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Principles of AI Planning

Exercise Sheet 6

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Exercise 6.1 - Delete Relaxation

(a) **Give the relaxation Π^+ of Π**

$$
\begin{aligned}
&A = \{haveCake, eatenCake, haveNoCake\}\\
&I = \{haveCake \to 0, eatenCake \to 0, haveNoCake \to 1\}\\
&O^+ = \{eatCake^+, bakeCake^+\}\\
&eatCake^+ = \langle haveCake, haveNoCake \land eatenCake\rangle\\
&bakeCake^+ = \langle haveNoCake, haveCake\rangle\\
&\gamma = haveCake \land eatenCake
\end{aligned}
$$

The negative effects of the operators were removed.

(b) **Give a sequence of π of operators (as short as possible) from O such that π is not a plan of Π but π^+ is a plan of Π^+.**

$$\pi^+ = \{bakeCake^+, eatCake^+\}$$

Why? We can simulate this plan in both planning tasks to demonstrate it.
States after $bakeCake^+$

$$S_1 = \{haveCake \to 1, eatenCake \to 0, haveNoCake \to 1\}$$

States after $eatCake^+$

$$S_2 = \{haveCake \to 1, eatenCake \to 1, haveNoCake \to 1\}$$

Goal accomplished.

On the other hand, if we run the same plan π in Π
States after $bakeCake$

$$S_1 = \{haveCake \to 1, eatenCake \to 0, haveNoCake \to 0\}$$

States after $eatCake$

$$S_2 = \{haveCake \to 0, eatenCake \to 1, haveNoCake \to 1\}$$

The goal is not accomplished as we don't have cake.

Exercise 6.2 - h^+ heuristic

before all we define the relaxed operator $move^+$ where:

$$move^+(t_m, p_{(i,j)}, p_{(k,l)}) = \langle at(t_m, p_{(i,j)}) \wedge empty(p_{(k,l)}), at(t_m, p_{(k,l)}) \wedge empty(p_{(i,j)}) \rangle$$

(a) **Show that $h^+(s) \geq h^{Mannhattan}(s)$ for each legal state s of a 15-puzzle planning task**
We know that $h^{Mannhattan}(s)$ is the sum of the distances in i and j to reach the goal for each tile. We also know that $h^+(s)$ is the minimum amount of operators to reach the goal state in the relaxed planning task.
We notice that a tile can only move to an adjacent position, and that position must be empty. If a tile where to move to the position it would occupy in the goal state, assumming all the places it would have to move over be empty then the number of operators required for that tile to reach the desired position would be exactly the Mannhattan distance, if its not possible to have a path such that the positions would be empty then we would require more movements or operators to reach the goal state. This means that the total sum of operators required to reach the goal state is greater or equal than the sum of Mannhattan distance for each operator.

(Should we mathematically formalize this type of answers in the exam or an explanation like the one above is sufficient proof?)

We can try to formalize this by defining the operator:

$$manhattanMove(tm, p_{(i,j)}, p_{(k,l)}) = \langle at(t_m, p_{(i,j)}), at(tm, p_{(k,l)}) \rangle$$

$$\text{where } (i = k \text{ and } |j - l| = 1) \text{ or } (j = l \text{ and } |i - k| = 1)$$

If we arrive to the goal by using these operators, then the optimal relaxed plan will be the manhattan distance, furthermore we can see that the *manhattanMove* dominates $move^+$ as every plan step applicable in $move^+$ is also applicable in $manhattanMove$ as it doesn't have the empty precondition, as $manhattanMove$ dominates $move^+$ then $h^+(s) \geq h^{Mannhattan}(s)$

(b) **Show that $h^+(s) > h^{Mannhattan}(s)$ for at least one state s of a 15-puzzle planning task**
Consider the following state s.

i\j	0	1	2	3
0	t_0	t_1	t_2	t_3
1	t_4	t_5	t_6	t_7
2	t_8	t_9	t_{10}	t_{11}
3	t_{12}	t_{14}	t_{13}	*empty*

To calculate the heuristic h^+ given the state s we first consider that the only tiles that can move are t_{11} and t_{13}. t_{11} is already in its desired position but t_{14} should be moved to $p_{(3,2)}$ and the precondition to do so requires that position to be empty first, then t_{13} should move at least once to reach the goal state. We start by applying operator $move(t_{13}, p_{(2,3)}, p_{(3,3)})$ in the relaxed planning task, this yields the following state:

i\j	0	1	2	3
0	t_0	t_1	t_2	t_3
1	t_4	t_5	t_6	t_7
2	t_8	t_9	t_{10}	t_{11}
3	t_{12}	t_{14}	$t_{13} \wedge empty$	$t_{13} \wedge empty$

It is important to recognize that because of the relaxation an state can be in several places at the time and that place can be "empty" as well. Now we can move t_{14} to its goal position by applying operator $move(t_{14}, p_{(3,1)}, p_{(3,2)})$ as the preconditions are now satisfied. This yields the state:

i\j	0	1	2	3
0	t_0	t_1	t_2	t_3
1	t_4	t_5	t_6	t_7
2	t_8	t_9	t_{10}	t_{11}
3	t_{12}	$t_{14} \wedge empty$	$t_{13} \wedge t_{14} \wedge empty$	$t_{13} \wedge empty$

and as t_{13} already *is* in $p_{(3,2)}$ we can simply move it to the desired position from there and we reach the goal state in the relaxed planning task.

i\j	0	1	2	3
0	t_0	t_1	t_2	t_3
1	t_4	t_5	t_6	t_7
2	t_8	t_9	t_{10}	t_{11}
3	t_{12}	$t_{13} \wedge t_{14} \wedge empty$	$t_{13} \wedge t_{14} \wedge empty$	$t_{13} \wedge empty$

Therefore for state s $h^+(s) = 3$ as we applied only 3 operators. However $h^{Mannhattan}$ would have estimated 2, because for t_{14} the Mannhattan distance is: $|3-3| + |2-1| = 1$, and for t_{13}: $|3-3| + |1-2| = 1$ thus $h^{Mannhattan}(s) = 2$. Finally we can conclude that for this s $h^+(s) > h^{Mannhattan}(s)$