1980A&A....82..322D

Astron. Astrophys. 82, 322–327 (1980)

ASTRONOMY AND ASTROPHYSICS

On Velocity Dispersions of Galaxies in Rich Clusters

L. Danese[1], G. De Zotti[2], and G. di Tullio[2]

[1] Osservatorio Astronomico, Vicolo dell'Osservatorio, 5, I-35100 Padova, Italy
[2] Unità di Ricerca di Padova del GNA-CNR, Vicolo dell'Osservatorio, 5, I-35100 Padova, Italy

Received February 23, 1979

Summary. We discuss the problem of evaluating the mean redshifts and the velocity dispersions of rich clusters on the basis of restricted samples of radial velocities. We describe a procedure for evaluating at any significance level the global uncertainty resulting from sampling errors, experimental errors, and errors in the projection factors. Using this procedure we have computed with their 68% confidence uncertainties, the mean redshifts and the velocity dispersions of 43 rich clusters. A strong indication that the velocity dispersion is correlated with the richness of the cluster comes out of our analysis.

Key words: clusters of galaxies — radial velocities — velocity dispersion-errors

1. Introduction

The possibility of exploiting correlations between velocity dispersion and X-ray luminosity of clusters of galaxies and between velocity dispersion and intergalactic gas temperature to test the various proposed models for X-ray emission (Solinger and Tucker, 1972; Yahil and Ostriker, 1973; Silk, 1976; Cowie and Binney, 1977; Mitchell et al., 1977) has stimulated spectroscopic studies of an increasing number of clusters. Of course such studies are of great importance in many other respects: they form the basis for understanding the cluster dynamics (Zwicky, 1957; King, 1966); provide important insights with regard to the problem of cluster formation (Gunn and Gott, 1972; Cavaliere et al., 1978); allow an estimate of the mean mass density in matter distributed like galaxies (Seldner and Peebles, 1977); clarify the interaction between galaxies and intergalactic medium (De Young, 1978); help the interpretation of radio tail galaxies in clusters (Jaffe and Perola, 1973; Pacholczyk and Scott, 1976), and so on.

The rapid increase in the number of clusters for which velocity data are available will conceivably allow, in the near future, increasingly sophisticated statistical tests of the various theoretical predictions. Of course, reliable estimates of errors are necessary before such tests can be performed. This point, though obvious, does not seem to have been generally appreciated; and indeed very little attention has been paid so far to the estimation of the errors in the cosmological redshifts and in the velocity dispersions of clusters.

These remarks have motivated the present work in which such a problem is examined. In Sects. 2 and 3, after a brief summary of the formulae to be used for arbitrarily distant clusters, we discuss the various error sources and derive simple rules for evaluating their contributions to the overall uncertainty. In Sect. 4 we compute the mean redshifts, the velocity dispersions, and their errors for 43 rich clusters. Sect. 5 is devoted to a discussion of the results and to the conclusions.

2. Cosmological Redshifts of Clusters

As it has been stressed by Harrison (1974), it is the product (and not the sum) of the contributions $1 + z_0 = [(c - v_0)/(c + v_0)]^{1/2}$ (due to the radial component of our own peculiar velocity), $1 + z_R$ (the cosmological redshift of the cluster) and $1 + z_G = [(c + v_{\|})/(c - v_{\|})]^{1/2}$ (due to the radial component of the peculiar velocity of the galaxy), which gives the observed redshift z:

$$1 + z = (1 + z_0)(1 + z_R)(1 + z_G). \tag{1}$$

Let us assume the redshifts to be independent of galaxy mass (an assumption that we shall keep throughout this paper and that we shall discuss in Sect. 5). Then, $\bar{z}_G = 0$ (the bar denotes the average value; weighting by masses is no longer necessary) and

$$z_R = (\bar{z} - z_0)/(1 + z_0). \tag{2}$$

In practice we estimate $\bar{z}$ by averaging a restricted set of observational values z_i. If the redshifts are normally distributed [as it is indicated by the work of Yahil and Vidal (1977)] the consequent sampling errors are easily evaluated. Let $\bar{z}'$ and σ'_z be the estimates, based on a random sample of size n, of the true mean redshift $\bar{z}$ and of the variance σ'_z. Then (cf. e.g. Hoel, 1971, p. 261) the quantity $T = (\bar{z} - \bar{z}')n^{1/2}/\sigma'_z$ possesses a Student's t distribution with $\nu = n - 1$ degrees of freedom. If $t_\nu(\alpha)$ represents the value of T such that the probability is α that $|T| \leqq t_\nu(\alpha)$, the 100α percent confidence interval for $\bar{z}$ is given by

$$\bar{z}' - t_\nu(\alpha)\sigma'_z n^{-1/2} < \bar{z} < \bar{z}' + t_\nu(\alpha)\sigma'_z n^{-1/2}. \tag{3}$$

An approximated analytic formula for $t_\nu(\alpha)$ is given by Zelen and Severo (1965). For large ν, $t_\nu(\alpha)$ approaches the

Send offprint requests to: L. Danese

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1980A&A....82..322D

"equivalent number if standard deviations" $x_p(\alpha)$ ($x_p = 1$, 1.645, 2, 3 for $\alpha = 0.68, 0.90, 0.95, 0.997$ respectively).

The additional uncertainty $(\Delta\bar{z})_0$, due to the observational errors ζ_i, is given by: $(\Delta\bar{z})_0^2 = \sum_i \zeta_i^2/n^2 = \overline{\zeta^2}/n$. Approximating the Student's t distribution by a gaussian with dispersion $t_\nu \sigma_z' n^{-1/2}$ we then obtain the 68% confidence uncertainty in z_R:

$$\Delta z_R = \Delta\bar{z}/(1+z_0) \simeq (t_\nu^2(0.68)\sigma_z'^2 + \overline{\zeta^2})^{1/2} n^{-1/2} \quad (4)$$

with $t_\nu(0.68) \to 1$ for $\nu \gg 10$; $z_0 \leqq 10^{-3}$ has been neglected.

3. Velocity Dispersion

After Eq. (1), the line-of-sight component of the velocity of a galaxy with respect to the cluster center of mass reads:

$$v_\parallel = (V_\parallel - \bar{V}_\parallel)/(1 + \bar{V}_\parallel/c) \quad (5)$$

where $V_\parallel = cz$ is the usually tabulated "radial velocity" *uncorrected* for the motion of the local observer and $\bar{V}_\parallel = c\bar{z}$. Note the frequently neglected factor $(1 + \bar{V}_\parallel/c)$. Note also that $v_\parallel$ retains its physical meaning independently of the value of z_R (provided that $z_G \ll 1$).

After subtracting the spurious systematic contribution due to the measurement errors $\delta_i = c\zeta_i$ (cf. Appendix), the sample estimate of the radial velocity dispersion $\sigma_\parallel = c\sigma_z'$ reads:

$$\sigma_\parallel^2 = \sum_i v_{\parallel i}^2/(n-1) - \overline{\delta^2}/(1 + \bar{V}_\parallel/c)^2. \quad (6)$$

The quantity $S = (n-1)\sigma_\parallel^2 \Sigma^{-2}$ (Σ = true velocity dispersion) is known to have, under the same hypotheses in the preceding section, a χ^2 distribution with $\nu = n - 1$ degrees of freedom (cf. e.g. Hoel, 1971, p. 254). Then, if χ_+^2 and χ_-^2 are two values of χ_ν^2 such that the probabilities are $(1+\alpha)/2$ and $(1-\alpha)/2$ that $\chi_\nu^2 < \chi_+^2$ and $\chi_\nu^2 < \chi_-^2$ respectively, the probability is α that:

$$\nu\sigma_\parallel^2/\chi_+^2 < \Sigma^2 < \nu\sigma_\parallel^2/\chi_-^2. \quad (7)$$

Note that χ_+^2 and χ_-^2 have been erroneously interchanged by several authors.

Values of $\chi_\pm^2$ for $\alpha = 0.68, 0.90, 0.95$, and 0.997 are listed in Table 1 for $9 \leqq \nu \leqq 30$. For large ν the following analytical approximations hold (Zelen and Severo, 1965):

$$\chi_\pm^2 \simeq \nu[1 - 2/9\nu \pm x_p(2/9\nu)^{1/2}]^3 \qquad \nu \geqq 30, \quad (8)$$

$$\chi_\pm^2 \simeq [\pm x_p + (2\nu - 1)^{1/2}]^2/2 \qquad \nu \geqq 100. \quad (9)$$

If $\alpha = 0.68$, Eq. (9) reduces to the well-known expression: $\chi_\pm^2(\alpha = 0.68) = \nu \pm (2\nu - 1)^{1/2}$, which has been frequently used also for small ν.

Taking into account the contribution due to the measurement errors δ_i (cf. Materne, 1974), the 68% confidence uncertainties $(\Delta\sigma_{\parallel\pm})^2$ in the velocity dispersion are written:

$$(\Delta\sigma_{\parallel\pm})^2 \simeq [(\nu/\chi_\mp^2)^{1/2} - 1]^2 \sigma_\parallel^2 + \overline{\delta_*^2}(1 + \overline{\delta_*^2}/2\sigma_\parallel^2)/n \quad (10)$$

Table 1. Values of $\chi_\pm^2/\nu$

	α							
ν	0.68		0.90		0.95		0.997	
9	0.547	1.454	0.369	1.880	0.292	2.145	0.138	3.010
10	0.568	1.433	0.394	1.831	0.317	2.077	0.158	2.879
11	0.587	1.414	0.416	1.789	0.339	2.020	0.178	2.768
12	0.603	1.397	0.436	1.752	0.359	1.970	0.196	2.673
13	0.618	1.382	0.453	1.720	0.377	1.927	0.213	2.590
14	0.631	1.369	0.469	1.692	0.394	1.889	0.229	2.518
15	0.643	1.357	0.484	1.666	0.410	1.854	0.244	2.454
16	0.654	1.346	0.498	1.644	0.428	1.824	0.258	2.397
17	0.664	1.336	0.510	1.623	0.437	1.796	0.272	2.345
18	0.673	1.327	0.522	1.604	0.450	1.771	0.285	2.299
19	0.681	1.319	0.533	1.587	0.461	1.748	0.297	2.256
20	0.689	1.311	0.543	1.571	0.472	1.727	0.308	2.218
21	0.696	1.304	0.552	1.556	0.482	1.707	0.319	2.182
22	0.703	1.297	0.561	1.542	0.492	1.689	0.330	2.149
23	0.709	1.291	0.569	1.529	0.501	1.672	0.340	2.118
24	0.715	1.285	0.577	1.517	0.509	1.656	0.349	2.090
25	0.721	1.279	0.584	1.506	0.518	1.642	0.358	2.064
26	0.726	1.274	0.592	1.496	0.525	1.628	0.367	2.039
27	0.731	1.269	0.598	1.486	0.533	1.615	0.375	2.016
28	0.736	1.264	0.605	1.476	0.540	1.602	0.383	1.994
29	0.740	1.260	0.611	1.468	0.546	1.591	0.391	1.973
30	0.745	1.255	0.616	1.459	0.553	1.580	0.399	1.954

For each value of α, the first column gives χ_-^2, the second χ_+^2; $\nu = n - 1$ is the number of degrees of freedom

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1980A&A....82..322D

Table 2. Mean redshifts and velocity dispersions of 43 clusters

Cluster	R	n	z_R	$\sigma_{\parallel}$ (km/s)	σ (km/s)
A 98	3	11	$0.1040\pm9\times10^{-4}$	798(+261,-134)	1383(+491,-302)
A 154	1	11	$0.0658\pm1\times10^{-3}$	843(+276,-142)	1461(+519,-319)
A 168	2	13	$0.0452\pm6\times10^{-4}$	571(+166,- 92)	988(+313,-202)
A 194	0	57	$0.0179\pm2\times10^{-4}$	396(+ 45,- 35)	686(+ 88,- 73)
A 262	0	38	$0.0167\pm3\times10^{-4}$	415(+ 63,- 48)	719(+121,- 98)
A 272	1	14	$0.0877\pm7\times10^{-4}$	711(+194,-109)	1232(+369,-241)
A 347	0	17	$0.0194\pm3\times10^{-4}$	394(+ 96,- 60)	683(+182,-128)
A 401	2	14	$0.0748\pm1.3\times10^{-3}$	1280(+352,-199)	2217(+666,-438)
A 426	2	113	$0.0181\pm4\times10^{-4}$	1282(+ 95,- 78)	2221(+189,-164)
Fornax	1	27	$0.0048\pm2\times10^{-4}$	240(+ 51,- 40)	416(95,- 78)
CA 0340-538	2	19	$0.0575\pm8\times10^{-4}$	1006(+222,-135)	1743(+425,-296)
A 576	1	20	$0.0389\pm1\times10^{-3}$	1211(+254,-158)	2097(+489,-346)
Cancer	-	21	$0.0149\pm4\times10^{-4}$	542(+115,- 76)	939(+220,-161)
A 754	2	11	$0.0539\pm1\times10^{-3}$	910(+297,-152)	1577(+560,-343)
A 779	0	12	$0.0231\pm5\times10^{-4}$	519(+161,- 87)	899(+303,-192)
A 1060	1	21	$0.0114\pm6\times10^{-4}$	777(+159,- 99)	1346(+306,-217)
A 1314	0	16	$0.0341\pm6\times10^{-4}$	644(+168,-107)	1116(+317,-224)
A 1367	2	68	$0.0214\pm3\times10^{-4}$	813(+ 81,- 63)	1408(+160,-133)
A 1452	0	15	$0.0630\pm5\times10^{-4}$	504(+138,- 87)	874(+261,-182)
Virgo	1	122	$0.0037\pm2\times10^{-4}$	673(+ 48,- 40)	1166(+ 95,- 84)
Centaurus	2	69	$0.0107\pm4\times10^{-4}$	870(+ 88,- 70)	1508(+173,-145)
Coma	2	207	$0.0232\pm2\times10^{-4}$	905(+ 49,- 43)	1567(+ 98,- 88)
IC 4329	-	10	$0.0144\pm7\times10^{-4}$	591(+208,-102)	1024(+391,-232)
A 1940	3	11	$0.1389\pm6\times10^{-4}$	534(+176,- 93)	925(+331,-206)
A 2029	2	18	$0.0775\pm1.3\times10^{-3}$	1522(+346,-206)	2637(+622,-455)
A 2065	2	16	$0.0717\pm1.0\times10^{-3}$	1108(+273,-158)	1919(+521,-349)
A 2147	1	14	$0.0377\pm1.0\times10^{-3}$	1074(+292,-162)	1860(+555,-361)
A 2151	2	22	$0.0367\pm6\times10^{-4}$	785(+158,-102)	1359(+304,-220)
A 2197	1	15	$0.0300\pm4\times10^{-4}$	395(+105,- 63)	684(+199,-135)
A 2199	2	46	$0.0306\pm4\times10^{-4}$	784(+100,- 75)	1358(+195,-157)
A 2247	0	14	$0.0391\pm4\times10^{-4}$	369(+104,- 63)	639(+197,-134)
A 2250	1	18	$0.0654\pm6\times10^{-4}$	694(+160,- 99)	1202(+306,-214)
A 2255	2	15	$0.0797\pm1.1\times10^{-3}$	1128(+296,-173)	1954(+562,-377)
A 2255 A	-	11	$0.0820\pm5\times10^{-4}$	385(+143,- 94)	668(+265,-187)
A 2256	2	15	$0.0600\pm1.2\times10^{-3}$	1254(+323,-182)	2172(+616,-406)
Zw 1809+50	-	10	$0.0508\pm8\times10^{-4}$	679(+240,-120)	1175(+451,-270)
A 2319	1	31	$0.0559\pm1\times10^{-3}$	1580(+249,-170)	2737(+484,-367)
A 2319 A	-	22	$0.0528\pm7\times10^{-4}$	848(+169,-107)	1469(+325,-233)
Zw 2247+11	0	14	$0.0260\pm5\times10^{-4}$	498(+148,- 96)	863(+277,-196)
Pegasus I	0	27	$0.0134\pm4\times10^{-4}$	655(+115,- 77)	1135(+222,-165)
A 2634	1	16	$0.0316\pm6\times10^{-4}$	705(+182,-114)	1221(+345,-241)
Klemola 44	0	16	$0.0276\pm4\times10^{-4}$	341(+106,- 80)	591(+195,-155)
A 2670	3	10	$0.0749\pm1.3\times10^{-3}$	1048(+370,-182)	1815(+693,-413)

A 98: Faber and Dressler (1977). *A 154:* Faber and Dressler (1977). *A 168:* Faber and Dressler (1977). *A 194:* Chincarini and Rood (1977). *A 262:* Moss and Dickens (1977) (33); Faber and Dressler (1977) (3: n. 7, 8, 10); Melnick and Sargent (1977) (2: n. 5, 9). *A 272:* Sargent (1972). *A 347:* Hintzen et al. (1978). *A 401:* Hintzen et al. (1977). *A 426:* Tifft (1978) (61); Chincarini and Rood (1971) (50); Sargent (1970) (1: Zw 297); de Vaucouleurs et al. (1976) (1: NGC 1233). *Fornax 1:* Welch et al. (1975) (group 53); richness from Sandage (1972). *CA 0340−538:* Havlen and Quintana (1978); richness from the same authors. *A 576:* Melnick and Sargent (1977). *Cancer:* Tifft et al. (1973). *A 754:* Faber and Dressler (1977). *A 779:* Hintzen et al. (1978). *A 1060:* Vidal and Peterson (1975) (15); Faber and Dressler (1977) (4: n. 5, 8, 10, 11); Sandage (1975) (2: Anon 8, Anon 9). *A 1314:* Coleman et al. (1976). *A 1367:* Tifft (1978) (54); Dickens and Moss (1976) (14: n. 17, 18, 19, 20, 21, 22, 26, 30, 35, 37, 43, 53, 59, 64). *A 1452:* Ulrich (1978). *Virgo:* Tammann (1972); richness from Sandage (1972). *Centaurus:* Dawe et al. (1977) (61); Sandage (1975) (4) (Anon 8, HB 276, HB 280, HB 288); Vidal and Wickramasinghe (1977) (3: O, P, R); richness from Sandage (1972). *Coma:* Gregory (1975). *IC 4329:* Sandage (1975). *A 1940:* Faber and Dressler (1977), *A 2029:* Faber and Dressler (1977). *A 2065:* Spinrad (1977). *A 2147:* Bautz (1972). *A 2151:* Burbidge and Burbidge (1959) (15); de Vaucouleurs et al. (1976) (4: NGC 6040 A, B; IC 1182, 1189); Arakelian et al. (1972) (2: MK 292, 299); Ulrich (1971) (1: MK 291). *A 2197:* Chincarini and Rood (1972a) (7); Chincarini and Rood (1972b) (4); de Vaucouleurs et al. (1976) (3: A 1626+41, A 1625+40, A 1625+41); Zwicky (1971) (1: III. Zw 82). *A 2199:* Tifft (1974) (34); Chincarini and Rood (1972a) (4); Chincarini and Rood (1972b) (2); Sargent (1970) (1: Zw 151); Minkowski (1961) (2: NGC 6166, n. 18); Kintner (1971) (3: the last 3 of his list). *A 2247:* Gregory and Connolly (1973). *A 2250:* Ulrich (1978). *A 2255:* Tarenghi and Scott (1976). *A 2256:* Faber and Dressler (1977) (14); Bridle and Fomalont (1976) (1: n. 7). *Zw 1809+50:* Hintzen et al. (1978). *A 2319:* Faber and Dressler (1977). *Zw 2247+11:* Scott et al. (1977); richness from Sandage (1972). *Pegasus I:* Chincarini and Rood (1976); richness from Sandage (1972). *A 2637:* Scott et al. (1977). *Klemola 44:* Chincarini et al. (1978) (12); Maccaccaro et al. (1977) (4: A, B, C, L); richness from Maccaccaro et al. (1977). *A 2670:* Oemler (1973).

where $\overline{\delta}_*^2 = \overline{\delta}^2/(1 + \overline{V}_{\parallel}/c)^2$ and the two tails of the χ^2 distribution have been approximated by the tails of two gaussians.

A further error source comes out when we try to infer from $\sigma_{\parallel}$ the physical velocity dispersion σ. An estimate of the uncertainty in the projection factor $F = (\sum_i v_{\parallel i}^2/\sum_i v_i^2)^{1/2}$, the v_i being the three-dimensional velocities with line-of-sight components $v_{\parallel i} = v_i \cos \vartheta_i$, can be readily obtained using a simple model. Let us consider an infinite number of spatial velocities having equal magnitudes and randomly oriented in a spherical volume. We then have $\overline{F} = 1/\sqrt{3}$ (Limber and Mathews, 1960) with a standard deviation $\sigma_F = \overline{F}/\sqrt{5}$. From the central limit theorem (cf. e.g. Hoel, 1971, p. 125) it follows that the probability distribution of $\overline{F}'$, based on a random sample of n velocities, approaches a gaussian with standard deviation

$$\Delta F/\overline{F} = (5n)^{-1/2} = 0.447n^{-1/2}. \tag{11}$$

In conclusion, the 68% confidence uncertainties of σ can be approximately estimated from:

$$(\Delta\sigma_{\pm})^2/\sigma^2 \cong (\Delta F/\overline{F})^2 + (\Delta\sigma_{\parallel\pm}/\sigma_{\parallel})^2. \tag{12}$$

4. Applications

We have collected from the literature a list, that we believe to be complete, of rich clusters with the redshifts of at least ten galaxies published by the end of December 1978. In many cases the data were gathered from several papers; nevertheless the non-homogeneity of the samples should not constitute a serious problem since, for each cluster, most redshifts come from a single source (in case of overlap the reference containing more data was preferred). As for the membership, we have relied on the original assignements and on Corwin's (1974) redshift catalog, save that we have removed from the samples, according to the criterion proposed by Yahil and Vidal (1977), those galaxies whose velocities deviate from the mean of the others by more than three standard deviations (also computed excluding the galaxy whose membership is being tested). The radial component of the velocity of the Sun has been computed using the standard formula $v_0 = 300 \sin l^{II} \cos b^{II}$ km s^{-1}. For each cluster we have evaluated the mean redshift, the velocity dispersion and their errors according to the rules discussed in the preceding sections.

Our results are displayed in Table 2 which gives: in Column 1, the name of the cluster; Column 2, the cluster richness (Abell, 1958); Column 3, the number of galaxies with measured

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1980A&A....82..322D

redshift; Columns 4, 5, 6, the mean redshift, the line of sight velocity dispersion, the physical velocity dispersion and their errors at the 68% confidence level. In the few cases when the experimental errors are not given $\delta = 100\ \mathrm{km\,s^{-1}}$ has been assumed. Note that in many cases observers give probable errors which must be multiplied by 1.483 to obtain the corresponding standard deviations δ_i.

In the footnotes we give, for each cluster, the references for the velocity data and for the richness classification of clusters not in Abell's (1958) catalog. When the velocities were collected from several sources we also give, in parentheses, how many and, if not obvious, which data were taken from each source.

5. Discussion and Conclusions

It must be emphasized that the errors given in Table 2, though considerably larger than those usually assumed, are likely to be under-estimates of the true errors since they do not include the non easily quantified contributions of several other sources of uncertainty that we shall briefly discuss in the following.

The main assumption in this paper is that radial velocities are independent of galaxy masses. Physically, this corresponds to a situation in which clusters have undergone violent relaxation (Lynden-Bell, 1967), which produces velocity equipartition, but are at the very beginning of the evolution (proceeding through two body interactions) towards energy equipartition. This picture is consistent, at least in first approximation, with observations which generally indicate that little two body relaxation has occurred except perhaps in the very center of clusters (cf. Bahcall, 1977). However, since just the brightest central galaxies are preferentially selected for redshift measurements, any partial equipartition would lead to an underestimate of velocity dispersion. On the other hand weighting velocities by luminosity masses would not be a good remedy for that in the present data situation: the mass to luminosity ratio M/L for ellipticals is uncertain by at least a factor of 3; for spirals M/L increases from early to late types by at least a factor of 5, and, moreover, the distribution functions of M/L for each type are probably strongly skewed and very broad (cf. Ozernoi and Reinhardt, 1976, and references therein).

Secondly, the velocity dispersion in clusters is expected to decrease with increasing distance from the cluster center (Peebles, 1970); and indeed Rood et al. (1972) and Dickens and Moss (1976) have found evidence for that in the Coma cluster (see also Gregory, 1975) and in A 1367, respectively. Thus σ should be evaluated within a fixed, physically significant radius. We do not have enough data to do this in a systematic way, however; judging from the measurements in Coma, uncertainties $\leqq$10–20% are expected for this reason.

In the third place, if virialization is incomplete and the velocities are non-randomly oriented, selection effects can come heavily into play. For example if the velocities were largely radial, redshift measurements near the projected center of the cluster would systematically select galaxies moving nearly in the line of sight (Abell, 1975).

Finally, failure of the membership criteria is always possible: contamination by field galaxies or clipping as accidentals of member galaxies whose velocities are in the tail of the distribution would result in an overestimate or in an underestimate, respectively, of the velocity dispersion.

The above considerations show that, in view of the large uncertainties, sophisticated statistical analyses of correlations between velocity dispersions and other cluster properties are perhaps premature. Nevertheless, it is worth noting that an inspection of our Table 2 reveals a distinct increase of the mean velocity dispersion of clusters as their richness R (Abell, 1958) increases from 0 to 2: we have $\bar{\sigma} = 820 \pm 64\ \mathrm{km\,s^{-1}}$ for $R = 0$, $\bar{\sigma} = 1400 \pm 210\ \mathrm{km\,s^{-1}}$ for $R = 1$ [$\bar{\sigma} = 1270 \pm 170\ \mathrm{km\,s^{-1}}$ if A 2319, which could be two clusters superposed along the line of sight (Faber and Dressler, 1977) is excluded] and $\bar{\sigma} = 1760 \pm 120\ \mathrm{km\,s^{-1}}$ for $R = 2$ [$\bar{\sigma} = 1750 \pm 130\ \mathrm{km\,s^{-1}}$ without A 2255 which suffers from the same problem as A 2319 (Tarenghi and Scott, 1976)]; the errors are given at the 68% confidence level.

If the richness is a measure of mass such an increase is indeed expected in the framework of the gravitational instability picture for the formation of clusters of galaxies (cf. Cavaliere et al., 1978); if the rich clusters are on the verge of virializing now, theory predicts $\sigma \propto M^{1/3}$.

The velocity dispersions of the three $R = 3$ clusters apparently conflict with the trend mentioned above. However, not only the smallness of the sample prevents from drawing any conclusion, but also the velocity dispersions and even more the richness classification of such objects must be regarded as very uncertain. As for the velocity dispersion, large errors can be expected, on account of the small number (10 or 11) of galaxies with measured redshift, not only because of an unfortunate selection of the observed galaxies (a possibility already accounted for by the error estimates in Table 2) but also because of the failure of the adopted membership criteria. Indeed possible high velocity members of A 98 and of A 1940 have been excluded on the basis of our velocity criterion; with such objects included, the velocity dispersions would be considerably higher, particularly in the case of A 98, for which we would obtain $\sigma = 2722\ \mathrm{km\,s^{-1}}$. As for richness, the works of Oemler (1973) on A 2670 and of Dressler (1978) on several clusters including A 98 and A 1940, have shown that Abell's richness classes may be unreliable when clusters are very rich; in particular all three clusters under examination turn out to be less rich than Coma, classified by Abell (1958) as $R = 2$.

The above mentioned difficulties should affect to a much smaller extent the results for the relatively closer $R \leqq 2$ clusters: not only are the samples of clusters bigger but also redshifts of a larger number of galaxies per cluster are generally available; also the richness classification should be easier. Therefore, in spite of the large uncertainties, we believe that the data strongly point to the existence of a correlation between velocity dispersion and richness.

Acknowledgements. We are indebted to Prof. F. Pesarin for advice on error analysis. One of us (G. di T.) acknowledges useful discussions with Dr. A. Dressler. We also wish to thank Mrs. C. Lista for carefully typing the manuscript. The work has been supported by Consiglio Nazionale delle Ricerche.

Appendix

It is known (cf. Jackson, 1973) that measurement errors introduce a fictitious contribution σ_F to the velocity dispersion. We give here an explicit derivation of a formula for σ_F. This

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1980A&A....82..322D

seems worth doing since, as far as it is known to us, no such derivation is available in the literature and, moreover, incorrect expressions for σ_F are sometimes used.

Let W_i be the true "radial velocity" of a galaxy with respect to a local observer, $V_i = W_i + \varepsilon_i$ its measured value, ε_i the observational error and $v_i = c(V_i - \sum_i V_i/n)/(c + \sum_i V_i/n)$ the velocity relative to the cluster center of mass [cf. Eq. (5)]; we have retained the convention of defining a fictitious velocity $V_i = cz_i$. Taking into account that $E(\varepsilon_i) = 0$, $E(\varepsilon_i^2) = \delta_i^2$ [$E(x)$ denotes the expected value of x] and neglecting $\sum_i \varepsilon_i/n \simeq 100\ \mathrm{km\ s^{-1}}$ with respect to $c + \sum_i W_i/n$ we obtain:

$$E[\textstyle\sum_i v_i^2/(n-1)]$$

$$= c^2 E\left[\frac{\sum_i [(W_i - \sum_j W_j/n)^2 + (\varepsilon_i - \sum_j \varepsilon_j/n)^2 + 2(W_i - \sum_j W_j/n)(\varepsilon_i - \sum_j \varepsilon_j/n)]}{(n-1)(c + \sum_j W_j/n)^2}\right]$$

$$= c^2 E\left[\frac{\sum_i [(W_i - \sum_j W_j/n)^2 + \varepsilon_i^2 + \sum_j \varepsilon_j^2/n^2 - 2\varepsilon_i^2/n]}{(n-1)(c + \sum_j W_j/n)^2}\right]$$

$$= \frac{c^2 \sum_i (W_i - \sum_j W_j/n)^2}{(n-1)(c + \sum_j W_j/n)^2} + \frac{c^2 \bar{\delta}^2}{(c + \sum_j W_j/n)^2}.$$

The second term on the right hand side is obviously σ_F^2. Note that the expression given by Jackson (1973) contains a wrong sign.

References

Abell, G. O.: 1958, *Astrophys. J. Suppl.* **3**, 211
Abell, G. O.: 1975, in *Stars and Stellar Systems*, Vol. 9, Eds. A. Sandage, M. Sandage, J. Kristian, University of Chicago Press, Chicago, p. 601
Arakelian, M. A., Dibay, E. A., Esipov, V. F.: 1972, *Astrofizica* **8**, 33 [*Astrophysics* **8**, 17 (1974)]
Bautz, P. L.: 1972, *Astron. J.* **77**, 331
Bahcall, N. A.: 1977, *Ann. Rev. Astron. Astrophys.* **15**, 505
Bridle, A. H., Fomalont, E. B.: 1976, *Astron. Astrophys.* **52**, 107
Burbidge, G. R., Burbidge, E. M.: 1959, *Astrophys. J.* **130**, 629
Cavaliere, A., Danese, L., De Zotti, G.: 1978, *Astrophys. J.* **221**, 399
Chincarini, G., Rood, H. J.: 1971, *Astrophys. J.* **168**, 321
Chincarini, G., Rood, H. J.: 1972a, *Astron. J.* **77**, 4
Chincarini, G., Rood, H. J.: 1972b, *Astron. J.* **77**, 448
Chincarini, G., Rood, H. J.: 1976, *Publ. Astron. Soc. Pacific* **88**, 388
Chinarini, G., Rood, H.J., *Astrophys. J.*, **214**, 351
Chincarini, G., Tarenghi, M., Bettis, C.: 1978, *Astrophys. J.* **221**, 34
Coleman, G. D., Hintzen, P., Scott, J. S., Tarenghi, M.: 1976, *Nature* **262**, 476
Corwin, H. G.: 1974, *Astron. J.* **79**, 1356
Cowie, L. L., Binney, J.: 1977, *Astrophys. J.* **215**, 723
Dawe, J. A., Dickens, R. J., Peterson, B. A.: 1977, *Monthly Notices Roy. Astron. Soc.* **178**, 675
De Young, D. S.: 1978, *Astrophys. J.* **223**, 47
Dickens, R. J., Moss, C.: 1976, *Monthly Notices Roy. Astron. Soc.* **174**, 47
Dressler, A.: 1978, *Astrophys. J.* **226**, 55
Faber, S. M., Dressler, A.: 1977, *Astron. J.* **82**, 187
Gregory, S. A.: 1975, *Astrophys. J.* **199**, 1
Gregory, S. A., Connolly, L. P.: 1973, *Astrophys. J.* **182**, 351
Gunn, J. E., Gott, J. R.: 1972, *Astrophys. J.* **176**, 1
Harrison, E. R.: 1974, *Astrophys. J. Letters* **191**, L51
Havlen, R. J., Quintana, H.: 1978, *Astrophys. J.* **220**, 14
Hintzen, P., Oergerle, W. R., Scott, J. S.: 1978, *Astron. J.* **83**, 478
Hintzen, P., Scott, J. G., Tarenghi, M.: 1977, *Astrophys. J.* **212**, 8
Hoel, P. G.: 1971, Introduction to Mathematical Statistics, Wiley, New York
Jackson, J. C.: 1973, *The Observatory* **93**, 19
Jaffe, V., Perola, G.: 1973, *Astron. Astrophys.* **26**, 423
King, I. R.: 1966, *Astron. J.* **71**, 64
Kintner, E. C.: 1971, *Astron. J.* **76**, 409
Limber, D. N., Mathews, W. G.: 1960, *Astrophys. J.* **132**, 286
Lynden-Bell, D.: 1967, *Monthly Notices Roy. Astron. Soc.* **136**, 101
Maccaccaro, T., Cooke, B. A., Ward, M. J., Penston, M. V., Haynes, R. F.: 1977, *Monthly Notices Roy. Astron. Soc.* **180**, 465
Materne, J.: 1974, *Astron. Astrophys.* **33**, 451
Melnick, J., Sargent, W. L. W.: 1977, *Astrophys. J.* **215**, 401
Minkowski, R.: 1961, *Astron. J.* **66**, 558
Mitchell, R. J., Ives, J. C., Culhane, J. L.: 1977, *Monthly Notices Roy. Astron. Soc.* **181**, 25P
Moss, C., Dickens, R. J.: 1977, *Monthly Notices Roy. Astron. Soc.* **178**, 701
Oemler, A.: 1973, *Astrophys. J.* **180**, 11
Ozernoy, L. M., Reinhardt, M.: 1976, *Astron. Astrophys.* **52**, 31
Pacholczyk, A. G., Scott, J. S.: 1976, *Astrophys. J.* **203**, 313
Peebles, P. J. E.: 1970, *Astron. J.* **75**, 13
Rood, H. J., Page, T. L., Kintner, E. C., King, I. R.: 1972, *Astrophys. J.* **175**, 627
Sandage, A.: 1972, *Astrophys. J.* **178**, 1
Sandage, A.: 1975, *Astrophys. J.* **202**, 563
Sargent, W. L. W.: 1970, *Astrophys. J.* **160**, 405
Sargent, W. L. W.: 1972, *Astrophys. J.* **176**, 581
Scott, T. S., Robertson, J. W., Tarenghi, M.: 1977, *Astron. Astrophys.* **59**, 23
Seldner, M., Peebles, P. J. E.: 1977, *Astrophys. J. Letters* **214**, L1
Silk, J.: 1976, *Astrophys. J.* **208**, 646
Solinger, A., Tucker, W.: 1972, *Astrophys. J. Letters* **175**, L107
Spinrad, H.: 1977, *Publ. Astron. Soc. Pacific* **89**, 116
Tammann, G. A.: 1972, *Astron. Astrophys.* **21**, 355
Tarenghi, M., Scott, J. S.: 1976, *Astrophys. J. Letters* **207**, L9
Tifft, W. G.: 1974, *Astrophys. J.* **188**, 221
Tifft, W. G.: 1978, *Astrophys. J.* **222**, 54
Tifft, W. G., Jewsbury, C. P., Sargent, T. A.: 1973, *Astrophys. J.* **185**, 115
Ulrich, M. H.: 1971, *Astrophys. J.* **163**, 447
Ulrich, M. H.: 1978, *Astrophys. J.* **221**, 422
Vaucouleurs, G., de, Vaucouleurs, A., de, Corwin, H. G.: 1976, Second Reference Catalogue of Bright Galaxies, University of Texas Press, Austin
Vidal, N. V., Peterson, B. A.: 1975, *Astrophys. J. Letters* **196**, L95
Vidal, N. V., Wickramasinghe, D. T.: 1977, *Monthly Notices Roy. Astron. Soc.* **180**, 305

© European Southern Observatory • Provided by the NASA Astrophysics Data System

1980A&A....82..322D

Welch, G. A., Chincarini, G., Rood, H. J.: 1975, *Astron. J.* **80**, 77
Yahil, A., Ostriker, J. P.: 1973, *Astrophys. J.* **185**, 787
Yahil, A., Vidal, N. V.: 1977, *Astrophys. J.* **214**, 347
Zelen, M., Severo, N. C.: 1965, in *Handbook of Mathematical Functions*, Eds. M. Abramowitz and I. A. Stegun, Dover, New York, p. 925
Zwicky, F.: 1957, Morphological Astronomy, Springer, Berlin
Zwicky, F.: 1971, Catalog of selected compact galaxies and of post-eruptive galaxies, Zurich

© European Southern Observatory • Provided by the NASA Astrophysics Data System