Cinemática de Manipuladores

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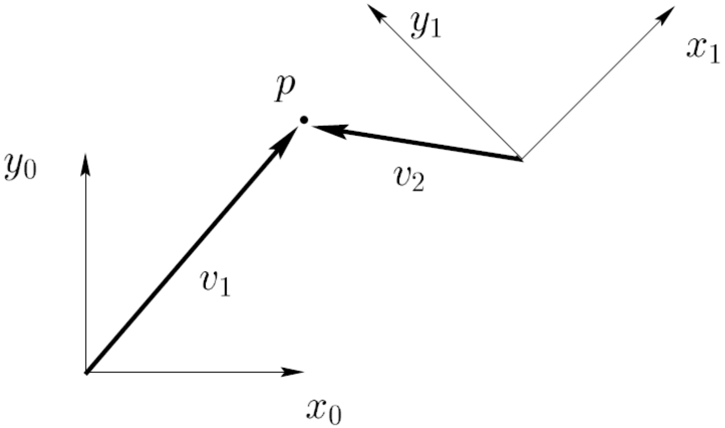
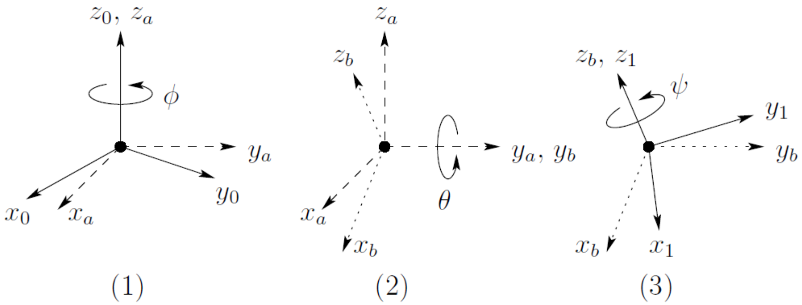
Organização do Documento

Posição de um Ponto



Orientação de um *Frame*



Matriz de Rotação

$$R_1^0 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix}$$

Ângulos de Euler

$$
\begin{aligned}
R_{ZYZ} &= R_{z,\phi}R_{y,\theta}R_{z,\psi} \\
&= \begin{bmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix} \begin{bmatrix} c_\psi & -s_\psi & 0 \\ s_\psi & c_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_\phi c_\theta c_\psi - s_\phi s_\psi & -c_\phi c_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta \\ s_\phi c_\theta c_\psi + c_\phi s_\psi & -s_\phi c_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta \\ -s_\theta c_\psi & s_\theta s_\psi & c_\theta \end{bmatrix}
\end{aligned}
$$

Roll, Pitch, Yaw

$$
\begin{aligned}
R_{XYZ} &= R_{z,\phi} R_{y,\theta} R_{x,\psi} \\
&= \begin{bmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\psi & -s_\psi \\ 0 & s_\psi & c_\psi \end{bmatrix} \\
&= \begin{bmatrix} c_\phi c_\theta & -s_\phi c_\psi + c_\phi s_\theta s_\psi & s_\phi s_\psi + c_\phi s_\theta c_\psi \\ s_\phi c_\theta & c_\phi c_\psi + s_\phi s_\theta s_\psi & -c_\phi s_\psi + s_\phi s_\theta c_\psi \\ -s_\theta & c_\theta s_\psi & c_\theta c_\psi \end{bmatrix}
\end{aligned}
$$

Matriz de Rotação

$$R_1^0 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix}$$

Transformação Homogênea: Pose

$$H = \left[\begin{array}{c|c} R_{3\times 3} & d_{3\times 1} \\ \hline f_{1\times 3} & s_{1\times 1} \end{array}\right] = \left[\begin{array}{c|c} Rotation & Translation \\ \hline perspective & scale\ factor \end{array}\right]$$

Parâmetros de Denavit Hartenberg

- θ_i: Rotação, em z_{i-1}, de x_{i-1} até x_i
- d_i: Translação, em z_{i-1}, de x_{i-1} até x_i
- a_i: Translação, em x_i, de z_{i-1} até z_i
- α_i: Rotação, em x_i, de z_{i-1} até z_i

Atribuição de *Frames*

1. Identificar os eixos z_i, de acordo com os sentidos de rotação da respectiva junta
2. Posicionar o_i na interseção de z_{i-1} com z_i; ou na interseção da perpendicular comum de z_{i-1} e z_i , com z_i . Se z_{i-1} e z_i forem paralelos, escolher de forma que fique mais simples.
3. Estabelecer x_i ao longo da perpendicular comum de z_{i-1} e z_i a partir de o_i; ou na direção normal ao plano de z_{i-1} e z_i, se eles se interceptarem.
4. Estabelecer y_i para que o sistema fique destrógiro.
5. Estabelecer o *frame* n final da ferramenta. Se tiver garra, usar a convenção de garra. Se não tiver ferramenta, repetir o *frame* $n-1$.
6. Estabelecer o *frame* 0 (zero) da base de forma que fique mais simples. Isso se esse *frame* já não tiver sido proposto.

Convenções de Denavit Hartenberg

Exigências da convenção

- DH1: x_i deve ser perpendicular a z_{i-1}
- DH2: x_i deve interceptar z_{i-1}

- Se não houver ferramenta, mas a repetição do *frame* $n-1$ não cumprir DH1 e DH2, repetir o eixo z_{n-1} e escolher x_n de forma a cumprir e poder aplicar D-H.
- Se houver alguma perda de *link* após o *frame* 0, ou a inclusão da ferramenta descumprir os requisitos DH1 e DH2, obtenha a transformação homogênea de um *frame* para o outro. [Spong et al., 2006].

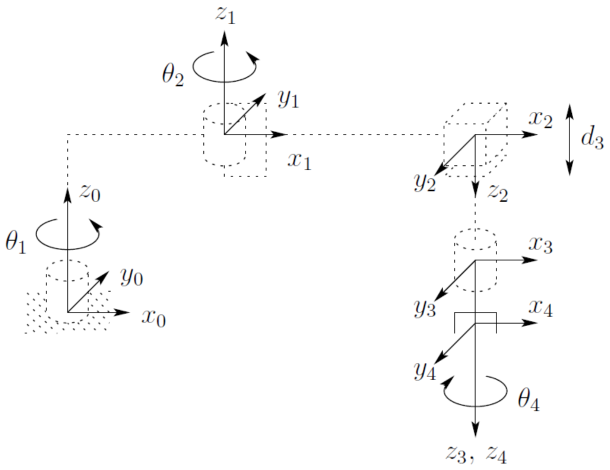
Matrizes A_i

$$
\begin{aligned}
A_i &= Rot_{z,\theta_i}\,\mathrm{Trans}_{z,d_i}\,\mathrm{Trans}_{x,a_i}\,Rot_{x,\alpha_i} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

MCD de um Manipulador

$$
\begin{aligned}
T_6^0 &= A_1 \cdots A_6 \\
&= \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Exemplo 1.1: *SCARA*



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\theta^\star$
2	a_2	180	0	$\theta^\star$
3	0	0	$d^\star$	0
4	0	0	d_4	$\theta^\star$

* joint variable

Exemplo 1.1: *SCARA*

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

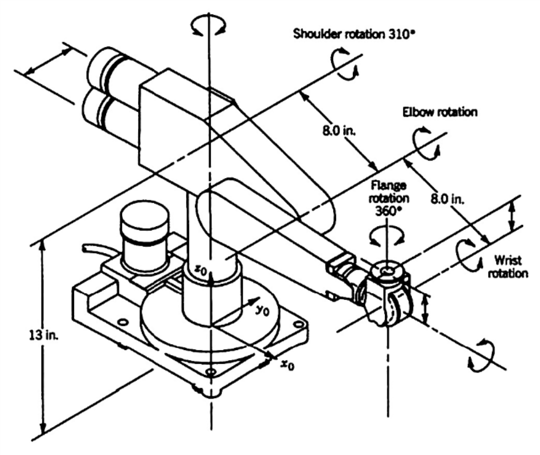
$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

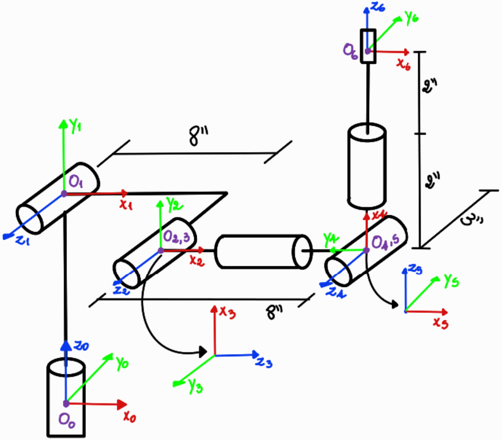
$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} T_4^0 &= A_1 \cdots A_4 \\ &= \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Exemplo 1.2: *PUMA 260*





Exemplo 1.2: *PUMA 260*

Tabela de DH - PUMA 260

i	θ_i	d_i	a_i	α_i
1	θ_1^*	13	0	90°
2	θ_2^*	3	8	0°
3	$\theta_3^* + 90°$	0	0	90°
4	θ_4^*	8	0	-90°
5	$\theta_5^* - 90°$	0	0	-90°
6	θ_6^*	4	0	0°

* variável

Exemplo 1.2: *PUMA 260*

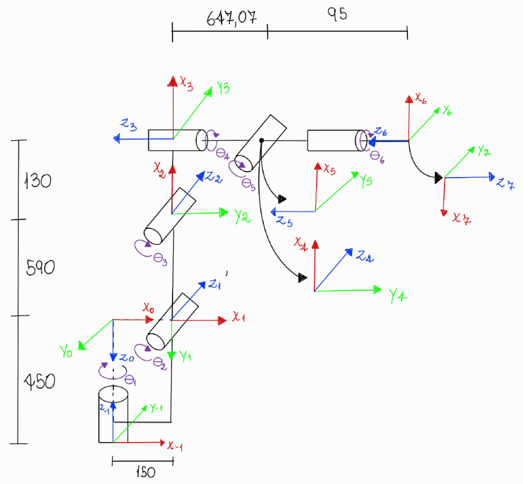
$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 13 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 8c_2 \\ s_2 & c_2 & 0 & 8s_2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} -s_3 & 0 & c_3 & 0 \\ c_3 & 0 & s_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} s_5 & 0 & c_5 & 0 \\ -c_5 & 0 & s_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 1.2: *PUMA 260*

$$H_6^0 = \begin{bmatrix} c_6\left(c_1c_{23}c_5 + s_5\left(-c_1c_4s_{23} + s_1s_4\right)\right) - s_6\left(c_1s_{23}s_4 + c_4s_1\right) & -c_6\left(c_1s_{23}s_4 + c_4s_1\right) - s_6\left(c_1c_{23}c_5 + s_5\left(-c_1c_4s_{23} + s_1s_4\right)\right) & -c_1c_{23}s_5 + c_5\left(-c_1c_4s_{23} + s_1s_4\right) & 8c_1c_2 - 4c_1c_{23}s_5 + 8c_1c_{23} + 4c_5\left(-c_1c_4s_{23} + s_1s_4\right) + 3s_1 \\ -c_6\left(-c_{23}c_5s_1 + s_5\left(c_1s_4 + c_4s_1s_{23}\right)\right) - s_6\left(-c_1c_4 + s_1s_{23}s_4\right) & -c_6\left(-c_1c_4 + s_1s_{23}s_4\right) + s_6\left(-c_{23}c_5s_1 + s_5\left(c_1s_4 + c_4s_1s_{23}\right)\right) & -c_{23}s_1s_5 - c_5\left(c_1s_4 + c_4s_1s_{23}\right) & -3c_1 + 8c_2s_1 - 4c_{23}s_1s_5 + 8c_{23}s_1 - 4c_5\left(c_1s_4 + c_4s_1s_{23}\right) \\ c_{23}s_4s_6 + c_6\left(c_{23}c_4s_5 + c_5s_{23}\right) & c_{23}c_6s_4 - s_6\left(c_{23}c_4s_5 + c_5s_{23}\right) & c_{23}c_4c_5 - s_{23}s_5 & 4c_{23}c_4c_5 + 8s_2 - 4s_{23}s_5 + 8s_{23} + 13 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 1.3: *Comau Smart5 SiX*



Exemplo 1.3: *Comau Smart5 SiX*

Tabela de DH - Comau Smart5 SiX

i	θ_i	d_i	a_i	α_i
1	θ_1^*	0	150	90°
2	$\theta_2^* - 90°$	0	590	180°
3	$\theta_3^* + 90°$	0	130	-90°
4	θ_4^*	-647,07	0	-90°
5	θ_5^*	0	0	90°
6	θ_6^*	-95	0	0°

$$H_0^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 450 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_7^6 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 1.3: *Comau Smart5 SiX*

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 150c_1 \\ -s_1 & 0 & c_1 & -150s_1 \\ 0 & -1 & 0 & 450 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_2 = \begin{bmatrix} s_2 & -c_2 & 0 & 590s_2 \\ -c_2 & -s_2 & 0 & -590c_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_3 = \begin{bmatrix} -s_3 & 0 & -c_3 & -130s_3 \\ c_3 & 0 & -s_3 & 130c_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & -647.07 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_6 = \begin{bmatrix} -c_6 & -s_6 & 0 & 0 \\ -s_6 & c_6 & 0 & 0 \\ 0 & 0 & -1 & -95 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

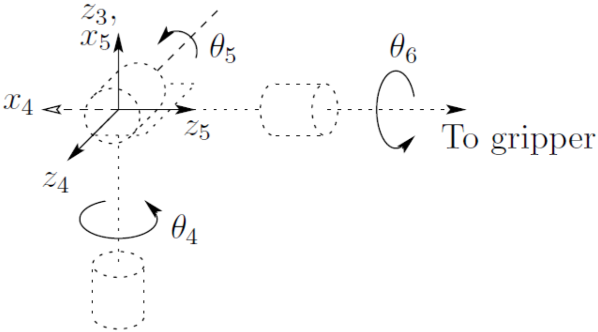
Exemplo 1.3: *Comau Smart5 SiX*

$$H_6^0 = \begin{bmatrix} -c_6\left(c_1s_5s_{(2-3)} + c_5\left(-c_1c_4c_{(2-3)} + s_1s_4\right)\right) - s_6\left(c_1c_{(2-3)}s_4 + c_4s_1\right) & c_6\left(c_1c_{(2-3)}s_4 + c_4s_1\right) - s_6\left(c_1s_5s_{(2-3)} + c_5\left(-c_1c_4c_{(2-3)} + s_1s_4\right)\right) & c_1c_5s_{(2-3)} - s_5\left(-c_1c_4c_{(2-3)} + s_1s_4\right) & 95c_1c_5s_{(2-3)} + c_1\left(-130.0c_{(2-3)} + 590.0s_2 + 647.07s_{(2-3)} + 150.0\right) - 95s_5\left(-c_1c_4c_{(2-3)} + s_1s_4\right) \\ -c_6\left(c_5\left(c_1s_4 + c_4c_{(2-3)}s_1\right) - s_1s_5s_{(2-3)}\right) + s_6\left(-c_1c_4 + c_{(2-3)}s_1s_4\right) & -c_6\left(-c_1c_4 + c_{(2-3)}s_1s_4\right) - s_6\left(c_5\left(c_1s_4 + c_4c_{(2-3)}s_1\right) - s_1s_5s_{(2-3)}\right) & -c_5s_1s_{(2-3)} - s_5\left(c_1s_4 + c_4c_{(2-3)}s_1\right) & -95c_5s_1s_{(2-3)} - s_1\left(-130.0c_{(2-3)} + 590.0s_2 + 647.07s_{(2-3)} + 150.0\right) - 95s_5\left(c_1s_4 + c_4c_{(2-3)}s_1\right) \\ -c_6\left(c_4c_5s_{(2-3)} + c_{(2-3)}s_5\right) + s_4s_6s_{(2-3)} & -c_6s_4s_{(2-3)} - s_6\left(c_4c_5s_{(2-3)} + c_{(2-3)}s_5\right) & -c_4s_5s_{(2-3)} + c_5c_{(2-3)} & 590.0c_2 - 95.0c_4s_5s_{(2-3)} + 95.0c_5c_{(2-3)} + 647.07c_{(2-3)} + 130.0s_{(2-3)} + 450.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Definição e Objetivos

- Deseja-se encontrar as soluções angulares que levam o manipulador à pose desejada
- Haverão várias soluções para uma mesma pose, a depender do manipulador em questão
- Métodos: Analítico X Geométrico X Numérico

O Punho Esférico



Link	a_i	α_i	d_i	θ_i
4	0	-90	0	θ_4^*
5	0	90	0	θ_5^*
6	0	0	d_6	θ_6^*

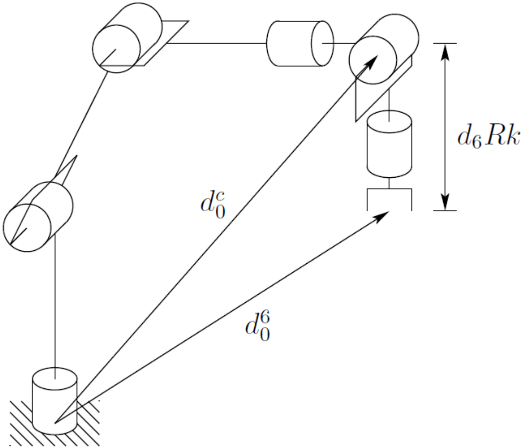
* variable

O Punho Esférico

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} T_6^3 &= A_4 A_5 A_6 \\ &= \begin{bmatrix} R_6^3 & o_6^3 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c_4 c_5 c_6 - s_4 s_6 & -c_4 c_5 s_6 - s_4 c_6 & c_4 s_5 & c_4 s_5 d_6 \\ s_4 c_5 c_6 + c_4 s_6 & -s_4 c_5 s_6 + c_4 c_6 & s_4 s_5 & s_4 s_5 d_6 \\ -s_5 c_6 & s_5 s_6 & c_5 & c_5 d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Desacoplamento Cinemático



Desacoplamento Cinemático

Para Posição:

$$o_c^0 = o - d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

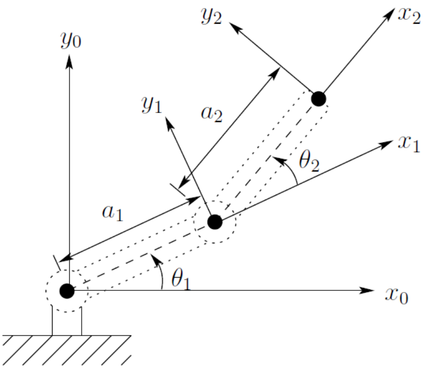
Para Orientação:

$$R_6^3 = (R_3^0)^{-1} R = (R_3^0)^T R$$

Função *atan*2()

- As soluções angulares serão da forma: $\theta = atan2(x, y) = Atan2(c_\theta, s_\theta)$
- O uso das funções *arccos* e *arcsin* poderiam gerar resposta incorreta para a configuração desejada, devido à ambiguidade de soluções para um mesmo valor de seno ou cosseno
- A função *arctan* também não é indicada, pois retorna o ângulo no intervalo $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right]$, por possuir apenas um argumento
- A função *atan*2() possui dois argumentos, retornando o Ângulo no intervalo $]-\pi, \pi]$

Exemplo 2.1.1: 2R Planar



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1^*
2	a_2	0	0	θ_2^*

* variable

Exemplo 2.1.1: 2R Planar

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} T_1^0 &= A_1 \\ T_2^0 &= A_1A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

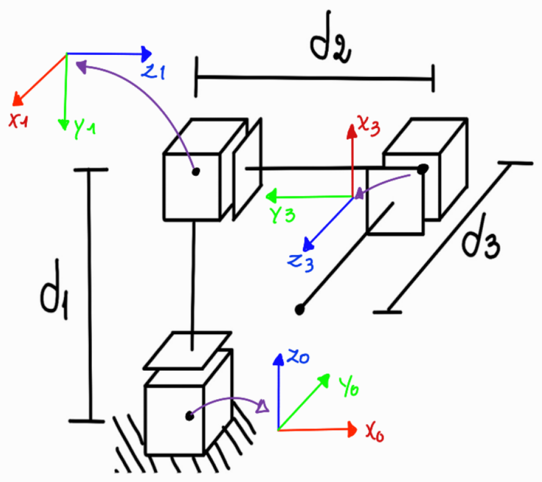
Exemplo 2.1.1: 2R Planar

$$\theta_1 = atan2(x, y) - atan2\left(\frac{a_2 s_2}{a_1 + a_2 c_2}\right)$$

$$c_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1 a_2}$$

$$\theta_2 = \pm atan2\left(c_2, \sqrt{1 - c_2^2}\right)$$

Exemplo 2.1.2: Manipulador Cartesiano



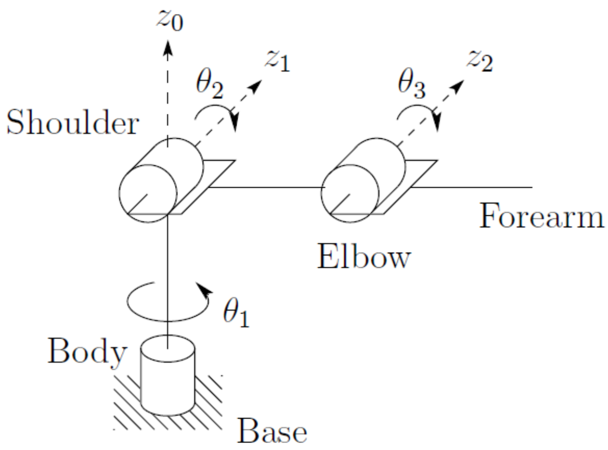
Exemplo 2.1.2: Manipulador Cartesiano

$$d_1 = z - l_1$$

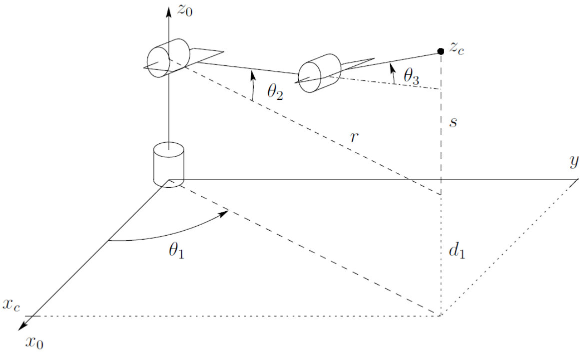
$$d_2 = x - l_2$$

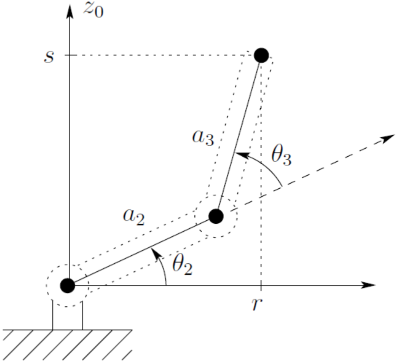
$$d_3 = -y - l_3$$

Exemplo 2.1.3: 3R Cotovelar

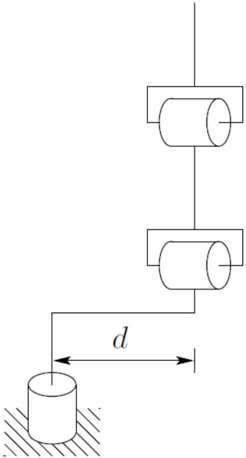


Exemplo 2.1.3: 3R Cotovelar

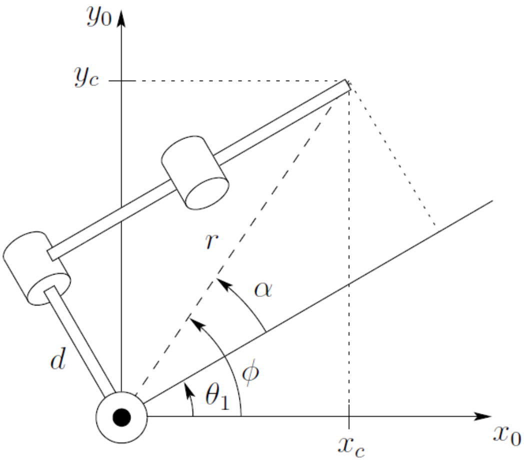




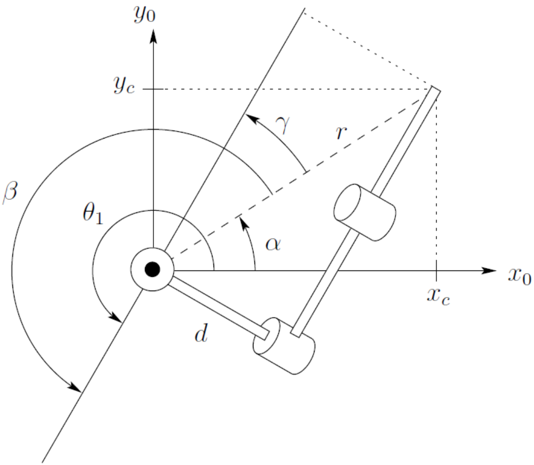
Exemplo 2.1.4: 3R Cotovelar + deslocamento em Y



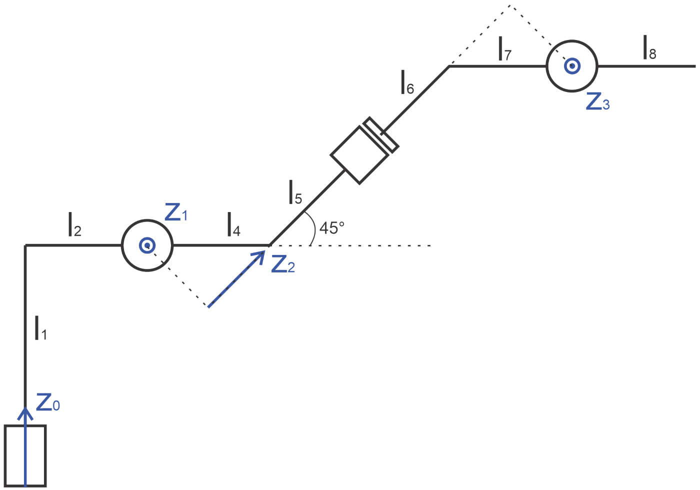
Exemplo 2.1.4: 3R Cotovelar + deslocamento em Y



Exemplo 2.1.4: 3R Cotovelar + deslocamento em Y



Exemplo 2.1.5: RRPR com *link* angulado

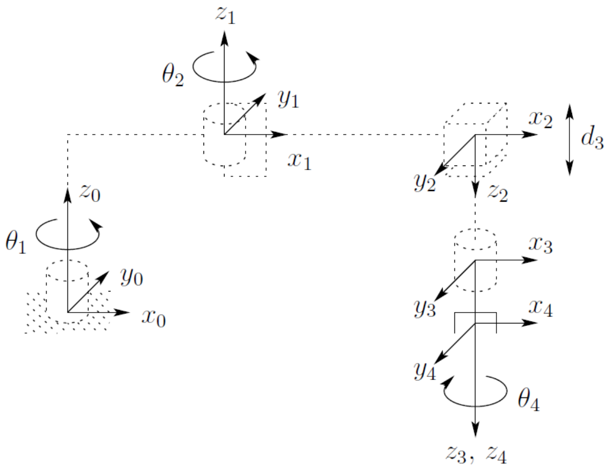


Orientação Generalizada pelo Punho Esférico

$$R_6^3 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix}$$

$$R_6^3 = (R_3^0)^T R$$

Exemplo 2.2.1: *SCARA*



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\theta^\star$
2	a_2	180	0	$\theta^\star$
3	0	0	$d^\star$	0
4	0	0	d_4	$\theta^\star$

* joint variable

Exemplo 2.2.1: *SCARA*

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} T_4^0 &= A_1 \cdots A_4 \\ &= \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Exemplo 2.2.1: *SCARA*

$$c_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

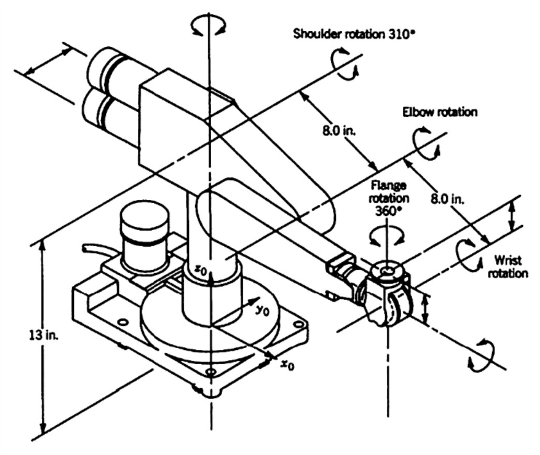
$$\theta_2 = atan2\left(c_2, \sqrt{1 - c_2^2}\right)$$

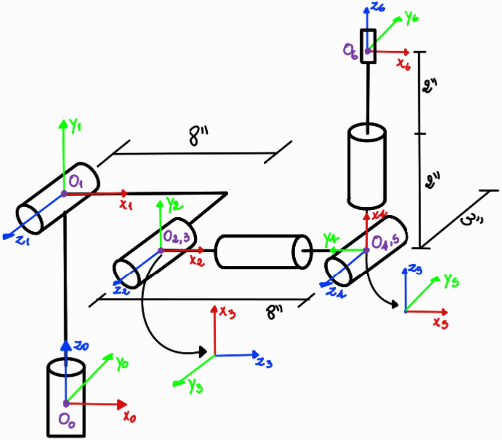
$$\theta_1 = atan2\left(x, y\right) - atan2\left(a_1 + a_2c_2, a_2s_2\right)$$

$$\theta_4 = \theta_1 + \theta_2 - atan2\left(r_{11}, r12\right)$$

$$d_3 = z - d4$$

Exemplo 2.2.2: *PUMA 260*





Exemplo 2.2.2: *PUMA 260*

Tabela de DH - PUMA 260

i	θ_i	d_i	a_i	α_i
1	θ_1^*	13	0	90°
2	θ_2^*	3	8	0°
3	$\theta_3^* + 90°$	0	0	90°
4	θ_4^*	8	0	-90°
5	$\theta_5^* - 90°$	0	0	-90°
6	θ_6^*	4	0	0°

* variável

Exemplo 2.2.2: *PUMA 260*

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 13 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 8c_2 \\ s_2 & c_2 & 0 & 8s_2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_3 = \begin{bmatrix} -s_3 & 0 & c_3 & 0 \\ c_3 & 0 & s_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_5 = \begin{bmatrix} s_5 & 0 & c_5 & 0 \\ -c_5 & 0 & s_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 2.2.2: *PUMA 260*

$$H_6^0 = \begin{bmatrix} c_6\left(c_1c_{23}c_5 + s_5\left(-c_1c_4s_{23} + s_1s_4\right)\right) - s_6\left(c_1s_{23}s_4 + c_4s_1\right) & -c_6\left(c_1s_{23}s_4 + c_4s_1\right) - s_6\left(c_1c_{23}c_5 + s_5\left(-c_1c_4s_{23} + s_1s_4\right)\right) & -c_1c_{23}s_5 + c_5\left(-c_1c_4s_{23} + s_1s_4\right) & 8c_1c_2 - 4c_1c_{23}s_5 + 8c_1c_{23} + 4c_5\left(-c_1c_4s_{23} + s_1s_4\right) + 3s_1 \\ -c_6\left(-c_{23}c_5s_1 + s_5\left(c_1s_4 + c_4s_1s_{23}\right)\right) - s_6\left(-c_1c_4 + s_1s_{23}s_4\right) & -c_6\left(-c_1c_4 + s_1s_{23}s_4\right) + s_6\left(-c_{23}c_5s_1 + s_5\left(c_1s_4 + c_4s_1s_{23}\right)\right) & -c_{23}s_1s_5 - c_5\left(c_1s_4 + c_4s_1s_{23}\right) & -3c_1 + 8c_2s_1 - 4c_{23}s_1s_5 + 8c_{23}s_1 - 4c_5\left(c_1s_4 + c_4s_1s_{23}\right) \\ c_{23}s_4s_6 + c_6\left(c_{23}c_4s_5 + c_5s_{23}\right) & c_{23}c_6s_4 - s_6\left(c_{23}c_4s_5 + c_5s_{23}\right) & c_{23}c_4c_5 - s_{23}s_5 & 4c_{23}c_4c_5 + 8s_2 - 4s_{23}s_5 + 8s_{23} + 13 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 2.2.2: *PUMA 260*

$$r^2 = x_c^2 + y_c^2 + 3^2$$

$$\theta_1 = atan2(x_c, y_c) \pm atan2(r, 3)$$

$$c_3 = \frac{r^2 + (z_c - 13)^2}{128} - 1$$

$$\theta_3 = \pm atan2(c_3, \sqrt{1 - c_3^2})$$

$$\theta_2 = atan2(r, (z_c - 13)) - atan2(8(1 + c_3), 8s_3)$$

$$s_5 = -c_1 c_{23} r_{13} - c_{23} s_1 r_{2,3} - s_{23}$$

$$\theta_5 = atan2(1 - \sqrt{1 - s_5^2})$$

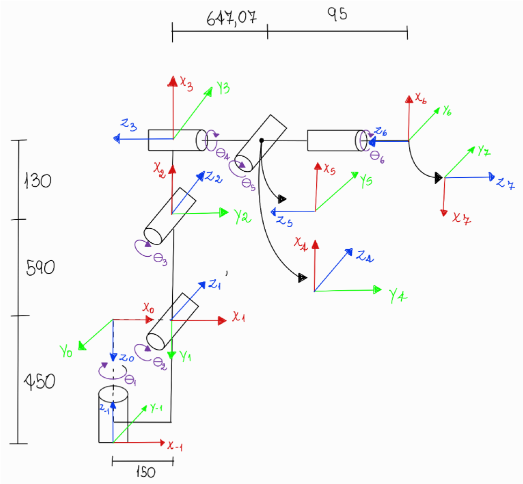
$$c_6 = \frac{c_1 c_{12} r_{11} + c_{23} s_1 r_{21} + s_{23} r_{31}}{c_5}$$

$$\theta_6 = atan2(c_6, \sqrt{1 - c_6^2})$$

$$s_4 = \frac{s_1 r_{13} - c_1 r_{23}}{c_5}$$

$$\theta_4 = atan2(1 - \sqrt{1 - s_4^2})$$

Exemplo 2.2.3: *Comau Smart5 SiX*



Exemplo 2.2.3: *Comau Smart5 SiX*

Tabela de DH - Comau Smart5 SiX

i	θ_i	d_i	a_i	α_i
1	θ_1^*	0	150	90°
2	$\theta_2^* - 90°$	0	590	180°
3	$\theta_3^* + 90°$	0	130	-90°
4	θ_4^*	-647,07	0	-90°
5	θ_5^*	0	0	90°
6	θ_6^*	-95	0	0°

$$H_0^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 450 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_7^6 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 2.2.3: *Comau Smart5 SiX*

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 150c_1 \\ -s_1 & 0 & c_1 & -150s_1 \\ 0 & -1 & 0 & 450 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} s_2 & -c_2 & 0 & 590s_2 \\ -c_2 & -s_2 & 0 & -590c_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} -s_3 & 0 & -c_3 & -130s_3 \\ c_3 & 0 & -s_3 & 130c_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & -647.07 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} -c_6 & -s_6 & 0 & 0 \\ -s_6 & c_6 & 0 & 0 \\ 0 & 0 & -1 & -95 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 2.2.3: *Comau Smart5 SiX*

$$H_6^0 = \begin{bmatrix} -c_6\left(c_1s_5s_{(2-3)} + c_5\left(-c_1c_4c_{(2-3)} + s_1s_4\right)\right) - s_6\left(c_1c_{(2-3)}s_4 + c_4s_1\right) & c_6\left(c_1c_{(2-3)}s_4 + c_4s_1\right) - s_6\left(c_1s_5s_{(2-3)} + c_5\left(-c_1c_4c_{(2-3)} + s_1s_4\right)\right) & c_1c_5s_{(2-3)} - s_5\left(-c_1c_4c_{(2-3)} + s_1s_4\right) & 95c_1c_5s_{(2-3)} + c_1\left(-130.0c_{(2-3)} + 590.0s_2 + 647.07s_{(2-3)} + 150.0\right) - 95s_5\left(-c_1c_4c_{(2-3)} + s_1s_4\right) \\ -c_6\left(c_5\left(c_1s_4 + c_4c_{(2-3)}s_1\right) - s_1s_5s_{(2-3)}\right) + s_6\left(-c_1c_4 + c_{(2-3)}s_1s_4\right) & -c_6\left(-c_1c_4 + c_{(2-3)}s_1s_4\right) - s_6\left(c_5\left(c_1s_4 + c_4c_{(2-3)}s_1\right) - s_1s_5s_{(2-3)}\right) & -c_5s_1s_{(2-3)} - s_5\left(c_1s_4 + c_4c_{(2-3)}s_1\right) & -95c_5s_1s_{(2-3)} - s_1\left(-130.0c_{(2-3)} + 590.0s_2 + 647.07s_{(2-3)} + 150.0\right) - 95s_5\left(c_1s_4 + c_4c_{(2-3)}s_1\right) \\ -c_6\left(c_4c_5s_{(2-3)} + c_{(2-3)}s_5\right) + s_4s_6s_{(2-3)} & -c_6s_4s_{(2-3)} - s_6\left(c_4c_5s_{(2-3)} + c_{(2-3)}s_5\right) & -c_4s_5s_{(2-3)} + c_5c_{(2-3)} & 590.0c_2 - 95.0c_4s_5s_{(2-3)} + 95.0c_5c_{(2-3)} + 647.07c_{(2-3)} + 130.0s_{(2-3)} + 450.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

O Jacobiano

$$\xi = J(q)\dot{q}$$

$$\xi = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad e \quad \dot{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} \quad e \quad J(q) = \begin{bmatrix} J_v(q) \\ J_\omega(q) \end{bmatrix}$$

Jacobiano de Velocidades Lineares

$$J_v(q) = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \cdots & \frac{\partial y}{\partial q_n} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \cdots & \frac{\partial z}{\partial q_n} \end{bmatrix}$$

Jacobiano de Velocidades Angulares

$$J_{\omega_i} = z_{i-1}, \text{ se a junta é rotacional.}$$

$$J_{\omega_i} = [0, 0, 0]^T, \text{ se a junta é prismática.}$$

Jacobiano Completo

$$J(q) = \begin{bmatrix} J_v(q) \\ J_\omega(q) \end{bmatrix}$$

$$J_i = \begin{cases} \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix} & \text{if joint i is revolute} \\ \\ \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} & \text{if joint i is prismatic} \end{cases}$$

Jacobiano Inverso e Velocidades Articulares

$$\dot{q} = J^{-1}(q)\xi$$

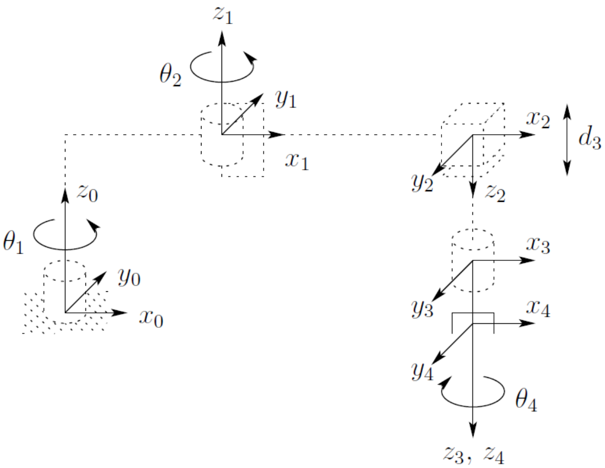
$$\xi = \begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad e \quad \dot{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} \quad e \quad J(q) = \begin{bmatrix} J_v(q) \\ J_\omega(q) \end{bmatrix}$$

Inversão de Matrizes

A partir de uma matriz A:

1. Calcular $det(A)$
2. Obter a matriz de cofatores C, onde $c_{ij} = (-1)^{i+j}|a_{ij}|$
3. Obter a matriz adjunta: $\bar{A} = C^T$
4. Obter a matriz inversa: $M^{-1} = \frac{1}{det(A)}\bar{A}$

Exemplo 3.1: *SCARA*



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\theta^\star$
2	a_2	180	0	$\theta^\star$
3	0	0	$d^\star$	0
4	0	0	d_4	$\theta^\star$

* joint variable

Exemplo 3.1: *SCARA*

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

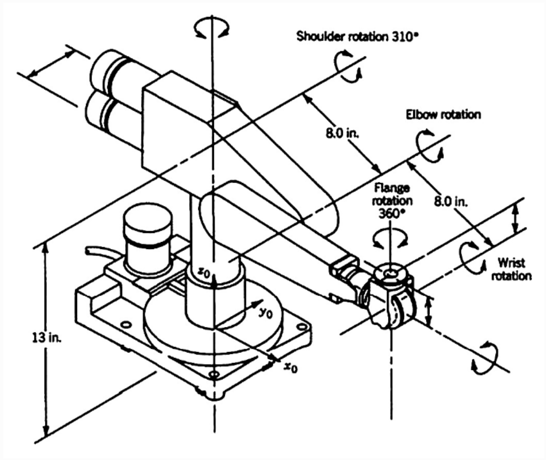
$$A_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

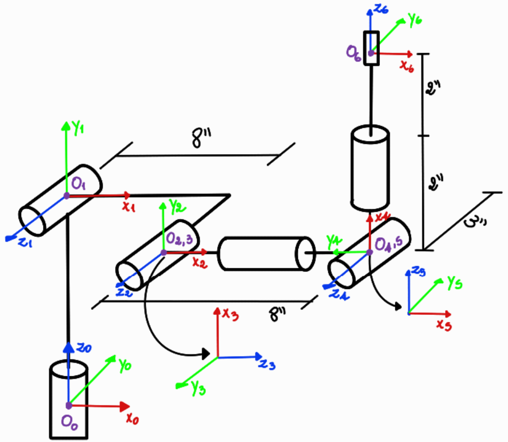
$$\begin{aligned} T_4^0 &= A_1 \cdots A_4 \\ &= \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Exemplo 3.1: *SCARA*

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

Exemplo 3.2: *PUMA 260*





Exemplo 3.2: *PUMA 260*

Tabela de DH - PUMA 260

i	θ_i	d_i	a_i	α_i
1	θ_1^*	13	0	90°
2	θ_2^*	3	8	0°
3	$\theta_3^* + 90°$	0	0	90°
4	θ_4^*	8	0	-90°
5	$\theta_5^* - 90°$	0	0	-90°
6	θ_6^*	4	0	0°

* variável

Exemplo 3.2: *PUMA 260*

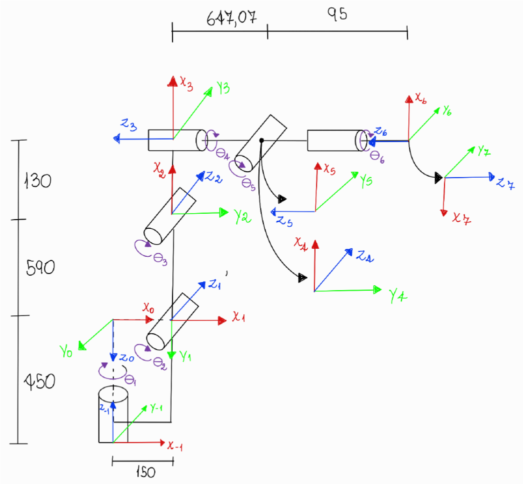
$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 13 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & 8c_2 \\ s_2 & c_2 & 0 & 8s_2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} -s_3 & 0 & c_3 & 0 \\ c_3 & 0 & s_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} s_5 & 0 & c_5 & 0 \\ -c_5 & 0 & s_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 3.2: *PUMA 260*

$$H_6^0 = \begin{bmatrix} c_6\left(c_1c_{23}c_5 + s_5\left(-c_1c_4s_{23} + s_1s_4\right)\right) - s_6\left(c_1s_{23}s_4 + c_4s_1\right) & -c_6\left(c_1s_{23}s_4 + c_4s_1\right) - s_6\left(c_1c_{23}c_5 + s_5\left(-c_1c_4s_{23} + s_1s_4\right)\right) & -c_1c_{23}s_5 + c_5\left(-c_1c_4s_{23} + s_1s_4\right) & 8c_1c_2 - 4c_1c_{23}s_5 + 8c_1c_{23} + 4c_5\left(-c_1c_4s_{23} + s_1s_4\right) + 3s_1 \\ -c_6\left(-c_{23}c_5s_1 + s_5\left(c_1s_4 + c_4s_1s_{23}\right)\right) - s_6\left(-c_1c_4 + s_1s_{23}s_4\right) & -c_6\left(-c_1c_4 + s_1s_{23}s_4\right) + s_6\left(-c_{23}c_5s_1 + s_5\left(c_1s_4 + c_4s_1s_{23}\right)\right) & -c_{23}s_1s_5 - c_5\left(c_1s_4 + c_4s_1s_{23}\right) & -3c_1 + 8c_2s_1 - 4c_{23}s_1s_5 + 8c_{23}s_1 - 4c_5\left(c_1s_4 + c_4s_1s_{23}\right) \\ c_{23}s_4s_6 + c_6\left(c_{23}c_4s_5 + c_5s_{23}\right) & c_{23}c_6s_4 - s_6\left(c_{23}c_4s_5 + c_5s_{23}\right) & c_{23}c_4c_5 - s_{23}s_5 & 4c_{23}c_4c_5 + 8s_2 - 4s_{23}s_5 + 8s_{23} + 13 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 3.3: *Comau Smart5 SiX*



Exemplo 3.3: *Comau Smart5 SiX*

Tabela de DH - Comau Smart5 SiX

i	θ_i	d_i	a_i	α_i
1	θ_1^*	0	150	90°
2	$\theta_2^* - 90°$	0	590	180°
3	$\theta_3^* + 90°$	0	130	-90°
4	θ_4^*	-647,07	0	-90°
5	θ_5^*	0	0	90°
6	θ_6^*	-95	0	0°

$$H_0^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 450 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_7^6 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 3.3: *Comau Smart5 SiX*

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 150c_1 \\ -s_1 & 0 & c_1 & -150s_1 \\ 0 & -1 & 0 & 450 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} s_2 & -c_2 & 0 & 590s_2 \\ -c_2 & -s_2 & 0 & -590c_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} -s_3 & 0 & -c_3 & -130s_3 \\ c_3 & 0 & -s_3 & 130c_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

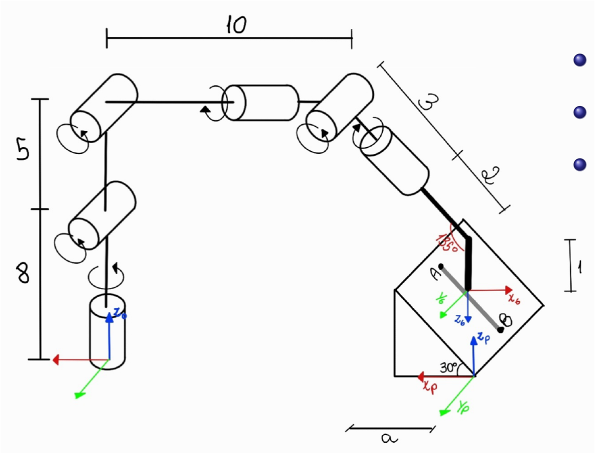
$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & -647.07 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} -c_6 & -s_6 & 0 & 0 \\ -s_6 & c_6 & 0 & 0 \\ 0 & 0 & -1 & -95 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exemplo 3.3: *Comau Smart5 SiX*

$$H_6^0 = \begin{bmatrix} -c_6\left(c_1 s_5 s_{(2-3)} + c_5\left(-c_1 c_4 c_{(2-3)} + s_1 s_4\right)\right) - s_6\left(c_1 c_{(2-3)} s_4 + c_4 s_1\right) & c_6\left(c_1 c_{(2-3)} s_4 + c_4 s_1\right) - s_6\left(c_1 s_5 s_{(2-3)} + c_5\left(-c_1 c_4 c_{(2-3)} + s_1 s_4\right)\right) & c_1 c_5 s_{(2-3)} - s_5\left(-c_1 c_4 c_{(2-3)} + s_1 s_4\right) & 95 c_1 c_5 s_{(2-3)} + c_1\left(-130.0 c_{(2-3)} + 590.0 s_2 + 647.07 s_{(2-3)} + 150.0\right) - 95 s_5\left(-c_1 c_4 c_{(2-3)} + s_1 s_4\right) \\ -c_6\left(c_5\left(c_1 s_4 + c_4 c_{(2-3)} s_1\right) - s_1 s_5 s_{(2-3)}\right) + s_6\left(-c_1 c_4 + c_{(2-3)} s_1 s_4\right) & -c_6\left(-c_1 c_4 + c_{(2-3)} s_1 s_4\right) - s_6\left(c_5\left(c_1 s_4 + c_4 c_{(2-3)} s_1\right) - s_1 s_5 s_{(2-3)}\right) & -c_5 s_1 s_{(2-3)} - s_5\left(c_1 s_4 + c_4 c_{(2-3)} s_1\right) & -95 c_5 s_1 s_{(2-3)} - s_1\left(-130.0 c_{(2-3)} + 590.0 s_2 + 647.07 s_{(2-3)} + 150.0\right) - 95 s_5\left(c_1 s_4 + c_4 c_{(2-3)} s_1\right) \\ -c_6\left(c_4 c_5 s_{(2-3)} + c_{(2-3)} s_5\right) + s_4 s_6 s_{(2-3)} & -c_6 s_4 s_{(2-3)} - s_6\left(c_4 c_5 s_{(2-3)} + c_{(2-3)} s_5\right) & -c_4 s_5 s_{(2-3)} + c_5 c_{(2-3)} & 590.0 c_2 - 95.0 c_4 s_5 s_{(2-3)} + 95.0 c_5 c_{(2-3)} + 647.07 c_{(2-3)} + 130.0 s_{(2-3)} + 450.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Questão de Prova: Soldagem em Plano Inclinado

Calcular as velocidades máximas de cada junta durante a soldagem do arco $\overline{AB}$.



- $o_p^0 = [-5, 3, 2]^T$
- O comprimento do arco de solda $\overline{AB} = \frac{3}{4}a$
- $a = 3$

Singularidades

- Singularidades representam configurações a partir das quais certas direções de movimento tornam-se restringidas.
- Em singularidades, as velocidades limitadas do efetor final podem corresponder a ilimitadas velocidades das juntas.
- Nas singularidades, as forças e torques limitados do efetor final podem corresponder a torques de junta ilimitados.
- Singularidades geralmente (mas nem sempre) correspondem a pontos no limite do espaço de trabalho do manipulador, ou seja, aos pontos de alcance máximo do manipulador.
- Singularidades correspondem a pontos no espaço de trabalho do manipulador que podem ser inacessível sob pequenas perturbações dos parâmetros do link, como comprimento, deslocamento, etc.
- Perto de singularidades não existirá solução única para a cinemática inversa problema. Em tais casos pode não haver solução ou pode haver infinitas soluções.

Desacoplamento de Singularidades

$$det\left(J(q)\right) = 0$$

$$J = [J_P \mid J_O] \quad = \quad \left[\begin{array}{c|c} J_{11} & J_{12} \\ \hline J_{21} & J_{22} \end{array}\right]$$

$$det(J) = det(J_{11})det(J_{22})$$

Tipos de Singularidades

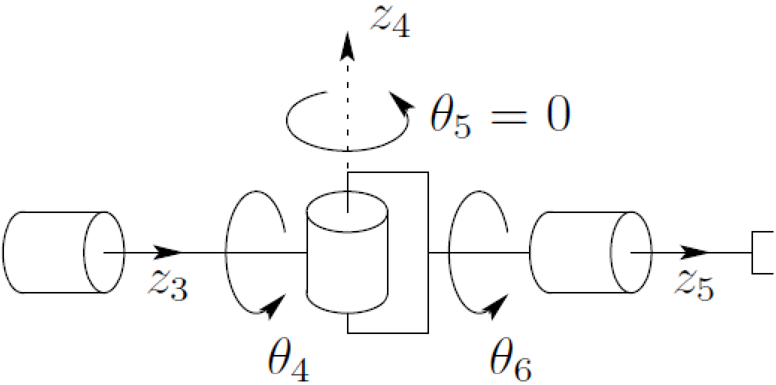
Singularidades Internas

Ocorre quando há diversas soluções para um mesmo ângulo, na cinemática inversa.

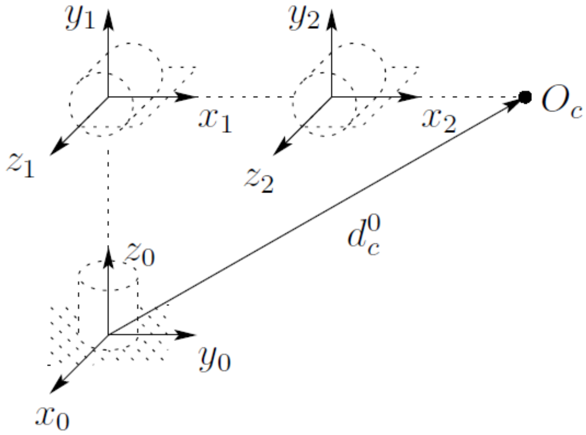
Singularidades Externas

Ocorre quando, a partir de uma posição atual, há alguma restrição de movimento, em alguma direção.

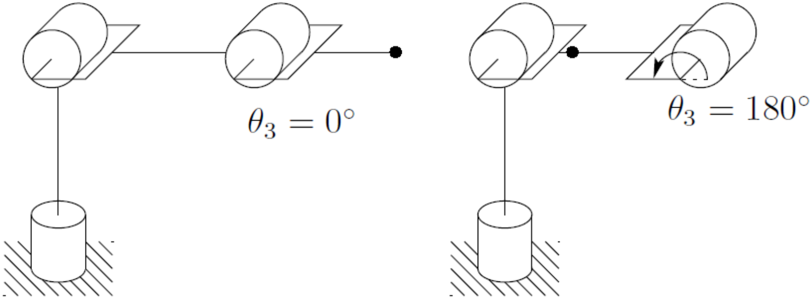
Singularidades de Punho



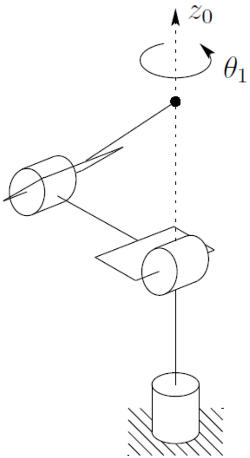
Singularidades de Braço



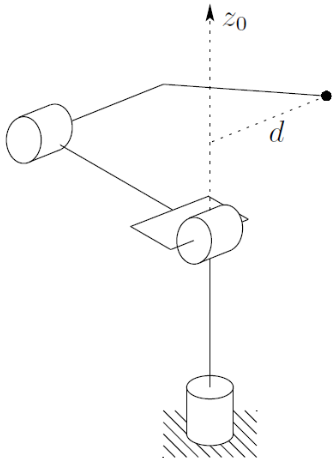
Singularidades em Manipuladores Cotovelares



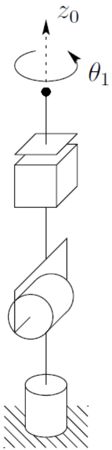
Singularidades em Manipuladores Cotovelares



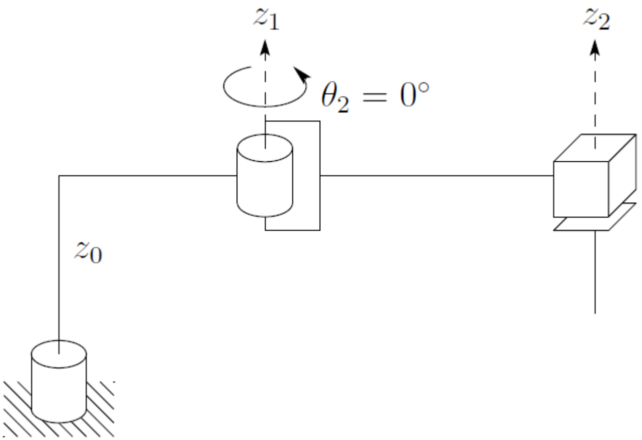
Singularidades em Manipuladores Cotovelares



Singularidades em Manipuladores Esféricos



Singularidades no Manipulador *SCARA*



Referências

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