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RESEARCH ARTICLE

# PID control of robot manipulators actuated by BLDC motors

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**ABSTRACT**
Few control schemes exist for brushless direct-current (BLDC) motors provided with formal global stability proofs. Moreover, most of these works are constrained to control velocity in single motors actuating on simple linear mechanical loads. Inspired by a recent proposal in the literature, we present a control scheme for direct drive BLDC motors, which is provided with a global stability proof, when actuating on n-degrees of freedom rigid robots. Thus, we solve the position control problem in n direct drive BLDC when actuating on a complex, nonlinear and highly coupled mechanical load. Another important feature of our proposal is its simple control law when compared to control schemes previously presented in the literature.

**KEYWORDS**
Brushless DC motor control; Robot manipulator control; PID position control; Lyapunov stability; Ultimate bound.



## 1. Introduction

Brushless direct-current (BLDC) motors are three-phase synchronous motors with a permanent magnet fixed at the rotor which does not present any saliency. The back electromotive force in BLDC motors has a trapezoidal shape because of uniformly distributed phase windings on the stator (Chiasson, 2005; Krishnan, 2001). This is the main difference with respect to the so called permanent magnet synchronous motors (PMSM) whose back electromotive force is sinusoidal because of sinusoidally distributed phase windings on the stator.

BLDC motors are an attractive alternative to induction motors (IM) because the employment of rare earth permanent magnets improves the motor performance and increases the power density. Moreover, BLDC motors can be controlled at a reduced cost with respect to IM's (Guerrero et al., 2017). On the other hand, BLDC motors have not any brushes reducing the operation cost with respect to use of standard brushed permanent magnet DC motors and its construction is simpler and cheaper than that of PMSM's (Gieras, 2010).

The standard control scheme for BLDC motors, which is conceived under steady

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state motor operation conditions (see Chiasson (2005)), consists in applying constant electric currents at each phase. These electric currents can be zero or not depending on the rotor position. Hall effect sensors have been traditionally used to determine when to switch-on or to switch-off the electric currents at each phase. These step commands for the desired electric currents allow to use inner linear PI electric current loops. If these PI controllers are well tuned, then it is possible to assume that the required torque (designed as the output of either standard PI velocity controllers or standard PID position controllers) is generated and that the control objective is accomplished (Chiasson, 2005; Krishnan, 2001).

Although successful in practice, the above described control scheme is based only on intuitive ideas and any formal stability study has not been presented to explain why it works well and, in particular, how to select the gains of the PI electric current controllers. Several recent control schemes have been proposed in the literature (Hajiaghasi et al., 2017; Salehifar & Moreno-Equilaz, 2016; Xia et al., 2015) which, however, are not provided with formal stability proofs.

A remarkable exception to the above described trend is the work by Guerrero et al. (2017). In that nice paper, a passivity-based control scheme is proposed to control velocity in BLDC motors which is provided with a global asymptotic stability proof. However, some important details have been forgotten. *i*) The time derivatives of the desired electric currents that are proposed are assumed to exist. However, because of the trapezoidal shape of torque generated by each phase, the time derivatives of the desired electric currents that are proposed by Guerrero et al. (2017) are not defined at some specific rotor positions. *ii*) The control scheme proposed by Guerrero et al. (2017) requires to feedback the complete expression for the time derivative of the desired electric currents. This is an important drawback because the number of required computations is very large which commonly results in performance deterioration because of numerical erros and noise amplification, as remarked by Ortega et al. (1998). Moreover, it is pointed out by Petrovic (2001) that the electric drives community is not enthusiastic with complex controllers. Thus, it is important to design controllers that are simple to implement but provided with formal stability proofs resulting in stability conditions useful to understand how the controller works. *iii*) Aside form point *i*), the expressions presented by Guerrero et al. (2017) for the time derivatives of the desired currents are incomplete, i.e. the required computations are even more complex. *iv*) The viscous friction coefficient and the load torque are assumed to be known by Guerrero et al. (2017). These are, perhaps, consequences of the facts that velocity measurements are not allowed and that a proportional velocity controller is considered instead of a more robust PI velocity controller. *v*) Velocity control of a single motor is considered, i.e. the position control problem is not solved.

Motivated by Guerrero et al. (2017), in the present paper we introduce a controller for position regulation in $n-$degrees-of-freedom rigid robots actuated by $n$ direct-drive BLDC motors. We present a formal stability proof which ensures global boundedness of the state and global convergence to a ball which can be rendered arbitrarily small. Our main contribution with respect to Guerrero et al. (2017) is described in the following items. 1) The desired electric currents that we propose are continuously differentiable for all rotor positions. 2) We dominate the time derivatives of the desired electric currents, i.e. we not feedback the time derivatives of the desired electric currents. This results in a much simpler control law. 3) The time derivatives of the desired currents that we present are complete. 4) We do not require the knowledge of the viscous friction coefficients. We must stress at this point, however, that the work in Guerrero et al. (2017) has the advantage of not requiring velocity measurements. 5)

We do not require either to know the load torque due to gravity. This is because we employ PID position controllers in the external loop. 6) Position control of a complex nonlinear and highly coupled mechanical load is accomplished, i.e. position control of a $n-$degrees-of-freedom rigid robot.

We consider that it is important to remark the difference between our proposal and that by Hernández-Guzmán et al. (2009). The latter paper is concerned with robots actuated by PMSMs instead of BLDC motors. A confusion in the nomenclature to designate these different classes of motors is present in Hernández-Guzmán et al. (2009). This confusion, however, is also present in the book by Dawson et al. (1998) and this seems to be the origin of such a confusion. Another difference between these works is made clear in remark 2.

The present paper is organized as follows. In section 2 we present the dynamic model that we consider as well as some useful mathematical tools. Our main result is presented in section 3. A numerical example is presented in section 4 and some conclusions are given in section 5.

Finally, some remarks on notation. Given a $x \in \mathcal{R}^n$, the 1-norm of $x$ is defined as $\|x\|_1 = \sum_{i=1}^{n} |x_i|$, where the symbol $|\cdot|$ stands for the absolute value of an scalar and the Euclidean norm is defined as $\|x\| = \sqrt{\sum_{i=1}^{n} x_i^2}$. A property that relates these norms is $\|x\| \leq \|x\|_1 \leq \sqrt{n}\|x\|$ (Khalil, 2002). Given a bounded matrix $A$ or a bounded vector $x$, the symbols $\|A\|_M$ and $\|x\|_M$ stand for the supreme values over the norm. The symbols $\lambda_m(B)$, $\lambda_M(B)$ stand for the smallest and the largest eigenvalues of the symmetric matrix $B$.

## 2. Dynamic model

BLDC motors have a star connected three-phase winding at the stator and a permanent magnet at the rotor. For the sake of simplicity and without any loss of generality we consider $n$ BLCD motors provided with one pole pair. According to Kirchhoff's and Faraday's laws, the electrical dynamics of the stator of the $i-$th motor is given as:

$$\dot{\psi}_i + R_i I_i = U_i, \ i = 1, \ldots, n, \tag{1}$$

where $I_i = [I_{i1}, I_{i2}, I_{i3}]^T$ and $U = [U_{i1}, U_{i2}, U_{i3}]^T$ are the three-phase electric currents through the stator and the three phase applied voltages, $R_i$ is a positive scalar standing for the phase windings resistance, $\psi_i \in \mathcal{R}^3$ is the flux linkages at each stator phase which is given as (Chiasson, 2005):

$$\psi_i = L_i I_i + \Gamma_i(q_i), \quad L_i = \begin{bmatrix} L_{si} & -M_i & -M_i \\ -M_i & L_{si} & -M_i \\ -M_i & -M_i & L_{si} \end{bmatrix}, \tag{2}$$

where $L_{si}$ and $M_i$ are the self-inductance and the mutual-inductance of the phases and they relate through $L_{si} = (7/3)M_i$. Using this relatioship it is easy to verify that $L_i$ is a positive definite matrix. Notice that the inductance matrix $L_i$ is constant because the rotor has not any saliency (Guerrero et al., 2017). $q_i$ is the rotor position and $\Gamma_i(q_i)$ is the, rotor position dependent, flux linkage due to the permanent magnet at the rotor. Replacing (2) in (1) we have:

$$L_i \dot{I}_i + \frac{d\Gamma_i(q_i)}{dq_i}\dot{q}_i + R_i I_i = U_i, \tag{3}$$

where the back electromotive force is given as (Chiasson, 2005; Guerrero et al., 2017):

$$\frac{d\Gamma_i(q_i)}{dq_i}\dot{q}_i = -E_{pi}E_{Ri}(q_i)\dot{q}_i, \tag{4}$$

where $E_{pi}$ is a constant known as the counter electromotive force constant and the vectorial function $E_{Ri}(q_i)$ is given as:

$$E_{Ri}(q_i) = \left[E_i(q_i), E_i\left(q_i - \frac{2\pi}{3}\right), E_i\left(q_i + \frac{2\pi}{3}\right)\right]^T, \tag{5}$$

where (see fig. 1):

$$E_i(q_i) = \begin{cases} \frac{6q_i}{\pi}, & -\frac{\pi}{6} \le q_i < \frac{\pi}{6} \\ 1, & \frac{\pi}{6} \le q_i < \frac{5\pi}{6} \\ -\frac{6(q_i-\pi)}{\pi}, & \frac{5\pi}{6} \le q_i < \frac{7\pi}{6} \\ -1, & \frac{7\pi}{6} \le q_i < \frac{11\pi}{6} \end{cases}. \tag{6}$$

The fact that each component of the vector $-E_{pi}E_{Ri}(q_i)\dot{q}_i$ has a trapezoidal shape when the motor velocity is constant, is the reason why this motor is also called *trapezoidal back electromotive force synchronous motor* (Chiasson, 2005).

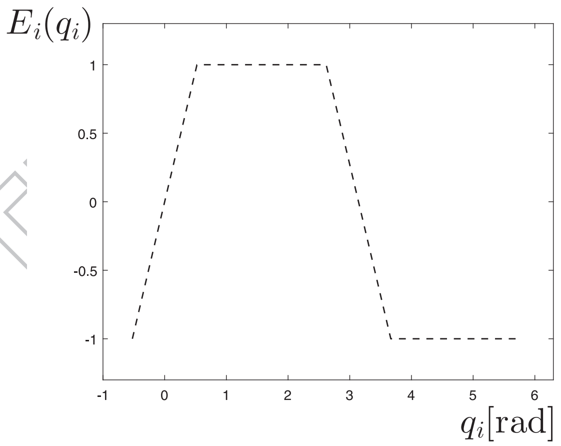



**Figure 1.** Plot of the function $E_i(q_i)$

On the other hand, according to D'Alembert's principle (Dawson et al., 1998), torque produced by the three phases together is given as:

$$\begin{aligned} \tau_i &= \frac{\partial}{\partial q_i}\left(\frac{1}{2}I_i^T L_i I_i + \Gamma^T(q_i) I_i\right), \\ &= \left(\frac{\partial \Gamma(q_i)}{\partial q_i}\right)^T I_i, \\ &= -\tau_{pi} E_{Ri}^T(q_i) I_i, \end{aligned} \tag{7}$$

according to (4), where $\tau_{pi} = E_{pi}$ is the torque constant and it is usual to use different symbols for these constants.

Using the above expressions, the dynamical model of an $n-$degrees of freedom rigid robot equipped with a direct driven BLDC motor at each joint is given as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau, \tag{8}$$

$$L_i \dot{I}_i = E_{pi} E_{Ri}(q_i)\dot{q}_i - R_i I_i + U_i, \tag{9}$$

$$\tau_i = -\tau_{pi} E_{Ri}^T(q_i) I_i, \tag{10}$$

where $i = 1, \ldots, n$, $q = [q_1, \ldots, q_n]^T$ is the joint positions vector, the $n \times n$ inertia matrix $M(q)$ is symmetric and positive definite, $C(q,\dot{q})\dot{q}$ is the Coriolis and centripetal effects term whereas $g(q)$ is the gravity effects term which is given as $g(q) = \frac{\partial \mathcal{U}(q)}{\partial q}$ where $\mathcal{U}(q)$ is the potential energy due to gravity, and, finally, $\tau = [\tau_1, \ldots, \tau_n]^T$ is the applied torques vector. The mechanical models of the $n$ direct drive BLDC motors are assumed to be included in the mechanical model of the robot. On the other hand, as it is by now well known, the following are some important properties of the mechanical part of the robot when all joints are revolute.

*Property 1.* (Kelly, 1995; Kelly et al., 2005). Matrices $M(q)$ and $C(q,\dot{q})$ satisfy $0 < \lambda_m(M(q))$, $\forall q \in \mathcal{R}^n$, and:

$$\begin{aligned} \dot{q}^T \left(\frac{1}{2}\dot{M}(q) - C(q,\dot{q})\right)\dot{q} &= 0, \quad \forall q, \dot{q} \in \mathcal{R}^n, \end{aligned} \tag{11}$$

$$\dot{M}(q) = C(q,\dot{q}) + C^T(q,\dot{q}), \qquad \forall q, \dot{q} \in \mathcal{R}^n,$$

where last expression only stands if $C(q,\dot{q})$ is defined using Christoffel symbols.

*Property 2.* (Kelly et al., 2005). There exists a positive constant $K'$ such that for all $q \in \mathcal{R}^n$, we have that $\|g(q)\| \le K'$. This means that every element of the gravity effects vector, i.e. $g_i(q)$, $i = 1, \ldots, n$, satisfies $|g_i(q)| \le K_i'$, $\forall q \in \mathcal{R}^n$, for some positive constants $K_i'$, $i = 1, \ldots, n$. Furthermore, there exists a positive constant $k_g$ such that:

$$\left\|\frac{\partial g(q)}{\partial q}\right\| \le k_g.$$

*P*roperty 3. (Kelly, 1995; Kelly et al., 2005; Tomei, 1991). There exists a positive constant $k_c$ such that for all $w, y, z \in R^n$, we have:

$$\|C(w,y)z\| \le k_c \|y\| \|z\| \tag{12}$$

Important for our purposes is the following class of saturation functions.

**Definition 2.1.** Given positive constants $L^*$ and $M^*$, with $L^* < M^*$, a function $\sigma : \mathcal{R} \to \mathcal{R} : \varsigma \mapsto \sigma(\varsigma)$ is said to be a strictly increasing linear saturation for $(L^*, M^*)$ if it is locally Lipschitz, strictly increasing, and satisfies (Zavala-Río & Santibanez, 2007):

$$\begin{aligned} \sigma(\varsigma) &= \varsigma, \text{when } |\varsigma| \leq L^*, \\ |\sigma(\varsigma)| &< M^*, \ \forall \varsigma \in \mathcal{R} \end{aligned}$$

Instrumental for the stability proof that we introduce in this paper is the following theorem.

**Theorem 2.2.** *(Theorem 4.18 in Khalil (2002), pp. 172). Consider the system:*

$$\dot{x} = f(t, x), \tag{13}$$

*where $f : [0, \infty) \times D \to \mathcal{R}^n$ is piecewise continuous in $t$ and locally Lipschitz in $x$ on $[0, \infty) \times D$, and $D \subset \mathcal{R}^n$ is a domain that contains the origin. Let $V : [0, \infty) \times D \to \mathcal{R}$ be a continuously differentiable function such that:*

$$\alpha_1(\|x\|) \leq V(t, x) \leq \alpha_2(\|x\|) \tag{14}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leq -W_3(x), \quad \forall \|x\| \geq \mu > 0 \tag{15}$$

*$\forall t \geq 0$ and $\forall x \in D$, where $\alpha_1$ and $\alpha_2$ are class $\mathcal{K}$ functions and $W_3(x)$ is a continuous positive definite function. Take $r > 0$ such that $B_r \subset D$ and suppose that:*

$$\mu < \alpha_2^{-1}(\alpha_1(r)) \tag{16}$$

*Then, there exists a class $\mathcal{KL}$ function $\beta$ and for every initial state $x(t_0)$, satisfying $\|x(t_0)\| \leq \alpha_2^{-1}(\alpha_1(r))$, there is $T \geq 0$ (dependent on $x(t_0)$ and $\mu$) such that the solution of (13) satisfies:*

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0), \quad \forall t_0 \leq t < t_0 + T, \tag{17}$$

$$\|x(t)\| \leq \alpha_1^{-1}(\alpha_2(\mu)), \quad \forall t \geq t_0 + T \tag{18}$$

*Moreover, if $D = \mathcal{R}^n$ and $\alpha_1$ belongs to class $\mathcal{K}_\infty$, then (17) and (18) hold for any initial state $x(t_0)$, with no restriction on how large $\mu$ is*

## 3. Main result

**Proposition 3.1.** *Consider the dynamic model (8)-(10) together with the following controller:*

$$U_i = R_i I_i^* - \alpha_{pi}\xi_i - K_q\|\dot{q}\|^2\xi_i - K_d\sqrt{\sum_{i=1}^{n}\|\xi_i\|^2}\ \xi_i, \tag{19}$$

$$I_i^* = \frac{\tau_i^*}{E'_{pi}\|E'_{Ri}(q_i)\|^2}E'_{Ri}(q_i), \tag{20}$$

$$\tau^* = K_P h(\tilde{q}) + K_D\dot{q} + K_I sat(z), \tag{21}$$

$$z = \int_0^t \left(\varepsilon\left(I_{n\times n} + \gamma K_P K_I^{-1}\right)h(\tilde{q}) + \left(I_{n\times n} + \varepsilon\gamma K_D K_I^{-1}\right)\dot{q}\right)dt, \tag{22}$$

*where, $i = 1,\ldots,n$, $I_i^* = [I_{i1}^*, I_{i2}^*, I_{i3}^*]^T$, $\tilde{q} = q - q^*$, with $q^* \in \mathcal{R}^n$ the vector of constant desired joint positions, $\xi_i = I_i - I_i^*$, $\tau^* = [\tau_1^*,\ldots,\tau_n^*]^T$ represents the vector of desired torques, and $I_{n\times n}$ is the $n\times n$ identity matrix. $E'_{pi} > 0$ is an estimate of $E_{pi} > 0$ whereas $E'_{Ri}(q_i)$ is an approximate of the function $E_{Ri}(q_i)$, introduced in (5), which is defined as:*

$$E'_{Ri}(q_i) = \left[E'_i(q_i), E'_i\left(q_i - \frac{2\pi}{3}\right), E'_i\left(q_i + \frac{2\pi}{3}\right)\right]^T, \tag{23}$$

*where:*

$$E'_i(q_i) = \begin{cases} -1 + r - \sqrt{r^2 - (-\frac{\pi}{6} - a - q_i)^2}, & -\frac{\pi}{6} \le q_i < -\frac{\pi}{6} - a + b \\ \frac{6q_i}{\pi}, & -\frac{\pi}{6} - a + b \le q_i < \frac{\pi}{6} + a - b \\ 1 - r + \sqrt{r^2 - (\frac{\pi}{6} + a - q_i)^2}, & \frac{\pi}{6} + a - b \le q_i < \frac{\pi}{6} + a \\ 1, & \frac{\pi}{6} + a \le q_i < \frac{5\pi}{6} - a \\ 1 - r + \sqrt{r^2 - (\frac{5\pi}{6} - a - q_i)^2}, & \frac{5\pi}{6} - a \le q_i < \frac{5\pi}{6} - a + b \\ -\frac{6(q_i - \pi)}{\pi}, & \frac{5\pi}{6} - a + b \le q_i < \frac{7\pi}{6} + a - b \\ -1 + r - \sqrt{r^2 - (\frac{7\pi}{6} + a - q_i)^2}, & \frac{7\pi}{6} + a - b \le q_i < \frac{7\pi}{6} + a \\ -1, & \frac{7\pi}{6} + a \le q_i < \frac{11\pi}{6} - a \\ -1 + r - \sqrt{r^2 - (\frac{11\pi}{6} - a - q_i)^2}, & \frac{11\pi}{6} - a \le q_i < \frac{11\pi}{6} \end{cases}, \tag{24}$$

$$\delta = \arctan\left(\frac{\pi}{6}\right), \quad b = r\cos(\delta), \quad \sigma = \frac{\pi}{2} - \delta, \quad a = r\tan\left(\frac{\sigma}{2}\right).$$

*We define the vectorial functions $h(\tilde{q}) = [s(\tilde{q}_1),\ldots,s(\tilde{q}_n)]^T$, and $sat(z) = [s(z_1),\ldots,s(z_n)]^T$, where $s(\cdot)$ is a strictly increasing linear saturation function for some $M^* > L^*$ (see definition 2.1). Furthermore, it is also required that function $s(\cdot)$ be continuously differentiable such that:*

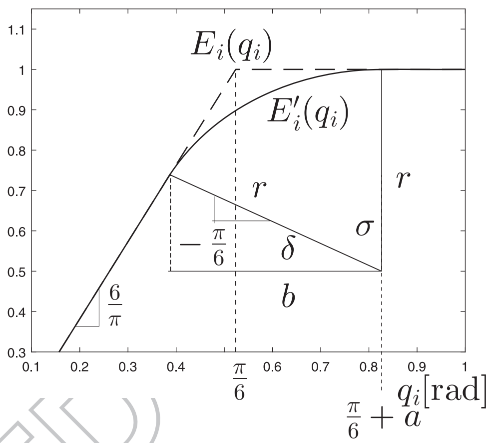
$$0 < \frac{ds(x)}{dx} \le 1,\ \forall x \in \mathcal{R}. \tag{25}$$

*There always exist diagonal positive definite $n\times n$ matrices $K_P$, $K_D$, $K_I$, and positive scalars $\varepsilon$, $\gamma$, $K_q$, $K_d$, $\alpha_{pi}$, $i = 1,\ldots,n$, and $r$, such that the whole state (which* *includes the position error $\tilde{q}$) remains bounded and it has an ultimate bound which can be rendered arbitrarily small by using a suitable choice of controller gains. This result stands when starting from any initial condition.*

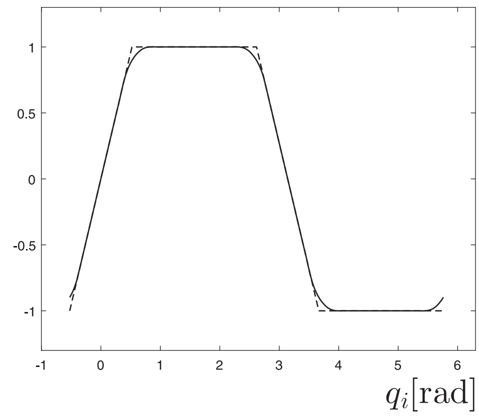
**Remark 1.** Notice that $E'_i(q_i)$, defined in (24), is exactly $E_i(q_i)$, defined in (6), excepting the use of some arcs of circles with radius $r$ to smooth $E_i(q_i)$ at the points where $E_i(q_i)$ is not differentiable. Thus, $E'_i(q_i)$ is continuously differentiable. This idea is illustrated in fig. 2, in particular fig. 2(a), at the point $q_i = \frac{\pi}{6}$. The functions $E'_i(q_i)$ and $E_i(q_i)$ are compared in fig. 2(b). An important observation for our purposes is that:

$$\sup |E_i(q_i) - E'_i(q_i)| \to 0, \text{ as } r \to 0. \tag{26}$$

Finally, an important observation by Guerrero et al. (2017) is that $\|E_{Ri}(q_i)\|^2$, although not constant, satisfies $2 \leq \|E_{Ri}(q_i)\|^2 \leq 3$, i.e. $\|E_{Ri}(q_i)\|^2 > 0$ is different from zero. According to (26), $\|E'_{Ri}(q_i)\|^2$ also satisfies all of these properties, which is easy to verify numerically.




(a) The function $E'_i(q_i)$ is obtained by smoothing the function $E_i(q_i)$.




(b) Comparison of functions $E_i(q_i)$ and $E'_i(q_i)$.

**Figure 2.** Definition of the function $E'_i(q_i)$ and comparison with the function $E_i(q_i)$. Continuous: $E'_i(q_i)$. Dashed: $E_i(q_i)$.

**Remark 2.** In fig. 3 we depict the control scheme in proposition 3.1. Notice that it is composed by a nonlinear proportional-integral-derivative (NPID) external loop intended to regulate position. An inner electric current loop is driven by a proportional controller intended to dominate some cross terms existing between the mechanical and the electrical subsystems. The term $R_i I_i^*$ is included to compensate for the presence of the windings electric resistances. Finally, the terms $-K_q\|\dot{q}\|^2\xi_i - K_d\sqrt{\sum_{i=1}^n \|\xi_i\|^2}\ \xi_i$ are included to dominate some terms involved in $\dot{I}_i^*$, i.e. for stability proof purposes.

Thus, it is concluded that the proposed control scheme is simple. One important factor contributing to this feature is that we are not required to feedback the expression for $\dot{I}_i^*$, which includes a large number of computations, but we just dominate such terms. Furthermore, notice that this is accomplished despite that velocity measurements are required by the PID position controller. We stress that this feature is instrumental to successfully complete the stability proof using a PID controller without relying on the viscous friction present in the mechanical subsystem of the robot. This is the drawback when trying to avoid velocity measurements in PID control of robot manipulators, see Hernández-Guzmán et al. (2009); Su et al. (2010, 2015), for instance.

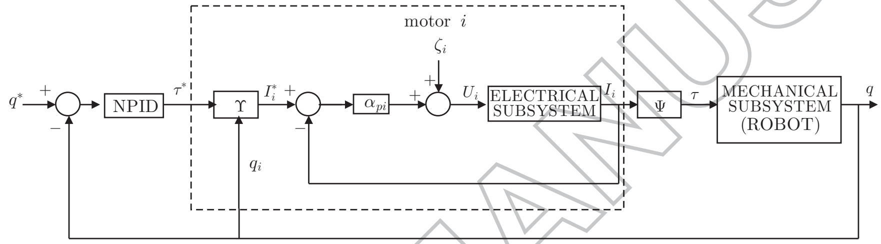



**Figure 3.** Control scheme in proposition 3.1. $\Upsilon$ is defined in (20) and $\Psi$ is defined in (10). $\zeta_i = R_i I_i^* - K_q\|\dot{q}\|^2\xi_i - K_d\sqrt{\sum_{i=1}^n \|\xi_i\|^2}\ \xi_i$.

### *Closed-loop dynamics*

Notice that there always exist $n$ constants $\epsilon_i > 0$, $i = 1, \ldots, n$, such that $E'_{pi} = \epsilon_i E_{pi}$. Hence, defining $K_{Pi} = \epsilon_i K'_{Pi}$, $K_{Di} = \epsilon_i K'_{Di}$, and $K_{Ii} = \epsilon_i K'_{Ii}$ we find that:

$$\begin{aligned} I_i^* &= \frac{\tau_i^{'*}}{E_{pi}\|E'_{Ri}(q_i)\|^2} E'_{Ri}(q_i), \\ \tau_i^{'*} &= K'_{Pi} h(\tilde{q}_i) + K'_{Di}\dot{q}_i + K'_{Ii} sat(z_i). \end{aligned} \tag{27}$$

On the other hand, from (10) we have:

$$\begin{aligned} \tau_i &= -\tau_{pi} E_{Ri}^T(q_i)\xi_i - \tau_{pi} E_{Ri}^T(q_i) I_i^*, \\ &= -\tau_{pi} E_{Ri}^T(q_i)\xi_i - \tau_{pi}[E_{Ri}^T(q_i) - (E'_{Ri})^T(q_i)] I_i^* - \tau_{pi}(E'_{Ri})^T(q_i) I_i^*, \end{aligned}$$

and replacing (27):

$$\tau_i = -\tau_{pi} E_{Ri}^T(q_i)\xi_i - \tau_{pi}\phi_i^T I_i^* - K'_{Pi} h(\tilde{q}_i) - K'_{Di}\dot{q}_i - K'_{Ii} sat(z_i),$$

where:

$$\phi_i = E_{Ri}(q_i) - E'_{Ri}(q_i). \tag{28}$$

Replacing again (27):

$$\begin{aligned}
\tau_i &= -\tau_{pi} E^T_{Ri}(q_i)\xi_i - \Phi_i(K'_{Pi}h(\tilde{q}_i) + K'_{Di}\dot{q}_i + K'_{Ii}sat(z_i)) \\
&\qquad\qquad\qquad -K'_{Pi}h(\tilde{q}_i) - K'_{Di}\dot{q}_i - K'_{Ii}sat(z_i), \\
\Phi_i &= \frac{\phi_i^T E'_{Ri}(q_i)}{\|E'_{Ri}(q_i)\|^2}. \qquad (29)
\end{aligned}$$

Hence, replacing $\tau = [\tau_1, \ldots, \tau_n]^T$ in (8) it is found:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau_e - (I_{n\times n} + \Phi)(K'_P h(\tilde{q}) + K'_D\dot{q} + K'_I x(z) - g(q^*)), \tag{30}$$

where:

$$\begin{aligned}
\Phi &= \mathrm{diag}\{\Phi_1, \ldots, \Phi_n\}, && (31)\\
\tau_e &= [-\tau_{p1}E^T_{R1}(q_1)\xi_1, \ldots, -\tau_{pn}E^T_{Rn}(q_n)\xi_n]^T, && (32)\\
x(z) &= sat(z) + (K'_I)^{-1}g(q^*). && (33)
\end{aligned}$$

On the other hand, replacing (19) in (9), and adding and subtracting the term $L_i\dot{I}_i^*$, we have that:

$$\begin{aligned}
L_i\dot{\xi}_i &= E_{pi}E_{Ri}(q_i)\dot{q}_i - (R_i + \alpha_{pi})\xi_i - K_q\|\dot{q}\|^2\xi_i - K_d\sqrt{\sum_{i=1}^n \|\xi_i\|^2}\,\xi_i - L_i\dot{I}_i^*, \qquad (34)\\
\dot{I}_i^* &= \left[\frac{d}{dq_i}\left(\frac{E'_{Ri}(q_i)}{E_{pi}\|E'_{Ri}(q_i)\|^2}\right)\right]\dot{q}_i[K'_{Pi}h(\tilde{q}_i) + K'_{Di}\dot{q}_i + K'_{Ii}sat(z_i)] \\
&\quad + \frac{E'_{Ri}(q_i)}{E_{pi}\|E'_{Ri}(q_i)\|^2}\left[K'_{Pi}\frac{dh(\tilde{q}_i)}{dq_i}\dot{q}_i + K'_{Di}\ddot{q}_i + K'_{Ii}\frac{d\,sat(z_i)}{dz_i}\dot{z}_i\right],
\end{aligned}$$

for $i = 1, \ldots, n$. Recall that $\dot{I}_i^*$ is continuous and defined for all $q_i \in \mathcal{R}$, according to the definition of $E'_{Ri}(q_i)$ in proposition 3.1. The closed loop dynamics is given by (34), (30), (33), (22).

***A positive definite and decrescent function***

Consider the following scalar function:

$$
\begin{aligned}
V(\tilde{q},\dot{q},z+(K_I')^{-1}g(q^*),\xi_1,\dots,\xi_n) &= \frac{1}{2}\sum_{i=1}^{n}\xi_i^T L_i\xi_i + V_q(\tilde{q},\dot{q},z+(K_I')^{-1}g(q^*)), \quad (35)\\
V_q(\tilde{q},\dot{q},z+(K_I')^{-1}g(q^*)) &= V_1(\dot{q},\tilde{q})+\mathcal{P}(\tilde{q})+V_2(\dot{q},z+(K_I')^{-1}g(q^*)),\\
V_1(\dot{q},\tilde{q}) = \frac{1}{4}\dot{q}^T M(q)\dot{q} &+ \varepsilon h^T(\tilde{q})M(q)\dot{q}+\varepsilon\int_0^{\tilde{q}} h^T(r)K_D' dr,\\
\mathcal{P}(\tilde{q}) &= \int_0^{\tilde{q}} h^T(r)K_P' dr+\mathcal{U}(q)-\mathcal{U}(q^*)-\tilde{q}^T g(q^*),\\
V_2(\dot{q},z+(K_I')^{-1}g(q^*)) = \frac{1}{4}\dot{q}^T M(q)\dot{q} &+ \varepsilon\gamma x^T(z)M(q)\dot{q}+\int_{-(K_I')^{-1}g(q^*)}^{z} x^T(r)K_I' dr,
\end{aligned}
$$

where $\varepsilon$ and $\gamma$ are some positive constants. The function $V_q(\tilde{q},\dot{q},z+(K_I')^{-1}g(q^*))$ was analyzed by Hernández-Guzman & Orrante-Sakanassi (2018). From that study and proceeding as in Hernández-Guzmán et al. (2013) it is concluded that there exist some small enough constant $c_1>0$ and some large enough constant $c_2>0$ such that the scalar function $V(\tilde{q},\dot{q},z+(K_I')^{-1}g(q^*),\xi_1,\dots,\xi_n)$, defined in (35) satisfies:

$$
\begin{aligned}
\alpha_1(\|y\|) &\le V(y)\le\alpha_2(\|y\|),\ \forall y=[\tilde{q},\dot{q},z+(K_I')^{-1}g(q^*),\xi_1,\dots,\xi_n]^T\in\mathcal{R}^{6n}, \quad (36)\\
\alpha_1(\|y\|) &= \begin{cases} c_1\|y\|^2, & \|y\|<1\\ c_1\|y\|, & \|y\|\ge 1\end{cases},\quad \alpha_2(\|y\|)=c_2\|y\|^2,
\end{aligned}
$$

if (27), (28), (32) in Hernández-Guzman & Orrante-Sakanassi (2018) are true and:

$$
L^* > \max_i\left\{\frac{K_i'}{K_{Ii}'}\right\},\quad i=1,\dots,n, \tag{37}
$$

where $K_{Ii}'$ is the $i-$th diagonal entry of matrix $K_I'$ whereas $K_i'$ is defined in Property 2.

***Time derivative of $V(y)$***

It is possible to verify, after some straightforward algebraic manipulations and the use of both expressions in (11), that the time derivative of $V(y)$, defined in (35), along the trajectories of the closed loop system (34), (30), (33), (22), can be upper bounded as:

$$
\begin{aligned}
\dot{V} \leq\ & -\dot{q}^T K'_D \dot{q} + \varepsilon \dot{q}^T \frac{dh(\tilde{q})}{d\tilde{q}} M(q)\dot{q} - \varepsilon h^T(\tilde{q})(g(q) - g(q^*)) \\
& -[\varepsilon h(\tilde{q}) + \dot{q} + \varepsilon\gamma x(z)]^T \Phi(K'_P h(\tilde{q}) + K'_D \dot{q} + K'_I x(z) - g(q^*)) \\
& +[\varepsilon h(\tilde{q}) + \varepsilon\gamma x(z)]^T \tau_e - \varepsilon h^T(\tilde{q}) K'_P h(\tilde{q}) \\
& +\varepsilon\gamma[\varepsilon(I_{n\times n} + \gamma K'_P (K'_I)^{-1}) h(\tilde{q}) + (I_{n\times n} + \varepsilon\gamma K'_D (K'_I)^{-1})\dot{q}]^T \frac{dx(z)}{dz} M(q)\dot{q} \\
& -\varepsilon\gamma x^T(z)(g(q) - g(q^*)) - \varepsilon\gamma x^T(z) K'_I x(z) + [\varepsilon h(\tilde{q}) + \varepsilon\gamma x(z)]^T C^T(q,\dot{q})\dot{q}, \\
& -\sum_{i=1}^{n} (R_i + \alpha_{pi}) \xi_i^T \xi_i + \sum_{i=1}^{n} \xi_i^T \left[ -K_q \|\dot{q}\|^2 \xi_i - K_d \sqrt{\sum_{i=1}^{n} \|\xi_i\|^2}\ \xi_i - L_i \dot{I}_i^* \right], \quad (38)
\end{aligned}
$$

where the facts that $K_P K_I^{-1} = K'_P (K'_I)^{-1}$ and $K_D K_I^{-1} = K'_D (K'_I)^{-1}$ have been employed. On the other hand, recall that given $w = [w_1, \ldots, w_n]^T$, then $|\ w_i| \leq \|w\|$, for $i = 1, \ldots, n$, and:

$$
\|\ddot{q}\| = \|M(q)^{-1}[-C(q,\dot{q})\dot{q} - g(q) + \tau_e - (I_{n\times n} + \Phi)(K'_P h(\tilde{q}) + K'_D \dot{q} + K'_I x(z) - g(q^*))]\|.
$$

Hence, according to (34) we can upper bound:

$$
\begin{aligned}
\|\dot{I}_i^*\| \leq\ & \Lambda_{i1} \|\dot{q}\| [\lambda_M(K'_P) M^* + \lambda_M(K'_D)\|\dot{q}\| + \lambda_M(K'_I) M^*] \\
& +\Lambda_{i2} \left[\lambda_M(K'_P)\|\dot{q}\| + \lambda_M(K'_I)\right. \\
& \times \left[\varepsilon\lambda_M\left(I_{n\times n} + \gamma K'_P (K'_I)^{-1}\right)\|h(\tilde{q})\| + \lambda_M\left(I_{n\times n} + \varepsilon\gamma K'_D (K'_I)^{-1}\right)\|\dot{q}\|\right]\Big], \\
& +\Lambda_{i2}\lambda_M(K'_D)\left[\frac{1}{\lambda_m(M(q))}[k_c\|\dot{q}\|^2 + \frac{k_{hg}}{k_a}\|h(\tilde{q})\| + \lambda_M((I_{n\times n} + \Phi)K'_P)\|h(\tilde{q})\|\right. \\
& \left. +\lambda_M((I_{n\times n} + \Phi)K'_D)\|\dot{q}\| + \lambda_M((I_{n\times n} + \Phi)K'_I)\|x(z)\| + \lambda_M(\Phi)K' + \|\tau_e\|]\right],
\end{aligned}
$$

where $K'$ is defined in Property 2 and:

$$
\begin{aligned}
\Lambda_{i1} &= \sup\left\|\frac{d}{dq_i}\left(\frac{E'_{Ri}(q_i)}{E_{pi}\|E'_{Ri}(q_i)\|^2}\right)\right\|, \quad \Lambda_{i2} = \sup\left|\frac{E'_{Ri}(q_i)}{E_{pi}\|E'_{Ri}(q_i)\|^2}\right|, \\
\|\tau_e\| &\leq \sqrt{\sum_{i=1}^{n} \tau_{pi}^2 \|E_{Ri}(q_i)\|^2\ \|\xi_i\|^2} \leq \max_i\{\tau_{pi}\|E_{Ri}(q_i)\|_M\} \sqrt{\sum_{i=1}^{n} \|\xi_i\|^2}.
\end{aligned}
$$

Thus, if we choose:

$$
K_q > n \max_i \left\{\lambda_M(L_i)\left(\Lambda_{i1}\lambda_M(K'_D) + \Lambda_{i2}\frac{k_c\lambda_M(K'_D)}{\lambda_m(M(q))}\right)\right\}, \quad (39)
$$

$$
K_d > n \max_i \left\{\lambda_M(L_i)\Lambda_{i2} \max_i\{\tau_{pi}\|E_{Ri}(q_i)\|_M\}\frac{\lambda_M(K'_D)}{\lambda_m(M(q))}\right\}, \quad (40)
$$

we can write:

$$
\begin{aligned}
\dot{V} &\leq -\bar{y}^T Q \bar{y} + \beta\|\bar{y}\|, \\
\beta &= \left[3(1+\varepsilon+\varepsilon\gamma) + n\max_i\{\Lambda_{i2}\}\frac{\lambda_M(K_D')}{\lambda_m(M(q))}\right]\lambda_M(\Phi)K' \\
\bar{y} &= [\|\dot{q}\|, \|h(\tilde{q})\|, \|x(z)\|, \|\xi\|]^T,
\end{aligned} \tag{41}
$$

if:

$$
\|\xi\| > \frac{n}{K_q}\max_i\left\{\lambda_M(L_i)\left(\Lambda_{i1}\lambda_M(K_D') + \Lambda_{i2}\frac{k_c\lambda_M(K_D')}{\lambda_m(M(q))}\right)\right\}, \tag{42}
$$

$$
\|\xi\| > \frac{n}{K_d}\max_i\left\{\lambda_M(L_i)\Lambda_{i2}\max_i\{\tau_{pi}\|E_{Ri}(q_i)\|_M\}\frac{\lambda_M(K_D')}{\lambda_m(M(q))}\right\}, \tag{43}
$$

where the vector $\xi$ is obtained by piling up $\xi_i$ for $i = 1, \ldots, n$, i.e. $\xi = \mathrm{col}(\xi_i)$. The entries of matrix $Q$ are given as:

$$
\begin{aligned}
Q_{11} &= \lambda_m(K_D') - \varepsilon\lambda_M(M(q)) - \varepsilon\gamma\lambda_M(M(q))\lambda_M(I_{n\times n} + \varepsilon\gamma K_D'(K_I')^{-1}) \\
&\quad -\lambda_M(\Phi K_D') - (\varepsilon\|h(\tilde{q})\|_M + \varepsilon\gamma\|x(z)\|_M)k_c, \\
Q_{22} &= \varepsilon\left(\lambda_m(K_P') - \lambda_M(\Phi K_P') - \frac{k_{hg}}{k_a}\right), \quad Q_{33} = \varepsilon\gamma(\lambda_m(K_I') - \lambda_M(\Phi K_I')), \\
Q_{44} &= \min_i\{R_i + \alpha_{pi}\}, \quad Q_{13} = Q_{31} = -\frac{\varepsilon\gamma}{2}\lambda_M(\Phi K_D') - \frac{1}{2}\lambda_M(\Phi K_I'), \\
Q_{12} &= Q_{21} = -\frac{\varepsilon^2\gamma}{2}\lambda_M(I_{n\times n} + \gamma K_P'(K_I')^{-1})\lambda_M(M(q)) - \frac{1}{2}\lambda_M(\Phi K_P') \\
&\quad -\frac{\varepsilon}{2}\lambda_M(\Phi K_D'), \\
Q_{23} &= Q_{32} = -\frac{\varepsilon\gamma}{2}\frac{k_{hg}}{k_a} - \frac{\varepsilon\gamma}{2}\lambda_M(\Phi K_P') - \frac{\varepsilon}{2}\lambda_M(\Phi K_I'), \\
Q_{14} &= Q_{41} = -\frac{n}{2}\max_i\{\Lambda_{i1}\}\lambda_M(K_P')M^* - \frac{n}{2}\max_i\{\Lambda_{i1}\}\lambda_M(K_I')M^* \\
&\quad -\frac{n}{2}\max_i\{\Lambda_{i2}\}\left[\lambda_M(K_P') + \lambda_M(K_I')\lambda_M\left(I_{n\times n} + \varepsilon\gamma K_D'(K_I')^{-1}\right)\right] \\
&\quad -\frac{n}{2}\max_i\{\Lambda_{i2}\}\frac{\lambda_M(K_D')}{\lambda_m(M(q))}\lambda_M((I_{n\times n} + \Phi)K_D'), \\
Q_{24} &= Q_{42} = -\frac{\varepsilon}{2}\max_i\{\tau_{pi}\|E_{Ri}(q_i)\|_M\} \\
&\quad -\frac{n\varepsilon}{2}\max_i\{\Lambda_{i2}\}\lambda_M(K_I')\lambda_M\left(I_{n\times n} + \gamma K_P'(K_I')^{-1}\right) \\
&\quad -\frac{n}{2}\max_i\{\Lambda_{i2}\}\frac{\lambda_M(K_D')}{\lambda_m(M(q))}\left[\frac{k_{hg}}{k_a} + \lambda_M((I_{n\times n} + \Phi)K_P')\right], \\
Q_{34} &= Q_{43} = -\frac{\varepsilon\gamma}{2}\max_i\{\tau_{pi}\|E_{Ri}(q_i)\|_M\} - \frac{n}{2}\max_i\{\Lambda_{i2}\}\frac{\lambda_M(K_D')}{\lambda_m(M(q))}\lambda_M((I_{n\times n} + \Phi)K_I'),
\end{aligned} \tag{44}
$$

where $k_{hg} \geq \frac{2K'}{s(\frac{2K'}{k_g})}$ (see Kelly et al. (2005)) and we have used $s(\|\tilde{q}\|) \leq \frac{\|h(\tilde{q})\|}{k_a}$ (see Hernández-Guzmán & Silva-Ortigoza (2011)) where $k_a$ is identical to parameter $\alpha$ introduced by Kelly (1998). The four principal minors of matrix $Q$ can always be rendered positive by choosing small enough $\varepsilon > 0$, $\gamma > 0$, $r > 0$, large enough positive definite matrices $K_D, K_P, K_I$, and large enough scalars $\alpha_{pi} > 0$, $i = 1\ldots, n$. Hence, matrix $Q$ is positive definite and $\lambda_m(Q) > 0$ is ensured.

Using some constant $0 < \Theta < 1$, we can rewrite (41) as:

$$
\begin{aligned}
\dot{V} &\leq -\bar{y}^T Q\bar{y} + \beta\|\bar{y}\|, \\
&\leq -(1-\Theta)\lambda_m(Q)\|\bar{y}\|^2 - \Theta\lambda_m(Q)\|\bar{y}\|^2 + \beta\|\bar{y}\|, \\
&\leq -(1-\Theta)\lambda_m(Q)\|\bar{y}\|^2, \ \forall\ \|\bar{y}\| \geq \frac{\beta}{\Theta\lambda_m(Q)}.
\end{aligned} \tag{45}
$$

Since $\lambda_M(\Phi) \to 0$, i.e. $\beta \to 0$, as $r \to 0$ (see (26), (28), (29), (31)), then the last inequality in (45) can always be forced to be satisfied by the linear parts of functions in $\bar{y}$. Moreover, according to (42) and (43), we can always choose some large enough $K_q$ and $K_d$ such that $\|\xi\|$ is arbitrarily small. Hence, it is always possible to choose a small radius $r > 0$ such that we can write:

$$
\dot{V} \leq -(1-\Theta)\lambda_m(Q)\|\bar{y}\|^2, \ \forall\ \|y\| \geq \mu_0 = \frac{\beta}{\Theta\lambda_m(Q)}. \tag{46}
$$

***Proof of Proposition 3.1***

Taking into consideration (36) and (46), we can invoke theorem 2.2 to conclude that given an arbitrary initial state $y(t_0) \in \mathcal{R}^{6n}$, we can always find controller gains such that the closed loop system state $y$ satisfies:

$$
\begin{aligned}
\|y(t)\| &\leq \beta_0(\|y(t_0)\|, t - t_0), \ \ \forall\ t_0 \leq t \leq t_0 + T, \\
\|y(t)\| &\leq \alpha_1^{-1}\left(\alpha_2\left(\mu_0\right)\right), \ \ \forall\ t \geq t_0 + T,
\end{aligned} \tag{47}
$$

where $\beta_0(\cdot,\cdot)$ is a $\mathcal{KL}$ function and $T \geq 0$ depends on $y(t_0)$ and $\mu_0$.

On the other hand, $\beta \to 0$ as $r > 0$ approaches to zero, hence, $\mu_0 > 0$ can be rendered arbitrarily small by choosing a small enough $r > 0$. Since $\alpha_1^{-1}(\alpha_2(\cdot))$ is a $\mathcal{K}_\infty$ function, then the ultimate bound in (47) tends to zero as $r > 0$ approaches to zero. Notice that, $r = 0$ would imply that $E'_{Ri}(q_i)$, for $i = 1, \ldots, n$, are not differentiable and, hence $\dot{I}_i^*$, for $i = 1, \ldots, n$, is not continuously differentiable, i.e. $\Lambda_{i1}$, for $i = 1, \ldots, n$, are not defined. Thus, $r > 0$ cannot be zero and, hence, $\beta > 0$ cannot be zero either. This means that the closed loop system has an ultimate bound which cannot be reduced to zero but can be rendered arbitrarily small by a suitable choice of controller gains. Recall that this result stands when starting from any initial condition. This completes the proof of proposition 3.1. Finally, we emphasize that the conditions to guarantee proposition 3.1 are summarized by (27), (28), (32) in Hernández-Guzman & Orrante-Sakanassi (2018), (37), (39), (40), in the present paper, the four principal minors of matrix $Q$ defined in (44) are positive, and some small constant $r > 0$.

**Remark 3.** Contrary to what happens in the standard control scheme for BLDC motors described in the introduction section, the desired electric currents defined in proposition 3.1 are not stepwise (see remark 1). Hence, the use of electric current controllers with integral parts are not recommended. This is the main reason why the terms $R_i I_i^* - \alpha_{pi}\xi_i$ are considered in (19) instead of a PI electric current controller.

Also notice that the only obstacle avoiding to demonstrate global asymptotic stability is the fact that functions $\phi_i$, $i = 1, \ldots, n$, defined in (28), are not zero because the difference $E_{Ri}(q_i) - E'_{Ri}(q_i)$ is not identically zero. However, according to the above arguments, it is not difficult to realize from (46) that global asymptotic stability is in fact accomplished if $E_{Ri}(q_i) - E'_{Ri}(q_i) = 0$ for $i = 1, \ldots, n$, at the equilibrium point. Furthermore, because of the integral action of the PID position controller, it is expected that the position errors always converge to zero.

## 4. Simulation study

We present next a simulation study using the two degrees of freedom ($n = 2$) robot manipulator reported by Campa et al. (2004). This robot has two revolute joints and two links which move on a vertical plane and point downwards when $q = [0, 0]^T$. We assume that this robot is equipped with two direct drive BLDC motors which are identical to the BLDC motor described in the simulations section of the work by Guerrero et al. (2017).

The controller parameters were chosen as $K_P = \text{diag}\{33.5,\ \ 4.8\}$, $K_D = \text{diag}\{9,\ \ 1\}$, $K_I = \text{diag}\{50,\ \ 1\}$, $\varepsilon = 5$, $\gamma = 5$, $\alpha_{p1} = 0.001$, $\alpha_{p2} = 0.001$, $K_q = 11$, $K_d = 11$, $r = 0.2$. The linear saturation function that has been employed is:

$$s(x) = \begin{cases} -L^* + (M^* - L^*)\tanh\left(\frac{x+L^*}{M^*-L^*}\right), & x < -L^* \\ x, & |x| \leq L^* \\ L^* + (M^* - L^*)\tanh\left(\frac{x-L^*}{M^*-L^*}\right), & x > L^* \end{cases},$$

with $M^* = 3$ and $L^* = 2.9$. Notice that $L^*$ is computed using (37) by employing the maximal torques required at each joint to compensate the gravity effects. We stress that it is straightforward to find experimentally approximate values of these maximal torques. On the other hand, $M^*$ is any constant larger than $L^*$. Finally, the desired positions were set to $q^* = [\pi/8,\ \ \pi/4]^T$ and all of the initial conditions were chosen to be zero.

The corresponding simulation results are presented in fig. 4. Notice that both joint position errors converge to zero in approximately 2 seconds instead of simple convergence to a small error, which is correctly predicted at the end of remark 3. We can also see that smooth torques are generated to achieve this task. Furthermore, these generated torques do not exceed the practical limits of $\pm 15$[Nm], for joint 1, and $\pm 4$[Nm], for joint 2, reported by Campa et al. (2004). Finally, we also present the electric currents flowing through the different motor phases as rotor (joint) positions increase. These simulation results corroborate our findings in proposition 3.1.

## 5. Conclusions

We have presented a simple control scheme to control robot manipulators actuated by direct drive BLDC motors. This is important to stress because several works in the literature uncorrectly call BLDC to motors that in fact are permanent magnet synchronous motors (PMSM's). Our proposal is simple: it consists of an external PID position loop, and an internal proportional electric current loop. Simplicity of our approach relies on the fact that we do not require to feedback the complex terms composing the time derivative of the desired electric current. It is important to highlight this point because most works in the literature on motor control become complex

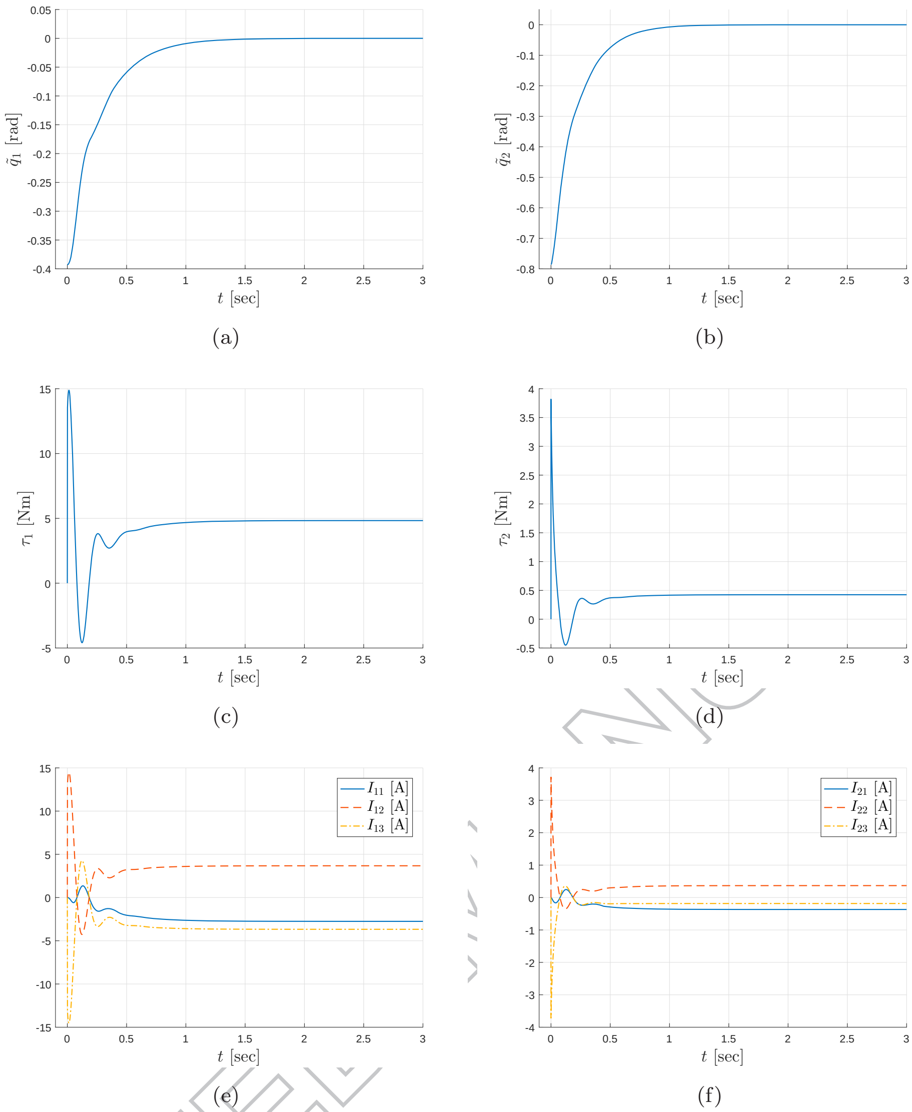



**Figure 4.** Simulation results.

because they require to feedback such complex terms. Moreover, they are forced to include position filtering to replace velocity measurements because, if not, complexity of controller increases further. On the contrary, simplicity of our approach is achieved despite velocity measurements are allowed. Furthermore, this is instrumental to successfully design a PID position controller without relying on natural friction that is present in the mechanical subsystem. It is the authors belief that this is the main reason why most works on electric motor control are constrained to control velocity in single motors actuating on linear mechanical loads. Thus, solving the position control problem in a complex, nonlinear and highly coupled mechanical load (a $n-$degrees of freedom robot) is an important contribution of our porposal.

## Declaration of interest statement

There is not any conflict of interest.

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