MATH 2605-G2

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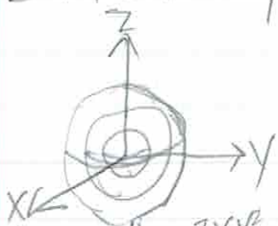
Level set: $x^2+y^2+z^2=c$



4) Does $\lim\limits_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^2}$ exist?

$\lim\limits_{x\to0} \frac{2xy^2}{x^2+y^2} = \frac{0}{y^2} = 0$

$\lim\limits_{y\to0} \frac{2xy^2}{x^2+y^2} = \frac{0}{x^2} = 0$

$\lim\limits_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^2} = \lim\limits_{x\to0} \frac{2x^3}{2x^2} = \lim\limits_{x\to0} x = 0$ for $x=y$

Same from all directions

However, the limit may be different from a path not yet tested.

Another way is to use polar coordinates ($r^2=x^2+y^2$, $x=r\cos\theta$, $y=r\sin\theta$).

$\lim\limits_{r\to0} \frac{2r\cos\theta\, r^2\sin^2\theta}{r^2} = \lim\limits_{r\to0} 2r\cos\theta\sin^2\theta = 0.$

5) $\lim\limits_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^2}$

$\lim\limits_{x\to0} \frac{xy^3}{x^4+y^2} = \frac{0}{y^2} = 0$

$\lim\limits_{y\to0} \frac{xy^3}{x^4+y^2} = \frac{0}{x^4} = 0$

$\lim\limits_{\substack{y\to x\\(x,y)\to(0,0)}} \frac{xy^3}{x^4+y^2} = \lim\limits_{x\to0} \frac{x^4}{x^4+x^2} = \lim\limits_{x\to0} \frac{x^4}{x^2(x^2+1)} = \lim\limits_{x\to0} \frac{x^2}{x^2+1} = 0$

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Ex.) $\lim\limits_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$

Through any line $y=kx$:

$\lim\limits_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2} = \lim\limits_{x\to0} \frac{2x^2(kx)}{x^4+(kx)^2} = \lim\limits_{x\to0} \frac{2kx^3}{x^2(x^2+k^2)} = \lim\limits_{x\to0} \frac{2kx}{x^2+k^2} = \frac{0}{k^2} = 0,\ k\neq0$

Does not include $y=0x=0$. For $y=0$:

$\lim\limits_{y\to0} \frac{2x^2y}{x^4+y^2} = \frac{0}{x^4} = 0$

However, along the parabola $y=x^2$:

$\lim\limits_{\substack{(x,y)\to(0,0)\\y=x^2}} \frac{2x^2y}{x^4+y^2} = \lim\limits_{x\to0} \frac{2x^2(x^2)}{x^4+(x^2)^2} = 1$

The method of path limits is not enough to prove the existence of a limit, and can only work to disprove by contradiction. Delta-epsilon is the surefire way.

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Continuity:

$f(x,y)$ is cont. at (x_0,y_0) if
1) $f(x,y)$ is defined at (x_0,y_0)
2) $\lim_{(x,y)\to(x_0,y_0)} f(x,y)$ exists
3) $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$

f is a cont. function if it is cont. at every pt. in its domain.
If $f(x,y)$ is cont. at (x_0,y_0) and $g(t)$ is cont., then $[g\circ f](x,y)=g(f(x,y))$ is cont. at (x_0,y_0).

Ex.) $\cos(x^2+y^2)$, e^{x-y}, and $\ln(1+x^2+y^2)$ are continuous.

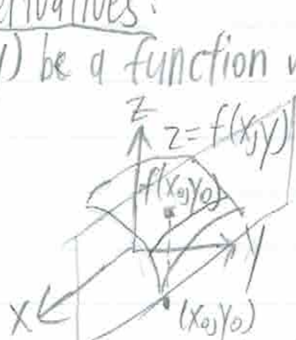
If R is a region in $\mathbb{R}^2$, (x,y) is an <u>interior pt.</u> of R if there is a solid disk around (x,y) that is contained in R. (x,y) is a <u>boundary pt.</u> of R if any disk around (x,y) contains pts. both inside and outside of R.

(x_0,y_0) is the only interior pt.
(x_1,y_1) and (x_2,y_2) are boundary pts.

For $\mathbb{R}^3$, use spheres instead of disks.
A set R is <u>open</u> if it consists of only interior pts.
A set R is <u>closed</u> if it contains all interior and boundary pts.

Partial derivatives:

Let $f(x,y)$ be a function with (x_0,y_0) in its domain and $z=f(x,y)$ as its surface.



The intersection between z and the plane $y=y_0$ is a curve $z'=f(x,y_0)=g(x)$.

We know: $g'(x_0)=\lim_{h\to 0}\frac{g(x_0+h)-g(x_0)}{h}=\lim_{h\to 0}\frac{f(x_0+h,y_0)-f(x_0,y_0)}{h}$. This is the slope of the tangent line to z' at $f(x_0,y_0)$.
Define: $g'(x_0)=\frac{\partial f}{\partial x}(x_0,y_0)$.

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Similarly (if x is held constant), $\frac{\partial f}{\partial y}(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0,y_0+h)-f(x_0,y_0)}{h}$.

Ex) Calculate the partial derivatives of $f(x,y)=2x^2y^3+x^4-y^2+3y\sin x$.

$\frac{\partial f}{\partial x}(x,y)=4xy^3+4x^3+3y\cos x$ $\qquad$ $\frac{\partial f}{\partial y}(x,y)=6x^2y^2-2y+3\sin x$

treat y as constant $\qquad$ treat x as constant