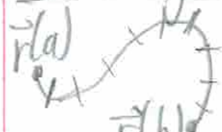
MATH 2605 - G2 10/06/14

Arc length:



Intuition: measure each segment as a straight line segment. Make the segments infinitely small.

$\vec{r}(t) = f(t)\hat{i} + g(t)\hat{j} + h(t)\hat{k}$

$L_k \approx \|\vec{r}(t_k) - \vec{r}(t_{k-1})\|$

$\approx \sqrt{[f(t_k)-f(t_{k-1})]^2 + [g(t_k)-g(t_{k-1})]^2 + [h(t_k)-h(t_{k-1})]^2}$

The curve is continuous, satisfying the mean value theorem. There exist $t_k^*, t_k^{**}, t_k^{***}$ such that

$L_k \approx \sqrt{[f'(t_k^*)(t_k - t_{k-1})]^2 + [g'(t_k^{**})(t_k - t_{k-1})]^2 + [h'(t_k^{***})(t_k - t_{k-1})]^2}$

$\approx \sqrt{[f'(t_k^*)]^2 + [g'(t_k^{**})]^2 + [h'(t_k^{***})]^2}\,(t_k - t_{k-1})$

Taking the limit of the sum of all segments $L = \sum L_k$ as $\Delta t \to 0$:

$L = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}\,dt$

$= \int_a^b \|\vec{r}'(t)\|\,dt$

Ex.) $\vec{r}(t) = (2+t)\hat{i} - (t+1)\hat{j} + t\hat{k},\ t \in [0,3]$

$L = \int_0^3 \|(1,-1,1)\|\,dt = \int_0^3 \sqrt{3}\,dt = 3\sqrt{3}$

Ex.) $\vec{r}(t) = 2\cos t\,\hat{i} + 2\sin t\,\hat{j} + t^2\hat{k},\ t \in [0, \frac{\pi}{2}]$

$\Rightarrow \vec{r}'(t) = -2\sin t\,\hat{i} + 2\cos t\,\hat{j} + 2t\hat{k}$

$\Rightarrow \|r'(t)\| = \sqrt{4\sin^2 t + 4\cos^2 t + 4t^2} = 2\sqrt{1+t^2}$

$\Rightarrow L = \int_0^{\pi/2} 2\sqrt{1+t^2}\,dt$

$= \left[\sqrt{1+t^2}\,t + \ln\left|\sqrt{1+t^2}+t\right|\right]_0^{\pi/2}$

$= \sqrt{1+\frac{\pi^2}{4}}\left(\frac{\pi}{2}\right) + \ln\left|\sqrt{1+\frac{\pi^2}{4}} + \frac{\pi}{2}\right|$

Recitation 10/07/14

1) $\vec{r}(t) = \cos 2t\,\hat{i} + 3\sin 2t\,\hat{j}$

$\vec{v}(t) = -2\sin 2t\,\hat{i} + 6\cos 2t\,\hat{j}$

*Use $v(t)$ to find direction of parametrization.

2) $\vec{r}(t) = te^t\hat{i} + e^t\hat{j} + \hat{k}$

$\int_0^1 \vec{r}(t) = \left(\int_0^1 te^t\,dt, \int_0^1 e^t\,dt, \int_0^1 dt\right)$

$u(t) = t, \quad dv(t) = e^t\,dt$

$du(t) = dt, \quad v(t) = \int dv(t) = e^t + C$

$\int u(t)\,dv(t) = te^t - \int e^t\,dt = te^t - e^t + C$

$\int_0^1 \vec{r}(t) = \left(te^t - e^t\Big|_0^1, e^t\Big|_0^1, t\Big|_0^1\right) = (1, e-1, 1)$

3) $\vec{v}(t) = \frac{te^{t^2}\sin^2 t}{\sqrt{1+t^{2014}}}\hat{i} + \cos t\, e^t\hat{j}$

$\int_{-1}^1 \vec{v}(t)\,dt = \left(0, \int_{-1}^1 \cos t\, e^t\,dt\right)$

why? Odd function

u	dv
$\cos t$	e^t
$-\sin t$	e^t
$-\cos t$	e^t

$\int \cos t\, e^t\,dt = \cos t\, e^t - [(-\sin t\, e^t) - \int (-\cos t) e^t\,dt]$

$2\int \cos t\, e^t\,dt = e^t \cos t + e^t \sin t$

$\int \cos t\, e^t\,dt = \frac{e^t(\cos t + \sin t)}{2}$

$\int_{-1}^1 \vec{v}(t)\,dt = \left(0, \frac{e(\cos 1 + \sin 1)}{2} - \frac{e^{-1}(\cos(-1) + \sin(-1))}{2}\right)$