MATH 2605-G2

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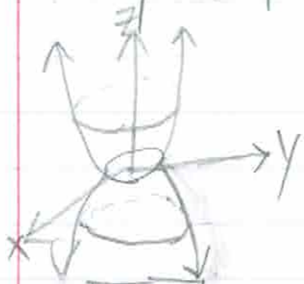

$\Rightarrow x^2+y^2=z^2+1$

$g=x^2+y^2-z^2-1$

$\nabla g=(2x, 2y, -2z)$

Tangent plane: $\nabla g(1,1,1)\cdot((x,y,z)-(1,1,1))=0$

$(2,2,-2)\cdot(x-1, y-1, z-1)=0$

$2x+2y-2z=2$

Normal vector: $x=1+2t \quad y=1+2t \quad z=1-2t$

or $(x,y,z)=(1,1,1)+t(2,2,-2)$

$z=\sqrt{x^2+y^2-1}$

$\nabla f=\nabla z=\left(\frac{x}{\sqrt{x^2+y^2-1}}, \frac{y}{\sqrt{x^2+y^2-1}}\right)$

$\frac{\partial f}{\partial x}=0$ at $x=0 \quad \frac{\partial f}{\partial y}=0$ at $y=0$

$z=\sqrt{0+0-1}$ is not a real number.

Therefore, the surface has no points of local extrema.

Find the linearization of f at the specified point.

1) $f(x,y)=e^{2y-x}$ at $(0,0)$

$\frac{\partial f}{\partial x}=-e^{2y-x} \quad \frac{\partial f}{\partial y}=2e^{2y-x}$

$\frac{\partial f}{\partial x}(0,0)=-1 \quad \frac{\partial f}{\partial y}(0,0)=2 \quad f(0,0)=1$

$f(x,y)\approx f(0,0)+\frac{\partial f}{\partial x}(0,0)(x-0)+\frac{\partial f}{\partial y}(0,0)(y-0)=1-x+2y$

<u>2nd Midterm is on Thursday, October 30.</u>

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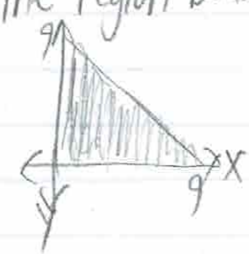
Ex.) Find the absolute extrema of $f(x,y)=2+2x+2y-x^2-y^2$ in the region bounded by $x=0$, $y=0$, and $x+y=9$.



Look for critical points in the interior:

$\frac{\partial f}{\partial x}=2-2x=0$

$\frac{\partial f}{\partial y}=2-2y=0$

$\Rightarrow (x,y)=(1,1)$ is the only critical point.

$f(1,1)=4$

Look for critical points on the boundary:

a) $g(x)=f(x,0),\ 0\le x\le 9$

$g(x)=2+2x-x^2 \Rightarrow g'(x)=2-2x=0 \Rightarrow x=1$

$g(1)=3 \quad \underbrace{g(0)=2 \quad g(9)=-61}_{\text{at boundary intersections}}$

b) $y=9-x \Rightarrow h(x)=f(x,9-x)=2+2x+2(9-x)-x^2-(9-x)^2,\ 0\le x\le 9$

$\Rightarrow h'(x)=2-2-2x+2(-x+9)=-4x+18=0 \Rightarrow x=\frac{9}{2}$

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$h(\frac{9}{2})=\frac{-41}{2}$ $h(0)=-61$

c) $k(y)=f(0,y)$, $0\le y\le 9$

Using symmetry, $k(1)=3$, $k(0)=2$, and $k(9)=-61$.

Absolute minima: -61 at $(9,0)$ and $(0,9)$

Absolute maximum: 4 at $(1,1)$

Review for test:

Ex) Find the max. of $d=2x+2y+5z$ in the surface $z=9-x^2-y^2$.

First step is to substitute for z: $d(x,y)=2x+2y+5(9-x^2-y^2)$

Ex) Draw the surface $9x^2=3y^2-4z^2+1$.

$x=0 \Rightarrow 3y^2-4z^2=-1$ hyperbola

$y=0 \Rightarrow 9x^2+4z^2=1$ ellipse

$z=0 \Rightarrow 9x^2-3y^2=1$ hyperbola