Solving the Heat Equation

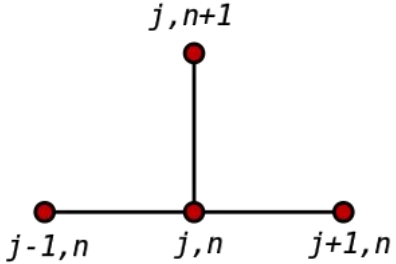
Erich Wanzek

The heat equation

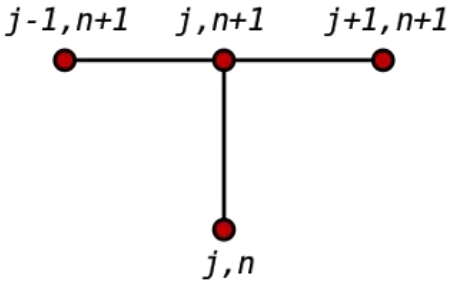
$$\frac{\partial u}{\partial t} = a\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Solution Method

$$\frac{u_j^{n+1} - u_j^n}{k} = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{h^2}.$$

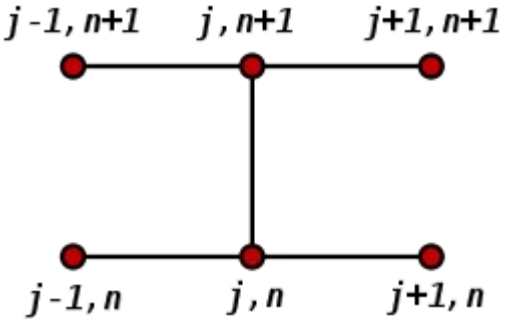


$$\frac{u_j^{n+1} - u_j^n}{k} = \frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{h^2}.$$



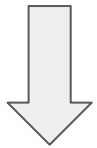
$$\frac{u_j^{n+1} - u_j^n}{k} = \frac{1}{2}\left(\frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{h^2} + \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{h^2}\right).$$

Crank-Nicolson Method



Solution Method

$$(1+2\mu)u_{i,j}^{n+1} - \frac{\mu}{2}\left(u_{i+1,j}^{n+1} + u_{i-1,j}^{n+1} + u_{i,j+1}^{n+1} + u_{i,j-1}^{n+1}\right)$$
$$= (1-2\mu)u_{i,j}^{n} + \frac{\mu}{2}\left(u_{i+1,j}^{n} + u_{i-1,j}^{n} + u_{i,j+1}^{n} + u_{i,j-1}^{n}\right).$$



$$\mathbf{A}\,T(t+h) = \mathbf{B}\,T(t)$$

The code

4 main sections/functions

Functions that set up initial conditions

Function that makes matrices A and B

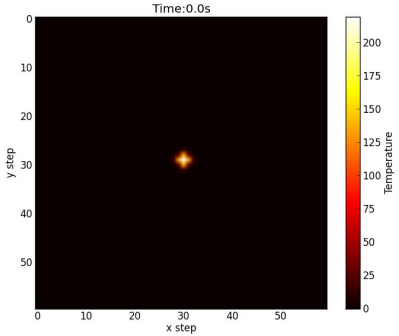
Function that solve the systems for each time step using numpy.linalg.solve

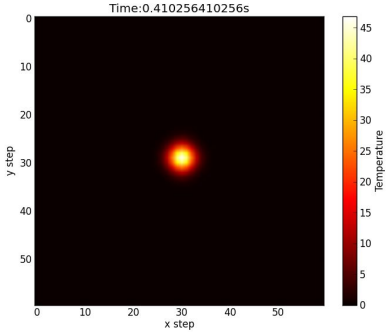
Function that makes heat maps for the solution

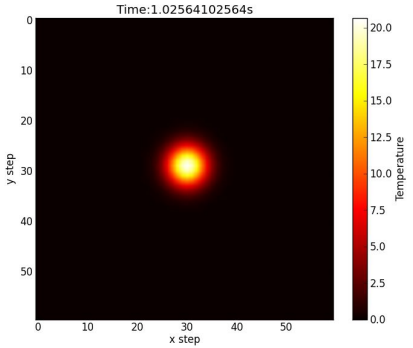
Results

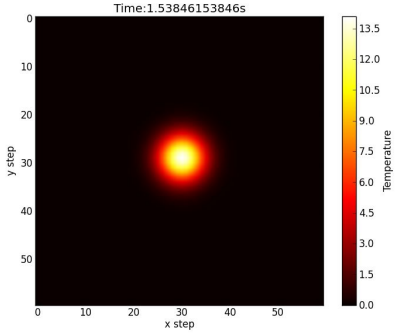
Information:

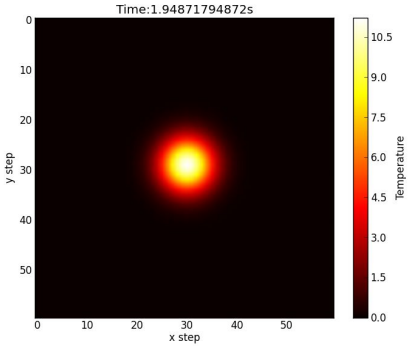
Number of grid steps(x and y) is equal to 60 steps. Times steps are equal to 40. The time interval is from 0 seconds 4 seconds.The dimension is equal to 1 dimensional Unit. Thermal diffusivity is equal 1*10^-3

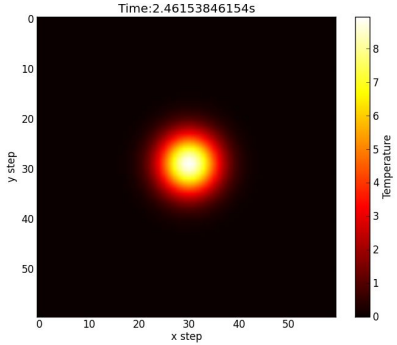


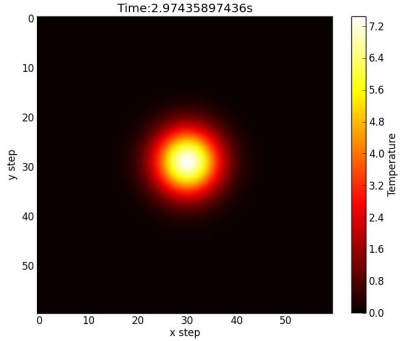


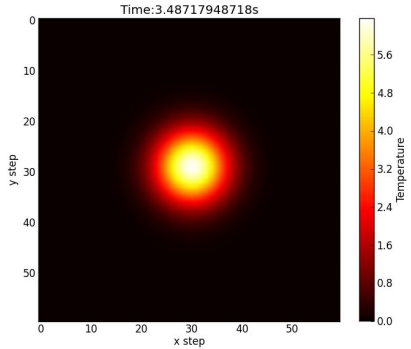












Results

Information:

Number of grid steps(x and y) is equal to 60 steps. Times steps are equal to 40. The time interval is from 0 seconds 4 seconds.The dimension is equal to 1 dimensional Unit. Thermal diffusivity is equal 1*10^-3