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Clicker Questions

1/26/15: Solve $y'=xy^3$ with the initial value $y(0)=\frac{1}{4}$.

$\int \frac{dy}{y^3}=\int x\,dx$

$\frac{-1}{2y^2}=\frac{x^2}{2}+C$

$y^2=\frac{-1}{2(\frac{x^2}{2}+C)}=\frac{1}{-x^2+C}$

$y=\pm(-x^2+C)^{-1/2}$

$\frac{1}{4}=+C^{-1/2}$

$C=16$

$y=(-x^2+16)^{-1/2}$ ← defined when $x^2-16<0 \Rightarrow -4<x<4$

C

Find $\vec{u}$ and $\vec{v}$ such that $(A-\lambda I)\vec{v}=0$ and $(A-\lambda I)\vec{u}=\vec{v}$ for $A=\begin{pmatrix}-5/2 & 3/2\\ -3/2 & 1/2\end{pmatrix}$.

$tr(A)=-\frac{5}{2}+\frac{1}{2}=-2$ $\quad$ $det(A)=-\frac{5}{2}(\frac{1}{2})+\frac{3}{2}(\frac{3}{2})=1$

$\lambda^2-(-2)\lambda+1=0 \Rightarrow \lambda=-1$

$A-I=\begin{pmatrix}-3/2 & 3/2\\ -3/2 & 3/2\end{pmatrix} \Rightarrow \vec{v}=\begin{pmatrix}1\\1\end{pmatrix}$

$\begin{pmatrix}-3/2 & 3/2\\ -3/2 & 3/2\end{pmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix} \Rightarrow \vec{u}=\begin{pmatrix}u_1\\ \frac{2}{3}+u_1\end{pmatrix}=\begin{pmatrix}-\frac{2}{3}+u_2\\ u_2\end{pmatrix}=\begin{pmatrix}-\frac{2}{3}\\0\end{pmatrix}$ for $u_1=0$

B

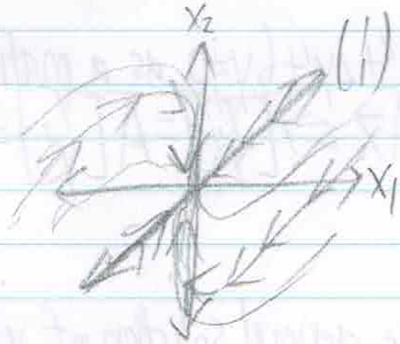
$\vec{X}_1=e^{-t}\begin{pmatrix}1\\1\end{pmatrix}$ $\quad$ $\vec{X}_2=te^{-t}\begin{pmatrix}1\\1\end{pmatrix}+e^{-t}\begin{pmatrix}-2/3\\0\end{pmatrix}$

Specific solution for $\vec{x}(0)=\begin{pmatrix}2\\1\end{pmatrix}$:

$\vec{x}(0)=c_1\begin{pmatrix}1\\1\end{pmatrix}+c_2\begin{pmatrix}-2/3\\0\end{pmatrix}=\begin{pmatrix}2\\1\end{pmatrix}$

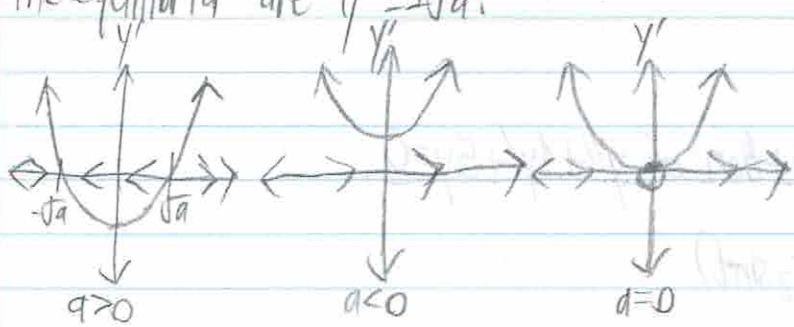
$\begin{cases}c_1-\frac{2}{3}c_2=2\\ c_1=1\end{cases}$ $\rightarrow c_2=-\frac{3}{2}$

$\vec{x}=e^{-t}\begin{pmatrix}1\\1\end{pmatrix}-\frac{3}{2}te^{-t}\begin{pmatrix}1\\1\end{pmatrix}-\frac{3}{2}e^{-t}\begin{pmatrix}-2/3\\0\end{pmatrix}$

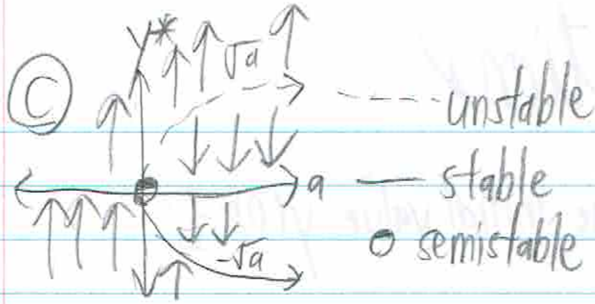


1/28/15: Draw the bifurcation diagram for $y'=y^2-a$.

The equilibria are $y^*=\pm\sqrt{a}$.



C



Find the integrating factor of $t^3y'+(4t^2+2t^3)y=Erf(x)$, x is a constant real number

$y'+(\frac{4}{t}+2)y=Erf(x)/t^3$

$\mu y'+(\frac{4}{t}+2)\mu y=\mu Erf(x)/t^3$

$\frac{d}{dt}(\mu y)=\mu y'+y(\frac{4\mu}{t}+2\mu) \Rightarrow \frac{d\mu}{dt}=\mu(\frac{4}{t}+2) \Rightarrow \int\frac{d\mu}{\mu}=\int(2+\frac{4}{t})dt$

$\frac{d}{dt}(\mu y)=\mu Erf(x)/t^3$ $\qquad \ln|\mu|=2t+4\ln|t|+C$

$\mu=Ce^{2t}t^4$

A

$\cancel{C}e^{2t}t^4y=\int\cancel{C}e^{2t}t\,Erf(x)dt$

$u=t \quad \int dv=\int e^{2t}dt$

$du=dt \quad v=\frac{1}{2}e^{2t}$

$\int e^{2t}t\,dt=\frac{1}{2}te^{2t}-\int\frac{1}{2}e^{2t}dt=\frac{1}{2}e^{2t}(t-\frac{1}{2})$

$y\cancel{e^{2t}}t^4=Erf(x)(\frac{1}{2}\cancel{e^{2t}})(t-\frac{1}{2})$

$y=Erf(x)(\frac{1}{t^3}-\frac{1}{2t^4})$

1/30/15: Patriots

Write $y''+2y'+6y=0$ as a matrix equation.

$\vec{x}'=A\vec{x} \Rightarrow \begin{bmatrix}y'\\y''\end{bmatrix}=A\begin{bmatrix}y\\y'\end{bmatrix}=\begin{bmatrix}0&1\\-6&-2\end{bmatrix}\begin{bmatrix}y\\y'\end{bmatrix}$ so $\vec{x}'=\begin{bmatrix}0&1\\-6&-2\end{bmatrix}\vec{x}$

B

2/2/15: Find the general solution of $y''+3y'+2y=0$.

$\lambda^2+3\lambda+2=0$

$\lambda_1=-1 \quad \lambda_2=-2$

$y_1=e^{-t} \quad y_2=e^{-2t}$

D $\quad y=c_1e^{-t}+c_2e^{-2t}$

Find the general solution of $y''+4y'+5y=0$.

$\lambda=-2\pm i$

C $\quad y=e^{-2t}(c_1\cos t+c_2\sin t)$

2/4/15: Write the guess for $y''+3y'+2y=te^{-t}$.

$t(A_1t+A_2)e^{-t}=(A_1t^2+A_2t)e^{-t}$

(C)

Write the guess for $y''-6y'+9y=e^{3t}$

Char. poly: $\lambda^2-6\lambda+9=0 \Rightarrow (\lambda-3)^2=0 \Rightarrow \lambda=3$ has multiplicity 2

Then $y_p=At^2e^{3t}$ (D)

$y_p'=2Ate^{3t}+3At^2e^{3t}$

$y_p''=9Ae^{3t}t^2+12Ae^{3t}t+2Ae^{3t}$

$A=\frac{1}{2}$

2/6/15: Find the gain function of $y''-2y'+5y$.

Guess $y_p=G(w)e^{iwt}$

$Ge^{iwt}(-w^2-2iw+5)=e^{iwt}$

$G=\frac{1}{5-w^2-2iw}\left(\frac{5-w^2+2iw}{5-w^2+2iw}\right)$

$=\frac{5-w^2+2iw}{(5-w^2)^2+4w^2}$

(B)

Compute the Wronskian of $y_1=te^t$ and $y_2=t$.

$\begin{vmatrix} te^t & t \\ te^t+e^t & 1 \end{vmatrix}=te^t-t^2e^t-te^t=-t^2e^t$

(D)

2/9/15: Compute the Wronskian of $y_1=te^{2t}$ and $y_2=e^{2t}$.

$\begin{vmatrix} te^{2t} & e^{2t} \\ 2te^{2t}+e^{2t} & 2e^{2t} \end{vmatrix}=-e^{4t}$ (D)

2/9/15: Solve $\vec{x}'=\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}\vec{x}+\begin{bmatrix} t \\ e^{3t} \end{bmatrix}$ using variation of parameters.

$\lambda_1=3$ $\lambda_2=-1$

$\vec{v}_1=\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\vec{v}_2=\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$X=\begin{bmatrix} e^{3t} & e^{-t} \\ 2e^{3t} & -2e^{-t} \end{bmatrix}$

Now we have $\begin{bmatrix} e^{3t} & e^{-t} \\ 2e^{3t} & -2e^{-t} \end{bmatrix}\begin{bmatrix} u_1' \\ u_2' \end{bmatrix}=\begin{bmatrix} t \\ e^{3t} \end{bmatrix}$.

$W=\begin{vmatrix} e^{3t} & e^{-t} \\ 2e^{3t} & -2e^{-t} \end{vmatrix}=-4e^{2t}$

$u_1'(t)=\dfrac{\begin{vmatrix} \vec{g}(t) & e^{-t} \\ & -2e^{-t} \end{vmatrix}}{W}=\dfrac{\begin{vmatrix} t & e^{-t} \\ e^{3t} & -2e^{-t} \end{vmatrix}}{W}=\dfrac{-2te^{-t}-e^{2t}}{-4e^{2t}}=\frac{1}{2}te^{-3t}+\frac{1}{4}$

$u_2'(t)=\dfrac{\begin{vmatrix} e^{3t} & t \\ 2e^{3t} & e^{3t} \end{vmatrix}}{W}=\dfrac{e^{6t}-2te^{3t}}{-4e^{2t}}=-\frac{1}{4}e^{4t}+\frac{1}{2}te^{t}$

$u_1(t)=\int u_1'(t)\,dt \quad u_2(t)=\int u_2'(t)\,dt$

$y=e^{3t}u_1(t)+e^{-t}u_2(t)$

2/11/15: Find the general solution of $t^2y''+2ty'-2y=3t$.

$y=\underbrace{c_1t^{-2}+c_2t}_{\text{complementary}}+\underbrace{t\ln t}_{\text{particular}}$

Find $\mathcal{L}(\sin(at))=\int_0^\infty e^{-st}\sin(at)\,dt$. Use $\mathcal{L}\{e^{at}\}=\frac{1}{s-a}$, $s>a$ and $\sin(at)=\frac{e^{iat}-e^{-iat}}{2i}$.

$\frac{a}{s^2+a^2}$, $s>\pm ia$ C

2/13/15: Solve $\sum_{n=1}^{\infty} c_n\left(-\frac{n^2\pi^2}{L^2}+1\right)\sin\left(\frac{n\pi t}{L}\right)=\sum_{n=1}^{\infty} a_n\sin\left(\frac{n\pi t}{L}\right)$ for c_n.

$c_n=\dfrac{a_n}{-\frac{n^2\pi^2}{L^2}+1}=\dfrac{L^2}{(L-n\pi)(L+n\pi)}$ B

2/16/15: Find $\mathcal{L}\{y''(t)\}$.

$s\mathcal{L}\{y'\}-y'(0)$

$=s^2Y(s)-sy(0)-y'(0)$

D

Find $Y(s)$ where $y''+3y'+2y=e^t$, $y(0)=1$, $y'(0)=-1$.

$s^2Y(s)-s+1+3(sY-1)+2Y=\frac{1}{s-1}$

$Y(s)=\frac{1}{(s-1)(s^2+3s+2)}+\frac{s+2}{s^2+3s+2}$ A

Use Partial Fraction Decomposition to solve for Y.

2/18/15: Decompose $\frac{5}{s^2-s-6}$.

$\frac{2}{5(s+2)}+\frac{3}{5(s-3)}$

Decompose $\frac{s^2+4s-1}{(s+1)(s+2)(s-3)}$.

$\frac{1}{s+1}-\frac{1}{s+2}+\frac{1}{s-3}$ D

Decompose $\frac{5s^2-12s+22}{(s^2-2s+10)(s+1)}$.

2/20/15: $5s^2-12s+22=(As+B)(s+1)+C(s^2-2s+10)$

$s=0$: $22=B+10C \Rightarrow B=-8$ $\quad$ $s=-1$: $39=13C \Rightarrow C=3$ $\quad$ $s=1$: $15=(A-8)(2)+3(9) \Rightarrow 2=A$

$\frac{2s-8}{s^2-2s+10}+\frac{3}{s+1}$

Decompose $\frac{2s^2+1}{(s-1)^2(s+2)}$.

$\frac{1}{s-1}+\frac{1}{(s-1)^2}+\frac{1}{s+2}$ $\quad$ B

2/23/15: Write the guess for the method of undetermined coefficients for $y''+2y'+2y=\sin t$.

$Y=A_0\sin t+B_0\cos t$ $\quad$ A

$y''+4y'+4y=te^{-2t}$

Solve $y''+4y'+4y=0$:

$(\lambda+2)^2=0$

$\lambda=-2$

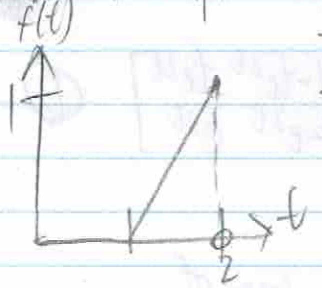
$\vec{v}=e^{-2t}$ ← repeated

$\vec{w}=te^{-2t}$ ← repeated

$Y=t^2(Ate^{-2t}+Be^{-2t})$ $\quad$ B

$=At^3e^{-2t}+Bt^2e^{-2t}$

2/27/15: Find the Laplace transform of $f(t)=\begin{cases}0, & t<1\\ t-1, & 1<t\le 2\\ 0, & t>2\end{cases}$.



$f_1=t-1 \quad t\to t+1$

$f_2=1-t \quad -1-t\to t+2$

$f(t)=(t-1)u_1(t)+(1-t)u_2(t)$

$=\mathcal{L}\{t\}e^{-s}+\mathcal{L}\{-1-t\}e^{-2s}$

$=\frac{1}{s^2}e^{-s}+\left(-\frac{1}{s}-\frac{1}{s^2}\right)e^{-2s}$

3/2/15: Find the Laplace transform of $f(t)=\begin{cases}t, & 0<t\le 2\\ 2, & 2<t\le 3\\ 0, & t>3\end{cases}$.

$\hat{f}_0=t \quad \hat{f}_1=2 \quad \hat{f}_2=0$

$f_0=t \quad f_1=\hat{f}_1(t+2)-\hat{f}_0(t+2)=2-(t+2)=-t \quad f_2=\hat{f}_2(t+3)-\hat{f}_1(t+3)=-2$

$F(s)=(\mathcal{L}\{f_0\}+\mathcal{L}\{u_2f_1\}+\mathcal{L}\{u_3f_2\})/(1-e^{-6s})$

$=\frac{\frac{1}{s^2}+e^{-2s}\left(-\frac{1}{s^2}\right)+e^{-3s}\left(-\frac{2}{s}\right)}{(1-e^{-6s})}$

period $T=6$

3/4/15: For $a<0$, $b>0$, $\int_a^b \delta(t)g(t)dt = ?$

$= \lim_{\varepsilon\to 0} \int_{-\varepsilon/2}^{\varepsilon/2} \frac{1}{\varepsilon} g(t)dt = \lim_{\varepsilon\to 0} \frac{G(t)}{\varepsilon}\Big|_{-\varepsilon/2}^{\varepsilon/2} = \lim_{\varepsilon\to 0} \frac{G(\frac{\varepsilon}{2})-G(-\frac{\varepsilon}{2})}{\varepsilon} = G'(0) = g(0)$ (B)

What is $\mathcal{L}\{y\}$ where $y'+ry=\delta(t-c)$?

$(s+r)Y(s) = e^{-sc}$

$Y(s) = \frac{e^{-sc}}{s+r}$, $y(t) = e^{-r(t-c)}u_c(t)$

3/6/15: Find $\mathcal{L}^{-1}\{\frac{s}{(s+1)(s^2+4)}\}$.

$\frac{s}{(s+1)(s^2+4)} = F(s)G(s)$

Let $F(s) = \frac{1}{s+1}$, $G(s) = \frac{s}{s^2+4}$

$\Rightarrow f(t) = e^{-t}$, $g(t) = \cos 2t$

$\int_0^t e^{-(t-\tau)}\cos(2\tau)d\tau$ (A) (equivalent to $\int_0^t e^{-\tau}\cos 2(t-\tau)d\tau$)

Solve $\phi(t) + \int_0^t \sin(t-\tau)\phi(\tau)d\tau = \cos t$.

Let $\mathcal{L}\{\phi(t)\} = \Phi(s)$.

$\Phi(s) + \frac{1}{s^2+1}(\Phi(s)) = \frac{s}{s^2+1}$

$\Phi(s) = \frac{s}{s^2+1} / (\frac{s^2+2}{s^2+1}) = \frac{s}{s^2+2}$

$\phi(t) = \mathcal{L}^{-1}\{\frac{s}{s^2+2}\} = \cos\sqrt{2}t$ (D)

3/9/15: Given $X(t) = \begin{bmatrix} t+1 & te^{2t} \\ e^t & e^{2t} \end{bmatrix}$, find e^{At}

$X^{-1}(t) = \frac{1}{(t+1)e^{2t} - te^{3t}} \begin{bmatrix} e^{2t} & -te^{2t} \\ -e^t & t+1 \end{bmatrix}$

$e^{At} = X(t)X^{-1}(0) = \begin{bmatrix} t+1 & te^{2t} \\ e^t & e^{2t} \end{bmatrix} \frac{1}{1} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} t+1-te^{2t} & te^{2t} \\ e^t - e^{2t} & e^{2t} \end{bmatrix}$ (D)

Given $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, find e^{At}

$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 0 & 2-\lambda \end{vmatrix} = (1-\lambda)(2-\lambda) = 0$

$\lambda_1 = 1$ $\lambda_2 = 2$

$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

one form of

$e^{At} = Te^{Dt}T^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} e^t & \\ & e^{2t} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \underbrace{\begin{bmatrix} e^t & e^{2t} \\ 0 & e^{2t} \end{bmatrix}}_{X(t)}\underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}}_{X^{-1}(0)} = \begin{bmatrix} e^t & e^{2t}-e^t \\ 0 & e^{2t} \end{bmatrix}$ (A)

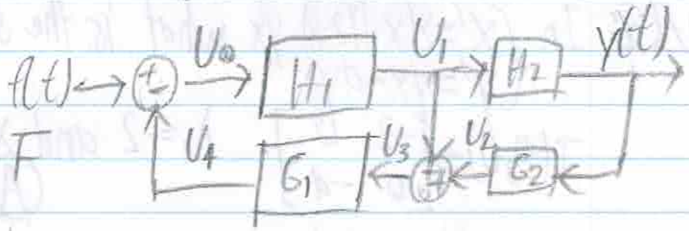
3/11/15: Find $\phi(t) = e^{At}$ for $A = \begin{bmatrix} 1 & 1 \\ -5 & -3 \end{bmatrix}$ using a Laplace transform.

$sI-A=\begin{bmatrix} s-1 & -1 \\ 5 & s+3 \end{bmatrix}$, $(sI-A)^{-1}=\frac{1}{(s-1)(s+3)+5}\begin{bmatrix} s+3 & 1 \\ -5 & s-1 \end{bmatrix}$

$$= \begin{bmatrix} \frac{s+1}{(s+1)^2+1}+\frac{2}{(s+1)^2+1} & \frac{1}{(s+1)^2+1} \\ \frac{-5}{(s+1)^2+1} & \frac{s+1}{(s+1)^2+1}-\frac{2}{(s+1)^2+1} \end{bmatrix}$$

$$\mathcal{L}^{-1}\{(sI-A)^{-1}\}=\begin{bmatrix} e^{-t}\cos t+2e^{-t}\sin t & e^{-t}\sin t \\ -5e^{-t}\sin t & e^{-t}\cos t-2e^{-t}\sin t \end{bmatrix}$$ D

3/13/15: Write the transfer function for



$$\begin{cases} U_0=F-U_4 \\ U_1=H_1U_0 \\ Y=H_2U_1 \\ U_2=G_2Y \\ U_3=U_1+U_2 \\ U_4=G_1U_3 \end{cases} \qquad Y=\underbrace{\frac{H_1H_2}{1+G_1G_2H_1H_2+G_1H_1}}_{H_T}F$$ B

Solve $\vec{x}'=\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}\vec{x}+\begin{bmatrix} 0 \\ e^{-2t} \end{bmatrix}$, $\vec{x}(0)=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\vec{x}(t)=\Phi(t)\vec{x}_0+\int_0^t \Phi(t-\tau)g(\tau)d\tau=\Phi(t)\begin{bmatrix} 1 \\ -1 \end{bmatrix}+\int_0^t \Phi(t-\tau)\begin{bmatrix} 0 \\ e^{-2\tau} \end{bmatrix}d\tau$

$\Phi(t)=e^{At}=\begin{bmatrix} 3e^{-t}-2e^{-2t} & -2e^{-t}+2e^{-2t} \\ 3e^{-t}-3e^{-2t} & -2e^{-t}+3e^{-2t} \end{bmatrix}$

3/23/15: Find e^{At} where $A=\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$.

$(\lambda-1)(\lambda+2)-4=0$

$\lambda^2+\lambda-6=0$

$\lambda_1=-3 \quad \lambda_2=2$

$\vec{v}_1=\begin{bmatrix} 1 \\ -4 \end{bmatrix} \quad \vec{v}_2=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A=\begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} -3 & \\ & 2 \end{bmatrix}\frac{1}{1(1)+1(4)}\begin{bmatrix} 1 & -1 \\ 4 & 1 \end{bmatrix}$

$e^{At}=\begin{bmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{bmatrix}\frac{1}{5}\begin{bmatrix} 1 & -1 \\ 4 & 1 \end{bmatrix}=\frac{1}{5}\begin{bmatrix} e^{-3t}+4e^{2t} & -e^{-3t}+e^{2t} \\ -4e^{-3t}+4e^{2t} & 4e^{-3t}+e^{2t} \end{bmatrix}$

Solve $\vec{x}'=\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}\vec{x}+\begin{bmatrix} 5e^{2t} \\ 0 \end{bmatrix}$, $\vec{x}_0=\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

$\vec{x}=\Phi(t)\vec{x}_0+\int_0^t \Phi(t-\tau)g(\tau)d\tau$

$=\frac{1}{5}\begin{bmatrix} -3e^{2t}-2e^{-3t} \\ 8e^{-3t}-3e^{2t} \end{bmatrix}+\frac{1}{5}\int_0^t \begin{bmatrix} 4e^{2(t-\tau)}+e^{-3(t-\tau)} & e^{2(t-\tau)}-e^{-3(t-\tau)} \\ 4e^{2(t-\tau)}-4e^{-3(t-\tau)} & e^{2(t-\tau)}+4e^{-3(t-\tau)} \end{bmatrix}\begin{bmatrix} 5e^{2\tau} \\ 0 \end{bmatrix}d\tau$

$\frac{1}{5}\begin{bmatrix} e^{2t}-e^{-3t}+20te^{2t} \\ -4e^{2t}+4e^{-3t}+20te^{2t} \end{bmatrix}$

$=\frac{1}{5}\begin{bmatrix} -2e^{2t}-3e^{-3t}+20te^{2t} \\ -7e^{2t}+12e^{-3t}+20te^{2t} \end{bmatrix}$ C

$\begin{cases} x' = 2x - 3xy - x^2 \\ y' = y - xy - y^2 \end{cases}$

3/27/15: In $\uparrow$, what type of point is $x = \frac{1}{2}, y = \frac{1}{2}$?
Saddle (C) (the eigenvalues of $J(\frac{1}{2}, \frac{1}{2})$ are positive and negative)

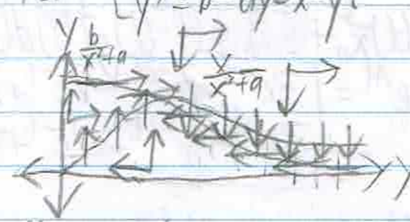
What type of point is $x = 0, y = 1$?
Stable node (A) (the eigenvalues of $J(0,1)$ are both -1)

3/30/15: In $\begin{cases} x' = x(x-1)(2-x) - yx \\ y' = yx - ay \end{cases}$ what is the stability of $(x,y) = (0,0)$? Assume $a > 0$.

$J(0,0) = \begin{bmatrix} -2 & 0 \\ 0 & -a \end{bmatrix}$ $\lambda_1 = -2$ and $\lambda_2 = -a$, so $(0,0)$ is stable.
(A)

What is the stability of $(x,y) = (2,0)$?
$J(2,0) = \begin{bmatrix} -2 & -2 \\ 0 & 2-a \end{bmatrix}$ $\lambda_1 = -2$ and $\lambda_2 = 2-a$, so stable if $a > 2$ and saddle if $a < 2$
(B)

4/1/15: Sketch the nullclines and phase plane for $\begin{cases} x' = -x + ay + x^2y \\ y' = b - ay - x^2y. \end{cases}$
x-nullcline: $ay + x^2y = x \Rightarrow y = \frac{x}{x^2+a}$
y-nullcline: $yx^2 + ay = b \Rightarrow y = \frac{b}{x^2+a}$



4/3/15: What is the conserved quantity in the system $y' = x - x^3$, $x' = y$?
$\frac{dx}{dt} = y$, $\frac{dy}{dt} = x - x^3$
$y \frac{dy}{dx} = x - x^3$ (C)
$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} - \frac{x^4}{4} + C \Rightarrow y^2 - x^2 + \frac{x^4}{2} = 2C$

When is $\frac{dx}{dt} = y - x^2$, $\frac{dy}{dt} = -x + q(x,y)$ conservative?
$\frac{dx}{dt} = y - x^2$ $\frac{dy}{dt} = -x + q(x,y)$
$0 - 2x + 0 + q_y(x,y) = 0$
$\Rightarrow q(x,y) = 2xy$ (C)

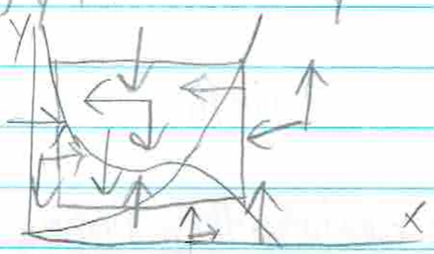
4/6/15: Compute r' for $x' = x - y - x(x^2 + 5y^2)$, $y' = x + y - y(x^2 + y^2)$.
$rr' = x(x - y - x(x^2 + 5y^2)) + y(x + y - y(x^2 + y^2))$
$= x^2 + y^2 - x^4 - 5x^2y^2 - x^2y^2 - y^4$ (D)
$= r^2(1 - r^2 - 4r^2\cos^2\theta \sin^2\theta) \Rightarrow r' = r(1 - r^2 - 4r^2\cos^2\theta \sin^2\theta)$

4/8/15:] Excused absence
4/10/15:

4/13/15: Sketch the nullclines and arrows for the Oregonator $(x, y>0)$.

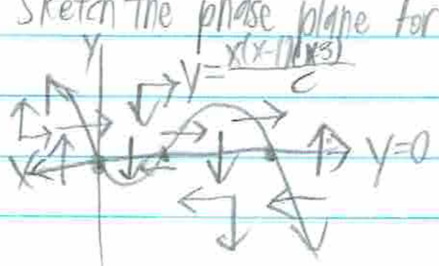
y-nullclines: $x=0$, $y=1+x^2$

x-nullclines: $y=\frac{(a-x)(1+x^2)}{4x}$



Ⓒ

4/15/15: Sketch the phase plane for $\begin{cases} x'=y \\ y'=x(x-1)(x-3)+cy, \ c>0. \end{cases}$



Classify all equilibria.

$J=\begin{bmatrix} 0 & 1 \\ 3x^2-8x+3 & c \end{bmatrix}$

$J(0,0)=\begin{bmatrix} 0 & 1 \\ 3 & c \end{bmatrix}$ has tr=c, det=-3 so (0,0) is a saddle.

$J(1,0)=\begin{bmatrix} 0 & 1 \\ -2 & c \end{bmatrix}$ has tr=c, det=2 so (1,0) is an unstable node or spiral.

$J(2,0)=\begin{bmatrix} 0 & 1 \\ 6 & c \end{bmatrix}$ has tr=c, det=-6 so (2,0) is a saddle.

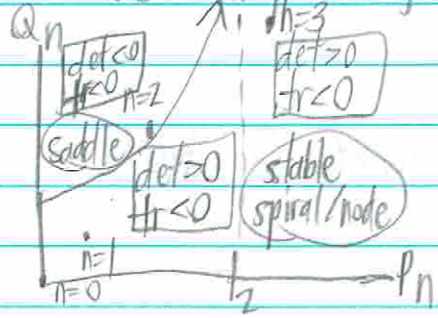
4/17/15: In the Turing instability system $\frac{\partial u}{\partial t}=2u(u-2v+1)+D_1\frac{\partial^2 u}{\partial z^2}$, $\frac{\partial v}{\partial t}=3v(u-v)+D_2\frac{\partial^2 v}{\partial z^2}$ with $D_1=D_2=0$, the equilibria (with $u, v>0$) are $(u^*, v^*)=(0,0)$ and $(u^*, v^*)=(1,1)$. What is the stability of (1,1)?

$tr(J(1,1))=-1<0$ $\quad$ $det(J(1,1))=6>0$ $\quad$ $(-1)^2-4(6)<0$

Stable spiral

Ⓑ

Sketch the stability of (1, 1) in the P_n-Q_n plane.



Ⓐ

4/20/15: Review of bifurcations and variation of parameters

4/22/15: Review of solving integral equations with convolution thm., finding matrix exponential with Laplace transform, variation of parameters with matrix exponential, solving ODE w/ convolution