Computation of the Output Torque, Power and Work of the Driving Motor for a Redundant Parallel Manipulator

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ABSTRACT

Inverse dynamic analysis of the 8-PSS redundant parallel manipulator is carried out in the exhaustive decoupled way. The required output of the torque, the power and the work of the driving motor are achieved. The whole actuating torque is divided into four terms which are caused by the acceleration, the velocity, the gravity, and the external force. It is also decoupled into the components contributed by the moving platform, the strut, the slider, the lead screw, the motor rotor-coupler, and the external force. The required powers contributed by the component of torque caused by the acceleration term, the velocity term, the gravity term, the external force term, and the powers contributed by the moving platform, the strut, the slider, the lead screw, and the motor rotor-coupler are computed respectively. For a prescribed trajectory, the required output work generated by the ith driving motor is obtained by the presented numerical integration method. Simulation for the computation of the driving motor's output torque, power and work is illustrated.

Keywords: *Inverse Dynamics, Power, Redundant Parallel Manipulator, Torque, Work*

INTRODUCTION

There are mainly two different types of redundancy for the parallel manipulators: a) kinematic redundancy and b) actuation redundancy. A parallel manipulator is said to be kinematically redundant manipulator when its mobility of the mechanism is greater than the required degrees of freedom of the moving platform. On the other hand, a parallel manipulator is called redundantly actuated manipulator when the number of actuators is greater than the mobility of the mechanism. It is believed that redundancy can improve the ability and performance of parallel manipulator (Kim, 2001; Merlet, 1996; Nokleby, 2005; Wang, 2004; Cheng, 2003; Müller, 2005; Mohamed, 2005; Ebrahimi, 2007; Zhao, 2009). There are some advantages for the redundant parallel manipulator such as avoiding kinematic singularities, increasing workspace, improving dexterity, enlarging load capability and so on. It is shown that the redundantly actuated parallel manipulator has a better dynamic characteristic than its non-redundant

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counterpart considering the presented dynamic index (Zhao, 2009). Redundant actuation in a parallel manipulator can be implemented by the following approaches. The first one is to actuate some of the passive joints within the branches of parallel manipulator. The second one is to add some additional branches beyond the minimum necessary to actuate the parallel manipulator. The last one can be the hybrid of the above two approaches.

The dynamics of the manipulator are of importance in the areas of simulation, control and dynamic optimum design. Many works have been done on the dynamics of parallel manipulators. Several approaches, including the Newton-Euler formulation (Carvalho, 2001; Dasgupta, 1998), the Lagrangian formulation (Lee, 1988; Miller, 1992), the Kane formulation (Ben-Horin, 1998; Liu, 2000) and the virtual work principle (Li, 2005; Sokolov, 2007; Wang, 1998; Tsai, 2000; Zhu, 2005; Staicu, 2009) have been applied to the dynamics analysis of parallel manipulators. In fact, the inverse dynamics of parallel manipulators involve almost all of the mechanics principles. Along with these mechanics principles, many mathematic methods such as screw theory (Gallardo, 2003), Lie algebra (Muller, 2003), natural orthogonal complement (Khan, 2005), motor algebra (Sugimoto, 1987), group theory (Geike, 2003), symbolic programming (Geike, 2003; McPhee, 2002), geometric approach (Selig, 1999), parallel computational algorithms (Gosselin, 1996) and system identification (Wiens, 2002) have also been adopted to the dynamics of parallel manipulators. However, the results computed by different methods have been shown to be equivalent.

Though much attention has been paid to the dynamics of the parallel manipulator, little work has been done on the dynamics of the redundant parallel manipulator (Cheng, 2003), especially in the exhaustive decoupled way. Furthermore, maybe no paper has dealt with the required output power and work of the driving motor when the redundant parallel manipulator moves on a prescribed trajectory. It is one of the motivations for this work. By taking the 8-PSS redundant parallel manipulator as an object of study, this paper presents the inverse dynamic analysis in the exhaustive decoupled way. The actuating torques caused by the following term: acceleration, velocity, gravity, external force, moving platform, strut, slider, lead screw and motor rotor-coupler are computed respectively. The required output powers generated by the motor corresponding to the above terms of torque are achieved. For a prescribed trajectory, the computation of the required output work generated by the *i*th driving motor is implemented by the numerical integration. The paper is organized as follows: the description and the dynamic model of the redundant parallel manipulator are presented in section two. The rigid dynamic model is decoupled in the exhaustive decoupled way. The computation of the required output power and work of the driving motor are given in section three. Investigations of the system dynamic characteristics through simulation and conclusions are presented in section four and section five, respectively.

SYSTEM DESCRIPTION AND DYNAMIC MODEL

System Description

As shown in Figure 1 and Figure 2, the 8-PSS redundant parallel manipulator consists of a moving platform and eight sliders. In each kinematic chain, the platform and the slider are connected via spherical ball bearing joints by a strut of fixed length. Each slider is driven by a DC motor via a linear ball screw. So the 8-PSS is an out actuated redundant parallel manipulator. The lead screws of B_1, B_2, B_3 and B_4 are vertical to the ground. The lead screws of B_5, B_6 and the lead screws of B_7, B_8 are parallel with the ground. They are orthogonal to each other.

For the purpose of analysis, the following coordinate systems shown in Figure 1 and Figure 2 are defined: the coordinate system $O-xyz$ is attached to the fixed base and an-

Figure 1. Diagram of an 8-PSS redundant parallel manipulator

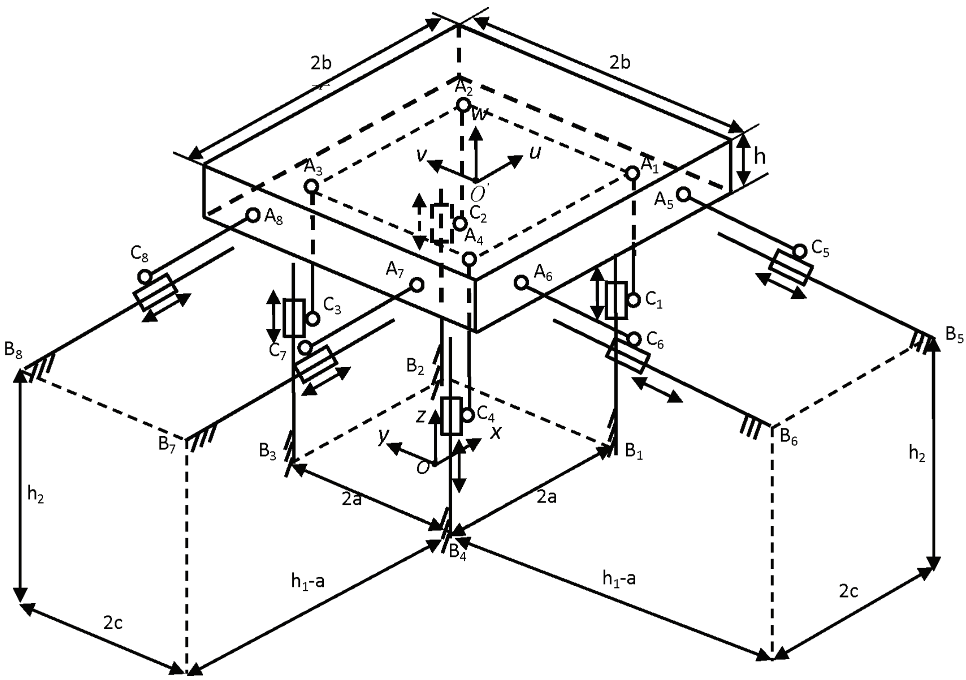


Figure 2. Vector diagram of a PSS kinematic chain

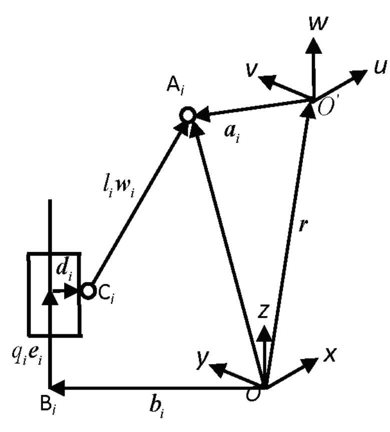
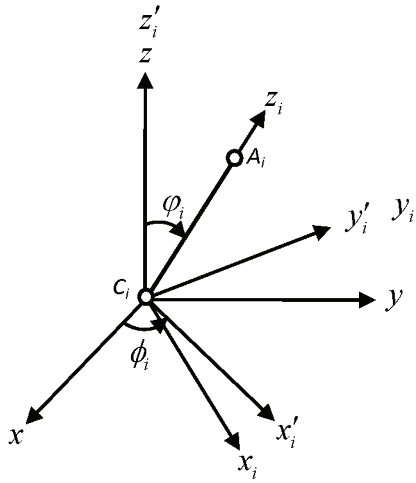


Figure 3. The local coordinate system of the ith strut



other moving coordinate frame $O' - uvw$ is located at the center of mass of the moving platform. The orientation of the moving frame with respect to the fixed frame is described by the matrix which is a function of three rotation angles ϕ_x, ϕ_y and ϕ_z about the fixed x, y and z axis. So the angular velocity of the moving platform is given by (Tsai, 2000)

$$\omega = \begin{bmatrix} \dot{\phi}_x & \dot{\phi}_y & \dot{\phi}_z \end{bmatrix}^T \tag{1}$$

The orientation of each kinematic strut with respect to the fixed base can be described by two Euler angles. As shown in Figure 3, the local coordinate system of the *i*th strut can be thought of as a rotation of ϕ_i about the z axis resulting in a $C_i - x_i'y_i'z_i'$ system followed by another rotation of φ_i about the rotated y_i' axis. So the rotation matrix of the *i*th strut can be written as

$$\mathbf{R}_i = \mathrm{Rot}(z, \phi_i)\,\mathrm{Rot}(y_i', \varphi_i) \tag{2}$$

The unit vector along the strut in the coordinate system $O - xyz$ is

$$\mathbf{w}_i = \mathbf{R}_i\,{}^i\mathbf{w}_i = \mathbf{R}_i \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{c}\,\phi_i\,\mathrm{s}\,\varphi_i \\ \mathrm{s}\,\phi_i\,\mathrm{s}\,\varphi_i \\ \mathrm{c}\,\varphi_i \end{bmatrix} \tag{3}$$

So the Euler angles ϕ_i and φ_i can be computed as follows

$$\begin{cases} \mathrm{c}\,\varphi_i = \mathbf{w}_{iz} \\ \mathrm{s}\,\varphi_i = \sqrt{\mathbf{w}_{ix}^2 + \mathbf{w}_{iy}^2},\ (0 \le \varphi_i < \text{À}) \\ \mathrm{s}\,\phi_i = \mathbf{w}_{iy} \,/\, \mathrm{s}\,\varphi_i \\ \mathrm{c}\,\phi_i = \mathbf{w}_{ix} \,/\, \mathrm{s}\,\varphi_i \\ \text{if } \varphi_i = 0, \text{ then } \phi_i = 0 \end{cases} \tag{4}$$

Dynamic Model

The rigid dynamic model of the 8-PSS redundant parallel manipulator had been formulated by means of the principle of virtual work and the concept of the link Jacobian matrices (Zhao, 2009) while considering the rotation inertia of the lead screw and the rotation inertia of motor rotor and coupler. There are eight unknown quantities in the six linear consistent equations for the 8-PSS redundant parallel manipulator. The most common strategy to solve this kind of problem with infinite solutions is by minimizing the Euclidean norm of the actuating forces. The rigid dynamic model of the two parallel manipulators can be expressed as

$$\begin{aligned}\tau &= -\mathbf{A}^{-T}\left(\mathbf{J}^T\right)^+(\mathbf{Q}_p + \sum_{i=1}^{8}\mathbf{J}_{iv\omega}^T\,{}^i\mathbf{Q}_i + \mathbf{J}^T\mathbf{f}_q) + \mathbf{I}_{LCM}\ddot{\theta} \\ &= -\mathbf{A}^{-T}\left(\mathbf{J}^T\right)^+(\mathbf{Q}_p + \sum_{i=1}^{8}\mathbf{J}_{iv\omega}^T\,{}^i\mathbf{Q}_i + \mathbf{J}^T\mathbf{f}_q) + \mathbf{I}_{LCM}\mathbf{A}\ddot{\mathbf{q}}\end{aligned} \tag{5}$$

The physical significance of Eq. (5) is that among the possible actuating torque vectors, the optimum solution is with minimum norm and least quadratic sum. A is the actuating torques, $\mathbf{J}^+$ is the Moore-Penrose inverse of the Jacobian matrix and when $\mathbf{J} \in \mathbf{R}_m^{m\times n}$, $\mathbf{J}^+ = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1}$, $\mathbf{J}_{iv\omega}$ is the *link Jacobian matrix* which maps the velocity of the moving platform into the velocity of the *i*th strut in the $C_i - x_iy_iz_i$ coordinate system, τ is the actuating torques. and can be expressed as:

$$\mathbf{J} = \mathrm{diag}\left(\frac{1}{\mathbf{w}_1^T\mathbf{e}_1}, \frac{1}{\mathbf{w}_2^T\mathbf{e}_2}, \cdots, \frac{1}{\mathbf{w}_i^T\mathbf{e}_i}, \cdots, \frac{1}{\mathbf{w}_n^T\mathbf{e}_n}\right) \begin{bmatrix} \mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_i, \cdots, \mathbf{w}_n \\ \mathbf{a}_1\times\mathbf{w}_1, \mathbf{a}_2\times\mathbf{w}_2, \cdots, \mathbf{a}_i\times\mathbf{w}_i, \cdots, \mathbf{a}_n\times\mathbf{w}_n \end{bmatrix}^T$$
$$i = 1, 2, \cdots, n;\ \text{or } 8 \tag{6a}$$

$$\mathbf{J}_{iv\omega} = \begin{bmatrix} \left[{}^i\mathbf{R}_o \quad -S({}^i\mathbf{a}_i)\,{}^i\mathbf{R}_o\right] + \frac{l_i}{2}S({}^i\mathbf{w}_i)\mathbf{J}_{i\omega} \\ \frac{1}{l_i}\left\{ \begin{matrix} \left[S({}^i\mathbf{w}_i)\,{}^i\mathbf{R}_o \quad -S({}^i\mathbf{w}_i)S({}^i\mathbf{a}_i)\,{}^i\mathbf{R}_o\right] - \\ ({}^i\mathbf{w}_i \times {}^i\mathbf{e}_i)\begin{bmatrix} \frac{\mathbf{w}_i^T}{\mathbf{w}_i^T\mathbf{e}_i} & \frac{(\mathbf{a}_i\times\mathbf{w}_i)^T}{\mathbf{w}_i^T\mathbf{e}_i} \end{bmatrix} \end{matrix} \right\} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{iv} \\ \mathbf{J}_{i\omega} \end{bmatrix} \tag{6b}$$

$$S({}^i\mathbf{w}_i) = \begin{bmatrix} 0 & -{}^iw_{iz} & {}^iw_{iy} \\ {}^iw_{iz} & 0 & -{}^iw_{ix} \\ -{}^iw_{iy} & {}^iw_{ix} & 0 \end{bmatrix} \tag{6c}$$

$$S({}^i\mathbf{a}_i) = \begin{bmatrix} 0 & -{}^ia_{iz} & {}^ia_{iy} \\ {}^ia_{iz} & 0 & -{}^ia_{ix} \\ -{}^ia_{iy} & {}^ia_{ix} & 0 \end{bmatrix} \tag{6d}$$

In Eq. (5), $\mathbf{Q}_P$ is the resultant of applied and inertia forces exerted at the center of mass of the moving platform

$$\mathbf{Q}_P = \begin{bmatrix} \mathbf{f}_P \\ \mathbf{n}_P \end{bmatrix} = \begin{bmatrix} \mathbf{f}_e + m_p\mathbf{g} - m_p\dot{\mathbf{v}} \\ \mathbf{n}_e - {}^o\mathbf{I}_p\dot{\omega} - \omega\times({}^o\mathbf{I}_p\omega) \end{bmatrix} \tag{7}$$

where $\mathbf{f}_e$ and $\mathbf{n}_e$ are the external force and moment exerted at the center of mass of the moving platform, ${}^o\mathbf{I}_p = {}^o\mathbf{R}_{o'}\,{}^{o'}\mathbf{I}_p\,{}^{o'}\mathbf{R}_o$ is the inertia matrix of the moving platform taken about the center of mass expressed in the $O - xyz$ coordinate system and m_p is its mass. $\mathbf{g}$ is the gravity acceleration. ω, $\dot{\mathbf{v}}$ and $\dot{\omega}$ are the angular velocity, the linear and angular acceleration of the moving platform, respectively. In Eq. (5), ${}^i\mathbf{Q}_i$ is the resultant of applied and inertia forces exerted at the center of the *i*th strut and expressed in the $C_i - x_iy_iz_i$ coordinate system

$${}^i\mathbf{Q}_i = \begin{bmatrix} {}^i\mathbf{f}_i \\ {}^i\mathbf{n}_i \end{bmatrix} = \begin{bmatrix} m_i\,{}^i\mathbf{R}_o\mathbf{g} - m_i\,{}^i\dot{\mathbf{v}}_i \\ -{}^i\mathbf{I}_i\,{}^i\dot{\omega}_i - {}^i\omega_i\times({}^i\mathbf{I}_i\,{}^i\omega_i) \end{bmatrix} \tag{8}$$

where ${}^i\mathbf{I}_i$ is the inertia matrix of the *i*th cylindrical strut about its center of mass expressed in the $C_i - x_iy_iz_i$ coordinate system and m_i is its mass. ${}^i\dot{\mathbf{v}}_i$, ${}^i\omega_i$ and ${}^i\dot{\omega}_i$ are the linear acceleration, the angular velocity and acceleration of the *i*th strut expressed in the $C_i - x_iy_iz_i$ coordinate system respectively, and can be given by:

$${}^i\omega_i = \mathbf{J}_{i\omega}\begin{bmatrix} \mathbf{v} \\ \omega \end{bmatrix} \tag{9}$$

$$ {}^{i}\dot{\omega}_{i} = \mathbf{J}_{i\omega}\begin{bmatrix}\dot{\mathbf{v}}\\ \dot{\omega}\end{bmatrix} + \frac{1}{l_{i}}(\Delta_{1} + \Delta_{2}) \tag{10} $$

$$ \Delta_{1} = -\frac{({}^{i}\mathbf{w}_{i} \times {}^{i}\mathbf{e}_{i})}{\mathbf{w}_{i}^{T}\mathbf{e}_{i}}((\mathbf{w}_{i}^{T}\omega)(\mathbf{a}_{i}^{T}\omega) - (\mathbf{w}_{i}^{T}\mathbf{a}_{i})(\omega^{T}\omega) + l_{i}\left|\omega_{i} \times \mathbf{w}_{i}\right|^{2}) \tag{11} $$

$$ \Delta_{2} = ({}^{i}\omega_{i}^{T}\,{}^{i}\mathbf{a}_{i})({}^{i}\mathbf{w}_{i} \times {}^{i}\omega_{i}) - ({}^{i}\omega^{T}\,{}^{i}\omega)({}^{i}\mathbf{w}_{i} \times {}^{i}\mathbf{a}_{i}) \tag{12} $$

$$ {}^{i}\dot{\mathbf{v}}_{i} = \mathbf{J}_{iv}\begin{bmatrix}\dot{\mathbf{v}}\\ \dot{\omega}\end{bmatrix} + S({}^{i}\omega_{i})S({}^{i}\omega_{i}){}^{i}\mathbf{a}_{i} + \frac{1}{2}S({}^{i}\mathbf{w}_{i})(\Delta_{1} + \Delta_{2}) - \frac{l_{i}}{2}S({}^{i}\omega_{i})S({}^{i}\omega_{i})\,{}^{i}\mathbf{w}_{i} \tag{13} $$

In Eq. (5), $\mathbf{f}_{q}$ is the resultant of applied and inertia forces exerted at the center of the slider expressed in the $O - xyz$ coordinate system:

$$ \mathbf{f}_{q} = \begin{bmatrix} f_{q1} & f_{q2} & f_{q3} & f_{q4} & f_{q5} & f_{q6} & f_{q7} & f_{q8} \end{bmatrix}^{T} \tag{14} $$

and

$$ f_{qi} = (m_{qi}\mathbf{g} - m_{qi}\ddot{\mathbf{q}}_{i})^{T}\mathbf{e}_{i} \tag{15} $$

where m_{qi} and $\ddot{\mathbf{q}}_{i}$ are the mass and the acceleration of the slider. $\ddot{q}_{i}$ is the acceleration of the slider:

$$ \ddot{q}_{i} = \frac{1}{\mathbf{w}_{i}^{T}\mathbf{e}_{i}}(\mathbf{w}_{i}^{T}\dot{\mathbf{v}} + (\mathbf{a}_{i} \times \mathbf{w}_{i})^{T}\dot{\omega} + \mathbf{w}_{i}^{T}(\omega \times (\omega \times \mathbf{a}_{i})) - \mathbf{w}_{i}^{T}(\omega_{i} \times (\omega_{i} \times l_{i}\mathbf{w}_{i}))) \tag{16} $$

In Eq. (5), $\mathbf{I}_{LCM}$ is the diagonal inertia matrix (for Equation 17, see Box 1):

$$ I_{LCMi} = I_{Li} + I_{Ci} + I_{Mi} \tag{18} $$

I_{Li}, I_{Ci}, and I_{Mi} are the rotary inertia of the lead screw, coupler and motor rotor, respectively.

In Eq. (5), $\ddot{\theta}$ is the diagonal matrix of the angular acceleration of the screw-coupler-rotor

$$ \begin{aligned} \ddot{\theta} &= \begin{bmatrix} \ddot{\theta}_{1} & \ddot{\theta}_{2} & \ddot{\theta}_{3} & \ddot{\theta}_{4} & \ddot{\theta}_{5} & \ddot{\theta}_{6} & \ddot{\theta}_{7} & \ddot{\theta}_{8} \end{bmatrix}^{T} \\ &= \mathrm{diag}\begin{pmatrix} \frac{2\pi}{p_{1}} & \frac{2\pi}{p_{2}} & \frac{2\pi}{p_{3}} & \frac{2\pi}{p_{4}} & \frac{2\pi}{p_{5}} & \frac{2\pi}{p_{6}} & \frac{2\pi}{p_{7}} & \frac{2\pi}{p_{8}} \end{pmatrix}\ddot{\mathbf{q}} \\ &= \mathbf{A}\ddot{\mathbf{q}} \end{aligned} \tag{19} $$

where $p_{i} = 0.05\mathrm{m}^{-1}$ is the lead of the linear ball screw. Other than the speed reduction caused by the pitch of the lead screws there is no speed reducer for the redundant parallel manipulator, otherwise the reduction ration should be included in Eq. (19).

Substituting Eq. (7), Eq. (8) and Eq. (14) into Eq. (5) yields (see Box 2), where $\mathbf{0} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{T}$. Simplifying the inverse dynamics model of the redundant parallel manipulator yields

$$ \begin{aligned} \tau &= \mathbf{M}(\mathbf{X})\ddot{\mathbf{X}} + \mathbf{V}(\mathbf{X},\dot{\mathbf{X}}) + \mathbf{g}(\mathbf{X}) - \mathbf{A}^{-T}\mathbf{J}^{-T}\begin{bmatrix}\mathbf{f}_{e}\\ \mathbf{n}_{e}\end{bmatrix} \\ &= \tau_{a} + \tau_{v} + \tau_{g} + \tau_{e} \end{aligned} \tag{21} $$

Box 1.

$$ \boldsymbol{I}_{LCM} = \mathrm{diag}\left(I_{LCM1} \quad I_{LCM2} \quad I_{LCM3} \quad I_{LCM4} \quad I_{LCM5} \quad I_{LCM6} \quad I_{LCM7} \quad I_{LCM8} \right) \tag{17} $$

Box 2.

$$\begin{aligned}
\tau = & -\mathbf{A}^{-T}\left(\mathbf{J}^{T}\right)^{+}\begin{bmatrix}\mathbf{f}_{e}\\ \mathbf{n}_{e}\end{bmatrix} - \mathbf{A}^{-T}\left(\mathbf{J}^{T}\right)^{+}\left\{\begin{bmatrix}m_{p}\mathbf{g}\\ \mathbf{0}\end{bmatrix} + \sum_{i=1}^{8}\mathbf{J}_{iv\omega}^{T}\begin{bmatrix}m_{i}\,{}^{i}\mathbf{R}_{o}\mathbf{g}\\ \mathbf{0}\end{bmatrix}\right.\\
& +\mathbf{J}^{T}\left[(m_{q1}\mathbf{g})^{T}\mathbf{e}_{1}\quad (m_{q2}\mathbf{g})^{T}\mathbf{e}_{2}\quad (m_{q3}\mathbf{g})^{T}\mathbf{e}_{3}\quad (m_{q4}\mathbf{g})^{T}\mathbf{e}_{4}\right.\\
& \left.\left.(m_{q5}\mathbf{g})^{T}\mathbf{e}_{5}\quad (m_{q6}\mathbf{g})^{T}\mathbf{e}_{6}\quad (m_{q7}\mathbf{g})^{T}\mathbf{e}_{7}\quad (m_{q8}\mathbf{g})^{T}\mathbf{e}_{8}\right]^{T}\right\}\\
& +\mathbf{A}^{-T}\left(\mathbf{J}^{T}\right)^{+}\left\{\begin{bmatrix}m_{p}\dot{\mathbf{v}}\\ {}^{o}\mathbf{I}_{p}\dot{\omega}\end{bmatrix} + \sum_{i=1}^{8}\mathbf{J}_{iv\omega}^{T}\begin{bmatrix}m_{i}\,{}^{i}\dot{\mathbf{v}}_{i}\\ {}^{i}\mathbf{I}_{i}\,{}^{i}\dot{\omega}_{i}\end{bmatrix}\right.\\
& \left.+\mathbf{J}^{T}\left[m_{q1}\ddot{q}_{1}\quad m_{q2}\ddot{q}_{2}\quad m_{q3}\ddot{q}_{3}\quad m_{q4}\ddot{q}_{4}\quad m_{q5}\ddot{q}_{5}\quad m_{q6}\ddot{q}_{6}\quad m_{q7}\ddot{q}_{7}\quad m_{q8}\ddot{q}_{8}\right]^{T}\right\} + \mathbf{I}_{LCM}\mathbf{A}\ddot{\mathbf{q}}\\
& +\mathbf{A}^{-T}\left(\mathbf{J}^{T}\right)^{+}\left\{\begin{bmatrix}\mathbf{0}\\ \omega\times({}^{o}\mathbf{I}_{p}\omega)\end{bmatrix} + \sum_{i=1}^{8}\mathbf{J}_{iv\omega}^{T}\begin{bmatrix}\mathbf{0}\\ {}^{i}\omega_{i}\times({}^{i}\mathbf{I}_{i}\,{}^{i}\omega_{i})\end{bmatrix}\right\}
\end{aligned} \tag{20}$$

where $\mathbf{X}$ is the pose and position of the moving platform and given by (for Equation 25 see Box 3):

$$\mathbf{X} = \begin{bmatrix}x & y & z & \phi_{x} & \phi_{y} & \phi_{z}\end{bmatrix}^{T} \tag{22}$$

$$\dot{\mathbf{X}} = \begin{bmatrix}\mathbf{v}\\ \omega\end{bmatrix} \tag{23}$$

$$\ddot{\mathbf{X}} = \begin{bmatrix}\dot{\mathbf{v}}\\ \dot{\omega}\end{bmatrix} \tag{24}$$

is the generalized inertia matrix of the manipulator which maps the acceleration of the moving platform into the actuating torques. $\mathbf{E}_{3}$ denotes the 3×3 unit matrix. $\tau_{a}, \tau_{v}, \tau_{g}$ and τ_{e} are the acceleration term, the velocity term, the gravity term and the external force term of the inverse dynamic equations respectively.

From Eq. (20), it is shown that the actuating torques are also determined by the inertia parameters of the moving platform, the strut, the slider, the lead screw, the motor rotor-coupler and the external forces. So Eq. (20) can be written as:

$$\tau = \tau_{mov} + \tau_{strut} + \tau_{slider} + \tau_{lead\ screw} + \tau_{motor\ rotor-coupler} + \tau_{e} \tag{26}$$

COMPUTATIONS OF THE REQUIRED OUTPUT POWER AND WORK OF THE DRIVING MOTOR

The instantaneous required power of the *i*th actuated joint is

$$P_{i} = \tau_{i}\dot{q}_{i} \tag{27}$$

Box 3.

$$\begin{aligned}
\mathbf{M}(\mathbf{X}) = & \mathbf{A}^{-T}\left(\mathbf{J}^{T}\right)^{+}\left(\begin{bmatrix}m_{p}\mathbf{E}_{3} & \\ & {}^{o}\mathbf{I}_{p}\end{bmatrix} + \sum_{i=1}^{8}\mathbf{J}_{iv}^{T}m_{i}\mathbf{J}_{iv} + \sum_{i=1}^{8}\mathbf{J}_{i\omega}^{T}\,{}^{i}\mathbf{I}_{i}\mathbf{J}_{i\omega}\right)\\
& +\mathbf{A}^{-T}\operatorname{diag}\left(m_{q1}\quad m_{q2}\quad m_{q3}\quad m_{q4}\quad m_{q5}\quad m_{q6}\quad m_{q7}\quad m_{q8}\right)\mathbf{J} + \mathbf{A}\mathbf{I}_{LCM}\mathbf{J}
\end{aligned} \tag{25}$$

Table 1. The parameters of the base platform (unit: m)

	1	*2*	*3*	*4*	*5*
x_{Bi} y_{Bi} z_{Bi}	0.400000 -0.400000 0.000000	0.400000 0.400000 0.000000	-0.400000 0.400000 0.000000	-0.400000 -0.400000 0.000000	0.400000 -2.000000 1.500000
6	*7*	*8*			
-0.400000 -2.000000 1.500000	-2.000000 -0.400000 1.500000	-2.000000 0.400000 1.500000			

Table 2. The parameters of the moving platform which are measured in the coordinate frame $O' - uvw$ (unit: m)

	1	*2*	*3*	*4*	*5*
x_{Ai} y_{Ai} z_{Ai}	0.400000 -0.400000 -0.166000	0.400000 0.400000 -0.166000	-0.400000 0.400000 -0.166000	-0.400000 -0.400000 -0.166000	0.400000 -0.681000 -0.037500
6	*7*	*8*			
-0.400000 -0.681000 -0.037500	-0.681000 -0.400000 -0.037500	-0.681000 0.400000 -0.037500			

Table 3. The length of the strut C_iA_i (unit: m)

	1	*2*	*3*	*4*	*5*
l_i	1.000000	1.000000	1.000000	1.000000	1.000000
6	*7*	*8*			
1.000000	1.000000	1.000000			

Table 4. The mass parameters of the manipulator (unit: kg)

	1	2	3	4	5	6	7	8
m_i	20	20	20	20	20	20	20	20
m_{qi}	50	50	50	50	50	50	50	50

where τ_i and $\dot{q}_i$ are the input torques and the angular velocity of the *i*th actuated joint.

Substituting Eq. (21) and Eq. (26) into Eq. (27) yields:

$$P_i = (\tau_{ia} + \tau_{iv} + \tau_{ig} + \tau_{ie})\dot{q}_i = P_{ia} + P_{iv} + P_{ig} + P_{ie} \tag{28}$$

and

$$P_i = \begin{pmatrix} \tau_{imov} + \tau_{istrut} + \tau_{islider} + \\ \boldsymbol{\tau}_{ilead\ screw} + \boldsymbol{\tau}_{imotor\ rotor-coupler} + \tau_{ie} \end{pmatrix} \dot{q}_i = P_{imov} + P_{istrut} + P_{islider} + P_{ilead\ screw} + P_{imotor\ rotor-coupler} + P_{ie} \tag{29}$$

where P_{ia}, P_{iv}, P_{ig} and P_{ie} are the required power of the *i*th actuated joint caused by the acceleration term, the velocity term, the gravity term and the external force term of the actuating torques, respectively. P_{imov}, P_{istrut}, $P_{islider}$, $P_{ilead\ screw}$ and $P_{imotor\ rotor-coupler}$ are the respective power term caused by the inertia of the moving platform, the strut, the slider, the lead screw and the motor rotor-coupler.

For the actuated joint, the sign of the required power is determined by the signs of the actuated joint's angular velocity and the torque which is exerted at the joint by the motor. However, the power generated by the driving motor should be regarded as positive since energy consumption is always necessary. So the required output work generated by the *i*th driving motor for a specific motion period can be achieved

$$W_i = \int_{t_s}^{t_e} \left|P_i\right| dt \tag{30}$$

where t_s and t_e are the start time and the end time of the motion.

The required output work generated by the *i*th driving motor can be implemented by the following numerical integration technique

$$W_i = \frac{\Delta t}{2}\left[\left|P_i(t_s)\right| + \left|P_i(t_e)\right| + 2\sum_{k=1}^{num-1}\left|P_i(t_k)\right|\right] \tag{31}$$

where

$$\Delta t = \frac{t_e - t_s}{num} \tag{32}$$

num is the number of the intervals. In general

$$num = 2^n \tag{33}$$

where n is a positive integral. The required approximation error of the computation of the output work should satisfy

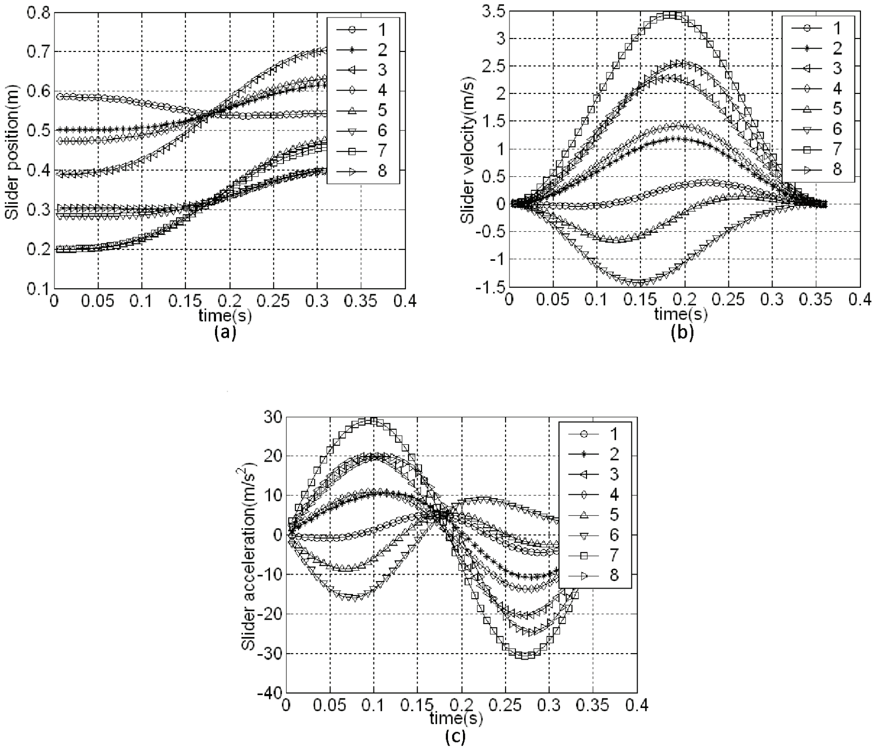
$$\left|W_i(num = 2^{n_1}) - W_i(num = 2^{n_1 - 1})\right| \le e \tag{34}$$

where e is the desired approximation error. n_1 is the positive integral adopted by the numerical integration.

SIMULATION

In this section, a numerical example for the inverse dynamics analysis of the 8-PSS redundant parallel manipulator in the exhaustive way is presented. The simulation is implemented by using the MATLAB software. The parameters

Figure 4. Variations of slider position (a), velocity (b) and acceleration (c) vs time



of the manipulator used for the simulation are given in Table 1 through Table 4.

The mass of the moving platform is $m_p = 200\text{kg}$. The inertia parameters used in the simulation are chosen as:

$$
{}^{o'}\mathbf{I}_p = \begin{bmatrix} 17.333 & 0 & 0 \\ 0 & 17.333 & 0 \\ 0 & 0 & 33.333 \end{bmatrix} \text{kg}\cdot\text{m}^2 ,
$$

$$
{}^{i}\mathbf{I}_i = \begin{bmatrix} 1.279 & 0 & 0 \\ 0 & 1.279 & 0 \\ 0 & 0 & 0.005 \end{bmatrix} \text{kg}\cdot\text{m}^2 ,
$$

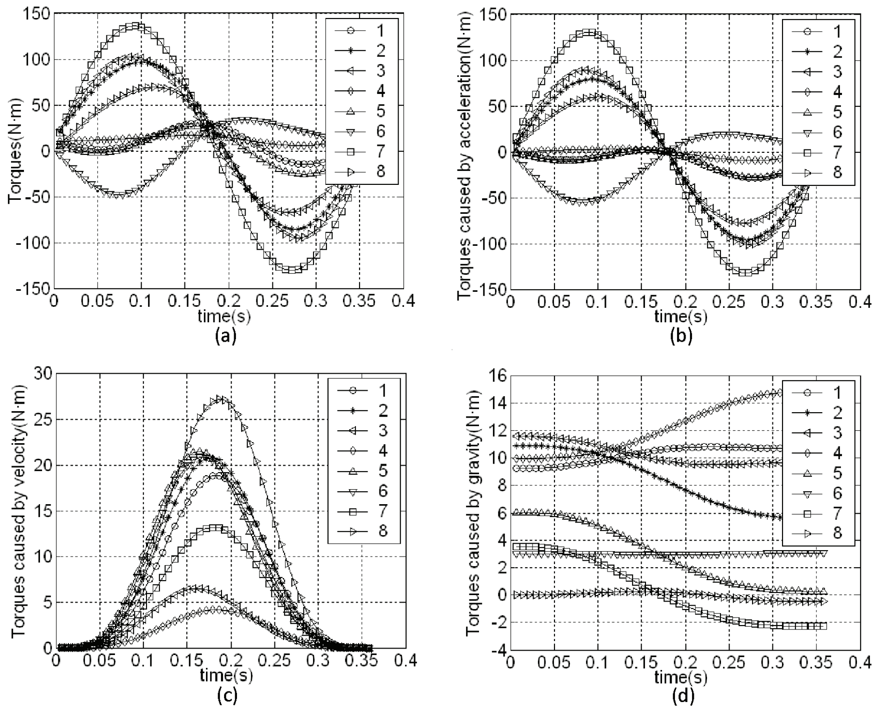
$$
I_{Li} = 10.5 \times 10^{-4}\text{kg}\cdot\text{m}^2 ,
$$

$I_{Ci} + I_{Mi} = 248 \times 10^{-4}\text{kg}\cdot\text{m}^2$. Another parameter used in the simulation is given as $d_i = 0.244\text{m}$.

The motion of the moving platform used in the numerical simulation is expressed as:

$$
\begin{cases}
x = -0.1 + \dfrac{a_{\max}T^2}{2\pi}\left(\tau - \dfrac{1}{2\pi}\sin\left(2\pi\tau\right)\right) \\
y = -0.1 + \dfrac{a_{\max}T^2}{2\pi}\left(\tau - \dfrac{1}{2\pi}\sin\left(2\pi\tau\right)\right) \\
z = 1.644 + \dfrac{a_{\max}T^2}{2\pi}\left(\tau - \dfrac{1}{2\pi}\sin\left(2\pi\tau\right)\right) \\
\phi_x = -0.1 + \dfrac{a_{\max}T^2}{2\pi}\left(\tau - \dfrac{1}{2\pi}\sin\left(2\pi\tau\right)\right) \\
\phi_y = -0.1 + \dfrac{a_{\max}T^2}{2\pi}\left(\tau - \dfrac{1}{2\pi}\sin\left(2\pi\tau\right)\right) \\
\phi_z = -0.1 + \dfrac{a_{\max}T^2}{2\pi}\left(\tau - \dfrac{1}{2\pi}\sin\left(2\pi\tau\right)\right)
\end{cases}
\tag{35}
$$

Figure 5. Variations of whole actuating torques (a), torques caused by acceleration term (b), torques caused by the velocity term (c) and torques caused by the gravity term (d) vs time



where $a_{\max} = 9.8\mathrm{m/s^2}, \tau = \dfrac{t}{T}, T = \sqrt{\dfrac{2\pi S}{a_{\max}}}\mathrm{s}$ and $S = 0.2\mathrm{m(rad)}$.

The position, velocity and acceleration of the sliders are shown in Figure 4.

The external force and moment exerted at the center of the moving platform are not considered since the redundant parallel manipulator is usually used for the motion simulator while not for the machine tool. According to Eq. (20) and Eq. (21), the whole actuating torques and the torques caused by the acceleration term, the velocity term and the gravity term can be computed respectively. The results shown in the Figure 5 indicate that the torques caused by the acceleration terms should be much bigger than the other two terms in the simulation.

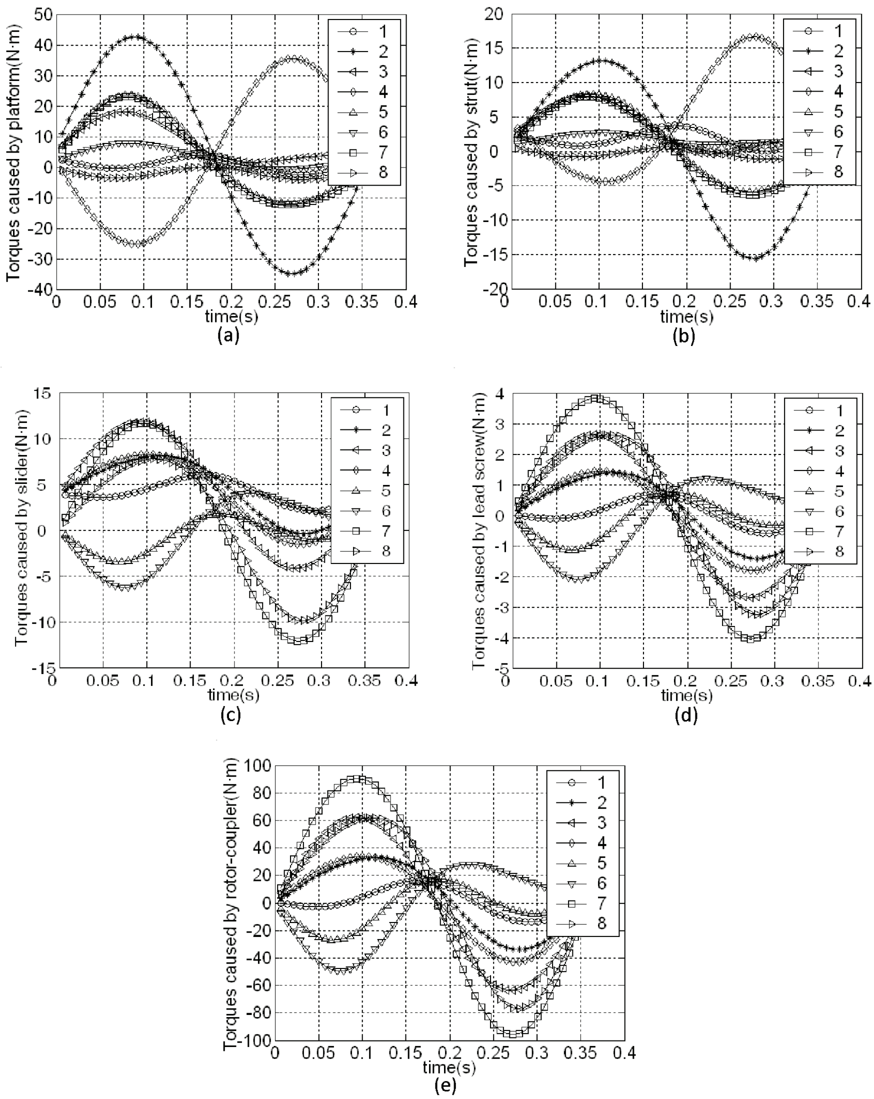
From Eq. (20) and Eq. (26), the torques caused by moving platform, strut, slider, lead screw, motor rotor and coupler are computed respectively. They are shown in the Figure 6.

According to Eq. (28), the required powers and the powers caused by the acceleration term, the velocity term and the gravity term of torques can be achieved respectively. The results of the simulation shown in Figure 7 indicate that the required powers are primarily dominated by the component caused by the acceleration term of torque.

According to Eq. (29), the required powers caused by the moving platform, the strut, the slider, the lead screw, the coupler and the motor rotor are computed respectively. The results of the simulation shown in Figure 8 indicate that the required powers caused by the rotation inertia of the motor rotor-coupler are much bigger than the others.

The results shown in the Figure 6 and Figure 8 turn out that the required torques and

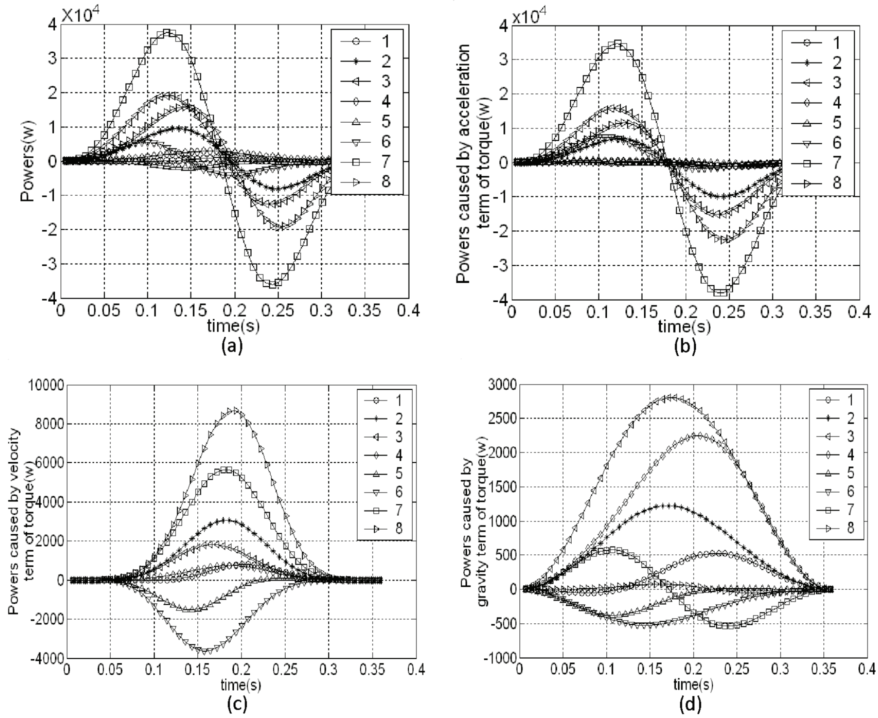
Figure 6. Variations of torques caused by the moving platform (a), torques caused by the strut (b), torques caused by the slider (c), torques caused by the lead screw (d) and torques caused by the motor rotor and coupler (e) vs time



powers caused by the rotation inertia of motor-coupler-screw should be included for the exact dynamic model of the out actuated parallel manipulator used for the design of control law or the estimation of servomotor parameters. However, the effects of the rotation inertia of motor-coupler-screw on the actuating torque and power have often been omitted in the inverse dynamics of the parallel manipulator in the past literatures.

For the prescribed trajectory of the moving platform expressed by Eq. (25), the required output work generated by the ith driving motor can be achieved by the numerical integration.

Figure 7. Variations of whole actuating powers (a), powers caused by acceleration term of torque (b), powers caused by the velocity term of torque (c) and powers caused by the gravity term of torque (d) vs time



The approximation error $e = 10^{-3}$ and $n_1 = 12$ are adopted for the numerical integration computation. The required output work generated by the *i*th driving motor and their sum for the prescribed motion is shown in Table 5. For the prescribed trajectory, the required output work generated by the seventh driving motor is biggest while the required output work generated by the first driving motor is smallest.

CONCLUSION

The inverse dynamic analysis of the 8-PSS redundant parallel manipulator in the exhaustive decoupled way has been presented in this paper. The whole actuating torques has been divided into four terms which are caused by the acceleration, the velocity, the gravity and the external force. It can also be decoupled into the components contributed by the moving platform, the strut, the slider, the lead screw, the motor rotor-coupler and the external force. The power contributed by the components of torque caused by the acceleration term, the velocity term, the gravity term, the external force term and the powers contributed by the moving platform, the strut, the slider, the lead screw, the motor rotor-coupler have been achieved respectively. For a prescribed trajectory, the required output work generated by the *i*th driving motor is obtained by the numerical integration technique in the simulation. The simulation results show that torques and powers caused by the rotation inertia of the motor rotor-coupler are bigger than those of caused by the other component inertia of the manipulator. So the rotation inertia of the motor rotor-coupler

Figure 8. Variations of powers caused by the moving platform (a), powers caused by the strut (b), powers caused by the slider (c), powers caused by the lead screw (d) and powers caused by the motor rotor and coupler (e) vs time

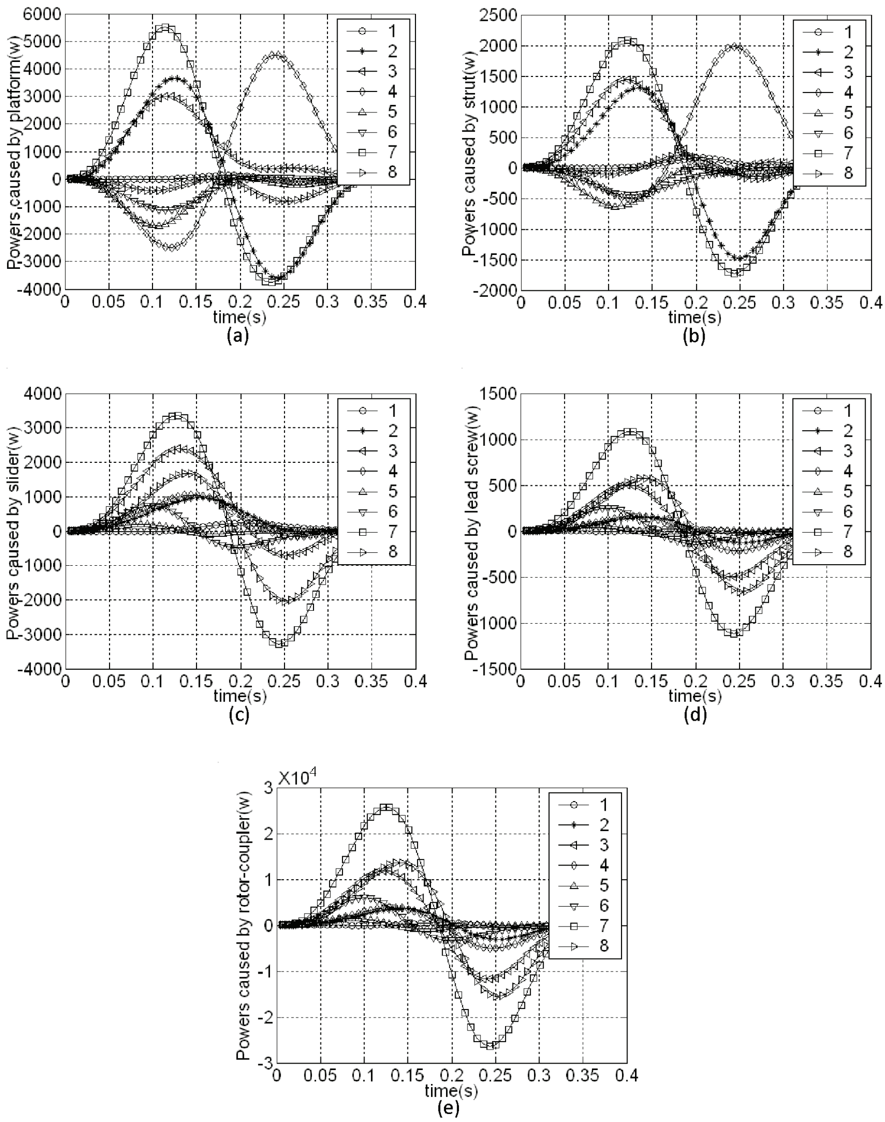


Table 5. The required output work generated by the ith driving motor and their sum for the prescribed trajectory (unit: J)

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*sum*
W_i	99.697	1544.119	2819.088	401.408	166.805	753.256	6495.371	2962.475	15242.219

should be included for the dynamic analysis of the out actuated parallel manipulator. The torque caused by the acceleration term is much bigger than the other two terms in the simulation. The required output power generated by the *i*th driving motor is primarily dominated by the component caused by the acceleration term of torque. The required output work generated by the seventh driving motor is biggest while the required output work generated by the first driving motor is smallest.

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REFERENCES

Ben-Horin, R., Shoham, M., & Djerassi, S. (1998). Kinematics, dynamics and construction of a planarly actuated parallel robot. *Robotics and Computer-integrated Manufacturing*, *14*(2), 163–172. doi:10.1016/S0736-5845(97)00035-5

Carvalho, J. C. M., & Ceccarelli, M. (2001). A closed-form formulation for the inverse dynamics of a Cassino parallel manipulator. *Multibody System Dynamics*, *5*(2), 185–210. doi:10.1023/A:1009845926734

Cheng, H., Yiu, Y. K., & Li, Z. X. (2003). Dynamics and control of redundantly actuated parallel manipulators. *IEEE Transactions on Mechatronics*, *8*(4), 483–491. doi:10.1109/TMECH.2003.820006

Dasgupta, B., & Mruthyunjaya, T. S. (1998). Closed-form dynamic equations of the Stewart platform through the Newton-Euler approach. *Mechanism and Machine Theory*, *33*(7), 993–1012. doi:10.1016/S0094-114X(97)00087-6

Ebrahimi, I., Carretero, J. A., & Boudreau, R. (2007). 3-PRRR redundant planar parallel manipulator: Inverse displacement, workspace and singularity analyses. *Mechanism and Machine Theory*, *42*(8), 1007–1016. doi:10.1016/j.mechmachtheory.2006.07.006

Gallardo, J., Rico, J. M., & Frisoli, A. (2003). Dynamics of parallel manipulators by means of screw theory. *Mechanism and Machine Theory*, *38*(11), 1113–1131. doi:10.1016/S0094-114X(03)00054-5

Geike, T., & McPhee, J. (2003). Inverse dynamic analysis of parallel manipulators with full mobility. *Mechanism and Machine Theory*, *38*(6), 549–562. doi:10.1016/S0094-114X(03)00008-9

Gosselin, C. M. (1996). Parallel computational algorithms for the kinematics and dynamics of planar and spatial parallel manipulators. *ASME Journal of Dynamic Systems Measurement and Control*, *118*(1), 22–28. doi:10.1115/1.2801147

Khan, W. A., Krovi, V. A., Saha, S. K., & Angeles, J. (2005). Recursive kinematics and inverse dynamics for a planar 3R parallel manipulator. *ASME Journal of Dynamic Systems Measurement and Control*, *127*(4), 529–536. doi:10.1115/1.2098890

Kim, J., Park, F. C., & Ryu, S. J. (2001). Design and analysis of a redundantly actuated parallel mechanism for rapid machining. *IEEE Transactions on Robotics and Automation*, *17*(4), 423–434. doi:10.1109/70.954755

Lee, K. M., & Shan, D. K. (1988). Dynamic analysis of a three-degrees-freedom in-parallel actuated manipulator. *IEEE Transactions on Robotics and Automation*, *4*(3), 361–367. doi:10.1109/56.797

Li, M., Huang, T., & Mei, J. P. (2005). Dynamic formulation and performance comparison of the 3-dof modules of two reconfigurable PKMs – the TriVariant and the Tricept. *ASME Journal of Mechanical Design*, *127*(6), 1129–1136. doi:10.1115/1.1992511

Liu, M. J., Li, C. X., & Li, C. N. (2000). Dynamics analysis of the Gough-Stewart platform manipulator. *IEEE Transactions on Robotics and Automation*, *16*(1), 94–98. doi:10.1109/70.833196

McPhee, J., Shi, P., & Piedboeuf, J. C. (2002). Dynamics of multibody systems using virtual work and symbolic programming. *Mathematical and Computer Modelling of Dynamical Systems*, *8*(3), 137–155. doi:10.1076/mcmd.8.2.137.8591

Merlet, J. P. (1996). Redundant parallel manipulators. *Laboratory Robotics and Automation*, *8*(1), 17–24. doi:10.1002/(SICI)1098-2728(1996)8:1<17::AID-LRA3>3.0.CO;2-#

Miller, K., & Clavel, R. (1992). The Lagrange-based model of Delta-4 robot dynamics. *Robotersysteme*, *8*(4), 49–54.

Mohamed, M. G., & Gosselin, C. M. (2005). Design and analysis of kinematically redundant parallel manipulators with configurable platforms. *IEEE Transactions on Robotics*, *21*(3), 277–287. doi:10.1109/TRO.2004.837234

Müller, A. (2005). Internal preload control of redundantly actuated parallel manipulators-its application to backlash avoiding control. *IEEE Transactions on Robotics*, *21*(4), 668–677. doi:10.1109/TRO.2004.842341

Muller, A., & Maißer, P. (2003). A Lie-group formulation of kinematics and dynamics of constrained MBS and its application to analytical mechanics. *Multibody System Dynamics*, *9*(4), 311–352. doi:10.1023/A:1023321630764

Nokleby, S. B., Fisher, R., & Podhorodeski, R. P. (2005). Force capabilities of redundantly-actuated parallel manipulators. *Mechanism and Machine Theory*, *40*(5), 578–599. doi:10.1016/j.mechmachtheory.2004.10.005

Selig, J. M., & McAree, P. R. (1999). Constrained robot dynamics II: Parallel machines. *Journal of Robotic Systems*, *16*(9), 487–498. doi:10.1002/(SICI)1097-4563(199909)16:9<487::AID-ROB2>3.0.CO;2-R

Sokolov, A., & Xirouchakis, P. (2007). Dynamics analysis of a 3-dof parallel manipulator with R-P-S joint structure. *Mechanism and Machine Theory*, *42*(5), 541–557. doi:10.1016/j.mechmachtheory.2006.05.004

Staicu, S. (2009). Power requirement comparison in the 3-RPR planar parallel robot dynamics. *Mechanism and Machine Theory*, *44*(5), 1045–1057. doi:10.1016/j.mechmachtheory.2008.05.009

Sugimoto, K. (1987). Kinematics and dynamic analysis of parallel manipulator by means of motor algebra. *ASME Journal of Mechanisms, Transmissions, and Automation in Design*, *109*(1), 3–7. doi:10.1115/1.3258783

Tsai, L. W. (2000). Solving the inverse dynamics of a Stewart-Gough manipulator by the principle of virtual work. *ASME Journal of Mechanical Design*, *122*(1), 3–9. doi:10.1115/1.533540

Wang, J., & Gosselin, C. M. (1998). A new approach for the dynamic analysis of parallel manipulators. *Multibody System Dynamics*, *2*(3), 317–334. doi:10.1023/A:1009740326195

Wang, J., & Gosselin, C. M. (2004). Kinematic analysis and design of kinematically redundant parallel mechanisms. *ASME Journal of Mechanical Design*, *126*(1), 109–118. doi:10.1115/1.1641189

Wiens, G. J., Shamblin, S. A., & Oh, Y. H. (2002). Characterization of PKM dynamics in terms of system identification. *Journal of Multi-body Dynamics. Part K*, *216*(1), 59–72.

Zhao, Y. J., & Gao, F. (2009). Dynamic performance comparison of the 8PSS redundant parallel manipulator and its non-redundant counterpart—the 6PSS parallel manipulator. *Mechanism and Machine Theory*, *44*(5), 991–1008. doi:10.1016/j.mechmachtheory.2008.05.015

Zhu, Z. Q., Li, J. S., Gan, Z. X., & Zhang, H. (2005). Kinematic and dynamic modeling for real-time control of Tau parallel robot. *Mechanism and Machine Theory*, *40*(9), 1051–1067. doi:10.1016/j.mechmachtheory.2004.12.024

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