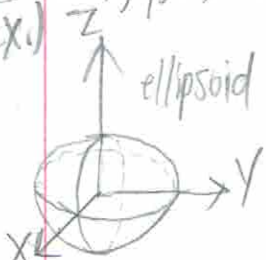
MATH 2605-G2

10/10/14

$f(x,y,z)=4x^2+y^2+9z^2$

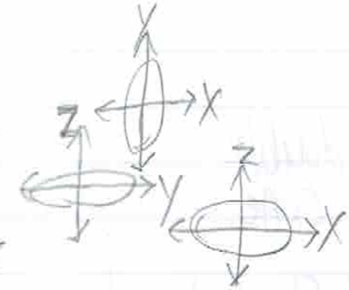
Ex.) $4x^2+y^2+9z^2=k$



If $z=0$: $4x^2+y^2=k$

If $x=0$: $y^2+9z^2=k$

If $y=0$: $4x^2+9z^2=k$

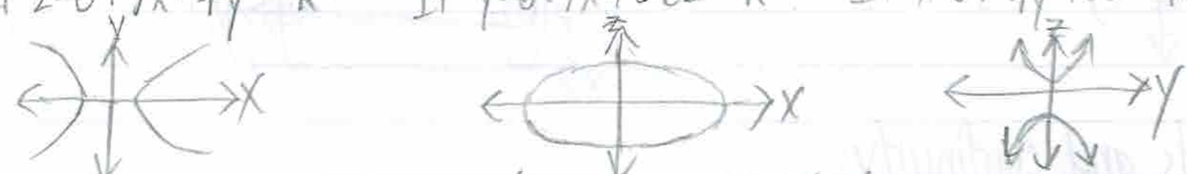


Ex.) Level surfaces of $f(x,y,z)=9x^2-4y^2+36z^2=k$

For $k>0$:

(Graph depends on sign of k.)

If $z=0$: $9x^2-4y^2=k$ If $y=0$: $9x^2+36z^2=k$ If $x=0$: $-4y^2+36z^2=k$



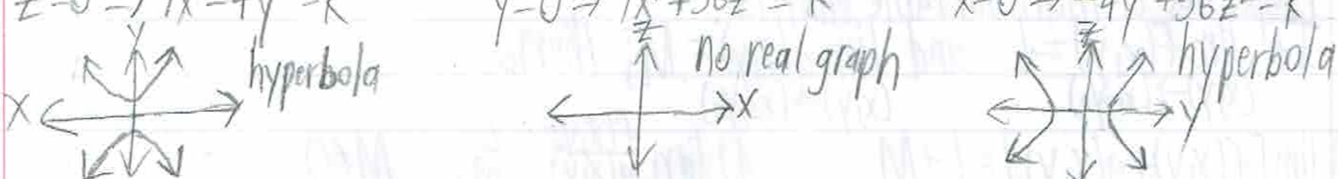
Graph of $9x^2+36z^2=k+4y^2$ (consider a constant $y=y_0$)

hyperboloid of one sheet

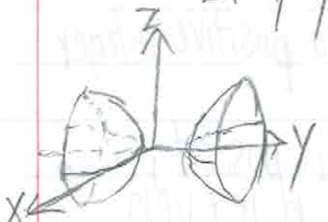
10/15/14

For $k<0$:

$z=0 \Rightarrow 9x^2-4y^2=k$ $y=0 \Rightarrow 9x^2+36z^2=k$ $x=0 \Rightarrow -4y^2+36z^2=k$



If $y=y_0$: $9x^2+36z^2=k+4y_0^2$ (Ellipse for $k+4y_0^2>0$)

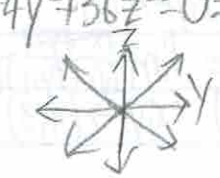


hyperboloid of two sheets

For $k=0$:

$z=0 \Rightarrow 9x^2=4y^2 \Leftrightarrow y=\pm\frac{3}{2}x$ $x=0 \Rightarrow -4y^2+36z^2=0 \Rightarrow z=\pm\frac{1}{3}y$

$y=y_0 \Rightarrow 9x^2+36z^2=4y_0^2$ (Ellipse)



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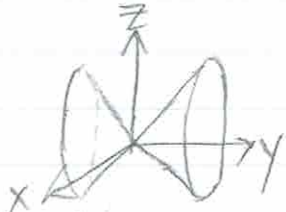
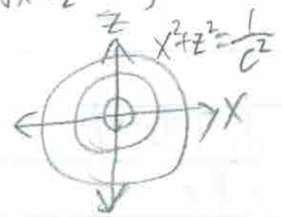


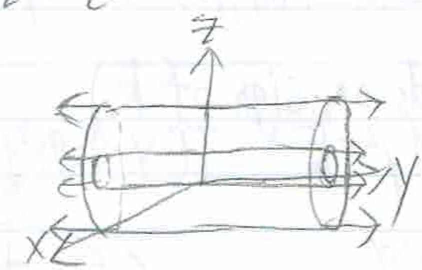
double cone

Ex.) Level surfaces of $f(x,y,z)=\frac{1}{\sqrt{x^2+z^2}}$

$\frac{1}{\sqrt{x^2+z^2}}=c,\ c>0 \Leftrightarrow \frac{1}{c}=\sqrt{x^2+z^2} \Leftrightarrow x^2+z^2=\frac{1}{c^2}$

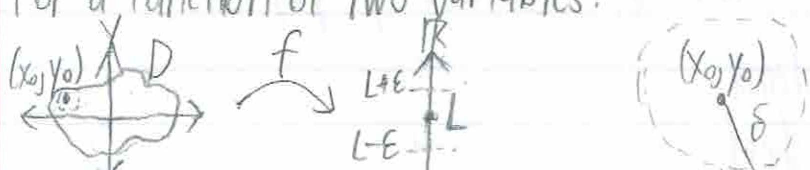


$y \in \mathbb{R} \implies$



Limits and continuity:

For a function of two variables:



$\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)=L$ iff for every $\varepsilon>0$, there exists $\delta>0$ such that for every point (x,y) in the domain of f,

continuous

$|f(x,y)-L|<\varepsilon$ whenever $\|(x,y)-(x_0,y_0)\|<\delta$.

Properties of multivariable limits:

If $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)=L$ and $\lim\limits_{(x,y)\to(x_0,y_0)} g(x,y)=M$, then

1) $\lim\limits_{(x,y)\to(x_0,y_0)}[f(x,y)+g(x,y)]=L+M$

2) $\lim\limits_{(x,y)\to(x_0,y_0)}[kf(x,y)]=kL$

3) $\lim\limits_{(x,y)\to(x_0,y_0)}[f(x,y)g(x,y)]=LM$

4) $\lim\limits_{(x,y)\to(x_0,y_0)}\frac{f(x,y)}{g(x,y)}=\frac{L}{M}$, $M\neq 0$

5) $\lim\limits_{(x,y)\to(x_0,y_0)}[f(x,y)]^n=L^n$, n is a positive integer

6) $\lim\limits_{(x,y)\to(x_0,y_0)}[f(x,y)]^{1/n}=L^{1/n}$, n is pos. int., $L>0$ if n is even

Ex.) $\lim\limits_{(x,y)\to(1,1)}\frac{x^2-y^2}{x-y}=\lim\limits_{(x,y)\to(1,1)}(x+y)=2$

Ex.) $\lim\limits_{(x,y)\to(4,3)}\frac{\sqrt{x}-\sqrt{y+1}}{x-y-1}=\lim\limits_{(x,y)\to(4,3)}\frac{\cancel{\sqrt{x}-\sqrt{y+1}}}{(\sqrt{x}+\sqrt{y+1})\cancel{(\sqrt{x}-\sqrt{y+1})}}=\frac{1}{\sqrt{4}+\sqrt{3+1}}=\frac{1}{4}$

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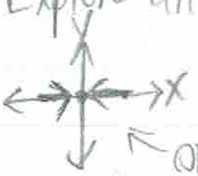
$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$$

Ex.) Explore different paths to go to (0,0):



$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \lim_{x\to 0} \frac{2x(0)}{x^2+0^2} = \lim_{x\to 0} \frac{0}{x^2} = 0$$

on the x-axis

Similarly, on the y-axis, $\lim_{y\to 0} f(x,y) = 0$.

But on the line $y = x$:

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{2x^2}{2x^2} = 1$$

Because the limit is not the same in every direction, it does not exist.