

Disassortative mixing in online social networks

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Abstract. - The conventional wisdom is that social networks exhibit an assortative mixing pattern, whereas biological and technological networks show a disassortative mixing pattern. However the recent research on the online social networks (OSN) modifies the wide-spread belief and many OSNs show a disassortative or neutral mixing feature. Especially we found that an OSN, *Wealink*, underwent a transition from degree assortativity characteristic of real social networks to degree disassortativity characteristic of many OSNs, and the transition can be reasonably elucidated by a simple network model we propose. The relations among network assortativity, clustering and modularity are also discussed in the paper.

A social network consists of all the people-friends, family and others-with whom one shares a social relationship, say friendship, commerce, or others. Traditional social network study can date back about half a century, focusing on interpersonal interactions in small groups, not structures of large and extensive networks due to the difficulty in obtaining large data sets [1]. The advent of modern database technology has greatly stimulated the statistical analysis of social networks. Novel network structures of human societies have been revealed.

Now the WWW is undergoing a landmark revolution from the traditional Web 1.0 to Web 2.0 characterized by social collaborative technologies. As a fast growing business, many social networking sites (SNS) have emerged in the Internet. The OSNs, constructed from the SNSs and embedded in Cyberspace, have attracted attentions of researchers from different disciplines, examples of which include *MySpace* [2], *Facebook* [3], *Pussokram* [4], etc. SNSs provide an online private space for individuals and tools for interacting with other people in the Internet. Thus the statistics and dynamics of these OSNs are of tremendous importance to researchers interested in human behaviors [5].

Assortativity coefficient of social networks. – A structural metric of great interest in the research of social networks, which characterizes the degree similarity of adjacent nodes, is the degree-degree correlation, that is “who is connected to who?” The correlation is characterized by the assortativity r and defined as the Pearson correlation

coefficient $r = (\langle ij \rangle - \langle i \rangle \langle j \rangle) / (\langle i^2 \rangle - \langle i \rangle^2)$, where i and j are the remaining degrees at the two ends of an edge and the $\langle \cdot \rangle$ notation represents the average over all links [6]. If a network’s assortativity coefficient is negative, a hub tends to be connected to non-hubs, and vice versa. When $r > 0$, we call the network to have an assortative mixing pattern, and when $r < 0$, disassortative mixing. An uncorrelated network exhibits the neutral degree-mixing pattern whose $r = 0$. While there is little systematic study of assortativity, there is a popular hypothesis that positive assortativity is a property of many socially generated networks, and contrasts with the opposite relationship that is more prevalent in technological and biological networks [7].

However recent extensive research on the OSNs provides many concrete counterexamples to the prevailing view. The assortativity coefficients for OSNs and real social networks are displayed in Table 1. It is noteworthy that many OSNs show disassortative or neutral mixing feature, which is in stark contrast to the significant assortative mixing for scientific, actor, and business collaboration networks.

The origins of obvious degree assortativity for real-world social networks are miscellaneous. From the perspective of sociology and psychology, in real life everyone would like to have intercourse with elites in a society; however the elites would rather communicate with the people with the same social status as theirs, which may lead to the assortative mixing pattern in the real-world social networks. For professional collaborations, such as scientific, actor, and business collaborations, the already big names preferably

Table 1: Degree assortativity coefficients of OSNs and real-life social networks. N indicates the number of nodes, r degree assortativity coefficient, and percentage in parenthesis sampling ratio.

Type	Network	N	r	References
Online social network	Cyworld	12, 048, 186	-0.13	[2]
	nioki	50, 259	-0.13	[4]
	All contacts in pussokram	29, 341	-0.05	[4]
	Messages in pussokram	21, 545	-0.06	[4]
	Friends in pussokram	14, 278	-0.04	[4]
	Flirts in pussokram	8, 186	-0.12	[4]
	MySpace	100, 000 (0.08%)	0.02	[2]
	orkut	100, 000 (0.30%)	0.31	[2]
	Xiaonei	396, 836	-0.0036	[8]
	Gnutella P2P(SN 6)	191, 679	-0.109	[9]
	Flickr	1, 846, 198 (26.9%)	0.202	[10]
	LiveJournal	5, 284, 457 (95.4%)	0.179	[10]
	YouTube	1, 157, 827	-0.033	[10]
	mixi	360, 802	0.1215	[11]
Real social network	ArXiv coauthorship	52, 909	0.36	[12]
	Cond-mat coauthorship	16, 264	0.18	[12]
	Mathematics coauthorship	253, 339	0.12	[6]
	Neuroscience coauthorship	205, 202	0.60	[13]
	Biology coauthorship	1, 520, 251	0.13	[12]
	Film actor collaboration	449, 913	0.21	[6]
	TV series actor collaboration	79, 663	0.53	[14]
	Company directors	7, 673	0.28	[6]

collaborate with other big names for success, reputation, influence and status. As indicated by Holme et al. [4], assortative mixing may be significant only to interaction in competitive areas. Another origin of degree assortativity in professional collaboration networks is the unsubstitutability for collaborators, which is usually decided by the similar research interests for scholars, act styles for actors or trade backgrounds for companies. Besides it appears that some of the degree correlation in real social networks could have real organizational origins. Generally the networks of collaborations between academics, actors, and businessmen are affiliation networks, in which people are connected together by membership of common groups (authors of a paper, actors in a film, researchers in a lab, etc.) [15].

OSNs differ from the real-life social networks in these regards. They break the invisible boundary among different communities or social estates in a society. In the virtual world elites will not refuse connections from anyone because they know that more connections show others they are elites. And unlike in real life, these links are not costly. Relationships in the real world have to be maintained and this requires continual effort. The basic difference could be the deciding difference between virtual and real world. Understanding the process, the generative mechanism, will supply a substantial comprehension of the formation and evolution of online virtual communities.

Assortativity transition of *Wealink*. — In the following we will focus on a virtual community - *Wealink* [16], which is one of the largest OSNs in China at present and whose users are mostly professionals, typically businessmen and office clerks. Each registered user of the SNS has a profile, including his/her list of friends. If we view the users as nodes V and friend relationships as edges E , an undirected social network $G(V, E)$ can be constructed from *Wealink*. For privacy reasons, the data, logged from 0:00:00 h on 11 May 2005 (the inception day for the SNS) to 15:23:42 h on 22 Aug 2007, include only each user's ID and list of contacts, and the establishment time for each friend relationship. The OSN is a dynamical evolving one with the new users joining in the community and new connections established between users.

We extract 27 snapshots of *Wealink* with an interval of one month from 11 Jun 2005 to 11 Aug 2007 and investigate the evolution of the network. Generally it is thought that real-world networks, man-made or naturally occurring, always belong to the same type over time, either assortative or disassortative. However as shown in Fig. 1, the *Wealink* underwent a transition from the initial assortativity characteristic of real social networks to subsequent disassortativity characteristic of many OSNs. To the best of our knowledge, this is the first real-world network observed which possesses the intriguing feature. For other SNSs, such as *Pussokram*, the assortativity coefficient of its guest book, friend and flirt

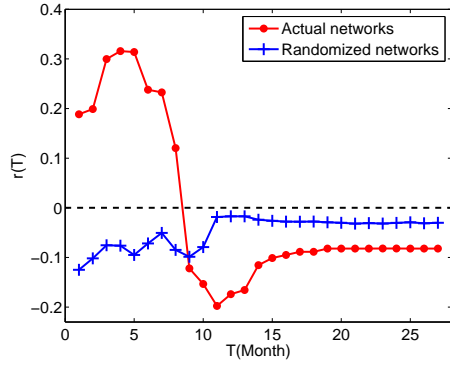


Fig. 1: Evolution of degree assortativity $r(T)$ for the *Wealink* and its randomized version.

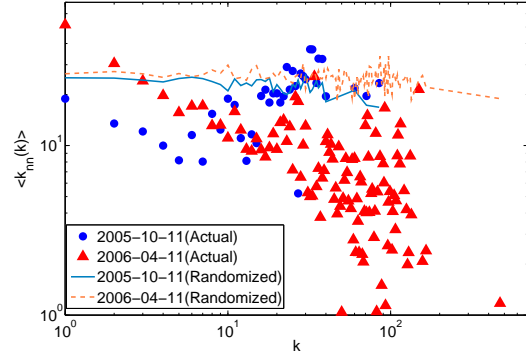


Fig. 2: $\langle k_{nn}(k) \rangle$ versus k at different time for the *Wealink* and its randomized version.

networks behaves similar to that of *Wealink* over time, however all the OSNs in *Pussokram* are disassortative and no assortativity-disassortativity transition occurs. There could be different evolving mechanisms for real-life and virtual social networks. A reasonable conjecture is that often the friendship relations in the beginning OSN are based on real-life interpersonal ones, that is *Wealink* users were linking to the other users who are their friends in the real world. In this case the OSN directly inherits the assortative structure of the underlying real-life social network. However at the later stage many online users of low degrees may preferentially establish connections with the network elites of high degrees, resulting in the disassortative mixing. As shown in Table 1, some OSNs have assortative mixing, however we cannot arbitrarily affirm that these networks are all in the initial stage of evolution and reflect only real-life interpersonal relationships. Obviously any OSN is the superposition of online and real-life interpersonal relationships and the different weights of both relations can regulate the assortativity coefficient of OSNs. SNSs of differing scopes, functions or purposes have distinct evolution pattern for the weights of online and real-life relations. Some OSNs can always be assortative or disassortative over time while few can undergo the assortativity transition.

Fig. 1 also shows the comparison of r between actual networks and reshuffled ones obtained by random degree-preserving rewiring of the original networks [17]. The randomized networks show disassortative or nearly neutral mixing feature, and all r 's are less than 0 and no transition appears. The comparison shows that the *Wealink* is strongly degree assortative at the beginning stage and disassortative at the later stage, suggesting that individuals indeed draw their partners from the users with degrees similar (beginning) or dissimilar (later) to theirs far more often than one would expect on the basis of pure chance. Fig. 2 shows the degree-degree correlation of *Wealink* on 11 Oct 2005 and 11 Apr 2006. Overall, $\langle k_{nn}(k) \rangle$ is an increasing function of k on 11 Oct 2005 and decreasing function of k on 11 Apr 2006, indicating degree assortativity

and disassortativity respectively. This can be validated by Fig. 1 and reveals the assortativity-disassortativity transition from a different point of view. With the increase of k , in Fig. 2 the $\langle k_{nn}(k) \rangle$ is almost flat for the randomized networks, suggesting nearly neutral mixing.

Degree mixing pattern has profound effect on network structure and behaviors, such as resistance to attacks [18], percolation [19], epidemic spreading [20], synchronization [21] and cooperation in games [22]. Online human interactions are driven by and can change social conventions. A major area of web science is to explore how a small technical innovation can launch a large social phenomenon. For the spreading phenomena in online communities, such as diffusion of opinions, technical innovations or gossip, one can expect the things to be spread to a larger segment of the population in disassortative networks than in assortative ones. Everything has two sides. The formation of online communities facilitates human interactions and information share in the Internet and speeds up the diffusion of good ideas and opinions, and if exploited correctly, the OSNs can also be a powerful medium for gauging the impact of a political initiative or the likely success of a product launch; however at the same time they also facilitate the spreading of vicious gossip.

A simple model based on the work of Catanzaro et al. [23] can reproduce the assortativity transition. Starting with a small random connected network, at every time step:

(1) With probability p a new node is added into the network, and it is linked to an old node i by Barabási-Albert (BA) preferential attachment rule [24], i.e. $pk_i / \sum_{j=1}^N k_j$, where k_i is the degree of node i and N is the number of nodes.

(2) With probability $(1-p)$ a new edge is added between two existing nodes (self-loops and multiple links are prohibited), which are chosen based on their degrees k_i, k_j . The probability of selecting the first node with degree k_i is $P_1(k_i)$, and the second node $P_2(k_j|k_i)$. Thus the probability of adding a new edge and connecting two old nodes is $(1-p)P_1(k_i)P_2(k_j|k_i)$. The first node is selected based on

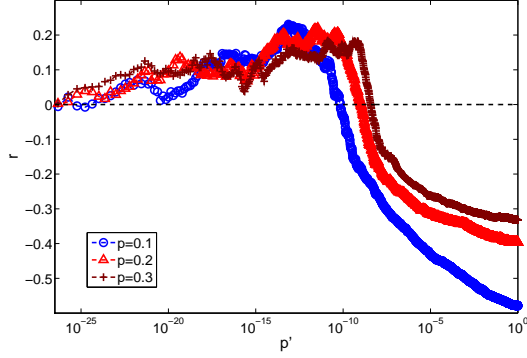


Fig. 3: Simulation result of r with the increase of p' . $e^{|k_i - k_j|}$ has a much stronger effect on $P_2(k_j|k_i)$ than $e^{-|k_i - k_j|}$, thus $r > 0$ occurs only for extremely small p' .

BA rule. The functional form of $P_2(k_j|k_i)$ can be chosen so as to favor links between similar or different degrees. We let $P_2(k_j|k_i) \propto (1-p')f_1(|k_i - k_j|) + p'f_2(|k_i - k_j|)$, where f_1 and f_2 are suitable decreasing functions and increasing functions of $|k_i - k_j|$ with positive range respectively, and $0 \leq p' \leq 1$ governs the weight of degree assortativity. By tuning p' from 0 to 1, the resulting network will undergo a gradual structural transition from assortativity to disassortativity.

The p modulates the relative role of growth and mixing. Generally $p < 0.5$ because the mixing is often more frequent than growth in social networks, i.e. the mean life of a node (a human or professional life) is longer than that of an edge (a business or social relation). Let $P_2(k_j|k_i) \propto (1-p')e^{-|k_i - k_j|} + p'e^{|k_i - k_j|}$, and $p = 0.1, 0.2$ and 0.3 , we generate three evolving networks, whose assortativity coefficients r are shown in Fig. 3. Indeed the model cannot reproduce the realistic and intricate evolving process of *Wealink* in many aspects since the real growth of the network will inevitably be affected by various exogenous and endogenous factors, however the qualitative agreement between Fig. 1 and Fig. 3 shows that the model could reproduce the macroscopical property of assortativity-disassortativity transition, including the microscopical mechanism of growth and mixing, and thus capture the basic aspect governing the evolution of r of the network.

In the above model, we assume that when a new node is added into the network, it is attached to an old node by BA rule, and when a new edge is added between two old nodes, the first node of creating link request also is selected based on BA rule. The linear preference can be validated by real network data [25]. Let k_i be the degree of user i . The probability that user i with degree k_i is chosen can be expressed as $\Pi(k_i) = k_i^\beta / \sum_j k_j^\beta$, where β is a constant. We can compute the probability $\Pi(k)$ that an old user of degree k is chosen, and it is normalized by the number of users of degree k that exist just before this step: $\Pi(k) =$

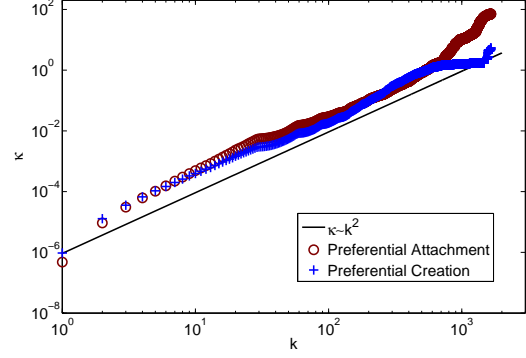


Fig. 4: Testing preference feature for *Wealink*.

$\sum_t [e_t = v \wedge k_v(t-1) = k] / \sum_t |\{u : k_u(t-1) = k\}| \sim k^\beta$, where $e_t = v \wedge k_v(t-1) = k$ represents that at time t the old user whose degree is k at time $t-1$ is chosen. We use $[\cdot]$ to denote a predicate (take value of 1 if expression is true, else 0). Generally $\Pi(k)$ has significant fluctuations, particular for large k . To reduce the noise level, instead of $\Pi(k)$ we study the cumulative function: $\kappa(k) = \int_0^k \Pi(k)dk \sim k^{\beta+1}$. Fig. 4 shows how the degree k of users is related to the preference metric κ . Approximately $\beta \approx 1$ for both preferential attachment and creation, which indicates that the linear preference hypothesis is reasonable.

Fig. 5 shows the evolution of clustering coefficient C [26] and modularity Q [27], in comparison with the same metric of randomized networks. C measures the strength of connections within individual neighborhood and Q measures the significance of community feature. The Q lies in $[0, 1]$ and a Q value above about 0.3 is a good indicator of significant community structure in a network. As shown by Fig. 5, the *Wealink* has significantly higher C and Q than those of randomized networks and both C and Q of actual networks are similar in growth trend to those of randomized ones. We find that the randomized networks still possess large Q , which may result from the structural constraint of degree sequence of original networks.

A social network might be divided up according to the location, affiliation, occupation, interests, and so forth, of its members. It is thought that clustering and assortativity in networks arise because the vertices are divided into groups or communities [7, 28] with a high density of edges between members of the same group, even though the density of edges in the network as a whole may be low; however the comparison between Fig. 1 and Fig. 5 shows that both clustering coefficient and degree assortativity are negatively correlative with modularity. Recent research also shows that the $r - C$ space obtained by successively rewiring pairs of edges of networks suggests that there exists positive correlation to some extent between assortativity coefficient r and clustering coefficient C [29], which is intuitively reasonable and is validated empirically by Figs. 1 and 5. However it should be quite cau-

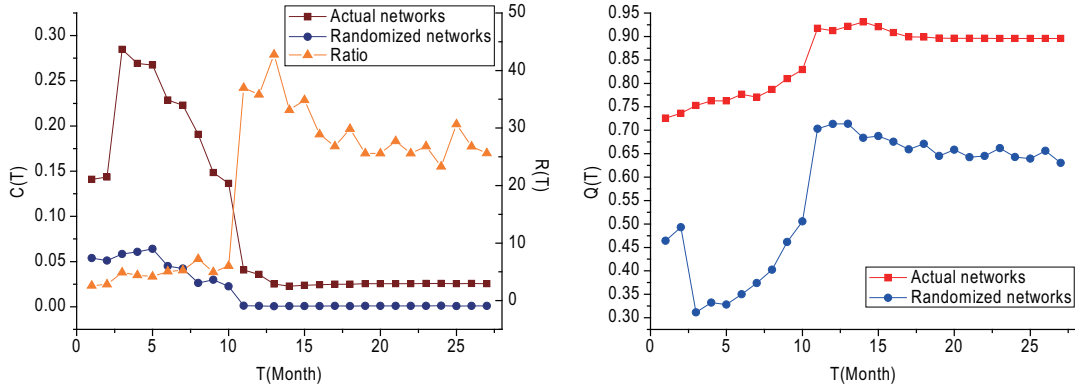


Fig. 5: Evolution of clustering coefficient $C(T)$ (left) and modularity $Q(T)$ (right) for the actual networks and randomized ones. The left scale of y-axis in the left panel is for clustering coefficient of *Wealink* and its randomized version and the right scale the ratio of C of actual networks to that of randomized ones.

tious to claim that there exists specific correlation between network metrics, and obviously under different conditions there may be quite different conclusions. In practice different network properties, such as modularity, clustering, assortativity, heterogeneity, synchronizability, etc., may constrain each other, or not be independent [30]. And the arbitrary claim about correlation usually may lead to unsound conclusions.

Summary. — In summary, we systematically study the assortativity of social networks. We find that, compared to real social networks, OSNs show diverse degree correlation pattern, including disassortative, assortative or nearly neutral mixing, which implies different evolving mechanisms for real world and virtual community. More interesting we have found that an online community, *Wealink*, underwent a transition from degree assortativity to disassortativity, which can be reasonably interpreted by a simple model we propose. As a rapidly developing field in interdisciplinary research, virtual community has attracted scholars of different backgrounds, mostly physicists and computer scientists. However the main body in the virtual world is still persons in real world, thus understanding the web community may also require insights from sociology and psychology every bit as much as from physics and computer science [31].

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