"Polynomial dendritic neural networks (2022) " uses the Generalized Dendrite module (°Ai-1) in "It may be time to improve the neuron of artificial neural network(2020)" without citing it.

In 2020, the Generalized Dendrite module (°A) was published. In the generalized Dendrite module, a special form (°X) named DD is published separately. On 22 February 2022, an paper "Polynomial dendritic neural networks" uses "Generalized Dendrite module" (WX°A)" and claims they propose dendrites in general form and call DD a special case.

• It may be time to improve the neuron of artificial neural network(2020),

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I just want them to point out "Generalized Dendrite module (WX°A i-1)" in their paper and cite "It may be time to improve the neuron of artificial neural network(2020)" in accordance with LICENCE: CC BY-NC-SA 4.0, but they ignored me.

Comparison between papers	
Polynomial dendritic neural networks (2022)	It may be time to improve the neuron of artificial neural network(2020)
	https://doi.org/10.36227/techrxiv.12477266
	(IEEE TechRxiv, This paper is the IEEE preprint Top 1 in yearly popularity. You Must cite it!)

Formally, an EPDN is defined as the following:

$$X_{l} = (W^{l}X^{l-1} + b^{l})_{o}(A_{l}X^{l-1} + t^{l})$$
(7)

where, in the following, the operation  $_{\circ}$  is the Hadamard product,  $b^{l}, t^{l} \in \mathbb{R}^{d_{l}}$  are biases, and  $W^{l}, A^{l} \in \mathbb{R}^{d_{l} \times d_{l-1}}$  are weight matrices. In this model,  $W^{l}, A^{l}, b^{l}$ , and  $t^{l}$  are parameters.

According to (7), the neurons in the layer k | < l - 1 are indirectly related to  $X^l$ . Starting with input  $X^0$ ,  $X^1$  will be a quadratic polynomial of  $X^0$ , and so on. Eventually, we obtain the following result:

2) Generalized Dendrite module ( $\circ A^i$ ): Figure 4 shows a generalized Dendrite module. The module is represented as follows.

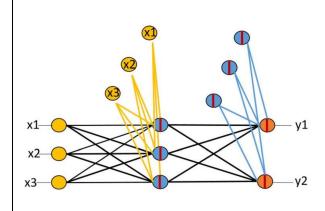
$$A^{i} = W^{i,i-1}A^{i-1} \circ A^{0|1|2|\dots|i-1}$$

Where  $A^{i-1}$  and  $A^i$  are the inputs and outputs of the module, respectively.  $A^{0|1|2|...|i-1}$  is any of A.  $\circ$  denotes Hadamard product.

It has been pointed out in the text that in matrix X, the element x0 is set to 1 in the 2020 paper. Thus, w0 corresponds to the bias in the 2022 paper.

Additionally, the 2020 paper said, "When the user wants to reduce or increase the dimension, just insert a linear module."

## Thus, The PDN called by the 2022 paper is the Generalized Dendrite module(2020).



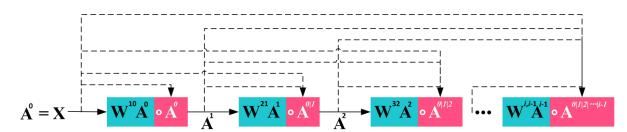
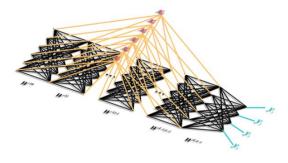


Fig. 4. Generalized Dendrite module. '|' denotes 'or'. The dotted line represents any of them.



Dendrite module (°X)

Both papers(2022 vs. 2020) are different drawings of the same schematic.

## Proposition 3 In (7), $X^l$ is a polynomial of $X^0$ with degree

Thus, the degrees increase quite fast. In ANNs, the layer  $k \in (0, l-1)$  is often considered as hidden layers when we focus on the layer l and the input layer 0. This suggests that we should consider the dependence of  $X^{l}$  on  $X^{0}$  and  $X^{l-1}$ , as demonstrated by Fig. 2.

**Proposition 4** In (8),  $X^l$  is a polynomial of  $X^0$  with degree l+1.

In this model,  $X^l$  is also a polynomial of  $X^0$ , but its degree increases in an asymptotic way as Proposition 4. This is why we call it the asymptotic polynomial dendritic neural network (APDN).

When  $A^{0|1|2|...|i-1} = A^{i-1}$ , the degree increases by i for one module. The degree increases with the number of modules exponentially. For a certain degree of network, the computational complexity is approximate  $log_2^n$ , where n denotes the number of modules. This is similar to exponentiation by squaring. However, exponentiation by squaring is only used for calculation, and the modules in this paper are used for approximation. The weight matrix in Fig. 4 can be solved for by error backpropagation.

## The degree claimed by the 2022 paper is also explained in the 2020 paper.

Proposition 7 The DD in [24] is a special case of APDN

Therefore, module (8) can be applied to more networks than the DD in [24]. Such generalization is necessary because the number of neurons at different layers usually is different. For example, [10] studied pyramidal neurons. In such cases, usually  $d_i \neq d_l$  for  $0 \leq i, j \leq L$ .

2) Generalized Dendrite module  $(\circ A^i)$ : Figure 4 shows a generalized Dendrite module. The module is represented as follows.

$$A^{i} = W^{i,i-1}A^{i-1} \circ A^{0|1|2|\dots|i-1}$$
(6)

Where  $A^{i-1}$  and  $A^i$  are the inputs and outputs of the module, respectively.  $A^{0|1|2|...|i-1}$  is any of A.  $\circ$  denotes Hadamard

We can adjust the promotion degree or the number of interactive variables added for the single module by  $A^{0|1|2|...|i-1}$ . When we add a module, the degree of network can increase by at least one and by i at most.

When  $A^{0|1|2|...|i-1} = A^0$ , the degree increases by 1 for one module, which is similar to Horner's method. The degree increases with the number of modules linearly. For a certain degree of network, the computational complexity is n-1, where n denotes the number of modules.

The 2022 paper said that DD is a special case, and the 2020 paper has pointed out that there is a paper with a special case of DD (the one for comparison in the 2022 paper).

I would like the authors in 2022 to point out the Generalized Dendrite module (°Ai-1) in a prominent place in the paper and cite it according to the LICENCE: CC BY-NC-SA 4.0.

The paper ( https://doi.org/10.36227/techrxiv.12477266 ) has been cited in multiple papers, such as IEEE Transactions on Cybernetics and IEEE Transactions on Neural Systems and Rehabilitation Engineering , on Google Scholar.