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$h(\frac{9}{2}) = \frac{-41}{2}$ $h(0) = -61$

c) $k(y) = f(0, y)$, $0 \leq y \leq 9$

Using symmetry, $k(1) = 3$, $k(0) = 2$, and $k(9) = -61$.

Absolute minima: -61 at $(9, 0)$ and $(0, 9)$

Absolute maximum: 4 at $(1, 1)$

Review for test:

Ex) Find the max. of $d = 2x + 2y + 5z$ in the surface $z = 9 - x^2 - y^2$.

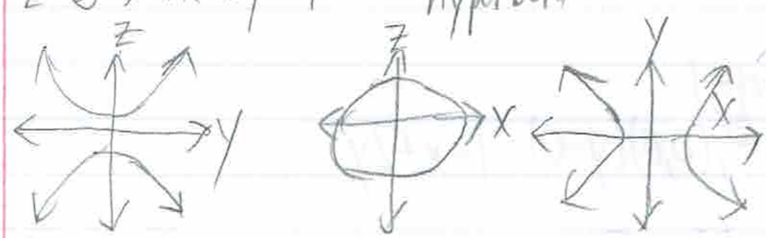
First step is to substitute for z: $d(x, y) = 2x + 2y + 5(9 - x^2 - y^2)$

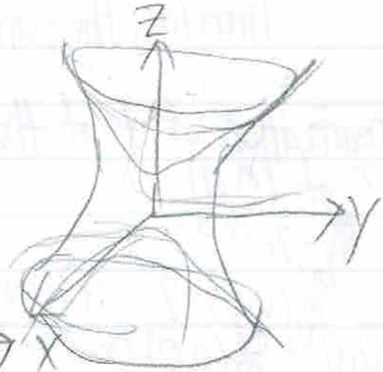
Ex) Draw the surface $9x^2 = 3y^2 - 4z^2 + 1$.

$x = 0 \Rightarrow 3y^2 - 4z^2 = -1$ hyperbola

$y = 0 \Rightarrow 9x^2 + 4z^2 = 1$ ellipse

$z = 0 \Rightarrow 9x^2 - 3y^2 = 1$ hyperbola





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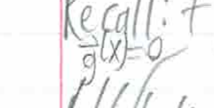
<u>Lagrange multipliers:</u>

Find the maximum of $f(x, y, z)$ subject to the constraint $g(x, y, z) = 0$.

Recall: f grows in the direction of ∇f.



Also, ∇g is normal to level set of g.

∇g (same direction as ∇f)

∇f and ∇g point in the same direction as maximum is reached:

$\nabla f(\vec{x}) = \lambda \nabla g(\vec{x})$, λ is <u>Lagrange multiplier</u>

Solve equation for $\vec{x}$ and λ.

Ex) A rectangular box with no lid has volume $V = 12$ and minimum surface area ($S.A. = xy + 2yz + 2xz$). Restriction: $V = xyz = 12$.

$\nabla f = \nabla S.A. = (y + 2z, x + 2z, 2x + 2y)$ $\nabla g = \nabla V = (yz, xz, xy)$

To solve $\nabla f = \lambda \nabla g$:

$\begin{cases} y + 2z = \lambda yz \end{cases}$

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$$\begin{cases} x+2z=\lambda xz \\ 2x+2y=\lambda xy \\ xyz=12 \end{cases}$$

Key: Multiply row 1 by x, row 2 by y

$$\left.\begin{array}{l} x(y+2z)=\lambda xyz \\ y(x+2z)=\lambda xyz \end{array}\right] \Rightarrow 2z(x-y)=0 \Rightarrow x=y$$

$$2(x+y)=\lambda xy \quad \Rightarrow \quad 4x=\lambda x^2 \Rightarrow x=y=\frac{4}{\lambda}$$

$$\frac{4}{\lambda}+2z=4z \Rightarrow \frac{4}{\lambda}=2z \Rightarrow z=\frac{2}{\lambda}$$

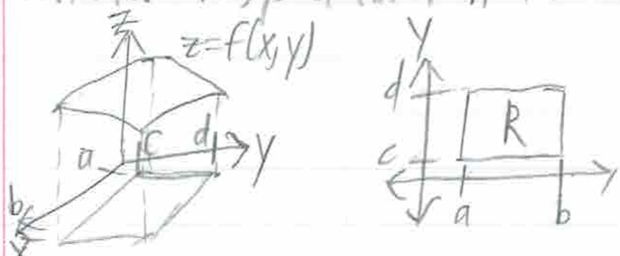
Then $V=\left(\frac{4}{\lambda}\right)\left(\frac{4}{\lambda}\right)\left(\frac{2}{\lambda}\right)=12 \Rightarrow \lambda^3=\frac{8}{3} \Rightarrow \lambda=\frac{2}{\sqrt[3]{3}}$

$\Rightarrow x=\frac{4}{2/\sqrt[3]{3}}=2\sqrt[3]{3}=y$

$\Rightarrow z=\frac{x}{2}=\sqrt[3]{3}$

Multiple Integrals:

Consider $f(x,y)$ defined on a rectangle: $R: a\le x\le b,\ c\le y\le d$



Divide R into rectangles R_k, and in each R_k choose a pt. (x_k, y_k).

$V_k \approx f(x_k,y_k)\,\text{Area}(R_k)$

We can construct a Riemann Sum

$$\sum_{k=1}^{n} V_k = \sum_{k=1}^{n} f(x_k,y_k)\,\text{Area}(R_k)$$

This is an approximation of the volume under the surface $z=f(x,y)$ and above the rectangle R. If the limit exists as $n\to\infty$ (more, smaller rectangles), we define

$$\iint_R f(x,y)\,dA=\iint_R f(x,y)\,dx\,dy.$$

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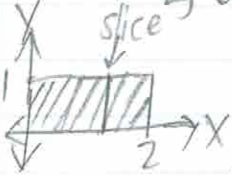
Ex.) Calculate the volume under the plane $z=4-x-y$ over the rectangle
$R: 0 \le x \le 2,\ 0 \le y \le 1$.



Consider a slice with a region perpendicular to x-axis. If the area of the slice for each fixed x is A, then the volume is
$V=\int_0^2 A(x)\,dx$.

Now, $A(x)=\int_0^1 f(x,y)\,dy$ for each fixed x.

Finally, $V=\int_0^2\left(\int_0^1 f(x,y)\,dy\right)dx$.