Authors:
Erick Rosete Beas — er165@uni-freiburg.de
Jessica Lizeth Borja Diaz — jb986@uni-freiburg.de

Principles of AI Planning
Exercise Sheet 7

13.12.2019

Exercise 7.1 - Innacuracy of h_{max}

Prove that the heuristic h_{max} is arbitrarily innacurate.
We need to prove that $c \cdot h_{max}(I) \leq h^+(I) \quad \forall c \in R^+$
Select an arbitrary c then we will construct a relaxed planning task Π where the previous equation holds.
The planning task is constructed as follows:

Given a constant c, we select an n such that $n \geq c$ where n is a natural number.

$$\Pi = \langle A, I, O^+, \gamma \rangle$$

$$A = \{a_i \mid 1 \leq i \leq n\} \cup \{b_i \mid 1 \leq i \leq n\}$$
$$I = \{a_i \mapsto 1 \mid 1 \leq i \leq n\} \cup \{b_i \mapsto 0 \mid 1 \leq i \leq n\}$$
$$O^+ = \{\langle a_i, b_i \rangle \mid 1 \leq i \leq n\}$$
$$\gamma = \bigwedge_{i=1}^{n} b_i$$

By solving this relaxed planning task we can see that

$$h_{max}(I) = 1$$

Because when we apply the parallel operators we can reach the goal state in one step as all b_i are turned true at the same time.
Whereas the minimal amount of sequential operators to be applied $h^+(I)$ will be equal to the amount of operators:

$$h^+(I) = n$$

Then:

$$c \cdot h_{max}(I) \leq h^+(I)$$
$$\iff c \cdot 1 \leq n$$
$$\iff n \geq c$$

Exercise 7.2 - Stability of h_{add}

Show that it is important to test for stability when computing h_{add} by giving an example where you get an unnecessairly high overestimation when not performing this test.
Consider the following relaxed planning task.

$$\Pi = \langle A, I, O^+, \gamma \rangle$$

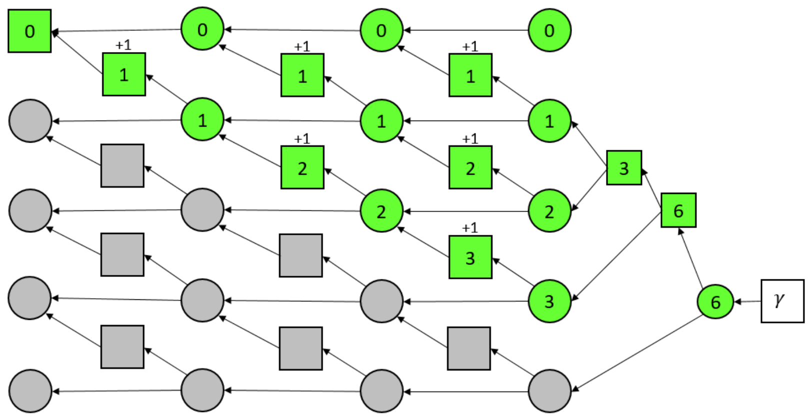
$$A = \{a_0, a_1, a_2, a_3, a_4\}$$

$$I = \{a_0 \mapsto 1, a_1 \mapsto 0, a_2 \mapsto 0, a_3 \mapsto 0, a_4 \mapsto 0\}$$

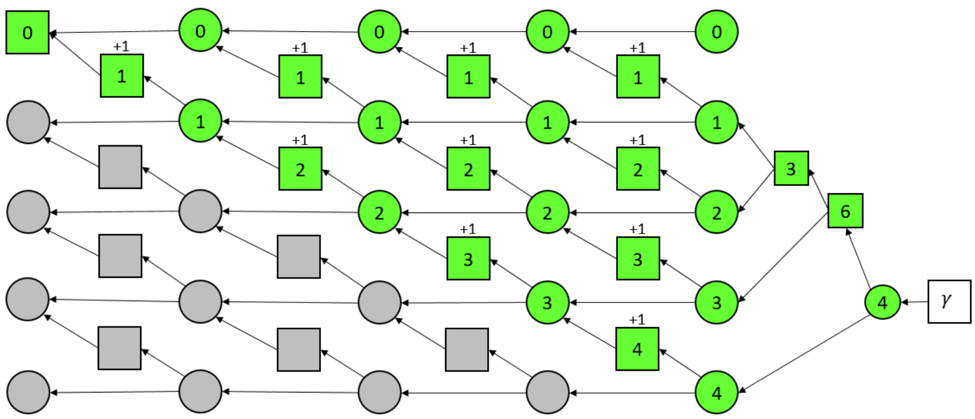
$$O^+ = \{\langle a_0, a_1 \rangle, \langle a_1, a_2 \rangle, \langle a_2, a_3 \rangle, \langle a_3, a_4 \rangle\}$$

$$\gamma = a_1 \wedge a_2 \wedge a_3 \vee a_4$$

Without a stability test we would get a planning graph as:



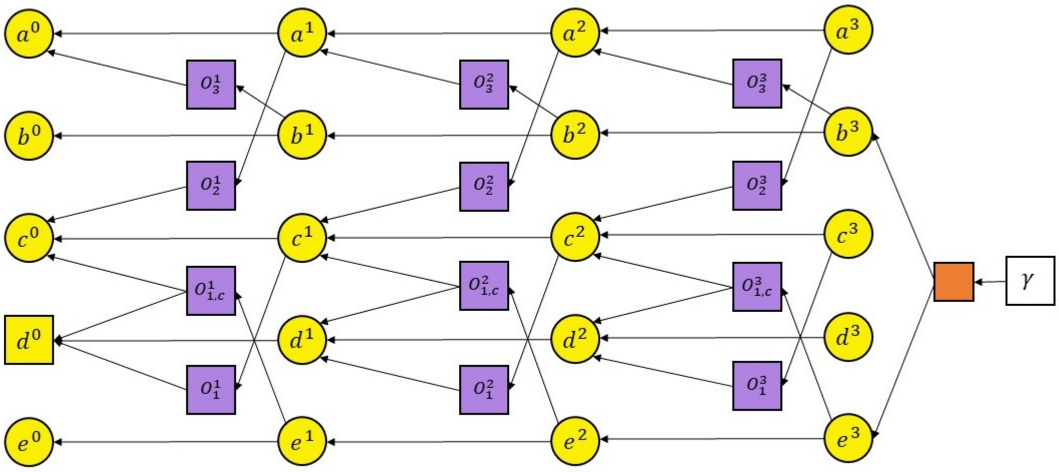
With stability test:



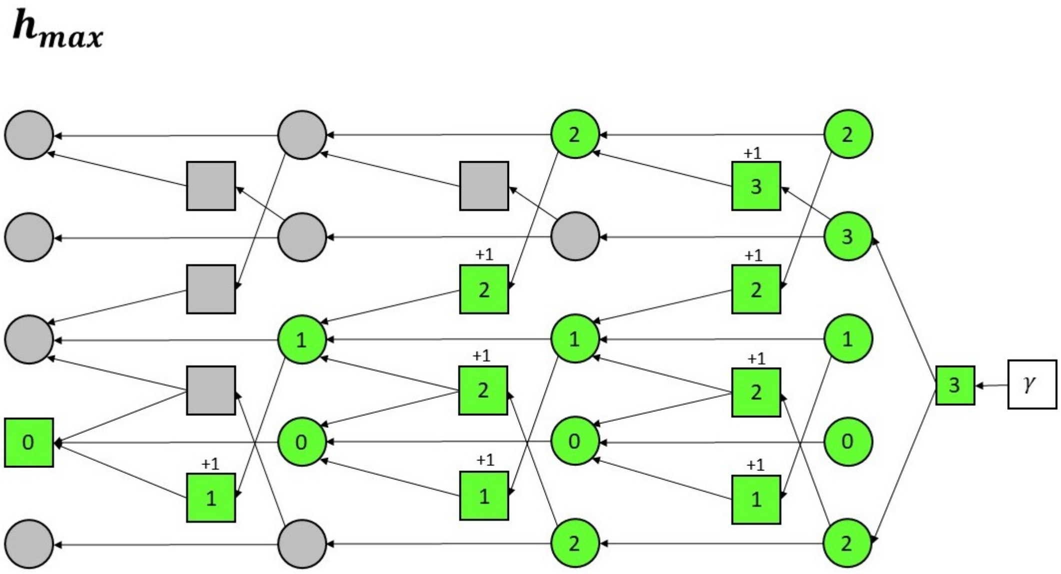
This means that if we do not wait for the planning graph to be stable we can overestimate the heuristic when using h_{add}.

Exercise 7.3 - Relaxed planning graph and heuristics

Consider the relaxed planning task Π^+ with variables $A = \{a, b, c, d, e\}$, operators $O = \{o_1, o_2, o_3\}$, $o_1 = \langle d, c \wedge (c \rhd e)\rangle$, $o_2 = \langle c, a\rangle$, $o_3 = \langle a, b\rangle$, goal $\gamma = b \wedge e$ and initial states $s = \{a \mapsto 0, b \mapsto 0, c \mapsto 0, d \mapsto 1, e \mapsto 0\}$. Solve the following by drawing the relaxed planning graph for the lowest depth k that is necessary to extract a solution



(a) **Calculate $h_{max}(s)$ for Π^+**



(b) **Calculate $h_{add}(s)$ for Π^+**