4. A. $A = U \Lambda U^T$

$Ax = \lambda x$

$x^T A x = \lambda x^T x$

$(x^T A x)^T = \bar{\lambda} x^T x$

$A = x \bar{\lambda} x^T = U \Lambda U^T$

B. $x^T A x \geq 0$

$\lambda_i \geq 0$

From problem 4A:

$\lambda x^T x = x^T A x$

$\lambda = \dfrac{x^T A x}{x^T x} = \dfrac{x^T A x}{||x||^2} \geq 0$

C. $x^T A x > 0$

$\lambda_i > 0$

$\lambda x^T x = x^T A x$

$\lambda = \dfrac{x^T A x}{||x||^2} > 0$

6 A: Calculate Bias of $\bar{X}$

$\text{Bias} = E[\hat{\theta}] - \theta$

$E[\bar{x}] = E[\frac{1}{n} \Sigma x_i]$

$= \frac{1}{n} E[\Sigma x_i]$

$= \frac{n}{n} E[x_1]$

$= \frac{n\mu}{n} =$

$\text{Bias} = \frac{n\mu}{n} - \mu = 0$

B: Find $Var[\bar{x}]$

$Var[\bar{x}] = Var[\frac{1}{n} \Sigma x_i]$

$= \frac{1}{n^2} Var[\Sigma x_i]$

$= \frac{1}{n} Var[x_i]$

$= \frac{\theta^2}{n}$

C. $MSE[\bar{x}] = \text{Bias}^2 + \text{Variance}$

$\text{Bias}^2 =$ [redacted] 0

$MSE =$ [redacted] $+ \frac{\theta^2}{n}$

D. $\text{Bias}[\hat{s}^2] = E[\hat{s}^2] - \theta^2$

$E[\hat{s}^2] = E[\frac{1}{n} \Sigma (x_i - \bar{x})^2]$

$= \frac{1}{n} E[\Sigma (x_i - \bar{x})^2]$ // Factor out $\bar{X}$

$= \frac{1}{n} E[\Sigma x_i^2 - n\bar{x}^2]$

$= \frac{1}{n} (n(\mu^2 + \theta^2) - n(\mu^2 + \frac{\theta^2}{n}))$

$= \mu^2 + \theta^2 - \mu^2 - \frac{\theta^2}{n}$

$= \theta^2 - \frac{\theta^2}{n}$

$\text{Bias} = -\frac{\theta^2}{n}$