Erick Scur

Lista - Derivadas da soma, produto e quociente

1)

a) $f(x) = 7x^4 - 2x^3 + 8x + 5$

$f'(x) = \frac{D}{Dx}(7x^4) + \frac{D}{Dx}(-2x^3) + \frac{D}{Dx}(8x) + \frac{D}{Dx}(5)$

$f'(x) = 28x^3 - 6x^2 + 8$

b) $y = 4x^3 + x^2$

$y' = \frac{D}{Dx}(4x^3) + \frac{D}{Dx}(x^2)$

$y' = 12x^2 + 2x$

c) $f(x) = \operatorname{sen} x + \cos x$

$f'(x) = \frac{D}{Dx}(\operatorname{sen}(x)) + \frac{D}{Dx}(\cos(x))$

$f'(x) = \cos(x) - \operatorname{sen}(x)$

2)

a) $f(x) = (3x^2 + x)(1 + x + x^3)$

$f'(x) = \frac{D}{Dx}\left((3x^2+x)(1+x+x^3)\right)$

$f'(x) = \frac{D}{Dx}(3x^2 + 3x^3 + 3x^5 + x + x^2 + x^4)$

$f'(x) = \frac{D}{Dx}(3x^5 + x^4 + 3x^3 + 4x^2 + x)$

$f'(x) = 15x^4 + 4x^3 + 9x^2 + 8x + 1$

b) $f(x) = x^2(x+x^4)(1+x+x^3)$

$f(x) = (x^3+x^6)(1+x+x^3)$

$f'(x) = \frac{D}{Dx}(x^3+x^4+x^6+x^6+x^7+x^9)$

$f'(x) = \frac{D}{Dx}(x^9+x^7+2x^6+x^4+x^3)$

$f'(x) = 9x^8+7x^6+12x^5+4x^3+3x^2$

c) $f(x) = (2+3x+x^2)^5$

$g = 2+3x+x^2$

$f'(x) = \frac{D}{Dx} = \frac{D}{Dg}(g^5) \cdot \frac{D}{Dx}(2+3x+x^2)$

$f'(x) = 5g^4 \cdot (2x+3)$

$f'(x) = 5(2+3x+x^2)^4 \cdot (2x+3)$

$f'(x) = (10x+15) \cdot (2+3x+x^2)^4$

d) $f(x) = (2x+3)^{52}$

$g = 2x+3$

$f'(x) = \frac{D}{Dg}(g)^{52} \cdot \frac{D}{Dx}(2x+3)$

$f'(x) = 52g^{51} \cdot 2$

$f'(x) = 52(2x+3)^{51} \cdot 2$

$f'(x) = 104(2x+3)^{51}$

e) $f(x) = x^3 \cdot e^x$

$f'(x) = \frac{D}{Dx}(x^3) \cdot e^x + x^3 \cdot \frac{D}{Dx}(e^x)$

$f'(x) = 3x^2 e^x + x^3 e^x$

f) $f(x) = x \cdot e^x + \cos x$

$f'(x) = \frac{D}{Dx}(x) \cdot e^x + x \cdot \frac{D}{Dx}(e^x) + \frac{D}{Dx}(\cos(x))$

$f'(x) = e^x + xe^x - \operatorname{sen}(x)$

g) $f(x) = x^4 \cdot a^{2x}$

$f'(x) = \frac{D}{Dx}(a^{2x}) \cdot x^4 + a^{2x} \cdot \frac{D}{Dx}(x^4)$

$f'(x) = \ln(a) \cdot a^{2x} \cdot 2x^4 + a^{2x} \cdot 4x^3$

$f'(x) = 2a^{2x} x^4 \cdot \ln(a) + 4a^{2x} x^3$

H) $f(x) = 3^{3x}$

$f'(x) = \ln(3) \cdot 3^{3x} \cdot 3$

$f'(x) = \ln(3) \cdot 3^{x+1}$

i) $f(x) = e^{5x+1}$

$g = 5x+1$

$f'(x) = \frac{D}{Dg}(e^g) \cdot \frac{D}{Dx}(5x+1)$

$f'(x) = e^g \cdot 5$

$f'(x) = 5e^g$

$f'(x) = 5e^{5x+1}$

d) $f(x) = \cos^5 x \qquad g = \cos x$

$f'(x) = \frac{D}{Dg}(g^5) \cdot \frac{D}{Dx}(\cos x)$

$f'(x) = 5g^4 \cdot -\text{sen } x$

$f'(x) = 5(\cos x)^4 \cdot -\text{sen } x$

$f'(x) = -5\cos^4 x \cdot \text{sen } x$

e) $f(x) = \text{sen}^7 x \cdot \cos^3 x$

$y = u^7$	$y = u^3$
$u = \text{sen } x$	$u = \cos x$
$y' = 7u^6$	$y' = 3u^2$
$u' = \cos x$	$u' = -\text{sen } x$
$f'(x) = 7(\text{sen}^6 x) \cdot \cos x$	$f'(x) = 3(\cos^3 x) \cdot -\text{sen } x$

$f'(x) = 7\,\text{sen}^6 x \cdot \cos^4 x \cdot \cos^3 x + \text{sen}^7 x \cdot 3\cos^2 x \cdot -\text{sen } x$

$f'(x) = 7\,\text{sen}^6 x \cdot \cos^4 x - 3\,\text{sen}^8 x \cdot \cos^2 x$

$f'(x)\ (\text{sen}^6 x \cdot \cos^2 x)(7\cos^2 x - 3\,\text{sen}^2 x)$

f) $f(x) = a \cdot \text{sen } x + B \cdot \cos x$

$f'(x) = \frac{D}{Dx}(a \cdot \sin(x)) + \frac{D}{Dx}(B \cdot \cos(x))$

$f'(x) = a \cdot \cos(x) + B \cdot -\text{sen}(x)$

$f'(x) = a \cdot \cos(x) - B \cdot \text{sen}(x)$

3)

a) $f(x) = \frac{2}{x^7}$

$f'(x) = \frac{-2 \cdot \frac{D}{Dx}(x^7)}{x^{7^2}}$

$f'(x) = -2 \cdot \frac{7x^6}{(x^7)^6}$ $\quad f'(x) = \frac{-14}{x^8}$

b) $f(x) = 3x^{-5}$

$f'(x) = 3 \cdot \frac{D}{Dx}(x^{-5})$

$f'(x) = 3 \cdot (-5x^{-6})$

$f'(x) = \frac{-15}{x^6}$

c) $f(x) = \frac{1}{x^2+x+1}$

$f'(x) = -\frac{\frac{D}{Dx}(x^2+x+1)}{(x^2+x+1)^2}$

$f'(x) = -\frac{2x+1+0}{(x^2+x+1)^2}$

D) $f(x) = \frac{x+1}{x-1}$

$f'(x) = \frac{\frac{D}{Dx}(x+1)(x-1) - (x+1) \cdot \frac{D}{Dx}(x-1)}{(x-1)^2}$

$f'(x) = \frac{1(x-1) - (x+1) \cdot 1}{(x-1)^2}$

$f'(x) = -\frac{2}{(x-1)^2}$

e) $f(x) = \frac{x+3}{x-1} + \frac{x+2}{x+1}$

$f'(x) = \frac{D}{Dx}\left(\frac{(x+1)(x+3) + (x-1)(x+2)}{(x-1)(x+1)}\right)$

$f'(x) = \frac{D}{Dx}\left(\frac{x^2+3x+x+3+x^2+2x-x-2}{x^2-1}\right)$

$f'(x) = \frac{D}{Dx}\left(\frac{2x^2+5x+1}{x^2-1}\right)$

$f'(x) = \frac{-5x^2-6x-5}{(x^2-1)^2}$

f) $f(x) = \frac{x^2+3x+1}{x-2}$

$f'(x) = \frac{\frac{D}{Dx}(x^2+3x+1).(x-2) - (x^2+3x+1).\frac{D}{Dx}(x-2)}{(x-2)^2}$

$f'(x) = \frac{(2x+3)(x-2) - (x^2+3x+1).1}{(x-2)^2}$

$f'(x) = \frac{x^2-4x-7}{(x-2)^2}$

g) $f(x) = \frac{x^2.\text{sen}x}{e^x}$

$f'(x) = \frac{\cancel{D}x(x^2.\text{sen}(x)).e^x - x^2.\text{sen}(x).\frac{D}{Dx}(e^x)}{(e^x)^2}$

$f'(x) = \frac{2x.\text{sen}(x) + x^2.\cos(x) - x^2.\text{sen}(x)}{e^x}$

H) $f(x) = \dfrac{\cos x}{x \cdot e^x}$

$$f'(x) = \frac{\frac{d}{dx}(\cos(x)) \cdot xe^x - \cos(x) \cdot \frac{d}{dx}(xe^x)}{(xe^x)^2}$$

$$f'(x) = \frac{xe^x \cdot \operatorname{sen}(x) + \cos(x) \cdot (e^x + xe^x)}{x^2 e^{2x}}$$

4)

a) $f(x) = \operatorname{cotg} x$

$f'(x) = -\operatorname{cossec}^2 x$

b) $f(x) = \sec x$

$f'(x) = \sec x \cdot \operatorname{tg} x$

c) $f(x) = \operatorname{cossec} x$

$f'(x) = -\operatorname{cotg} x \cdot \operatorname{cossec} x$

D) $f(x) = \operatorname{tg}^2 x$

$f'(x) = 2 \cdot \operatorname{tg} \cdot \sec^2 x$

e) $f(x) = \sec x - \operatorname{tg} x$

$f'(x) = \sec x \cdot (\operatorname{tg} x - \sec x)$

f) $f(x) = (x^2+1) \cdot \operatorname{tg} x$

$f'(x) = 2x \cdot \operatorname{tg} x + (x^2+1) \cdot \sec^2 x$

g) $f(x) = \dfrac{\operatorname{tg}}{\operatorname{sen} x + \cos x}$

$$f'(x) = \frac{\operatorname{sen}^2 x (\operatorname{sen} x + \cos x) - \operatorname{tg} x \cdot (\cos x - \operatorname{sen} x)}{(\operatorname{sen} x + \cos x)^2}$$

H) $\left(\frac{e^x}{tgx}\right)^2 = \frac{2 \cdot e^x}{tg\, x^3}$

$\frac{2 \cdot e^x}{tg\, x^3} \cdot (tgx \cdot sec^2 x)$