MATH 2605-G2 9/29/14

Say $\vec{b}=\begin{pmatrix}6\\8\end{pmatrix}$.

$\left(\begin{array}{cc|c}2 & -1 & 6\\ -1 & 2 & 8\end{array}\right) \Rightarrow \left(\begin{array}{cc|c}1 & -2 & -8\\ 0 & 3 & 22\end{array}\right) \Rightarrow \left(\begin{array}{cc|c}1 & 0 & 20/3\\ 0 & 1 & 22/3\end{array}\right)$

so $\vec{x}^*=\begin{pmatrix}20/3\\22/3\end{pmatrix}$.

Now use Jacobi:

Define $\begin{pmatrix}u_0\\v_0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$.

$\begin{cases} u_1=\frac{1}{2}v_0+\frac{1}{2}b_1=0+\frac{1}{2}(6)=3 \\ v_1=\frac{1}{2}u_0+\frac{1}{2}b_2=\frac{1}{2}(0)+\frac{1}{2}(8)=4 \end{cases}$

$\begin{cases} u_2=\frac{1}{2}v_1+\frac{1}{2}b_1=\frac{1}{2}(4)+\frac{1}{2}(6)=5 \\ v_2=\frac{1}{2}u_1+\frac{1}{2}b_2=\frac{1}{2}(3)+\frac{1}{2}(8)=\frac{11}{2} \end{cases}$

$\vdots$

Process continues until $\|\vec{x}_{n+1}-\vec{x}_n\|<\varepsilon$, a tolerance. Ideally $\varepsilon=0$.

Theorem: If A is dominant:

$|a_{ii}|>\sum_{i\neq j} a_{ij}$

and both iterative methods converge.

In general, eigenvalues of $S^{-1}T$ determine convergence.

9/30/14

Recitation

Householder:

$P(x)=(I-2\vec{v}\vec{v}^T)\cdot\vec{x}=\|\vec{x}\|\vec{e}_1$

$\vec{x}-2\vec{v}\vec{v}^T\vec{x}=\|\vec{x}\|\vec{e}_1$

$\vec{v}=\dfrac{\vec{x}-\|\vec{x}\|\vec{e}_1}{2\vec{v}^T\vec{x}}$

$A=(\vec{x}_1|\vec{x}_2|\vec{x}_3)$

$P_2P_1A=R$

$A=P_1^{-1}P_2^{-1}R=QR$

Use this for #1 and #2 in Week 6 Homework.

1) $\vec{x}=\begin{pmatrix}11\\2\end{pmatrix}$

$\vec{v}=\vec{x}-\alpha\vec{e}_1=\vec{x}+\|\vec{x}\|\vec{e}_1=\begin{pmatrix}11\\2\end{pmatrix}+5\sqrt{5}\begin{pmatrix}1\\0\end{pmatrix}$

$\vec{q}=\frac{\vec{v}}{\|\vec{v}\|}$

$M=I-2\vec{q}\vec{q}^T$

3) a) $A=\begin{pmatrix}3 & 5\\ 4 & 2\end{pmatrix}$

$Q_1A=\begin{pmatrix}\alpha & Q_1\cdot x_2\\ 0 & \end{pmatrix}$

$\begin{pmatrix}3\\4\end{pmatrix}=\alpha\begin{pmatrix}1\\0\end{pmatrix}$, $\alpha=-\|\vec{x}\|$

$\vec{v}=\vec{x}+\|\vec{x}\|\vec{e}_1$