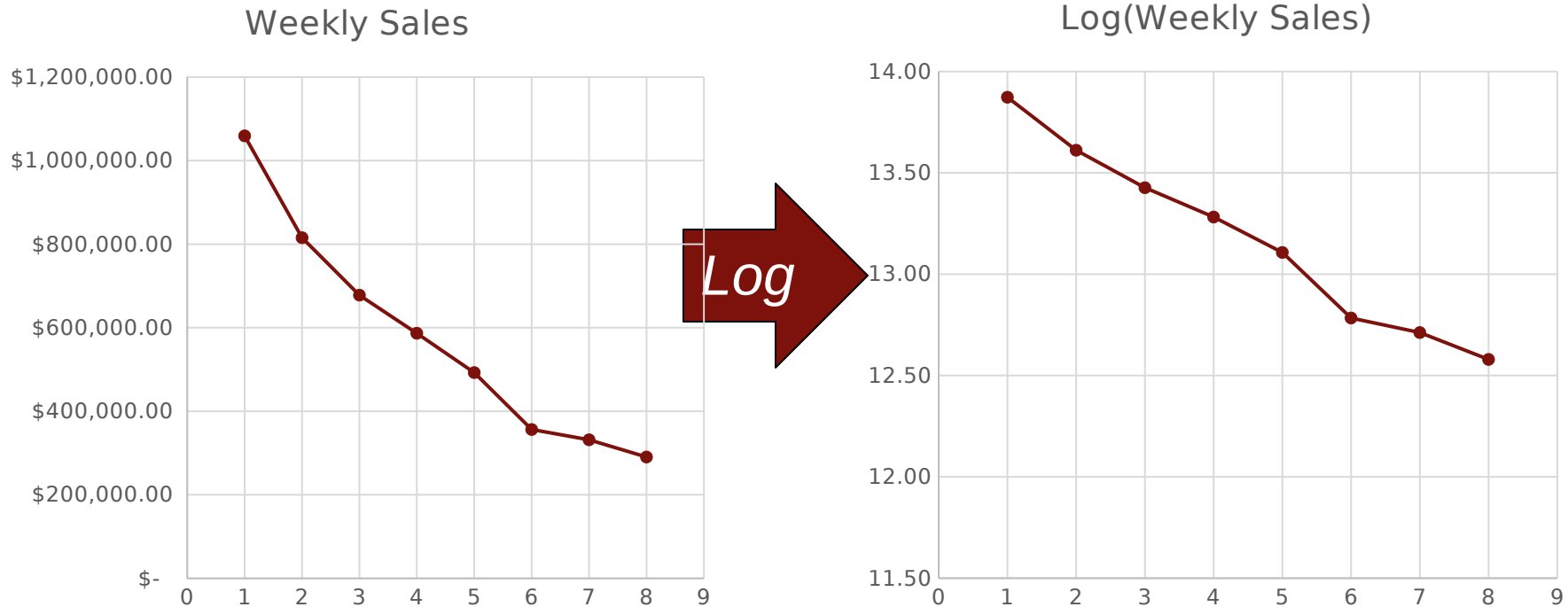
- Let's examine sales of a new technology product.
- Remember, graph the data first.

Week	Cumulative Sales (\$)	Weekly Sales
1	\$ 1,059,204.63	\$ 1,059,204.63
2	\$ 1,874,630.54	\$ 815,425.91
3	\$ 2,552,518.85	\$ 677,888.31
4	\$ 3,139,080.54	\$ 586,561.69
5	\$ 3,631,538.93	\$ 492,458.39
6	\$ 3,987,823.02	\$ 356,284.09
7	\$ 4,319,397.99	\$ 331,574.97
8	\$ 4,609,827.33	\$ 290,429.34

Sales of a New Electronics Product



Types of Models: Multiplicative



Types of Models: Multiplicative *Example 4*

- Log transform can be accomplished with the =LN function.
- Once we are done, we can recover our coefficients with the =EXP function (exponential, the inverse of log).
- How to tell if we need to use a multiplicative model?
Examine the plot of data and the residual plot.