Authors:
Erick Rosete Beas — er165@uni-freiburg.de
Jessica Lizeth Borja Diaz — jb986@uni-freiburg.de

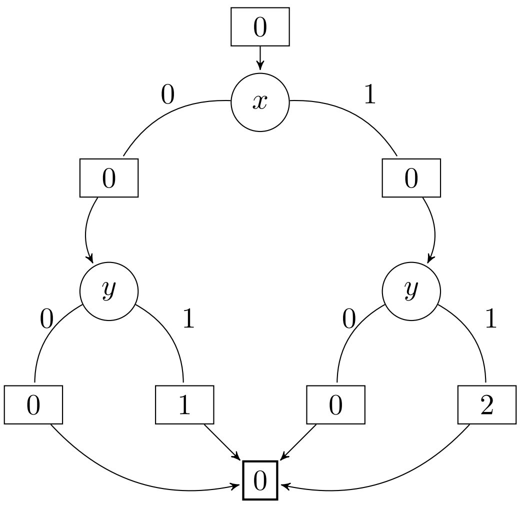
Principles of AI Planning

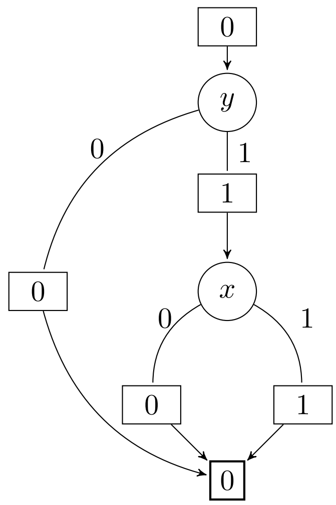
Exercise Sheet 13

07.02.2020

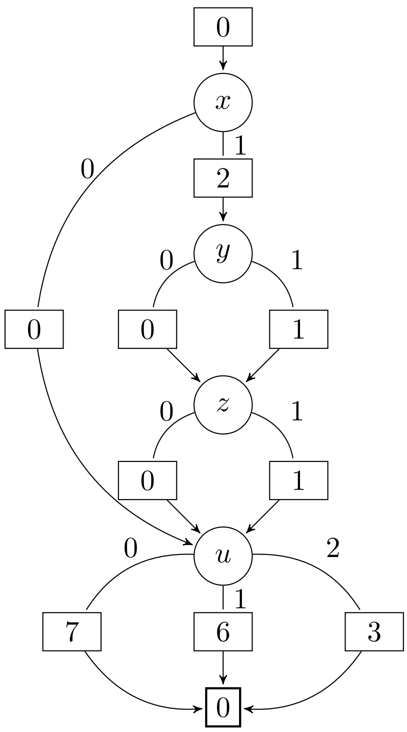
Exercise 13.1 - EVMDDs

(a)





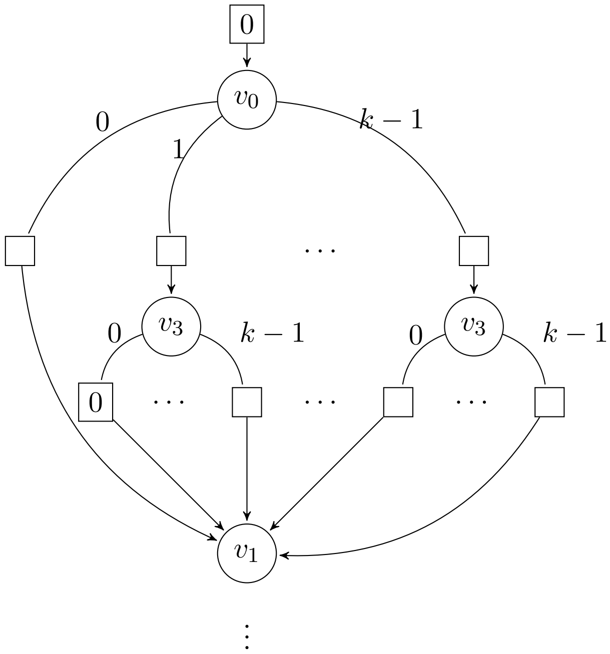
(b)



Exercise 13.2 - EVMDD sizes and variable orders

Let $v_0, ..., v_{2n-1}$ be variables with domains $\mathcal{D}_{v_i} = \{0, ..., k-1\} \forall i = 0, ..., 2n-1$, let $\pi := \{0, ..., 2n-1\} \rightarrow \{0, ..., 2n-1\}$ be a permutation of the variables, let $k_j \in \mathbb{N}, j = 0, ..., n-1$, be natural numbers and let $c = \Sigma_{j=0}^{n-1} k_j v_{\pi(2j)} v_{\pi(2j+1)}$ be an arithmetic function over $v_0, ..., v_{2n-1}$.Intuitively, c is a weighted sum of products of two variables each, such that no variable occurs in more than one product subterm. Show that there exists a variable order for $v_0, ..., v_{2n-1}$ such that there exists an EVMDD with that order that represents the function c and that has a size (number of edges) in the order of nk^2.

Hint: Consider the example $c = 2v_0v_3 + 6v_1v_5 + 4v_2v_4$. How should the variables be ordered to minimize the size of the EVMDD.

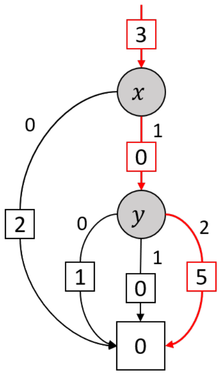


for $c = 2v_0v_3 + 6v_1v_5 + 4v_2v_4$ we consider the diagram above which is a EVMDD for c. v_0 can take values from 0 to $k-1$ hence that node has k outcoming edges. And for $k-1$ edges that branch out from v_0 there is a node v_3 with k more edges. Then for each pair $v_{\pi(2j)}v_{\pi(2j+1)}$ there exists $(k-1)k+1$ edges which is in order $O(k^2)$, and we have n variable pairs which means the EVMDD has in total $O(n * k^2)$ edges.

Given arbitrary n and k, using as variable order the order on which the variables show in the cost function there exists an EVMDD with that order and represents the cost function with $O(n * k^2)$ edges using the same construction as in the previous example.

Exercise 13.3 - Evaluating states with EVMDDs

Consider a cost function represented by the EVMDD on the right.
Let s be a state with $s(x) = 1$ and $s(y) = 2$. To which value does the EVMDD evaluate for state s?
$cost(s) = 3 + 0 + 5 = 8$



Exercise 13.4 - EVMDD-based action compilation

Consider again the EVMDD from Exercise 13.3. Assume it encodes the cost c_{o_1} of operator $o_1 = \langle z = 1 \wedge u = 1, x := 0\rangle$.

a) Give the EVMDD-based action compilation of o_1 using this EVMDD.

$$
\begin{array}{ll}
O_1^{z=1\wedge u=1} = \langle z = 1 \wedge u = 1 \wedge \sigma = 0 \wedge \alpha_{o_1} = 0, \sigma := 1 \wedge \alpha_{o_1} := 1\rangle & cost = 3 \\
O_1^{1,x=0} = \langle \alpha_{o_1} = 1 \wedge x = 0, \alpha_{o_1} := 3\rangle & cost = 2 \\
O_1^{1,x=1} = \langle \alpha_{o_1} = 1 \wedge x = 1, \alpha_{o_1} := 2\rangle & cost = 0 \\
O_1^{2,y=0} = \langle \alpha_{o_1} = 2 \wedge y = 0, \alpha_{o_1} := 3\rangle & cost = 1 \\
O_1^{2,y=1} = \langle \alpha_{o_1} = 2 \wedge y = 1, \alpha_{o_1} := 3\rangle & cost = 0 \\
O_1^{2,y=2} = \langle \alpha_{o_1} = 2 \wedge y = 2, \alpha_{o_1} := 3\rangle & cost = 5 \\
O_1^{x:=0} = \langle \alpha_{o_1} = 3, x := 0 \wedge \sigma := 0 \wedge \alpha_{o_1} := 0\rangle & cost = 0
\end{array}
$$

b) Let $\Pi = \langle V, I, O, \gamma, (c_o)_{o \in O}\rangle$ with $V = \{x, y, z, u\}$, $\mathcal{D}_x = \mathcal{D}_z = \mathcal{D}_u = \{0, 1\}$ and $\mathcal{D}_y = \{0, 1, 2\}$, initial state I with $I(x) = I(y) = I(z) = I(u) = 1$, operators $O = \{o_1, o_2\}$ with o_1 as above and $o_2 = \langle x = 0, z := 0\rangle$ with cost function $c_{o_2} = 1$ and goal formula $\gamma = (z = 0)$. Give an optimal plan π for Π and an optimal plan π for the EVMDD-based action compilation of Π and their respective costs.
Optimal plan π for Π

$$
\begin{array}{l}\{x \mapsto 1,\\ y \mapsto 1,\\ z \mapsto 1,\\ u \mapsto 1\}\end{array}
\xrightarrow[cost = 3]{o_1}
\begin{array}{l}\{x \mapsto 0,\\ y \mapsto 1,\\ z \mapsto 1,\\ u \mapsto 1\}\end{array}
\xrightarrow[cost = 1]{o_2}
\begin{array}{l}\{x \mapsto 0,\\ y \mapsto 1,\\ z \mapsto 0,\\ u \mapsto 1\}\end{array}
$$

Optimal plan π for $\Pi' = EAC(\Pi)$

$$\begin{tikzcd}
\begin{array}{l}\{x \mapsto 1,\\ y \mapsto 1,\\ z \mapsto 1,\\ u \mapsto 1,\\ \sigma \mapsto 0,\\ \alpha_{o_1} \mapsto 0,\\ \alpha_{o_2} \mapsto 0\}\end{array} \arrow[r, "o_1^{z=1\wedge u=1}", "cost = 3"'] &
\begin{array}{l}\{x \mapsto 1,\\ y \mapsto 1,\\ z \mapsto 1,\\ u \mapsto 1,\\ \sigma \mapsto 1,\\ \alpha_{o_1} \mapsto 1,\\ \alpha_{o_2} \mapsto 0\}\end{array} \arrow[r, "o_1^{1,x=1}", "cost = 0"'] &
\begin{array}{l}\{x \mapsto 1,\\ y \mapsto 1,\\ z \mapsto 1,\\ u \mapsto 1,\\ \sigma \mapsto 1,\\ \alpha_{o_1} \mapsto 2,\\ \alpha_{o_2} \mapsto 0\}\end{array} \arrow[r, "o_1^{2,y=1}", "cost = 0"'] &
\begin{array}{l}\{x \mapsto 1,\\ y \mapsto 1,\\ z \mapsto 1,\\ u \mapsto 1,\\ \sigma \mapsto 1,\\ \alpha_{o_1} \mapsto 3,\\ \alpha_{o_2} \mapsto 0\}\end{array} \arrow[d, "cost = 0"', "o_1^{x:=0}"] \\
&
\begin{array}{l}\{x \mapsto 0,\\ y \mapsto 1,\\ z \mapsto 0,\\ u \mapsto 1,\\ \sigma \mapsto 0,\\ \alpha_{o_1} \mapsto 0,\\ \alpha_{o_2} \mapsto 0\}\end{array} &
\begin{array}{l}\{x \mapsto 0,\\ y \mapsto 1,\\ z \mapsto 1,\\ u \mapsto 1,\\ \sigma \mapsto 1,\\ \alpha_{o_1} \mapsto 0,\\ \alpha_{o_2} \mapsto 1\}\end{array} \arrow[l, "o_2^{z:=0}"', "cost = 0"] &
\begin{array}{l}\{x \mapsto 0,\\ y \mapsto 1,\\ z \mapsto 1,\\ u \mapsto 1,\\ \sigma \mapsto 0,\\ \alpha_{o_1} \mapsto 0,\\ \alpha_{o_2} \mapsto 0\}\end{array} \arrow[l, "o_2^{x=0}"', "cost = 1"]
\end{tikzcd}$$

We can notice the cost of both plans is the same, 3.