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Then $\frac{d\vec{T}}{dt} = -\cos t\,\hat{i} - \sin t\,\hat{j}$

and $\left\|\frac{d\vec{T}}{dt}\right\| = \sqrt{\cos^2 t + \sin^2 t} = 1.$

Finally:

$K = \frac{1}{\|\vec{r}'\|}\left\|\frac{d\vec{T}}{dt}\right\| = \frac{1}{a}.$

Larger circle has higher curvature.

In the special case of $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$ (curve in the plane), K becomes

$$K = \frac{|x'y'' - y'x''|}{[(x')^2 + (y')^2]^{3/2}}.$$

Principle unit normal:

$$\vec{N} = \frac{1}{K}\frac{d\vec{T}}{ds} = \frac{1}{\|d\vec{T}/ds\|}\frac{d\vec{T}}{ds} = \frac{d\vec{T}/ds}{\|d\vec{T}/ds\|} = \frac{d\vec{T}/dt}{\|d\vec{T}/dt\|}$$

Ex.) Helix is given by $\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\,\hat{k}$. Calculate K, $\vec{N}$, and $\vec{T}$.

$\vec{r}'(t) = -a\sin t\,\hat{i} + a\cos t\,\hat{j} + b\hat{k} = \vec{v}$

$\|\vec{r}'(t)\| = \sqrt{a^2+b^2} = \|\vec{v}\|$

$\vec{T} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{a^2+b^2}}(-a\sin t\,\hat{i} + a\cos t\,\hat{j} + b\hat{k})$

$$K = \frac{1}{\|\vec{v}\|}\left\|\frac{d\vec{T}}{dt}\right\| = \frac{1}{\sqrt{a^2+b^2}}\left\|\frac{1}{\sqrt{a^2+b^2}}(-a\cos t\,\hat{i} - a\sin t\,\hat{j})\right\|$$

$$= \frac{1}{a^2+b^2}\sqrt{a^2(\cos^2 t + \sin^2 t)}$$

$$= \frac{a}{a^2+b^2}$$

$$\vec{N} = \frac{d\vec{T}/dt}{\|d\vec{T}/dt\|} = \frac{1}{a/\sqrt{a^2+b^2}}\left(\frac{1}{\sqrt{a^2+b^2}}\right)(-a\cos t\,\hat{i} - a\sin t\,\hat{j})$$

$$= \frac{1}{a}(-a\cos t\,\hat{i} - a\sin t\,\hat{j})$$

$$= -\cos t\,\hat{i} - \sin t\,\hat{j}$$

Points toward z-axis.

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Recitation

Parametrize and draw the surface.

1) $x+y=z \Rightarrow (x, y, x+y)$

2) $x^2+y^2=z^2 \Rightarrow (x, y, \sqrt{x^2+y^2})$

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3) $x^2-y^2=z^2 \Rightarrow (x, y, \sqrt{x^2-y^2})$

Also: $x^2=y^2+z^2$

double cone

4) $x^2-y^2=z \Rightarrow (x, y, x^2-y^2)$ Also: $x^2=y^2+z^2$

Hyperbolic paraboloid (hard to draw)

5) $x^2+y^2=z+1 \Rightarrow (x, y, x^2+y^2-1)$

6) $x^2+y^2=z^2+1 \Rightarrow (x, y, \sqrt{x^2+y^2-1})$

"Hourglass"

7) Find T, N, and K of $r(t)=(\cos t+t\sin t)\hat{i}+(\sin t-t\cos t)\hat{j}$.

$r'(t)=(-\sin t+t\cos t+\sin t)\hat{i}+(\cos t+t\sin t-\cos t)\hat{j}$

$\|r'(t)\|=\sqrt{t^2\cos^2 t+t^2\sin^2 t}=t$

$T=\frac{r(t)}{\|r(t)\|}=\frac{t\cos t\hat{i}+t\sin t\hat{j}}{t}=\cos t\hat{i}+\sin t\hat{j}$

$N=\frac{dT/dt}{\|dT/dt\|}=\frac{-\sin t\hat{i}+\cos t\hat{j}}{1}$

$K=\frac{1}{\|r(t)\|}\left\|\frac{dT}{dt}\right\|=\frac{1}{t}(1)=\frac{1}{t}$