

Least-Squares Filter Applied to a Temperature Control System using Simulink

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Abstract—Accurate temperature monitoring is crucial in numerous daily and industrial applications. To address this, the utilization of a least-squares filter to effectively mitigate noise in the measured variable is explored in this paper.

For the study, a heating system was simulated using Simulink, incorporating Gaussian white noise into the output to mirror real-world disturbances. The system maintains a setpoint of 25°C, which represents the ambient temperature, and features an input pulse that activates the heating mechanism.

The findings indicate that the least-squares estimation technique successfully approximates the actual temperature with a high degree of accuracy, achieving results that closely match the true values.

Keywords—Control System, Least Squares Filter, Matlab, Simulink, Temperature

I. INTRODUCTION

I-A. Heating and Temperature Control

Heating and temperature control systems have played a crucial role since the end of the 19th century. These systems have a wide range of applications from day-to-day tasks to industrial applications.

In 1895, Johnson made a significant breakthrough in temperature control with an automatic multi-zone temperature control system, designed to regulate the temperature in individual rooms or apartments [1]. Subsequently, a variety of other applications have emerged. In the field of biotechnology, applications like Polymerase Chain Reaction (PCR)—such [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]—and Temperature Gradient Focusing (TGF) [17, 18] for Electrophoresis requires tight temperature control [19]. In biological science, precise temperature control is crucial, such as maintaining specific temperatures for cell viability or for PCR temperature cycling [19, 20]. Also, in the food supply chain is important to ensure product quality and customer satisfaction, in transportation modes like sea, land, and railway, and in the logistic clusterization process can significantly reduce costs [21]. On the other hand, in the automotive industry, thermal management in vehicle electrification enhances vehicle efficiency and battery performance [22, 23] and in specific areas like thermal management of lithium-ion batteries [24], electric machines [25], and novel thermal management systems for batteries [26].

In critical applications such as mentioned before, accurate temperature monitoring is imperative. Consequently, the utilization of a least-squares filter to effectively mitigate noise in the target variable is explored.

I-B. Least-Squares Filter

Least-squares estimation theory was introduced by Gauss and further developed by Kalman, focusing on minimizing the sum of the squares of the difference between observed and estimated values [27]. This methodology finds applications across a broad spectrum of categories, such as data curve fitting, parameter identification, and the realization of system models [28]. This technique is versatile, with applications spanning various domains. Examples include calculating the damping properties of a fluid-filled damper based on temperature, identifying aircraft dynamic and static aerodynamic coefficients, determining orbit and attitude, locating position using triangulation, identifying modes of vibratory systems, and modern control strategies like some adaptive controllers where least-squares method is used to refine model parameters within the control system [28].

Assuming a set or a batch of measured values, \tilde{y}_j , of a process $y(t)$, taken at known discrete instants of time t_j :

$$\{\tilde{y}_1, t_1; \tilde{y}_2, t_2; \dots; \tilde{y}_m, t_m; \} \quad (1)$$

and a proposed mathematical model of the form

$$y(t) = \sum_{i=1}^n x_i h_i(t), \quad m \geq n \quad (2)$$

where

$$h_i(t) \in \{h_1(t), h_2(t), \dots, h_n(t)\} \quad (3)$$

are a set of independently specified basis functions where the measurements \tilde{y}_j and the estimated output \hat{y}_j can be related to the true and the estimated x-values leading to the Eq 4 and 5 [28]:

$$\tilde{y}_j \equiv \tilde{y}(t_j) = \sum_{i=1}^n x_i h_i(t_j) + v_j, \quad j = 1, 2, \dots, m \quad (4)$$

$$\hat{y}_j \equiv \hat{y}(t_j) = \sum_{i=1}^n \hat{x}_i h_i(t_j), \quad j = 1, 2, \dots, m \quad (5)$$

where v_j is the measurement error. This leads to the following identity:

$$\tilde{y}_j = \sum_{i=1}^n \hat{x}_i h_i(t_j) + e_j, \quad j = 1, 2, \dots, m \quad (6)$$

where residual error e_j is defined by

$$e_j \equiv \tilde{y}_j - \hat{y}_j \quad (7)$$

and this can be rewritten in compact matrix form as:

$$\tilde{\mathbf{y}} = H\hat{\mathbf{x}} + \mathbf{e} \quad (8)$$

where [28]

$\tilde{\mathbf{y}} = [\tilde{y}_1 \ \tilde{y}_2 \ \dots \ \tilde{y}_m]$ = measured y-values

$\mathbf{e} = [e_1 \ e_2 \ \dots \ e_m]$ = residual errors

$\hat{\mathbf{x}} = [\hat{x}_1 \ \hat{x}_2 \ \dots \ \hat{x}_m]$ = estimated x-values

In similar way, Eq 9 and 10 can be represented using its compact matrix form as [28]:

$$\tilde{\mathbf{y}} = H\mathbf{x} + \mathbf{v} \quad (9)$$

$$\hat{\mathbf{y}} = H\hat{\mathbf{x}} \quad (10)$$

where

$\mathbf{x} = [x_1 \ x_2 \ \dots \ x_m]$ = true x-values

$\mathbf{v} = [v_1 \ v_2 \ \dots \ v_m]$ = measurements errors

$\hat{\mathbf{y}} = [\hat{y}_1 \ \hat{y}_2 \ \dots \ \hat{y}_m]$ = estimated y-values

$\tilde{\mathbf{y}} = [\tilde{y}_1 \ \tilde{y}_2 \ \dots \ \tilde{y}_m]$ = measured y-values

I-B1. Linear Least Squares: The least squares principle selects particular $\hat{\mathbf{x}}$ that minimizes the sum square of the residual errors as an optimum choice for the unknown parameters, given by [28]:

$$J = \frac{1}{2} e^T e \quad (11)$$

By substituting Eq 6 e into the Eq 11, applying the gradient of $\nabla_{\hat{\mathbf{x}}} J$ and equaling to zero, the explicit solution for the optimal estimate is obtained:

$$\hat{\mathbf{x}} = (H_T H)^{-1} H_T \tilde{\mathbf{y}} \quad (12)$$

I-B2. Weighted Least Squares: In cases where measurements are taken with varying degrees of precision, the approach of assigning equal weight appears to be logically flawed. To incorporate proper weighting, a least squares criterion of the form is established:

$$J = \frac{1}{2} e^T W e \quad (13)$$

producing the solution for $\hat{\mathbf{x}}$ given by

$$\hat{\mathbf{x}} = (H_T W H)^{-1} H_T W \tilde{\mathbf{y}} \quad (14)$$

where W is an $m \times m$ symmetric matrix [28].

When data is not available for batch processing—like in numerous real-world applications—but it is sequentially available, one might find it advantageous to calculate fresh estimates by considering all past measurements. For such cases, the covariance recursion form is used [28]:

$$\hat{x}(k+1) = \hat{x}(k) + K(k+1) [\tilde{y}(k+1) - H(k+1)\hat{x}(k)] \quad (15)$$

where

$$K(k+1) = P(k)H^T(k+1)[H(k+1)P(k)H^T(k+1) + W_{-1}(k+1)]^{-1} \quad (16)$$

$$P(k+1) = [I - K(k+1)H(k+1)]P(k) \quad (17)$$

with initial values of

$$P(0) = \left[\frac{1}{\alpha^2} I + H_T(0)W(0)H(0) \right]^{-1} \quad (18)$$

$$\hat{x}(0) = P(0) \left[\frac{1}{\alpha} \beta + H_T(0)W(0)\hat{y}(0) \right] \quad (19)$$

I-C. System Model

Given a basic heating house system that considers the outdoor and indoor temperature, the dynamics of the systems are described by a first-order differential equation with a time constant τ [29]:

$$\dot{T}(t) = \frac{1}{\tau} (T_{ambient} - T(t)) + \frac{K_p}{\tau} \cdot u(t) \quad (20)$$

where

- τ is the time constant of the system.
- $T_{ambient}$ is the temperature outside the house.
- $T(t)$ is the temperature in the house at time t .
- K_p is the proportional gain or the heat rating of the system.
- $u(t)$ is the switch, = 1 if the system is on and = 0 if the system is off.

In the continuous-time state-space representation of a first-order system, the dynamics are described by the following first-order differential equation [30]:

III. RESULTS AND DISCUSSION

The Simulink-aided implemented design can be observed in Fig 1. The output of the system, denoted as x_k , is connected to a display for real-time visualization of the parameter estimation (ϕ and Γ). Additionally, the input function (sum of unit step functions), the ambient temperature readings, and the prediction of the system (x_k) are connected for waveform analysis and verification.

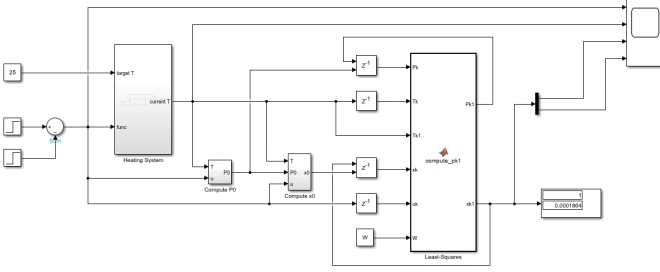


Figure 2.

The initial scenario involves a solitary measurement of the temperature. Fig 3 shows the corresponding waveforms. The first column illustrates the input function alongside the recorded temperature. Meanwhile, the second column depicts the estimation of the filter.

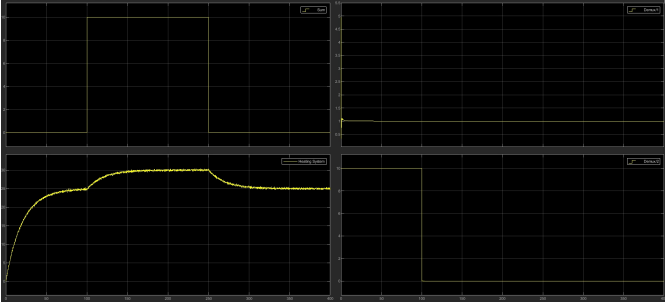


Figure 3. In the first column are the input function and measured temperature. In the second column are the filter estimations

In Fig 4 presents the estimation of the parameters. The system estimates $\phi = 1$ and $\Gamma = 0.0001864$, reflecting a relative error of $\phi_{error} = 0.5\%$ and $\Gamma_{error} = 92\%$. Despite the relatively high error in Γ , the system's overall accuracy for a single sample remains commendable.

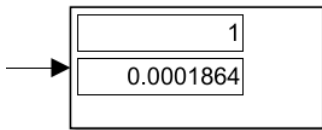


Figure 4. ϕ and Γ estimation using just one sample of the temperature

In the second scenario, leveraging multiple temperature samples significantly enhances system's estimation capabilities. Fig 5 illustrates the estimation of the parameters based on this expanded dataset. In this scenario, the system estimates $\phi = 0.9975$ and $\Gamma = 0.002558$ resulting in a relative error of $\phi_{error} = 3\%$ and $\Gamma_{error} = 9\%$. These findings underscore the system's enhanced accuracy as it benefits from a richer dataset, showcasing its capacity to improve with increased data input.

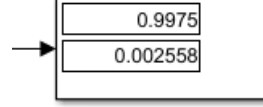


Figure 5. ϕ and Γ estimation using multiple samples of the temperature

IV. CONCLUSIONS

Heating and temperature control systems play a crucial role in enhancing both energy efficiency and occupant comfort. Accurately controlling indoor temperatures is indispensable across several fields, including but not limited to biotechnology, biology, food supply chain, transportation, automotive industries. These systems ensure optimal thermal comfort while providing a high degree of customization and flexibility to accommodate diverse user requirements and building characteristics. By minimizing energy wastage, they facilitate cost savings and contribute significantly to environmental sustainability efforts.

The least-squares filter exhibits remarkable robustness in handling data susceptible to random errors or fluctuations, showcasing its adaptability to diverse degrees of data complexity. While it excels in delivering accurate parameter estimations, particularly with large data, it does entail heightened computational demands as dataset size increases, necessitating careful resource allocation. Nevertheless, its versatility renders it invaluable across a wide range of fields and disciplines, underscoring its universal utility and relevance.

Simulink presents a robust platform for multidomain modeling and simulation, empowering engineers and researchers to design and simulate complex systems that span multiple domains. Its intuitive graphical user interface and extensive library of pre-built blocks streamline the integration of diverse components and subsystems, facilitating accelerated prototyping and comprehensive system-level analysis and validation. With its user-friendly block diagram interface, users can quickly iterate on system designs, fine-tune parameters, and evaluate performance in real-time, thereby trimming development timelines and expenses. Furthermore, Simulink's seamless integration with MATLAB provides a cohesive environment for learning and applying computational techniques across various engineering and scientific disciplines.

REFERENCES

- [1] W. S. Johnson, "Heat regulating apparatus," U.S. Patent 542 733, July 16, 1895. [Online]. Available: <https://patents.google.com/patent/US542733A>
- [2] K. B. Mullis and F. A. Faloona, "[21] Specific synthesis of DNA in vitro via a polymerase-catalyzed chain reaction," in *Recombinant DNA Part F*, ser. Methods in Enzymology. Academic Press, 1987, vol. 155, pp. 335–350. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/0076687987550236>
- [3] R. K. Saiki, D. H. Gelfand, S. Stoffel, S. J. Scharf, R. Higuchi, G. T. Horn, K. B. Mullis, and H. A. Erlich, "Primer-Directed Enzymatic Amplification of DNA with a Thermostable DNA Polymerase," *Science*, vol. 239, no. 4839, pp. 487–491, 1988. [Online]. Available: <https://www.science.org/doi/abs/10.1126/science.2448875>
- [4] J. M. S. Bartlett and D. Stirling, *A Short History of the Polymerase Chain Reaction*. Totowa, NJ: Humana Press, 2003, pp. 3–6. [Online]. Available: <https://doi.org/10.1385/1-59259-384-4:3>
- [5] C. D. Ahrberg, A. Manz, and B. G. Chung, "Polymerase chain reaction in microfluidic devices," *Lab Chip*, vol. 16, pp. 3866–3884, 2016. [Online]. Available: <http://dx.doi.org/10.1039/C6LC00984K>
- [6] G. Maltezos, M. Johnston, K. Taganov, C. Srichantaramee, J. Gorman, D. Baltimore, W. Chantratita, and A. Scherer, "Exploring the limits of ultrafast polymerase chain reaction using liquid for thermal heat exchange: A proof of principle," *Applied physics letters*, vol. 97, p. 264101, 12 2010.
- [7] T. Mcknight, S. Jacobson, L. Waters, R. Foote, and J. Ramsey, "Integrated System for Rapid PCR-Based DNA Analysis in Microfluidic Devices," *Analytical chemistry*, vol. 72, pp. 2995–3000, 08 2000.
- [8] J. Yang, Y. Liu, C. B. Rauch, R. L. Stevens, R. H. Liu, R. Lenigk, and P. Grodzinski, "High sensitivity PCR assay in plastic micro reactors," *Lab Chip*, vol. 2, pp. 179–187, 2002. [Online]. Available: <http://dx.doi.org/10.1039/B208405H>
- [9] Z. Hua, J. L. Rouse, A. E. Eckhardt, V. Srinivasan, V. K. Pamula, W. A. Schell, J. L. Benton, T. G. Mitchell, and M. G. Pollack, "Multiplexed real-time polymerase chain reaction on a digital microfluidic platform," *Analytical chemistry*, vol. 82, no. 6, pp. 2310–2316, 2010.
- [10] S. Mahjoob, K. Vafai, and N. R. Beer, "Rapid microfluidic thermal cyler for polymerase chain reaction nucleic acid amplification," *International Journal of Heat and Mass Transfer*, vol. 51, no. 9-10, pp. 2109–2122, 2008.
- [11] M. P. Dinca, M. Gheorghe, M. Aherne, and P. Galvin, "Fast and accurate temperature control of a PCR microsystem with a disposable reactor," *Journal of Micromechanics and Microengineering*, vol. 19, no. 6, p. 065009, 2009.
- [12] K.-Y. Lien, S.-H. Lee, T.-J. Tsai, T.-Y. Chen, and G.-B. Lee, "A microfluidic-based system using reverse transcription polymerase chain reactions for rapid detection of aquaculture diseases," *Microfluidics and nanofluidics*, vol. 7, pp. 795–806, 2009.
- [13] X. Qiu, M. G. Mauk, D. Chen, C. Liu, and H. H. Bau, "A large volume, portable, real-time PCR reactor," *Lab on a Chip*, vol. 10, no. 22, pp. 3170–3177, 2010.
- [14] T.-M. Hsieh, C.-H. Luo, F.-C. Huang, J.-H. Wang, L.-J. Chien, and G.-B. Lee, "Enhancement of thermal uniformity for a microthermal cyler and its application for polymerase chain reaction," *Sensors and Actuators B: Chemical*, vol. 130, no. 2, pp. 848–856, 2008.
- [15] K. Shen, X. Chen, M. Guo, and J. Cheng, "A microchip-based PCR device using flexible printed circuit technology," *Sensors and Actuators B: Chemical*, vol. 105, no. 2, pp. 251–258, 2005.
- [16] J.-H. Wang, L.-J. Chien, T.-M. Hsieh, C.-H. Luo, W.-P. Chou, P.-H. Chen, P.-J. Chen, D.-S. Lee, and G.-B. Lee, "A miniaturized quantitative polymerase chain reaction system for DNA amplification and detection," *Sensors and Actuators B: Chemical*, vol. 141, no. 1, pp. 329–337, 2009.
- [17] T. Matsui, J. Franzke, A. Manz, and D. Janasek, "Temperature gradient focusing in a PDMS/glass hybrid microfluidic chip," *Electrophoresis*, vol. 28, no. 24, pp. 4606–4611, 2007.
- [18] D. Ross and L. E. Locascio, "Microfluidic temperature gradient focusing," *Analytical chemistry*, vol. 74, no. 11, pp. 2556–2564, 2002.
- [19] V. Miralles, A. Huerre, F. Malloggi, and M.-C. Jullien, "A Review of Heating and Temperature Control in Microfluidic Systems: Techniques and Applications," *Diagnostics*, vol. 3, no. 1, pp. 33–67, 2013. [Online]. Available: <https://www.mdpi.com/2075-4418/3/1/33>
- [20] P. Hung, P. Lee, P. Sabounchi, R. Lin, and L. Lee, "Continuous perfusion microfluidic cell Culture array for high-throughput cell-based assays," *Biotechnology and bioengineering*, vol. 89, pp. 1–8, 01 2005.
- [21] Baskutis, Saulius and Navickas, Valentinas and Gruzauskas, Valentas and Olencevičiūtė, Dalia, "The temperature control impact to the food supply chain," *Mechanika 2015 : proceedings of the 20th international scientific conference*, pp. 42 – 47, 01 2015.
- [22] L. C. Casals, E. Martinez-Laserna, B. A. García, and N. Nieto, "Sustainability analysis of the electric vehicle use in Europe for CO2 emissions reduction," *Journal of cleaner production*, vol. 127, pp. 425–437, 2016.
- [23] G. Previati, G. Mastinu, and M. Gobbi, "Thermal Management of Electrified Vehicles—A Review," *Energies*, vol. 15, no. 4, 2022. [Online]. Available: <https://www.mdpi.com/1996-1073/15/4/1326>
- [24] G. Karimi and X. Li, "Thermal management of lithium-ion batteries for electric vehicles," *International Journal of Energy Research*, vol. 37, no. 1, pp. 13–24, 2013.
- [25] Y. Yang, B. Bilgin, M. Kasprzak, S. Nalakath, H. Sadek, M. Preindl, J. Cotton, N. Schofield, and A. Emadi, "Thermal management of electric machines," *IET Electrical Systems in Transportation*, vol. 7, no. 2, pp. 104–116, 2017.
- [26] M. Al-Zareer, I. Dincer, and M. A. Rosen, "A review of novel thermal management systems for batteries,"

- International Journal of Energy Research*, vol. 42, no. 10, pp. 3182–3205, 2018.
- [27] H. W. Sorenson, “Least-squares estimation: from Gauss to Kalman,” *IEEE Spectrum*, vol. 7, no. 7, pp. 63–68, 1970.
 - [28] J. Crassidis and J. Junkins, *Optimal Estimation of Dynamic Systems*. CRC Press, 01 2004.
 - [29] G. Franklin, J. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems (What’s New in Engineering)*, 8th ed. Pearson, 07 2021.
 - [30] M. Ayomoh and M. Ajala, “Neural network modeling of a tuned pid controller,” *European Journal of Scientific Research*, vol. Vol. 71, pp. Pp. 283–297, 02 2012.
 - [31] D. F. Griffiths and D. J. Higham, *Euler’s Method*. London: Springer London, 2010, pp. 19–31. [Online]. Available: https://doi.org/10.1007/978-0-85729-148-6_2
 - [32] M. Suleiman, Z. Ibrahim, and A. Rasedee, “Solution of higher-order odes using backward difference method,” *Mathematical Problems in Engineering*, vol. 2011, 01 2011.
 - [33] C. R. Phillips and N. T. Nagle, “Digital control system analysis and design,” *IEEE Transactions on Systems, Man, and Cybernetics*, vol. SMC-15, no. 3, pp. 452–453, 1985.