Zeige $\{1, \omega_4^2, (\omega_4^2)^2, (\omega_4^2)^3\} = \{1, \omega_2\}$

Wir wissen: $\omega_4 = \exp\left(\frac{2\pi i}{4}\right) = \exp\left(\frac{\pi i}{2}\right)$

$$\omega_2 = \exp\left(\frac{2\pi i}{2}\right) = \exp(\pi i)$$

Damit folgt

$$\omega_4^2 = \exp\left(\frac{2\pi i}{2}\right) = \exp(\pi i) = \omega_2$$

$$(\omega_4^2)^2 = \exp(2\pi i) = \cos(2\pi) + i\sin(2\pi) = 1 + 0 = 1$$

verwende $\exp(i\varphi) = \cos(\varphi) + i\sin(\varphi) \quad \forall \varphi \in \mathbb{R}$

$$(\omega_4^2)^3 = \exp(3\pi i) = \underbrace{\exp(2\pi i)}_{=1}\,\underbrace{\exp(\pi i)}_{=\omega_2} = \omega_2$$