Econ 272/Mgtecon 607 - Section 1

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Stanford University

Spring 2025

Outline

1. General Info & Logistics
2. Discussion Questions
3. Intro to Causality
4. Completely Randomized Experiments
5. Stratified/Paired Experiments
6. Power Analyses
7. Practice Problems

General Info

- Email: amar.venugopal@stanford.edu
- Section: Thursdays 4:30-6:20pm in STLC 114
- Office Hours: Wednesdays 2-4pm in Econ 149
- Section Questions: Question Form
- Section Feedback: Feedback Form
- Note: please use office hours to ask questions about the material, rather than over email.

For the entirety of the course, section materials are drawn heavily from those prepared by Lea Bottmer in past years.

Announcements

- Remember to email me about whether you wish to take the final exam, and if you require any special accommodations for it
  - Unless you are an Econ PhD, in which case you have no choice :)
  - If you are a predoc and wish to waive the class next year, you must take the exam as well

Section Norms

In order to foster a healthy, inclusive classroom environment, we will all abide by the following **community norms**:

- One microphone: we will allow everyone to finish their thought without interruption
- Everyone got accepted to Stanford and is here for a reason. There is no need to prove anything to anyone.
- Asking questions is a great way to deepen your understanding and resolve any confusion you may have. It is not a signaling device.
- There is no such thing as a stupid or dumb question
- Questions asked during section will be answered by the TA

Homework

- First problem set due **Sunday April 6, 11pm**. No late submissions are accepted.
- All submissions should consist of 2 files:
  - A **typed** writeup, which should be self-contained (the grader should not have to view any code)
  - A .zip file with all your code and user-created programs
- Please make sure your writeup is easy to read, and code should be well-commented
- For empirical questions: try to make economic sense of your results, or explain why they may not make sense
- For theoretical questions: make sure to explain steps clearly (cite theorems, references, etc) for full credit
- For problem set-related questions, come to my office hours or reach out to me (cc Guido on all problem set-related emails)
  - I **will not** respond to problem set emails sent on the due date

Hints for Problem Set 1

- The lecture notes (on Canvas) may be helpful if you get stuck at estimating homoskedastic variance

Lecture Feedback

Guido and I are both committed to making lecture and section as helpful/instructive as possible.

Feel free to discuss or email either of us with any feedback you might have.

Can also you section feedback google form if you prefer anonymous submission of feedback (just note in your response that the feedback pertains to lecture).

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Discussion Questions - Randomized Experiments

1. What are the roles of *sampling*, *potential outcomes*, and the *assignment mechanism* in the analysis of randomized experiments?
2. Why is the Stable Unit Treatment Value Assumption (SUTVA) important?
3. What is a *sharp* null hypothesis? Why does this allow us to calculate exact Fisher p-values? What are some examples of sharp hull hypotheses in the content of potential outcomes?
4. How can we calculate/approximate the exact p-value? What test statistics can we think of?
5. How is the unbiased estimator of the average treatment effect calculated?
6. What is the formula for the variance of the unbiased estimator of the average treatment effect? What is the primary challenge in estimating this? What do we need to do to make the variance calculable and does this make standard errors more or less conservative?

Discussion Questions - Stratified Randomized Experiments and Power Analyses

1. How does one use a stratified randomized experiment to calculate average treatment effects? (formulas)
2. What are the (dis-) advantages of a stratified, paired, fully randomized design?
3. When/how should a researcher re-randomize units to treatment and control?
4. What do we mean by power calculations? How do we perform power calculations based on the difference in means estimator?

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Key question: what is the effect of changing someone's treatment status on some outcome?

3 key areas of interest:

1. Point estimation ($\hat{\tau}$)
2. Inference ($\hat{V}$)
3. Hypothesis testing

Fundamental problem of causal inference:

We never observe counterfactuals

Notation

i denotes unit

- Y_i: Outcome
- W_i: Treatment → $W_i \in \{0, 1\}$
- $Y_i(w)$: Potential outcome at $W_i = w$
- X_i: Covariates

Note: Via Neymans's repeated sampling approach, PO are assumed to be *fixed* (unlike observed outcomes)

$$Var(Y_i) = Var(W_i Y_i + (1 - W_i) Y_i) = Var(W_i Y_i) + Var((1-W_i)$$

$$= Var(W_i Y_i(1)) \quad (Cov$$

$$= Y_i(1)^2 \, Var(W_i)$$

Assumptions

- **Known treatment assignment** $(Y_i(0), Y_i(1)) \perp W_i$
- **Stable Unit Treatment Value Assumption** $Y_i(\vec{w}) = Y_i(w_i)$

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Point Estimation

- Estimand: $\tau^{ATE} = \mathbb{E}\left[Y_i(1) - Y_i(0)\right]$ $\quad \tau^{ATT} = \mathbb{E}\left[Y(1) - Y(0) \mid W=1\right]$
- Estimator: $\hat{\tau}^{DiM} = \bar{Y}_1^{obs} - \bar{Y}_0^{obs}$
- Properties: Unbiased $\quad \mathbb{E}\left[\hat{\tau}^{DiM}\right] = \tau^{ATE}$

Inference

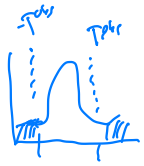
- True Variance:
- Estimator:
- Properties:
- Confidence Intervals:

Hypothesis Testing: Fisher Exact p-Values

H_0: $Y_i(1) = a\,Y_i(0) + b$

4-step procedure:

1. Define a sharp null H_0
   is H_0: $Y_i(0), Y_i(1)$ under H_0
2. Define test statistic, T $\quad (T = \bar{Y}_1 - \bar{Y}_0)$
3. Calculate distribution of T (exact numerical)
4. Calculate p-value $\quad p = \dfrac{\sum_{a \in A} 1\{|T^a| \geq |T^{obs}|\}}{|A|}$



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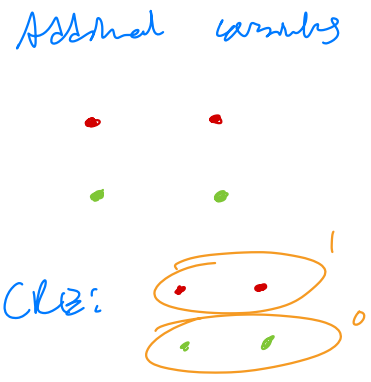
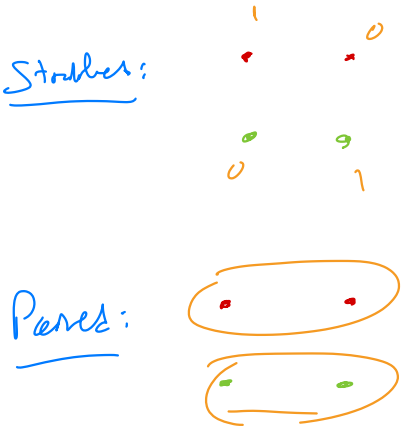
What is the problem?





- Estimand: $\tau^{Strat} = E\left[Y_i(1) - Y_i(0)\right] = E\left[\tau(X_i)\right]$
- Estimator: $\hat{\tau}^{Strata} = \sum_{g=1}^{G} \frac{N_g}{N} \hat{\tau}_g$

$$Y_i^{obs} = \alpha + \tau W_i + \beta_1 S_{6i2g} + \varepsilon_i$$

$$\hat{\tau}^{RM} = \bar{Y}_1^{obs} - \bar{Y}_0^{obs}$$

Inference

- True Variance: $Var(\hat{\tau}^{strata}) = \sum_g \left(\frac{N_g}{N}\right)^2 Var(\hat{\tau}_g)$
- Estimator: $\widehat{Var}(\hat{\tau}^{strata}) = \sum_g \left(\frac{N_g}{N}\right)^2 \widehat{Var}(\hat{\tau}_g)$

Completely Randomized vs. Stratified vs. Paired

Design

	CRE	Strat	Pair
Pro	Simple, easy	Lower MSE vs CRE Heterogeneity	Lowest MSE in finite sample Higher power in large samples
Cons	Doesn't use potentially useful covariates		Cannot estimate variance of ATE (within strata)

$$V_C - V_S = A\left(\underbrace{n(X) \cdots}_{>0} - \underbrace{\text{Var}(\tilde{\tau})^2}_{\to >0}\right) \geq 0$$

Practical Recommendations

If sample large + predictive covs. → shrinkage or pool

Generally, small shrink > pool

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Power Calculations

Power of a test (β): $P(\text{reject} \mid H_0 \text{ false})$

Size of a test (α): $P(\text{reject} \mid H_0 \text{ true})$

General procedure to get required sample size for given α, β:

1. Specify rejection criterion (e.g. $|T| \geq \Phi^{-1}(1-\alpha/2)$)
2. Specify probability $= \beta$
3. Solve for N

$\downarrow$

$P(\quad \cdot \quad) = \beta$

$\uparrow$

N

Useful Formulae

To determine minimum sample size when...

- Testing a mean against 0 (assuming random sample from normal distribution):

$$N = \left(\frac{\Phi^{-1}(\beta) + \Phi^{-1}(1 - \alpha/2)}{\mu_0/\sigma} \right)^2$$

- Testing DiM with unequal sample sizes and equal variances:

$$N = \frac{(\Phi^{-1}(\beta) + \Phi^{-1}(1 - \alpha/2))^2}{(\tau_0^2/\sigma^2)\gamma(1 - \gamma)}$$

where γ = share of treated units

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Randomization Test: Fixed M vs. Random M

Assume we observe $\left(W_i, Y_i(W_i)\right)$ for all i in the population of four individuals in a randomized experiment. Consider the following test statistic and data:

$$T = \frac{\sum_{i=1}^{n} W_i Y_i}{\sum_{i=1}^{n} W_i} - \frac{\sum_{i=1}^{n} (1 - W_i) Y_i}{\sum_{i=1}^{n} (1 - W_i)}$$

i	W_i	Y_i^{obs}	$Y_i(0)$	$Y_i(1)$
1	0	0		
2	0	6		
3	1	18		
4	1	42		

1. Perform the randomization test that was discussed in lecture (fixing the number of treated units at two). What is the Fisher exact p-value implied by the data when testing the sharp null hypothesis of no treatment effect for any individual?
2. Now perform the randomization test that would result if you let the number of treated units vary stochastically according to a binomial distribution with success probability of $\frac{1}{2}$. What exact p-value (for the sharp null of no treatment effect for any individual) is implied?

Consider a completely randomized experiment with N_0 control units and N_1 treated units.

(a) Describe how you would test the null hypothesis that $Y_i(1) = 2 \times Y_i(0) + 3$ for all i.

(b) Suppose you want to estimate the difference in medians by treatment status. Describe how you would estimate this. Explain why we might want to use this statistic over the difference in means.

Randomized Experiment Variance Calculation (Final Exam 2018, #1)

Consider a completely randomized experiment with N_0 control units and N_1 treated units. Let $Y_i(0)$, $Y_i(1)$ denote the potential outcomes, Y_i denote the realized outcome, and $W_i \in \{0, 1\}$ the treatment. The exact variance for the simple difference in means estimator

$$\bar{Y}_1 - \bar{Y}_0 = \frac{1}{N_1} \sum_{i=1}^{N} W_i Y_i - \frac{1}{N_0} \sum_{i=1}^{N} (1 - W_i) Y_i$$

is

$$\frac{S_0^2}{N_0} + \frac{S_1^2}{N_1} - \frac{S_{01}^2}{N}$$

(a) Calculate the exact, finite sample, variance of $\bar{Y}_1$.
(b) Calculate the exact, finite sample, covariance of $\bar{Y}_1$ and $\bar{Y}_0$.

Randomized Experiment (Final Exam 2020, #1)

Suppose we conduct a randomized experiment on a random sample of the US population. We assign the treatment randomly at the individual level. The dataset observed by the econometrician includes individual-level outcomes, location (state) for each individual, and a binary treatment indicator.

(a) Suppose you estimated the average treatment effect as the difference in means by treatment status, give an expression for the variance of the estimator, and how you could estimate the unknown components of that variance? You can ignore the fact that the population is finite.

(b) How would you use the location information to obtain a more efficient estimator? What would its variance be?

Randomized Experiments (Final Exam 2022, #1)

Suppose we have an infinitely large population. We randomly sample N units from this population. We consider an experiment where we randomly assign each unit to a binary treatment (independently for each unit), with the probability of being in the treatment group equal to p. Let $W_i \in \{0, 1\}$be the binary treatment. Let $Y_i(0)$ and $Y_i(1)$ be the potential outcomes (assuming SUTVA, or no-interference), and let $\tau = \mathbb{E}[Y_i(1) - Y_i(0)]$ be the population average effect of the treatment, and let $\tau_s = \frac{1}{N}\sum_{i=1}^{N}(Y_i(1) - Y_i(0))$ be the sample average treatment effect. Consider two estimators

$$\hat{\tau}^1 = \frac{1}{N}\sum_{i=1}^{N}\{\frac{W_i Y_i}{p} - \frac{(1 - W_i)Y_i}{1 - p}\}$$

and

$$\hat{\tau}^2 = \frac{1}{\sum_{i=1}^{N} W_i}\sum_{i=1}^{N} W_i Y_i - \frac{1}{\sum_{i=1}^{N}(1 - W_i)}\sum_{i=1}^{N}(1 - W_i)Y_i$$

(if one of the denominators is equal to zero, the estimator is defined to be zero).

1. What is the exact finite variance of the two estimators, viewed as estimators of the sample average treatment effect? (For the second estimator you can condition on the value of $\sum_{i=1}^{N} W_i$, and you can focus on the case where this value is between 1 and $N - 1$).
2. What is the asymptotic variance of the two estimators, viewed as estimators of the population average treatment effect?
3. Suppose I also observe some covariates (but the assignment is still completely random). Describe how you can improve over the second estimator by exploiting the covariates.

Randomized Experiment (Final Exam 2023, #1)

Suppose we have an infinitely large population. We randomly sample N units from this population. We consider an experiment where we randomly assign each unit to a binary treatment (independently for each unit), with the probability of being in the treatment group equal to p. Let $W_i \in \{0, 1\}$ be the binary treatment. Let $Y_i(0)$ and $Y_i(1)$ be the potential outcomes (assuming SUTVA, or no interference) and let $\tau = \mathbb{E}[Y_i(1) - Y_i(0)]$ be the population average effect of the treatment and let $\tau_s = \sum_{i=1}^{N}(Y_i(1) - Y_i(0))/N$ be the sample average treatment effect.

(a) Show that

$$Y_i \frac{W_i - p}{p(1-p)}$$

conditional on the potential outcomes is unbiased for the unit-level treatment effect $\tau_i = Y_i(1) - Y_i(0)$.

(b) Is the average

$$\frac{1}{N} \sum_{i=1}^{N} Y_i \frac{W_i - p}{p(1-p)}$$

a good estimator for τ?

Power Calculations

Suppose we plan to test whether the mean of a population is different from 0 using a t-statistic, with a $\alpha = 0.05$ significance level. Assume that the variance of the observations is known to be $\sigma^2 = 100$. How many (independent) observations N do we need to have power $\beta = 0.9$ when the true mean is $\mu = 1$? (Let $\Phi^{-1}(0.8) = 0.8416, \Phi^{-1}(0.9) = 1.2816, \Phi^{-1}(0.95) = 1.6449$ and $\Phi^{-1}(0.975) = 1.9600$.)

Power Calculations

Suppose we want to run a completely randomized experiment on $N = 3000$ individuals. We plan to test whether the average treatment effect is different from 0 using the difference in means estimator, at a $\alpha = 0.05$ significance level. I want to achieve a power of $\beta = 0.8$ when the true treatment effect is $\tau = 1$.

(a) What should the variance of my outcome be for me to achieve my goal? (Let $\Phi^{-1}(0.8) = 0.8416, \Phi^{-1}(0.9) = 1.2816, \Phi^{-1}(0.95) = 1.6449$ and $\Phi^{-1}(0.975) = 1.9600$.)

(b) Suppose I observe some pre-treatment data and find that $\sigma^2 = 64$. Based on your answer to (a), will I be able to achieve the desired power in my experiment? Explain why or why not.