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Principles of AI Planning

Exercise Sheet 10

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Exercise 10.1 - Affecting labels vs. orthogonality

Recall: For a transition system $\mathcal{A}$ and a label l of $\mathcal{A}$, we say that l affects $\mathcal{A}$ if $\mathcal{A}$ has a transition $\langle s, l, t\rangle$ with $s \neq t$.
Prove the following: Let $\mathcal{A}_i$ be an abstraction of some transition system $\mathcal{T}$ with abstraction mapping α_i for $i \in \{1, 2\}$. If no label of $\mathcal{T}$ affects both $\mathcal{A}_1$ and $\mathcal{A}_2$, then α_1 and α_2 are orthogonal.

Take an arbitrary label $l \in \mathcal{T}$, by the premise we know that this label can affect at most one abstraction $\mathcal{A}_i$ for $i \in \{1, 2\}$. If it doesn't affect any abstraction $\mathcal{A}_i$ then there is no transition $\langle s, l, t\rangle \in \mathcal{T}$ such that $\alpha_1(s) \neq \alpha_1(t)$ or $\alpha_2(s) \neq \alpha_2(t)$.

If the label affects only one abstraction, we take the affected abstraction $\mathcal{A}_i$, we know there is at least one $\langle s, l, t\rangle \in \mathcal{T}$ such that $\alpha_i(s) \neq \alpha_i(t)$, but in the other abstraction $\mathcal{A}_j$ where $j \in \{1, 2\}$ and $j \neq i$ for each transition $\langle s, l, t\rangle$ of $\mathcal{T}$ we have $\alpha_j(s) = \alpha_j(t)$ as the label does not affect $\mathcal{A}_j$.

Therefore no matter which label l is selected there is no transition $\langle s, l, t\rangle \in \mathcal{T}$ where $\alpha_i(s) \neq \alpha_i(t)$ for both $i \in \{1, 2\}$ then by definition α_1 and α_2 are orthogonal.

Exercise 10.2 - Potential heuristics: consistency constraints

Let $\Pi = \langle V, I, O, \gamma\rangle$ be an SAS^+ planning task in transition normal form, and let $\mathcal{F} = \{f_{v=d} \mid v \in V, d \in \mathcal{D}_v\}$ be the set of all atomic features over Π. Finally, let:

$$h(s) = \sum_{fact\ v=d} w_{v=d} \cdot f_{v=d}(s)$$

be the potential heuristic with potentials $w_{v=d}$ for all $v \in V, d \in \mathcal{D}_v$, such that for all $o \in O$, the following constraint is satisfied:

$$\sum_{fact\ v=d \text{ consumed by } o} w_{v=d} - \sum_{fact\ v=d \text{ produced by } o} w_{v=d} \leq cost(o)$$

Prove: Then h is consistent, i. e., $h(s) - h(t) \leq cost(o)$ for all transitions (s, o, t) in $\mathcal{T}(\Pi)$.

Taking an arbitrary transition (s, o, t) in $\mathcal{T}(\Pi)$ as it is consistent

$$h(s) - h(t) \leq cost(o)$$

$$\iff \sum_{fact\ v=d} w_{v=d} \cdot f_{v=d}(s) + \sum_{fact\ v=d} w_{v=d} \cdot f_{v=d}(t) \leq cost(o)$$

$$f_{v=d}(s) = \begin{cases} 0, & \text{if } s \not\models v = d \\ 1, & \text{if } s \models v = d \end{cases} \qquad f_{v=d}(t) = \begin{cases} 0, & \text{if } t \not\models v = d \\ 1, & \text{if } t \models v = d \end{cases}$$

$$\iff \sum_{\substack{fact\ v=d \\ s \models v=d}} w_{v=d} + \sum_{\substack{fact\ v=d \\ t \models v=d}} w_{v=d} \leq cost(o)$$

We know that in the transition (s, o, t) the $vars(t) = vars(s) -$ consumed $vars$ by o + produced $vars$ by o, therefore:

$$\iff \sum_{\substack{fact\ v=d \\ s \models v=d}} w_{v=d} - \left[\sum_{\substack{fact\ v=d \\ s \models v=d}} w_{v=d} - \sum_{\substack{fact\ v=d \\ \text{consumed by o}}} w_{v=d} + \sum_{\substack{fact\ v=d \\ \text{produced by o}}} w_{v=d} \right] \leq cost(o)$$

$$\iff \sum_{\substack{fact\ v=d \\ \text{consumed by o}}} w_{v=d} - \sum_{\substack{fact\ v=d \\ \text{produced by o}}} w_{v=d} \leq cost(o)$$