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Principles of AI Planning

Exercise Sheet 9

10.01.2020

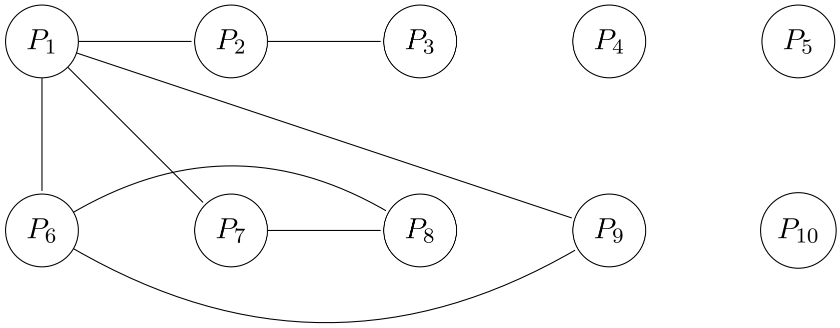
Exercise 9.1 - Additive patterns and canonical heuristic

(a) **Specify the compatibility graph of $\mathcal{C}$ and determine its maximal cliques**
Process:
To determine the compatibility graph, first we avoid drawing connections with the patterns with the same variables.
$\{P_1, P_3, P_8\}$ share variable *at-goal*$_{s2}$
$\{P_2, P_4, P_6, P_7, P_{10}\}$ share variable *at-goal*$_{s1}$
$\{P_2, P_4\}$ share variable *position*$_{s1}$
$\{P_4, P_5\}$ share variable *position*$_p$

Then we analyzed which operations affect two variables in different patterns and we find the followings:

- *move* always affects *position*$_p$ but it can also affect $content_x = nothing$ to $content_x := p$ or viceversa, therefore $\{P_4, P_5\}$ are not orthogonal with $\{P_6, P_7, P_8, P_9, P_9, P_{10}\}$ (no line from first set to second set)
- *push* affects *position*$_p$ and *position*$_{s1}$ or *position*$_{s2}$, therefore $\{P_4, P_5\}$ are not orthogonal with $\{P_2, P_3\}$
- *push* affects *position*$_{s1}$ and *content*$_x$, analogously with *position*$_{s2}$ therefore $\{P_2, P_3, P_4, P_5\}$ are not orthogonal with $\{P_6, P_7, P_8, P_9, P_10\}$
- When we *push* box $s1$ to the goal, *position*$_{s1}$, *position*$_p$ and *at-goal*$_{s1}$ are affected, analogously with $s2$, therefore:
  - $\{P_2, P_4, P_5\}$ are not orthogonal with $\{P_2, P_4, P_6, P_7, P_{10}\}$ because of *position*$_{s1}$ and *at-goal*$_{s1}$
  - $\{P_3\}$ are not orthogonal with $\{P_1, P_3, P_8\}$ because of *position*$_{s2}$ and *at-goal*$_{s2}$
  - $\{P_4, P_5\}$ are not orthogonal with $\{P_1, P_2, P_3, P_4, P_6, P_7, P_8, P_{10}\}$ because of *position*$_p$ and *at-goal*$_{s1}$ or *at-goal*$_{s2}$
- We can *push* a box from Q to a goal position, therefore, affecting *content*$_Q$ and *at-goal*$_{s1}$ or *at-goal*$_{s2}$, therefore $\{P_{10}\}$ are not orthogonal with $\{P_1, P_2, P_3, P_4, P_6, P_7, P_8\}$
- We can *push* a box from K to Q with the agent position in E, *content*$_E$ and *content*$_Q$ are affected, therefore $\{P_9\}$ and $\{P_{10}\}$ are not orthogonal.
- We can *push* a box from F to E with the agent position in G, *content*$_G$ and *content*$_E$ are affected, therefore $\{P_7\}$ and $\{P_9\}$ are not orthogonal.
- We can *move* between D and E, affecting *content*$_D$ and *content*$_E$, therefore $\{P_8\}$ and $\{P_9\}$ are not orthogonal.
- We can *move* between G and H, affecting *content*$_G$ and *content*$_H$, therefore $\{P_6\}$ and $\{P_7\}$ are not orthogonal.

Finally we connect the orthogonal abstractions to find the maximal cliques:



Maximal Cliques		
$\{P_1, P_2\}$	$\{P_1, P_6, P_9\}$	$\{P_1, P_7\}$
$\{P_7, P_8\}$	$\{P_6, P_8\}$	$\{P_2, P_3\}$
$\{P_4\}$	$\{P_5\}$	$\{P_{10}\}$

(b) **Determine the canonical heuristic $h^{\mathcal{C}}$ and simplify it as much as possible**
We will obtain the canonical heuristic value for the initial state given this patterns and maximal cliques.

i	h^{P_i}
1	1
2	5
3	4
4	13
5	0
6	1
7	1
8	1
9	0
10	1

Cliques heuristics

Clique	$h^{\mathcal{C}}$
$\{P_1, P_2\}$	6
$\{P_1, P_6, P_9\}$	2
$\{P_1, P_7\}$	2
$\{P_7, P_8\}$	2
$\{P_6, P_8\}$	2
$\{P_2, P_3\}$	9
$\{P_4\}$	13
$\{P_5\}$	0
$\{P_{10}\}$	1

$h^{\mathcal{C}} = 13$

(c) **Which patterns in $\mathcal{C}$ can be omitted and why?**

The patterns that don't include any variable from the goal condition as they are in the same abstraction mapping α that a goal statement the heuristic value will be 0, therefore these pattern heuristics are not informative at all, these are P_5 and P_9.

(d) **What would the canonical heuristic look like if we omitted those patterns before even constructing the compatibility graph**

As the canonical heuristic obtains the sum of all pattern heuristics in cliques, and this pattern heuristic is always 0, the resultant canonical heuristic won't change.

Exercise 9.2 - Orthogonality and pairwise orthogonality

Prove the following: $\alpha_1, ..., \alpha_n$ are orthogonal if and only if they are pairwise orthogonal.

1) if $\alpha_1, ..., \alpha_n$ are orthogonal then they are pairwise orthogonal

If $\alpha_1, ..., \alpha_n$ are orthogonal, then by definition we know that for all transitions $\langle s, l, t\rangle$ of $\mathcal{T}$, we have $\alpha_i(s) \neq \alpha_i(t)$ for at most one $i \in \{1, ..., n\}$. For the sake of contradiction, let's assume that $\alpha_1, ..., \alpha_n$ are not pairwise orthogonal, this implies that there is at least one abstraction mapping pair $\{\alpha_j \text{ and } \alpha_k \mid \quad j, k \in \{1, ..., n\}, j \neq k\}$ which is not orthogonal.

This means that we have at least one transition $\langle s, l, t\rangle$ of $\mathcal{T}$ where $\alpha_j(s) \neq \alpha_j(t)$ and $\alpha_k(s) \neq \alpha_k(t)$, but this cannot be true because the entire system is orthogonal, which is a contradiction, therefore if the abstraction mappings are orthogonal then they are pairwise orthogonal as well.

2) if $\alpha_1, ..., \alpha_n$ are pairwise orthogonal then they are orthogonal

If $\alpha_1, ..., \alpha_n$ are pairwise orthogonal, then by definition we know that for all $j, k \in \{1, ..., n\}$ with $j \neq k$, mappings α_j and α_k are orthogonal. For the sake of contradiction let's assume that the abstraction mappings are not orthogonal, this implies that there are at least two abstraction mappings $\{\alpha_j \text{ and } \alpha_k \mid \quad j, k \in \{1, ..., n\}, j \neq k\}$ in the system where in one or more transitions $\langle s, l, t\rangle$ of $\mathcal{T}$, we have $\alpha_i(s) \neq \alpha_i(t)$ for $i \in \{1, ..., n\}$.

Then α_j and α_k are not orthogonal, but all the abstraction mappings are pairwise orthogonal, which is a contradiction, therefore the if the abstraction mappings are pairwise orthogonal then they are also orthogonal.