Proceedings of the 2000 IEEE
International Conference on Robotics & Automation
San Francisco, CA • April 2000

Experimental identification of robot dynamics for control*

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Abstract

This paper discusses the experimental identification of dynamic robot models for their application in model based robot control, e.g. computed torque control. The accuracy of these controllers relies highly on the ability of the robot model to accurately predict the required actuator torques. The paper shows how this application reflects on the choices that have to be made in the different steps of the identification procedure, and consequently on the accuracy of the obtained model parameters and actuator torque prediction.

1 Introduction

Experimental robot identification is the only efficient way to obtain accurate robot models as well as indications on their accuracy, confidence and validity. The dynamic model parameters provided by robot manufacturers are insufficient, inaccurate, or often non-existing, especially those dealing with friction and compliance characteristics. Direct measurement of the physical parameters is unrealistic, because of the complexity of most robots.

Experimental robot identification deals with the problem of estimating the robot model parameters from the response measured during a robot experiment.

A typical experimental robot identification procedure consists of the three steps : **(1)** the generation of an identifiable dynamic robot model, **(2)** the generation of optimized excitation trajectories (the experiment design step), and **(3)** the estimation of the model parameters. For each of these steps the user has to make an optimal choice between different options, depending on the application of the model. In this paper, the envisaged application is advanced model based control, e.g. computed torque control. The accuracy of these controllers relies highly on the ability of the model to accurately calculate/predict the required actuator torques/forces, i.e. the feedforward signals.

This paper discusses the different experimental robot identification steps in relation to the envisaged application of the model, i.e. model based robot control. The importance of the selection of the experiment design criterion and its influence on the envisaged model accuracy is illustrated by means of experiments on a KUKA IR 361 industrial robot.

2 Generation of dynamic robot models

The generation of a dynamic robot model is based on the kinematic structure of the robot (e.g. link lengths, transformation matrices between the local coordinate systems of the different links,...). The model relates system inputs and outputs. Based on the type of inputs and outputs, two different models can be distinguished : **external** and **internal** models. External models relate robot motion (inputs) to reaction forces and torques measured at the base of the robot (outputs). Internal models relate the robot motion (inputs) to actuator torques or forces (outputs).

2.1 Internal and external models

The robot is considered as a set of n rigid bodies interconnected by one d.o.f. joints which can be either prismatic or revolute. We assume that the structure is a topological tree, excluding the presence of any closed loops in the system. If we denote the n joint coordinates by $\boldsymbol{q}$, the equations of motion, obtained by the Potential Power Principle, take the form :

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{g}) = \boldsymbol{\tau}_i \qquad (1)$$

*Acknowledgement This text presents research results of the Belgian programme on Interuniversity Poles of attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by its authors.

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where, $\mathbf{M}(\mathbf{n}\times\mathbf{n})$ is the symmetric positive definite mass matrix; $\mathbf{C}(\mathbf{n}\times\mathbf{1})$ includes the Coriolis, centrifugal, gyroscopic, gravity terms, and terms resulting from joint friction; $\boldsymbol{\tau}_i(n\times 1)$ is the vector of the generalized forces associated with $\boldsymbol{q}$, i. e. the forces and torques applied at the joints by the actuators. It is common to model friction in robot joints by means of viscous and Coulomb friction [11]. Without loss of generality, the sequel of this paper will be formulated to the context of revolute joints only to ease the burden of reading.

In these equations, the internal model of the robot is characterized by a set of inertial parameters describing the mass distribution of the links, and friction parameters. The inertial parameters could be considered individually for each body, but it is well-known that some of them combine together [7] to form the so-called barycentric parameters.

It can be shown [2] that equation (1) (*the internal model*) can be written as:

$$\boldsymbol{\Psi}_i(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\delta}_i = \boldsymbol{\tau}_i \tag{2}$$

where $\boldsymbol{\delta}_i(n_i\times 1)$ is the vector containing the barycentric and friction parameters, and $\boldsymbol{\Psi}_i(n\times n_i)$ is the associated identification matrix.

The *external model* consists in a reformulation of the system dynamics relating the motion of the robot to the reaction forces and torques on its bedplate. It can be easily obtained by projecting the force and torque vectors at the first joint on the axes of the inertial reference frame attached to the bedplate. The external model equations have thus a form similar to equation (2):

$$\boldsymbol{\Psi}_e(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\delta}_e = \boldsymbol{\tau}_e \tag{3}$$

where $\boldsymbol{\delta}_e(n_e\times 1)$ contains the n_e barycentric parameters associated with the external model, $\boldsymbol{\Psi}_e(6\times n_e)$ is the associated identification matrix, and $\boldsymbol{\tau}_e(6\times 1)$ is the vector of the reaction forces and torques. Note here that the dimension 6 in $\boldsymbol{\Psi}_e$ and $\boldsymbol{\tau}_e$ represents the 3 components of forces and torques at the base of the robot.

It is shown in [3] that all the barycentric parameters appearing in equation (2) are also present in equation (3).

It is assumed that, in order to ensure the identifiability of the system, both sets of barycentric parameters are minimal sets, i.e. that a model reduction procedure has already been applied [3, 5, 9].

Equations (2) and (3) are both linear in the parameters. This simplifies the parameter estimation as it reduces to solving an overdetermined set of linear equations, in e.g. a least squares sense (see section 3). In the classical identification approach [4, 8, 11], the parameters are estimated from motion data and actuator torques, both measured by "internal" measurement devices (i. e. joint encoders for motion data and actuator current measurements for forces and torques) using the internal model. The classical approach suffers however from an important drawback: the torques applied to the **links** are not directly available so that they are corrupted by friction torque modelling errors and by the low precision of the actuator torque constants.

An alternative approach makes use of the external model of the robot [10]. Since this model relates the motion of the robot to the reaction forces and torques on its bedplate (measured by means of an external force/torque platform) it is totally independant from internal torques such as joint friction, allowing a accurate estimate of the inertial parameters. This approach however suffers from the fact that joint friction parameters cannot be estimated. These parameters are important for accurate actuator torque prediction which is used in computed torque control.

2.2 Combining internal and external robot models

Combining the internal and external models into one single identification scheme, requires the determination of the parameter set involved in such combined model. The total combined robot model can be formulated as follows [2]:

$$\boldsymbol{\Phi}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\theta} = \boldsymbol{\tau} \tag{4}$$

with

$$\boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{\tau}_i \\ \boldsymbol{\tau}_e \end{bmatrix} \tag{5}$$

an $(n+6)\times 1$ column vector. The total parameter vector $\boldsymbol{\theta}$ consists of 2 subvectors:

$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\delta}_i \\ \boldsymbol{\delta}_e \setminus \boldsymbol{\delta}_i \end{bmatrix} \tag{6}$$

$\boldsymbol{\delta}_e \setminus \boldsymbol{\delta}_i$ is the set of external parameters $\boldsymbol{\delta}_e$ except the barycentric parameters of $\boldsymbol{\delta}_i$.

Combining both models and identification approaches yields more accurate parameter estimates (smaller parameter uncertainties) and therefore more accurate actuator torque/force estimates (see section 3). [2] shows this by means of simulated experiments. The considered test case is a KUKA IR 361 industrial robot (see also section 5). It is shown that the RMS actuator torque prediction errors for the first three axes resulting from the combined model are between 20% and 50% smaller than those resulting from the internal model only. This result has a natural explanation. Combining both models results in only a slight increase of the number of model parameters with respect to the internal model (e.g. for first three links of a KUKA IR 361 (see section 5), 35 parameters instead of 28 [2]), and a considerable increase of the number of force/torque data involved (3 times more data for the considered example).

Remark: The mentioned models depend on the barycentric parameters, which differ from the standard set of interial parameters describing the mass distribution of each link. These barycentric parameters have no direct physical meaning compared with these classical inertial parameters and cannot be transformed to them. The knowledge of these inertial parameters is not important if the model is used to estimate the required actuator torque. If applications are envisaged where the knowledge of the real inertial parameters is important, model (1) forms that basis for the identification. The estimation of these parameters is however much more cumbersome since this model in nonlinear in the parameters which complicates their estimation considerably (see section 3). Moreover, some of these parameters are not identifiable by the methods considered here.

3 Parameter estimation

As models (2) and (3) are linear in the unknown parameters, most researchers proposed the *linear least-squares method* for the estimation of the parameters values [6, 13]. [1] proposed to use a *weighted linear least-squares* when the noise on the force/torque measurements varies over the different axes. These methods consider noise free joint angle measurements.

[11] presents a maximum likelihood estimation of the dynamical parameters. This approach is based on a statistical framework aiming at estimating the robot model parameters with minimal uncertainty, i.e. with minimal parameters covariance matrix. The maximum likelihood estimate $\boldsymbol{\theta}_{ml}$ of the parameter vector $\boldsymbol{\theta}$ is given by the value of $\boldsymbol{\theta}$ which maximizes the likelihood of the measurements. The minimization of such a likelihood function is a nonlinear least squares minimization problem even if the model is linear in the parameters. If the measured joint angles are free of noise and the model is linear in the parameters, this minimization problem simplifies to the *Markov estimate*, i.e. the *weighted linear least squares estimate* for which the weighting function is the reciprocal of the standard deviation of the noise on the measured force/torque data [11]. This simplification is often justified since the noise level on the joint angle measurements is much smaller than the noise level on the force/torque measurements [11].

3.1 Minimal uncertainty

Maximum likelihood estimators are unbiased and efficient, which means that they yield parameter estimates with minimal covariance matrix corresponding to the Cramèr-Rao bound [11]. Minimal uncertainty on the parameter estimates also yields minimal uncertainty on the estimates of the actuator torques. This can easily be seen as follows. Consider two unbiased estimates of $\boldsymbol{\theta}$: $\hat{\boldsymbol{\theta}}^*$ and $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\theta}}^*$ being an efficient estimate and $\hat{\boldsymbol{\theta}}$ not. This means that $\boldsymbol{P}^* \leq \boldsymbol{P}$ with $\boldsymbol{P}^*$ and $\boldsymbol{P}$ their respective covariance matrices, which correspond to $\boldsymbol{P}^* - \boldsymbol{P}$ being a negative definite matrix, i.e. $\forall \boldsymbol{x} \neq 0$, $\boldsymbol{x}^T(\boldsymbol{P}^* - \boldsymbol{P})\boldsymbol{x} \leq 0$ or $\boldsymbol{x}^T\boldsymbol{P}^*\boldsymbol{x} \leq \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x}$. Since the considered models are linear in the parameters, estimates of the actuator torques correspond to taking a linear combinations of the parameters, i.e. $\boldsymbol{x}^T\theta$, with $\boldsymbol{x}^T$ the row of the matrices $\boldsymbol{\Psi}_i$ (equation (2)) or $\boldsymbol{\Phi}$ (equation (4)) corresponding to considered the joint. The variance on this estimate, which represents the uncertainty, equals $\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x}$. From the abovementioned conclusion, it follows that efficient parameter estimators yield models that estimate actuator torques more accurately. Maximum likelihood or Markov estimators, which are efficient, are therefore to be prefered.

Remark: If the applied models are non-linear in the parameters, e.g. models which are based on standard inertial parameters, all these identification approaches (least-squares, maximum likelihood, etc...) result in non-linear (least-squares) minimization problems which are more complex to solve: they are solved iteratively and therefore require an initial guess of the parameters. In addition, convergence to the global minimum cannot be guaranteed.

4 Design of optimized excitation trajectories

It is well recognized that reliable, accurate, and efficient robot identification requires **specially designed experiments**. When designing an identification experiment for a robot manipulator, it is essential to consider whether the excitation is sufficient to provide accurate and fast parameter estimation in the presence of disturbances such as measurement noise and actuator disturbances.

The generation of an optimal robot excitation trajectory involves nonlinear optimization with motion constraints (i.e. constraints on joint angles, velocities, and accelerations, and on the robot end effector position in the cartesian space in order to avoid collisions).

Several approaches have been presented. They all use a different trajectory parameterization. These parameters are the degrees of freedom of the optimization problem. Armstrong [1] describes an approach in which the degrees of freedom are the points of a sequence of joint accelerations. This approach is the most general one, but it results in a large number of degrees of freedom, such that optimization is cumbersome. The optimization is done by maximizing the minimum singular value of the square of the identification matrix.

Gautier [6] optimizes a linear combination of the condition number and the equilibrium of the mentioned identification matrix. The degrees of freedom are a finite set of joint angles and velocities separated in time. The actual trajectory is continuous and smooth, and is calculated by interpolating a fifth-order polynomial between the optimized points, assuming zero initial and final acceleration. Only a very small part, namely the finite set of joint angles and velocities, of the final trajectory is optimized. As a result, the total smooth trajectory cannot be guaranteed to satisfy all motion constraints nor to be optimal with respect to the considered criterion.

All these design approaches are (implicitly) based on a deterministic framework, since the excitation trajectory design does not consider uncertainties on the measurements or the parameter estimates.

[11] describes a design method based on a statistical framework. It differs from all other methods in two aspects: the parameterization of the trajectory and the optimization criterion. These aspects are discussed in the following sections.

4.1 Periodic excitation

The excitation trajectory for each joint is a finite sum of harmonic sine and cosine functions, i.e. a finite Fourier series. The angular position q_i, for joint i of a n-degrees-of-freedom robot are written as:

$$q_i(t) = \sum_{l=1}^{N_i} \frac{a_l^i}{\omega_f l} \sin(\omega_f l t) - \frac{b_l^i}{\omega_f l} \cos(\omega_f l t) + q_{i\,0} \quad (7)$$

with ω_f the fundamental pulsation of the Fourier series. This Fourier series specifies a periodic function with period $T_f = 2\pi/\omega_f$. The fundamental pulsation is common for all joints, in order to preserve the periodicity of the overall robot excitation. Each Fourier series contains $2 \times N_i + 1$ parameters, that constitute the degrees of freedom for the optimization problem: a_l^i, and b_l^i, for $l = 1$ to N_i, which are the amplitudes of the cosine and sine functions, and $q_{i\,0}$ which is the offset on the position trajectory. The offset determines the robot configuration around which excitation will occur.

This approach guarantees bandlimited periodic trajectories and therefore allows:

- time domain data averaging, which improves the signal-to-noise ratio of the experimental data. This is extremely important since motor current (torque) measurements are very noisy.
- estimation of the characteristics of the measurement noise [11]. This information is valuable in case of maximum likelihood parameter estimation [11].
- specification of the bandwidth of the excitation trajectories, such that excitation of the robot flexibility can be either completely avoided or intentionally brought about.
- calculation of joint velocities and accelerations from the measured response in an analytic way [11].

None of the abovementioned robot excitation methods possesses these interesting features.

4.2 d-optimal experiment design

The design criterion for these trajectory parameters is the underbound on uncertainty on the parameter estimates, which is the Cramèr-Rao bound on the covariance matrix of the parameter estimates. The covariance matrix of an efficient parameter estimate, such as the maximum likelihood and Markov estimates, equals the Cramèr-Rao bound. In the case of the Markov estimate, the covariance matrix depends only on the robot trajectory via the identification matrix, which depends on the motion data via $\mathbf{\Phi}$ (4), and the variance of the noise on the force/torque measurements. It does not depend on the model parameters.

For this optimization, the covariance matrix or its lower bound is replaced by a representative scalar measure, i.e. its determinant, yielding the so called *d-optimality* criterion. This optimization is a complex nonlinear optimization problem with the abovementioned motion constraints.

This experiment design criterion in combination with the maximum likelihood parameter estimation is

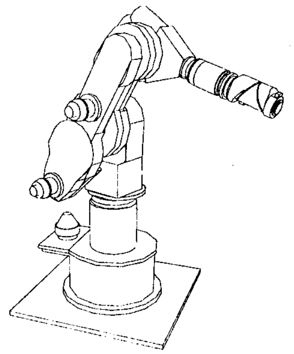

Figure 1: Schematic representation of a KUKA IR 361 robot

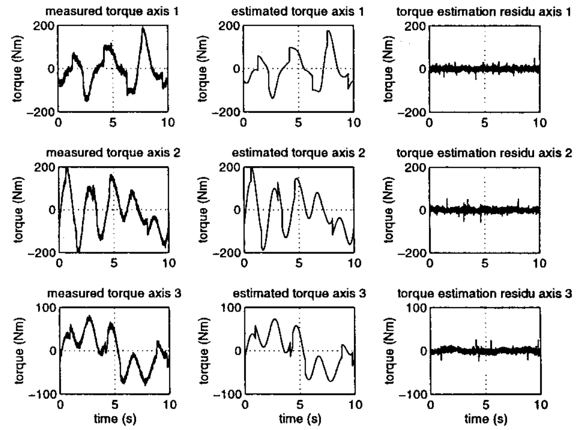

Figure 2: Measured torque, estimated torque and estimation residu, for trajectory and model 2

the appropriate approach for the envisaged application of the identified model. Maximum likelihood (or Markov) parameter estimates have a minimum covariance which equals the Cramèr-Rao bound. This bound is minimized by the d-optimal experiment design approach for given levels of noise. As it is explained in section 3, minimum parameter covariance yields minimum uncertainty on the estimation of the actuator forces/torques.

5 Experimental verification

This section illustrates the influence of the experiment design criterion on the accuracy of the model parameters and consequently on the accuracy of the estimated actuator torque estimates.

The test case is a KUKA IR 361 robot (figure 1) of which only the first three robot axes are considered. The experimental identification is based on the minimal internal model set as described in [11].

5.1 Description of the experiments

Two different excitation trajectories are considered: **(1)** a trajectory which is optimized according to the condition number of the normalized identification matrix [11], and **(2)** a trajectory which is optimized according to the d-optimal criterion. Criterion two is based on the assumption that the measured joint angles are free of noise.

Trajectory 1 yields a condition number equal to 47.3 and a determinant equal to $5.26\,10^{-46}$. Trajectory 2 yields a condition number equal to 111.7 and a determinant equal to $1.85\,10^{-56}$.

The models obtained using trajectories 1 and 2 are referred to as model 1 and 2 respectively. The motion constraints, the sampling frequency (150 Hz), the number of trajectory parameters (11 per joint) and the fundamental frequency of the excitation trajectory (0.1 Hz) are the same for both experiments. The estimated variances are $1.1\,10^{-10}rad^2$, $8.3\,10^{-11}rad^2$, $2.9\,10^{-10}rad^2$ for the position of respectively axes 1, 2 and 3 and $25N^2m^2$, $26N^2m^2$ and $10N^2m^2$ for the torque of respectively axes 1, 2 and 3.

Figure 2 shows the measured and estimated actuator torques, and the estimation residu for model 2. The motor torque measurements are averaged over 16 periods. The estimation residues for model 1 are comparable. However, we must bear in mind that this comparison is not completely justified since the two trajectories are not the same.

The peaks in the estimation residu occur at low joint angular velocity, which indicates that the assumed friction model, which includes viscous and Coulomb friction, is too simple. It can be expected that including more advanced friction and gear models, as described e.g. in [12], results in smaller estimation residus.

Due to these modelling errors, the mean values of the squared estimation residu ($81N^2m^2$, $89N^2m^2$ and $23N^2m^2$ for respectively axes 1, 2 and 3) are larger than the noise variances of the measured torques. Despite modelling errors (stiction, backlash and flexibility in the transmissions, kinematic errors) the estimated models are accurate but biased.

5.2 Model validation

The accuracy of the obtained parameter estimates can be verified for a different validation trajectory by comparing the measured torques and an estimate of these torques based on the model and the measured position data. The validation trajectory goes through 20 points randomly chosen in the workspace of the robot. The robot moves with maximum acceleration and deceleration between these points, and comes to full stop in each point.

The mean squared torque estimation residu for the validation trajectory is $45N^2m^2$, $54N^2m^2$ and $12N^2m^2$ for the axes 1, 2 and 3. Model 1 yields mean squared torque estimation residus which are approximately 20% larger than the mentioned values, indicating that model 2 is more accurate than model 1 with respect to its ability to predict the motor torques based on joint angular position measurements.

6 Conclusions

This paper discusses the different steps of a robot identification procedure: the generation of a robot model, the design and optimization of a robot excitation, and the parameter estimation. The choice between the different design options in each of these steps must be guided by the application of the model. This paper discusses and illustrates these choices for the model based robot control application. The appropriate choices are: internal or combined models which are linear in the parameters, the d-optimal experiment design criterion, and maximum likelihood or Markov estimation.

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