



## Problem H

### Countable Rational Numbers

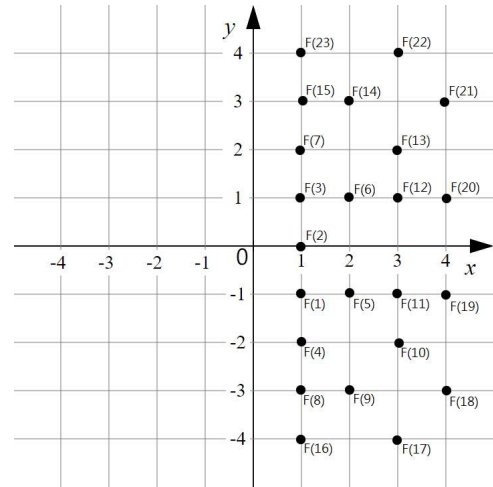
(Time Limit: 3 seconds)

The set of rational numbers is described as  $Q = \left\{ \frac{y}{x} \mid x, y \in Z, x > 0, \gcd(x, y) = 1 \right\}$  where  $Z$  is the set of integers and  $\gcd(x, y)$  is the greatest common divisor of  $x$  and  $y$ . For examples,  $\frac{1}{2}, \frac{-1}{2}, \frac{3}{1}, \frac{-3}{1}, \dots$  are rational numbers while  $\frac{2}{4}, \frac{1}{-2}, \frac{8}{2}, \frac{-3}{-2}, \dots$  are syntactically not rational numbers since they violate the constraints  $x > 0$  or  $\gcd(x, y) = 1$ . It is well-known that the cardinality of rational numbers is countable infinity. However how to prove that is to find a bijection from natural numbers  $N$  to rational numbers  $Q$ .

Consider two rational numbers,  $\frac{y_1}{x_1}$  and  $\frac{y_2}{x_2}$ . Let  $m_1 = \max(x_1, \text{abs}(y_1))$ ,  $m_2 = \max(x_2, \text{abs}(y_2))$ . We define  $\frac{y_1}{x_1} \triangleleft \frac{y_2}{x_2}$  if one of following constraints is satisfied.

1.  $m_1 < m_2$
2.  $m_1 = m_2$  and  $y_1 < y_2$
3.  $m_1 = m_2$  and  $y_1 = y_2$  and  $x_1 \times y_1 > x_2 \times y_2$

According to the ordering  $\triangleleft$ , we can build a bijection  $F$  from  $N$  to  $Q$  by enumerating elements of  $Q$  from the smallest to larger as  $\frac{-1}{1} \triangleleft \frac{0}{1} \triangleleft \frac{1}{1} \triangleleft \frac{-2}{1} \triangleleft \frac{-1}{2} \triangleleft \frac{1}{2} \triangleleft \frac{2}{1} \triangleleft \frac{-3}{1} \triangleleft \frac{-3}{2} \triangleleft \frac{-2}{3} \triangleleft \frac{-1}{3} \triangleleft \frac{1}{3} \triangleleft \frac{2}{3} \triangleleft \frac{3}{2} \triangleleft \frac{3}{1} \dots$ , that is  $F(1) = \frac{-1}{1}, F(2) = \frac{0}{1}, F(3) = \frac{1}{1}, F(4) = \frac{-2}{1}, \dots$ . For each rational number



$F(n) = \frac{y}{x}$ , we can plot it at  $(x, y)$  on  $x - y$  plane as the above picture. On the picture, we can see that there is no rational numbers on the left part since the value of  $x$  has to be larger than zero. We can also understand that there is no rational number at  $(4, 2)$  since  $\frac{2}{4}$  is not a rational number because  $\gcd(4, 2) = 2 \neq 1$ .



## 第九屆全國私立大專校院程式競賽

### National Contest for Private Universities (NCPU), 2019

For any given integers  $y$  and  $x$ , can you find an integer  $n$  such that  $F(n) = \frac{y}{x}$ .

#### Input Format

The first line is a number indicating the number of test cases. Each test case has two integers  $y$  and  $x$ . (Remark: "+" and "-0" do not appear in test cases.)

#### Output Format

For each test case, please output  $n$  if there exists an integer  $n$  such that  $F(n) = \frac{y}{x}$  and please output 0 if  $\frac{y}{x}$  is syntactically not a rational number.

#### Technical Specification

- There are at most 10 test cases.
- $-1,000,000 < x < 1,000,000$
- $-1,000,000 < y < 1,000,000$

#### Example

Sample Input:	Sample Output:
5	0
4 2	11
-1 3	13
2 3	0
1 -3	23
4 1	