ECE 2331 – Programming Assignment 4
The Kessel Run

The problem

Your job is to create an algorithm that can plot a course through a black hole cluster. You will load a series of black hole positions and masses:

$$X = [x_1, x_2, x_3, \cdots, x_N]$$
$$Y = [y_1, y_2, y_3, \cdots, y_N]$$
$$M = [m_1, m_2, m_3, \cdots, m_N]$$

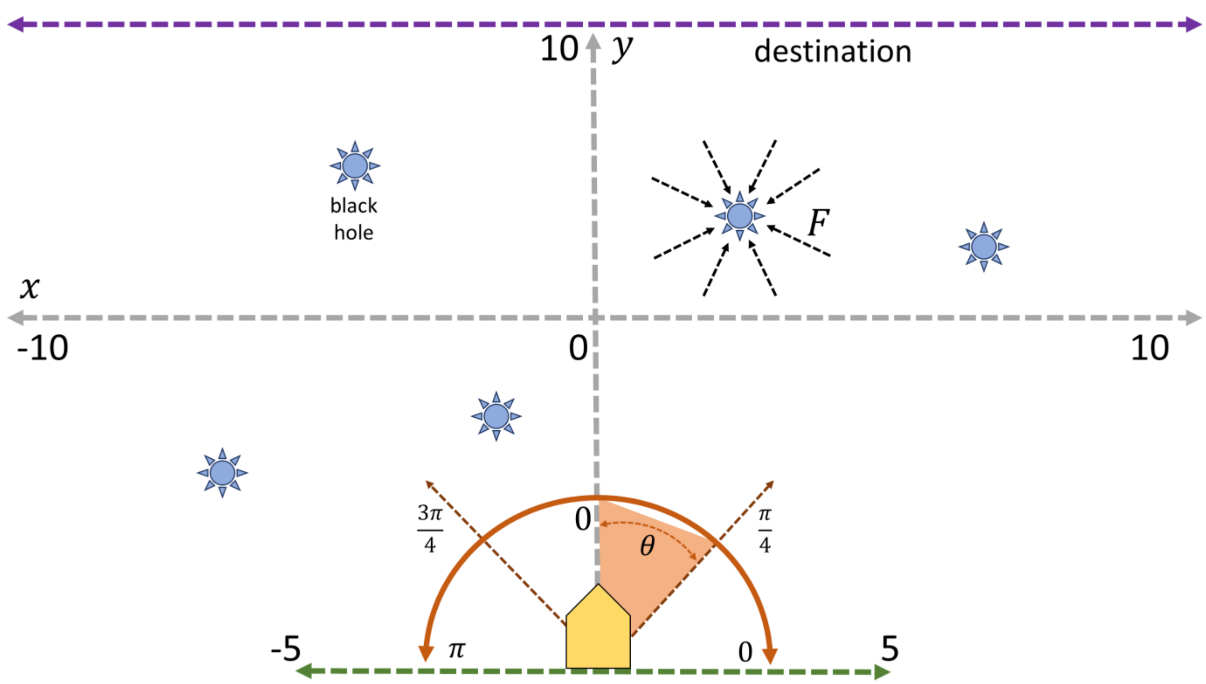
where (x_i, y_i) is the position of gravity well i and m_i is its mass. These black holes will be placed on a 20x20 parsec plane with the origin at the center: ($[-10, 10] \times [-10, 10]$). The starting point p_0 can be anywhere along a 10 parsec line centered at the bottom of the playing field:

$p_0 = (p_x, p_y)$, where $p_x \in [-5, 5]$ and $p_y = -10$.

The starting velocity $v(t_0) = \boldsymbol{v_0}$ is given by the direction vector $\overline{\boldsymbol{v}}_{\boldsymbol{0}}$ with magnitude $|\boldsymbol{v_0}|$:

$$\boldsymbol{v_0} = |\boldsymbol{v_0}| \cdot \overline{\boldsymbol{v_0}}$$

The starting trajectory $\overline{\boldsymbol{v_0}}$ is specified by a normal distribution with a standard deviation of $\frac{\pi}{4}$ radians from the positive y-axis: $[0, 1]$. The initial state of the playing field is shown in the following figure:



Software

Your software will compute the shortest path through the black hole cluster and return the optimal start point p_0 and velocity v_0 to make the trip. This optimal start state will be estimated using a Monte-Carlo simulation. Each start state will be tested using a physically-based model of the black hole cluster using an explicit Runge-Kutta integration method (ex. Euler's method).

Output

You are required to use Matlab, and may use any functions available in the standard distribution. Turn your program in as a single *.m file. Display the following using a scatter plot:

- The gravity well positions (files of positions will be provided)
- The shortest path found as a color-mapped curve (blue = start, red = end)
- The longest path found as a color-mapped curve (blue = start, red = end)

Write your code to perform the following functions:

For each Monte-Carlo sample----------------

1. Select a starting position $p(t_0) = p_0$ for your ship
   a. The start position is a randomly selected position at $y = -10$ and $x \in [-5,\ 5]$
   b. Draw the random position from a uniform distribution
2. Select a starting velocity $v(t_0) = \boldsymbol{v_0} = |\boldsymbol{v_0}| \cdot \overline{\boldsymbol{v_0}}$
   a. The orientation vector $\overline{\boldsymbol{v_0}}$ is a normalized trajectory from the start point
      i. Select $\overline{\boldsymbol{v_0}}$ to be pointing outward from p_0
      ii. Draw this trajectory from a normal distribution with a standard deviation of $\frac{\pi}{4}$
      iii. The mean of this distribution is along the y-axis: usually expressed as $\theta = \frac{\pi}{2}$ in polar coordinates
   b. The scalar magnitude $|\boldsymbol{v_0}|$ is the starting speed of your ship relative to the start point
      i. Select $|\boldsymbol{v_0}| \in [2, 5]$
      ii. Draw this random speed from a uniform distribution
3. Simulate the passage of the ship
   a. For each time step
      i. Determine the force incident on the ship (see below)
         1. Terminate if the maximum net force exceeds $F = 4$
      ii. Update the ship's velocity using an explicit method (ex. Euler's method)
      iii. Update the ship's position using an explicit method (ex. Euler's method)
         1. Terminate (successfully) if the ship reaches its destination: $y > 10$

System of Differential Equations

In step (3), you are solving for the ship position as a function of time $p(t)$ using the following system of differential equations:

$$\frac{dp}{dt} = v(t)$$

$$\frac{dv}{dt} = a(t)$$

$$\sum_{i=1}^{N} F_i = m_s \cdot a(t)$$

The net force F applies an acceleration to your ship. The net force applied is the sum of forces applied by all black holes. The force applied by a black hole is proportional to the distance between your ship and the black hole. The vector from your ship and a black hole is given by:

$$r_i = p_s - (x_i, y_i)$$

where p_s is the position of your ship and i is the index of the black hole. The distance between your ship and the black hole is $|r_i|$, where the vector magnitude is given by the Euclidean norm:

$$|b| = \sqrt{b_x^2 + b_y^2}$$

The force that a single black hole applies to your ship is:

$$F_i = \frac{r_i}{|r_i|} \cdot \frac{m_s m_i}{|r_i|^2} = \frac{r_i m_s m_i}{|r_i|^3}$$

Where m_s is the mass of your ship and m_i is the mass of the ith black hole. Therefore, the force applied by all of the black holes is given by:

$$F = -\sum_{i=1}^{N} \frac{r_i m_s m_i}{|r_i|^3}$$

Programming Assignment 4 – Runge-Kutta Methods (Matlab)

Name____________________________

Correct result		**__________ / 30**
shortest path found	__________ / 20	
longest path found	__________ / 10	
Physics		**__________ / 25**
evaluation of force	__________ / 10	
force in differential equations	__________ / 5	
comments and readability	__________ / 10	
Euler Integration		**__________ / 45**
correct formulas for velocity	__________ / 10	
correct formulas for position	__________ / 15	
code is correct / robust	__________ / 10	
comments and readability	__________ / 10	
Total		**__________ / 100**