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Principles of AI Planning

Exercise Sheet 10

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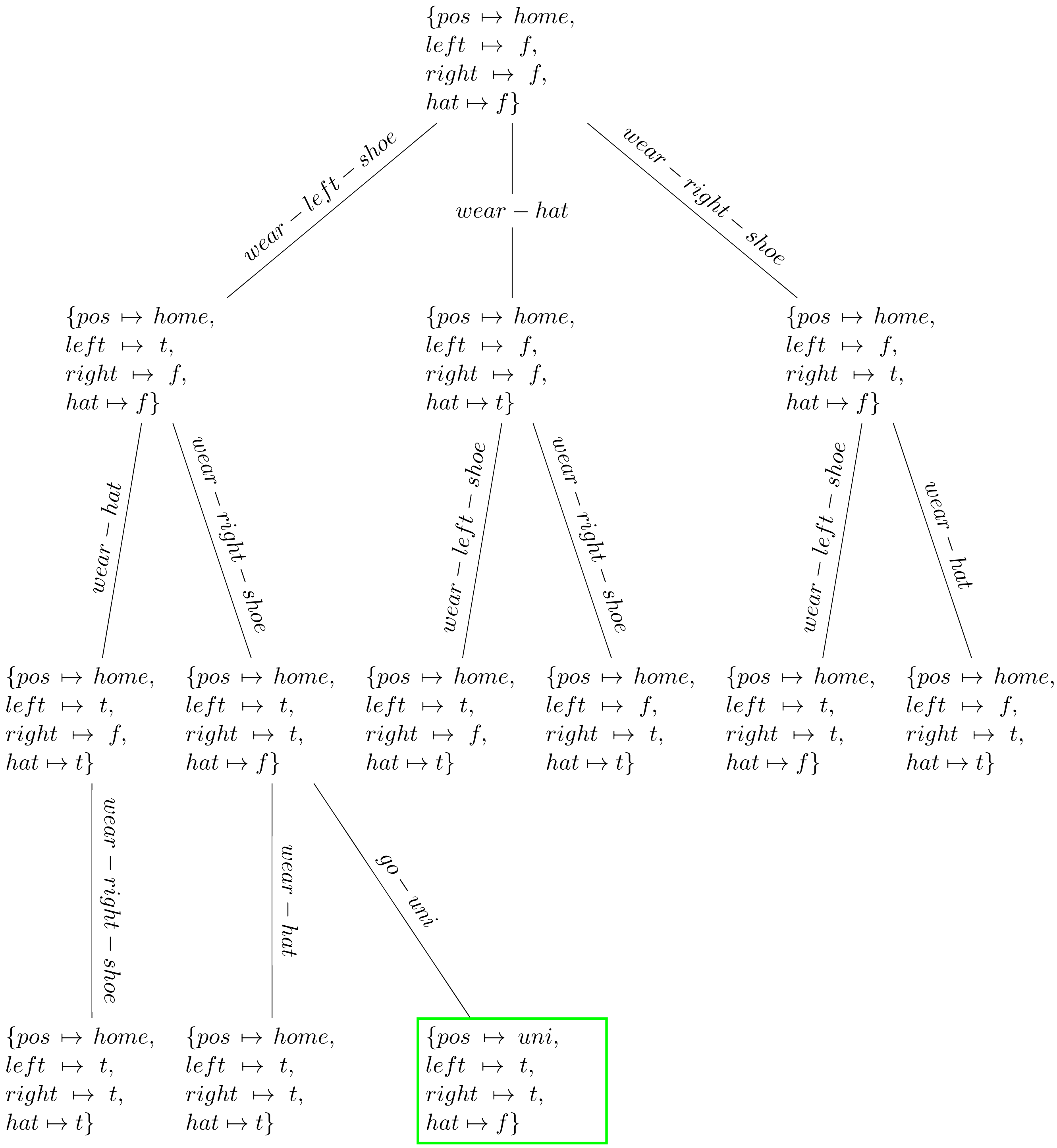
Exercise 11.1 - Strong stubborn sets

Consider the SAS^+ planning task Π with variables $V = \{pos, left, right, hat\}$, $\mathcal{D}_{pos} = \{home, uni\}$ and $\mathcal{D}_{left} = \mathcal{D}_{right} = \mathcal{D}_{hat} = \{t, f\}$. The initial state $I = \{pos \mapsto home, left \mapsto f, right \mapsto f, hat \mapsto f\}$ and the goal specification is $\gamma = \{pos \mapsto uni\}$. There are four operators in O, namely

$$
\begin{aligned}
\text{wear-left-shoe (WLS)} &= \langle pos = home \wedge left = f, left := t\rangle \\
\text{wear-right-shoe (WRS)} &= \langle pos = home \wedge right = f, right := t\rangle \\
\text{wear-hat (WH)} &= \langle pos = home \wedge hat = f, hat := t\rangle \\
\text{go-to-university (GU)} &= \langle pos = home \wedge left = t \wedge right = t, pos := uni\rangle
\end{aligned}
$$

(a) Draw the breadth-first search graph (with duplicate detection) for planning task Π without any form of partial-order reduction.



(b) Draw the breadth-first search graph (with duplicate detection) for planning task Π using strong stubborn set pruning. For each expansion of a node for a state s, specify in detail how T_s (and thus $T_{app(s)}$) are computed, i.e., explain how the initial disjunctive action landmark is chosen and how operators are iteratively added to T_s as part of necessary enabling sets or interfering operators, respectively. Break ties in favor of *wear-left-shoe* over *wear-right-shoe.*

How many node expansion do you save with strong stubborn sets compared to plain breadthfirst search? What about the lengths of the resulting solutions?

$$
\begin{array}{c}
\{pos \mapsto home,\ left \mapsto f,\ right \mapsto f,\ hat \mapsto f\} \\
| \\
WLS \\
| \\
\{pos \mapsto home,\ left \mapsto t,\ right \mapsto f,\ hat \mapsto f\} \\
| \\
WRS \\
| \\
\{pos \mapsto home,\ left \mapsto t,\ right \mapsto t,\ hat \mapsto f\} \\
| \\
GU \\
| \\
\boxed{\{pos \mapsto uni,\ left \mapsto t,\ right \mapsto t,\ hat \mapsto f\}}
\end{array}
$$

Exercise 11.2 - Weak vs. strong stubborn sets

show that *weak* stubborn sets admint exponentially more pruning than *strong* stubborn sets.

Let $\Pi = \langle V, I, O, \gamma \rangle$ be an SAS^+ planning task in transition normal form, and let $\mathcal{F} = \{f_{v=d} \mid v \in V, d \in \mathcal{D}_v\}$ be the set of all atomic features over Π. Finally, let:

$$h(s) = \sum_{fact\ v=d} w_{v=d} \cdot f_{v=d}(s)$$

be the potential heuristic with potentials $w_{v=d}$ for all $v \in V, d \in \mathcal{D}_v$, such that for all $o \in O$, the following constraint is satisfied:

$$\sum_{fact\ v=d \text{ consumed by } o} w_{v=d} - \sum_{fact\ v=d \text{ produced by } o} w_{v=d} \leq cost(o)$$

Prove: Then h is consistent, i. e., $h(s) - h(t) \leq cost(o)$ for all transitions (s, o, t) in $\mathcal{T}(\Pi)$.

Taking an arbitrary transition (s, o, t) in $\mathcal{T}(\Pi)$ as it is consistent

$$h(s) - h(t) \leq cost(o)$$

$$\iff \sum_{fact\ v=d} w_{v=d} \cdot f_{v=d}(s) + \sum_{fact\ v=d} w_{v=d} \cdot f_{v=d}(t) \leq cost(o)$$

$$f_{v=d}(s) = \begin{cases} 0, & \text{if } s \not\models v=d \\ 1, & \text{if } s \models v=d \end{cases} \qquad f_{v=d}(t) = \begin{cases} 0, & \text{if } t \not\models v=d \\ 1, & \text{if } t \models v=d \end{cases}$$

$$\iff \sum_{\substack{fact\ v=d \\ s \models v=d}} w_{v=d} + \sum_{\substack{fact\ v=d \\ t \models v=d}} w_{v=d} \leq cost(o)$$

We know that in the transition (s, o, t) the $vars(t) = vars(s) -$ consumed $vars$ by o + produced

$vars$ by o, therefore:

$$\iff \sum_{\substack{fact\ v=d \\ s \models v=d}} w_{v=d} - \left[\sum_{\substack{fact\ v=d \\ s \models v=d}} w_{v=d} - \sum_{\substack{fact\ v=d \\ \text{consumed by o}}} w_{v=d} + \sum_{\substack{fact\ v=d \\ \text{produced by o}}} w_{v=d} \right] \leq cost(o)$$

$$\iff \sum_{\substack{fact\ v=d \\ \text{consumed by o}}} w_{v=d} - \sum_{\substack{fact\ v=d \\ \text{produced by o}}} w_{v=d} \leq cost(o)$$