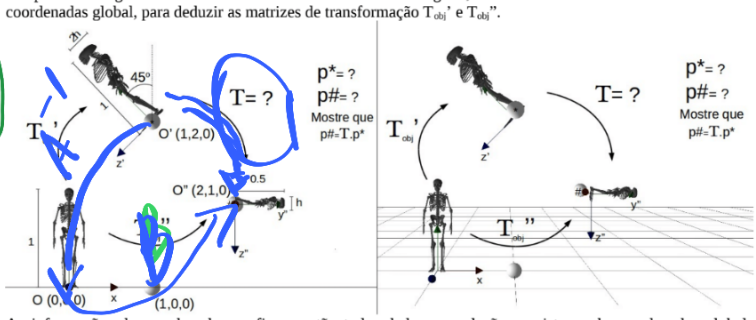
1- **[Composição de transformações] (5,0 pontos)** Baseado nas figuras seguintes, gere a matriz T correspondente à composição das transformações que levam o modelo inicial' ao modelo final''. Primeiramente, compare as imagens dos modelos ' e '' com a do modelo original, inicialmente alinhado com o sistema de coordenadas global, para deduzir as matrizes de transformação T_{obj}' e T_{obj}''.



As informações de coordenadas na figura estão todas dadas em relação ao sistema de coordenadas global. Verifique ainda se sua matriz gerada está correta mostrando que ela transforma as coordenadas globais do ponto p* (representado pela esfera *) nas coordenadas globais do ponto p# (representado pela esfera #). Observe que as esferas * e # (assim como os modelos ' e '') correspondem à esfera original transformada por T_{obj}' e T_{obj}'', respectivamente. Por último, escreva o código OpenGL equivalente para gerar a matriz T.

$AB \neq BA$

CG

LIFO

$(AB)C = A(BC)$

$$A \Rightarrow T\begin{pmatrix}1\\2\\0\end{pmatrix} \cdot \underbrace{R_y(-90°)}_{1°} \cdot \underbrace{R_z(45°)}_{2°} \cdot S\begin{pmatrix}1\\1\\2\end{pmatrix}$$

$$A \Rightarrow T\begin{pmatrix}1\\2\\0\end{pmatrix} \cdot R_z(45°) \cdot R_y(-90°) \cdot S\begin{pmatrix}1\\1\\2\end{pmatrix}$$

$$A^{-1} \Rightarrow S\begin{pmatrix}1\\1\\1/2\end{pmatrix} \cdot \underbrace{R_y(90°) \cdot R_z(-45°)}_{\alpha} \cdot T\begin{pmatrix}-1\\-2\\0\end{pmatrix}$$

α'

$$\alpha = R_y(90) \cdot R_z(-45°)$$

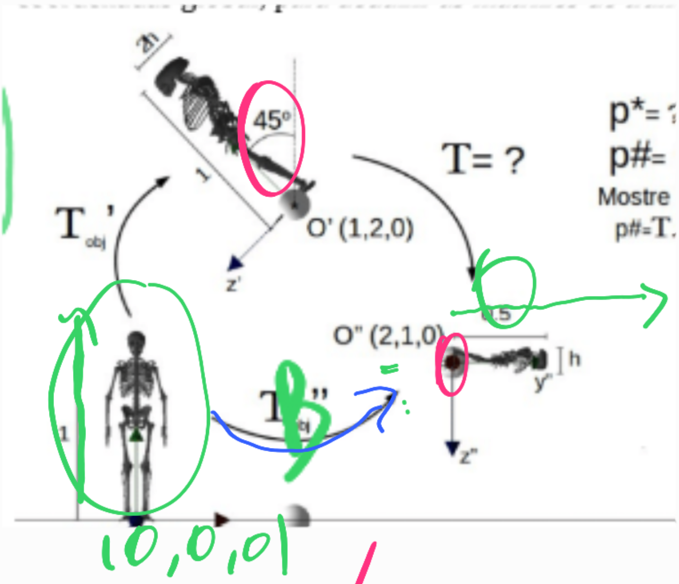
$$\alpha = \begin{bmatrix}0 & 0 & 1\\0 & 1 & 0\\-1 & 0 & 0\end{bmatrix} \cdot \begin{bmatrix}\sqrt{2}/2 & \sqrt{2}/2 & 0\\-\sqrt{2}/2 & \sqrt{2}/2 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\-\sqrt{2}/2 & \sqrt{2}/2 & 0\\-\sqrt{2}/2 & -\sqrt{2}/2 & 0\end{bmatrix}$$

$$\underline{A^{-1} = S\begin{pmatrix}1\\1\\1/2\end{pmatrix} \cdot \alpha \cdot T\begin{pmatrix}-1\\-2\\0\end{pmatrix}}$$

$$\alpha T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ -\sqrt{2}/2 & -\sqrt{2}/2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} - \frac{2\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & \frac{3\sqrt{2}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} - \frac{2\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & \frac{3\sqrt{2}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} - \sqrt{2} \\ -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & 0 & \frac{3\sqrt{2}}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} - \sqrt{2} \\ -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & 0 & \frac{3\sqrt{2}}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$B = T\begin{pmatrix}2\\1\\0\end{pmatrix} \cdot \underbrace{R_x(90^\circ)}_{1^\circ} \cdot \underbrace{R_y(90^\circ)}_{2^\circ} \cdot S\begin{pmatrix}1\\1/2\\1\end{pmatrix}$$

$$B = \underbrace{T\begin{pmatrix}2\\1\\0\end{pmatrix}}_{X} \cdot \underbrace{\underbrace{R_y(90^\circ) \cdot R_x(90^\circ)}_{T \quad \lambda} \cdot \underbrace{S\begin{pmatrix}1\\1/2\\1\end{pmatrix}}_{X}}$$

$$\lambda = R_y(90) \cdot R_x(90)$$

$$\lambda = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_2$$

$$T_2 S = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1/2 & 0 & 2 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = B \quad \checkmark$$

$$\begin{bmatrix} 0 & 1/2 & 0 & 2 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$T = B \cdot A^{-1}$$

$$X \longrightarrow X$$

$$\left(\frac{\sqrt{2}}{2} - \sqrt{2}\right) \cdot \frac{1}{2} \Rightarrow \frac{\sqrt{2}}{4} - \frac{\sqrt{2}}{2} \Rightarrow \frac{\sqrt{2}}{4} - \frac{2\sqrt{2}}{4}$$

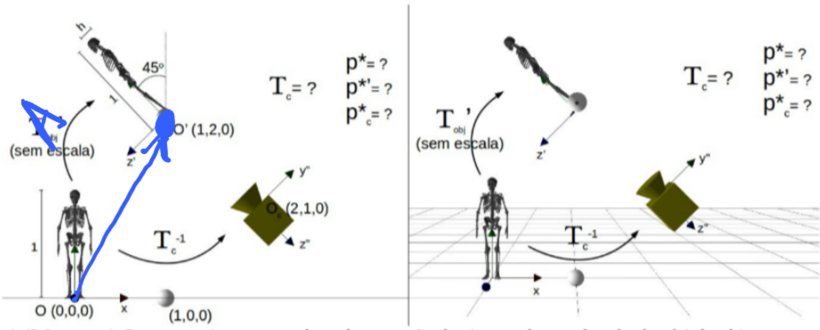
Π

$$\begin{bmatrix} 0 & 1/2 & 0 & 2 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} - \sqrt{2} \\ -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & 0 & \frac{3\sqrt{2}}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}}{4} + 2 \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & -\frac{3\sqrt{2}}{4} + 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}}{4} + 2 \\ \sqrt{2}/4 & \frac{\sqrt{2}}{4} & 0 & -3\sqrt{2}/4 + 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cancel{-\frac{\sqrt{2}}{4}} + \cancel{\frac{2\sqrt{2}}{4}} - \cancel{\frac{\sqrt{2}}{4}} + 2 \\ \cancel{\sqrt{2}/4} + \cancel{\frac{2\sqrt{2}}{4}} - \cancel{\frac{3\sqrt{2}}{4}} + 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

2- [Conversão entre sistemas de coordenadas] (5,0 pontos) Baseado nas figuras seguintes, trabalhe com mudança de sistemas de coordenadas.



a) (2,0 pontos) Gere a matriz correspondente à conversão do sistema de coordenadas local ' do objeto para o sistema de coordenadas global e mostre que ela é equivalente à matriz de composição de transformações T_{obj}'. Primeiramente, compare as imagens do modelo ' com a do modelo original, inicialmente alinhado com o sistema de coordenadas global, para deduzir a matriz de composição de transformações T_{obj}' (observe que nesse caso nenhuma transformação de escala foi aplicada). Para obter os vetores i', j' e k' (em coordenadas globais), correspondentes à base ortonormal do sistema de coordenadas local ', considere j'=$(O'-O_c)_u$ e i'=(0,0,1).
Os vetores i', j' e k' correspondem a transformar os vetores canônicos i, j e k usando parte da própria matriz T_{obj}'.
As informações de coordenadas na figura estão todas dadas em relação ao sistema de coordenadas global.

$$A \Rightarrow T\begin{pmatrix}1\\2\\0\end{pmatrix} \cdot \underbrace{R_y(-90) \cdot R_x(45°)}_{\alpha} \quad T \cdot R_z R_y$$

$$A^{-1} = R_x(-45°) \cdot R_y(+90°) \cdot T\begin{pmatrix}-1\\-2\\0\end{pmatrix}$$

$$\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

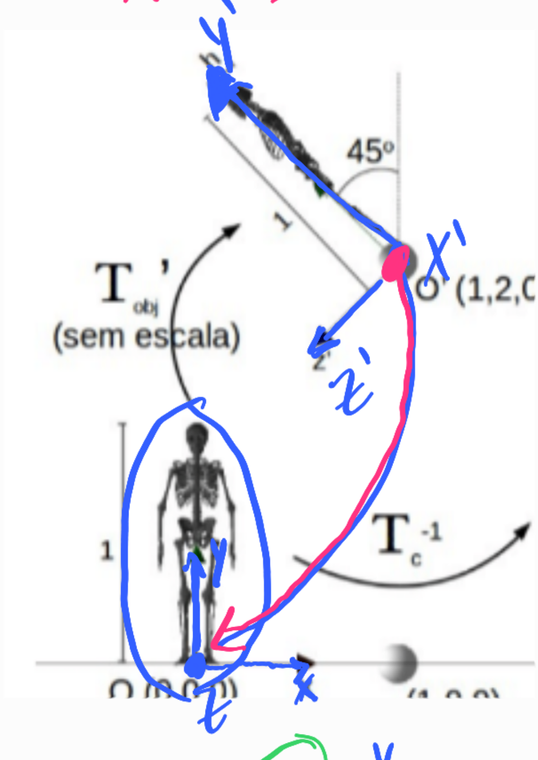
$$\alpha = \begin{bmatrix} 0 & 0 & 1 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & -\sqrt{2}/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ -\sqrt{2}/2 & -\sqrt{2}/2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & -\frac{3\sqrt{2}}{2} \\ -\sqrt{2}/2 & -\sqrt{2}/2 & 0 & 3\sqrt{2}/2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$P_g = B'' \cdot P_L$$

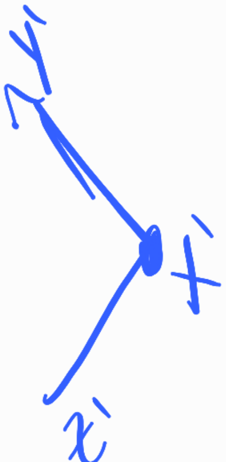
$$P_L = B''^{-1} \cdot P_g$$

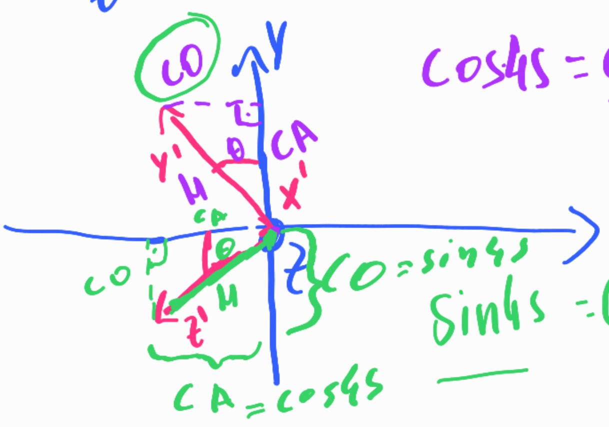


$i' = (0, 0, 1)$

$j' = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$

$k' = (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0)$





$\cos 45 = \frac{CA}{\cancel{H}\ 1}$

$\sin 45 = \frac{CO}{1}$

$i' = (0, 0, 1)$

$j' = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$

$k' = (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0)$

$P_g \to P_L$ | $P_L \to P_g$

$$\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} R^T & R^T * (-T) \\ 0 & 1 \end{bmatrix}$$

$$T\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$\overbrace{\begin{bmatrix} 0 & 0 & 1 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & -\sqrt{2}/2 & 0 \end{bmatrix}}^{R^T} \cdot \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{3\sqrt{2}}{2} \\ \frac{3\sqrt{2}}{2} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & -\frac{3\sqrt{2}}{2} \\ -\sqrt{2}/2 & -\sqrt{2}/2 & 0 & \frac{3\sqrt{2}}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & -\frac{3\sqrt{2}}{2} \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$