Cálculo Diferencial e Integral I
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Atividade 05

1) $\lim_{x \to 2} f(x) = 4$, $\lim_{x \to 2} g(x) = -2$, $\lim_{x \to 2} H(x) = 0$

a) $\lim_{x \to 2} [f(x) + 5g(x)] = \lim_{x \to 2} f(x) + 5 \cdot \lim_{x \to 2} g(x)$

$= 4 + 5 \cdot -2 = 4 - 10 = -6$

b) $\lim_{x \to 2} [g(x)]^3 = [\lim_{x \to 2} g(x)]^3 = (-2)^3 = -8$

c) $\lim_{x \to 2} \sqrt{f(x)} = \sqrt{\lim_{x \to 2} f(x)} = \sqrt{4} = 2$

d) $\lim_{x \to 2} \frac{3f(x)}{g(x)} = \frac{3 \cdot \lim_{x \to 2} f(x)}{\lim_{x \to 2} g(x)} = \frac{3 \cdot 4}{-2} = \frac{12}{-2} = -6$

e) $\lim_{x \to 2} \frac{g(x)}{H(x)} = \frac{\lim_{x \to 2} g(x)}{\lim_{x \to 2} H(x)} = \frac{-2}{0}$ não existe, pois é uma indeterminação matemática

f) $\lim_{x \to 2} \frac{g(x) H(x)}{f(x)} = \frac{\lim_{x \to 2} g(x) \cdot \lim_{x \to 2} H(x)}{\lim_{x \to 2} f(x)} = \frac{-2 \cdot 0}{4} = \frac{0}{4} = 0$

2)

a) $\lim_{x \to 2} x = 2$

b) $\lim_{x \to 2} x^3 = 2^3 = 8$

c) $\lim_{x \to 1} 2x = 2 \cdot 1 = 2$

d) $\lim_{x \to 1} x^6 = 1^6 = 1$

e) $\lim_{x \to 3} (x^2 - x) = (3^2 - 3) = 9 - 3 = 6$

f) $\lim_{x \to 3} \frac{x+1}{x^2} = \frac{4}{9}$

g) $\lim_{x \to 2} (3x + x^2) = (3.2 + 2^2) = 6 + 4 = \underline{10}$

h) $\lim_{x \to 1} \frac{x^2+2x}{3x-1} = \frac{1^2+2.1}{3.1-1} = \frac{3}{2}$

i) $\lim_{x \to 3} (4x^2 - 2x + 1) = (3^2 - 2.3 + 1) = (9 - 6 + 1) = 4$

j) $\lim_{x \to 1} (2x^3 + 3x^2 - x + 3) = (2.1^3 + 3.1^2 - 1 + 3) = 2 + 3 - 1 + 3 = 7$

l) $\lim_{x \to 1} (3x^2 - 2x - 1) = (3.1^2 - 2.1 - 1) = (3 - 2 - 1) = 0$

m) $\lim_{x \to 2} \frac{x+1}{x^2} = \frac{2+1}{2^2} = \frac{3}{4}$

n) $\lim_{x \to 0} (4x^3 + 2x^2 + x + 2) = (4.0^3 + 2.0^2 + 0 + 2) = 2$

o) $\lim_{x \to 1} (x^4 - x^3 + x^2 + x + 1) = (1^4 - 1^3 + 1^2 + 1 + 1) = 3$

p) $\lim_{x \to -1} (-2x^2 - x + 2) = (-2.(-1)^2 - (-1) + 2) = (-2.1 + 1 + 2)$
$= -2 + 3 = 1$

q) $\lim_{x \to 2} (x^3 - x^2 - 3)^{10} = (2^3 - (2)^2 - 3)^{10} = (8 - 4 - 3)^{10} = 1^{10} = 1$

r) $\lim_{x \to 1} (x+2)^5 = (3)^5 = 243$

s) $\lim_{x \to 3} \frac{x^3 + x^2 + x + 1}{x^2 - 1} = \frac{3^3 + 3^2 + 3 + 1}{3^2 - 1} = \frac{27 + 9 + 3 + 1}{9 - 1} = \frac{40}{8} = 5$

t) $\lim_{x \to 0} \frac{x^2 + x}{x^3 + x^2 + x + 1} = \frac{0^2 + 0}{0^3 + 0^2 + 0 + 1} = \frac{0}{1} = 0$

u) $\lim_{x \to 0} \frac{3}{2x+1} = \frac{3}{2.1+1} = \frac{3}{3} = 1$

v) $\lim_{x \to -1} \frac{3x^2 + x + 1}{2} = \frac{3.(-1)^2 + (-1) + 1}{2} = \frac{3 - 1 + 1}{2} = \frac{3}{2}$

x) $\lim_{x \to 0} \frac{x^3+2x^2+3x+2}{2x^3+x^2+2x+4} = \frac{0^3+2\cdot 0^2+3\cdot 0+2}{2\cdot 0^3+0^2+2\cdot 0+4} = \frac{2}{4} = \frac{1}{2}$

3)

a) $\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = x+3 = 3+3 = 6$

b) $\lim_{x \to -7} \frac{49-x^2}{x+7} = \lim_{x \to -7} \frac{(-x+7)\cancel{(x+7)}}{\cancel{x+7}} = -x+7 = -(-7)+7 = 14$

c) $\lim_{x \to 5} \frac{5-x}{25-x^2} = \lim_{x \to 5} \frac{\cancel{5-x}}{\cancel{(5-x)}(5+x)} = \frac{1}{5+x} = \frac{1}{10}$

d) $\lim_{x \to 0} \frac{x^2+x}{x^2-3x} = \lim_{x \to 0} \frac{\cancel{x}(x+1)}{\cancel{x}(x-3)} = \frac{0+1}{0-3} = \frac{1}{-3}$

e) $\lim_{x \to 0} \frac{x^3}{2x^2-x} = \lim_{x \to 0} \frac{\cancel{(x)}(x)(x)}{\cancel{x}(2x-1)} = \frac{x^2}{2x-1} = \frac{0}{-1} = 0$

f) $\lim_{x \to -7} \frac{49+14x+x^2}{7+x} = \lim_{x \to -7} \frac{(7+x)^2}{7+x} = 7+x = 7-7 = 0$

g) $\lim_{x \to 3} \frac{x^2-6x+9}{x-3} = \lim_{x \to 3} \frac{(x-3)^2}{(x-3)} = (x-3) = 3-3 = 0$

H) $\lim_{x \to 1} \frac{x^2-4x+3}{x-1} = \lim_{x \to 1} \frac{1(x-3)\cancel{(x-1)}}{(x-1)} = x-3 = 1-3 = -2$

$a(x-x')(x-x'')$ $\quad 1(x-3)(x-1)$

$\Delta = -4^2 - 4 \cdot 1 \cdot 3$ $\quad x = \frac{4 \pm 2}{2}$

$\Delta = 16 - 12$

$\Delta = 4$ $\quad x' = \frac{6}{2} = 3$

$x'' = \frac{2}{2} = 1$

i) $\lim_{x \to 4} \frac{x^2-7x+12}{x-4} = \lim_{x \to 4} \frac{1(x-4)(x-3)}{x-4} = x-3 = 4-3 = 1$

$a(x-x')(x-x'')$

$\Delta = 49-4.1.12 \quad x = \frac{7 \pm 1}{2}$

$\Delta = 49-48$

$\Delta = 1 \quad x' = \frac{8}{2} = 4 \rightarrow 1(x-4)(x-3)$

$x'' = \frac{6}{2} = 3$

d) $\lim_{x \to 1} \frac{x-1}{x^2-3x+2} = \lim_{x \to 1} \frac{x-1}{1(x-2)(x-1)} = \frac{1}{x-2} = \frac{1}{1-2} = \frac{1}{-1} = -1$

$a(x-x')(x-x'') = 1(x-2)(x-1)$

$\Delta = 9-4.1.2 \quad x = \frac{3 \pm 1}{2}$

$\Delta = 9-8$

$\Delta = 1 \quad x' = \frac{4}{2} = 2$

$x'' = \frac{2}{2} = 1$

l) $\lim_{x \to 1} \frac{x^2-2x+1}{x-1} = \lim_{x \to 1} \frac{(x-1)^2}{x-1} = \lim_{x \to 1} x-1 = 1-1 = 0$

m) $\lim_{x \to 2} \frac{x-2}{x^2-4} = \lim_{x \to 2} \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2} = \frac{1}{2+2} = \frac{1}{4}$

4) a) $\lim_{x \to 6^+} \frac{4}{x-6} = +\infty$

b) $\lim_{x \to 6^-} \frac{4}{x-6} = -\infty$

c) $\lim_{x \to 1^+} \frac{3}{1-x} = -\infty$

d) $\lim_{x \to 1^-} \frac{3}{1-x} = +\infty$

e) $\lim_{x \to 0^+} \frac{x+5}{x} = +\infty$

f) $\lim_{x \to 0^+} \frac{x+5}{x} = -\infty$

g) $\lim_{x \to 1^+} \frac{x^2}{x-1} = +\infty$

H) $\lim_{x \to 1^-} \frac{x^2}{x-1} = -\infty$

i) $\lim_{x \to 0^+} \frac{-1}{x^2} = -\infty$

j) $\lim_{x \to 0^-} \frac{-1}{x^2} = +\infty$

5)

a) $\lim_{x \to +\infty} (2x+7) = \lim_{x \to +\infty} 2x + \lim_{x \to +\infty} 7 = +\infty + 7 = \underline{+\infty}$

b) $\lim_{x \to -\infty} (-4x+1) = \lim_{x \to -\infty} -4x + \lim_{x \to -\infty} 1 = +\infty + 1 = \underline{+\infty}$

c) $\lim_{x \to +\infty} (3x^6 + 2x^3 - x + 4) = \lim_{x \to +\infty} x^6 \left(3 + \frac{2}{x^3} - \frac{1}{x^5} + \frac{4}{x^6}\right) =$

$\lim_{x \to +\infty} x^6 \cdot \lim_{x \to +\infty} \left(3 + \frac{2}{x^3} - \frac{1}{x^5} + \frac{4}{x^6}\right) = +\infty (2 + 0 - 0 + 0) =$

$\underline{+\infty}$

D) $\lim_{x \to -\infty} (4x^7 + 2x^2 + \sqrt{3x}) = \lim_{x \to -\infty} x^7 \left(4 + \frac{2}{x^5} + \left(\frac{3}{x^6}\right)^{\frac{1}{2}}\right) =$

$\lim_{x \to -\infty} x^7 \cdot \lim_{x \to -\infty} \left(4 + \frac{2}{x^5} + \left(\frac{3}{x^6}\right)^{\frac{1}{2}}\right) = -\infty (4 + 0 + 0) =$

$\underline{-\infty}$

e) $\lim_{x \to +\infty} \frac{8x+1}{4x-5} = \lim_{x \to +\infty} \left(\frac{x \cdot \left(8+\frac{1}{x}\right)}{x \cdot \left(4-\frac{5}{x}\right)} \right) = \frac{8+0}{4-5 \cdot 0} = \frac{8}{4} = 2$

f) $\lim_{x \to -\infty} \frac{3x+2}{x^2-5x+6} = \lim_{x \to -\infty} \left(\frac{x \left(\frac{3}{x} + \frac{2}{x^2} \right)}{x^2 \left(1 - \frac{5}{x} + \frac{6}{x^2} \right)} \right) = \frac{0+0}{1-0+0} = 0$

g) $\lim_{x \to +\infty} \sqrt{2x^3 - x^2 + x + 1} = \lim_{x \to +\infty} x^3 \left(2 - \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} \right)^{\frac{1}{2}} =$

$\lim_{x \to +\infty} x^3 \cdot \lim_{x \to +\infty} \left(2 - \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} \right)^{\frac{1}{2}} = +\infty \cdot (2-0+0+0)^{\frac{1}{2}} = +\infty$

H) $\lim_{x \to +\infty} \sqrt{\frac{x^2-3x}{x^2-1}} = \lim_{x \to +\infty} = \frac{\frac{\sqrt{x^2-3x}}{\sqrt{x^2}}}{\frac{\sqrt{x^2-1}}{\sqrt{x^2}}} = \lim_{x \to +\infty} \frac{\sqrt{1-3x}}{\sqrt{1-1}} =$

$\frac{1-\infty}{0} = -\infty$

i) $\lim_{x \to +\infty} \left(\frac{6x-1}{2x+3} \right)^2 = \lim_{x \to +\infty} \left(\frac{x\left(6-\frac{1}{x}\right)}{x\left(2+\frac{3}{x}\right)} \right)^2 = \left(\frac{6-0}{2+0} \right)^2 = 3^2 = 9$

j) $\lim_{x \to +\infty} \sqrt{x^2-5x+7} - x = \sqrt{x^2 \left(1 - \frac{5}{x} + \frac{7}{x^2} - \frac{1}{x} \right)} =$

$\lim_{x \to +\infty} x \cdot \lim_{x \to +\infty} \left(1 - \frac{5}{x} + \frac{7}{x^2} - \frac{1}{x} \right) = +\infty \cdot (1-0+0-0) = +\infty$

K) $\lim_{x \to +\infty} \frac{5x^2-3x+1}{2x^2+4x-7} = \lim_{x \to +\infty} \left(\frac{x^2 \left(5 - \frac{3}{x} + \frac{1}{x^2} \right)}{x^2 \left(2 + \frac{4}{x} - \frac{7}{x^2} \right)} \right) = \frac{5-0+0}{2+0-0} = \frac{5}{2}$

L) $\lim_{x \to -\infty} \frac{4-7x}{2+3x} = \frac{x\left(-7+\frac{4}{x}\right)}{x\left(3+\frac{2}{x}\right)} = \frac{-7+0}{3+0} = \frac{-7}{3}$

m) $\lim_{x \to +\infty} \frac{-x^3+2x}{2x^2-3} = \lim_{x \to +\infty} \frac{\cancel{x^2}\left(-1+\frac{2}{x^2}\right)}{\cancel{x^2}\left(\frac{2}{x}-\frac{3}{x^2}\right)} = \frac{-1+0}{0+0} = -1$

n) $\lim_{x \to -\infty} \frac{2-x^2}{x+3} = \frac{\cancel{x^2}\left(-1+\frac{2}{x^2}\right)}{\cancel{x^2}\left(\frac{1}{x}+\frac{3}{x^2}\right)} = \frac{-1+0}{0+0} = -1$

o) $\lim_{x \to \infty} \sqrt{\frac{8+x^2}{x(x+1)}} = \lim_{x \to \infty} \frac{\sqrt{\cancel{x^2}\left(1+\frac{8}{x^2}\right)}}{\sqrt{x^2+1}} = \frac{1+0}{1} = \frac{1}{1} = 1$

p) $\lim_{x \to \infty} \frac{3-2x}{5x+1} = \lim_{x \to \infty} \frac{\cancel{x}\left(-2+\frac{3}{x}\right)}{\cancel{x}\left(5+\frac{1}{x}\right)} = \frac{-2+0}{5+0} = \frac{-2}{5}$

q) $\lim_{x \to -\infty} \frac{4x-3}{3x+2} = \lim_{x \to -\infty} \frac{\cancel{x}\left(4-\frac{3}{x}\right)}{\cancel{x}\left(3+\frac{2}{x}\right)} = \frac{4+0}{3+0} = \frac{4}{3}$

r) $\lim_{x \to \infty} \frac{x^2-4}{x+1} = \lim_{x \to \infty} \frac{\cancel{x^2}\left(1-\frac{4}{x^2}\right)}{\cancel{x^2}\left(\frac{1}{x}+\frac{1}{x^2}\right)} = \frac{1-0}{0+0} = 1$

s) $\lim_{x \to -\infty} \frac{x^3-1}{x^2+1} = \frac{\cancel{x^3}\left(1-\frac{1}{x^3}\right)}{\cancel{x^3}\left(\frac{1}{x}-\frac{1}{x^3}\right)} = \frac{1-0}{0+0} = 1$

t) $\lim_{x \to \infty} \frac{x^2-3x+4}{3x^3+5x^2-6x+2} = \lim_{x \to \infty} \frac{\cancel{x^3}\left(\frac{1}{x}-\frac{3}{x^2}+\frac{4}{x^3}\right)}{\cancel{x^3}\left(3+\frac{5}{x}-\frac{6}{x^2}+\frac{2}{x^3}\right)} = \frac{0-0+0}{3+0+0+0} = 0$

u) $\lim_{x \to -\infty} \frac{x^2+4}{x+1} = \frac{\cancel{x^2}\left(1+\frac{4}{x^2}\right)}{\cancel{x^2}\left(\frac{1}{x}+\frac{1}{x^2}\right)} = \frac{1+0}{0+0} = 1$

v) $\lim_{x \to -\infty} \frac{x^2+x+1}{(x+1)^3-x^3} = \lim_{x \to -\infty} \frac{x^2+x+1}{3x^2+3x+1} = \lim_{x \to -\infty} \left(\frac{x^2\left(1+\frac{1}{x}+\frac{1}{x^2}\right)}{x^2\left(3+\frac{3}{x}+\frac{1}{x^2}\right)}\right) =$

$\frac{1+0+0}{3+0+0} = \frac{1}{3}$

w) $\lim_{x\to\infty} \frac{(2x-3)^3}{x(x+1)(x+2)} = \lim_{x\to\infty} \left(\frac{8x^3-36x^2+54x-27}{x^3+3x^2+2x}\right) =$

$\lim_{x\to+\infty} \left(\frac{\cancel{x^3}\left(8-\frac{36}{x}+\frac{54}{x^2}-\frac{27}{x^3}\right)}{\cancel{x^3}\left(1+\frac{3}{x}+\frac{2}{x^2}\right)}\right) = \frac{8-0+0-0}{1+0+0} = \frac{8}{1} = 8$

x) $\lim_{x\to\infty} \frac{(3x+2)^3}{2x(3x+1)(4x-1)} = \lim_{x\to\infty} \left(\frac{27x^3+54x^2+36x+8}{24x^3+2x^2-2x}\right) =$

$\lim_{x\to\infty} \left(\frac{\cancel{x^3}\left(27+\frac{54}{x}+\frac{36}{x^2}+\frac{8}{x^3}\right)}{\cancel{x^3}\left(24+\frac{2}{x}-\frac{2}{x^2}\right.}\right) = \frac{27}{24} = \frac{9}{8}$

y) $\lim_{x\to\infty} \frac{(2x-3)^3(3x-2)^2}{x^5} = \frac{72x^5-420x^4+950x^3-1035x^2+540x-108}{x^5} =$

$\lim_{x\to\infty} = \frac{\cancel{x^5}\left(72-\frac{420}{x}+\frac{950}{x^2}-\frac{1035}{x^3}+\frac{540}{x^4}-\frac{108}{x^5}\right)}{\cancel{x^5}(1)} = 72$

z) $\lim_{x\to-\infty} \frac{(x+2)^4-(x-1)^4}{(2x+3)^3} = \lim_{x\to-\infty} \left(\frac{12x^3+18x^2+36x+15}{(8x^3+36x^2+54x+27)}\right) =$

$\lim_{x\to-\infty} \left(\frac{\cancel{x^3}\left(12+\frac{18}{x}+\frac{36}{x^2}+\frac{15}{x^3}\right)}{\cancel{x^3}\left(8+\frac{36}{x}+\frac{54}{x^2}+\frac{27}{x^3}\right)}\right) = \frac{12}{8} = \frac{3}{2}$