Controlling chaos in mechanical systems

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The problem of controlling chaos, that is to convert the chaotic behaviour to a periodic time dependence is discussed. We described a number of effective controlling methods in the context of mechanical systems.

1. INTRODUCTION

Chaos occurs widely in mechanical systems; historically it has usually been regarded as a nuisance and designed out if possible. It has been noted only as irregular or unpredictable behaviour, and often attributed to random external influences. More recently there are examples of the potential usefulness of chaotic behaviour, and we remark on some of the potential usefulness of chaotic behaviour, and we remark on some of these in the final section of the paper.

The mathematical modelling of mechanical systems often comprises a set of coupled oscillators, each of which may bear some resemblance to the paradigm examples of the Duffing and Van der Pol equations, representing nonlinearity in the stiffness and damping respectively. Extensive research on these equations has of course revealed exceedingly complex behaviour (Moon: 1986, Thompson and Steward: 1986, Guckenheimer and Holmes: 1983, Kapitaniak: 1991), including numerous regions in parameter space in which a chaotic response occurs. Similar regions of chaotic behaviour occur in coupled systems; an example is that presented by True (True: 1993) in this issue. Similar unpredictable behaviour, if not strictly chaotic, may appear in fluid systems (Tritton: 1986, Infeld and Rowlands: 1990, Jimenez: 1991) or coupled fluid/solid systems, and electromechanical systems (Ottino: 1991).

In this paper we review a number of methods by which undesirable chaotic behaviour may be controlled or eliminated. More speculatively, we indicate ways in which the existence of chaotic behaviour may be directly beneficial or exploitable.

We can divide the approaches into two broad categories; firstly those in which the actual trajectory in the phase space of the system is monitored, and some **feedback** process is employed to maintain the trajectory in the desired mode, and secondly **nonfeedback** methods in which some other property or knowledge of the system is used to modify or exploit the chaotic behaviour.

In the first category, the situation most investigated as yet is that in which a trajectory is held close to one of the infinitely many periodic orbits embedded in a chaotic attractor (Grebogi et al.: 1988). We discuss this approach in the context of mechanical systems in Section 2. The second category, and that which we cover in rather more detail, is discussed in Section 3 and 4, in which we describe respectively how system design or operating conditions might be utilised to optimise system behaviour. A key feature here is the avoidance of the need for feedback mechanisms, which may, in mechanical systems, be complex and costly. These methods do, however, depend on a knowledge of the dynamics of the system.

Finally in Section 5 we discuss the potential usefulness of the various approaches in mechanical systems and add some remarks on the possible direct exploitation of chaotic behaviour.

2. CONTROL THROUGH FEEDBACK

Ott, Grebogi and Yorke (Ott et al.: 1990, 1991, Romeiras et al.: 1992) have, in an important series of papers, proposed and developed a method by which chaos can always be suppressed by shadowing one of the infinitely many unstable periodic orbits (or perhaps steady states) embedded in the chaotic attractor. The idea is to start with any initial condition, wait until the trajectory falls into a target region around the desired periodic orbit, and then apply feedback control. More specifically, the procedure is to first identify some suitable orbit then to apply some (small) control so as

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to stabilize this orbit. The effectiveness of OGY methods is increased if it is combined with an appropriate targetting procedure (Shinbrot et al.: 1990, 1992) in which the actual trajectory is directed into the neighbourhood of appropriate unstable periodic orbit.

Later developments of the OGY approach have established connections between control magnitude and length of chaotic transient preceding steady periodicity, and have indicated the important (and possibly damaging) effects of low levels of noise which may occasionally provoke large excursions from the desired orbit. They present results for the control of the double rotor map, a four-dimensional system that describes the particular impulsively periodically forced mechanical system.

The OGY approach has stimulated a good deal of research activity, both theoretical and experimental. The efficiency of the technique has been demonstrated by Ditto (Ditto et al.: 1991) in a periodically forced system, converting its chaotic behaviour into period-one and period-two orbits, and the application of the method to stabilise higher periodic orbits in a chaotic diode resonator has been demonstrated by Hunt (Hunt: 1991). Another interesting application of the method is the generation of a desired aperiodic orbit (Mehta and Henderson: 1991), and again Tel (Tel: 1991) has been able to demonstrate controlled transient chaos. Related work by Nitche and Dressler (Nitche and Dressler: 1992) uses time delay techniques to control chaos.

Though the OGY theory has been proposed in the context of low dynamical systems, and most of the experimental or observation investigations have been concerned with clearly low order mechanical or electrical contexts, the interesting experiments by Singer (Singer et al.: (1990) demonstrate its potential for fluid (and perhaps fluid-solid) mechanical phenomena. The experiments succeeded in achieving regular laminar flow in previously unstable thermal convection loops by use of a thermostat-type feedback.

Generally experimental application of the OGY method requires a permanent computer analysis of the state of the system. The changes of the parameters, however, are discrete in time since the method deals with the Poincare map. This leads to some serious limitations. The method can stabilize only those periodic orbits whose maximal Lyapunov exponent is small compared to the reciprocal of the time interval between parameter changes. Since the corrections of the parameter are rare and small, the fluctuation noise leads to occasional bursts of the system into the region far from the desired periodic orbit especially in the presence of noise. To avoid these problems Pyragas (Pyragas: 1992) proposed the method of a time-continuous control. This method is based on the construction of a special form of a time-continuous perturbation, which does not change the form of the desired unstable periodic orbit, but under certain conditions can stabilize it. Two feedback controlling loops shown in Fig.1, have been proposed. A combination of feedback and periodic external force is used in the first method - Fig.1.a. The second method does not require any external source of energy and it is based on a self controlling delayed feedback.

Alternative methods based on the classical feedback controlling methods have been described in (Jackson: 1990a, 1990b, Bandyopadhyay et al.: 1992, Chen and Dong: 1992, Hartley and Mossayebi: 1992, Rajesekar and Lakshmanan: 1992).

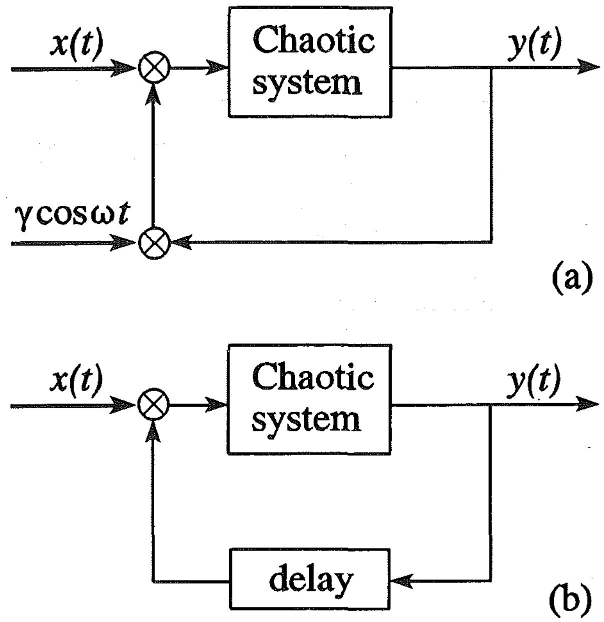

FIG.1 Feedback loops for continuous control of chaos

3. CONTROL THROUGH OPERATING CONDITIONS

Virtually all mechanical systems are subjected to external forcing during operation. This forcing will contain (and hopefully be dominanated by) planned and intentional components; it will also almost invariably contain unintentional "noise". Judicious design and control of this forcing is often able to annihilate or shift to a harmless region of parameter space unwanted chaotic behaviour (in some circumstances, as we remark in section 5, exactly the reverse process may be desirable, so that we may wish to produce chaotic behaviour.

We can illustrate this idea in the context of the Duffing's oscillator:

$$\ddot{x}+a\dot{x}+bx+c^3=B_0+B_1\cos\Omega t \tag{1}$$

where a, b, c, B_0, B_1 and Ω are constant.

It is well-known that eq.(1) shows chaotic behaviour for certain values of the parameters (Ueda: 1979, 1991, Sato et al.: 1983, Kapitaniak: 1991). In many cases it can be shown that chaotic behaviour is obtained via a period doubling bifurcation (Ueda: 1991, Sato et al.: 1983, Kapitaniak: 1991). Recently, there have been some attempts to create an analytical criterion which allows us to estimate the chaotic domain in the parameter space (Szemplińska-Stupnicka: 1989, Kapitaniak: 1990, 1991). Boundaries of the chaotic zone have been obtained using classical approximate theory of nonlinear oscillations, by examining approximate

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periodic solutions and studying particular types of higher order instabilities which precede the destruction of a periodic attractor in the variational Hill's type equation (Hayashi: 1964). Now we adopt a similar procedure (particularly harmonic balance method) to control chaotic behaviour.

First consider the first approximate solution in the form:

$$x(t)=C_0+C_1\cos(\Omega t+\zeta) \tag{2}$$

where C_0, C_1 and ζ are constants. Substituting eq.(2) into eq.(1) it is possible to determine these constants (Kapitaniak: 1991, Hayashi: 1964). To study the stability of the solution (2) a small variational term $\delta x(t)$ is added to eq.(2) as

$$x(t)=C_0+C_1\cos(\Omega t+\zeta)+\delta x(t) \tag{3}$$

After some algebraic manipulations, the linearized equation with periodic coefficients for $dx(t)$ is obtained

$$\delta\ddot{x}+a\delta\dot{x}+\delta x[\lambda_0+\lambda_1\cos\Xi+\lambda_2\cos 2\Xi]=0 \tag{4}$$

where: $\lambda_0=3C_0^2+(3/2)C_1^2$, $\lambda_1=6C_0C_1$, $\lambda_2=(3/2)C_1^2$, $\Xi=\Omega t+\zeta$. In the derivation of eq.(4), for simplicity it was assumed without loss of generality that $b=0$. As we have a parametric term of frequency Ω - $\lambda_1\cos\Xi$, the lowest order unstable region is that which occurs close to $\Omega/2\approx\surd\lambda_0$ and at its boundary we have the solution:

$$\delta x=b_{1/2}\cos\left(\frac{\Omega}{2}t+\zeta\right) \tag{5}$$

To determine the boundaries of the unstable region we insert eq.(5) into eq.(4), and the conditions of nonzero solution for $b_{1/2}$ lead us to the following criterion to be satisfied at the boundary:

$$\left(\lambda_0-\frac{\Omega^2}{4}\right)^2+a^2\frac{\Omega^2}{4}-\frac{\lambda_1^2}{4}=0 \tag{6}$$

From eq.(6) one obtains the interval $(\Omega_1^{(2)}, \Omega_2^{(2)})$ within which period-two solutions exist. Further analysis shows that at Ω_2 we have a stable period-doubling bifurcation for decreasing Ω and at Ω_1 an unstable period-doubling bifurcation for increasing Ω (Szemplińska-Stupnicka: 1989). In this interval we can consider the period two solution of the form:

$$x(t)=A_0+A_{1/2}\cos\left(\frac{\Omega}{2}t+\eta\right)+A_1\cos\Omega t \tag{7}$$

where A_0, $A_{1/2}$, A_1 and η are constants to be determined.

Again to study the stability of the period two solution we have to consider a small variational term $\delta x(t)$ added to eq.(7). The linearized equation for $\delta x(t)$ has the following form:

$$\delta\ddot{x}+a\delta\dot{x}+\delta x\left[\lambda_0^{(2)}+\lambda_{1/2c}\cos\frac{\Omega}{2}t+\lambda_{1c}^{(2)}\cos\Omega t+\right.$$
$$\left.+\lambda_{3/2}\cos\frac{\Omega}{2}\theta+\lambda_{1s}^{(2)}\sin\Omega t+\lambda_2^{(2)}\cos 2\Omega t\right]=0 \tag{8}$$

where:

$$\lambda_0^{(2)}=3\left(A_0^2+\frac{1}{2}A_{1/2}^2+\frac{1}{2}A_1^2\right),\ \lambda_{1/2c}=3A_{1/2}(2A_0+A_1)\cos\eta,$$

$$\lambda_{1/2s}=3A_{1/2}(A_1-2A_0)\sin\eta,\ \lambda_{3/2}=3A_1A_{1/2},$$

$$\lambda_{1/c}^{(2)}=6A_0A_1+\frac{3}{2}A_{1/2c}^2\cos 2\eta,\ \lambda_{1s}^{(2)}=-\frac{3}{2}A_{1/2s}\sin 2\eta,\ \lambda_2^{(2)}=\frac{3}{2}A_1^2.$$

The form of eq.(8) enables us to find the range of existence of a period four solution, represented by:

$$\delta x=b_{1/4}\cos\left(\frac{\Omega}{4}t+\eta\right)+b_{3/4}\cos\left(3\frac{\Omega}{4}t+\eta\right) \tag{9}$$

After inserting eq.(9) into eq.(8) the condition of nonzero solution for $b_{1/2}$ and $b_{3/4}$ gives us the following set of nonlinear algebraic equations for Ω, $\cos\eta$, and $\sin\eta$ to be satisfied for existence:

$$\begin{aligned}&(\lambda_{1/2s}+\lambda_{1s}^{(2)})-0.5(\lambda_{1/2c}-\lambda_{1c}^{(2)})(-a\Omega/2+\lambda_{1/2s}-\lambda_{3/2}\sin\eta)=0,\\&\frac{9}{8}\Omega^2+0.5\lambda_0+\lambda_{3/2}\cos\eta-0.5(\lambda_{1/2c}+\lambda_{1c}^{(2)})(\lambda_{1s}^{(2)}+\lambda_{1/2c})=0\\&-\frac{3}{2}a\Omega-\lambda_{3/2}\sin\eta-0.5(\lambda_{1/2c}+\lambda_{1c}^{(2)})(\lambda_{1s}^{(2)}+\lambda_{1/2c})=0.\end{aligned} \tag{10}$$

Solving eq.(10) by a numerical procedure it is possible to obtain $\Omega_1^{(4)}$ and $\Omega_2^{(4)}$, the frequencies of stable and unstable period four bifurcations.

Now we assume that the Feigenbaum model (Feigenbaum: 1978) of period doubling is valid for our system i.e.:

$$\lim_{n\to\infty}\frac{\Omega_{1,2}^{2^n}-\Omega_{1,2}^{2^{n-1}}}{\Omega_{1,2}^{2^{n+1}}-\Omega_{1,2}^{2^n}}\to\delta \tag{11}$$

where $\delta=4.669...$ is the universal Feigenbaum constant and $n=1,2,....$

Although it has not been proved that all period-doubling bifurcations fulfill Feigenbaum model, there are examples where this model can be taken as a good approximation to the real phenomena (Isomaki: 1986, Steeb and Louw: 1986).

To obtain approximate values of the limits of period-doubling bifurcations (accumulation points) we replaced the limit in (11) by an equality. After it let us indicate

$$\Delta\Omega_{1,2}^{2^n}=\Omega_{1,2}^{2^n}-\Omega_{1,2}^{2^{n-1}}.$$

Now it is easy to show that $\Delta\Omega_{1,2}^{2n},\Delta\Omega_{1,2}^{2n-1},...$ form infinite

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geometrical series with a ratio $1/\delta$. With both stable and unstable period two and period four boundaries using the above described approximation one obtains:

$$\Omega_1^\infty = \Omega_1' + \frac{\Delta\Omega_1}{1-\frac{1}{\delta}}$$
$$\Omega_2^\infty = \Omega_2' + \frac{\Delta\Omega_2}{1-\frac{1}{\delta}} \tag{12}$$

where: $\Delta\Omega_1 = \Omega_1^{(4)} - \Omega_1^{(2)}$, $\Delta\Omega_2 = \Omega_2^{(2)} - \Omega_2^{(4)}$.

The domain where chaotic behaviour can occur is proposed to be between the limits of unstable and stable period-doubling cascades, in the interval $(\Omega_1^{(\infty)}, \Omega_2^{(\infty)})$ and of course to expect chaos one must have:

$$\Omega_1^{(\infty)} < \Omega_2^{(\infty)}.$$

More details about this method can be found in (Kapitaniak: 1990, 1991). This approach besides estimating chaotic region in parameter space evaluates analytically the approximate unstable orbits or at least regions in parameter space where they exist. The above analysis can be used to control eq.(1) by changing parameter Ω in the range $\Omega \in [\Omega\text{-}\Omega^*, \Omega\text{-}\Omega^*]$. It should be noted here that the frequency Ω is this parameter which can be easily changed in real experimental systems modelled by eq.(1). From Fig.2 one can find that by changing Ω by $\Omega^* < 0.12$ one can obtain different types of periodic behaviour; from period-one to theoretically period-2^n ($n=1,2,\ldots$). Periodic orbits with higher order $n>4$ are difficult to obtain as the Ω intervals of the existence of these solutions are very small. The example of controlling a few of periodic orbits are shown in Fig.3. We plot coordinate x of the Poincaré map as a function of discrete time $\hat{t} = 2\pi n/(\Omega+\Omega^*)$, $n=1,2,\ldots$. The frequency perturbations were programmed to control successively four different periodic orbits. The times at ehich we switched the control from stabilizing one periodic orbit to stabilize another are labelled by arrows in Fig.3.

Recently it has been shown that the small additive quasiperiodic noise of the form:

$$h(t) = \sum_{i=1}^{N} A_i \cos(\nu_i t + \phi_i), \tag{13}$$

where $A_i \ll B_{0,1}$ are constant, ν and ϕ are time independent random variables, shifts the period doubling bifurcation point decreasing the zone of existence of each period-2^n solution and can even eliminate chaos (Kapitaniak: 1990).

This phenomenon shows that we can reduce the range in which the control parameter is allowed to vary by adding simultaneously noise into the system i.e.

$$\ddot{x} + a\dot{x} + bx + cx^3 = B_0 + B_1\cos((\Omega+\Omega^*)t) + h(t). \tag{14}$$

Quasiperiodic noise given by eq.(13) is an approximation of the realization of the band-limited white noise stochastic

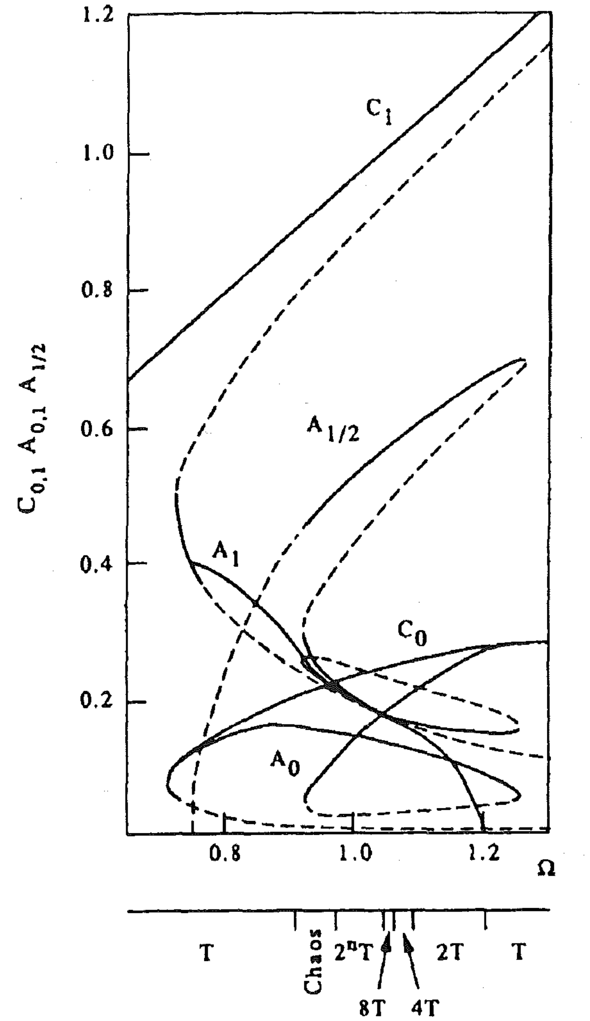

FIG.2. Parameters of period-one and period-two approximate solutions (2) and (7), and the Ω intervals of existence of period-four and period-eight solutions; $a=0.05$, $b=0$, $c=1$, $B_0=0.03$, $B_1=0.16$, solid line indicates stable solutions while broken line indicates unstable solutions

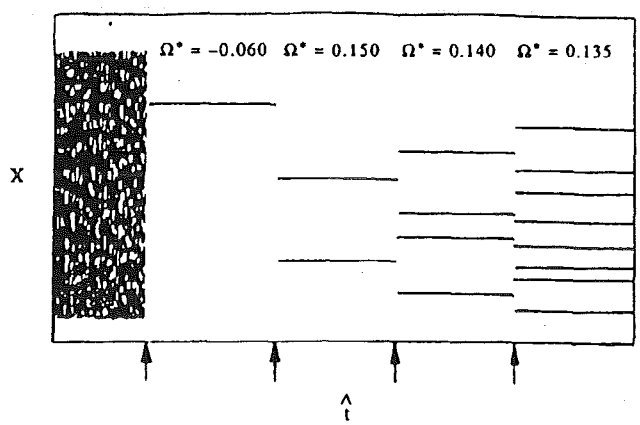

FIG.3 Successive control of period-one, -two, -four, eight orbits; the arrows indicate time switching: $a=0.05$, $b=0$, $c=1$, $B_0=0.03$, $B_1=0.16$, $\Omega=0.97$

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process with zero mean and a spectral density:

$$s(\nu) = \begin{cases} \dfrac{s}{\nu_{max}-\nu_{min}}, & \nu \in [\nu_{min}, \nu_{max}] \\ 0, & \nu \notin [\nu_{min}, \nu_{max}] \end{cases}$$

where s is the intensity of noise and $[\nu_{min}, \nu_{max}]$ is the interval of considered frequencies (Kapitaniak: 1990) and can be easily simulated experimentally.

The example of this type of control is shown in Fig.4. In this case as an effect of control we obtain solutions of an appropriate periods perturbed by noise (13).

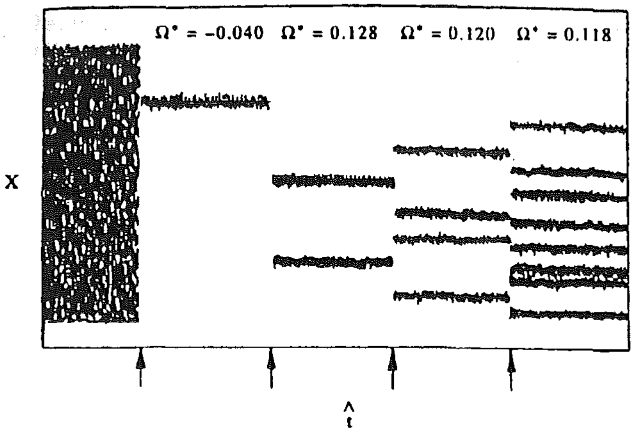

FIG.4. Successive control of period-one, -two, -four, -eight orbits with noise added; the arrows indicate the times of switching; a=0.05, b=0, c=1, B_0=0.03, B_1=0.16, Ω=0.97, A_i=0.004, ν_{min}=0.9, ν_{max}=1.1, N=200

4. CONTROL BY SYSTEM DESIGN

In this section we explore the idea of modifying or removing chaotic behaviour by appropriate system design. It is clear that, to a certain extent, chaos may be "designed out" of a system by appropriate modification of parameters, perhaps corresponding to modification of mass or inertia of moving parts. Equally clearly, there exist strict limits beyond which such modifications can not go without seriously affecting the efficiency of the system itself.

More usefully, it may be possible to affect the control by joining the chaotic system with some other (a small) system. The idea of this method is similar to that of the so-called dynamical vibration absorber. A dynamical vibration absorber is a one-degree of freedom system, usually a mass on a spring (sometimes viscous damping is also added), which is connected to the main system as shown in Fig.5. The additional degree of freedom introduced shifts resonance zones, and in some cases can eliminate oscillations of the main mass. Although such a dynamical absorber can change the overall dynamics substantially, it need usually only be physically small in comparison with the main system, and does not require an increase of excitation force. It can be easily added to the existing system without major changes of design or construction. This contrasts with devices based on feedback control, which can be large and costly.

To explain the role of dynamical absorbers in controlling chaotic behaviour let us consider the Duffing oscillator, coupled with an additional linear system:

$$\ddot{x}+a\dot{x}+bx+cx^3+d(x-y)=B_0+B_1\cos\Omega t \tag{15a}$$

$$\ddot{y}+e(y-x)=0 \tag{15b}$$

where a,b,c,d,e,B and Ω are constant. Here d and e are the characteristic parameters for the absorber, and we take e as control parameter. The parameters of equations (15) are related to those of Fig.5 in the following way: $a=c/m\Omega$, $b=k/m\Omega^2$, $c=k_c/m\Omega^2$, $d=k_a/m\Omega^2$, $e=k_a/m_a\Omega^2$, $B_0=F_0/m\Omega$ and $B_1=F_1/m\Omega$. It should be noted here that parameters d and e are related to each other thought the absorber stiffness k_a. For simplicity in the rest of this paper we assume that d is constant and consider e as control parameter, i.e., we take constant stiffness k_a and allow the absorber mass, m_a, to vary.

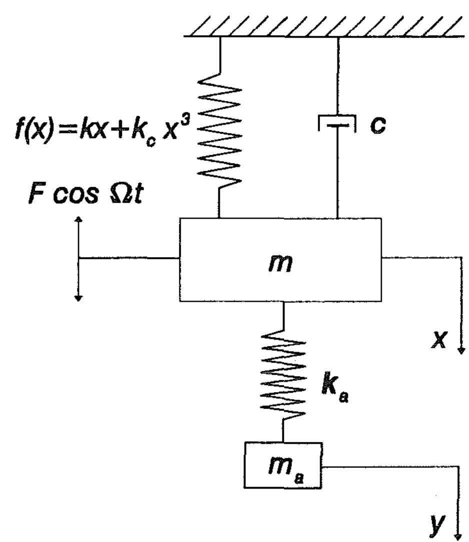

FIG.5 Schematic diagram of the main system and the dynamical absorber

It is well-known that the uncoupled eq.(15.a) (i.e., without the dynamical absorber) shows chaotic behaviour for certain parameter regions (Ueda: 1991, Kapitaniak: 1991). As it was mentioned in the previous section in many cases the route to chaos proceeds via a sequence of period-doubling bifurcations, and in such cases our method provides an easy way of switching between chaotic and periodic behaviour.

To analyze the system with the absorber (present d, $e\neq0$), we first assume that all parameters of eq.(15), excluding the forcing frequency Ω, are constant, and estimate

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the Ω-domain where chaos exists. The application of the harmonic balance method enables us to determine the stability domain of appropriate $2\pi/\Omega=T$ periodic solutions:

$$\begin{aligned} x &= C_0+C_1\cos(\Omega t+\psi) \\ y &= D_0+D_1\cos(\Omega t+\gamma) \end{aligned} \tag{16}$$

and $2T$ periodic solutions:

$$\begin{aligned} x &= A_0+A_{1/2}\cos\left(\frac{\Omega}{2}t+\rho\right)+A_1\cos\Omega t \\ y &= E_0+E_{1/2}\cos\left(\frac{\Omega}{2}t+\beta\right)+E_1\cos\Omega t \end{aligned} \tag{17}$$

where C_0, C_1, D_0, D_1, A_0, $A_{1/2}$, A_1, A_0, $E_{1/2}$, E_1, ψ, γ, ρ and β are constants which are determined by substituting eqs.(16) or (17) into eqs.(15). Approximate boundaries of stability as functions of forcing frequency Ω for each solution can be estimated by adding small perturbations dx and dy to x and y, and considering an appropriate Hill's equation. The whole procedure is similar to one described in the previous section, so we omit details here. Knowing the period-doubling bifurcation values Ω_1' and Ω_2' at which we have bifurcation from $T\rightarrow 2T$ periodic solutions, and Ω_1'' and Ω_2'' at which we have bifurcation from $2T\rightarrow 4T$ periodic solutions, we can obtain approximate values for the accumulation points Ω_1^∞ and Ω_2^∞ from eq.(12) and the interval $[\Omega_1^\infty,\Omega_2^\infty]$ can be considered as an approximation of the Ω frequency domain for which chaos exists.

The above procedure can be easily performed using any symbolic algebra system (we used Mathematica) and by following it for different values of e we are able to obtain a map of behaviour of eqs.(15) as a function of two parameters: the frequency Ω and the dynamical absorber control parameter e, as shown for example in Fig.6 (solid lines). The other parameters of eqs.(15) have been fixed at the values $a=0.077$, $b=0$, $c=1.0$, $B_0=0.045$ and $B_1=0.16$. This plot is in good agreement with numerically obtained behaviour domains as shown in Fig.6 (broken lines). Numerical results were obtained using a fourth order Runge-Kutta method with a time step $\pi/200\Omega$, and to determine chaotic behaviour the Lyapunov exponents were calculated using the algorithm of Wolf et al. (Wolf et al.: 1985).

From Fig.6 it is clear that, for fixed Ω, we can obtain different types of periodic behaviour by making slight changes in e. As an example, consider a system with $\Omega=0.98$. For $e<0.09$, the system is chaotic, but by changing e from 0.01 to 0.16 it is possible to obtain easily T, $2T$, $4T$, $8T$ periodic orbits. Theoretically orbits of higher periods are also possible, but their narrow range of existence makes them difficult to find either experimentally or numerically. What is of vital significance is that values of the parameter $e\in[0.01,0.16]$ can be obtained with an absorber mass m_a approximately 100 times smaller than the main mass (Fig.5).

To show the effectiveness of our method in real experimental conditions we have considered the effect of quasiperiodic noise given by eq.(13) on our system (15).

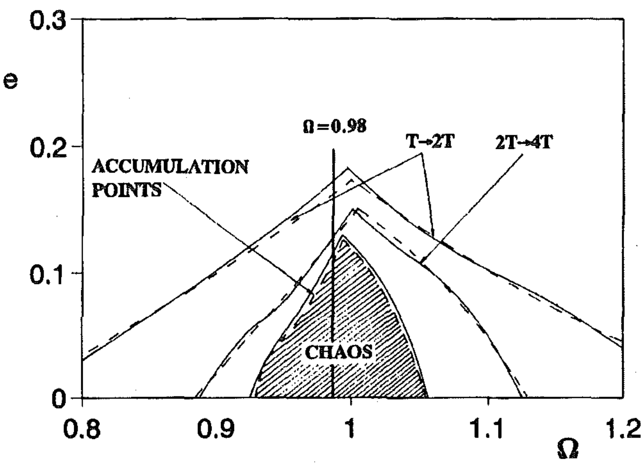

FIG.6 Behaviour of eqs.(15): $a=0.077$, $b=0$, $c=1.0$, $B_0=0.045$ and $B_1=0.16$; analytical approximation: solid line, numerical simulation: broken line

Considering the perturbed system

$$\ddot{x}+a\dot{x}+bx+cx^3+d(x-y)= \\ =B_0+B_1\cos(\Omega t)+\sum_{i=1}^{N}A_i\cos(\nu_i t+\phi_i) \tag{18.a}$$

$$\dot{y}+e(y-x)=0 \tag{18.b}$$

we have found the interesting property that the presence of noise reduces the magnitude of e necessary to obtain an appropriate periodic solution. This property is summarized in Fig.7, where we compare the behaviour of the system (15) for different noise intensity.

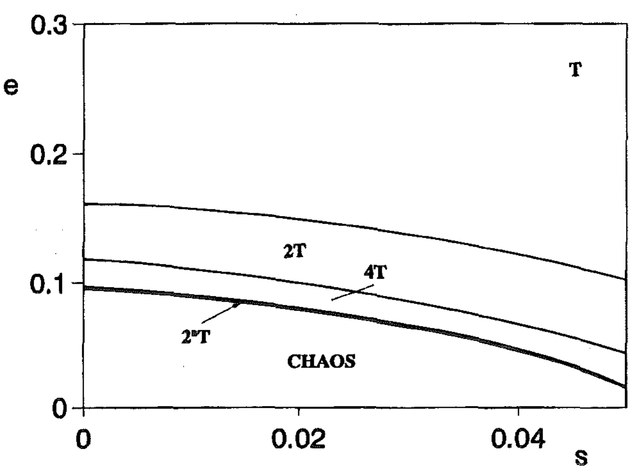

FIG.7 Effect of noise on the behaviour of eqs.(1): $\Omega=0.98$, other parameter values as in Fig.5

A similar controlling effect can be obtained by varying the absorber stiffness i.e., by simultaneous changes of parameters d and e.

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5. DISCUSSION

We have described several methods by which chaotic behaviour in a mechanical system may be modified, displaced in parameter space or removed. The OGY method is extremely general, relying only on the universal property of chaotic attractors that they have embedded within them infinitely many unstable periodic orbits (or even static equilibria). On the other hand the method requires us to follow the trajectory and employ a feedback control system which must be highly flexible and responsive; such a system in the mechanical configuration may be large and expensive. It has the additional disadvantage that small amounts of noise may cause occasional large departures from the desired operating trajectory.

The nonfeedback approach is inevitable much less flexible, and requires more prior knowledge of equations of motion. On the other hand, to apply such a method, we do not have to follow the trajectory. The control procedures can be applied at any time and we can switch from one periodic orbit to another without returning to the chaotic behaviour, although after each switch transient chaos may be observed. The lifetime of this transient chaos strongly depends on initial conditions. Moreover in a nonfeedback method we do not have to wait until the trajectory is close to an appropriate unstable orbit; in some cases this time can be quite long. The dynamic approach can be very useful in mechanical systems, where feedback controllers are often very large (sometimes larger than the control system). In contrast, a dynamical absorber having a mass of order 1% of that of the control system is able, as we have shown in the example of section 4, to convert chaotic behaviour to periodic one over a substantial region of parameter space. Indeed the simplicity by which chaotic behaviour may be changed in this way, and possibility of an easy access to different periodic orbits, may actually motivate the search for and exploitation of chaotic behaviour in practical mechanical systems. This prompts us to pose a final question -how can we exploit chaos in mechanical systems? The OGY method, at least in theory, gives access to the wide range of possible behaviour encompassed by the unstable periodic (and other) orbits embedded in a chaotic attractor. Moreover, the sensitivity of the chaotic regime to both initial conditions and parameter values means that the desired effects may be produced by fine tuning. Thus we might actually wish to design chaos into a system, in order to exploit this adaptability. Nonfeedback methods can, in principle, give us advice on the design, whether we wish to design chaos out or in. Additionally, they enable us to choose regions of design parameter space or operating parameter space within which chaos will occur and be acceptable. An example of practical use might be the minimisation of metal fatigue by switching from a necessary strictly periodic operation of the fully loaded conditions, which repeated stresses applied at the same places, to a noisy periodicity (rather like a healthy heartbeat) under idling conditions.

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