MATH 2605-G2

11/25/14

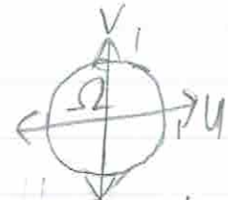
Recitation

1) Find the area enclosed by $x^2+xy+y^2=1$.

$\left(x^2+xy+\left(\frac{y}{2}\right)^2\right)+\frac{3}{4}y^2=1$

$\left(x+\frac{y}{2}\right)^2+\left(\frac{\sqrt{3}y}{2}\right)^2=1$

$u=x+\frac{y}{2}$ $\quad v=\frac{\sqrt{3}y}{2}$ $\quad \Rightarrow u^2+v^2=1$



$x=u-\frac{v}{\sqrt{3}} \leftarrow y=\frac{2v}{\sqrt{3}}$

area of circle

$A=\iint_{\Omega}\left|\begin{matrix}\frac{dx}{du} & \frac{dx}{dv}\\ \frac{dy}{du} & \frac{dy}{dv}\end{matrix}\right|du\,dv=\iint_{\Omega}\left|\begin{matrix}1 & -\frac{1}{\sqrt{3}}\\ 0 & \frac{2}{\sqrt{3}}\end{matrix}\right|du\,dv=\frac{2}{\sqrt{3}}\iint_{\Omega}du\,dv=\frac{2\pi}{\sqrt{3}}$

2) Find the volume of $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{6}=1$.

volume of sphere

$u=\frac{x}{2},\ v=\frac{y}{\sqrt{5}},\ w=\frac{z}{\sqrt{6}} \Rightarrow u^2+v^2+w^2=1$

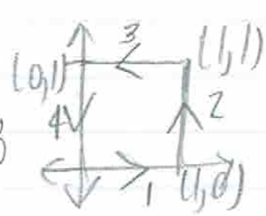
$V=\iiint_{\Omega}\left|\begin{matrix}2 & & \\ & \sqrt{5} & \\ & & \sqrt{6}\end{matrix}\right|du\,dv\,dw=2\sqrt{30}\iiint_{\Omega}du\,dv\,dw=2\sqrt{30}\left(\frac{4\pi}{3}\right)=\frac{8\sqrt{30}\pi}{3}$

3) Find the line integral with respect to arc length of $\int_C (x+y)\,ds$, where C is the line segment from $(0,1)$ to $(1,0)$.

$(x,y)=(t,1-t)$ for $t\in[0,1]$

$\int_0^1 (t+(1-t))\cdot\|\vec{r}'(t)\|\,dt=\int_0^1\sqrt{2}\,dt=\sqrt{2}$

4) $\int_C \frac{ds}{x^2+y^2+1}$, C:



Line 1: $r(t)=(t,0),\ 0\le t\le 1 \Rightarrow x=t,\ y=0$

$\int_0^1 \frac{1}{t^2+1}\cdot\|r'(t)\|\,dt$

$=\tan^{-1}t\big|_0^1$

$=\frac{\pi}{4}$

Add up line integrals from lines 2, 3 and 4 as well.