MATH 2605-G2

10/27/14

(0,0) is the only critical point.

$\frac{\partial^2 f}{\partial x^2} = y^2 e^{xy}$ $\quad$ $\frac{\partial^2 f}{\partial x \partial y} = yxe^{xy} + e^{xy}$ $\quad$ $\frac{\partial^2 f}{\partial y^2} = x^2 e^{xy}$

$A = 0$ $\quad$ $B = 0 + 1 = 1$ $\quad$ $C = 0$

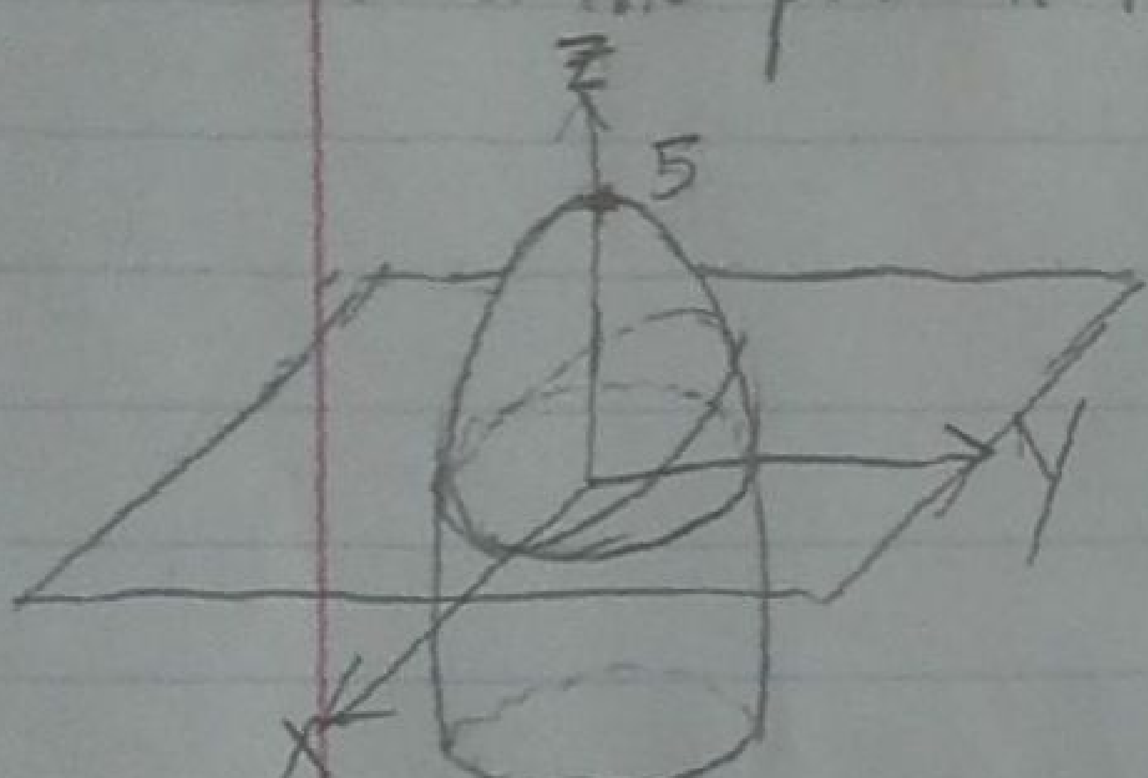
$D = AC - B^2 = -1$

Since $D < 0$, $(0,0)$ is a saddle point.

Absolute max and min.:

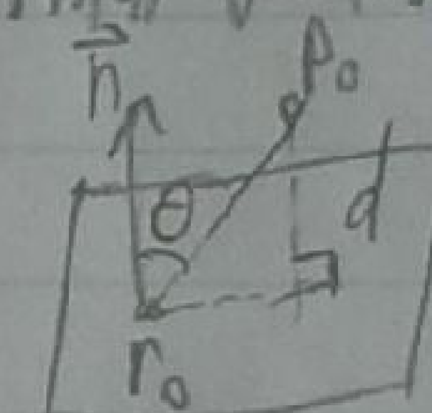
If f is continuous on a closed, bounded set D, then f takes on an absolute maximum and an absolute minimum in D.

Ex.) Find the point (x_0, y_0, z_0) in the surface $z = 5 - x^2 - y^2$ that is above and farthest from the plane $x + 4y + 8z = 0$



Review: the distance between a point P_0 and a plane with a point r_0 and a unit normal vector $\vec{n}$ is

$$d = \|P_0 - r_0\| \cos\theta = \|P_0 - r_0\| \|\vec{n}\| \cos\theta = (P_0 - r_0) \cdot \vec{n}$$



The vector $(1, 4, 8)^T$ is normal to the plane.

$$\vec{n} = \frac{(1,4,8)}{\sqrt{1^2+4^2+8^2}} = \frac{1}{9}(1,4,8)$$

$r_0 = (0,0,0)$ is on the plane.

$$d = (x_0, y_0, z_0) \cdot \frac{1}{9}(1,4,8)$$
$$= \frac{1}{9}(x_0 + 4y_0 + 8(5 - x_0^2 - y_0^2))$$

$\frac{\partial d}{\partial x_0} = \frac{1}{9} - \frac{16x_0}{9} = 0 \Rightarrow x_0 = \frac{1}{16}$

$\frac{\partial d}{\partial y_0} = \frac{4}{9} - \frac{16y_0}{9} = 0 \Rightarrow y_0 = \frac{1}{4}$

$z_0 = 5 - (\frac{1}{16})^2 - (\frac{1}{4})^2$

Therefore, the point is $(\frac{1}{16}, \frac{1}{4}, 5 - \frac{1}{16^2} - \frac{1}{4^2})$.

Recitation

10/28/14

Draw the surface. Find the tangent plane and parametrize the normal vector at (1,1,1). Then, taking $z = f(x,y)$, find ∇f and draw the level set of ∇f. Find the local extrema.

1) $x^2 + y^2 - z^2 = 1$