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Adaptive control for an uncertain robotic manipulator with input saturations

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Abstract

In this paper, we address the control problem of an uncertain robotic manipulator with input saturations, unknown input scalings and disturbances. For this purpose, a model reference adaptive control like (MRAC-like) is used to handle the input saturations. The model reference is input to state stable (ISS) and driven by the errors between the required control signals and input saturations. The uncertain parameters are dealt with by using linear-in-the-parameters property of robotic dynamics, while unknown input scalings and disturbances are handled by non-regressor based approach. Our design ensures that all the signals in the closed-loop system are bounded, and the tracking error converges to the compact set which depends on the predetermined bounds of the control inputs. Simulation on a planar elbow manipulator with two joints is provided to illustrate the effectiveness of the proposed controller.

Keywords: Robotic manipulator, adaptive control, uncertain parameter, input saturation, trajectory tracking

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1 Introduction

The control of robotic manipulator has gained great attentions in the field of automatic control and robotics communities due to its advantages of applications [1–5]. Dynamic model of robots is high nonlinearity and unknown exactly due to the payloads. Furthermore robots are usually restricted by physical conditions, and the input scalings are uncertain due to the transmission mechanisms [6–9]. Neglect of the uncertainties or violation of the restrictions may result in undesired performance or system damage. In this paper, we address the control

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problem of uncertain robotic manipulator with taking into account the input saturations, unknown input scalings and disturbances.

The first specification of the above problem is to handle the input saturations. The existing approach based on solving an optimal control problem with the input constraints was proposed in [10]. Alternative approach is based on the concept of the saturation functions in [11, 12]. As shown in these works, for the trajectory tracking or the output tracking problem, the initial conditions are required to lie within a bounded region to ensure the satisfaction of input saturations. This means that the resulted system can only operate locally. The global stability is achieved only for the set-point tracking problem, i.e., the position tracking problem as shown in [13].

The second specification is to deal with the uncertain parameters of the system. Due to the coupling of the variables and uncertain parameters in the model, in particular, the inertia and Coriolis and centrifugal forces matrices, the formers proposed the regressor approach to estimate the uncertain parameter in [14, 15]. Based on this approach, several adaptive controls have been developed such that adaptive inverse dynamics control in [16], adaptive passivity-based control in [17] and intermediate between the two approaches in [18]. However, when the input saturations are taken into account, handling simultaneously both uncertain parameters and input constraints is problematic. The existing approaches [19–22] may not be directly applied due to constraints. Furthermore, when the input scalings are unknown and disturbances effect the system, the control problem becomes much more challenging.

Motivated by above consideration, we propose here a solution for the control problem of an uncertain robotic manipulator with taking into account the input saturations, unknown input scalings and disturbances. To deal with input saturation, our approach is to transfer the system with input saturation to the corresponding system with free control. This can be accomplished by using the MRAC-like, which is ISS and driven by the errors between the required control signals and saturations. The uncertain parameters are estimated by using regressor matrix, while the unknown input scalings and disturbances are handled by non-regressor based approach [23]. It is shown that the violation of input saturations is prevented, all the signals in the closed-loop system are bounded and the tracking error converges to the compact set which depends on the input saturations. Our design here is the extension of the work in [9].

The rest of the paper is organized as follows. In Section 2, the dynamic model of the robotic manipulator and the control problem are provided. Section 3 presents the control design. Simulation results are carried out to illustrate the effectiveness of the proposed control in Section 4. The paper concludes in Section 5.

Notation $\mathbb{R}^n$ denotes the Euclidean space with $n-$dimension. $A \in \mathbb{R}^{n\times n}$ denotes the $n \times n-$ matrix. $\|\cdot\|$ is the Euclidean norm of vector "$\cdot$". $\lambda_{\min}(\cdot)(\lambda_{\max}(\cdot))$ is the minimum (maximum) eigenvalue of the matrix "$\cdot$".

2 Problem formulation

In this paper, we study the robotic manipulator described by the following equations [12]:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Psi u + w, \tag{1}$$

$$|u_i| \leqslant \bar{u}_i, \quad i = 1,\dots,n, \tag{2}$$

where $q \in \mathbb{R}^n$ is a vector of the generalized coordinates; $u \in \mathbb{R}^n$ is a vector of control inputs satisfying constraint (2); $\bar{u}_i > 0, i = 1,\dots,n$ is the positive number; $D \in \mathbb{R}^{n\times n}$ is the symmetric positive inertia matrix; $C \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal forces matrix; $G \in \mathbb{R}^n$ is the gravity force; $\Psi \in \mathbb{R}^{n\times n}$ is a matrix of uncertain input scalings; and $w \in \mathbb{R}^n$ is a disturbance vector, but known bound, i.e., there exists a positive number $\bar{w}$ such that $\|w\| \leqslant \bar{w}$. In this paper, we assume that all the states of the system are available for feedback.

The control objectives of this paper are: 1) to track a desired trajectory $q_{\mathrm{d}}(t) \in \mathbb{R}^n$ while all the signals in the closed-loop system are bounded and 2) the constraint (2) is satisfied.

It is noticed that the control problem here differs from the problem in [8]. Since the control u subjects to constraint (2), and the disturbance w is added. In comparison with the existing works in [11,24,25], the unknown input scaling Ψ makes the problem here more challenging. Before we move on the control design to deal with the arisen challenges, the following properties of the robotic manipulator (1) are needed.

Property 1 [26] There exist the positive constants d_{m} and d_{M} such that:

i) $d_{\mathrm{m}} \leqslant \|D(q)\| \leqslant d_{\mathrm{M}}$.

ii) Matrix $\dot{D}(q) - 2C(q,\dot{q})$ is skew-symmetric if the matrix $C(q,\dot{q})$ is defined by the Christoffel symbols, i.e.,

$$z^{\mathrm{T}}(\dot{D}(q) - 2C(q,\dot{q}))z = 0, \quad \forall z \in \mathbb{R}^n.$$

iii) The left-hand side of the (1) can be rewritten as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\theta, \tag{3}$$

where $\theta \in \mathbb{R}^{\ell}$ is the vector of parameters, $Y(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times\ell}$ is the known function *regressor*.

3 Control design

This section presents the control design of an adaptive bounded control for systems (1) and (2). Toward this end, we define the tracking error

$$p = q - q_{\mathrm{d}}, \tag{4}$$

and the filtered velocity variable

$$r = \dot{p} + \Lambda p, \tag{5}$$

where $\Lambda \in \mathbb{R}^{n\times n}$ is a positive diagonal matrix. It is clear that if $\|r\|$ is bounded then $\|(p,\dot{p})\|$ is bounded. Furthermore, $\|r\| \to 0$ implies $\|(p,\dot{p})\| \to 0$. Accordingly, taking the time derivative of (5) along (1), we obtain the dynamic equation of r as

$$\begin{aligned} D(q)\dot{r} =& -C(q,\dot{q})r + \Psi u + w - D(q)(\ddot{q}_{\mathrm{d}} - \Lambda\dot{p}) \\ & - C(q,\dot{q})(\dot{q}_{\mathrm{d}} - \Lambda p) - G(q). \end{aligned} \tag{6}$$

The terms $D(q), C(q,\dot{q})$ and $G(q)$ in (6) are unknown, we deal with these uncertainties by using the regressor matrix as follows. Denoting

$$\varpi = \dot{q}_{\mathrm{d}} - \Lambda p, \tag{7}$$

with the use of item iii) of Property 1, we have

$$Y(q,\dot{q},\varpi)\theta = D(q)\dot{\varpi} + C(q,\dot{q})\varpi + G(q), \tag{8}$$

where $\theta \in \mathbb{R}^{\ell}$ is the vector of unknown parameters and $Y(q,\dot{q},\varpi) \in \mathbb{R}^{n\times\ell}$ is the known function regressor. Then we rewrite (6) as

$$D(q)\dot{r} = -C(q,\dot{q})r - Y\theta + \Psi u + w. \tag{9}$$

In view of (9), because Ψ is unknown and u is restricted by (2), therefore the conventional adaptive approaches in [8, 19, 20] cannot be directly applied. To deal with this situation, we firstly handle the uncertainty of Ψ by separating Ψ as

$$\Psi = \Psi_0 + \Psi_\Delta, \tag{10}$$

where $\Psi_0 \in \mathbb{R}^{n\times n}$ and $\Psi_\Delta \in \mathbb{R}^{n\times n}$ are the known nominal and uncertain values of Ψ, respectively.

Assumption 1 There exists a positive number $\psi_{\max} > 0$ such that

$$0 \leqslant \|\Psi_\Delta\| \leqslant \psi_{\max}, \text{ and } 0 < \Psi_0 + \Psi_\Delta. \tag{11}$$

Since even we do not know the exact value of Ψ, but we can estimate the bounds of Ψ. Furthermore, $\Psi > 0$ is the assumption as usual in the existing works [8, 20]. Therefore, Assumption 1 is reasonable.

Under Assumption 1, we rewrite (9) as

$$D(q)\dot{r} = -C(q,\dot{q})r - Y\theta + \Psi_0 u + w_1, \tag{12}$$

where

$$w_1 = \Psi_\Delta u + w, \tag{13}$$

and it is bounded by

$$\|w_1\| \leqslant \|\Psi_\Delta\|\,\|u\| + \|w\| \leqslant \psi_{\max}\Big(\sum_{i=1}^{n} \bar{u}_i^2\Big)^{\frac{1}{2}} + \bar{w}. \tag{14}$$

Consequently, we deal with the constraint (2). To do this, we use the control of the form [27]

$$u_i(t) = \begin{cases} \mathrm{sgn}(v_i(t)), & |v_i(t)| \geqslant \bar{u}_i, \\ v_i(t), & |v_i(t)| < \bar{u}_i, \end{cases} \tag{15}$$

for $i = 1,\dots,n$ where v_i is the ith control to be designed, and $\mathrm{sgn}(\cdot)$ is denoted a signum function. Furthermore, we introduce the following reference model for (12)

$$D_0(q)\dot{\eta} = -(C_0(q,\dot{q}) + K)\eta + \Psi_0\delta, \ \ \eta(0) = 0, \tag{16}$$

where $\eta \in \mathbb{R}^n$ is the auxiliary state; $D_0(q)$ and $C_0(q,\dot{q})$ are the priori estimations of $D(q)$ and $C(q,\dot{q})$, respectively, with fixed parameters [16]; $K \in \mathbb{R}^{n\times n}$ is a positive matrix; and $\delta \in \mathbb{R}^n$ is the corresponding input defined as

$$\delta = u - v. \tag{17}$$

The following lemma shows that the model (16) is ISS.

Lemma 1 Consider the system described by (16), if the matrix $\dot{D}_0(q) - 2C_0(q,\dot{q})$ is skew symmetric, i.e.,

$$\eta^{\mathrm{T}}(\dot{D}_0(q) - 2C_0(q,\dot{q}))\eta = 0, \ \ \forall \eta \in \mathbb{R}^n, \tag{18}$$

then (16) is ISS with η as the state and δ as the input. Furthermore, if $\eta(0) = 0$ then $\eta(t)$ is bounded by

$$\|\eta(t)\| \leqslant \sqrt{\frac{d_{\mathrm{M}}\lambda_{\max}(\Psi_0)}{d_{\mathrm{m}}(2\lambda_{\min}(K) - \lambda_{\max}(\Psi_0))}}\|\delta\|, \quad \forall t \geqslant 0 \quad (19)$$

with $2\lambda_{\min}(K) > \lambda_{\max}(\Psi_0)$.

Proof See the appendix. □

Having Lemma 1, we denote

$$z = r - \eta, \quad (20)$$

and subtract (16) from (12) with the use of $r = z + \eta$, then we obtain the dynamic equation of z as

$$\begin{aligned} D(q)\dot{z} = &- C(q,\dot{q})z - Y\theta + K\eta + \Psi_0 v + w_1 \\ &+ D_0\ddot{\eta} - D(q)\ddot{\eta} + C_0(q,\dot{q})\eta - C(q,\dot{q})\eta. \end{aligned} \quad (21)$$

Let

$$P = -D(q)\dot{\eta} - C(q,\dot{q})\eta + D_0(q)\dot{\eta} + C_0(q,\dot{q})\eta + w_1, \quad (22)$$

and (21) becomes

$$D(q)\dot{z} = -C(q,\dot{q})z - Y\theta + P + K\eta + \Psi_0 v. \quad (23)$$

Obtaining (23), we have transferred (12) with the bounded control u to (23) with the free control v. The prise of the transformation is that (23) is driven by the additional auxiliary state η of (16).

The goal now is to design the free control v for (23) such that $z(t)$ converges to a neighborhood of the origin. To this end, we need to estimate the bound of P by the non-regressor approach in the following lemma.

Lemma 2 [23] Let $\Phi(\eta,\dot{\eta},\dot{q})$ be a scalar function defined by

$$\Phi(\cdot) = b(\|\dot{\eta}\| + \|\dot{q}\|\,\|\eta\|), \quad (24)$$

where $b \geqslant 1$ is a known real number. Then the lumped nonlinearity and uncertainty P in (22) obeys

$$\|P\| \leqslant \zeta(\rho + \Phi(\eta,\dot{\eta},\dot{q})), \quad (25)$$

with ζ is a "normalized" unknown constant and ρ is a known constant.

Proof See [23]. □

In view of system (23) and (25), θ and ζ are the current unknown parameters. Let $\hat{\theta}$ and $\hat{\zeta}$ be the estimations of θ and ζ, respectively, and we propose the control of the form

$$v = -\Psi_0^{-1}(Kr - Y\hat{\theta} + \frac{\hat{\zeta}\Phi z}{\|z\| + \varepsilon}), \quad (26)$$

where ε is a scale positive function ($\varepsilon(t) > 0, \forall t \geqslant 0$) to be specified, and the update laws are

$$\dot{\hat{\theta}} = -\Gamma(Y^{\mathrm{T}}z + k_2\hat{\theta}), \quad (27)$$

$$\dot{\hat{\zeta}} = \frac{k_1\|z\|^2\Phi}{\|z\| + \varepsilon} - k_2k_3\hat{\zeta}, \quad (28)$$

where $\Gamma \in \mathbb{R}^{\ell\times\ell}$ is a positive diagonal matrix, and k_1, k_2 and k_3 are the positive numbers.

We have the following results.

Theorem 1 Consider the system described by (1) and constraint (2) under Assumption 1, the control u obtained by (15), and v obtained by (26) with the update laws $\hat{\theta}$ and $\hat{\zeta}$ described by (27) and (28), respectively. Then, the constraint (2) is satisfied and the following statements hold

1) The signal $z(t)$ converges to a neighborhood of the origin which can be made arbitrarily small, i.e.,

$$\lim_{t\to\infty}\|z(t)\| \leqslant \sqrt{\frac{2\gamma}{d_{\mathrm{m}}\chi}}, \quad (29)$$

where χ and γ are the positive numbers and γ can be made arbitrarily small.

2) All the signals in the closed-loop system are bounded. In addition, the tracking errors $p(t)$ and $\dot{p}(t)$ converge to the compact sets Ω_1 and Ω_2, respectively, defined as

$$\Omega_1 = \{p \in \mathbb{R}^n \,\big|\, \|p\| \leqslant \lambda_{\min}(\Lambda)^{-1}(\sqrt{\frac{2\gamma}{d_{\mathrm{m}}\zeta}} + \alpha\|\delta\|)\}, \quad (30)$$

$$\Omega_2 = \{\dot{p} \in \mathbb{R}^n \,\big|\, \|\dot{p}\| \leqslant (1 + \frac{\lambda_{\max}(\Lambda)}{\lambda_{\min}(\Lambda)})(\sqrt{\frac{2\gamma}{d_{\mathrm{m}}\zeta}} + \alpha\|\delta\|)\}, \quad (31)$$

where α is a positive constant.

Proof 1) Consider the candidate Lyapunov function

$$V_1 = \frac{1}{2}z^{\mathrm{T}}D(q)z + \frac{1}{2}\tilde{\theta}\Gamma^{-1}\tilde{\theta} + \frac{1}{2k_1}\tilde{\zeta}^2, \quad (32)$$

its time derivative along (23) is

$$\begin{aligned} \dot{V}_1 = &z^{\mathrm{T}}(Y\theta + P + K(r - z) + \Psi_0 v) \\ &- \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\theta}} - \frac{1}{k_1}\tilde{\zeta}\dot{\hat{\zeta}}. \end{aligned} \quad (33)$$

Substituting (26)–(28) into (33), we have

$$\dot V_1 = -z^{\mathrm T}Kz + k_2\tilde\theta^{\mathrm T}\hat\theta + z^{\mathrm T}P - \frac{z^{\mathrm T}\hat\zeta\Phi z}{\|z\|+\varepsilon} - \frac{\tilde\zeta\|z\|^2\Phi}{\|z\|+\varepsilon} + k_3\tilde\zeta\hat\zeta.$$

Using the estimation of P in (25) of Lemma 2, yields

$$\dot V_1 \leqslant -z^{\mathrm T}Kz + k_2\tilde\theta^{\mathrm T}\hat\theta + k_3\tilde\zeta\hat\zeta + \frac{\|z\|\zeta\Phi\varepsilon}{\|z\|+\varepsilon}. \tag{34}$$

Choosing $\varepsilon = k_4/(\Phi + k_5)$ where $k_5 > 0$ and using the following inequalities:

$$k_2\tilde\theta\hat\theta \leqslant \frac{k_2}{2}\theta - \frac{k_2}{2}\tilde\theta;\;\; k_3\tilde\zeta\hat\zeta \leqslant \frac{k_3}{2}\zeta - \frac{k_3}{2}\tilde\zeta,$$
$$\frac{\|z\|}{\|z\|+\varepsilon} < 1;\;\; \frac{\Phi}{\Phi+k_5} < 1,$$

then, (34) becomes

$$\dot V_1 \leqslant -z^{\mathrm T}Kz - \frac{k_2}{2}\tilde\theta - \frac{k_3}{2}\tilde\zeta + \frac{k_2}{2}\theta + \frac{k_3}{2}\zeta + k_4\zeta. \tag{35}$$

Let us denote

$$\chi = \frac{\min\left\{\lambda_{\min}(K), k_2/2, k_3/2\right\}}{\max\{\|D(q)\|, \|\Gamma^{-1}\|/2, 1/2k_1\}}, \tag{36}$$

$$\gamma = \frac{k_2}{2}\theta + \frac{k_3}{2}\zeta + k_4\zeta, \tag{37}$$

and rewrite (35) as

$$\dot V_1 \leqslant -\chi V_1 + \gamma. \tag{38}$$

Multiplying (38) by $\mathrm e^{\chi t}$, yields

$$\frac{\mathrm d}{\mathrm dt}(V_1(t)\mathrm e^{\chi t}) \leqslant \gamma \mathrm e^{\chi t}. \tag{39}$$

Integrating both size of the above inequality, we have

$$V_1(t) \leqslant V_1(0)\mathrm e^{-\chi t} + \frac{\gamma}{\chi},\;\; t \geqslant 0. \tag{40}$$

From (32), we have $\|z(t)\| \leqslant \sqrt{2V_1(t)/d_{\mathrm m}}$, then we obtain (29) as $t \to \infty$. Furthermore, γ can be made arbitrarily small by decreasing k_2, k_3 and k_4 in (37).

2) We need to show that r is bounded. From (20) we have

$$\|r(t)\| \leqslant \|z(t)\| + \|\eta(t)\|. \tag{41}$$

Using (40) with the denotation

$$\mu = V_1(0) + \gamma/\chi, \tag{42}$$

we obtain

$$\|z(t)\| \leqslant \sqrt{\frac{2\mu}{d_{\mathrm m}}}. \tag{43}$$

On the other hand, choosing $\eta(0) = 0$ for (16) we obtain the bound of r from Lemma 1 as

$$\|r(t)\| \leqslant \sqrt{\frac{2\mu}{d_{\mathrm m}}} + \alpha\|\delta\|, \tag{44}$$

where

$$\alpha = \sqrt{\frac{d_{\mathrm M}\lambda_{\max}(\Psi_0)}{d_{\mathrm m}(2\lambda_{\min}(K) - \lambda_{\max}(\Psi_0))}} > 0. \tag{45}$$

Having the bound of r in (44), we show that p and $\dot p$ are bounded as follows. From (5) we have

$$\dot p = -\Lambda p + r. \tag{46}$$

Solving (46), we obtain

$$p(t) = \mathrm e^{-\Lambda t}(p(0) - \Lambda^{-1}r) + \Lambda^{-1}r. \tag{47}$$

Using (44), $p(t)$ is bounded by

$$\|p(t)\| \leqslant p(0) + \lambda_{\min}(\Lambda)^{-1}\Big(\sqrt{\frac{2\mu}{d_{\mathrm m}}} + \alpha\|\delta\|\Big). \tag{48}$$

Consequently, from (46) the bound of $\dot p$ is

$$\|\dot p(t)\| \leqslant \|r(t)\| + \lambda_{\max}(\Lambda)\|p(t)\|,\;\; t \geqslant 0 \tag{49}$$

or

$$\|\dot p(t)\| \leqslant \lambda_{\max}(\Lambda)p(0) + \Big(1 + \frac{\lambda_{\max}(\Lambda)}{\lambda_{\min}(\Lambda)}\Big)\Big(\sqrt{\frac{2\mu}{d_{\mathrm m}}} + \alpha\|\delta\|\Big),\;\; \forall t \geqslant 0. \tag{50}$$

Thus, (44), (48) and (50) show that all the signals in the closed-loop system are bounded. Furthermore, from (29), (44), (47) and (49) we have

$$\lim_{t\to\infty}\|p(t)\| = \lambda_{\min}(\Lambda)^{-1}\Big(\sqrt{\frac{2\gamma}{d_{\mathrm m}\zeta}} + \alpha\|\delta\|\Big), \tag{51}$$

$$\lim_{t\to\infty}\|\dot p(t)\| = \Big(1 + \frac{\lambda_{\max}(\Lambda)}{\lambda_{\min}(\Lambda)}\Big)\Big(\sqrt{\frac{2\gamma}{d_{\mathrm m}\zeta}} + \alpha\|\delta\|\Big). \tag{52}$$

Thus $p(t)$ and $\dot{p}(t)$ converge to the sets Ω_1 and Ω_2 defined as (30) and (31), respectively. □

Remark 1 The compact sets Ω_1 and Ω_2 in (30) and (31), respectively, still depend on Ψ_0 and δ. In practice, the matrix Ψ_0 may be chosen as identity matrix I. The bound $\bar{u}_i$ of the control is needed to be large enough. Since, the large $\bar{u}_i$, for $i = 1, \dots, n$, implies the small δ, and hence the small Ω_1 and Ω_2 as well. Clearly, free u results in $\delta = 0$, then p and $\dot{p}$ can be made arbitrarily small by choosing small γ in (37).

Remark 2 In comparison with the existing work [8], the proposed control in this paper handles the situation of unknown input scalings Ψ in different direction. Namely, we consider the error Ψ_Δ between the uncertain and nominal values of input scalings as a bounded disturbance. Then, the non-regressor approach is used to deal with this disturbance as well as bounded external disturbances. Furthermore, the proposed control here also differs from the control design in [27, Chapter 11].

4 Simulation results

To illustrate the effectiveness of the proposed control, we carry out the simulation on a Planar Elbow Manipulator with two revolute joints as depicted in Fig. 1 (see [16, Example 2.1]). The dynamics of the manipulator are described by

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \Psi u + w, \tag{53}$$

where

$$D(q) = \begin{bmatrix} \theta_1 + 2\theta_2 \cos q_2 & \theta_3 + \theta_2 \cos q_2 \\ \theta_3 + \theta_2 \cos q_2 & \theta_3 \end{bmatrix},$$

$$C(q,\dot{q}) = \begin{bmatrix} -\theta_3 \dot{q}_2 \sin q_2 & -\theta_3(\dot{q}_1 + h\dot{q}_2)\sin q_2 \\ \theta_3 \dot{q}_1 \sin q_2 & 0 \end{bmatrix},$$

$$G(q) = \begin{bmatrix} \theta_4 \mathrm{g} \cos q_1 + \theta_5 \mathrm{g} \cos(q_1 + q_2) \\ \theta_5 \mathrm{g} \cos(q_1 + q_2) \end{bmatrix},$$

$$\Psi = \begin{bmatrix} \psi_1 & 0 \\ 0 & \psi_2 \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}.$$

and $\theta = [\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5]^{\mathrm{T}}$ is the vector of uncertain parameters. Moreover, from (8) the matrix $Y(q,\dot{q},\varpi) \in \mathbb{R}^{2\times 5}$ has the form of (54). The values of the parameters used in the simulation are given in Table 1.

$$Y(\cdot) = \begin{bmatrix} -\dot{\varpi}_1 & Y_{12} & -\dot{\varpi}_2 & -\mathrm{g}\cos q_1 & -\mathrm{g}\cos(q_1+q_2) \\ 0 & -(\cos q_2 \cdot \dot{\varpi}_1 + \sin q_2 \cdot \dot{q}_1 \varpi_2) & -(\dot{\varpi}_1 + \dot{\varpi}_2) & 0 & -\mathrm{g}\cos(q_1+q_2) \end{bmatrix}, \tag{54}$$
$$Y_{12} = -(2\cos q_2 \cdot \dot{\varpi}_1 + \cos q_2 \cdot \dot{\varpi}_2 - \sin q_2 \cdot \dot{q}_2 \varpi_1 - \sin q_2 \cdot (\dot{q}_1 + \dot{q}_2)\varpi_2).$$

The general expression for the desired trajectory is [8]

$$q_{\mathrm{d}}(t, t_{\mathrm{d}}) = q_0 + (-2\frac{t^3}{t_{\mathrm{d}}^3} + 3\frac{t^2}{t_{\mathrm{d}}^2})(q_{\mathrm{f}} - q_0), \tag{55}$$

where q_0 and q_{f} are the initial and final positions, and t_{d} is the time at which the desired arm trajectory reaches the desired final position.

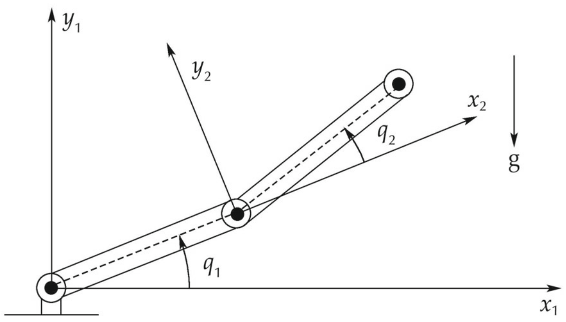


Fig. 1 The Planar Elbow Manipulator with two revolute joints.

Table 1 The parameters of the planar elbow manipulator.

Parameters	Value
$[\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5]$	$[0.42\ 0.08\ 0.12\ 0.70\ 0.20]\ \mathrm{kg\cdot m^2}$
$[\psi_1\ \psi_2]$	$[1.3\ 1.3]$
$[\bar{u}_1\ \bar{u}_2]$	$[13\ \ 4]\ \mathrm{N\cdot m}$

In the simulation, the following values are chosen: $q(0) = [0\ \ 0]^{\mathrm{T}}$ rad, $\dot{q}(0) = [0\ \ 0]^{\mathrm{T}}$ rad/s, $t_{\mathrm{d}} = 1$ s, $q_{\mathrm{d}}(0) = [0\ \ 0]^{\mathrm{T}}$ rad, $q_{\mathrm{d}}(t_{\mathrm{d}}) = [1\ \ 2]^{\mathrm{T}}$ rad, $\hat{\theta}(0) = [0.21\ 0.04\ 0.06\ 0.35\ 0.10]^{\mathrm{T}}\ \mathrm{kg\cdot m^2}$, $\Psi_0 = \mathrm{diag}\{1,1\}$, $\Lambda = \mathrm{diag}\{1,1\}$, $K = \mathrm{diag}\{15,3\}$, $\Gamma = \mathrm{diag}\{0.2, 0.2\}$. The control parameters are set as: $k_1 = 1, k_2 = 0.03, k_3 = 5, \rho = 3, b = 2, \varepsilon = 2/(\Phi + 0.1)$. For (53), w gets the random values in $[-0.01\ 0.01]$. For the model reference (16), we assume $D_0(q) = 0.75D(q)$ and $C_0(q,\dot{q}) = 0.75C(q,\dot{q})$.

For the comparison, we carry out the simulations of our proposed controller and the controller [28] with assuming exact input scalings and saturations. Fig. 2 illustrates the positions q_1 and q_2 while the angular velocities $\dot{q}_1$ and $\dot{q}_2$ are depicted in Fig. 3. The controls u_1 and u_2 are shown in Fig. 4. We can see that the tracking performance can be obtained well by proposed controller, while the static errors and big tracking errors appear under the controller with saturating the control inputs. The estimations of uncertain parameters are illustrated in Fig. 5. The filtered tracking error r and the auxiliary state η are shown in Fig. 6.

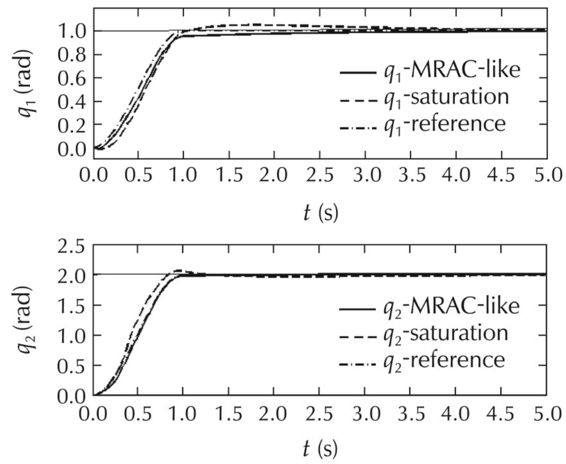


Fig. 2 Positions q_1 and q_2.

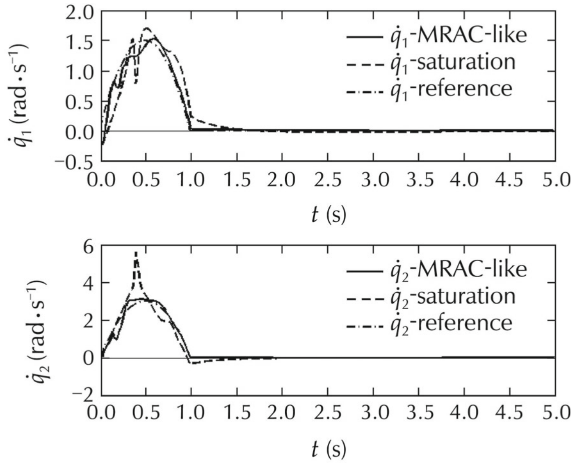
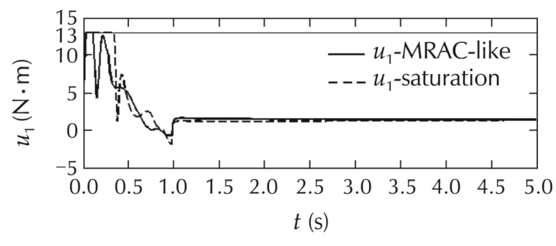


Fig. 3 Angular velocities $\dot{q}_1$ and $\dot{q}_2$.



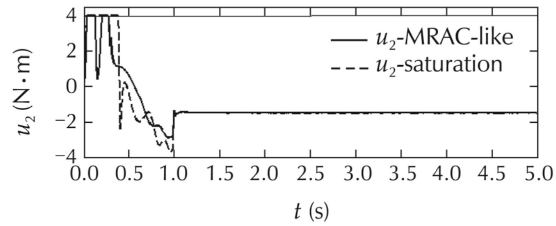


Fig. 4 Controls u_1 and u_2.

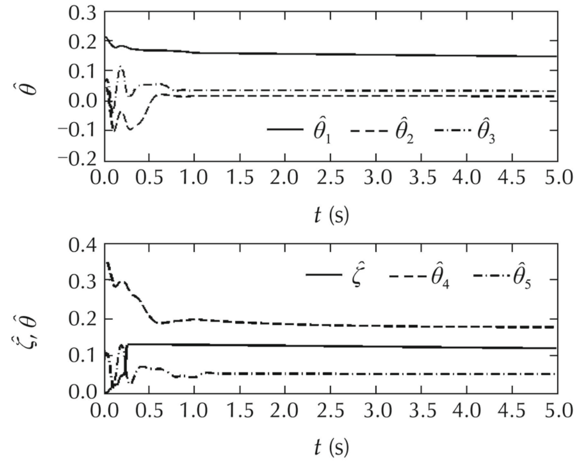


Fig. 5 Estimations of $\hat{\theta}$ and $\hat{\zeta}$.

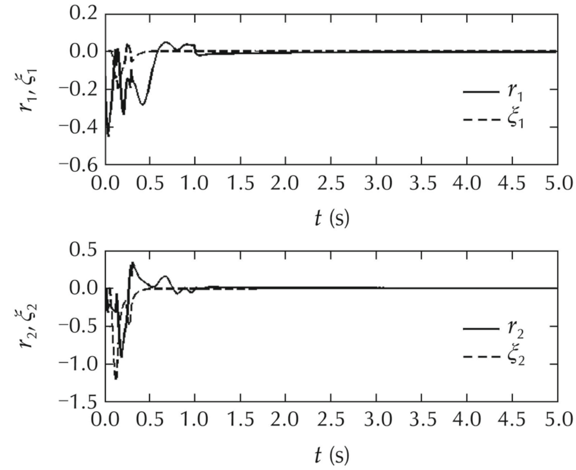


Fig. 6 The filtered velocity tracking error r and the auxiliary state η.

5 Conclusions

We have proposed the solution for the uncertain robotic manipulator with input saturations, unknown input scalings and external disturbances. Firstly, the uncertain dynamics of robot are handled by the use of the known function regressor. Secondly, we introduced the MRAC-like to deal with the input saturations. Finally, the unknown input scalings and external disturbances were rejected by the non-regressor approach. In the results, the controller satisfied the input saturations, and achieved the convergence of the tracking errors to the region which depends on the input saturations. The simulation shows that the control objectives were completed. The future work is to design an output feedback controller for such robotic manipulator.

References

[1] F. L. Lewis, D. M. Dawson, C. T. Abdallah. *Robot Manipulator Control: Theory and Practice*. 2nd ed. New York: Marcel Dekker Inc., 2004.

[2] M. W. Spong, S. Hutchinson, M. Vidyasagar. *Robot Modeling and Control*. New York: John Wiley & Sons, 2005.

[3] W. He, S. S. Ge, B. V. E. How, et al. Robust adaptive boundary control of a flexible marine riser with vessel dynamics. *Automatica*, 2011, 47(4): 722 – 732.

[4] W. He, S. S. Ge, Y. N. Li, et al. Neural network control of a rehabilitation robot by state and output feedback. *Journal of Intelligent & Robotic Systems*, 2015, 80(1): 15 – 31.

[5] W. He, S. Zhang, S. S. Ge. Robust adaptive control of a thruster assisted position mooring system. *Automatica*, 2014, 50(7): 1843 – 1851.

[6] H. Berghuis, R. Ortega, H. Nijmeijer. A robust adaptive robot controller. *IEEE Transactions on Robotics and Automation*, 1993, 9(6): 825 – 830.

[7] E. V. Panteley, A. A. Stotsky. Adaptive trajectory/force control scheme for constrained robot manipulators. *International Journal of Adaptive Control and Signal Processing*, 1993, 7(6): 489 – 496.

[8] S. S. Ge. Adaptive control of robots having both dynamical parameter uncertainties and unknown input scalings. *Mechatronics*, 1996, 6(5): 557 – 569.

[9] T. T. Tran, S. S. Ge, W. He. Adaptive control for a robotic manipulator with uncertainties and input saturations. *IEEE International Conference on Mechatronics and Automation* (*ICMA*), Beijing: IEEE, 2015: 1525 – 1530.

[10] M. W. Spong, J. S. Thorp, J. M. Kleinwaks. The control of robot manipulators with bounded input. *IEEE Transactions on Automatic Control*, 1986, 31(6): 483 – 490.

[11] W. E. Dixon, M. S. De Queiroz, F. Zhang, et al. Tracking control of robot manipulators with bounded torque inputs. *Robotica*, 1999, 17(2): 121 – 129.

[12] A. Loría, H. Nijmeijer. Bounded output feedback tracking control of fully actuated Euler-Lagrange systems. *Systems & Control Letters*, 1998, 33(3): 151 – 161.

[13] A. Zavala-Río, V. Santibáñez. Simple extensions of the pd-with-gravity-compensation control law for robot manipulators with bounded inputs. *IEEE Transactions on Control Systems Technology*, 2006, 14(5): 958 – 965.

[14] P. K. Khosla, T. Kanade. Parameter identification of robot dynamics. *Proceedings of the 24th IEEE Conference on Decision and Control*, Fort Lauderdale: IEEE, 1985: 1754 – 1760.

[15] C. H. An, C. G. Atkeson, J. M. Hollerbach. Estimation of inertial parameters of rigid body links of manipulators. *Proceedings of the 24th IEEE Conference on Decision and Control*, Fort Lauderdale: IEEE, 1985: 990 – 995.

[16] M. W. Spong, R. Ortega. On adaptive inverse dynamics control of rigid robots. *IEEE Transactions on Automatic Control*, 1990, 35(1): 92 – 95.

[17] J. J. E. Slotine, W. Li. On the adaptive control of robot manipulators. *The International Journal of Robotics Research*, 1987, 6(3): 49 – 59.

[18] R. Kelly, R. Carelli, R. Ortega. Adaptive motion control design of robot manipulators: an input-output approach. *International Journal of Control*, 1989, 50(6): 2563 – 2581.

[19] M. Krstic, I. Kanellakopoulos, P. Kokotovic. *Nonlinear and Adaptive Control design*. New York: John Wiley & Sons, 1995.

[20] S. S. Ge, C. C. Hang, T. H. Lee, et al. *Stable Adaptive Neural Network Control*. 1st ed. Berlin: Springer, 2001.

[21] W. He, S. S. Ge. Robust adaptive boundary control of a vibrating string under unknown time-varying disturbance. *IEEE Transactions on Control Systems Technology*, 2012, 20(1): 48 – 58.

[22] W. He, S. S. Ge, B. V. E. How, et al. *Dynamics and Control of Mechanical Systems in Offshore Engineering*. London: Springer, 2014.

[23] Y. D. Song. Adaptive motion tracking control of robot manipulators non-regressor based approach. *International Journal of Control*, 1996, 63(1): 41 – 54.

[24] W. E. Dixon. Adaptive regulation of amplitude limited robot manipulators with uncertain kinematics and dynamics. *IEEE Transactions on Automatic Control*, 2007, 52(3): 488 – 493.

[25] D. J. López-Araujo, A. Zavala-Río, V. Santibáñez, et al. Output-feedback adaptive control for the global regulation of robot manipulators with bounded inputs. *International Journal of Control, Automation and Systems*, 2013, 11(1): 105 – 115.

[26] R. Ortega, M. W. Spong. Adaptive motion control of rigid robots: A tutorial. *Automatica*, 1989, 25(6): 877 – 888.

[27] J. Zhou, C. Y. Wen. *Adaptive Backstepping Control of Uncertain Systems, Nonsmooth Nonlinearities, Interactions of Time-Variations*. Berlin: Springer, 2008.

[28] N. Sadegh, R. Horowitz. Stability and robustness analysis of a class of adaptive controllers for robotic manipulators. *The International Journal of Robotics Research*, 1990, 9(3): 74 – 92.

[29] H. K. Khalil. *Nonlinear Systems*. 3rd ed. Upper Saddle River: Prentice Hall, 2002.

Appendix

Proof of Lemma 1 The proof is the same line with one of Lemma 1 in [9]. Consider the positive function

$$V_2(\eta) = \frac{1}{2}\eta^{\mathrm{T}} D_0(q)\eta, \tag{a1}$$

and its time derivative along (16) is given by

$$\begin{aligned}\dot{V}_2 =& \frac{1}{2}\eta^{\mathrm{T}}\dot{D}_0(q)\eta + \eta^{\mathrm{T}} D_0(q)\dot{\eta} = -\eta^{\mathrm{T}} K\eta + \eta^{\mathrm{T}}\Psi_0\delta \\ \leqslant& -\lambda_{\min}(K)\|\eta\|^2 + \lambda_{\max}(\Psi_0)\|\eta\|\,\|\delta\|. \end{aligned}\tag{a2}$$

Let $0 < \varrho < 1$ and (a2) satisfies

$$\dot{V}_2 \leqslant -\lambda_{\min}(K)(1-\varrho)\|\eta\|^2, \quad \forall \|\eta\| \geqslant \frac{\lambda_{\max}(\Psi_0)\|\delta\|}{\lambda_{\min}(K)\varrho}, \tag{a3}$$

which shows that (16) is ISS from Theorem 4.19 in [29]. To estimate η when $\eta(0) = 0$, from (a2) we have

$$\begin{aligned}\dot{V}_2 &\leqslant -\lambda_{\min}(K)\|\eta\|^2 + \frac{1}{2}\lambda_{\max}(\Psi_0)(\|\eta\|^2 + \|\delta\|^2)\\ &\leqslant -c_1V_2 + c_2,\end{aligned} \tag{a4}$$

where $c_1 = (2\lambda_{\min}(K) - \lambda_{\max}(\Psi_0))/d_{\mathrm{M}}$, and $c_2 = \lambda_{\max}(\Psi_0)\|\delta\|^2/2$. Multiplying (a4) by $\mathrm{e}^{(c_1t)}$, yields

$$\frac{\mathrm{d}}{\mathrm{d}t}(V_2\mathrm{e}^{(c_1t)}) \leqslant c_2\mathrm{e}^{(c_1t)}. \tag{a5}$$

Taking integral of (a5) over the time $[0, t]$ with using $\eta(0)=0$ (i.e., $V_2(0) = 0$), we obtain

$$V_2(t) \leqslant V_2(0) + c_2/c_1 \leqslant c_2/c_1. \tag{a6}$$

Furthermore, we have $\|\eta(t)\|^2 \leqslant 2V_2(t)/d_{\mathrm{m}}$, then we obtain (19). $\square$

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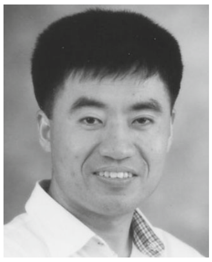

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