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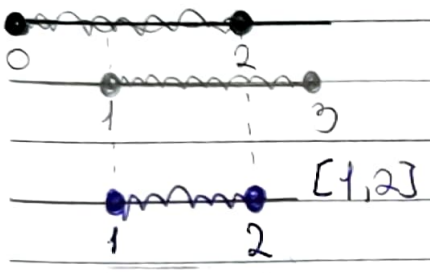
Calculo Diferencial e Integral I

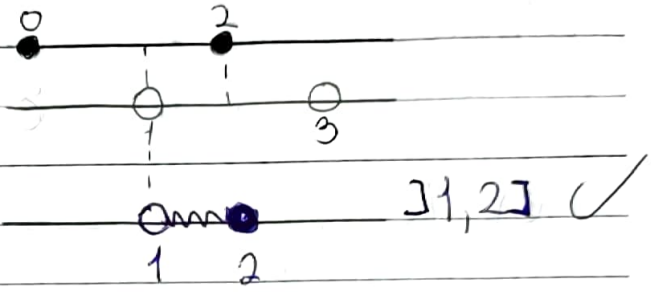
Lista 2 - Conceitos de Função e intervalos

1) • Descreva os seguintes conjuntos:

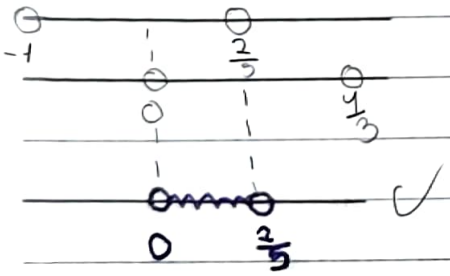
a) $[0,2] \cap [1,3]$



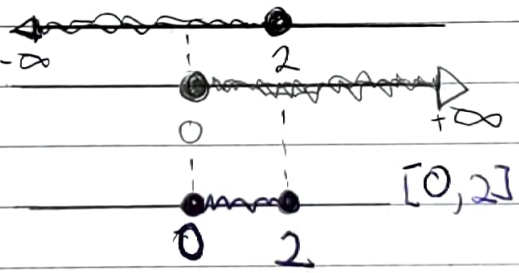
b) $[0,2] \cap]1,3[$



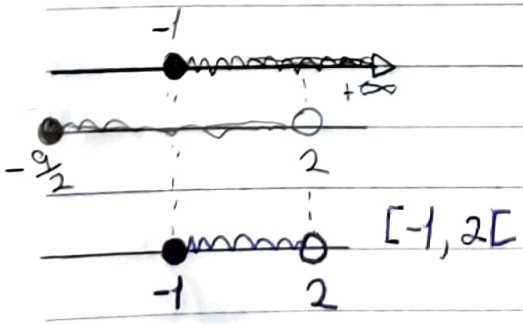
c) $]-1, \frac{2}{5}[\cap]0, \frac{4}{3}[$



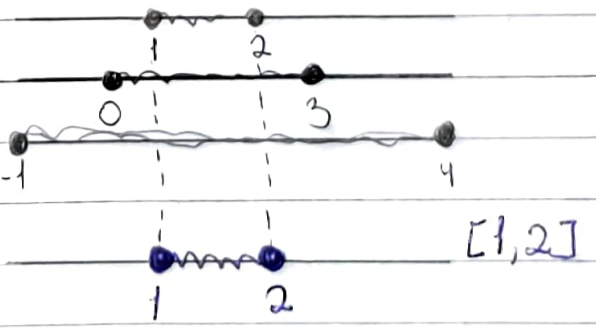
d) $]-\infty, 2] \cap [0, +\infty[$



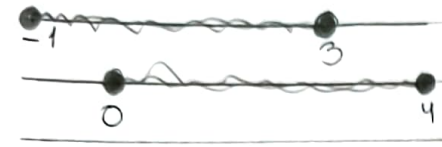
e) $[-1, +\infty[\cap [-\frac{9}{2}, 2[$



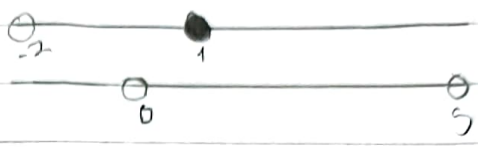
f) $[1,2] \cap [0,3] \cap [-1,4]$



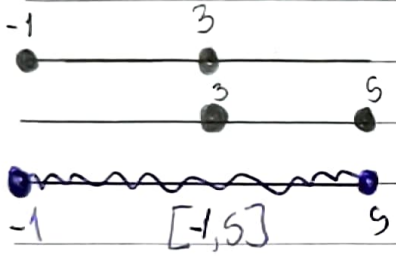
a) $[-1,3] \cup [0,4]$



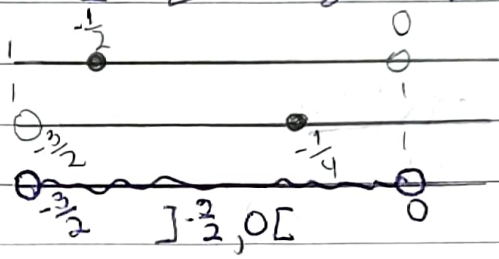
B) $]-2,1] \cup]0,5[$



C) $[-1,3] \cup [3,5)$



D) $\left[-\frac{1}{2}, 0\right[\cup \left]-\frac{3}{2}, -\frac{1}{4}\right]$



2)

a) $\{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$

B) $\{x \in \mathbb{R} \mid 0 < x < 3\}$

C) $\{x \in \mathbb{R} \mid x \leq 0 \text{ ou } x > 2\}$

$]-\infty, 0] \cup]2, \infty[$

D) $\{x \in \mathbb{R} \mid -1 < x < 0 \text{ ou } x \geq 3\}$

$]-1,0[\cup [3,+\infty[$

3)

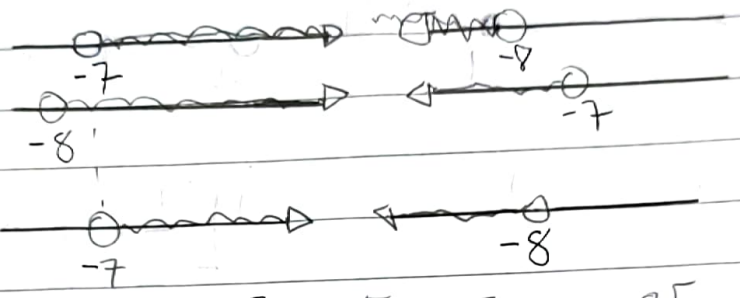
a) $\frac{1}{x+7} > -1 \rightarrow$ $1 > -1(x+7)$ $\quad 1 < -1(x+7)$

$x+7 \neq 0$ $\quad 1 > -x-7$ $\quad 1 < -x-7$

$x \neq -7$ $\quad 8 > -x$ $\quad 8 < -x$

$x > -7$ $\quad x > -8$ $\quad x < -8$

$x < -7$

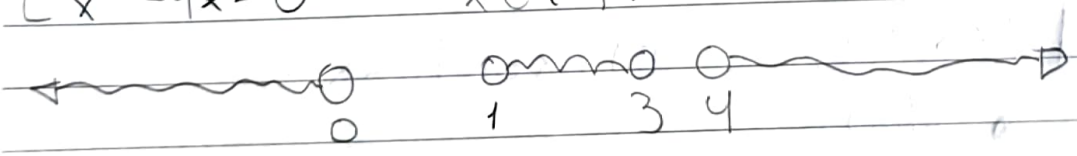


$]-7, +\infty[\cup]-\infty, -8[$

B) $\frac{x^2-4x+3}{x^2-4x} \geq 0$

$\begin{cases} x^2-4x+3>0 \\ x^2-4x>0 \end{cases}$ $x \in (-\infty,1) \cup (3,+\infty)$ $\quad x \in (-\infty,0) \cup (4,+\infty)$

$\begin{cases} x^2-4x+3<0 \\ x^2-4x<0 \end{cases}$ $x \in (1,3)$ $\quad x \in (0,4)$



C) $|x+4| \leq |2x-6|$

$(x+4)^2 \leq (2x-6)^2$

$x^2+8x+16 \leq 4x^2-24x+36$

$x^2-4x^2+8x+24x+16-36 \leq 0$

$-3x^2+32x-20 \leq 0$

$\Delta = b^2-4.a.c$

$\Delta = 1024-4.-3.-20$

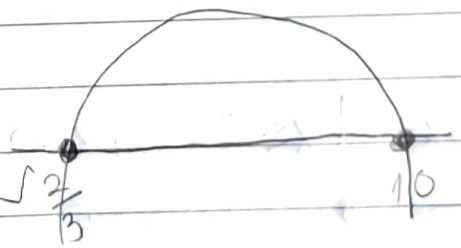
$\Delta = 1024-240$

$\Delta = 784$

$x'' = \frac{-b \pm \sqrt{\Delta}}{2a}$

$x' = \frac{-32+28}{-6} = \frac{-4}{-6} = +\frac{2}{3}$

$x'' = \frac{-32-28}{-6} = \frac{-60}{-6} = 10$



$S = (-\infty, \frac{2}{3}] \cup [10, +\infty)$

4)

a) $|5x-3|=12$

$5x-3=12$ $\quad$ $5x-3=-12$

$x=3$ $\quad$ $x=-\frac{9}{5}$ $\quad$ $S=\{-\frac{9}{5}, 3\}$

b) $|-4+12x|=7$

$-4+12x=7$ $\quad$ $-4+12x=-7$ $\quad$ $S=\{-\frac{1}{4}, \frac{11}{12}\}$

$x=\frac{11}{12}$ $\quad$ $x=-\frac{1}{4}$

c) $|2x-3|=|7x-5|$

$2x-3=7x-5$ $\quad$ $2x-3=-(7x-5)$

$-5x=-2$ $\quad$ $9x=8$

$x=\frac{2}{5}$ $\quad$ $x=\frac{8}{9}$ $\quad$ $S=\{\frac{2}{5}, \frac{8}{9}\}$

d) $\left|\frac{x+2}{x-2}\right|=5$

$\frac{x+2}{x-2}=5$ $\quad$ $\frac{x+2}{x-2}=-5$ $\quad$ $S=\{\frac{4}{3}, 3\}$

$x-2$ $\quad$ $x-2$

$x=3$ $\quad$ $x=\frac{4}{3}$

$x\neq 2$

5) a) $|x+12|<7$

$\begin{cases} x+12<7 \\ -x+12\geq 0 \end{cases}$ $\quad$ $x<-5$ $\quad$ $x\geq -12$ $\quad$ $x\in(-19,-5)$

$\begin{cases} -(x+12)<7 \\ x+12<0 \end{cases}$ $\quad$ $x>-19$ $\quad$ $x<-12$ $\quad$ $]-19,-5[$

b) $|3x-4| \leq 12$

$\begin{cases} 3x-4 \leq 2 \\ 3x-4 \geq 0 \end{cases}$ $\quad x \leq 2$, $x \geq 4/3$ $\quad x \in (\frac{3}{3}, 2)$ $[\frac{3}{3}, 2]$

$\begin{cases} -(3x-4) \leq 2 \\ 3x-4 < 0 \end{cases}$ $\quad x \geq 3/3$, $x < 4/3$

c) $|5-6x| \geq 9$

$\begin{cases} 5-6x \geq 9 \\ 5-6x \geq 0 \end{cases}$ $\quad x \leq -\frac{2}{3}$, $x \leq 5/6$ $\Rightarrow x \in (-\infty, -\frac{2}{3})$

$\begin{cases} -(5-6x) \geq 9 \\ 5-6x > 0 \end{cases}$ $\quad x \geq 7/3$, $x > 5/6$ $\Rightarrow x \in (\frac{7}{3}, +\infty)$

$]-\infty, -\frac{2}{3}] \cup [\frac{7}{3}, +\infty[$

d) $|2x-5| > 3$

$\begin{cases} 2x-5 > 3 \\ 2x-5 \geq 0 \end{cases}$ $\quad x > 4$, $x \geq 5/2$ $\Rightarrow x \in (4, +\infty)$

$\begin{cases} -(2x-5) > 3 \\ 2x-5 < 0 \end{cases}$ $\quad x < 1$, $x < 5/2$ $\Rightarrow x \in (-\infty, 1)$

$]-\infty, 1[\cup]4, +\infty[$

e) $|6+2x| < |4-x|$

$6+2x < 4-x$ $\quad 6+2x > -4+x$

$3x < -2$ $\quad x > -10$

$x < -\frac{2}{3}$

$S =]-10, -\frac{2}{3}[$

f) $|x+4| \leq |2x-6|$

$x+4 \leq 2x-6$ $\quad x+4 \geq -2x+6$

$-x \leq -10$ $\quad 3x \geq 2$

$x \geq 10$ $\quad x \geq \frac{2}{3}$

6)

a) $f(x) = |x| + |2x-1| + |x-1|$

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$|2x-1| = \begin{cases} 2x-1, & x \geq 0, \ 2x-1 \geq 0, \ 2x \geq 1, \ x \geq \frac{1}{2} \\ -(2x-1), & x < 0, \ 2x-1 < 0, \ 2x < 1, \ x < \frac{1}{2} \end{cases}$

$S_1 = -x - (2x-1) - (x-1)$

$S_1 = -x - 2x + 1 - x + 1$

$S_1 = -4x + 2 \rightarrow x < 0$

$S_2 = x - 2x + 1 - x + 1$

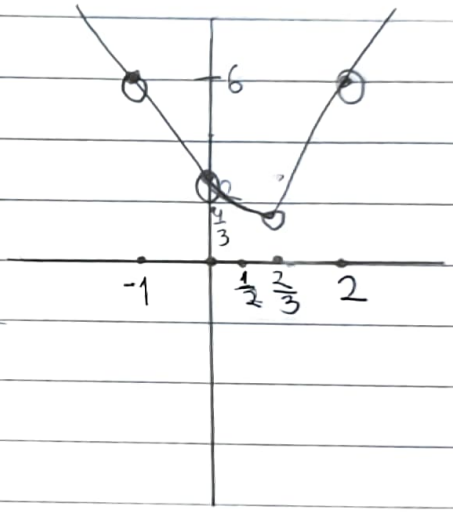
$S_2 = -3x + 2 \rightarrow 0 \leq x < \frac{1}{2}$

$S_3 = x + 2x - 1 - x + 1$

$S_3 = 2x \rightarrow \frac{1}{2} < x < 1$

$S_4 = x + 2x - 1 + x - 1$

$S_4 = 4x - 2 \rightarrow x \geq 1$



$\frac{2}{3} \cdot 2 = \frac{4}{3}$

b) $f(x) = |9 - x^2|$

$|9-x^2| = \begin{cases} 9-x^2 \geq 0, & -x^2 \geq 9, \ -x \geq 3, \ x \leq -3 \\ -(9-x^2) < 0, & -x^2 < 9, \ -x < 3, \ x > 3 \end{cases}$

$S_1 = -9 + x^2 \rightarrow x \leq -3$

$S_2 = +9 - x^2 \rightarrow x > 3$

$-9+(3)^2 = 0$

$-9+(-2)^2 =$